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PHYSICS DIVISION
SUMMARY REPORT

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Louis A. Turner, Division Director

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1-8 The Argonne Fast Neutron Velocity Selector (5200-01)

Lowell M. Bollinger, Robert E. Cote'
Duane A. Dahlberg, James M. LeBlanc
and George E. Thomas
Reported by Lowell M. Bollinger

a) Chopper M II

No important changes have been made in the velocity selector that is now in use since it was installed at CP-5 in late July 1954.

b) Chopper M III

The design of the new chopper is essentially completed. Its construction is about 2/3 complete: the inlet collimator is completed, the driving motors have been delivered; one rotor, the rotor housing, the chopper base and the outlet collimator are nearing completion.

It is expected that initial spin tests will begin some time in March.

c) 1024-Channel Time Analyzer

R. W. Schumann of the Electronics Division has undertaken the design and construction of a 1024-channel time analyzer to be used with the new fast chopper. The development of the analyzer has not yet progressed far enough to discuss its characteristics in any detail. The line of development that is being attempted, however, is to measure the time between a zero time and a detector pulse by counting oscillator cycles with a fast binary scaler, to stop the scaler when a detector pulse is received, to transfer the

count into a matrix of magnetic cores for permanent storage, to start the scaler up again in synchronism with its initial zero time, and finally to read the information out of the storage matrix at the end of a run.

The new time analyzer is expected to be delivered in about 4 months.

d) 25-Meter Detector Station

The construction of a small building to house the scattering detector discussed in section 2-7 is about 9/10 completed. The detector will be installed at such a position so as to give a flight path of 25 meters.

2-8 Liquid Scintillation Neutron Detector (5200-01)

Lowell M. Bollinger, Harold E. Jackson and
George E. Thomas
Reported by Lowell M. Bollinger

a) Small Detectors for Transmission Measurements

A boron-loaded liquid scintillation neutron detector having an active surface area of 37 in² has been constructed of sheet steel. It has not yet been tested.

b) Scattering Detector

The large, general purpose scintillation detector that was described in ANL 5412, 2-7, has been constructed and is being assembled. Two of the outside sections have been tested and found satisfactory. The installation of the system for use with the chopper will start in about 3 weeks.

Lowell M. Bollinger, Robert E. Cote', Duane A.
Dahlberg, Harold E. Jackson and James M. LeBlanc
Reported by Robert E. Cote'

Neutron Resonances in Uranium²³⁸ and Thorium

Analysis of the transmission measurements gives the following parameters.

<u>Thorium</u>				
E_0 (ev)	σ_0 (b)	$\sqrt{\Gamma}$ (ev)	$\sigma_0/\sqrt{\Gamma}^2$ (b-ev)	$\frac{\sqrt{\Gamma_0}}{\sqrt{\Gamma}}$
69.1	18,200	0.081	119	0.49
59.6	7,700	0.026	5.3	0.18

Uranium²³⁸

The uranium data are presented in Table I. These data show that $\sqrt{\Gamma}_\gamma$ remains fairly constant from one resonance to another in that nucleus.

$\sqrt{\Gamma}_\gamma^0/\bar{D}$ for Slow Neutron Resonances

Neutron resonance structure has been observed in Sc^{45} , V^{51} , Co^{59} , Cu^{63} , Cu^{65} , and As^{75} from which values of $\sqrt{\Gamma}_\gamma^0/\bar{D}$ have been calculated. Figure 1 shows these results along with data¹ on

1. Private communications from L. M. Bollinger, D. A. Dahlberg, and R. R. Palmer.

Table I
Neutron Resonance Parameters

Uranium ²³⁸

E_0 ev	σ_0 b	Γ ev	$\sigma_0 \Gamma^2$ b-ev	Γ_0 ev	Γ_0/Γ	Γ $\times 10^3$	$\Gamma/\sqrt{E_0}$ $\times 10^4$	ΣR barns
6.67	21,500	.027	15.7	.0255	.055	1.49	5.8	129 $\pm$ 10
20.8	39,000	.0318	39.4	.0219	.31	9.9	21.6	64.5 $\pm$ 3
36.6	38,000	.0625	149	.029	.54	33.8	56.0	44.6 $\pm$ 5
66.0	16,500	.0535	47	.031	.42	22.5	27.5	12.2 $\pm$ 1
80.5	1,550	.038	2.24	.036	.05	.1.9	2.1	1.1
102.4	18,700	.100	187.	.0265*	.735	73.5	72.6	7.6 $\pm$ 2
117	10,700	.052	29	.0265*	.48	25	23.1	3.8
189	11,500	.17	332	.0265*	.84	143	104	2.5

265.3 $\pm$ 12

+21 $\pm$ 10

286 $\pm$ 20

$$\frac{\Gamma}{2} \sum \left(\frac{\sigma_0 \Gamma}{E_0} \right)_n =$$

$$\bar{\Gamma}_0 = 3.0 \times 10^{-3} \text{ ev}^{1/2}$$

$$\bar{D} = 18 \text{ ev}$$

$$\bar{\Gamma}_0 / \bar{D} = 1.6 \times 10^{-4} \text{ ev}^{-1/2}$$

* Assumed values of Γ_γ

Figure 1

Mn⁵⁵, Zn⁶⁷, Ga⁷¹, Sb¹²¹, and Sb¹²³. The solid curve was calculated using the "cloudy crystal ball"² model of the nucleus. A paper on this work is being prepared for publication in the Physical Review and an extension of the program to include data in the mass region between 60 and 110 is under-way.

Publications

The following 10-minute papers were given at meetings of the American Physical Society.

"Neutron Transmission Measurements at the New Argonne Reactor, CP-5" by L. M. Bollinger, D. A. Dahlberg, R. E. Cote', H. E. Jackson, and G. E. Thomas, Chicago 1954.

" $\tilde{\Gamma}_h/\bar{D}$ for slow Neutron Resonances" by R. Cote' and L. M. Bollinger, New York, 1955.

6-7 Spiral Velocity Selector for Slow Neutrons (5200-01)

G. R. Ringo

The shop work on the selector including balancing of the rotor has been essentially completed. It is now waiting a high speed test.

2. Feshbach, Porter and Weisskopf, Phys. Rev. 96, 448 (1954).

1. Textolite Damage - Reported by F. Paul Mooring

During the middle of October the Van de Graaff was down for two weeks because of additional textolite damage. Scraping, vacuuming, and shellacking the interior of the textolites again restored the machine to usefulness. It was decided to replace all the textolite structural members with new ones designed differently in the hope of avoiding similar trouble in the future. (See the report of Jack Wallace).

2. The 90° Electrostatic Analyzer - Reported by R. R. Weeks

Beams in excess of 15 μ a (max. 19) have been successfully passed through the 90° analyzer and utilized for total neutron cross section work.

The improvement in the operation of the 90° analyzer is not the result of any changes in the analyzer. A wire that supplied the return path for the focus voltage on the HV shell was broken loose when the machine was being worked on. The break was hidden and was not discovered until violent changes up to 50% in beam current and working of the beam focus were observed. After this break was repaired there was a remarkable improvement in the reliability of operation of the analyzer and steadiness of the beam current. It may be that this lead had been making poor connection and causing fluctuations for many months.

A comparison may be made between narrow resonances in neutron total cross sections made with the proton beam through the 90°

analyzer (for production of neutrons and control of the Van de Graaff) with slit widths corresponding to resolving power 1:3000 and previous work using the molecular hydrogen beam through the 25° analyzer (for control of the Van de Graaff only).

<u>Element</u>	<u>Energy E_0</u>	<u>Previous Width</u>	<u>New</u>
F	≈ 27 kev	1.5 kev	≈ 0.4 kev
Bi	≈ 2.2 kev	700 ev	≈ 250 ev

Perhaps still better resolution may be achieved in this type of work as more experience is gained in using the equipment. Erratic behavior in the potentiometer to which the analyzer high voltage is compared has been traced to a noisy switch in the battery current supply. Target thicknesses have been reduced to ≈ 0.7 kev (apparent thickness at 120° is about 1/2 of this).

Larger beam currents increase the amount of stray beam available to cause secondary plate-to-plate currents. Very clean surfaces of the plates associated with a vacuum $\approx 5 \times 10^{-6}$ mm keeps the secondary currents low, while increased regulation in the power supply may reduce the correction in the energy determination to a negligible value.

3. Targets - Reported by Jack Wallace

Parts have been made for another rotating target to be installed in the east 25° position. This will make available four positions where neutrons may be had.

4. New Structural Textolites for Van de Graaff - Reported by Jack Wallace

Much thought and discussion has been devoted to the damage to the textolites of last summer (see report of Paul Mooring - (ANL5412) Quarterly May 7 to July 30). After much study and conferences with Professor Ray Herb, University of Wisconsin, a possible improved design has been worked out. The principal design changes are:

1) The corona rings will be vertical to the axis of the textolite member instead of at a slight angle to the vertical.

2) The corona rings will be held in place by a slight squeeze on the outside of the textolite instead of by small locating pins drilled into the textolite members.

3) The ends have been redesigned to provide for seals and fittings for forced circulation of dry gas through textolite members in their installed position.

The above changes have been incorporated into a new set of drawings. The machine shop has constructed a complete new set of structural members according to the above-mentioned changes. Also plans have been made for moving the diagonal structural member farther from the belt in the middle section. These members were the closest to the charging belt and it was thought desirable to increase this separation. Replacement of otherwise inaccessible small parts will be done where needed as the generator is reassembled.

The entire operation of disassembly and reassembly has been planned so that it can be accomplished in the shortest possible time

and with the most efficient use of the available manpower.

5. Operational Experience - Reported by Jack Wallace

During the period (July 31, 1954 to January 31, 1955) the Van de Graaff generator has been operating at various energies up to 3.0 Mev and with proton beam currents up to 15 microamps as required by experimenters. There was also a period of operation with deuterons as the charged particle.

Following is a list of experiments using the Van de Graaff with the number of hours the generator was operated for each. These times are not representative of the total machine time allotted to the various experimenters. There has been no effort made to show the efficiency factor of the time allotted to the time the generator was used.

1. Proportional counter	G. Perlow	23.6 hours
2. Magnetic spectrometer	Holland, Inglis, Lee, Mooring	175.5
3. Plate irradiations	Krohn, Ringo	58.5
4. Total cross sections	Hibdon, Weeks	227.1
5. Self detection	Mooring, Monahan, Hibdon	108.1
6. Angular distribution	Lane, Langsdorf	88.4
7. Checking machine	Van de Graaff group	72.4
		<hr/> 753.5 hours
Time available (99 days x 15 hours, 27 days x 8 hours)		1701.0 hours
Start up and daily maintenance (126 days x 0.5 hours)		63.0
Experimental set up which ties up generator		227.7

Time generator available but not used

247.2

20

Generator out of action

409.6

This is the first report since the night shift of two technicians has been in effect. Although it is difficult to compare two periods of generator operation I believe one can make the following comparison. During the present period the operational hours of the generator is higher per hour of time available than in past periods. This seems to indicate that the double shift makes possible a more efficient use of the generator. There are other gains from the longer operation period that improve the efficiency of the generator and those associated experimenters, such as generator improvements, improved vacuum plumbing, filtered cooling water for vacuum pumps, auxiliary drier for insulating gas in the generator, installation of new target assembly, maintenance of auxiliary equipment, repair and improvement of experimental equipment to make it ready for experimenters. These things were accomplished by the extra shift at times when the generator was not scheduled for use in the evening. The generator was also repaired and returned to operation more quickly and efficiently.

14-7

Pulsed Neutron Beams for Van de Graaff Generator

(5200-01)

Robert Holland

Further work on pulsing the output of the ion source at the high voltage and of the machine has been discontinued for the present in favor of the simpler method of pulsing the beam after it has left the machine. A power oscillator and deflection plates for sweeping the beam past a hole have been built and shown to operate satisfactorily at the Van de Graaf generator.

15-4

Cross Sections for Stopping of Heavy Ions (5200-02)

Merle T. Burgy

The two precision slit arrangements and the support framework for the gas cell, its pumping system and gas supply have been completed and many more components of the final experimental set up have been designed and built.

The 25° electrostatic analyzer has been equipped with new field-defining slits and also slits for defining the beam and for alignment. These additional slits were added as a consequence of experience with the 90° analyzer. The large reduction of pumping speed introduced with these slits has been compensated by adding pumping by passes around these slits.

The parts needed to provide flexible connections among the analyzer, the precision slits, the photomultiplier, the gas cell and the two large cold traps have been built as well as two vacuum valves according

to a design by R. Holland.

In order to minimize the set-up and take-down time at the accelerator, two carriages and special hoisting jigs were built. With these, the equipment can be transported and put in place as two units, with only minor connections remaining to be made at the accelerator.

The electrical circuits needed for the diffusion pumps and vacuum gauges have been built or obtained. The principal electrical equipment still to be made is an adjustable high voltage supply for the 25° analyzer.

There remains some shop construction to be done on connections between the gas cell and the 90° electrostatic analyzer system and on adjustable mountings for the photomultiplier and the precision slits.

16-1

A New Neutron Counting System (5200-01)

Carl T. Hibdon, Alexander Langsdorf, Jr.
James E. Monahan and F. Paul Mooring
Reported by F. Paul Mooring

In the low energy, high resolution work and in the self detection experiments, the large shielded annular counter¹ designed by A. Langsdorf has been used. It has become increasingly evident that new neutron detection equipment is desirable. Some preliminary calculations of the proper counting geometry have been made by J. W. Butler of the Reactor Engineering Division. Experiments to determine

1. Hibdon, Langsdorf, and Holland, Phys. Rev. 85, 595 (1952)

the best practical counting geometry are being considered. As yet designs are tentative; only rather general features of the proposed counting system have been considered.

17-1

The Energy Spread of Neutrons from Lithium Targets

(5200-01)

Alexander Langsdorf, Jr., James E. Monahan
and F. Paul Mooring
Reported by F. Paul Mooring

During the work on self detection¹ it became evident that the lithium targets used as a neutron source were probably non-uniform in such a manner that the usual method of determining neutron energy spread² from the rise curve gives too small a value. To check this conclusion the following experiment was performed. A thin lithium fluoride scattering sample was placed at the center of the large shielded annular counter³ known locally as the elephant. A study was then made of the effect of the thickness of the lithium layer on the shape of the neutron scattering peak arising from the narrow 27-kev resonance in F¹⁹.

The results are shown in Figure 2, neutron counts per microcoulomb of proton charge being plotted as a function of incident

-
1. Monahan's report for this quarter.
 2. Hanson, Taschek, and Williams, Rev. Mod. Phys. 21, 635 (1949)
 3. Hibdon, Langsdorf and Holland, Phys. Rev. 85, 595 (1952)

Figure 2

proton energy. The width of the scattering peaks at half maximum is a measure of the thickness of the lithium layer and its shape is a measure of the uniformity of the layer. The table in Figure 1 gives for the various curves the thickness of the lithium target in kev (not including the spread in energy of the incident proton beam) that was thought to have been evaporated and also the observed width of the scattering peak at half maximum. If the incident proton energy spread is combined with the "evaporated target width", a neutron energy spread is obtained in each case that is about one third that needed to account for the observed width of the scattering peaks. The large assymetries exhibited by the peaks can be easily explained by a non-uniform lithium layer.

We have corresponded with some other laboratories that use lithium targets for monoergic neutron sources. Most use and accept the rise curve method for measuring target thickness and are unaware of any difficulty. W. D. Allen, at Harwell, has recognized that he is having similar difficulties and has embarked on a program of developing a procedure for producing uniform targets. Though he reports considerable progress, as yet he can offer no sure fire recipe for producing a uniform lithium layer. In good resolution work it is very desirable that good uniform lithium targets be used, hence a further study of this problem will be made.

20-8

Energy States of Light Nuclei from Charged Particle
Reactions (5200-01)

Linwood L. Lee, David R. Inglis, Robert E.
Holland and F. Paul Mooring
Reported by David R. Inglis

In the course of continued work on the reactions $B^{10}(d, \alpha) Be^8$ and $B^{11}(p, \alpha) Be^8$ during the summer, a short circuit developed in the windings of the magnetic spectrograph which limited observation for some time to alpha particles of low energy. Late in the fall, the magnet was disassembled and the trouble was found to have developed at the points where the rather sharp edges of the iron yoke exerted pressure on the cambric cloth and the plastic tape insulation wound around the heavy-current "pancakes", which had experienced some thermal and magnetic working. Since completion of the repair (by Lee) the work on the above-mentioned reactions has been essentially completed with the observation of the former reaction at a forward angle, with no indication of the "extra" excited states in Be^8 . Observations have been made on the reaction $B^{11}(d, \alpha) Be^9$, at 1.5 Mev and three angles, which indicate the existence of excited states at about 1.8, 2.4 and 3.1 Mev, a group of three low excited states of Be^9 reported tentatively from Oak Ridge and expected theoretically as mentioned elsewhere in this report (V-1-8). Somewhat less conclusive evidence is found for states at 4.7 and 5.0 Mev.

Publications presenting the data on the status of Be^8 and of Be^9 are in preparation.

The Decay of Ce^{143} (33-Hr.) (5200-01)

S. Bradley Burson, James M. Cork (University of Michigan) and David W. Martin
Reported by David W. Martin

The 33-hr. beta emitter in cerium was first observed by Pool and Kurbatov.¹ Their assignment of the activity to Ce^{143} has been confirmed². Gamma rays of about 57, 290, 350, 660 and 720 kev have been resolved in conversion- and secondary- electron spectra.^{3,4,5} Additional gamma rays, one of 126 kev, and a second very weak one of ~ 160 kev seen only in coincidence with the former, have been reported⁵ from early scintillation spectra at low resolution. The beta-ray spectrum has been resolved into at least three components of about 1.38, 1.09, and 0.71 Mev, with indications of other still lower branches.^{4,5} Three different decay schemes^{3,4,5} have been proposed for the activity, each based mainly on energies without adequate support from coincidence experiments. Study of the activity was undertaken with the ten-channel coincidence scintillation spectrometer and with magnetic spectrographs in hopes of clarifying the decay scheme. Cerium enriched in mass 142 was on hand as cerium oxide.

The study has revealed the presence of five gamma

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1. M. L. Pool and J. D. Kurbatov, Phys. Rev. 63, 463 (1943)
 2. M. L. Pool and N. L. Krisberg, Phys. Rev. 73, 1035 (1948)
 3. H. B. Keller and J. M. Cork, Phys. Rev. 84, 1079 (1951)
 4. E. Kondiah, Phys. Rev. 83, 471 (1951)
 5. W. H. Burgus, Phys. Rev. 88, 1129 (1952)

rays, and has provided some revision of the previously reported energy values. In all, ten gamma rays of energies 0.0574 ± 0.0002 , 0.232 ± 0.001 , 0.294 ± 0.001 , 0.351 ± 0.001 , 0.493 ± 0.002 , 0.565 ± 0.005 , 0.668 ± 0.002 , 0.724 ± 0.002 , 0.861 ± 0.005 , and 1.10 ± 0.01 Mev were found to decay with the 33-hr period in the NaI (Tl) pulse-height distribution. Conversion lines are seen in the spectrographs for all of these except the 0.565- and 1.10-Mev transitions. The former is in fact very weak, and it is seen only in certain coincidence distributions. No evidence could be found for the reported gamma rays of 126 or 160 kev.

The decay scheme shown in Figure 3 is consistent with these energies of the first three beta-ray components, and is supported by the following coincidence data.

The gamma-ray pulse spectrum in coincidence with the 0.057-Mev peak is identical with the total non-coincidence spectrum except for the conspicuous absence of the 0.232-, 0.351-, 0.484-, and 0.724-Mev peaks. The intense 0.294-Mev peak is coincident only with the 0.057- and the very weak 0.565-Mev peaks. Coincidences are observed between the 0.232- and 0.493-Mev peaks. Further, the transitions of 1.10, 0.861, and 0.724 Mev are found not to be coincident with anything > 100 kev. On the other hand, the 0.660-Mev peak is apparently coincident with something > 100 kev, but only so feebly that the coincident radiation is probably only bremsstrahlung or a very weak unobserved transition of 0.2-0.4 Mev.

27

Figure 3

Decay Scheme of Ce^{143} and Pr^{143}

In beta-gamma coincidence absorption measurements, the beta rays coincident with the 0.057-Mev gamma ray absorb out indistinguishably from the total beta spectrum with a half-thickness of 59 mg/cm^2 of aluminum. The 0.294- and 0.351-Mev peaks absorb out together at 51 mg/cm^2 , verifying that both are fed by the same beta ray. Likewise, the 0.668- and 0.724-Mev peaks absorb out together at 28 mg/cm^2 , with the same conclusion. These half-thickness values are consistent with the reported energies of the first three beta-ray components.

In a different experiment, the distribution of beta-ray pulses from an anthracene crystal coincident with various gamma rays was examined with the ten-channel analyzer. The distribution coincident with the 0.057-Mev peak was again indistinguishable from the total spectrum. Lower-energy distributions interpretable as the next two reported components are observed to be coincident with the 0.294- and 0.668-Mev peaks, respectively. In addition, two more distinct distributions with endpoints corresponding to about 0.5 and 0.3 Mev are found in coincidence with the 0.861- and 1.10-Mev peaks, respectively. While these observations do not constitute "measurements" of new beta-ray energies of any precision, they are at least evidence for the existence of components of 0.52 and 0.28 Mev required by the decay scheme.

It is believed that these data establish the given decay scheme uniquely, except for the order of the 0.232- and 0.493- Mev transitions. The intense beta ray of the highest reported energy proceeds

to the 0.057-Mev level rather than to the ground state as previously assumed. It can be asserted that any additional transition directly to the ground state must be very much less intense. The data are considered to be essentially complete, although further analysis is intended in the hope of establishing the character of some of the major transitions. The material is being prepared for presentation before the American Physical Society (Washington, April 1955) and for subsequent publication.

29-1 Decay of Nd¹⁵¹ (12 min.) and Pm¹⁵¹ (27 hr.) (5200-01)

S. Bradley Burson, James M. Cork (University of Michigan) and David W. Martin
Reported by David W. Martin

A 12-min. period resulting from the irradiation of Nd with pile neutrons was first observed by Marinsky and Glendenin¹ and assigned tentatively to Pm¹⁵¹. It was believed that this activity must be the daughter of a Nd¹⁵¹ activity of lifetime too short to be seen at that time. Later Rutledge et. al.² showed that the 12-min. activity was in Nd¹⁵¹ and was followed by a new 27-hr. activity in Pm¹⁵¹. Maximum beta-ray energies of 1.93 and 1.1 Mev were measured for the two activities respectively by absorption techniques². Six gamma rays were found associated with the 12-min. decay, and sixteen with the 27-hr. decay, by internal-conversion spectra and some scintillation measurements². Some relatively inconclusive coincidence data were also obtained by absorption methods². Two different almost equally acceptable level schemes could be constructed for the Pm¹⁵¹ decay, based upon the observed transition

1. J. A. Marinsky and L. E. Glendenin, NNES IV-9, 1264 (1952)

2. W. C. Rutledge, J. M. Cork, and S. B. Burson, Phys. Rev. 86, 775 (1952)

energies². Study of the activities was undertaken with the ten-channel coincidence scintillation spectrometer in hopes of clarifying the decay schemes.

Nd¹⁵¹ (12 min.)

Peaks are seen in the NaI (Tl) pulse-height distribution corresponding to all six of the gamma rays previously reported,² although the 110- and 117-kev peaks are not resolved. An additional peak of considerable intensity at about 250 kev appears on cursory examination to decay with this period, although this has not been carefully determined. The peak corresponding to the reported² gamma ray of 725 kev is very broad and is perhaps not a single transition. No coincidence measurements have been performed as yet. Further progress in this study must await clarification of the details of the daughter activity, which is visible in the spectrum and interferes even after a very short irradiation.

Pm¹⁵¹ (27 hr.)

Many of the gamma rays in this activity are too closely spaced in energy to be clearly resolved in the scintillation spectra. Three sets of three transitions each, and one other pair of transitions, as reported by Rutledge², each appear as single peaks. All peaks observed to decay with this period correspond to one or more of the reported transition energies except two. These two, at about 0.45 and 0.67 Mev, respectively, represent new gamma rays to be associated with the activity.

Some preliminary gamma-gamma coincidence experiments

have been performed. As might be expected, the multiple peaks mentioned above are more or less in coincidence with nearly every peak of the spectrum, making analysis difficult. The only salient fact that has so far emerged is that the 715-kev transition is definitely in coincidence with X rays. In both of the alternative decay schemes proposed previously, a level of 715 kev was supposed to be the highest one occurring, with the transition proceeding directly to the ground state. The above result shows that this is not the case.

There remains some doubt that unambiguous determination of the level scheme will be possible with present techniques, in view of the special problems involved. It is expected that work will continue on this activity in the coming period.

30-8 Scintillation Coincidence Spectrometer (5200-01)

S. Bradley Burson

A new high-voltage supply (Atomic Instrument Company "High Voltage supply, Model 312") was procured for the probe of the ten-channel analyzer resulting in a considerable reduction in the energy calibration drift with temperature.

The single-channel analyzer originally supplied with the equipment was found to have a dead-time of the order of fifty microseconds. This component has recently been replaced with a different analyzer having a dead-time of five microseconds. This change improves the counting-rate capacity and renders results more reliable without the need for extensive corrections.

The first attempts at utilizing "sum-peaks" for decay-scheme analysis were not particularly promising. The simple expedient of tying the photo-multiplier anodes proved to be too simple to be useful. A sufficient attenuation in gain resulted so that excessively high photo-cathode voltages were necessary. A more elaborate electronic "summing" circuit must be built if this method is to be employed.

In view of the fact that the inherent defects in this equipment were not readily rectified and that there is hope for its early replacement, it no longer seems advisable to publish a report on the instrument itself in one of the journals.

31-8

The 1.42-Minute Isomeric State in ^{192}Ir (5200-01)

S. Bradley Burson, Bernard Hamermesh,
Virginia Hummel, Willard C. Jordan
Reported by S. Bradley Burson

Thinner sources of iridium were prepared and the spectrum in the neighborhood of 25 kev was examined with the scintillation spectrometer. This radiation was found enhanced relative to the 57-kev gamma ray peak when compared with the powdered metal sources used earlier.

Since the work of the Los Alamos experimenters which was reported at the 1954 Washington meeting of the American Physical Society is confirmed by these experiments and since their results are in agreement with our earlier work, no further study will be made of this activity.

35-8

(Decay Schemes of Long-Lived Rare Earth Isotopes)
(5200-01)

Eugene L. Church

The report originally submitted on this program has been withdrawn, to be superseded by one appearing in the following quarterly report.

37-6

Decay Schemes of the Transuranic Elements
(5200-01)

Eugene L. Church

(Reports on this work have previously appeared in the classified supplement to the quarterly report. The last one appeared in ANL-5318 .)

Pu^{238} is a 90-year even-even alpha emitter decaying to U^{234} . The conversion-electron spectrum of this activity has been previously examined, and transitions of 43.6 ± 0.3 and 100.0 ± 0.4 kev reported in uranium.¹ These transitions are below the threshold for K conversion, and convert only in the L, M and higher shells. The present experiments, conversion lines corresponding to the various L and M sub-shells were resolved, and both transitions found to convert almost entirely in the II and III sub-shells.

Because of the high atomic number and low energies involved, the character of these transitions is most easily determined from their relative conversion properties. The possibility that they are magnetic is excluded at the outset by the observed relative importance of

1. ANL-5141, p. 6

the L_{II} conversion in the L shell.² This is in agreement with the observation that transitions between states of even-even nuclei populated by alpha decay must be electric. On the other hand, the conspicuous lack of L_I conversion requires these transitions to be other than electric dipole, while the exclusive conversion in the M_{II} and M_{III} sub-shells indicates that they are electric quadrupole.³ Higher-order electric transitions should convert more strongly in the M_{IV} and M_V sub-shells, as well as exhibiting detectable lifetimes.

By measuring delayed coincidences between alpha particles and conversion electrons, Sunyar,⁴ at Brookhaven, has found the lifetime of the 43.6-kev transitions to be $< 5 \times 10^{-10}$ second. This measurement further excludes higher-order transitions and corresponds to a gamma lifetime at least an order of magnitude less than the corresponding single-particle estimate for an E2 transition.⁵ The lifetime of the 100-kev transition could not be measured because of the low abundance of its preceding alpha group.

The alpha fine structure of Pu^{238} has been studied by Asaro and Perlman.⁶ The energies of the alpha groups indicate that the

2. ANL-5273, p. 95.

3. ANL-5174, p. 71.

4. A. W. Sunyar, private communication (1954). Also, BNL-2157 (to be published).

5. V. F. Weisskopf, Phys. Rev. 83, 1073 (1951).

6. F. Asaro and I. Perlman, Phys. Rev. 94, 381 (1954).

two transitions observed in U^{234} form a cascade leading to the ground state. Assuming these two transitions to be electric quadrupole, and the ground state of U^{234} to be $0+$, the first excited state must be $2+$, and the second excited state $0+$ or $4+$. However, the observed alpha-branching ratios and the lack of a cross-over transition argue in favor of the $4+$ and against the $0+$ assignment for this level.

The expected alpha branching for various spin sequences have been previously computed for this activity on the basis of the Gamow theory.¹ For convenience these results are reproduced in Table II along with the experimentally observed values. Of the three spin possibilities, 000 , 020 and 024 , the last lies closest to the observed ratios. In this case the relative population of the ground and first excited states is in good agreement with experiment, although the expected branching to the $4+$ state is about 20 times that observed. The reason for this extra inhibitedness is unknown, but is undoubtedly due in part to the fact that heavy even-even nuclei have large intrinsic quadrupole moments leading to electrostatic forces which are neglected in the simple Gamow theory. The theoretical aspects of this situation have been considered by Rasmussen⁷ and Christy⁸.

7. J. O. Rasmussen, UCRL-2431.

8. R. F. Christy, Bul. Am. Phys. Soc. 30, 1, ZA7.

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Table II

ALPHA BRANCHING IN PERCENT

SPINS	GROUND STATE	FIRST EXCITED	SECOND EXCITED
0-0-0	59	33	8
0-2-0	68	23	9
0-2-4	73	25	2
OBS	72	28	0.1

If the second excited state of U^{234} were $O+$, it could be de-populated by a radiationless or "zero-zero" transition leading directly to the ground state. The theoretical lifetime⁹ of such a transition is $\sim 10^{-11}$ second, which is $> 10^5$ times shorter than the corresponding single-particle estimate of the lifetime of the competing 100-kev E2 transition. The fact that no K-"conversion" electrons are observed for the 144-kev cross-over transition is a possible argument against the $O+$ assignment of the second excited state.

The present results are summarized in the tentative decay scheme shown in Fig. 4.

The most interesting aspect of the present decay scheme is its interpretation in terms of rotational states of excitation of the U^{234} nucleus. Bohr and Mottelson¹⁰ have predicted that the lowest states of excitation of even-even nuclei far from magic numbers should be those of "rotation", having an energy-level spectrum of the form:

$$E_I = \frac{\hbar^2}{2\mathcal{I}} I(I+1),$$

where $I = O+, 2+, 4+$, etc. are the spins and parities of the successive excited states, and $\mathcal{I}$ is the effective nuclear moment of inertia.

9. E. L. Church and J. E. Monahan, accompanying unclassified progress report.

10. A. Bohr and B. R. Mottelson, Det Kongelige Danske Videnskabernes Selskab, 27, 16 (1953)

40
PROPOSED DECAY SCHEME OF $\text{Pu}^{238} \rightarrow \text{U}^{234}$

Figure 4

PHG-817
 1-24-85
 B.G.

In the case of U^{234} , the ratio of the observed transition energies agrees with this simple formula to within 2%. However, this small deviation is real and may be interpreted in terms of a rotation-vibration interaction. The two measured energies can then be used to compute both the parameter $\hbar^2/2\mathcal{J}$ and a mean nuclear vibrational energy. In this case the vibration energy turns out to be ~ 1.0 Mev, in fortuitously good agreement with the estimate of ~ 0.9 Mev based on Bohr and Mottelson's hydrodynamical or incompressible nuclear model.¹⁰

The effective nuclear moment of inertia can be related to the intrinsic nuclear quadrupole moment, Q_0 . This turns out to be about 30 barns for U^{234} . This quantity can be compared with that computed from the observed electrostatic quadrupole moments of adjacent odd-mass nuclei. Sluis and McNally¹¹ have estimated a value of the quadrupole moment of U^{235} , which corresponds to an intrinsic quadrupole moment of about 22 barns. The agreement between these two values is quite reasonable and is much better than in the rare-earth region, where the intrinsic quadrupole moments determined from mechanical moments are 4 to 6 times greater than those determined by direct measurement.⁴

Because they involve collective nuclear motion, the lifetimes of the electric-quadrupole transitions between rotational states should be appreciably shorter than the corresponding single-particle

11. K. L. Vander Sluis and J. R. McNally, Jr., Jr. Opt. Soc. Amer. 44, 87 (1954).

estimates.⁵ In principle, this increase in transition probability could also be used to determine the intrinsic quadrupole moment.¹⁰ However, in U^{234} , the values of Q_0 determined from the moment of inertia, or the measurements on U^{235} , correspond to speed-up factors of the order of 1500. In the case of the 43.6-kev transition, this would lead to an observed lifetime of the order of 50 times shorter than the upper limit of 5×10^{-10} second set by the present experiments.

The above discussion represents a modified version of the text of a paper presented at the New York Meeting of the American Physical Society, January, 1955.¹² Since immediate publication of these results is not anticipated, they are included here in their entirety. No further reports will be made.

41-6 (The Decay Scheme of Cd^{117} and In^{117}) (5200-01)

S. Bradley Burson, James M. Cork (University of Michigan) and James M. LeBlanc
Reported by James M. LeBlanc

No further results have been obtained since the previous report was made.

43-5 (The Decay Scheme of Sb^{122m}) (5200-01)

S. Bradley Burson, James M. Cork (University of Michigan) and James M. LeBlanc
Reported by James M. LeBlanc

A paper covering this study is to appear in the March 1st issue of the Physical Review. No further work is contemplated.

12. E. L. Church and A. W. Sunyar, Bul. Amer. Phys. Soc. 30, 1, S5.

45

(The Short-Lived Activities of Zn) (5 200-01)

S. Bradley Burson, James M. Cork (University
of Michigan), James M. LeBlanc
Reported by James M. LeBlanc

A paper covering this study is to appear in the
February 1st issue of the Physical Review. No further work is
contemplated.

46-5

(The Decay of $_{32}\text{Ge}^{77}$ (12 hour) and $_{32}\text{Ge}^{77m}$ (52 sec)) (5200-01)

S. Bradley Burson, Willard C. Jordan, James
M. LeBlanc
Reported by S. Bradley Burson

These programs have been completed and were
published in The Physical Review, Volume 96, No. 6, December 15,
1954, page 1555, under the title "Decay of $_{32}\text{Ge}^{75m}$ (49 sec), $_{32}\text{Ge}^{77m}$
(52 sec) and $_{32}\text{Ge}^{77}$ (12 hr).

47-4

(The Decay Scheme of 30-Min. Pt^{199}) (5200-01)

S. Bradley Burson, James M. Cork (University
of Michigan) James M. LeBlanc
Reported by James M. LeBlanc

No further results have been obtained since the
previous report was made.

S. Bradley Burson, Willard C. Jordon,
James M. LeBlanc
Reported by S. Bradley Burson

When our report was submitted to The Physical Review for publication, it was pointed out to us by Dr. S. A. Goudsmit that it was in conflict with certain unpublished work done at Brookhaven. The disagreement concerned the existence of a possible ground-state beta transition. As a result of this communication, some of the beta-gamma coincidence experiments were re-done more exhaustively. The plot of the ratio of beta - 0.32 gamma coincidence to beta ray count was again found to be statistically flat as far out as measurements were feasible, that is, to about 0.67 gm/cm^2 of Al. As a result, we reaffirm our earlier conclusion that the maximum energy observed beta ray of 2.3 Mev is in coincidence with the 0.32-Mev gamma ray. Further, the upper limit on the intensity of an additional transition to the ground state, previously given as 30%, is now revised down to not more than 15%.

After appropriate efficiency corrections, we estimate the intensities of the three gamma rays, 0.32, 0.61 and 0.92 Mev, to be in the ratios of 100:1:5. Assuming the correctness of the decay scheme, the log (ft) values of the 2.3- and 1.8-Mev beta branches are then estimated to be 5.0 and 5.7, respectively, indicating that both are "allowed".

The ground state of V^{51} has been measured to be $7/2^{1,2}$.

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1. Bleaney, Ingram and Scovil, Proc. Phys. Soc. (London) A64, 601 (1951)
 2. Kikuchi, Sirvetz and Cohen, Phys. Rev. 88, 149 (1952); 92, 109 (1953)

According to the single-particle model, possible assignments for the levels of V^{51} are f 7/2 for the ground state, p 3/2 and f 5/2 for the 0.31 and 0.92-Mev states, respectively. The log (ft) values are consistent with the assignment of p 3/2 to the ground state of Ti^{51} . Thus, a beta transition to the ground state of V^{51} would be second-forbidden.

This report has been published in The Physical Review
96, 1582 (1954).

49-1 The Decay of $_{50}Sn^{125}$ (9.7 d and 9.4 min) (5200-01)

S. Bradley Burson, James M. LeBlanc and
David W. Martin
Reported by S. Bradley Burson

9.7-Day Activity:

In 1939, Livingood and Seaborg¹ observed a beta activity with a half-life of about ten days upon the irradiation of tin with deuterons and neutrons and assigned the activity to that element. The

1. J. J. Livingood and G. T. Seaborg, P. R. 55, 667 (1939).

activity has been assigned to Sn^{125} .^{6,7} Absorption measurements showed the predominant beta ray to have an energy of about 2.5 Mev;^{2,3,4,6} later independent spectrometer measurements assigned values of 2.33⁷ and 2.37⁸ Mev and showed the shape for $\Delta I=2$, yes transition. One of these investigators reported evidence for a weak low-energy branch of about 0.41⁸ Mev. Its existence was confirmed by beta-gamma coincidence measurements.¹¹ The observed weak gamma radiation^{3,4} has been variously reported, from absorption measurements at 1.5⁵ and 1.67¹¹ Mev, and from a scintillation spectrometer measurement at 1.90 Mev.⁹ We have found no references to other quantum radiation associated with the activity.

Enriched Sn^{124} was irradiated in CP-5 to provide sources of Sn^{125} . Without chemical purification after irradiation, a sufficient amount of the Sb^{125} (2.7 yr.) daughter of Sn^{125} was present in the sources to prevent analysis of the gamma-ray spectrum below 0.7 Mev. In these sources,

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2. J. Seiler, NNES-PPR Vol. 9B, paper 7.26.1 (1946)
 3. Plutonium Project, J. Am. Chem. Soc. 68, 2411 (1946)
 4. K. D. Coleman and M. L. Pool, P. R. 72, 1070 (1947)
 5. A. S. Newton and W. R. McDonell, UCRL-395 (unclassified) (1949)
 6. J. C. Lee and M. L. Pool, P. R. 76, 606 (1949)
 7. B. H. Ketelle, C. M. Nelson, and G. E. Boyd, P. R. 79, 242 (1950);
C. M. Nelson, B. H. Ketelle, and G. E. Boyd, ORNL-828 (unclassified) (1950)
 8. Raymond W. Hayward, P. R. 79, 409 (1950)
 9. G. E. Boyd and B. H. Ketelle, ORNL-1953 (1951) (classified)
or NBS Circular No. 499, Supplement No. 2 (1951)
 10. J. A. Seiler, NNES-PPR 9, 910 (1951)
 11. R. J. Mendenhall, C. E. Mandeville, E. Shapiro, E. R. Zucker, and
G. L. Conklin, P. R. 87, 203 (1952); C. E. Mandeville, E. Shapiro,
R. J. Mendenhall, E. R. Zucker, and G. L. Conklin, P. R. 88, 554 (1952)

however, peaks were resolved which correspond to gamma rays having energies of 0.81, 0.90, 1.07, 1.41, and 1.97 Mev. Although the peak at 1.07 Mev is many times stronger than that at 1.97 Mev, the latter is the only radiation which can be associated with anything previously reported in the literature. These peaks were all seen to decay with the 9.7-day period. After chemical purification* which removed the antimony daughter, five other peaks remained in the spectrum, three of which were associated with Sn^{125} , the other two to activities in other tin isotopes still present in the source material. In addition to the X-ray peak, two gamma rays of 0.34 and 0.47 Mev were detected.

The energies of these seven radiations were sufficient to suggest the decay scheme indicated in Figure 5. From gamma-gamma coincidence measurements, the following six pairs of gamma rays were seen to be in coincidence: (0.34-0.47), (0.34-1.07), (0.47-1.41), (0.47-1.07), (0.81-1.07), and (0.90-1.07) (Mev). Search was made for all the other possible combinations and no others were observed. Examination of the decay scheme will show that these six include all combinations consistent with the scheme.

Beta-gamma coincidence measurements were carried out using the anthracene crystal for beta detection with the single-channel analyzer. Aluminum absorption curves were obtained for the beta rays in coincidence with selected gamma rays.

* The chemical purifications were kindly performed for us by Dr. E. P. Steinberg and K. Flynn of the Argonne Chemistry Division.

ENERGY LEVELS OF $^{125}_{51}\text{Sb}$ IN THE DECAY OF $^{125}_{50}\text{Sn}$

PH6-849
4-28-55
BG

Figure 5

The intense ground-state transition of about 2.35 Mev was not found to be in coincidence with any of the gamma rays. With the 1.97-Mev gamma ray, a single beta ray was found to be in coincidence; its half-value thickness in aluminum corresponded to an end-point energy of about 0.4 Mev, which is in agreement with earlier reports.^{8,11} Two components are found to be in coincidence with the 1.07-Mev gamma ray. The 0.4-Mev branch is seen and, in addition, a component with an energy of about 1.35 Mev. These results are seen to be consistent with and in support of the level scheme.

The presence of four excited states in the Sb^{125} nucleus is deduced from these experiments; they have energies of 1.07, 1.41, 1.88, and 1.97 Mev. There is no evidence in the 9.7-day activity for either beta or gamma transitions leading to the excited state at 0.326 Mev seen in the 9.4-minute period.

9.4-minute Activity

The 9.4-min beta emitter in Sn was first produced by irradiation of Sn with deuterons and neutrons by Livingood and Seaborg¹ and tentatively assigned to Sn^{125} . This assignment has been confirmed.^{6,7} Early reports of the maximum beta energy vary^{12,6}, but two recent reports^{13,7} agree on a value of about 2.05 Mev for this component. They also agree on the existence of a branch of about 0.5 Mev. One of these

12. W. H. Sullivan and H. E. Wyatt, Plut. Proj. Rep. Mon. No. 243, 3 (1947)

13. R. B. Duffield and L. M. Langer, P. R. 77, 143 (1950)

investigators¹³ resolved an additional component of 1.17 Mev in a spectrometer measurement. An intense gamma ray has been measured by its internal-conversion and secondary electrons to have an energy of 0.326 Mev¹³ and shown to be in coincidence with the highest-energy beta ray.⁷ Higher-energy gamma radiation was first reported at 1.86 Mev from absorption measurement¹⁴.

Studies of the scintillation spectrum thus far show, in addition to the very intense 0.326-Mev gamma ray, a weak peak at 1.41 Mev which also decays with this period. This portion of the study is not yet complete, but there is some indication that others of very low intensity may also be present.

Absorption studies of the beta-gamma coincidences show that the highest energy beta ray, which is the predominant mode of decay, leads to the excited state at 0.326 Mev. The (beta-gamma) beta plot shows no perceptible "fall-off" near the end point, indicating that little or no decay takes place directly to the ground state. A possible slight rise in this curve in the region of thin absorbers is consistent with the existence of a very weak lower-energy branch, feeding the 1.41-Mev level. No absorption curves have yet been attempted on the coincidences which should exist between such a branch and the 1.41-Mev gamma ray. The excited state of Sb^{125} nucleus at 1.41 Mev is thus far the only one found common to both activities.

14. G. E. Boyd, ORNL-870 (1951) (Classified) or NBS Circular No. 499, Supplement No. 2 (1951)

This study is continuing, and the results thus far will be presented at the Washington meeting of the American Physical Society, to be held in April, 1955.

51-8 Characteristics of a Scintillation Spectrometer (5200-01)

Bernard Hamermesh and Carroll C. Trail
Reported by Carroll C. Trail

The pulse height discriminating circuit of the twenty-channel analyzer has been modified for more stable operation. The window width is now nearly independent of pulse height.

The coincidence circuit was modified so that delay times could be inserted in the input channels.

52-8 Capture Gamma-Ray Spectra Using a Scintillation Spectrometer (5200-01)

Bernard Hamermesh and Carroll C. Trail
Reported by Carroll C. Trail

The gamma-ray spectra from neutron capture by Li^7 and F^{19} have been studied. No lines were resolved. A search has been made for a 30-kev gamma-ray from neutron capture by Al^{27} , corresponding to a 30-kev level in Al^{28} . No consistent results have been obtained.

60-8

7. 7-Meter Bent Crystal Spectrometer (5200-01)

Bernard Hamermesh and David Rose
Reported by Bernard Hamermesh

During the past six months, most of the machining operations were completed. The lead baffle-collimator has been tested and after several restackings was found to have a transmission of about 27.5% out of a possible 50%. The track sections have proven difficult to position but that operation is now half completed.

The crystal table drive which seemed to be satisfactory upon preliminary study was found to operate in an erratic manner. Several of the drive components were found to have been made unsatisfactorily. These are being reworked.

The delivery date on the quartz crystal was delayed several times. While work on the quartz is well advanced the completion date is unknown at the present time. By the time the crystal arrives the clamp blocks, which only require final lapping, should be completed.

It is hoped that the spectrometer will be put into operation during the next quarter.

70-8

Measurement of Nuclear Moments by Angular Correlation (5200-01)

Victor E. Krohn and Sol Raboy
Reported by Victor E. Krohn

A. Pb^{204m}

A letter reporting the existence of a third gamma ray in the Pb^{204m} cascade was published in The Physical Review 95, 354 (1954). This result has now been confirmed by other investigators.

An article concerning the gyromagnetic ratio of the 3×10^{-7} -sec state, the spin assignments of the levels, and the multipole mixing of the gamma rays in the $\text{Pb}^{204\text{m}}$ decay scheme has been accepted for publication in the Physical Review. Also, a paper on this subject was presented at the November, 1954 meeting of the American Physical Society.

The decay scheme must be consistent with the angular correlation and internal conversion data. Hence, the 374-kev transition must be an E2 transition with an admixture of 0.5% M3. This is unexpected on the basis of the single particle and/or collective models. The same is true for the one percent admixture of M6 in the 912-kev transition. In the near future an attempt will be made to measure the angular correlation of the prompt (374-899 kev) gamma rays. The decay scheme and mixing parameters of the aforementioned article enable one to predict the anisotropy of the prompt cascade.

B. Hf^{181}

A letter describing the measurement of the 10^{-8} -sec state of Ta^{181} which appears following beta decay of Hf^{181} was published in the Physical Review 95, 1689 (1954).

This measurement may be repeated with the techniques developed for the Pb^{204} measurement when faster electronic circuits are available.

C. W^{187}

The first attempt to measure the gyromagnetic ratio of the 5×10^{-7} -sec state of Re^{187} which occurs in W^{187} beta decay was

unsuccessful because the anisotropy of the 480- and 135-kev gamma rays was too small. Preliminary results indicate that it may be possible to use the 480- and 70-kev gamma rays for this measurement, but the measurement will be tedious. Work on this decay scheme has been temporarily discontinued.

D. Am²⁴¹

Two papers concerning this decay scheme were presented at the January, 1955 meeting of the American Physical Society. The first described the time-dependent attenuation of the α - γ angular correlation in liquid film sources. The second paper gave a preliminary result for the gyromagnetic ratio of the 7×10^{-8} -sec state of Np²³⁷.

The study of this decay scheme is being continued.

T. B. Novey of the ANL Chemistry Division is collaborating in the study of Am²⁴¹.

E. General

A paper entitled "A Theorem on Angular Correlation" has been accepted for publication in the Physical Review.

80-8

Molecular Beam Studies (5200-01)

John A. Dalman, Leonard S. Goodman, Sol Wexler
Reported by Leonard S. Goodman

The direct hyperfine transition, $\Delta F = 1, \Delta m_F = \pm 1$, of Cs^{134m} was measured as $\Delta\nu = 3684.5 \pm .5$ Mc/sec. The sign of the nuclear magnetic moment has thus been established as positive by comparison.

of this new measured value to the values predicted ($3684.3 \pm .5$ Mc/sec for a positive, and 3695.0 ± 5 Mc/sec for a negative moment) from previous low frequency, $\Delta F = 0, \Delta m_F = \pm 1$, measurements.

A letter to the Editor giving these results appeared in Phys. Rev. 97, 242 (Jan. 1, 1955).

A paper describing our work on $\text{Cs}^{134\text{m}}$ has been cleared by the Unclassified Areas Committee and will soon be submitted to the Physical Review.

Work has been started on the radioactive isotope 50-d. $\text{In}^{114\text{m}}$.

Numerous problems have presented themselves in connection with this experiment:

1. The system used for inserting the samples in the $\text{Cs}^{134\text{m}}$ experiments proved to be impractical because of outgassing of the Apiezon grease on the sliding oven tube and lack of sufficient space for radiation shields. A bulk-head with a sliding "O" ring valve was sealed between the oven and differential pumping chambers so that the oven chamber could be let up to atmospheric pressure and evacuated separately from the rest of the system.
2. A new oven mount with quick action sealing features was designed, fabricated, tested and aligned. Much time was spent tracking down a high temperature binding of sliding parts in the oven mount.
3. Because the molybdenum oven emits electrons at the temperature of operation (Ca. 1100°C), the electron bombardment

system could no longer be used with self rectification.

A rectified high voltage supply was designed and installed by Ed Rathbun of the Electronics Division. Mr. Rathbun also built an emission current regulator for the electron bombardment system.

4. Because the deflection of indium atoms is smaller than that of cesium atoms (In has a smaller g_j and higher velocity) for the same magnetic field and gradient, the 10-mil slits for the cesium experiment were reduced to 3 mils. This makes the lineup of the apparatus much more difficult and sensitive.

5. Because the atomic ionization potential of indium is higher than the work function of a clean tungsten surface., the surface ionization detector used for cesium was very insensitive to an indium beam.

Research on a surface ionization detector sensitive to indium was instituted. After trying recipes from several people in the business, a resonable scheme adaptable to our experiment was evolved with much trial and error, and we now have some confidence in the surface ionization detector for indium.

Looking ahead to other problems, experiments have been started on using an electron multiplier detector and mass spectrometer for the inactive beam. This system will be very important if studies on halogens are attempted.

One of the problems which cut down efficiency in the past has been the use of a single counter for measuring the radioactivity

deposited on the cold copper collection disks. The time taken to count all eight samples with good enough statistics to identify a resonance took almost as much time as it took to collect the samples.

A bank of ten gas flow counters has been designed, fabricated and tested. These will allow the collection of data in 1/8 the time and will almost double the amount of data which can be collected per loading of a radioactive sample.

To obviate the necessity of buying signal generators and frequency measuring equipment in the frequency range from 50 to 300 Mc/sec, a multiplier and power amplifier has been constructed by Ed Rathbun of the Electronics Division. For the frequency range 300-1500 Mc/sec, a war surplus radar-jammer T85-APT5 is being adapted for our use.

90-8

Cross Sections for 14-Mev Neutrons (5200-01)

Harvey Casson, Leonard S. Goodman,
Jack Haugsnes, Louis A. Rayburn
Reported by Harvey Casson

Since the last report, the operation of the deuteron accelerator has been satisfactory, enabling us to check our equipment and proceed with experimental work. By magnetic separation of the accelerated ion beam into its components, it was possible to estimate that about 80% consisted of monatomic ions, with the remainder divided between diatomic and triatomic ions.

With a deuteron energy of 50 kev and beam currents of about 10 microamperes, we have obtained neutron source strengths of the order of 10^7 neutrons/sec. Although we can easily increase the neutron yield by raising the deuteron energy and/or beam current, biological hazards have deterred us from going beyond about 10^8 neutrons/sec. The neutron yield from a given target has been found to decrease with time (probably as a result of carbon deposits collecting on its surface). By removing the target and polishing its surface, we have been able to bring the yield back to normal.

The shape of the neutron cone has been investigated. (By the neutron cone, we mean the distribution in space of neutrons which are in coincidence with the alpha particles arriving at the alpha detector). The shape of the neutron cone depends on the geometry of the alpha detector, the deuteron energy and the amount of air scattering of the neutrons. Neutron cone geometries of the order of 10^{-4} to 10^{-5} are being used.

Both the neutron and alpha detectors are composed of scintillating plastic. Pulse height distributions for 14-Mev neutrons were run on the neutron detector indicating a relatively flat distribution.

Two peaks are observable in the alpha detector pulse-height distribution: one due to 3.5-Mev alpha particles and the other due to protons from the D-D reaction. The latter is of much lower intensity than the former, and the pulses do not produce real coincidences with the neutron detector.

Measurements of room scattering of non-coincident neutrons have indicated that accidental coincidences would seriously interfere with our planned determination of the angular distributions of elastically-scattered 14-Mev neutrons, using our present coincidence circuits. We have, therefore, postponed work on this phase of the project pending the arrival of faster (5-millimicrosecond resolving time) circuits. The latter are being developed by the Electronics Division.

A method has been devised for the determination of total cross sections for 14-Mev neutrons which shows promise of greater accuracy than the usual methods, since it does not require the usual uncertain corrections for (a) the finite solid angle of the detector and (b) multiple scattering.

The method may be described as follows: Using a small geometry for the alpha detector, and hence for the neutron cone (say, 10^{-5}), the scatterer is located very close to the target where it subtends a large solid angle. Two positions are specified for the neutron detector, P_1 inside the neutron cone, and P_2 at a small angular distance away (of the order of 1 degree or less) but partly outside the neutron cone in order to give a reduced coincidence counting rate. The following four measurements are made:

C_1 , the coincidence counting rate at P_1 without the scatterer in position

C_{1s} , the coincidence rate at P_1 with the scatterer

C_2 , the coincidence rate at P_2 without the scatterer

C_{2s} , the coincidence rate at P_2 with the scatterer.

We may then write $C_1 = N$, $C_2 = fN$, $C_{1s} = N e^{-\lambda t} + S_I$, $C_{2s} = fN e^{-\lambda t} + S_I$, where f is the ratio of the coincidence counting rates at P_1 and P_2 , t is the thickness of the scatterer, λ the attenuation of the beam per unit length of scatterer and S_I is the inscattering. The term S_I which is taken to be the same in both C_{1s} and C_{2s} includes single elastic scattering, multiple scattering and inelastic scattering. The assumption made is that in view of the small angular range between P_1 and P_2 , these effects would give the same contribution to the counting rates at the two points.

The true attenuation factor, $e^{-\lambda t}$, from which the total cross-section is determined is seen to be equal to $\frac{C_{1s} - C_{2s}}{C_1 - C_2}$.

Measurements using this method are in progress.

91-8

Neutron Transport Cross Sections, Angular
Distribution of Scattered Neutrons (5200-04)

Raymond O. Lane and Alexander Langsdorf, Jr.
Reported by Alexander Langsdorf, Jr.

The apparatus described in ANL 5317, pgs. 37-40, has been completed and operated successfully. The first complete data were taken September 28 through October 7, 1954. After several days of testing, twelve elements were measured at 18 energy settings and

Figure 6

Differential scattering cross section in barns per steradian for beryllium plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

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Figure 7

Differential scattering cross section in barns per steradian for carbon plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 8

Differential scattering cross section in barns per steradian for aluminum plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 9

Differential scattering cross section in barns per steradian for titanium plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 10

Differential scattering cross section in barns per steradian for iron plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 11

Differential scattering cross section in barns per steradian for nickel plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 12

Differential scattering cross section in barns per steradian for copper plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 13

Differential scattering cross section in barns per steradian for zinc plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 14

Differential scattering cross section in barns per steradian for zirconium plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 15

Differential scattering cross section in barns per steradian for silver plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 16

Differential scattering cross section in barns per steradian for tin plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

Figure 17

Differential scattering cross section in barns per steradian for lead plotted against incident neutron energy and cosine of the laboratory scattering angle. $\sigma_t(\theta)$ includes elastic and inelastic scattering. The curves on the right are for various incident neutron energies.

five angles. Since reduction of such a mass of data takes much more time than the experiments, steps are being taken to program future computations on the Avidac.

The initial program, of which the first run was the beginning, is intended to survey many elements over a broad energy region with broad energy resolution. Results of the first run are presented on accompanying graphs. Data was taken with about 80-kev energy spread (from 100 kev at low energy to 60 kev at high energy) in about 80-kev steps from below 100 kev up to 1.5 Mev neutron energy. Data was obtained at five angles: 22° , 54° , 92° , 112° , 144° . All cross sections are normalized to the known total cross section of carbon and are corrected for attenuation but not for plural scattering. By attenuation is meant that the actual scattering varies as $(1 - e^{-N\sigma_t})$ rather than simply linearly.* Since the samples only scatter 10% to 15% of the incident neutrons, the correction is small. The more complex plural scattering corrections are expected to be only about 10%. Statistical accuracy is generally better than 5%.

Two checks on our results are available. First, the results of Walt and Barschall¹ for 1-Mev neutrons are comparable after adding to their elastic scattering distributions, the inelastic contribution assuming the latter is isotropic. When this is done, agreement is within their stated uncertainty of 15%. Second, when our differential scattering distributions, which are total elastic plus inelastic, are integrated to give the total scattering cross section, the

¹. Scattering of 1-Mev Neutrons by Intermediate and Heavy Nuclei, M. Walt and H. H. Barschall Phys. Rev. 93, 1062 (1954).

* Here σ_t is the total cross section (assuming also that there is no capture so σ_t is entirely scattering) and N is the number of atoms per cm^2 .

results are in fair agreement with prior measured total cross sections, such as are presented in AECU-2040. Disagreements are usually less than 10%, occasionally 20%, but is of such a nature as to make it doubtful that the total cross section data is itself reliable enough for a closer comparison than this.

Details of the computational techniques will be presented in publication expected to be prepared this summer. The only remaining problem in computation, not yet worked out, is that of making plural scattering corrections by analysis. However, if an analytical approach is too difficult, the empirical approach of extrapolation to zero thickness of data on a series of sample thicknesses can be used.

Since the first run was completed a great deal of effort has been expended preparing samples for a second run. In particular, the "canning" of alkali metals required great effort, and has been successfully completed.

Note: The second run was made in February 1955. It gave data on 16 additional elements: H (as CH_2), Li, F (as CF_2), Na, Mg, K, Co, Mo, Cd, Ta, W, Pt, Au, Bi, Th, U, plus reruns on Be, C, Al, Fe, Ag, and Pb, plus some 18-8 stainless steel.

Some comments on the apparatus:

The collimation of the beam emerging from the source collimator seems to be excellent, as is the shielding of the five detector systems, up to the 1.5-Mev neutron energy so far employed. Probably conditions will be satisfactory at least up to 3 Mev. As was expected,

a large part of the background counting rate is due to scattering by air in the vicinity of the scattering sample position.

Most of the factors in the geometry of the system which need to be evaluated to make highly refined analyses of the angular data can be determined by practical auxiliary experiments, many of which are planned for execution.

The apparatus is flexible in its uses. It will be well adapted to polarization studies. It is also easily used to study transport properties (diffuse reflection and transmission) by thick scattering plates. Such experiments are pertinent to reactor shielding, etc.

98-7

Total Neutron Cross Sections in the Kev Region

(5200-01)

Carl T. Hibdon and Raymond R. Weeks
Reported by Carl T. Hibdon

1. Resolution of the 90° Electrostatic Analyzer.

Measurements of the neutron total cross sections in the kev energy region as described in ANL-5080, have been continued but with considerably improved resolution accomplished by passing the proton beam through the 90° electrostatic analyzer. Energy variations of the Van de Graaff generator are controlled from the output slits of the analyzer. Narrow entrance and exit slits about 30 mils wide define the proton beam to a very small energy spread. In order to determine the effective resolution of this arrangement it would be desirable to re-measure

the neutron cross section of some resonances known to be so narrow that the measured width gives the effective resolution. The thickness of the lithium target used for the neutron source is also a factor in the resolution. For best resolution only a very thin lithium layer is deposited on the tantalum backing. Layers less than a kev thick, as measured by the rise curve method, have been used. The following results were obtained.

(a) Resonances of Bismuth.

To determine the resolution at very low energies the narrow resonance of Bi at 2.1 kev was studied using lithium targets less than one kev thick. A width of 250 ev at half the maximum height of the experimental peak was observed, to be compared with a previous value of 700 ev.

A second resonance of Bi at 12 kev was also studied by a single run. The width observed at half the maximum height was 0.75 kev, to be compared with a previous value of 1.2 kev. This resonance is undoubtedly narrower than 0.75 kev but more thorough study will be necessary to resolve it fully. During the present experiments a well-defined minimum before this resonance was observed (s-wave scattering).

(b) 27-kev Resonance of Fluorine.

This resonance shows a half width of 400 ev at half the maximum height of the experimental peak, instead of the previously observed value of 1.5 kev. The resonance is symmetrical, shows no dip before the resonance, and gives a measured peak cross section

greater than the maximum possible value for a value of $J = 0$. Thus, $\ell > 0$ and $J > 0$. The detailed theory and application of it to the experimental data will be given in a subsequent report.

2. The 98-Kev Resonance of Fluorine.

Because of discrepancies in both resonance width and peak height between former experimental results and calculations made by Monahan from results obtained by the poor-resolution, self-detection technique, it appeared desirable to re-measure the cross section in the 98-kev region with good resolution.

Measurements were first made by flat detection over the energy range from 81 to 110 kev using a Li target about one kev thick. The results obtained still indicate a total width of 12 kev with no cross section greater than 25 barns. Contrary to the earlier experiments, there did not appear to be a single peak or single well-rounded maximum, but rather an apparent flattening of the top with a number of dips and peaks. Most of these features repeated in successive runs but were not of a magnitude sufficiently outside probable errors to be certain.

This resonance was also studied by self-detection using teflon scattering samples and synthetic LiF transmission samples. The measurements were repeated using natural LiF scattering samples. For both measurements essentially the same results were obtained as for flat-detection with the peaks and dips being more prominent.

All of these measurements indicate the possibility of the presence of more than one resonance. The results are, however,

inconclusive because of assorted difficulties with the machine and targets.

3. Total Cross Section of Magnesium

A survey of the transmission cross section of Mg over the energy range from 2 to 22 kev was made using Mg metal of 99.9% purity obtained from the Dow Chemical Co. Lucite was used as a flat scattering sample in the neutron detector. Points were taken at kev intervals up to 19 kev and above this energy at 500 ev intervals using a lithium target 0.65 kev thick (as measured by the rise curve method). An average cross section of about 3.75b was observed in this energy region, with the exception of the resonance at 20 kev, which had a maximum of 8 barns. On the basis of this peak cross section this resonance cannot be assigned to Mg^{24} (78.8% abundant) but must be attributable to either Mg^{25} or Mg^{26} , each of which has an isotopic abundance of the order of 10%. Further measurements at higher energies are planned.

Neutron Cross Section Measurements by the
Self-Indication Technique (5200-01)

Carl T. Hibdon, Alexander Langsdorf, Jr.
James E. Monahan and F. Paul Mooring
Reported by James E. Monahan

The analysis of a single scattering resonance by "poor resolution" self-indication methods depends on an independent measurement of the resolution width, Δ , of the incident neutron beam. An application of this method to the 98-kev resonance in F^{19} indicated that (i) the resolution given by the conventional "rise-curve" method is in error by a factor of about three, and/ or (ii) this is a multiple resonance. A standard "good-resolution" experiment also implied a possibility of this being a multiple resonance. Further experiments showed that Δ , as usually determined, is indeed in error by a factor of the order of magnitude mentioned above. It is believed that this latter effect is due, at least partially, to a non-uniformity of the lithium target. Both of these experiments are discussed more fully in other sections of this report. (Cf. Carl T. Hibdon, The 98-kev Resonance of F^{19} , and F. Paul Mooring, The Possibility of Non-Uniform Lithium Layers.)

In the light of these developments it was felt that another method of obtaining the width, Δ , would be desirable. By adapting the self-indication technique to a "good-resolution" experiment, such a method is found. More explicitly, by obtaining both the flat and self-detection cross sections for at least three points within the

resonance region, it is possible to obtain the value of Δ as well as values for the Breit-Wigner parameters of the resonance. Such an experiment gave the following results for the narrow 27-kev resonance in F^{19} : $\Delta = (0.60 \pm 0.29)$ kev, $\Gamma = (0.55 \pm 0.24)$ kev, $\sigma^{(r)} = (18 \pm 21)$ barns, and $\sigma_p = (5 \pm 1)$ barns. Here Γ is the resonance width, and $\sigma^{(r)}$ is the peak resonance cross section above the background of cross section σ_p . The errors quoted are standard deviations and are due mainly to uncertainties in the energy setting of the Van de Graaff machine. It might be mentioned that since less than one-quarter of the available data was used in the calculation, these errors can probably be reduced by a factor of about two.

An analysis of experimental data pertinent to this method is being coded for the AVIDAC.

Two abstracts on this subject were presented before the New York meeting of the American Physical Society.

108-2 An He^3 Neutron Spectrometer (5200-04)

F. Lynch

Preliminary experiments to determine the properties of He in the proportional region at high pressure were made using a large cylindrical counter 4" in diameter and 18" long. With 99% He and 1% A at 90 p. s. i., pulses from Cm^{244} alpha particles have a distribution at 1400 volts showing a width at half maximum of 15%. At 300 p. s. i., the resolution was much worse. Although the counter

was filled by slowly filtering through an activated charcoal trap at liquid nitrogen temperature, it is apparent that the gas purity was not adequate. Variations in pulse height and rise time indicate the presence of electronegative gas, probably oxygen, which interferes with electron collection. The shortest rise time was about 10 μ sec. A system has been started for continuous recirculation of the filling gas at 300 p. s. i. through a hot calcium purifier and cold trap.

Titanium hydride and silver solder have been obtained in powdered form for making a metal to ceramic seal for constructing a very small high pressure proportional counter suitable for filling with He³.

110-8

The Neutron Crystal Spectrometer (5200-04)

Sol Raboy, G. R. Ringo, Carroll C. Trail
Reported by Sol Raboy

The neutron crystal spectrometer was installed at port 13 of CP-5. The port provides an 8-inch high beam with a collimation of 20 minutes. A beryllium crystal 2-5/8" x 1/2" x 11/16" and oriented to provide lattice spacing from 0.7 to 1.2 Å is available. The shield for the spectrometer was mounted and tested with satisfactory results. This completes the installation of the spectrometer.

Gilbert J. Perlow

Development continued on the double proportional counter neutron spectrometer described in previous reports.

Improved amplifier circuits were constructed and the electronic system altered to utilize a 10-channel pulse-height analyzer which was purchased.

New proton recoil collimators were constructed by assembling approximately 1200 silver-plated monel tubes in close-packed array and furnace brazing the assembly. Collimators were then sliced off the unit, ground to size and finished by manipulations under a microscope. The counters were filled to various pressures with purified methane and sealed off.

Data were taken with the new counters and electronic system in December using neutrons from $\text{Li}^7(p,n)\text{Be}^7$ at the Van de Graaff generator. The anti-fold-over circuit appeared to operate, although not as satisfactorily as might be hoped. By and large the data were comparable to those obtained with the original makeshift apparatus. Widths at half maximum of the peaks for monoenergetic neutrons varied from about 22 to 12% depending on the energy. This is considerably wider than what would be predicted from the collimator dimensions. The possibility cannot be disregarded that the lithium targets are thicker than the traditional forward-angle yield curve measurement predicts.

An additional unpleasant circumstance is the appearance of a low-energy tail on the distribution. Calculations have been made on various possibilities to account for it. This work is still in progress. It is clear that at the higher gas pressures (~ 1 Atm) the tail is mainly due to double scattering; that is, an incident neutron scatters twice, once in each chamber. With the present geometry this appears to set a limit on the useful maximum energy of somewhat below 2 Mev.

In the next trials, a collimator of greater angular aperture is to be used on the assumption that this cannot appreciably worsen the resolution but can definitely improve the ratio of true to spurious events.

115-4

Boron Trimethyl (5200-02)

Louis Basile

A considerable amount of time was spent on the purification of this gas. The gas was definitely identified as being an etherate, complexed with diethyl ether. The ordinary methods of fractional condensation would not separate this gas from the impurity. As a result a long tedious procedure was developed to purify the gas and the following is a brief summary of the procedure.

The crude material was allowed to react with ammonia so that the boron trimethyl, and any other boron compound, would form an amine addition product.

The amine addition product is a solid at room temperature and not very volatile. All of the volatile impurities were then pumped off of the adduct. A slight deficiency of anhydrous hydrogen chloride was added to the adduct to liberate the boron trimethyl gas.

Any other volatile boron compounds will be liberated also. To eliminate these undesirable compounds and also any hydrogen chloride that might have been carried over, the gas was bubbled through water. The water was removed by slow fractional condensation. The product was characterized by its vapor pressure which checked satisfactorily with the literature value.

The purified gas was then turned over to Dr. John Simpson of the Institute of Nuclear Studies for testing purposes. The gas appeared to be satisfactory as a counter gas but not quite as good as expected. It was felt that there still was a trace of impurity which affected the characteristics of the gas as a counter gas. Very little more could be done to further purify this gas by the available techniques. A fresh supply of the boron trimethyl, with normal isotopic boron, was ordered and we are awaiting shipment of this material.

The following are assorted papers and memoranda that were gotten out on the indicated dates. Copies were circulated among the members of the Midwest Accelerator Conference. (A technical group, most of its members being from universities in MURA (Midwest Universities Research Association).)

A. Coupling of Betatron and Phase Oscillations in a Synchrotron-August, 1954.

Edwin A. Crosbie and Morton Hamermesh

ABSTRACT

The coupled equations for radial position, momentum, and phase oscillations in a synchrotron are treated by use of difference equations. The effect of a radial variation in the accelerating radio-frequency voltage is included in the treatment; any resultant damping of one type of oscillation is shown necessarily to be accompanied by equal anti-damping of oscillations of the other type. A simple treatment of the adiabatic variation of parameters for systems of linear differential or difference equations is given.

Coupling of Betatron and Phase Oscillations in a Synchrotron

The equations of motion in a synchrotron have been treated by many authors.¹ The accelerating radio-frequency voltage,

1. R. O. Twiss and N. H. Frank, RSI 20, (1949)

which is applied across one or more narrow gaps, is Fourier-analyzed and only the synchronous component considered. In addition, the separation of the motion into betatron and synchrotron oscillations appears rather arbitrary. We felt it worthwhile to treat the problem in a more direct fashion. We consider, for simplicity, a machine in which the accelerating rf voltage is applied across a narrow gap at some azimuth, say $\theta = 0$. The applied voltage varies sinusoidally in time, but can also vary radially. The magnetic field B_0 at the fixed "equilibrium" radius R_0 and the radio frequency ω_s , will be tracked to fit an ideal particle which is continuously accelerated so that its momentum p_s (speed v_s and total energy $\mathcal{E}_s$) just match to give a circular orbit of radius R_0 at all times. Since the particles in the actual machine are given impulses at $\theta = 0$, none of the actual particles will follow the ideal circular orbit.

Except at $\theta = 0$, the equations of motion are just those of free oscillation:

$$\frac{d^2 x}{d\theta^2} + (1-n)x = \frac{\Delta p}{p_s} \quad (1)$$

$$\Delta p = p - p_s \quad (1a)$$

where x is the ratio of the radial displacement from the equilibrium orbit to R_0 , n is the field index, and p is the momentum of the particle. We have limited ourselves to the linear approximation. We also have for the speed:

$$v = r \dot{\theta} \quad (2)$$

$$\dot{\theta} = \frac{\nu}{r} = \frac{\nu_s + \Delta\nu}{R_c(1+\chi)} \approx \frac{\nu_s}{R_c} \left(1 + \frac{\Delta\nu}{\nu_s} - \chi\right) \quad (2a)$$

The radio frequency will be tracked to equal $\omega_s = \frac{\nu_s}{R_o}$ so that the ideal particle always reaches the gap at the same phase ψ . From 2a:

$$\omega_s dt = d\theta \left(1 - \frac{\Delta\nu}{\nu_s} + \chi\right) = d\theta \left(1 - \frac{\epsilon_0^2}{\epsilon_s^2} \frac{\Delta p}{p_s} + \chi\right) \quad (3)$$

where ϵ_0 is the rest energy. Integrating Equation 3 over θ from 0 to 2π , the left side gives $\delta\psi$, the increase in phase ψ for one turn:

$$\delta\psi = 2\pi + \int_0^{2\pi} d\theta \left(\chi - \frac{\epsilon_0^2}{\epsilon_s^2} \frac{\Delta p}{p_s}\right) \quad (4)$$

We denote x and $x' = \frac{dx}{d\theta}$ just after passing the gap on the γ' th turn by $x_{\gamma'}$, $x'_{\gamma'}$. By integrating Equation 1, we find their values after one turn around the machine, $x_{\gamma'+1}$, $x'_{\gamma'+1}$, which are linear functions of $x_{\gamma'}$, $x'_{\gamma'}$, and $\left(\frac{\Delta p}{p_s}\right)_{\gamma'}$. $\delta\psi = \psi_{\gamma'+1} - \psi_{\gamma'}$ is given by Equation 4. Using for x the solution of Equation 1 for the γ' th turn, $\psi_{\gamma'+1}$ is expressible linearly in terms of $x_{\gamma'}$, $x'_{\gamma'}$, $\left(\frac{\Delta p}{p_s}\right)_{\gamma'}$, $\psi_{\gamma'}$. Finally, we must obtain an equation for the momentum. The particle will receive an increment of energy after it has completed the γ' th turn. The amount it then receives will depend on the radius $x_{\gamma'+1}$ at which it crosses the gap, and the phase $\psi_{\gamma'+1}$. Up to linear terms in the displacement, this energy increment is

$$e(V_0 + V_c' x_{\gamma'+1}) \sin \psi_{\gamma'+1}$$

where V_0 and V_0' are the voltage and voltage gradient at R_0 .

Meanwhile, the ideal particle receives the energy increment $e V_0 \sin \varphi_s$.

Thus the change in the energy error $\Delta \varepsilon = \varepsilon - \varepsilon_s$ is:

$$(\Delta \varepsilon)_{\nu+1} - (\Delta \varepsilon)_\nu = e (V_0 + V_0' \chi_{\nu+1}) \sin \varphi_{\nu+1} - e V_0 \sin \varphi_s. \quad (5)$$

Expanding the sin function (we assume small oscillations, $\varphi_\nu - \varphi_s \ll 1$)

and dropping products of small quantities, we have:

$$(\Delta \varepsilon)_{\nu+1} - (\Delta \varepsilon)_\nu = (e V_0 \cos \varphi_s) (\varphi_{\nu+1} - \varphi_s) + (e V_0' \sin \varphi_s) \chi_{\nu+1}. \quad (5a)$$

Measuring φ from φ_s , and using $\Delta \varepsilon = \frac{c^2 p_s^2}{\varepsilon_s} \frac{\Delta p}{p_s}$ we can write:

$$\left(\frac{c^2 p_s^2}{\varepsilon_s} \frac{\Delta p}{p_s} \right)_{\nu+1} - \left(\frac{c^2 p_s^2}{\varepsilon_s} \frac{\Delta p}{p_s} \right)_\nu = (e V_0 \cos \varphi_s) \varphi_{\nu+1} + (e V_0' \sin \varphi_s) \chi_{\nu+1}. \quad (6)$$

Our discussion shows that we obtain four coupled linear difference equations for the four quantities x_ν , x'_ν , φ_ν and

$$p_\nu = \left(\frac{\Delta p}{p_s} \right)_\nu. \quad (7)$$

We now do the detailed problem for the ordinary synchrotron, where n is independent of azimuth. The solutions of Equation 1

$$\frac{d^2 x}{d\theta^2} + (1-n)x = \frac{\Delta p}{p_s} = p \quad (8)$$

have the betatron frequency $\sqrt{1-n}$. The general solution of Equation 8 for initial conditions x_v, x'_v is

$$x = \frac{p_v}{1-n} + (x_v - \frac{p_v}{1-n}) \cos \sqrt{1-n} \theta + \frac{x'_v}{\sqrt{1-n}} \sin \sqrt{1-n} \theta ; \quad (9)$$

setting $\theta = 2\pi$, $\psi = 2\pi\sqrt{1-n}$ we have

$$x_{v+1} = \frac{1-\cos\psi}{1-n} p_v + \cos\psi x_v + \frac{\sin\psi}{\sqrt{1-n}} x'_v \quad (10)$$

$$x'_{v+1} = \frac{\sin\psi}{\sqrt{1-n}} p_v - \sqrt{1-n} \sin\psi x_v + \cos\psi x'_v . \quad (11)$$

Substituting Equation 9 in Equation 4 and integrating

$$\varphi_{v+1} - \varphi_v = \left[2\pi \left(\frac{1}{1-n} - \frac{\epsilon_z^2}{\epsilon_s^2} \right) - \frac{\sin\psi}{(1-n)^{3/2}} \right] p_v + \frac{\sin\psi}{\sqrt{1-n}} x_v + \frac{(1-\cos\psi)}{1-n} x'_v . \quad (12)$$

Equations 6, 10, 11, 12 are the coupled difference equations describing the system. They can be simplified by the following scale changes.

Replace

$$\sqrt{1-n} x_v \rightarrow x_v$$

$$\frac{p_v}{\sqrt{1-n}} \rightarrow p_v$$

$$\varphi_v (1-n) \rightarrow \varphi_v .$$

Our set of equations becomes

$$x_{v+1} = \cos\psi x_v + \sin\psi x'_v + (1-\cos\psi) p_v \quad (13)$$

$$X'_{v+1} = -\sin \psi X_v + \cos \psi X'_v + \sin \psi P_v \quad (14)$$

$$\left(\frac{c^2 P_s^2}{\epsilon_s} P\right)_{v+1} - \left(\frac{c^2 P_s^2}{\epsilon_s} P\right)_v = \frac{e V_0' \sin \psi_s}{1-n} X_{v+1} + \frac{e V_0 \cos \psi_s}{(1-n)^{3/2}} \varphi_{v+1} \quad (15)$$

$$\varphi_{v+1} - \varphi_v = \sin \psi X_v + (1 - \cos \psi) X'_v + C_v P_v \quad (16)$$

where

$$C_v = 2\pi \gamma_v (1-n)^{3/2} - \sin \psi \quad (17)$$

$$\gamma_v = \left[\frac{1}{1-n} - \frac{\epsilon_0^2}{\epsilon_s^2} \right]_v \quad (17a)$$

First we neglect the slow variation of the coefficients in Equations 15-17, drop the subscript on C_v and replace 15 by

$$P_{v+1} = A X_{v+1} + P_v + B \varphi_{v+1} \quad (15a)$$

where

$$A = \frac{e V_0' \sin \psi_s \epsilon_s}{c^2 P_s^2 (1-n)}, \quad B = \frac{e V_0 \cos \psi_s \epsilon_s}{c^2 P_s^2 (1-n)^{3/2}} \quad (18)$$

Both A and B are of order energy gain per turn/total energy $\ll 1$. We find the eigenfrequencies and eigenvectors of the system 13, 14, 15a, 16.

Let

$$\chi_\nu = \xi \lambda^\nu, \chi'_\nu = \xi' \lambda^\nu, \rho_\nu = \pi \lambda^\nu, \varphi_\nu = f \lambda^\nu \quad (19)$$

where ξ, ξ', π, f are the components of the eigenvector belonging to the eigenvalue λ , and substitute in the equations. The secular matrix is

$$\begin{pmatrix} \cos \psi - \lambda & \sin \psi & 1 - \cos \psi & 0 \\ -\sin \psi & \cos \psi - \lambda & \sin \psi & 0 \\ A \lambda & 0 & 1 - \lambda & B \lambda \\ \sin \psi & 1 - \cos \psi & C & 1 - \lambda \end{pmatrix} \quad (20)$$

Since A and B are small, we can find the roots of the secular equation by successive approximation. If we set $A = B = 0$, we find $\lambda = 1, 1, e^{\pm i\psi}$. For the roots in the neighborhood of the double root 1, we set $\lambda = 1 + \epsilon$ in the secular equation. Keeping only the lowest power of ϵ, A, B , we find

$$\epsilon^2 + \epsilon^3 - \epsilon(A + 2BC') - BC' = 0 \quad (21)$$

where

$$C' = C + \sin \psi = 2\pi \delta(1-n)^{\frac{3}{2}}, \quad (22)$$

The lowest order equation would be

$$\epsilon^2 = BC', \quad \epsilon = \pm \sqrt{BC'} \quad (23)$$

Thus the oscillations will be stable if $BC' < 0$, i. e., $\gamma \cos \varphi_s < 0$.

Substituting 23 back into 21, we find the next approximation:

$$\begin{aligned} \epsilon^2 - \epsilon(A + BC') - BC' &= 0 \\ \epsilon &= \frac{A + BC'}{2} \pm \sqrt{\frac{(A + BC')^2}{4} + BC'} \simeq \pm \sqrt{BC'} + \frac{A + BC'}{2} \end{aligned} \quad (24)$$

or

$$\lambda \simeq 1 \pm \sqrt{BC'} + \frac{BC'}{2} + \frac{A}{2} \simeq \exp \left[\pm \sqrt{BC'} + \frac{A}{2} \right] \quad (25)$$

From 18 and 22,

$$|BC'| = \left| \frac{e V_0 \cos \varphi_s E_s}{c^2 p_s^2} 2\pi x \right| = \Omega^2 \quad (26)$$

where Ω is the synchrotron frequency. Thus the eigensolutions near

$\lambda = 1$ correspond to synchrotron oscillations in the limit of zero coupling.

From 25 it would appear that by choosing $A < 0$, these oscillations could

be damped exponentially. However, we shall now show that the oscilla-

tions for the other pair of eigenfrequencies will be anti-damped by the

same factor. For the roots near $e^{\pm i\psi}$ we let $\lambda = e^{\pm i\psi} (1 + \epsilon)$. Again

expanding the secular equation to lowest terms, we find $\epsilon = 1 A/2 \pm iB/2$

so that the roots are

$$\lambda = \exp \left[\pm i \left(\psi + \frac{B}{2} \right) - \frac{A}{2} \right] \quad (27)$$

From Equations 25 and 27 we conclude that a radial variation of the

accelerating voltage (which leads to the term $\pm A/2$) will cause damping of one pair of oscillations and an equal anti-damping of the other pair. This effect was first discussed by Garren, et al.,² From 27 we see that the second pair of roots correspond in the limit of zero coupling to the betatron oscillations.

The components of the eigenvectors are easily found from Equation 20:

$$\begin{aligned} \xi : \begin{vmatrix} \sin \psi & 1 - \cos \psi & 0 \\ \cos \psi - \lambda & \sin \psi & 0 \\ 1 - \cos \psi & C & 1 - \lambda \end{vmatrix} &= \xi' : - \begin{vmatrix} \cos \psi - \lambda & 1 - \cos \psi & 0 \\ -\sin \psi & \sin \psi & 0 \\ \sin \psi & C & 1 - \lambda \end{vmatrix} \\ = \pi : \begin{vmatrix} \cos \psi - \lambda & \sin \psi & 0 \\ -\sin \psi & \cos \psi - \lambda & 0 \\ \sin \psi & 1 - \cos \psi & 1 - \lambda \end{vmatrix} &= f : - \begin{vmatrix} \cos \psi - \lambda & \sin \psi & 1 - \cos \psi \\ -\sin \psi & \cos \psi - \lambda & \sin \psi \\ \sin \psi & 1 - \cos \psi & C \end{vmatrix} \end{aligned} \quad (28)$$

Evaluating the determinants:

$$\frac{\xi}{(1 - \cos \psi)(1 - \lambda^2)} = \frac{\xi'}{-\sin \psi (1 - \lambda)^2} = \frac{\pi}{(1 - \lambda)(\lambda^2 - 2\lambda \cos \psi + 1)} = \frac{f}{\left[\frac{C(\lambda^2 - 2\lambda \cos \psi + 1)}{+ 2\lambda \sin \psi (1 - \cos \psi)} \right]} \quad (29)$$

For the roots near unity we set $\lambda = 1 + \epsilon$. Keeping only the leading terms, we find

$$\xi = \frac{\epsilon f}{c'} ; \xi' = \frac{\epsilon^2 \sin \psi}{2 c' (1 - \cos \psi)} f ; \pi = \frac{\epsilon}{c'} f \quad (30)$$

For $\epsilon \ll 1$, we see that only the phase has sizable oscillation amplitude;

i. e., we have a synchrotron oscillation. Similarly for

$$\lambda = e^{\pm i \psi} (1 + \epsilon)$$

we find:

$$\xi = \pm i f ; \xi' = -f ; \pi = \left(\mp i - \frac{\sin \psi}{1 - \cos \psi} \right) \epsilon f \quad (31)$$

so that the betatron oscillations are necessarily accompanied by comparable changes in ψ .

We must now take account of the slow variation of the coefficients in our coupled equations. The WKB method can be extended easily to systems of linear differential or difference equations. The derivations are given in the Appendix. Applying this technique, we find

(a) for the roots near unity

$$\chi_\nu = \frac{\epsilon \varphi_\nu}{c'_\nu} , \quad \chi'_\nu = \frac{\epsilon^2 \sin \psi}{2 c'_\nu (1 - \cos \psi)} \varphi_\nu , \quad p_\nu = \frac{\epsilon}{c'_\nu} \varphi_\nu \quad (32)$$

$$\varphi_\nu = \sqrt{\Omega_{\nu+1}} \prod_{j=1}^{\nu} \left(1 \pm i \Omega_j + \frac{A_j - \Omega_j^2}{2} \right) \quad (33)$$

with
$$\epsilon = \pm i \Omega + \frac{A - \Omega^2}{2}$$

$$\Omega_{j'} = -i \sqrt{B_{j'} c'_{j'-1}} \quad (34)$$

(b) for the roots near $e^{\pm i \psi}$

$$\chi_v = \pm i' \mathcal{Q}_v, \quad \chi_v' = -\mathcal{Q}_v, \quad P_v = \left(\mp i' - \frac{\sin \gamma}{1 - \cos \gamma} \right) \varepsilon \mathcal{Q}_v \quad (35)$$

$$\mathcal{Q}_v = e^{\pm i' \gamma \gamma} \prod_{j=1}^{\gamma} \left(1 - \frac{A_j'}{2} \pm i' \frac{B_j'}{2} \right) \quad (36)$$

with

$$\varepsilon = -\frac{A}{2} \pm i' B \zeta' \quad (37)$$

APPENDIX

WKB Method for systems of linear differential or difference equations.

1. Differential Equations.

Consider the system

$$\dot{y}_i = \sum_j a_{ij} y_j \quad (A.1)$$

where the coefficients a_{ij} are slowly varying functions of t . Let

$$y_i = x_i e^{i' \int \gamma dt} \quad (A.2)$$

where x_i and γ are slowly varying. Differentiating.

$$\dot{y}_i = (\dot{x}_i + i' \gamma x_i) e^{i' \int \gamma dt} \quad (A.2a)$$

Substituting in A.1:

$$\dot{x}_i = \sum_j a_{ij} x_j - \lambda Y x_i = \sum_j (a_{ij} - \lambda Y \delta_{ij}) x_j, \quad (\text{A.3})$$

First we neglect the time derivatives $\dot{x}_i$:

$$\sum_j (a_{ij} - \lambda Y \delta_{ij}) x_j = 0, \quad (\text{A.4})$$

The Y and x_i determined from this first approximation will be denoted by

$\bar{Y}$, $\bar{x}_i$, i.e.,

$$\sum_j (a_{ij} - \lambda \bar{Y} \delta_{ij}) \bar{x}_j = 0.$$

Thus $\bar{Y}$ is a root of the equation

$$|a_{ij} - \lambda \bar{Y} \delta_{ij}| = |A - \lambda \bar{Y} I| = D(\bar{Y}) = 0 \quad (\text{A.5})$$

where $A = (a_{ij})$, I is the unit matrix and $||$ means determinant = D .

The $\bar{x}_i$ proportional to the cofactors of any row, say the k 'th:

$$\bar{x}_j = f M_{kj} \quad (\text{A.6})$$

where M_{kj} is the cofactor of the kj element in $D(\bar{Y})$. Differentiating

$$\text{A.6} \quad \frac{\dot{\bar{x}}_j}{\bar{x}_j} = \frac{\dot{f}}{f} + \frac{\dot{M}_{kj}}{M_{kj}} \quad (\text{A.7})$$

Now we use this first approximation for the x_i in A.3 and find

$$\sum_j (a_{ij} - \lambda \bar{Y} \delta_{ij} - \frac{\dot{f}}{f} \delta_{ij} - \frac{\dot{M}_{kj}}{M_{kj}} \delta_{ij}) \bar{x}_j = 0 \quad (\text{A.8})$$

Let $Y = \bar{Y} + \epsilon$. Then

$$\sum_j (a_{ij} - i\bar{Y}\delta_{ij} - \delta_{ij} \{i\epsilon + \dot{f}/f + \dot{M}_{kj}/M_{kj}\}) \chi_j = 0. \quad (\text{A. 8a})$$

The secular equation for ϵ is:

$$|a_{ij} - i\bar{Y}\delta_{ij} - \delta_{ij}(i\epsilon + \dot{f}/f + \dot{M}_{kj}/M_{kj})| = 0. \quad (\text{A. 9})$$

ϵ , $\dot{f}/f$ and $\dot{M}_{kj}/M_{kj}$ are small quantities; expanding to lowest order we get:

$$|a_{ij} - i\bar{Y}\delta_{ij}| - \sum_j (i\epsilon + \dot{f}/f + \dot{M}_{kj}/M_{kj}) M_{jj} = 0. \quad (\text{A. 10})$$

The first term is zero according to Equation A. 5. Solving for ϵ :

$$\epsilon = \left[\frac{\dot{f}}{f} + \sum_j \frac{\dot{M}_{kj}}{M_{kj}} M_{jj} / \sum_j M_{jj} \right]. \quad (\text{A. 11})$$

$Y = \bar{Y} + \epsilon$. Substituting 11, and 6 in Equation A2:

$$\begin{aligned} y_i &= f M_{ki} \exp i \left[\int dt (\bar{Y} + i \{ \frac{\dot{f}}{f} + \sum_j \frac{\dot{M}_{kj}}{M_{kj}} M_{jj} / \sum_j M_{jj} \}) \right] \\ &= M_{ki} \left[\exp i \int \bar{Y} dt \right] - \int dt \left\{ \sum_j \frac{\dot{M}_{kj}}{M_{kj}} M_{jj} / \sum_j M_{jj} \right\}. \end{aligned} \quad (\text{A. 12})$$

Note that the factor f drops out. The result is independent of the index k , since a change of k is equivalent to multiplying by a common factor f .

As examples, we treat some simple cases. We omit comment, but label equations with their numbers in the general derivation.

$$\dot{y}_1 = -a_{12} y_2, \quad \dot{y}_2 = a_{21} y_1 \quad (1)$$

$$\dot{x}_1 = -i \gamma x_1 - a_{12} x_2 \quad (3)$$

$$\dot{x}_2 = a_{21} x_1 - i \gamma x_2$$

$$(\bar{\gamma})^2 = a_{12} a_{21}; \quad \bar{\gamma} = \pm \sqrt{a_{12} a_{21}} \quad (5)$$

Using the minors of the first row in 3,

$$\bar{x}_1 = a_{12}, \quad \bar{x}_2 = -i \bar{\gamma} = \mp i \sqrt{a_{12} a_{21}} \quad (6)$$

$$\dot{\bar{x}}_1 / \bar{x}_1 = \dot{a}_{12} / a_{12}; \quad \dot{\bar{x}}_2 / \bar{x}_2 = \frac{1}{2} \left(\frac{\dot{a}_{12}}{a_{12}} + \frac{\dot{a}_{21}}{a_{21}} \right) \quad (7)$$

$$- \left(\frac{\dot{a}_{12}}{a_{12}} + i \gamma \right) x_1 - a_{12} x_2 = 0 \quad (8)$$

$$a_{21} x_1 - \left(i \gamma + \frac{1}{2} \left\{ \frac{\dot{a}_{12}}{a_{12}} + \frac{\dot{a}_{21}}{a_{21}} \right\} \right) x_2 = 0$$

$$\gamma = \bar{\gamma} + \epsilon; \quad - \left(\frac{\dot{a}_{12}}{a_{12}} + i \epsilon + i \bar{\gamma} \right) x_1 - a_{12} x_2 = 0 \quad (8a)$$

$$a_{21} x_1 - \left(i \bar{\gamma} + i \epsilon + \frac{1}{2} \left\{ \frac{\dot{a}_{12}}{a_{12}} + \frac{\dot{a}_{21}}{a_{21}} \right\} \right) x_2 = 0$$

Expanding the determinant,

$$i \epsilon (2 i \bar{\gamma}) + i \bar{\gamma} \left(\frac{3}{2} \frac{\dot{a}_{12}}{a_{12}} + \frac{1}{2} \frac{\dot{a}_{21}}{a_{21}} \right) = 0 \quad (10)$$

$$\epsilon = i \left[\frac{3}{4} \frac{\dot{a}_{12}}{a_{12}} + \frac{1}{4} \frac{\dot{a}_{21}}{a_{21}} \right] \quad (11)$$

$$\begin{aligned}
 y_1 &= a_{12} \exp i' \int dt \left[\pm \sqrt{a_{12} a_{21}} + i \left(\frac{3}{4} \frac{\dot{a}_{12}}{a_{12}} + \frac{1}{4} \frac{\dot{a}_{21}}{a_{21}} \right) \right] \\
 &= \left(\frac{a_{12}}{a_{21}} \right)^{1/4} \exp \pm i' \int dt \sqrt{a_{12} a_{21}}
 \end{aligned} \tag{12}$$

$$y_2 = \mp i' \left(\frac{a_{21}}{a_{12}} \right)^{1/4} \exp \pm i' \int dt \sqrt{a_{12} a_{21}}$$

Another example

$$\dot{y}_1 = a_{11} y_1 + a_{12} y_2; \quad \dot{y}_2 = a_{21} y_1 + a_{22} y_2 \tag{1}$$

$$\begin{aligned}
 \dot{x}_1 &= (a_{11} - i' \gamma) x_1 + a_{12} x_2 \\
 \dot{x}_2 &= a_{21} x_1 + (a_{22} - i' \gamma) x_2
 \end{aligned} \tag{3}$$

$$(a_{11} - i' \bar{\gamma})(a_{22} - i' \bar{\gamma}) - a_{12} a_{21} = 0 \tag{5}$$

$$\bar{\gamma} = -i' T/2 \pm \sqrt{d - T^2/4}$$

where

$$T = a_{11} + a_{22}; \quad d = a_{11} a_{22} - a_{12} a_{21}.$$

$$\bar{x}_1 = -a_{12} = M_{21}; \quad \bar{x}_2 = a_{11} - i' \bar{\gamma} = M_{22} \tag{6}$$

$$\frac{\dot{\bar{x}}_1}{\bar{x}_1} = \frac{\dot{M}_{21}}{M_{21}}; \quad \frac{\dot{\bar{x}}_2}{\bar{x}_2} = \frac{\dot{M}_{22}}{M_{22}} \tag{7}$$

Substitute in 3 to get

$$(a_{11} - i' \bar{\gamma} - i' \epsilon - \dot{M}_{21}/M_{21}) x_1 + a_{12} x_2 = 0$$

$$a_{21} x_1 + (a_{22} - i' \bar{\gamma} - i' \epsilon - \dot{M}_{22}/M_{22}) x_2 = 0 \tag{8a}$$

Expand.

$$\epsilon = i' \frac{(a_{22} - i' \bar{\gamma}) \frac{\dot{M}_{21}}{M_{21}} + (a_{11} - i' \bar{\gamma}) \frac{\dot{M}_{22}}{M_{22}}}{a_{11} + a_{22} - 2 i' \bar{\gamma}} \tag{11}$$

Substitute in 1':

$$(1 + i \gamma_v) \chi_{v+1}^{i'} - \chi_v^{i'} = \sum_j a_v^{ij'} \chi_v^{j'} \quad (\text{A3}')$$

$$(1 + i \gamma_v) (\chi_{v+1}^{i'} - \chi_v^{i'}) = \sum_j a_v^{ij'} \chi_v^{j'} - i \gamma_v \chi_v^{i'} = \sum_j (a_v^{ij'} - i \gamma_v \delta^{ij'}) \chi_v^{j'} \quad (\text{A3a}')$$

Neglect the first differences.

$$\sum_j (a_v^{ij'} - i \gamma_v \delta^{ij'}) \chi_v^{j'} = 0 \quad (\text{A4}')$$

Call the root $\bar{\gamma}_v$ and the minors M_v^{kj} :

$$|A_v - i \bar{\gamma}_v I| = 0 \quad (\text{A5}')$$

$$\chi_v^{j'} = M_v^{kj} \quad (\text{A6}')$$

$$\frac{\chi_{v+1}^{i'} - \chi_v^{i'}}{\chi_v^{i'}} = \frac{M_{v+1}^{i'} - M_v^{i'}}{M_v^{i'}} \quad (\text{A7}')$$

Substitute $\bar{\gamma}$ and A7' on the left of A3a':

$$\sum_j \left[a_v^{ij'} - (i \gamma_v + (1 + i \bar{\gamma}_v) \left(\frac{M_{v+1}^{i'} - M_v^{i'}}{M_v^{i'}} \right)) \delta^{ij'} \right] \chi_v^{j'} = 0 \quad (\text{A8}')$$

$$\gamma_v = \bar{\gamma}_v + \epsilon_v$$

$$|a_v^{ij'} - \{ i \bar{\gamma}_v + i \epsilon_v + (1 + i \bar{\gamma}_v) \left(\frac{M_{v+1}^{i'} - M_v^{i'}}{M_v^{i'}} \right) \} \delta^{ij'}| = 0 \quad (\text{A9}')$$

ε_v and $\frac{M_{v+1}^{k_j'} - M_v^{k_j'}}{M_v^{k_j'}}$ are small quantities. Expanding to first order and using A5':

$$\sum_j \left[i\varepsilon_v + (1 + i\bar{Y}_v) \left(\frac{M_{v+1}^{k_j'} - M_v^{k_j'}}{M_v^{k_j'}} \right) \right] M_v^{j,j} \quad (\text{A10'})$$

$$\varepsilon_v = i(1 + i\bar{Y}_v) \left[\sum_j \frac{M_{v+1}^{k_j'} - M_v^{k_j'}}{M_v^{k_j'}} M_v^{j,j} / \sum_j M_v^{j,j} \right] \quad (\text{A11'})$$

$$Y_v = \bar{Y}_v + \varepsilon_v$$

$$1 + iY_v = 1 + i\bar{Y}_v + \varepsilon_v = (1 + i\bar{Y}_v) \left(1 - \sum_j \frac{M_{v+1}^{k_j'} - M_v^{k_j'}}{M_v^{k_j'}} / \sum_j M_v^{j,j} \right) \quad (\text{A12'})$$

Again, we take an example:

$$\begin{cases} y'_{v+1} - y'_v = -a_v'^2 y_v'^2 \\ y_v'^2 - y_v^2 = a_v'^2 y'_v \end{cases} \quad (1')$$

$$\begin{cases} (1 + iY_v) x'_{v+1} - x'_v = -a_v'^2 x_v'^2 \\ (1 + iY_v) x_v'^2 - x_v^2 = a_v'^2 x'_v \end{cases} \quad (3')$$

$$\begin{cases} (1 + iY_v)(x'_{v+1} - x'_v) = -iY_v x'_v - a_v'^2 x_v'^2 \\ (1 + iY_v)(x_v'^2 - x_v^2) = a_v'^2 x'_v - iY_v x_v'^2 \end{cases} \quad (3a')$$

Neglecting first differences:

$$\bar{Y}_v = \pm \sqrt{a_v'^2 a_v^{2'}} \quad (A5')$$

$$\frac{\chi_v'}{\chi_v^2} = \frac{i a_v'^2}{\bar{Y}_v} = \pm i \sqrt{a_v'^2 / a_v^{2'}}$$

Choose the factor f so that

$$\chi_v' = \pm i \sqrt{a_v'^2}, \quad \chi_v^2 = \sqrt{a_v^{2'}} \quad (6')$$

$$\begin{aligned} \frac{\chi_{v+1}' - \chi_v'}{\chi_v'} &= \sqrt{\frac{a_{v+1}'^2}{a_v'^2}} - 1 = \sqrt{1 + \frac{a_{v+1}'^2 - a_v'^2}{a_v'^2}} - 1 \\ &\approx \frac{1}{2} \frac{a_{v+1}'^2 - a_v'^2}{a_v'^2} \end{aligned} \quad (7')$$

Similarly

$$\frac{\chi_{v+1}^2 - \chi_v^2}{\chi_v^2} = \frac{1}{2} \frac{a_{v+1}^{2'} - a_v^{2'}}{a_v^{2'}}$$

Substitute from 5' and 7' on the left side of 3a':

$$-(i Y_v + \{i \bar{Y}_v + 1\} \frac{1}{2} \left\{ \frac{a_{v+1}'^2 - a_v'^2}{a_v'^2} \right\}) \chi_v' - a_v'^2 \chi_v^2 = 0$$

$$a_v^{2'} \chi_v' - (i Y_v + \{i \bar{Y}_v + 1\} \frac{1}{2} \left\{ \frac{a_{v+1}^{2'} - a_v^{2'}}{a_v^{2'}} \right\}) \chi_v^2 = 0$$

$$Y_v = \bar{Y}_v + \varepsilon_v \quad (8')$$

Expanding the determinant,

$$\varepsilon_v = \frac{i}{4} (1 + i \bar{Y}_v) \left(\frac{a_{v+1}'^2 - a_v'^2}{a_v'^2} + \frac{a_{v+1}^{2'} - a_v^{2'}}{a_v^{2'}} \right) \quad (11')$$

$$\begin{aligned} 1 + i Y_v &= 1 + i \bar{Y}_v + i \varepsilon \\ &= (1 + i \bar{Y}_v) \left(1 - \frac{i}{4} \left\{ \frac{a_{v+1}'^2 - a_v'^2}{a_v'^2} + \frac{a_{v+1}^{2'} - a_v^{2'}}{a_v^{2'}} \right\} \right) \\ &\approx (1 + i \bar{Y}_v) \left(\frac{a_{v+1}'^2}{a_v'^2} \frac{a_{v+1}^{2'}}{a_v^{2'}} \right)^{-1/4} \end{aligned}$$

$$\prod_{n=0}^{v-1} (1 + i Y_n) = \left[\prod_{n=0}^{v-1} (1 + i \bar{Y}_n) \right] \left(\frac{a_0'^2 a_0^{2'}}{a_v'^2 a_v^{2'}} \right)^{1/4}$$

Omitting constant factors,

$$y_v' = \pm i \left(\frac{a_v'^2}{a_v'^2} \right)^{1/4} \prod_n^{v-1} (1 + i \bar{Y}_n)$$

$$y_v'' = \left(\frac{a_v'^2}{a_v'^2} \right)^{1/4} \prod_n^{v-1} (1 + i \bar{Y}_n)$$

(12')

B. Contributions to FFAG Theory - December 27, 1954

The Accelerator Group at Argonne has been working this past month on more detailed design of FFAG (Fixed Frequency Alternating Gradient) machines. The focussing effect of magnet edges has been included in our formulas which have been coded for computation on the Argonne computer AVIDAC. Machine calculations have started and should be completed in the next few weeks.

The following is a compilation of formulas we have derived for various properties of the FFAG machine. The notation is, in general, that adopted by the MURA group (e.g., MURA-LWJ-7), except that all symbolic quantities are essentially positive.

The fringing field of a magnet is taken to be equivalent to a thin lens of focal length f , where

$$f = \pm \rho / \tan \Theta$$

(1)

ρ = radius of curvature of trajectory within the magnet;

θ = angle between the particle trajectory and the normal to the magnet edge. The lens has equal and opposite focal lengths in the radial and vertical directions.

For simplicity, we assume that the magnet edges are sloped symmetrically, so that the focussing effect of a pair of edges in adjacent magnets is given by the focal length F , where

$$\frac{1}{F} = \tan \theta \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) = \frac{1}{2} \left(\frac{\rho \frac{ds_1}{dr} - H_1 s_1}{\rho^2} \right) (H_1 + H_2). \quad (2)$$

The equations of motion for a machine in which the fields increase radially outward ("ordinary" machine) are:

$$\rho = H_1 \rho_1 = H_2 \rho_2 \quad (3)$$

$$\text{Radial} \quad \begin{cases} \frac{d^2 x_1}{ds^2} + K_1^2 x_1 = 0 \\ \frac{d^2 x_2}{ds^2} - K_2^2 x_2 = 0 \end{cases}$$

$$\text{Vertical} \quad \begin{cases} \frac{d^2 z_1}{ds^2} - \omega_1^2 z_1 = 0 \\ \frac{d^2 z_2}{ds^2} + \omega_2^2 z_2 = 0 \end{cases} \quad (4)$$

$$K_1^2 = \frac{n_1 + 1}{\rho_1^2} = \frac{U_1^2}{S_1^2} \quad \omega_1^2 = \frac{n_1}{\rho_1^2} = \frac{\gamma_1^2}{S_1^2}$$

$$K_2^2 = \frac{n_2 - 1}{\rho_2^2} = \frac{U_2^2}{S_2^2} \quad \omega_2^2 = \frac{n_2}{\rho_2^2} = \frac{\gamma_2^2}{S_2^2}$$

(5)

$$\frac{\gamma_1^2}{s_1^2} = \frac{f_1 H_1'}{H_1 f_1^2} = \frac{H_1'}{\rho}$$

$$\frac{\gamma_2^2}{s_2^2} = \frac{f_2 H_2'}{H_2 f_2^2} = \frac{H_2'}{\rho} \quad (6)$$

where the prime means differentiation with respect to r .

$$\varphi_1^2 = \gamma_1^2 + \frac{s_1^2}{f_1^2} = \gamma_1^2 + \frac{H_1^2 s_1^2}{\rho^2}$$

$$\varphi_2^2 = \gamma_2^2 - \frac{s_2^2}{f_2^2} = \gamma_2^2 - \frac{H_2^2 s_2^2}{\rho^2} \quad (7)$$

The eigenvalues of the transfer matrix are given by:

$$\begin{aligned} \cos \sigma_1 = & \cos \varphi_1 \cosh \varphi_2 + \frac{1}{2} \left(\frac{\varphi_2 s_1}{\varphi_1 s_2} - \frac{\varphi_1 s_2}{\varphi_2 s_1} + \frac{s_1 s_2}{\varphi_1 \varphi_2 F^2} \right) \sinh \varphi_1 \sinh \varphi_2 \\ & - \frac{s_1}{\varphi_1 F} \sinh \varphi_1 \cosh \varphi_2 - \frac{s_2}{\varphi_2 F} \cosh \varphi_1 \sinh \varphi_2 \end{aligned} \quad (8)$$

$$\begin{aligned} \cos \sigma_2 = & \cos \gamma_2 \cosh \gamma_1 + \frac{1}{2} \left(\frac{\gamma_1 s_2}{\gamma_2 s_1} - \frac{\gamma_2 s_1}{\gamma_1 s_2} + \frac{s_1 s_2}{\gamma_1 \gamma_2 F^2} \right) \sinh \gamma_2 \sinh \gamma_1 \\ & + \frac{s_2}{\gamma_2 F} \sinh \gamma_2 \cosh \gamma_1 + \frac{s_1}{\gamma_1 F} \cosh \gamma_2 \sinh \gamma_1 \end{aligned}$$

The quantity $\beta_{\max}$, which is needed for calculating the vertical aperture

(c.f., EDC/HSS-1 or notes by M. Hamermesh), is given by

$$\begin{aligned}
 B_{\max} \sin \sigma_2 = & \frac{s_2}{\psi_2} \sin \psi_2 \cosh \psi_1 + \left(\frac{s_2}{\psi_2} \right)^2 \frac{\psi_1}{s_1} \sin^2 \frac{\psi_2}{2} \sinh \psi_1 \quad (9) \\
 & + \frac{s_1}{\psi_1} \cos^2 \frac{\psi_2}{2} \sinh \psi_1 + \frac{s_1 s_2}{\psi_1 \psi_2 F} \sin \psi_2 \sinh \psi_1 \\
 & + \frac{2}{F} \left(\frac{s_2}{\psi_2} \right)^2 \sin^2 \frac{\psi_2}{2} \cosh \psi_1 + \frac{s_1 s_2^2}{\psi_1 \psi_2^2 F^2} \sin^2 \frac{\psi_2}{2} \sinh \psi_1
 \end{aligned}$$

The momentum compaction α , is given by:

$$K_1^2 \beta_1^2 \alpha = \frac{1}{\varphi_1 (1 + \frac{\tau}{\sqrt{J}}) \sqrt{J}} \left[\varphi_1 \left(\sqrt{J} - \frac{\tau}{J} \frac{\beta_1^2}{\beta_2^2} \right) + \right. \\
 \left. + \frac{2}{J} \frac{(\frac{\beta_1}{\beta_2} - J)^2 - \frac{1}{K_1 F} (J^{\frac{3}{2}} \coth \frac{\tau \varphi_1}{2} - \frac{\beta_1^2}{\beta_2^2} \cot \frac{\varphi_1}{2})}{\coth \frac{\tau \varphi_1}{2} - \sqrt{J} \cot \frac{\varphi_1}{2} + \frac{1}{K_1 F} \cot \frac{\varphi_1}{2} \coth \frac{\tau \varphi_1}{2}} \right] \quad (10)$$

where

$$\tau = \varphi_2 / \varphi_1 \quad ; \quad J = K_2^2 / K_1^2 \quad (11)$$

Inside-Out Machine

For a machine in which the fields decrease

as we move radially outward, the corresponding equations are:

Equations of motion:

$$\begin{aligned}
 \text{Radial} \quad & \begin{cases} \frac{d^2 x_1}{ds^2} - \bar{K}_1^2 x_1 = 0 \\ \frac{d^2 x_2}{ds^2} + \bar{K}_2^2 x_2 = 0 \end{cases} & \text{Vertical} \quad & \begin{cases} \frac{d^2 z_1}{ds^2} + \bar{\omega}_1^2 z_1 = 0 \\ \frac{d^2 z_2}{ds^2} - \bar{\omega}_2^2 z_2 = 0 \end{cases} \quad (4a)
 \end{aligned}$$

$$\begin{aligned}
 \bar{K}_1^2 &= \frac{n_1 - 1}{f_1^2} = \frac{\bar{\varphi}_1^2}{s_1^2} & \bar{\omega}_1^2 &= \frac{n_1}{f_1^2} = \frac{\bar{\psi}_1^2}{s_1^2} \\
 \bar{K}_2^2 &= \frac{n_2 + 1}{f_2^2} = \frac{\bar{\varphi}_2^2}{s_2^2} & \bar{\omega}_2^2 &= \frac{n_2}{f_2^2} = \frac{\bar{\psi}_2^2}{s_2^2}
 \end{aligned} \tag{5a}$$

$$\bar{\varphi}_1^2 = \bar{\psi}_1^2 - \frac{H_1^2 s_1^2}{\rho^2} \quad ; \quad \bar{\varphi}_2^2 = \bar{\psi}_2^2 + \frac{H_2^2 s_2^2}{\rho^2} \tag{6a}$$

The eigenvalues are given by:

$$\begin{aligned}
 \cos \bar{\sigma}_x &= \cos \bar{\varphi}_2 \cosh \bar{\varphi}_1 + \frac{1}{2} \left(\frac{\bar{\varphi}_1}{\bar{\varphi}_2} \frac{s_2}{s_1} - \frac{\bar{\varphi}_2}{\bar{\varphi}_1} \frac{s_1}{s_2} + \frac{s_1 s_2}{\bar{\varphi}_1 \bar{\varphi}_2 F^2} \right) \sin \bar{\varphi}_2 \sinh \bar{\varphi}_1 \\
 &\quad - \frac{s_2}{\bar{\varphi}_2 F} \sin \bar{\varphi}_2 \cosh \bar{\varphi}_1 - \frac{s_1}{\bar{\varphi}_1 F} \cos \bar{\varphi}_2 \sinh \bar{\varphi}_1 \\
 \cos \bar{\sigma}_2 &= \cos \bar{\psi}_1 \cosh \bar{\psi}_2 + \frac{1}{2} \left(\frac{\bar{\psi}_2}{\bar{\psi}_1} \frac{s_1}{s_2} - \frac{\bar{\psi}_1}{\bar{\psi}_2} \frac{s_2}{s_1} + \frac{s_1 s_2}{\bar{\psi}_1 \bar{\psi}_2 F^2} \right) \sin \bar{\psi}_1 \sinh \bar{\psi}_2 \\
 &\quad + \frac{s_1}{\bar{\psi}_1 F} \sin \bar{\psi}_1 \cosh \bar{\psi}_2 + \frac{s_2}{\bar{\psi}_2 F} \cos \bar{\psi}_1 \sinh \bar{\psi}_2 .
 \end{aligned} \tag{8a}$$

The results for the two types of machines are related by the substitutions:

$$\begin{aligned}
 \sigma_x &\rightarrow \bar{\sigma}_2 & \psi_1 &\rightarrow \bar{\varphi}_1 & \varphi_1 &\rightarrow \bar{\psi}_1 \\
 \sigma_2 &\rightarrow \bar{\sigma}_x & \psi_2 &\rightarrow \bar{\varphi}_2 & \varphi_2 &\rightarrow \bar{\psi}_2 & F &\rightarrow -\bar{F}
 \end{aligned} \tag{12}$$

The quantity $\bar{\beta}_{\max}$ for the inside-out machine is:

$$\begin{aligned}\bar{\beta}_{\max} \sin \bar{\sigma}_2 = & \frac{s_1}{\bar{\gamma}_1} \sin \bar{\gamma}_1 \cosh \bar{\gamma}_2 + \left(\frac{s_1}{\bar{\gamma}_1} \right)^2 \frac{\bar{\gamma}_2}{s_2} \sin^2 \frac{\bar{\gamma}_1}{2} \sinh \bar{\gamma}_2 \\ & + \frac{s_2}{\bar{\gamma}_2} \cos^2 \frac{\bar{\gamma}_1}{2} \sinh \bar{\gamma}_2 + \frac{s_1 s_2}{\bar{\gamma}_1 \bar{\gamma}_2 \bar{F}} \sin \bar{\gamma}_1 \sinh \bar{\gamma}_2 \\ & + \frac{2}{\bar{F}} \left(\frac{s_1}{\bar{\gamma}_1} \right)^2 \sin^2 \frac{\bar{\gamma}_1}{2} \cosh \bar{\gamma}_2 + \frac{s_1^2 s_2}{\bar{\gamma}_1^2 \bar{\gamma}_2 \bar{F}^2} \sin^2 \frac{\bar{\gamma}_1}{2} \sinh \bar{\gamma}_2\end{aligned}$$

(9a)

and the momentum compaction is given by:

$$\bar{K}_1^2 s_1^2 \alpha = - \frac{1}{\bar{\varphi}_1 (1 + \frac{\gamma}{\sqrt{3}}) \sqrt{3}} \left[\begin{aligned} & \bar{\varphi}_1 (\sqrt{3} - \frac{\gamma}{3} \frac{s_1^2}{s_2^2}) \\ & + \frac{2}{3} \frac{(\sqrt{3} - s_1/s_2)^2 + \frac{1}{\bar{K}_1 \bar{F}} (\sqrt{3} \cot \frac{\gamma \bar{\varphi}_1}{2} - \frac{s_1^2}{s_2^2} \coth \frac{\bar{\varphi}_1}{2})}{\cot \frac{\gamma \bar{\varphi}_1}{2} - \sqrt{3} \coth \frac{\bar{\varphi}_1}{2} - \frac{1}{\bar{K}_1 \bar{F}} \coth \frac{\bar{\varphi}_1}{2} \cot \frac{\gamma \bar{\varphi}_1}{2}} \end{aligned} \right]$$

(10a)

where

$$\gamma = \bar{\varphi}_2 / \bar{\varphi}_1 \quad ; \quad \beta = \bar{K}_2^2 / \bar{K}_1^2$$

C. Detailed Computations of Mark II on AVIDAC - January 13, 1955

Our purpose was to check the conjecture that effects of edges on focussing will cancel the effect of variation in sector length, so that Mk II machines with a circumference factor less than that of Mk I will not give a satisfactory design. The computations were carried out on the Argonne digital computer, AVIDAC, using the equations of ANL-Progress Report No. 1. These equations are exact in the linear approximation. The effects of the fringing field is calculated neglecting the radial gradient of H and taking for the equivalent thin lens

$$f = \frac{\rho}{\tan \theta} \quad \text{where } f = \text{focal length}$$

ρ = radius of curvature within the magnet

θ = angle of trajectory relative to the normal to the magnet edge.

The slope, b , of the magnet edges was taken to be a constant; unless otherwise stated, we used the value $N = 100$. The integration method was checked to insure that the interval size was sufficiently small to give accurate results. The initial values of H_1 and H_2 are unity.

$r_0 = 1$ = radius of machine at the highest energy

r_f = radius at which $p = p_f$

$p_0 = H \rho$ at the highest energy

$p_f = H \rho$ when H_2 changes sign or n_2 goes below 1.

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We first chose the operating point of Mk 1

$$\sigma_y = \frac{5\pi}{6} \quad \sigma_z = \frac{\pi}{6}$$

and tried various C's. Let $b = \frac{ds_1}{dr}$ the slope of the magnet edge.

	b = 20	10	6	2	0	-2	-6	-8	-10
C = 3	$r_f = .9987$	.9988	.9988	.9988	.9987	.9987	.5986	.9986	.9985
$p_o = .33$	$p_f = .146$	.157	.159	.160	.151	.150	.139	.137	.125
	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$	$H_2 < 0$

These computations were stopped when H_2 reversed direction. The momentum in all cases is reduced by only a factor of ~ 2 . To check what happens if we continue beyond the point where $H_2 = 0$, we tried:

$$\left| \begin{array}{l} r_f = .9965 \\ p_f = .054 \\ H_1 = -.596 \\ H_2 = -.379 \\ n_2 < 1 \end{array} \right|$$

We have succeeded in reducing the momentum by a factor of 6. We stop our calculations when $n_2 < 1$, because the functional forms have altered and provision had not been made in the code for this situation. In any case, H_1 has already reversed and reached half its maximum value. From the computed values of $\frac{dH_1}{dr}$ we would expect H_1 to reach its peak value at about $r = .9950$ where the momentum would have decreased by at most a factor of 25. (With output energy 10 Bev this would mean input energy 100 Mev.) For a short interval, where H_1 is + and H_2 is -, the machine is AG with both fields in the same direction, but with the orbit radius allowed to vary with momentum. Then, when H_1 reverses, the machine reverts to a Mk, II, where sector 2 is radially focussing and sector 1 is radially defocussing.

	b = 20	10	6	2	0	-2	-6	-8	-10
C = 4		$r_f = .9969$	.9973		.9980		.9970		
$p_o = .25$		$p_f = .045$	.053		.077		.036		
		$H_2 < 0$	$H_2 < 0$		$H_2 < 0$		$H_2 < 0$		
The computation for b = was carried beyond the stop at $H_2 < 0$:			$r_f = .9950$						
			$p_f = .021$						
			$H_1 = .-43$						
			$H_2 = .-10$						
			$n_2 = < 1$						

H_2 reverses when the momentum has been reduced by a factor of 3 $\rightarrow$ 7. For b = 6 the computation was continued; n_2 went below 1 when H_1 had reversed and reached $\frac{4}{10}$ peak value. Extrapolation indicates that H_1

would be best, reach its peak for $r = .9900$. This case might lead to a sufficiently low final momentum before H_1 reaches its peak in the reverse direction. We are considering code changes to carry the computation beyond the stop at $n_2 < 1$. In any case, magnet designs in which the fields run from peak to peak would require magnet cores on both inside and outside radii of the vacuum chamber, and would make the machine very heavy, expensive and inaccessible.

	$b = 2$	0	-2
$C = 5$	$r_f = .9930$	.9925	.9950
$p_o = .2$	$p_f = .0028$	.0019	.0095
	$n_2 = 1$	$n_2 = 1$	$n_2 = 1$

$C = 5$, $b = 0$ corresponds closely to Mk I (we have included the small edge effects and have not neglected 1 compared to n in our computation). Our computations with $b = 0$ achieved a reduction in momentum by a factor of 100 before the stop at $n_2 < 1$. We shall return to this case later.

	$b = 0$
$C = 6$	$r_f = .9960$
$p_o = .167$	$p_f = .013$
	$n_2 = 1$

The radial extension for the same % change in p is about the same as for $C = 5$, while the circumference is increased by 20%. Thus $C = 6$ is not as good as $C = 5$.

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Next we tried other points in the stability diagram:

C	P_o	b	σ_x	σ_z	r_f	P_f	Stop	P_o/P_f
4	.25	0	π	0	.9967	.027	$H_2 < 0$	10
5	.2	0	π	0	.9955	.0079	$n_2 < 1$	25
5	.2	0	π	$\pi/6$	.9945	.0048	$n_2 < 1$	40
5	.2	0	π	$\pi/2$	.9978	.081	$H_2 < 0$	2.5
5	.2	0	$5\pi/6$	$\pi/3$	.9967	.040	$H_2 < 0$	5
5	.2	0	$5\pi/6$	$\pi/2$	.9979	.095	$H_2 < 0$	2
5	.2	0	$2\pi/3$	$\pi/6$	.9955	.028	$H_2 < 0$	7
5	.2	0	$2\pi/3$	$\pi/2$	.9981	.133	$H_2 < 0$	0.7
5	.2	0	$2\pi/3$	$\pi/3$	.9973	.075	$H_2 < 0$	2.5
5	.2	0	$\pi/2$	$\pi/2$	.9983	.186	$H_2 < 0$	1.1

Even at $C = 5$, one must choose σ_x close to π and σ_z near 0 to obtain a satisfactory design.

We now resume our discussion of Mk I. The calculation for $C = 5$, $\sigma_x = 5\pi/6$, $\sigma_z = \pi/6$, $b = 0$ gave a reduction in $H\rho$ by a factor of 100. The design parameters for various energies would be:

$E_{\max}$	$H\rho_{\max}$	$r_{\max}$	$E_{\min}$	$H_{\min}$	$\triangle r$	Amplitudes of oscillation (in ft.) for injection with .001 radian half-angle.	
Bev	kg ft	ft	Mev	kg ft	ft	radial	vertical
10	1210	302.5	6	11.63	2.27	.031	.063
20	2300	575	21.5	22.1	4.31	.058	.120
30	3400	850	47	32.7	6.38	.086	.177

The minimum energies shown are those at which the code stop $n_2 < 1$ ended our computation. The minimum energies can be reduced below the values given. Using the equations of ANL Progress Report No. 1, we find

$$\beta_{\max.\text{radial}} = .1014 \qquad \beta_{\max.\text{vertical}} = .2080$$

which lead to the amplitude figures given in the table above, since the amplitude is the product of $\beta_{\max}$, the radius, and the half-angle of injection. The amplitudes are small, but it should be remembered that other factors (finite beam diameter, gas scattering, synchrotron oscillations, misalignments, etc.) will increase the necessary doughnut dimensions.

We plan to do the following:

1. Alter code to carry computations beyond the stop at

$$n_2 < 1.$$

2. Increase the number, N, of section pairs, in order to try to decrease the doughnut width.
3. Compute inside-out machines.

D. Effects of Gas Scattering in Alternating Gradient Synchrotrons -
January 17, 1955

The problem of gas scattering in ordinary synchrotrons has been treated by Blachman and Courant (Phys. Rev. 74, 140, 1948; we shall refer to this paper as A) and by Courant (RSI 24, 836, 1953; reference B). The probability of survival of a proton during the acceleration process is given by equations 1 and 4 of B as:

$$P(\eta) = 1 - \left(\frac{2\pi}{\eta}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2\eta}\right) \text{ for } \eta < 0.2 \quad (1)$$

$$P(\eta) = 2 \sum_{s=0}^{\infty} \left[J_1(\lambda_s) \right]^{-1} \exp(-\lambda_s^2 \eta) \text{ for } \eta > 0.2 \quad (2)$$

where J_0 and J_1 are Bessel functions, λ_s is the s^{th} root of $J_0(x)$ and

$$\eta = \frac{1}{4} \frac{\langle b^2 \rangle}{A^2} \quad (3)$$

$\langle b^2 \rangle$ is the mean square amplitude of vertical oscillations induced by gas scattering; A is the vertical semi-aperture. In terms of parameters of the accelerator,

$$\eta = \frac{\pi^2 N R^3 Z^2 e^4}{4 \eta A^2 T_i eV} \left[\ln \left(\frac{\theta_i^2}{\gamma^2} \right) - 1 \right] \quad (4)$$

$$\theta_i = A n^{1/2} / R \quad (5)$$

$$\chi = 1.20 \frac{\lambda Z^{1/3}}{a_0} (1 + 3.33 \gamma^2)^{1/2} \quad (6)$$

N = no. of scattering atoms per cc.

Z = atomic no. of scattering gas

T_i = injection energy

R = radius of accelerator

eV = energy gain per turn

n = field index

$a_0 = 5.3 \times 10^{-9}$ cm.

In equation 6, $2\pi\lambda$ is the proton wave length and $\gamma = \frac{Ze^2}{\hbar v}$, both to be evaluated at twice the injection energy. We have omitted the effect of straight sections from the equations.

In attempting to extend the treatment to alternating

gradient machines, two problems must be considered. First, the function $P(\gamma)$ in equation 2 was derived in reference A from the Fokker-Planck equation. The coefficients in that equation (Reference A, equation 14), which determine the form of $P(\gamma)$, depend on the relation

$$\langle \Delta B \rangle = \frac{\langle (\Delta B)^2 \rangle}{2B} \quad (7)$$

where B is the oscillation amplitude and ΔB is its change per unit path length (Reference A, just below equation 14). The relation 7 was derived in A for the simple harmonic oscillator. We shall show later that the same relation holds for a general periodic restoring force. Secondly, the parameters n and R of the ordinary synchrotron in equations 4, 5 must be replaced by corresponding quantities for the general machine.

We shall treat the second problem first and obtain a simple, approximate transcription of the equations which is applicable to alternating gradient machines. Then we shall verify our statement about the first problem, and check the approximate answer for a Mk I FFAG machine. The parameters R and n of the ordinary synchrotron appear in equations 4 and 5. If we follow the derivation given in reference A, we see that one factor R in equation 4 comes from the

relation between distance traveled and energy gained

$$\frac{dT}{dx} = \frac{eV}{2\pi R} \quad (8)$$

so in general for a machine of circumference C we must replace one R by $\frac{C}{2\pi}$. The remaining machine factor $\frac{R^2}{n}$ in front of the bracket in equation 4 comes from the derivation (reference A, equation 7) of the mean-square amplitude. $\frac{n}{R^2}$ is just the spring constant of the ordinary synchrotron (for vertical oscillation. The radial oscillations are handled similarly.) An approximate average spring constant can be introduced in the alternating gradient problem. If we neglect the modulation of the betatron oscillations with the period of the magnet sections, then the motion can be described as a simple oscillation which is multiplied by a factor $\exp(\pm i \sigma_z)$ if we move across one sector-pair. If the number of sector-pairs around the circumference C is ν , then the space dependence is $\exp(\pm i \frac{\sigma_z}{C} \nu s)$, so that the spring constant is $(\frac{C}{\sigma_z \nu})^2$ which replaces $\frac{R^2}{n}$. Similarly, equation 5 for G_1 was derived by calculating the deflection which gives rise to a displacement equal to the vertical half-aperture. Again we must replace $\frac{n}{R}$ by $\frac{\sigma_z \nu}{C}$. From previous use of this approximation for other problems (cf. Brookhaven report EDC-HSS/1 and others) we know that the actual amplitudes reached are about a factor 2 greater than given by the approximation. Carrying out the transcription according to this recipe

we obtain for γ :

$$\gamma = \frac{\pi}{8} Z^2 \frac{e^4 N C \mu^2}{T_i \text{ eV}} \left[2 \ln \left(\frac{1}{\mu \chi} \right) - 1 \right] \quad (9)$$

where

$$\mu = \frac{2 C}{A \nu \sigma_z} \quad (10)$$

Putting in all the numerical values we obtain, for a nitrogen filling at pressure p in 10^{-5} mm Hg:

$$\gamma = 2.82 \times 10^{-13} \frac{p C \mu^2}{T_i \text{ (eV)}} \left[2 \ln \left(\frac{1}{\mu \chi} \right) - 1 \right] \quad (11)$$

and

$$\chi = 1.396 \times 10^{-4} T_i^{-\frac{1}{2}} \left[1 + 2.04 T_i^{-1} \right]^{\frac{1}{2}} \quad (12)$$

where circumference C is in cm, T_i and (eV) are in Mev, A is the half-height of the chamber at twice the injection energy. ν is the number of sector-pairs, and σ_z is the engenvalue for the vertical oscillations. (For the radial oscillations produced by gas scattering, replace σ_z by σ_r and let A be the radial half-width.)

Figure 1 shows the variation of γ with A for a Mk.

I FFAG ($\sigma_2 = \pi/6$) at 10 Bev output energy, $p = 10^{-5}$ mm Hg, $\gamma = 96$, $\frac{C}{\gamma} = 594$ cm, $eV = .1$ Mev. The maximum magnetic field is taken as 20 kg. Curves are given for $T_i = 3, 6, 10$, and 50 Mev. The recipes for scale changes of T_i , eV , p , and the output energy are given in the legend. The function $P(\gamma)$ of equation 2 is plotted in reference A. For $\gamma = .116$, $P(\gamma) = .90$; for $\gamma = .199$, $P(\gamma) = .50$; $\gamma = .5$ gives $P(\gamma) = .1$.

For the 10 Bev Mk. I, with $p = 1$, $T_i = 3$, $eV = .1$,

$$P(\gamma) = .9 \text{ for } A = 3.1 \text{ cm.}$$

$$P(\gamma) = .5 \text{ for } A = 2.2 \text{ cm.}$$

Doubling the pressure would give

$$P(\gamma) = .9 \text{ at } A = 4.9 \text{ cm.}$$

$$P(\gamma) = .5 \text{ at } A = 3.1 \text{ cm.}$$

More than 90% of the particles will be lost at $p = 5$, unless A is made greater than 3.5 cm.

For higher energies it will certainly be convenient to keep C/γ constant so that the lengths of magnet sections are kept fixed. On this assumption a 30 Bev machine with $p = 1$, $T_i = 3$, $eV = .1$ would give

$$P(\gamma) = .9 \text{ at } A = 6.1 \text{ cm.}$$

$$P(\gamma) = .5 \text{ at } A = 4.3 \text{ cm.}$$

Raising the energy gain per turn to 280 kev would restore the values given above for 10 Bev, $p = 1$, $T_i = 3$.

In general, we see that gas scattering does not impose any serious requirements on the aperture. However, it might lead to trouble if one tries too hard to increase the current output of the machine by having groups of particles of different energies circulating in the machine and operating on harmonics. In such cases the energy gain per turn is small at the start of the acceleration cycle, and it is precisely this initial value of eV which appears in the survival equations.

Next we shall give a rigorous treatment of the problem. We consider a circular machine with spring constant periodic in azimuth, and try to parallel the steps in the derivation of reference A. We must calculate the change in amplitude which results from a sequence of deflections through angles ϕ_i at arc lengths S_i . (For the notation and some of the theorems used, see EDC-HSS/1 or notes by Hamermesh).

We suppose that we have a reference oscillation with $y = 0$, $\frac{dy}{ds} = a$ at $s = 0$, chosen as the point at which β is a maximum. (Then $a = 0$ at $S = 0$.)

Thus the initial amplitude of oscillation is

$$\left[y^2 + (\alpha y + \beta y')^2 \right]_{S=0}^{1/2} = \beta_{\max} a \quad (13)$$

The matrix which carries us from $S = 0$ to the point $S = S_1$, we call $M(S_1)$

$$M(S_1) = \begin{pmatrix} M_{11}(S_1) & M_{12}(S_1) \\ M_{21}(S_1) & M_{22}(S_1) \end{pmatrix} \quad (14)$$

If a deflection φ_i occurs at s_i , this means the addition to the already present $\gamma(s_i), \gamma'(s_i)$ of $\Delta\gamma=0, \Delta\gamma'=\varphi_i$. We transfer this back to $s=0$, i.e., we form

$$M^{-1}(s_i) \begin{pmatrix} 0 \\ \varphi_i \end{pmatrix}$$

where

$$M^{-1}(s_i) = \begin{pmatrix} M_{22}(s_i) & -M_{12}(s_i) \\ -M_{21}(s_i) & M_{11}(s_i) \end{pmatrix}$$

So the disturbance $\begin{pmatrix} 0 \\ \varphi_i \end{pmatrix}$ at s_i is equivalent to a disturbance

$$\begin{pmatrix} -M_{12}(s_i) \varphi_i \\ M_{11}(s_i) \varphi_i \end{pmatrix}$$

The sequence of disturbances φ_n at s_n is therefore equivalent to a disturbance

$$\begin{pmatrix} -\sum_n M_{12}(s_n) \varphi_n \\ \sum_n M_{11}(s_n) \varphi_n \end{pmatrix} \quad \text{at } s=0$$

Combining with the initial oscillation $\begin{pmatrix} 0 \\ a \end{pmatrix}$ gives

$$\begin{pmatrix} -\sum_n M_{12}(s_n) \varphi_n \\ a + \sum_n M_{11}(s_n) \varphi_n \end{pmatrix}$$

so the new amplitude is

$$\begin{aligned}
 & \left[\beta_{\max}^2 (a + \sum M_{11} U)^2 + (\sum M_{12} U)^2 \right]^{\frac{1}{2}} \\
 &= \beta_{\max} a \left[1 + \frac{2 \sum M_{11} U}{a} + \frac{(\sum M_{11} U)^2}{a^2} + \frac{(\sum M_{12} U)^2}{\beta_{\max}^2 a^2} \right]^{\frac{1}{2}} \\
 &\approx \beta_{\max} a \left[1 + \frac{\sum M_{11} U}{a} + \frac{1}{2} \frac{(\sum M_{11} U)^2}{a^2} + \frac{(\sum M_{12} U)^2}{2 \beta_{\max}^2 a^2} - \frac{1}{8} \left(\frac{2 \sum M_{11} U}{a} \right)^2 \right] \\
 &= \beta_{\max} a + \beta_{\max} \sum M_{11} U + \frac{(\sum M_{12} U)^2}{2 \beta_{\max} a}
 \end{aligned}$$

Thus the change in amplitude is

$$\Delta B = \beta_{\max} \sum_n M_{11}(S_n) U_n + \frac{(\sum_n M_{12}(S_n) U_n)^2}{2 \beta_{\max} a} \quad (15)$$

where $\beta_{\max} a = B$ is the original amplitude. The positions S_n at which the deflections U_n occur, are random, and uncorrelated, so we must average over the S_n , and later, over U . Since

$$\begin{aligned}
 \overline{U_n U_{n'}} &= \overline{U_n^2} \quad \text{for } n = n' \\
 &= 0 \quad \text{for } n \neq n' \\
 \overline{U_n} &= 0
 \end{aligned}$$

we have

$$\langle \Delta B \rangle = \frac{\tau}{2 \beta_{\max} a} \overline{M_{12}^2} \overline{U^2}$$

$$\langle (\Delta B)^2 \rangle = \tau \beta_{\max}^2 \overline{M_{11}^2} \overline{U^2}$$

where τ = the total no. of deflections,

so that

$$\frac{\langle \Delta B \rangle}{\langle (\Delta B)^2 \rangle} = \frac{\overline{M_{12}^2}}{\beta^2 \overline{M_{11}^2}} \cdot \frac{1}{2\beta} \quad (16)$$

For an ordinary synchrotron with constant field index n

$$\frac{d^2 z}{ds^2} + \frac{n}{R^2} z = 0$$

The transfer matrix is

$$M(s_1) = \begin{pmatrix} \cos \frac{n^{\frac{1}{2}} s_1}{R} & \frac{R}{n^{\frac{1}{2}}} \sin \frac{n^{\frac{1}{2}} s_1}{R} \\ -\frac{n^{\frac{1}{2}}}{R} \sin \frac{n^{\frac{1}{2}} s_1}{R} & \cos \frac{n^{\frac{1}{2}} s_1}{R} \end{pmatrix} \quad (17)$$

$$M_{11}(s_1) = \cos \frac{n^{\frac{1}{2}} s_1}{R} \quad M_{12} = \frac{R}{n^{\frac{1}{2}}} \sin \frac{n^{\frac{1}{2}} s_1}{R} \quad (18)$$

Also, β is the same for all S and

$$\beta = \frac{R}{n^{\frac{1}{2}}} \quad (19)$$

Thus

$$\overline{M_{11}^2} = \frac{1}{2}, \quad \overline{M_{12}^2} = \frac{\beta^2}{2}$$

so that

$$\frac{\overline{M_{12}^2}}{\beta^2 \overline{M_{11}^2}} = 1$$

and

$$\frac{\langle \Delta B \rangle}{\langle (\Delta B)^2 \rangle} = \frac{1}{2B} \quad (20)$$

This same relation must hold if we want the Fokker-Planck differential equation for the general periodic machine to agree with that for the ordinary synchrotron. We now show that this is the case.

For the ordinary synchrotron the averaging was easy because we had simple periodic functions (sine and cosine). In the general case, unless we are at resonance, the motion is not strictly periodic. However, suppose we know $M(s_1)$ from $s_1 = 0$ to $s_1 = \ell$ where ℓ is the length of repeating magnet section. Then

$$M(\ell + s_1) = M(s_1) M(\ell) \quad (21)$$

and

$$M(s_1 + \tau \ell) = M(s_1) M^T(\ell) \quad (22)$$

where

$$M^T(\ell) = \begin{pmatrix} \cos \tau \sigma & B_{\max} \sin \tau \sigma \\ -\frac{1}{B_{\max}} \sin \tau \sigma & \cos \tau \sigma \end{pmatrix}$$

so

$$M_{11}(s_1 + \tau \ell) = \cos \tau \sigma M_{11}(s_1) - \frac{1}{B_{\max}} \sin \tau \sigma M_{12}(s_1) \quad (23)$$

$$M_{12}(s_1 + \tau \ell) = B_{\max} \sin \tau \sigma M_{11}(s_1) + \cos \tau \sigma M_{12}(s_1) \quad (24)$$

To average M_{12}^2 we must take

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\tau=0}^N \langle M_{12}^2(s_1 + \tau l) \rangle_{\text{av. over } s_1 \text{ from } 0-l}$$

or

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\tau=0}^N \left[B_{\max}^2 \sin^2 \tau \sigma \langle M_{11}^2 \rangle_{0-l} + B_{\max} \sin 2\tau \sigma \langle M_{11} M_{12} \rangle_{0-l} + \cos^2 \tau \sigma \langle M_{12}^2 \rangle_{0-l} \right]$$

but

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\tau=0}^N \frac{\sin^2}{\cos^2} \{ \tau \sigma \} = \frac{1}{2} \quad (25)$$

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{\tau=0}^N \sin 2\tau \sigma = 0 \quad (26)$$

Thus

$$\langle M_{12}^2 \rangle = \frac{1}{2} \left[B_{\max}^2 \langle M_{11}^2 \rangle_{0-l} + \langle M_{12}^2 \rangle_{0-l} \right] \quad (27)$$

Similarly evaluating $\langle M_{11}^2 \rangle$ from 23, 25, 26

$$\text{we find } \langle M_{12}^2 \rangle = B_{\max}^2 \langle M_{11}^2 \rangle \quad (28)$$

and equations 16 and 20 are identical, so the probability functions calculated for the ordinary synchrotron apply to the general periodic machine.

However, $\langle (\Delta B)^2 \rangle$ will have a numerical value depending on the details of the machine. Per deflection,

$$\langle (\Delta B)^2 \rangle = B_{\max}^2 \langle M_{11}^2 \rangle \langle \psi^2 \rangle = \langle M_{12}^2 \rangle \langle \psi^2 \rangle$$

and we must evaluate $\langle M_{12}^2 \rangle$. For the ordinary synchrotron

$$M_{12} = \frac{R}{n^{1/2}} \sin \frac{n^{1/2} s}{R} \quad \text{so} \quad \langle M_{12}^2 \rangle = \frac{R^2}{2n}$$

So the numerical coefficient which replaces $\frac{R^2}{2n}$ is to be found by calculating

$$\langle M_{12}^2 \rangle = \frac{1}{2\ell} \int_0^\ell ds \left[\beta_{\max}^2 M_{11}^2(s) + M_{12}^2(s) \right] \quad (29)$$

We have calculated the integrals for the FFAG machine omitting focussing effect of edges. We omit the details and give the final result

$$\begin{aligned} \langle M_{12}^2 \rangle = & \frac{1}{8\ell} \beta_{\max}^2 \left[\frac{s_2}{\gamma_2} (\gamma_2 + \sinh \gamma_2) + \frac{s_1}{\gamma_1} (\gamma_1 + \sinh \gamma_1) \cos^2 \frac{\gamma_2}{2} \right. \\ & + \left(\frac{s_1}{\gamma_1} \right)^3 \left(\frac{\gamma_2}{s_2} \right)^2 (\sinh \gamma_1 - \gamma_1) \sin^2 \frac{\gamma_2}{2} \\ & \left. - \left(\frac{s_1}{\gamma_1} \right)^2 \frac{\gamma_2}{s_2} (\cosh \gamma_1 - 1) \sin \gamma_2 \right] \\ & + \frac{1}{8\ell} \left[\left(\frac{s_1}{\gamma_1} \right)^3 (\gamma_2 - \sinh \gamma_2) + \left(\frac{s_2}{\gamma_2} \right)^2 \frac{s_1}{\gamma_1} (\gamma_1 + \sinh \gamma_1) \sin^2 \frac{\gamma_2}{2} \right. \\ & \left. + \left(\frac{s_1}{\gamma_1} \right)^3 (\sinh \gamma_1 - \gamma_1) \cos^2 \frac{\gamma_2}{2} + \frac{s_2}{\gamma_2} \left(\frac{s_1}{\gamma_1} \right)^2 (\cosh \gamma_1 - 1) \sin \gamma_2 \right] \end{aligned} \quad (30)$$

where $\beta_{\max}$ is

$$\begin{aligned} \beta_{\max} \sin \sigma_2 = & \frac{s_2}{\gamma_2} \sin \gamma_2 \cosh \gamma_1 + \left(\frac{s_2}{\gamma_2} \right)^2 \frac{\gamma_1}{s_1} \sin^2 \frac{\gamma_2}{2} \sinh \gamma_1 \\ & + \frac{s_1}{\gamma_1} \cos^2 \frac{\gamma_2}{2} \sinh \gamma_1 \end{aligned} \quad (31)$$

For the Mk. I FFAG,

$$\frac{s_1}{\gamma_1} = \frac{s_2}{\gamma_2} = \frac{s}{\gamma} \quad (32)$$

$$\beta_{\max} \sin \sigma_z = \frac{s}{\gamma} [\sin \gamma_2 \cos \gamma_1 + \sin \gamma_1] \quad (33)$$

Eq. 30 becomes

$$\langle M_{12}^2 \rangle = \frac{1}{2(s_1 + s_2)} \left(\frac{s}{\gamma} \right)^3 \left(\frac{\sin \gamma_2 \cos \gamma_1 + \sin \gamma_1}{\sin^2 \sigma_z} \right) (\gamma_2 \sin \gamma_2 \cos \gamma_1 + \gamma_1 \cos \gamma_2 \sin \gamma_1 + 2 \sin \gamma_2 \sin \gamma_1) \quad (34)$$

Substituting the values

$$\frac{s_1}{s_2} = 1.5, s_1 + s_2 = \frac{c}{\gamma}, \gamma_1 = 2.01, \gamma_2 = 1.34, \sigma_z = \frac{\pi}{6}$$

we find

$$\langle M_{12}^2 \rangle = 5.43 \left(\frac{c}{\gamma} \right)^2 \quad (35)$$

whereas our previous approximation method gave

$$\frac{2}{\sigma_z^2} \left(\frac{c}{\gamma} \right)^2 = 7.1 \left(\frac{c}{\gamma} \right)^2 \quad (36)$$

Thus the numerical factor in Equation 11 should be lowered to 2.17. (This change has not been included in the graphs of Figure 18, since the value of the correction depends on the position of the operating point in the stability diagram and will vary from one machine to another.)

The second point which requires correction is our expression

for θ_1 as

$$\theta_1 = \frac{A \sigma_z \gamma}{2 c} \quad (37)$$

Figure 18

The factor 2 was introduced as a rough correction. In any case this term appears under the logarithm sign, so changing the numerical factor from 2 will hardly be noticeable.

135-7 Electron-Neutron Interaction (5200-04)

G. R. Ringo

A measurement of the coherent scattering amplitude for neutrons of krypton and xenon is to be made by reflection of neutrons from the surface of liquid krypton and xenon. The apparatus for this experiment has been completed. Only one problem of any difficulty was encountered in building the apparatus. This was to maintain the tightness of the seal between the observation window and the mirror pan when the latter was cooled to about - 150° C. (The observation window was necessary to make sure there was only liquid krypton or xenon present rather than a mixture of liquid and solid krypton or xenon). The problem was solved by adding two 12" arms to the mirror chamber on the end of which were placed windows sealed to the arms with the aid of flanges kept warm by heaters.

140-4 A Gaseous Scintillation Counter (5200-02)

Charles Egglar and Charles M. Huddleston
Reported by Charles M. Huddleston

In earlier models of the gas cell plastic color shifters have been used to convert the ultra-violet fluorescent light of argon or helium into radiation of longer wave length. It now appears that nitrogen gas may prove an effective color shifter with none of the noxious vapor quenching properties of plastics.

Hanle¹ has studied argon-nitrogen mixtures excited by electrons and deduced that the energy transfer comes about non-radiatively by collisions between metastable argon atoms and nitrogen atoms. Our results with alpha-particle excitation are consistent with that explanation. Kallman and Furst² give a quite general expression for energy transfer from the atoms of one component of a homogeneous mixture to the atoms of another component:

$$J = k_1 c \cdot \frac{1}{1 + k_2 c} + \frac{1}{1 + k_3 c}$$

where c is the concentration of the radiating molecules and the k 's depend on the type of process under consideration. In the present case, we have, simply

$$J \sim \frac{c}{1 + c} + \frac{1}{1 + k_3 c}$$

where the first term gives the initial light intensity, and the second term represents a competition between quenching and energy transfer. The above expression is consistent with our data.

An additional effect, not yet fully understood, is that the output pulse height of the photomultiplier depends inversely on the total gas pressure. Experiments designed to study this pressure effect are being carried out. Also studies are in progress to determine whether any impurities are tending to suppress the light output. Since large pulse heights are desirable, efforts are being made to obtain more

1. W. Hanle, *Physikalsche Blatter*, 565, 12/1954.

2. H. Kallman and M. Furst, *Phys. Rev.* 79, 857 (1950).

transparent optical coupling between the gas and the photomultiplier tube.

1 41-2

Uranium Fluorescence Photometer (5200-02)

Frank Lynch

The third instrument was delivered to L. Ross of Chemical Engineering. The report entitled "A New Fluorescence Photometer" is scheduled for publication in the March issue of R. S. I. Operation and service notes also were written. No further reports will be made.

142-2

100-KV Electron Accelerator (5200-02)

Warren L. Buck, Robert K. Swank
H. Bruce Phillips
Reported by H. Bruce Phillips

Since the previous report was submitted, work on the accelerator has slowed because a necessary transformer is not yet on hand. The electron gun requires power supplies as well as trigger and sweep circuits, all of which are at 100 KV with respect to ground; a high-voltage isolation transformer is therefore required. It was originally intended to use an x-ray type filament transformer for this purpose, but the transformer on hand proved entirely inadequate. An order was therefore placed for a special H. V. isolation transformer. It should be delivered in February. In the meantime several other necessary components have been completed:

1. H. V. blocking condenser. In order to synchronize the electron bursts with the photomultiplier pulses and the display oscilloscope sweep, a timing pulse is fed into the electron gun circuit

which is at high voltage. A blocking condenser for doing this was made by modifying one of the lucite insulators which support the H. V. cage. A long axial hole was drilled into the insulator, and a smooth metal rod was inserted into the hole. A metal sleeve was fitted over the closed end of the insulator, thereby forming a coaxial condenser with lucite as the dielectric. This condenser has a capacitance of about 35 mmf, and should be capable of withstanding 150 KV.

2. 24-KV power supply. A high voltage, low current power unit has been constructed for supplying the electron gun with the appropriate accelerating potentials. The unit consists of a Tektronic type 420 transformer-rectifier unit, driven by audio oscillators. The circuit supplies - 4kv, + 6.6 kv, + 13.3 kev, and + 20 kv, simultaneously.

3. Photomultiplier mount. A light-tight box for mounting the photomultiplier scintillation detector was made of brass and bakelite. A light shutter was installed so that the photomultiplier voltages need not be removed while fluor samples are being changed. Two of these probe assemblies will be used, one for dc measurements, and one for pulse measurements, the main difference between them being the wiring arrangements inside the boxes.

4. Elevating table. The probe assemblies must be precisely and reproducibly positioned with respect to the electron accelerator exit window. Accordingly, a mounting table has been made for mounting the probes and moving them into position. The table has a fine adjustment for elevation, and the entire probe moves on rollers into position

below the accelerator. Leveling adjustments are also provided.

143-2

Plastic Scintillators

(5200-02)

Louis Basile

Boron-Loaded Plastic Scintillators

Attempts have been made to polymerize various boron-containing compounds with vinyl toluene. As reported in our last report B-trichloroborazole was one of the boron-containing compounds being investigated. Below 100°C samples of B-trichloroborazole in vinyl toluene will not polymerize satisfactorily. Above this temperature, the substance decomposes. Because of this no further work with B-trichloroborazole was attempted.

Several samples containing diborane in vinyl toluene were prepared. Since diborane is such a hazardous substance to handle only small quantities were used. At 125° C polymerization did proceed, although not completely. There are indications that the substance may polymerize completely at higher temperatures. We have not attempted to do this with our present set up but may do so in the future.

The most promising results at the present stage seem to be with the use of N-triethylborazole as the boron additive. Two samples containing N-triethylborazole in vinyl toluene were prepared. At 125° C the samples were almost completely polymerized with no indication of decomposition. It may even be feasible to go to a still higher temperature for polymerization. Our main difficulty in working

with N-triethylborazole is preparing large enough samples of the borazole to work with. We are continuing work along these lines.

Heavy Element-Loaded Plastic Scintillators:

In our last report mention was made that samples of vinyl iodide in vinyl toluene were in the constant temperature bath undergoing polymerization. After several days at temperature the solutions decomposed forming a deep red-colored solution. This red color was due to iodine liberated by decomposition of vinyl iodide. The samples were discarded.

Attention has been turned towards the addition of organo-metallic compounds in vinyl toluene. The compounds chosen for investigation were triphenylstibine, $(C_6H_5)_3 Sb$, triphenylbismuthine, $(C_6H_5)_3 Bi$, and diphenylmercury, $(C_6H_5)_2 Hg$. These compounds were chosen because they are colorless, readily available, and should be fairly soluble in vinyl toluene. Preliminary work indicated that it seemed feasible to add samples of the above organo-metallic compounds to vinyl toluene. The commercial organo-metallic compounds available have to be carefully purified several times before they can be used. We are now in the process of preparing test samples and hope to be able to report on them in the near future.

144-2

Scintillation Counter Techniques (5200 -02)

Robert K. Swank and John S. Moenich
Reported by Robert K. Swank

The standard photomultiplier assembly has been modified to use DC-200 silicone fluid for optical coupling between

the Lucite light pipe and the photomultiplier. Polyvinylacetate, which was previously used for this purpose, became excessively brittle at low temperatures. Since the DC-200 fluid is a liquid at room temperature, a liquid-tight seal must be made around the edge of the light pipe. This seal has been successfully accomplished with rubber sealer type EC-801. The temperatures of one test model have been varied from -15°C to $+60^{\circ}\text{C}$ many times over a four-month period with no apparent deterioration of the seal or optical coupling.

The use of the DC-200 fluid for the optical coupling between sodium iodide scintillation crystals and the window of their hermetically sealed can has proved unsatisfactory because of a gradual absorption of this fluid by the MgO reflector. A solution of Monsanto HB-40 in polystyrene was previously tried, following a suggestion found in the literature, but the scintillation pulse size was substantially reduced by this material. During this period this formula was again tried, using HB-40 which was first vacuum distilled. Now the pulse size is only attenuated $\sim 8\%$ and the pulse height resolution is not affected measurably. This wetting agent can be tailor-made to any desired viscosity and is much less readily absorbed by MgO.

Several cylindrical sodium iodide crystals were obtained which were $2\frac{1}{2}$ inches in diameter x $2\frac{1}{8}$ inches long. These were canned by our standard technique and mounted on 3" diameter DuMond photomultiplier tubes. The pulse height resolution was slightly better than that previously obtained with the $2\frac{1}{4}$ " cubical crystals

made for the bent crystal DuMond-type spectrometer. These cylindrical crystals are very much easier to can than the cubical ones and it is hoped that their availability will eliminate much of the demand for the cubical ones.

During this period seven sodium iodide crystals were canned and twenty-four photomultiplier tubes were potted.

145-1

Organic Crystal Scintillators (5200-02)

Robert K. Swank and Warren L. Buck
Reported by Robert K. Swank

A study was begun of the scintillation properties of organic crystals containing small amounts of fluorescent impurities. Although some previous work has been done elsewhere with this type of scintillator, the work was limited in scope and accuracy. Such scintillators may have advantages over plastic or liquid scintillators in regard to pulse size and the possibility of incorporating heavy elements.

This investigation was facilitated by the invention of a technique whereby a number of single-crystal specimens containing a wide range of impurity concentrations could be made in one operation. This is accomplished by the procedure illustrated in Fig. 19. A long tube containing compacted crystalline material is lowered through a zone-melting oven. The unmelted solid, U, enters the oven and drips into the liquid pool, M, as it melts. At the lower boundary of the oven the liquid freezes to form the solid crystal, C. When the tube is started through the oven the fluorescent impurity (activator) is in the bottom and is completely dissolved by the initial pool of liquid. As the tube is lowered, the liquid, containing

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Figure 19

$c\%$ activator, freezes to form a solid crystal containing $kc\%$ activator, where k is the "solubility constant." Since activated crystal is being removed from the zone and pure crystal added, the amount of activator in the zone diminishes, and it can be shown readily that the activator concentration in the zone at any time is

$$c = c_0 e^{-\frac{kx}{d}}$$

where c_0 is the initial concentration and x and d are the distances shown in Fig.19. By adjusting the length of the molten zone and of the tube, one can obtain any desired range of activator concentrations in the boule (below the maximum imposed by solubility). The boule may then be cut up into thin transverse sections and the variation of efficiency of the mixed crystal with activator concentration determined.

4- methylbiphenyl was selected as the material from which the initial activated crystals would be made. Approximately 500 gms. of this chemical, in a rather impure state, were obtained commercially. Before use the material was purified by vacuum distillation, followed by recrystallization from ethanol, passage through an activated-alumina adsorption column, and a further recrystallization from ethanol. Purity of the material was checked by melting-point determinations and by u. v. absorption measurements, the latter proving to be the more sensitive test.

The material from which a crystal is to be grown is introduced by vacuum-distillation into a long, Pyrex glass tube. This process is carried out in a special oven designed with a movable horizontal partition or baffle. During the distillation this baffle is kept at the bottom of the oven. Forced circulation of the heated air in the space above

the baffle and of hot water through the condenser maintains the distillate in a molten condition. When the tube is filled to the desired depth with molten distillate, distillation is stopped, and the baffle is raised slowly. A hole in the baffle allows the tube, in effect, to pass through into the space below the baffle, where forced circulation of cool air causes the contents of the tube to freeze slowly from the bottom up. This results in a compact filling, free of voids.

The filled tube is removed from the oven and a small amount of the desired activator fluor is added on top of the filling. Then, after being sealed off under vacuum a short distance above the filling, the tube is inverted and lowered through the zone-melting oven.

The zone-melting oven consists of a length of large-diameter Pyrex glass tubing, the ends of which are closed by Transite discs. A heating element of nichrome wire is wound spirally along the axis of the oven, the diameter of the spiral being slightly greater than that of the tube to be passed through. Holes are drilled at the centers of the Transite discs to allow passage of the tube through the oven. The tube is suspended by a fine wire and lowered slowly through the oven by means of a clock motor.

A scintillator tester comprising a photomultiplier tube and a vacuum tube electrometer circuit was designed and built for the purpose of making measurements of the relative light output from different samples of scintillator. Power for this tester is supplied by self-contained batteries, and provision is made for interposing a shutter between the scintillator and the source of excitation without exposing the photomultiplier to light.

The equipment and procedures described above have been used successfully to grow a boule of 4-methylbiphenyl approximately 25 cm. long and 1 cm. in diameter containing tetraphenylbutadiene as the activator. Results of tests on this mixed organic crystal scintillator will be reported later.

146-2

A Rotating Condenser Electrometer (5200-02)

Frank Lynch

The rotating condenser converter was completed. The maximum capacity is $30\mu\mu\text{f}$ and the minimum capacity is $21\mu\mu\text{f}$. This should give a conversion efficiency of about 14%. It seems possible to lower the minimum capacity to about $10\mu\mu\text{f}$ by a change in design, thereby doubling the conversion efficiency. The rotor runs with no appreciable electrical noise synchronously at 7.5 r.p.s. generating a 75 cps signal. In order to remedy the excessive contact potential, the condenser plates will be coated with gold by electrolysis and evaporation. Provision has been made to chop a light beam with the rotating condenser plates; this signal to be used to synchronize the detector circuit. A low noise low power amplifier and synchronous detector will be constructed to test the rotating condenser as an electrometer.

147-1

A Reliable Alpha Air Proportional Counter (5200-02)

Frank Lynch

Instruments using this detector have been in use for several years. Because of its good performance and reliability, a

technical report on the development is being prepared for publication in Rev. Sci. Instr. "Notes". In connection with this report some measurements on negative ion formation and humidity-induced corona discharges are being made.

The counter of large area uses air in the proportional amplification region. It has an efficiency of 38% of the total disintegrations and a 200-volt plateau having a slope of 4% per 100 volts. It is shown that for good performance the slow component of the pulses due to negative oxygen ions must be amplified as well as the fast component due to electrons. A guard ring insulator eliminates spurious pulses due to insulator failure even when the relative humidity is 99%.

148-2

Electrical Conduction in Dielectrics (5200-03)

Raymond J. Munick

Measurements were continued of the electric current in circuits containing one-eighth inch thick specimens of the plastic insulating materials Lucite, Fluorothene, Polyethylene, Polystyrene, and Teflon. These can be grouped under time-dependent currents, steady currents, and potential distribution.

The measurements of the time dependent currents consisted of three parts: (1) a comparison of the current at 49° C with that at 25° C, (2) a number of tests of the Hopkinson superposition principle, and (3) an extension of the measurements to shorter times, of the order of milliseconds.

The results obtained at 25 and 49°C are summarized in Table III. They indicate a difference in the mechanisms of the time dependent and steady currents. The effect of temperature on the time dependent currents is small enough to justify depending on the air-conditioning for temperature control in many of the measurements.

Table III
Comparison of Current at 49° C to that at 25° C

Material	Time Dependent		Steady*
	M_{49} / M_{25}	I_{49} / I_{25}	I_{49} / I_{25}
Lucite	1	1	18
polystyrene	1	1	7
Telfon	1	2	8
polyethylene	1	2	27
Fluorothene	0.75	1 at about 5 min.	7

M = slope of log current vs. log time curve.

I = magnitude of current

* Data taken before July 31, 1954.

One group of these measurements was to test the Hopkinson superposition principle,¹ which is the generalization concerning

¹. M. F. Manning and M. E. Bell, "Electrical Conduction and Related Phenomena in Solid Dielectrics", Revs. Mod. Phys. 12, 215 (1940); H. Frohlich, The Theory of Dielectrics (At the Clarendon Press, Oxford, 1949), pp. 6-7.

these time-dependent currents in dielectrics. This principle states that the current density at a given time is simply the sum of the current densities produced by each change in field acting independently or that

$$J = \sum_{k=1}^n \Delta E_k f(t - t_k), \quad (1)$$

where J = current density at time t ,

ΔE_k = change in electric field at time t_k ,

and $f(t)$ = current decay function for unit field change at time $t = 0$.

Three groups of tests were made. First, field E was applied at $t = 0$, which should give $J = E f(t)$. When J had practically reached a steady state, E was removed at $t' = 0$. This should yield $J = -E f(t')$. Secondly, field $2E$ was applied at time $t = -t_1$ and then decreased to E at $t = 0$. This should give $J = 2E f(t + t_1) - E f(t)$. For $t > 0$, the current should start negatively, increase to a positive maximum, and then approach the values for field E applied at $t = 0$. Thirdly, equal field changes were made at suitably long time intervals. With proper allowance made for the steady state currents, the time dependent curves should be equivalent, regardless of the total field.

The results of all these tests were that although Lucite, Fluorothene, and polystyrene do obey the superposition principle, Teflon and polyethylene do not.

The superposition principle implies linear interactions and, therefore, in the simplest case a simple exponential decay of current with respect to time. In this present investigation over the time range of

10 to 10^4 seconds, $f(t)$ is of the form At^{-n} , where n is near one, as has been reported by others for other insulating materials. Theoreticians have postulated that there is a distribution of relaxation times, which yields approximately At^{-n} over a limited time interval. To determine whether this power law breaks down over longer time intervals and whether the current approaches a finite value near zero time, a method, which employs a d. c. oscilloscope, has been developed to extend the measurements to times from 10^{-3} to 10^{-2} seconds. Crude data have been obtained for Lucite, Fluorothene, Teflon, and polyethylene, which indicate that the power law holds over the extended range of times.

The temperature range of the steady current measurements was extended as high as 70°C for polystyrene, 80°C for Lucite, and 90°C for Fluorothene. One significant result was that the apparently steady current decreased to a lower steady value when maintained at the elevated temperature. At 70°C the steady current was just barely measurable for Lucite and Fluorothene and could not be detected for polystyrene. Another significant result was that the activation energy indicated by currents for Fluorothene at 70 , 80 , and 90°C , was decidedly greater, between three and four times, than that found previously between 25 and 49°C .

One school of thought attributes the decay of current with respect to time to the formation of a space charge field within the insulator. The direct evidence for this consists of determinations² with a conducting probe of the potential distribution on a surface parallel to the applied

2. J. Evans, Phys. Rev. 57, 47 (1940); A. F. Joffe', The Physics of Crystals (McGraw-Hill Book Company, Inc., New York, 1928), pp. 102-125.

electrical field. Because such determinations have yielded a non-uniform field for parallel plane geometry, an attempt was made to determine the potential of the mid-plane within a Lucite disk, or , what seemed equivalent, the potential of a brass disk sandwiched between two Lucite disks. A high voltage vacuum tube electrometer recorded the potential of the brass disk as a function of time from change of electromotive force across the sandwich. The brass disk behaved like an insulated conductor and not like a connection between two resistances, that is, its potential depended on the potentials of the other conductors (as indicated by changing the electromotive force) and upon the grid current of the electrometer tube (as indicated by gradual changes). This result suggests what improvements should be made to perform a conclusive experiment, namely to apply a greater electromotive force and to use an electrometer of higher impedance.

II. MASS SPECTROSCOPY

1-8 (Mass Spectrometric Techniques) (5200-02)

Mark G. Inghram

No results to report for this quarter.

2-8 Construction and Operation of MA-17 (5200-02)

Henry E. Stanton

Work on the construction of the machine has been progressing more slowly than formerly because of a larger proportion of effort being devoted to other phases of the work.

A new control panel has been installed. This panel supplies voltages to the various plates in the source and to the two plates in the electrostatic ion deflecting condenser. The ion accelerating potential, i.e., from the point of ion formation to the electrical midpoint of the condenser, and the potentials of the condenser plates with respect to this midpoint are supplied with well-regulated power supplies which are manually adjustable. An automatic sweep in the condenser potentials is also available for sweeping, i.e., "tuning" in ions of different energies.

A new 10-kv precision bleeder panel has been built and installed for use when the machine is operated as a double-focussing instrument. The control panel will not supply suitable voltages for this arrangement since its output is limited to about 500 v.

Although some leaking around the source has been stopped,

and the pressure there has been reduced by an order of magnitude, not much effort has been devoted to this problem, since while pressures are undesirably high, experiments in progress can be carried out satisfactorily as is.

10-8

A Search for Rare Naturally Occurring Isotopes

(5200-01)

David C. Hess

Several elements have been examined during the last six months, No new isotopes have been discovered. Limits on occurrence of various isotopes in nature are:

Kr	76	<	4×10^{-8}	Xe	119	<	10^{-8}
	77	<	3×10^{-7}		120	<	10^{-8}
	79	<	2×10^{-7}		121	<	10^{-8}
	81	<	10^{-7}		122	<	10^{-8}
	85	<	5×10^{-7}		123	<	10^{-8}
	87	<	10^{-7}		125	<	2×10^{-8}
	88	<	10^{-8}		127	<	2×10^{-8}
	89	<	2×10^{-8}		133	<	6×10^{-8}
Ru	86	<	2.5×10^{-5}		135	<	6×10^{-8}
Ru	92	<	10^{-4}		137	<	3×10^{-8}
	93	<	10^{-4}		138	<	2×10^{-8}
	94	<	3×10^{-5}		139	<	3×10^{-8}
	95	<	10^{-4}		140	<	2×10^{-8}
	97	<	3×10^{-4}		141	<	2×10^{-8}
	103	<	1.5×10^{-4}				

Ru	105	<	4×10^{-4}		
	106	<	2×10^{-5}		
	107	<	10^{-4}		
	108	<	10^{-5}		
	109	<	3×10^{-5}		
Zr	84	<	3×10^{-6}	Y	85 < 1×10^{-6}
	85	<	2×10^{-6}		86 < 1×10^{-6}
	86	<	2×10^{-6}		87 < 10^{-6}
	87	<	2×10^{-6}		88 < 5×10^{-6}
	88	<	2×10^{-6}		90 < 5×10^{-6}
	89	<	10^{-4}		91 < 2×10^{-6}
	93	<	2×10^{-5}		92 < 2×10^{-6}
	95	<	10^{-5}		93 < 5×10^{-6}
	97	<	10^{-5}		94 < 5×10^{-6}
	98	<	10^{-5}		95 < 3×10^{-6}
	99	<	10^{-5}		
	100	<	5×10^{-6}		
	101	<	10^{-5}		

All limits are in terms of the total natural element.

Most of the high values are due to interference by an impurity, e. g. ,

Y interferences at Zr⁸⁹.

Much time was spent in working on improvement of MA-16.

Stabilization of the magnets by continuous excitation was tried and worked well except for the difficulty that the old magnet regulators used batteries which did not have a very long life under the operating conditions.

Regulators of a new type which use no batteries and are capable of 168 hrs, a week's operation, were installed. Another source of instability was grid current drawn by the cathode follower voltage divider. This circuit was replaced by a three-stage feedback amplifier.

With these improvements it appears possible to run the machine without the excessive variations in discrimination described in the last report.

While the gas source was in the machine it was felt that some gases should be searched, so arrangements were made to bake the sample lead and source end. This successfully reduced the machine background (but did not eliminate it as at present no provision is made to bake the whole tube).

In adjusting the machine it was found that a wide range of masses could not be swept with good sensitivity, e.g., Kr^{++} could be seen at one adjustment but not Kr^+ . This effect was found to be due to lack of tracking of the beam correction voltages in the intermediate region of the machine. A special "power supply" whose outputs are proportional to the ion accelerating voltage was used to replace the battery-operated circuit and the range of masses with good sensitivity is now over two to one. Hg^{+++} and Hg^+ can not be seen by changing only the high voltage, as for Hg^+ the beam voltage falls to too low a value for the source arrangement if the voltage for Hg^{+++} is "safe", i.e., about 9 kv.

12-8

(Double Beta Disintegration of Te^{130}) (5200-01)

Mark G. Inghram and Richard J. Hayden
Reported by Richard J. Hayden

No progress has been made on this problem
this quarter.

14-8

Fission Yield Determinations (5200-01)

Ellis P. Steinberg (Chemistry), Lawrence
E. Glendenin (Chemistry), Mark C. Inghram
and Richard J. Hayden
Reported by Mark G. Inghram

The results reported in the last quarterly report,
ANL-5412, II 14-7 have appeared in the Physical Review 95, 867
(1954) under the title "Fine Structure in U^{233} Fission". It is
noted therein that Steinberg and Glendenin have also observed a fine-
structure peak in the light group of fission products from spontaneous
fission of Cm^{242} at the mass number complementary to the 82-neutron
nucleus of the heavy group, as is found in the slow neutron-induced
fission in U^{235} and U^{233} .

Long-Lived Activities (5200-01)

David C. Hess

A sample of aluminum which has received a heavy neutron bombardment has been decontaminated by George Reed of the Chemistry Division but has not yet been measured on the mass spectrometer.

Preliminary investigations of paper work and of feasibility of obtaining heavy neutron bombardments of a number of elements in the MTR have been started. Some of the work proposed would extend some of our previous results in the rare earth region.

Adsorption of Mixtures of He^3 and He^4 (5200-02)

Mark G. Inghram, Earl A. Long (U. of Chgo.)
and Lothar Meyer (U. of Chgo.)
Reported by Mark G. Inghram

The distribution of He^3 between the vapor phase and dilute mixtures of He^3 in He^4 adsorbed on jewellers' rouge (Fe_2O_3) has been measured between 1.6° K and 2.3° K, for saturations in the range 15 percent to 99 percent, corresponding to film thicknesses of 1.3 to 34 layers. The ratio of the mole fraction of He^3 in the vapor phase to that in the adsorbed phase is practically independent of the thickness of the adsorbed film, and essentially the same as for the bulk liquid case. The onset of superfluidity in the adsorbed films does not influence the He^3 concentration in samples taken by simple desorption techniques.

The foregoing is the abstract of a paper that has been submitted to the Physical Review for publication.

29-2

Gaseous Species in Equilibrium with Metal-Metal
Oxide Systems (5200-02)

R. F. Porter (University of Chgo.), William A. Chupka, and Mark G. Inghram
Reported by Mark G. Inghram

In experiments made with the Knudsen cell-mass spectrometer apparatus it was found that at temperatures ranging from 1345° K to 1463° K the vapor in equilibrium with Si - SiO₂ contains both SiO and Si₂O₂ molecules. At temperatures of 1800° K to 1900° K, SiO, SiO₂, O₂ and O were found over SiO₂ near neutral oxidizing conditions. Measurements of the temperature dependence of the SiO₂⁺ and Si₂O₂⁺ ions show that these ions are formed by direct ionization of effusing SiO₂ and Si₂O₂ gases and are not formed by some second order mechanism involving SiO⁺. The heat of sublimation of SiO₂ at 298° K is found to be 136 ± 8 K cal/mole.

These results are reported in the Journal of Chemical Physics, 23, 216, (1955). Similar studies are being made of the gases in equilibrium with BaO, SiO and MgO.

30-8

The Heat of Vaporization of Carbon (5200-02)

William A. Chupka and Mark G. Inghram
Reported by Mark G. Inghram

The heats of vaporization of C, C₂ and C₃ as determined from the vapor pressures and from the temperature variation of these

pressures are given in the following table.

Heat of Vaporization of Carbon Species

<u>Species</u>	<u>ΔH (O° K) k cal/mole</u>	
	<u>From pressure</u>	<u>From temperature effect</u>
C	170 $\pm$ 7	171 $\pm$ 10
C ₂	197 $\pm$ 7	183 $\pm$ 10
C ₃	199 $\pm$ 7	200 $\pm$ 10

The value for C strongly supports the spectroscopic value of 170.4 k cal/mole that could be inferred from the spectroscopic data by other possible interpretations of those data.

The value for the heat of dissociation of the C₂ molecule that is consistent with the above heats of vaporization of C and C₂ is 150 k cal/mole.

The assignment of the mass 60 peak to C₅⁺ can not be considered as well-established without further study.

These experiments are reported in a paper that has been submitted to the Journal of Physical Chemistry (59, 100, 1955).

The earlier results appeared in a letter to the Journal of Chemical Physics 22, 1472 (1954).

A criticism that possibly an appreciable part of the mass 36 peak is to be attributed to (H₂O)₂⁺ ions rather than C₃⁺ ions has been answered and demolished in a letter to the Journal of Chemical Physics, 23, 203 (1955).

William A. Chupka and Henry E. Stanton
Reported by William A. Chupka

Appearance Potentials

In order to determine appearance potentials with the necessary accuracy it has become apparent that the conventional electron guns are inadequate due to the Maxwellian distribution of electron energies from hot filaments. The high energy "tail" of the distribution is especially troublesome. Two approaches are being attempted to solve this problem. An electrostatic analyzer has been designed to provide energy selection of the electron beam. Also, an electron gun employing a field emission source has been built and tested. The field emission electrons are slowed down and refocussed to a point by cylindrical electrostatic lenses. At present, a transmission of 20-39% has been attained with electrons which have been slowed down to about 300 volts. Transmitted currents are of the order of 100 microamperes. An additional focussing stage will be required to retard the electrons down to the energies employed for appearance potentials. The energy distribution of these electrons has a relatively sharp high energy cut off which can be made appreciably sharper by cooling the field emission tip to liquid N₂ temperatures. In addition, the focussing system should be of value in future studies of field emission of positive ions.

Richard J. Hayden and Gerald J. Wasserburg
(Univ. of Chgo.)
Reported by Richard J. Hayden

Work on this project has been delayed by the construction of a new gas extraction line and by a series of vacuum and electronic difficulties on MA-18, the gas analysis spectrometer. The new line has now been finished and the mass spectrometric difficulties overcome. A new set of A^{38} tracers has been made and calibrated. A series of east coast feldspars and micas is being analyzed.

Meteoric xenon has been examined. A limit on the existence of Xe^{129} in the Beardsley chondritic meteorite has been set. This limit, coupled with the assumptions that (1) I^{129} and I^{127} were originally formed in equal abundance, (2) I^{127} is present in the meteorite to 1 ppm, (3) the half life of I^{129} is 1.7×10^7 yrs, and (4) no xenon has been lost from the meteorite leads to the result that the time interval between the formation of the elements and the last crystallization of the meteorite is at least 4.1×10^8 yrs. Details of this work are in a letter which has been sent to Nature.

35-4

Studies of Negative Ion Emission (5200-02)

William A. Chupka and Henry E. Stanton
Reported by Henry E. Stanton

Recently MA -17 has been adjusted for negative ions.

In order to study the electron affinities of various atoms and molecules, particularly oxygen, the usual gas source was not sufficient.

A new source was installed in which the negative ions are formed in a substantially field-free region, or with a known electrical gradient there. The source of electrons is a tantalum plate heated by electron bombardment from a hot filament. The electron source is also located in a region where the electric field is accurately controllable.

By these means, it is hoped to measure the electron affinity of oxygen, CO, HO, etc. of this group.

36-4

Field Emission Studies

Mark Inghram and R. Gomer (University of
Chicago
Reported by Mark Inghram

A Letter to the Editor on the subject of this experimental technique in Mass Spectroscopy will appear in Jour. Am. Chem. Soc. 77, 500, 1955.

V. THEORETICAL PHYSICS, GENERAL

1-8

Interpretation of the Energy States of Light Nuclei

(5200-01)

David R. Inglis

A note to appear in the Review of Modern Physics indicates that Be^9 is expected to have a group of three low excited states, rather than a lone state as suggested in the article in the Reviews of Modern Physics for April, 1953. Experimental confirmation appears elsewhere in this report, (I-20-8).

A paper on "Dynamics of Nuclear Deformation" to appear in the Physical Review, as a sequel to a recent paper on "Nuclear Rotation Properties", discusses the problem of justifying liquid-drop vibrations by starting with nucleon wave functions for a closed-shell nucleus. It is shown that the vibrational deformation is so rapid that the adiabatic approximation is not valid and the concept of such a vibration in a closed-shell nucleus is questionable. The very recent indication (G. Scharff-Goldhaber and Weneser, unpublished) that even-even nuclei with unfilled shells below $A = 150$ display vibrational excited states is probably to be reconciled with this result by considering that the Gallone-Salvetti "spilling over" between shells during deformation

slows down the vibration sufficiently.

The success of Dennison in attributing the excited states of O^{16} to an alpha model is probably to be understood in terms of a nascent clustering of the nucleons more nearly describable as a surface wave of tetrahedral symmetry. The objections raised in the article in the Reviews of Modern Physics for April, 1953 to the use of the alpha model in C^{12} do not apply so clearly to this modification of the model, and the question arises whether this model might not account for the maverick 7.7-Mev state of C^{12} .

2-8

Excited States of Light Nuclei (5200-01)

Dieter Kurath

The computation of the energy matrix elements for all configurations within the 1p shell has been completed. The routine for diagonalization of the matrices constructed from the above elements has been checked out on the AVIDAC by Ruth Freshour of the computing section. The case of three nucleons was run, giving results that agree with the hand calculation done previously.

The first run of the four nucleon case which gives the energy levels of Be^8 and C^{12} was also made. The 86 energy levels resulting from a given set of parameters took 45 minutes of computation. Such a time is short enough that it should be possible to get results for

considerable variation of parameters in a reasonable length of time.

The routine for obtaining magnetic dipole moments and electric quadrupole moments has been checked out and tested on the three nucleon case. The routine for extracting the lowest eigenfunction from the diagonalized matrices is in process of being checked out. This will complete the coding of the entire problem.

4-8 Investigation of Nuclear Shell Structure (5200-01)

Maria G. Mayer

Miscellaneous calculations were made for the Unified Nuclear Model but no major progress was made.

Calculations of shapes were also made for various beta ray spectra.

5-8 Book on Nuclear Shell Structure (5200-01)

Maria G. Mayer

The proof has been corrected during the period.

7-8

(Collective Description of Nucleon Interactions) (5211-01)

Melvin Ferentz, Murray Gell-Mann (Univ. of Chgo.) and David Pines (Princeton Univ.)
Reported by Melvin Ferentz

There are no new results to report at this time.

9-6

(Calculation of Neutron Cross Sections by the Weisskopf-Porter [Clouded Crystal Ball] Method)

Bernard I. Spinrad

Reporting on this topic in these quarterly reports is discontinued because of the transfer of Dr. Spinrad to the Reactor Engineering Division.

10-4

(The Effect of Inelastic Scattering in the Analysis of Scattering Resonances) (5200-01)

James E. Monahan

No work was done on this subject during the past quarter.

12-1

Investigation of Neutron Scattering Under the Feshbach-Porter-Weisskopf Model of the Nucleus
(5200-01)

Morton Hamermesh and Jack Sokoloff
Reported by Jack Sokoloff

An investigation has been undertaken to determine the validity of the neutron low-energy differential scattering cross sections computed under the Feshbach-Porter-Weisskopf model of the nucleus.¹

1. H. Feshbach, C. E. Porter, and V. F. Weisskopf, Phys. Rev., 96, 448 (1954)

Initially, the curves will be generated using the parameters adopted by FPW (namely $V_0 = 42$ Mev, $\beta = 0.03$, and $R = 1.45 \times 10^{-13} A^{1/3}$ cm). The theoretical curves will then be compared with the neutron differential scattering data obtained by Alexander Langsdorf and his group.

The calculation of the differential shape elastic scattering, $\frac{d\sigma_{se}}{d\Omega}$, and the cross section for compound nucleus formation, σ_c , has been set up for the AVIDAC and is being programmed by Joseph Cook at present. Some of the elements of the calculation have been hand computed.

Preliminary calculations by James Monahan indicate that the σ_c for 0.5-Mev neutrons on a target of $A = 240$ using the FPW model with the previously mentioned parameters is slightly smaller than the σ_c calculated under the assumption of strong coupling (which is a limiting case of the FPW model). However, the penetrabilities, I_ℓ , which are proportional to the $\sigma_c(\ell)$, are quite different under the two models. This indicates that the angular distribution of the sum of the compound elastic scattered neutrons and the inelastically scattered neutrons may be quite different for the two cases.

13-1

Multipolarity of Neutron-Capture Gamma Rays

(5200-01)

Morton Hamermesh and Jack Sokoloff
Reported by Jack Sokoloff

A preliminary investigation has been made to determine whether the multipolarity of gamma rays emitted in the neutron capture

process can be fixed on the basis of the intensity of the spectrum. A start in this direction has been made by Kinsey and Bartholomew¹ using the spectrum obtained with a pair spectrometer. The spectrum to be obtained on the Bent Crystal Spectrometer of B. Hamermesh and D. Rose would complement that of Kinsey and Bartholomew, and between the two instruments the complete spectrum of the neutron capture gamma rays should be obtained.

Kinsey and Bartholomew have attempted to fix multipolarities on the basis of competition, using transition probabilities calculated by Weisskopf.² Weisskopf used some rather crude approximations in the independent particle model to find the transition probabilities. K and B have applied those probabilities to certain capture processes in which there seems to be evidence for the independent particle picture as an adequate description of the behavior of the product nucleus. In addition, they have used the stripping data of Holt and Marsham³ to assign the orbital angular momentum and parity of some of the excited states.

We have used the data of Kinsey and Bartholomew and Holt and Marsham to see what the spectra in the region of 3.5 Mev down might be. Among the nuclei investigated are Mg^{25} , S^{33} , Mg^{26} , and Si^{29} .

-
1. B. B. Kinsey and G. A. Bartholomew, Phys. Rev. 93, 1260 (1954)
 2. V. F. Weisskopf, Phys. Rev. 83, 1073 (1951)
 3. H. R. Holt and T. M. Marsham, Proc. Phys. Soc. (London) A65, 763, (1952); A66, 249 (1953)

It appears that the bent crystal spectrum could fix the intensities of both the low level and high level spectra. (Some of the K and B intensity assignments are fairly speculative.) Also the spins of some of the intermediate levels for some of the nuclei may be assigned on the basis of competition, subject to the following criteria:

(1) The adequacy of the independent particle model in describing the transition.

(2) The validity of the transition probabilities calculated by

Weisskopf under extreme assumptions. Especially important is the competition between M1 and E2 radiation. These long have been believed to be equally probable. However, Weisskopf⁴ predicted a ratio of about 10^4 in the transition probabilities of magnetic radiation of order ℓ and electric radiation of order $\ell + 1$ for light quanta of energy between 0.1 and 1 Mev, the magnetic being highly favored.

(3) The intensity of the spectrum below 3.5 Mev. It is important that the level feeding the low lying excited states be reasonably well excited so that the statistics allow a prediction on the basis of competition.

4. Blatt and Weisskopf, "Theoretical Nuclear Physics", p. 629

Eugene L Church and James E. Monahan
Reported by Eugene L. Church

1. Threshold Values of K-Shell Internal-Conversion Coefficients .

In a previous report,¹ absolute non-relativistic K-shell internal-conversion coefficients (ICC's) were computed at threshold for comparison with the analogous relativistic results of Spinrad and Keller.² However, as was pointed out at that time, the results of the latter authors were in error, so that no valid comparison could be made. Spinrad³ has subsequently re-evaluated his expressions (limiting forms of those used by Rose et. al.⁴), leading to the results shown in Fig. 20.

That the indicated ratios become unity in the limit $Z \rightarrow 0$ indicates the correctness of the non-relativistic formulation. The exact origin of the deviations for higher atomic numbers is not clear, but is due in part to the difference between the relativistic and non-relativistic binding energies, $0.5(\alpha Z)^2$ and $1 - \sqrt{1 - (\alpha Z)^2}$, respectively. The possible effects of screening have been neglected in all cases, since for a given nuclear transition energy they should be small. However, screening seriously lowers the "threshold" energy relative to its unscreened value. In general $E_{\text{rel}} > E_{\text{n-rel}} > E_{\text{phys}}$, a distinction which must be made when interpreting data of this sort. For $Z = 75$, for example, these quantities have the values 83.32, 76.53 and 71.67 kev, respectively.

1. ANL -5174, p. 69.

2. B. I. Spinrad and L. B. Keller, Phys. Rev. 84, 1056 (1951).

3. B. I. Spinrad, private communication (February 1955).

4. Rose, Goertzel, Spinrad, Harr and Strong, Phys. Rev. 83, 79 (1951).

Figure 20

Comparison of relativistic and non-relativistic K-shell ICC's at their respective thresholds. Upper half: non-relativistic to relativistic for electric transitions. Lower half: relativistic to non-relativistic for magnetic transitions. Numbers indicate multiple orders.

2. Threshold Conversion in the L and M Sub-Shells

Non-relativistic estimates of the relative internal conversion of electric transitions in the L and M sub-shells have been previously reported.⁵ These results have now been extended to magnetic transitions in the Pauli approximation.

As pointed out by Berestetskii⁶ and Drell⁷, the radial matrix elements for magnetic transitions consist of both "volume" and "surface" terms. The magnitudes of the surface terms are found to be considerably greater than the volume terms, although they contribute only in consecutive odd sub-shells (I, III, V, . . . etc.), where the total number of such sub-shells equals the multipole order of the transition considered. This effect is particularly noticeable for M1 radiations, which are known to convert almost exclusively in the I sub-shells.

The relative internal conversion of M1 transitions in the five M sub-shells is shown in Table IV. In this case the total "volume" contribution in any sub-shell is less than 10% of the total conversion, in agreement with results for the L shell.⁵ In the latter case, however, the contribution of these terms is found to decrease rapidly with increasing multipole order. For simplicity, therefore, only the surface terms have been considered in the calculation of the relative M sub-shell conversion coefficients of higher magnetic multipoles. These results, along with the analogous but complete results for the L shell,⁵ are given in Table V.

5. ANL-5174, p. 71

6. V. B. Berestetskii, Zhur. Exptl. i Teoret. Fiz. 18, 1070 (1948)

7. S. D. Drell, Phys. Rev. 75, 132 (1949)

TABLE IV

Relative internal conversion of M1 transitions in the M sub-shells for vanishing electron momentum.

M_I	M_{II}	M_{III}	M_{IV}	M_V
1	0.104	0.005	0.010	4×10^{-4}

TABLE V

Relative internal conversion of magnetic transitions in the L and M sub-shells at their respective thresholds. The indicated results for the M shell include only the contributions of the "surface" terms.

	M1	M2	M3	M4	M5
L_I	1	1	0.100	0.021	0.007
L_{II}	0.096	0.053	0.004	5×10^{-4}	1×10^{-4}
L_{III}	0.040	0.757	1	1	1
M_I	1	1	0.084	0.018	0.002
M_{III}	0	0.889	1	1	0.381
M_V	0	0	0.042	0.556	1

The above results, as well as a general discussion of the conversion properties of low-energy transitions in heavy elements, have been prepared and submitted for publication in the Physical Review (December 1954).

3. Non-Relativistic Calculation of Internal-Conversion Coefficients

This project has been abandoned because of the lack of IBM machine time before its deactivation (September 1954). Because of the imminent appearance of the exact relativistic results of Rose et. al.⁸, programming this problem for the AVIDAC does not appear worth while.

4. The Properties of Radiationless (E-0) Nuclear Transitions

As a consequence of the finite nuclear size, atomic electrons spend a fraction of their time within the nuclear volume. During this time they experience an interaction with the transforming nucleus different from the ordinary external radiation field. This interaction may lead to the direct ejection of atomic electrons, which, in conjunction with gamma-ray emission and internal conversion, provides a third competitive mode of nuclear de-excitation. This process is most probable when it involves $\ell = 0$ electrons carrying off zero units of angular momentum, a possibility forbidden in all other processes. Since this can only occur if there is no change in parity, such transitions are called electric monopole, or E0 transitions. The analogous M0 transitions, involving parity change, are forbidden in all first-order processes.⁹

8. Rose, Goertzel and Swift, to be published.

9. R. G. Sachs, Phys. Rev. 57, 194 (1940)

Electric monopole transitions represent the only first-order process allowed between two nuclear states having zero spin and the same parity. A relativistic expression for the K-ejection lifetime of such "zero-zero" transitions has been given by Thomas.¹⁰ For the usual case of $\alpha Z W/P > 1/2$, his results may be written in the form:

$$T_{1/2} = \frac{(W^2 - 1)^{1-\gamma}}{W(W + \gamma)} g(Z), \quad (1)$$

where $W = \sqrt{P^2 + 1}$ is the total energy of the ejected electron in units of mc^2 , and $\gamma \equiv \sqrt{1 - (\alpha Z)^2}$. The function $g(Z)$ is diagrammed in Fig. 21.

In evaluating this quantity the "single-particle" estimate

$$\sum \langle i | R_p^2 | f \rangle \sim \frac{3}{5} R_0^2, \quad R_0 = 1.2 \times 10^{-13} A^{1/3} \text{ cm}, \quad (2)$$

has been employed. The W -dependent factor appearing in (1) is always less than unity and does not vary greatly providing the nuclear transition energy is $< mc^2$ and not too close to threshold. Under these conditions, $T_{1/2} \sim (1/2 - 1/5) g(Z)$.

Non-relativistically, the lifetimes of E0 transitions vary approximately as $1/Z^4$. However, the very rapid decrease in lifetime indicated in Fig. 21 is due mainly to the relativistic divergence of the $S_{1/2}$ electronic wave functions at the origin. Experimental lifetimes¹¹ of the proposed zero-zero transitions in O^{16} , Ge^{72} and Po^{214} correspond to matrix elements $(1/2 - 1/4)$ times those given by (2), indicating the essential validity of the above considerations.

10. R. Thomas, Phys. Rev. 58, 714 (1940)

11. S. D. Drell, ORNL-792.

Figure 21
K-Shell Lifetime Factor Versus Atomic Number

Pure E0 transitions may be most easily identified by their "infinite" absolute conversion coefficients. Their expected relative "conversion" in the K, L_I and M_I shells may be derived from Fig.22 by inclusion of the corresponding ratio of energy factors appearing in (1). However, this dependence is usually slight, and the indicated results may be taken as a direct measure of the corresponding ratios. Conversion in the remaining sub-shells should be negligible. The observed¹² K/L and L/M conversion ratios of the 1.42-Mev transition in Po^{214} are in agreement with these predictions. Following Drell,¹¹ the above results were obtained neglecting the small components of the electronic wave functions, because of their expected attenuation within the nuclear charge distribution.

The extremely short lifetimes of E0 transitions between zero spin states in heavy nuclei suggests the possible importance of such transitions between two states of spin $I \neq 0$. Examination of the general properties of possible E0 + M1 mixtures, as well as the related effects of the finite charge distribution on the usual calculation of internal-conversion coefficients,⁴ is now under consideration.

12. D. E. Alburger and A. Hedgran, Arkiv for Fysik, 7, 423, (1954)

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Figure 22
Ratios of K , L_I and M_I Lifetime Factors versus Atomic Number

R. H. Bryan and K. Tanaka
Reported by K. Tanaka

In quantum mechanics, one has the commutation relation between coordinate and momentum

$$[x_i, p_j] = i \hbar \delta_{ij}$$

The commutator $[x'_\mu, p'_\nu] = i \hbar [\delta_{\mu\nu} + (\ell^2 / \hbar^2) p_\mu p_\nu]$ has been proposed by several authors independently as one which implies that the eigenspectrum of the space coordinate (x_i) is discrete. The quantity ℓ is the fundamental length.

Since the latter commutator appears to be the simplest generalization of the former, we are investigating the implication of this commutator on the equations of motion and its application to electromagnetic field.

Katsumi Tanaka

The calculations become prohibitively laborious even for the simplest maxwellian particles (inverse 5th power law of force). There will therefore be no further reports since there appears to be no way to make sufficient progress to justify the labor involved.

39-6

Applications of the Theory of Angular Correlation
(5200-01)

Melvin Ferentz and Norbert Rosenzweig
Reported by Norbert Rosenzweig

Tabulation of F Coefficients. (A paper having this title was given at the APS meetings in Chicago, November 1954).

The combination F of Clebsch-Gordan and Racah coefficients,¹

$$F_k(l l' j j) = (-1)^{j' - j - 1} \left[(2l+1) (2l'+1) (2j+1) \right]^{1/2} C_{1-1}^{l l' k} W(j j l l'; k j')$$

occurs in the angular correlation functions for $\gamma - \gamma$, $\alpha - \gamma$, and $\beta - \gamma$ cascades. Biedenharn and Rose¹ give a short table of the F coefficients which has been useful, but which is not extensive enough to meet the needs at this Laboratory and elsewhere. Using the AVIDAC, we are preparing a table which will contain the following range of arguments:

1. Integral and half-integral values of j and j' up to 12.
2. $l' = l, l+1, l+2, l+4, l+6$ subject to the limitation
3. $k \text{ (even)} \leq 8.$

$$l' \leq \begin{cases} |j - j'| + 6 \\ 10 \end{cases}$$

This amounts to an extension of the Biedenharn and Rose table in two important respects. First, it encompasses the much higher spin values as observed, for example, by Krohn and Raboy, T. Novey, H. Frauenfelder, and others. Secondly, it includes the high multipolarities and "interference" coefficients which enter in a decay.

The AVIDAC computing program has been coded and debugged largely by R. Freshour and production has started with the help of J. Nelson. The quoted range results in a table of about 13,000 entries which will require in excess of 60 effective AVIDAC hours.

¹. L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. 25, 729 (1954).

Fritz Coester, Morton Hamermesh

Katsumi Tanaka

Reported by Katsumi Tanaka

In the interaction representation Heisenberg's S matrix takes $\leftarrow$ the form $U(\infty, -\infty)$. The operators $U(t, -\infty)$ and $U(\infty, t)$ may be defined either by adiabatic switching on of the interaction or by a certain average over initial and final times or by insertion of a small imaginary part in the energy denominator of the time independent scattering theory. The equivalence of these different formal limiting processes employed in the theory of scattering is shown.

A paper entitled "Limiting Processes in the Formal Theory of Scattering" appeared in the November 15, 1954 issue of the Physical Review.

Katsumi Tanaka

A relativistic system may be described in a 4-dimensional Bethe-Salpeter formalism or a 3-dimensional old and new Tamm-Dancoff formalism and also their covariant formalisms. However, these methods are far from satisfactory for the study of the system of two interacting particles.

The intimate relationship between the various formalisms for one particle states have been studied. The analysis has been confined to the second order in the coupling constant. A similar analysis is under investigation for the two particle states.

F. Coester (State University of Iowa) and
K. Tanaka
Reported by K. Tanaka

Lee (Phys. Rev. 95, 1329 (1954)) has treated some simple problems of interacting fields that are both renormalizable and solvable in closed form in order to obtain some deeper insight as to the nature of the renormalization procedure without the use of perturbation method. Lee considered the reactions $V \rightleftharpoons N + \theta$ and $N + \theta \rightleftharpoons V$ where V and N are neutral Fermion fields and θ is a neutral scalar Boson field.

In order to see whether his conclusions that all divergent quantities can be removed by both mass and coupling constant renormalizations, we investigated the two particle reaction $V + N \rightleftharpoons V + N$. The eigenstate that represents this process may be written as

$$\Psi = \sum_{p,k} \left[a(\underline{p}) \left| \underline{N}(-\underline{p}) \underline{V}(\underline{p}) \right\rangle + 1/2 b(\underline{p},k) a^+(k) \left| \underline{N}(-\underline{p}) \underline{N}(\underline{p}-k) \right\rangle \right] \quad (1)$$

where $\left| \underline{N}(-\underline{p}) \underline{V}(\underline{p}) \right\rangle$ is the functional representing the presence of physical N and V particles with momentum $(-\underline{p})$ and $(\underline{p})$, respectively, and $a(\underline{p})$ is the probability amplitude for the particular configuration Ψ . The second term represents a state in which, in addition, a θ particle of momentum $\underline{k}$ is present. We substitute Ψ into the Schrodinger equation

$$H \Psi = (m_v + m_n - B) \Psi$$

(2)

where the binding energy B is chosen to be positive, if there are bound states. Solving for $\underline{a}(\underline{p})$ and expressing the equation in terms of the renormalized coupling constant g_c , one obtains the following integral equation for the binding energy

$$\begin{aligned} B \underline{a}(\underline{p}) & \left[1 - g_c^2 B \sum_{\underline{k}} (2w_{\underline{k}})^{-1} (w + m_n - m_v + B)^{-1} (w + m_n - m_v)^{-2} \right] = \\ & = -g_c^2 \sum_{\underline{k}} \underline{a}(\underline{k}-\underline{p}) (2w_{\underline{k}})^{-1} (w + m_n - m_v + B)^{-1} \end{aligned} \quad (3)$$

We introduce the fourier transformer of $\underline{a}(\underline{p})$

$$\underline{U}(\underline{s}) = \int d^3 p e^{i \underline{p} \cdot \underline{s}} \underline{a}(\underline{p}) \quad (4)$$

and obtain

$$\begin{aligned} B \underline{U}(\underline{s}) & \left[1 - g_c^2 B \sum_{\underline{k}} (2w_{\underline{k}})^{-1} (w + m_n - m_v + B)^{-1} (w + m_n - m_v)^{-2} \right] \\ & = -g_c^2 \underline{U}(-\underline{s}) \sum_{\underline{k}} e^{i \underline{k} \cdot \underline{s}} / (2w_{\underline{k}})(w + m_n - m_v + B) \end{aligned} \quad (5)$$

This is the integral equation to determine the binding energy B . An abstract entitled "On the Lee Model" will be submitted to the Washington meeting (1955) of the American Physical Society.

7-3

(Lectures on Neutron Transport Theory)

Bernard I. Spinrad

Reporting on this program is discontinued because
of the transfer of Dr. Spinrad to the Reactor Engineering Division.

VII. ELECTRONIC DIGITAL COMPUTERS

1-8

(Report on Design of the AVIDAC)

Raymond F. Kramer

Nothing has been written on this report since the time has been occupied with the design of the Computer GEORGE.

2-8

The AVIDAC

1. The Use of the AVIDAC - Jean F. Hall

The AVIDAC was in use during this period for both checking out new codes and running programs for processing data and solving problems.

Two data-processing programs were in extensive use. One of these takes counting data from the neutron chopper and calculates transmissions and other quantities. The computation time is barely noticeable so that the time required to run this program is the total of the read-in-time (200 digits/sec.) and the punch-out time (20 digits/sec.).

The other data-processing program handles counting data from the fast exponential experiment. Flux plots are fitted to either cosines or exponentials, each with three free parameters. A startling efficient iterative procedure is used. Each set of data is processed in from three to five minutes.

Two reactor criticality programs were active. One of these is a three-group Serber-Wilson calculation for a 2-region spherical

reactor. The program iterates on the source to compute the critical radius of the core. The computing time varies greatly depending on the parameters of the problem from 20 minutes to 2 hours.

The other is a multi-group diffusion calculation of a spherical reactor. Inelastic scattering is considered and the diffusion equations are solved by iteration. The source is varied to criticality if desired. The computing time runs around 1 to 2 hours. Because of the small internal memory on the AVIDAC paper tape is used as an auxiliary memory. The actual computing time is barely noticeable, the time being practically all spent in reading tape. This program is also coded for the AEC UNIVAC at NYU so that the AVIDAC is used only for a few calculations when it is inconvenient to go to New York.

A series of problems in reactor kinetics have been run. This series has been of an experimental nature for developing methods of solving the kinetics equations both accurately and in a reasonable amount of time. Because at the beginning very little was known of the ranges of intermediate numbers the AVIDAC floating point routine was used to handle scale factors. This increased the running time enormously.

Considerable computation was done on Kurath's calculation of the Excited States of Light Nuclei. A calculation of internal conversion was done for B. I. Spinrad.

Miscellaneous work included calculations of tables of unpleasant integrals for various members of the Laboratory.

2. Preventive Maintenance

J. C. Chu, R. F. Kramer, D. H. Jacobsohn and C. J. Bergstrom
Reported by C J. Bergstrom

a) A continuing incidence of tube troubles, due to internal shorting of elements, caused an intensive effort to find the cause and some effective remedy. The troubles were particularly disturbing in that they were usually intermittent and took an excessive amount of time to find.

Several types of tubes were tried as substitutes in an effort to see whether the failure could be attributed to some particular type. This was only partially effective. Although certain types have a slightly lower frequency of failure, no type was completely satisfactory.

The procedure which gives the most promise of markedly increasing tube life is the operation of filaments at less than manufacturers' ratings. Experiments were made on batches of tubes using filament voltages lowered in steps to 5.0 volts from a nominal 6.3. These tests showed that safe operation of the tube could be achieved if filaments were operated within $\pm$ or $-$ 5% of 5.6 volts in the AVIDAC. A trial period of operation using these standards was next tried on the machine. The results were entirely satisfactory in all but one case. One particular tube type (5687) would not function under this condition. By running this type at its normal rating and all others at the new reduced rating, marked reduction of the number of internal shorts has been experienced. The investigation is being continued.

b) A tube file system has been instituted in order that complete data on the life and history data of each tube can be recorded. A filing chest with a cell for every tube in the AVIDAC has been built and stocked with a duplicate set of tubes. Thus, tubes are now being checked and re-used in the exact location where they were first installed.

3. Sub-Routine Library - Ruth Freshour

There has been added to the AVIDAC library of sub-routines one for the numerical computation of the characteristic values of a real symmetric matrix. It was proposed by Wallace Givens in ORNL-1574. The program will handle a 16 by 16 matrix. It is being used in a slightly different form in the work of D. Kurath.

7.-8

The Auxiliary Memories for the AVIDAC and ORACLE (4520-02)

Raymond F. Kramer, Leslie C. Merrill
Reported by Leslie C. Merrill

The AVIDAC auxiliary memory unit was finished and tested. Since the recording heads and clutches were the chief subject of tests on the ORACLE unit, close attention was given to the wind-up reel servomechanisms on the AVIDAC unit. A few minor modifications were made for the sake of reliable operation. Descriptions of the changes were sent to the ORNL computer group.

13-5

Computer GEORGE

(4520-02)

J. C. Chu, Leslie Merrill, David Jacobsohn
Carl Bergstrom and Raymond F. Kramer
Reported by Raymond F. Kramer

1. The Magnetic Core Memory.

The contract with International Telemeter Corporation for the construction of a magnetic core memory of 4096 words each of 40 bits has been signed. The electrical design is to be similar to a memory being built by Telemeter Corporation for Rand Corporation. Construction of the ANL memory was started late in this report period. Delivery was to be made on February 28, 1955; however, delay in completing the Rand memory made it necessary to delay delivery of the ANL memory until June 1, 1955.

This memory, a new arithmetic unit, control console, and a magnetic tape auxiliary memory are the principal units of the new digital computer-----"GEORGE". The auxiliary memory was originally built for the "AVIDAC".

2. Arithmetic Unit and Control.

The control unit and arithmetic unit design proposed in the last report has been adopted for this machine. The logical design for these units has been completed. The electrical design of the registers, adder, M selector and drivers is complete and construction is under way. Tests

conducted on models indicate that the adder carry time for 40 stages will be about 2.5μ secs; shifting time in the registers will be about 2.5μ secs. The electrical design of the control units is about 50% complete. The electrical design makes considerable use of germanium diodes. This will result in a reduced heat loss, smaller size, and should increase the reliability of the computer.

Mechanical construction of the arithmetic unit frame and components is about 60% complete.

3. The new computer will be located in Room C-221 of Building D-203. Modifications of the room and its ventilation are necessary to accommodate the machine.

The design for these room changes, ventilation, air conditioning, and power facilities has been completed. Invitations to bid on this work are in the hands of contractors.

The DC power supplies and part of the paper tape data preparation equipment for the computer, also have been purchased.

13-3

REAC Problems

Nye F. Morehouse, Jr.

(1) The integral $\int_0^x e^{-\frac{a}{2+1}} d\xi$ was computed on the REAC for

$$0 \leq x \leq y; \quad 0 \leq a \leq 6.$$

(2) An attempt was made to solve the Painlevé differential equation

$$\frac{d^2 \omega}{dz^2} = 6\omega^2 + \lambda z \quad \lambda = 0, 1, 2, 3, 4$$

$$\omega(0) = 1$$

$$\omega'(0) = 0$$

It was found, however, that covering the desired range of ω required an initial condition near enough to the noise level of the machine to make an accurate solution impossible.

(3) The equation

$$K'' = \left(\frac{1-\beta}{L} K - \frac{\beta}{L} \right) K' - r \left(\frac{1-\beta}{L} K - \frac{\beta}{L} \right)$$

$$K(0) = 0$$

$$K'(0) = r$$

This equation has both stable and unstable solutions depending on the values chosen for the parameters. It was found that the solutions well inside of a region of stability in the parameter space were accurate but

that operation in the neighborhood of the boundary of a stable region was impossible because the noise in the computer caused the solution to cross over into the region of instability.

(4) The integrals $\int_0^x e^{-\beta(x-x_i)} x \sin Kx \, dx$ were evaluated.