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HYDRODYNAMICS OF A NEW CONCEPT
OF PRIMARY CONTAINMENT
BY ENERGY ABSORPTION

by

H. C. Sorensen and S. H. Fistedis

Reactor Engineering Division

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HYDRODYNAMICS OF A NEW CONCEPT OF PRIMARY CONTAINMENT BY ENERGY ABSORPTION

by

H. C. Sorensen and S. H. Fistedis

ABSTRACT

To determine the effect that the destructive component of a nuclear accident produces on a primary containment structure, a fluid dynamical analysis is made for an idealized reactor system having spherical symmetry. This primary containment structure reflects a new concept developed at Argonne National Laboratory.¹ It consists of the utilization of the concrete biological shield as an energy-absorbing device. For this purpose, unbonded, unstressed, high-strength steel strands are embedded in the biological shield. The new containment concept is based on the energy-absorbing capability of the strands.

The partial differential equations describing the fluid motion are based upon the von Neumann-Richtmeyer artificial-viscosity technique and are presented in Lagrangean form. A numerical solution based upon finite-difference approximations of these partial differential equations is obtained by the use of a CDC-3600 digital computer. All computer programs used in obtaining the numerical solution are written in FORTRAN IV computer language.

Two solutions for an example problem are given. The solution for an ideal reactor system that does not include the reactor vessel is given in Part One, and the solution for the same ideal reactor system but with the reactor vessel included is given in Part Two. In each case, the effect that the destructive component produces on the concrete containment structure is determined from the computer results and is discussed. The results of the two solutions are then compared. Suggestions for improvements in the analysis and suggestions for future work are also mentioned.

PREFACE

During the power excursion that results from the occurrence of a hypothetical core meltdown (accident) in a power reactor, a large amount of energy is deposited as heat in the materials of the core.* At the end of the excursion, the core is left at a very high temperature and pressure. This condition is the beginning of core disintegration. The material immediately surrounding the core is compressed by the high pressure and is accelerated outward. The core expands in volume, accompanied by a rapid decrease in the core pressure. At the same time, a pressure wave forms and moves out into the surrounding medium. This pressure wave, after moving a short distance through the compressible medium, becomes a shock wave; i.e., it is characterized by a sharp front. As the shock wave moves through the compressible medium, there is an irreversible compression at the front, which continually deposits some of the energy of the shock wave in the material, raising the temperature slightly. This process causes an attenuation of the shock wave, so that it has less destructive capability when it finally reaches some important part of the containment structure.

For many hypothetical nuclear accidents, the local pressure produced by the energy deposited in the core is comparable to the pressure produced by the detonation of a TNT charge. Thus, it is common practice to analyze the destructiveness of a hypothetical nuclear accident on the basis of an equivalent TNT explosion. The size of the TNT charge is chosen such that the amount of energy released in its detonation is equivalent to that calculated for the accident.

Many people have investigated explosives, explosions, and shock waves. The list of investigators in these areas includes W. J. M. Rankine, H. Hugoniot, Lord Rayleigh, H. Jones, A. R. Miller, G. I. Taylor, W. G. Penney, P. W. Bridgman, H. K. Dasgupta, J. G. Kirkwood, H. A. Bethe, S. R. Brinkley, Jr., R. H. Cole, M. H. Rice, J. M. Walsh, J. von Neumann, R. D. Richtmeyer, P. D. Lax, H. L. Brode, L. D. Landau, K. P. Stanyukovich, R. Courant, and K. O. Friedrichs.

The analyses of shock waves in fluids have used a variety of mathematical methods and techniques. Penney²¹ and later Penney and Dasgupta²² used numerical methods to solve the differential equations for finite-amplitude spherical waves in TNT. They reduced the governing differential equations to a form convenient for step-by-step integration by the use of Riemann functions. This technique is called the method of characteristics.

Kirkwood and Bethe¹⁹ developed an analytical approach to shock-wave propagation in water. They considered the solution of the fundamental equations by the introduction of the enthalpy associated with the fluid.

*This account of what happens in a nuclear accident is condensed from Ref. 2.

Kirkwood and Brinkley²⁰ developed an approach to the solution for shock waves of the basic hydrodynamic equations that is particularly convenient for extending experimentally determined shock-wave data obtained at a single distance, or over a limited range, to other distances. They obtained the parameters of their theory from experimental data. These data were obtained at relatively large distances from the explosive charge where the pressures are relatively low. Then they used their formulation to extrapolate for the pressures at points nearer the charge.

Von Neumann and Richtmeyer¹⁶ developed another method for the analysis of fluid-dynamical problems involving shock waves. In this method, the equations of hydrodynamics are modified by the inclusion of additional terms that greatly simplify the procedures necessary for the stepwise numerical solution of the equations. The quantitative influence of these terms can be made as small as one wishes by choice of a sufficiently fine mesh for the numerical integration. The method uses an artificial viscosity as a mechanism for avoiding shock-front discontinuities, and it takes care of shocks automatically, whenever and wherever they occur. Brode^{3,4} used the von Neumann-Richtmeyer method for numerical solutions of spherical blast waves in air. This is the method employed in this work.

The concept of primary containment by energy absorption treated herewith was originally developed by S. H. Fistedis. Part One of this report constitutes the thesis of H. C. Sorensen in partial fulfillment of the requirements of the degree of Doctor of Philosophy at the University of Nebraska. Part Two was subsequently completed. All work was performed at the Reactor Engineering Division, Argonne National Laboratory.

PART ONE

SOLUTION FOR THE IDEAL REACTOR SYSTEM WITHOUT A REACTOR VESSEL

I. INTRODUCTION

The increasing demand for power has influenced and stimulated the development of atomic reactors. At present, electricity produced by atomic reactors is, in most cases, not competitive with that produced from fossil fuels. To become competitive, power reactors must accommodate larger and larger cores, thereby increasing their electrical output. As the production capability of the reactor facility increases, the net cost per unit of output decreases.

Many difficult design problems are associated with the development of larger reactor cores. One such problem is the requirement for rendering the facility safe with respect to the radiation exposure to the surrounding public. This requirement must be satisfied for normal operation of the facility as well as for abnormal reactor behavior.

To protect against radiation exposure to the reactor site personnel during normal operation, a radiation shield (also called a biological shield) is provided which prevents radioactive particles from escaping from the reactor cavity. The safety requirement for this type of reactor performance is easily satisfied.

Abnormal reactor behavior requires safety provisions in which those for the maximum credible accident (MCA) are included. The MCA is that particular accident (one of a large number of potential accidents examined in considering the safety of the reactor system) for which the consequences, as measured by the radiation exposure to the surrounding population, would not be exceeded by any other accident for which the occurrence during the lifetime of the facility would appear to be credible. The safety requirement for this type of reactor performance is not easily satisfied. If the MCA would occur in a power reactor, an enormous amount of energy would be released in a very short time. This energy release could cause serious damage to the surrounding structure. As reactor cores become larger to produce more electricity, the amount of energy released by the MCA increases. Thus, the control of radiation exposure becomes more difficult. The control of the radiation released by a nuclear accident is referred to as containment.

Reactor containment is a general term which is defined⁶ to include all structures, systems, mechanisms, and devices that can be provided to attain, with a high degree of reliability, some specified attenuation in the radioactivity presumed to be released from the primary system in a reactor accident and which otherwise might be released to the surrounding environment.

Containment is accomplished in two phases: The first is referred to as primary containment; the second is referred to as secondary containment. The objective of the first phase is to attenuate the destructive component, i.e., the energy release, of a nuclear accident and is performed by the primary containment system. The objective of the second phase is to control, within acceptable limits, the release to the environment of the radioactive products of a nuclear accident and is performed by the outer shell of the reactor plant.

In general, present design methods do not guarantee primary containment. However, safeguards are built into the reactor system to prevent rupture of the outer shell of the reactor plant, thereby insuring secondary containment. The outer shell is normally designed to withstand a certain static pressure, which is correlated to the magnitude of the MCA by P-V-T relations.

Every reactor requires a biological shield for normal operation. This biological shield is generally constructed of concrete (4-6 ft thick) and envelops the reactor cavity. The reactor cavity contains the reactor vessel, which in turn contains the reactor core. The nuclear accident begins in the core.

Primary containment is often achieved by placing concentric steel cylinders around the reactor vessel with a layer of crushable filler material separating adjacent steel cylinders. This primary-containment device is placed entirely within the reactor cavity. The destructive component of the accident is thereby attenuated by the subsequent stretching of the steel cylinders and crushing of the filler material. A multitude of concentric cylinders can be an effective energy-absorbing system. One serious objection to this method of energy absorption is the substantial expense involved in the construction. Another objection is the lack of ascertainable margins of safety in design, since a local imperfection in a cylinder could initiate a tear in that cylinder; thus, failure of that cylinder would occur without significant energy absorption.

To reduce the cost of primary nuclear containment and to provide reliable safety factors for credible nuclear accidents, a new approach for primary nuclear containment by energy absorption¹ has been proposed. This new approach uses high-strength steel strands placed in the concrete biological shield forming a basketlike configuration around the reactor cavity (see Fig. 1). The strands are placed in conduits to prevent bond with the concrete and are not prestressed. The energy-absorbing process is accomplished by the plastic deformation that the strands are capable of exhibiting. This approach eliminates the necessity of the expensive multiple steel cylinders in the reactor cavity and extends the primary containment system to include the concrete biological shield, which heretofore has only played an effective role as a radiation shield.

Fig. 1. Primary Containment Structure Showing the New Concept of Energy Absorption¹

The feasibility of this new approach for primary containment has been established on the basis of the mechanical properties of the strands measured under static conditions. This was done by equating the total energy released by the MCA to the total allowable strain energy of the strands. Equating the energies is a conservative basis for establishing the new concept because of the energy that dissipates during its transport to the containment structure. However, one factor necessary to establish the application of the new concept for primary containment is the rate of strain to which the strands are subjected. This is important because the total strain energy developed by the strands is dependent upon the strain rate.

The purpose of this investigation is to make a fluid dynamical analysis of the destructive component, i.e., the energy release, of a nuclear

accident. This destructive component is assumed equivalent to that produced by the detonation of a charge of TNT. It is desired from the analysis to:

1. Obtain general information as to how the destructive component propagates and changes with time.
2. Determine the effect that the destructive component produces on the concrete primary-containment structure so that the required rate of strain for the strands can be calculated.

The system analyzed in this investigation is an idealized reactor system having spherical symmetry. It consists only of a reactor core, a liquid coolant surrounding the core, a layer of inert gas, and a concrete biological shield (see Fig. 2). The high-strength steel strands (omitted from Fig. 2) are assumed to be embedded in the concrete. This configuration results in a system of one space variable.

Fig. 2. Idealized Reactor System

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \bar{v}) = 0,$$

Continuity (1)

$$\rho \frac{\partial \bar{v}}{\partial t} + \text{grad } P = 0,$$

Motion (2)

and

$$\frac{dE}{dt} - \frac{P}{\rho^2} \frac{d\rho}{dt} = 0,$$

Energy (3)

For an energy release due to the detonation of TNT inside a fluid, the destructive component can be separated into three elements. These three elements, in the order of appearance on the time scale, are:

1. The shock wave.
2. The fluid momentum due to the passage of the shock wave.
3. The expanding gas bubble at high temperature and pressure pushing on the fluid.

The Eulerian form of the basic equations⁵ describing the motion of an inviscid fluid (with no shock waves present) are

where the velocity vector $\bar{v}$ has the components u , v , and w in Cartesian coordinates. The density, ρ , is usually expressed in terms of the pressure, P , to obtain an additional equation called the equation of state of the fluid.

The front of the shock wave that is transmitted to the fluid medium surrounding the core is verified by experiment to be so steep that it can be considered a discontinuity. When a fluid particle is traversed by a shock

Fig. 3. Schematic Profile of a Shock Wave

wave, it suffers an almost discontinuous change in its thermodynamic state. The shock front propagates with a velocity, U_s , which is supersonic relative to the undisturbed medium; and a particle velocity, U_p , is imparted to the fluid behind the shock front (see Fig. 3). The basic equations for fluid flow do not apply in the region of the discontinuity.

The equations that do apply to such a discontinuity are obtained by considering regions immediately ahead of and behind the shock front. These equations,* which are based on the conservation laws, are

$$V_1/V_0 = (U_s - U_p)/U_s, \quad \text{Mass (4)}$$

$$P_1 - P_0 = U_s U_p / V_0, \quad \text{Momentum (5)}$$

and

$$E_1 - E_0 = \frac{1}{2}(P_1 + P_0)(V_0 - V_1), \quad \text{Energy (6)}$$

where P_0 , V_0 , and E_0 and P_1 , V_1 , and E_1 denote pressure, specific volume, and specific internal energy ahead of and behind the shock front, respectively.

Taken together, Eqs. 1-6 completely describe the fluid motion considered in this investigation. These equations can be solved by finite-difference techniques.

* These three relations, called the Rankine-Hugoniot equations, are derived in any text on shock-wave hydrodynamics. See, for example, Ref. 7.

II. DEVELOPMENT OF THE THEORY*

A. Introduction

Finite-difference methods for the calculation of time-dependent fluid flows have mostly been based on either the Eulerian or the Lagrangean form of the equations. In the Eulerian form, the independent space variables refer to a coordinate system fixed in space and through which the fluid is thought of as moving. The flow is characterized by a time-dependent velocity field, which is to be found by solving the initial value problem. In the Lagrangean form, the independent space variables refer to a coordinate system fixed in the fluid and undergoing all the motion and distortion of the fluid. Therefore the particles of the fluid are permanently identified by their Lagrangean variables, while their actual positions in space are among the dependent quantities to be solved. Although these forms are essentially equivalent, the Lagrangean form gives more information (it tells where each bit of fluid came from originally) and has the virtue that conservation of mass is automatic and exact, even in the finite-difference approximations. In some problems, this results in considerably greater accuracy. For these reasons, the Lagrangean form is generally preferred for problems in one space variable.

Fluid flows are frequently characterized by internal discontinuities, such as shocks, on which special boundary conditions (jump conditions) are required. These special boundary conditions are provided by the well-known Rankine-Hugoniot equations, but their application in practice is encumbered by the following difficulties: (1) The surfaces on which the conditions are to be applied are in motion through the fluid, and (2) the motion of these surfaces is not known in advance, but is determined by the equations themselves (the differential equations plus the jump conditions). Thus, in numerical work, a highly implicit situation prevails. The differential equations that apply to the smooth part of the flow, as for problems without shocks or for the flow between shocks, will be described first.

B. The Lagrangean Equations

It is assumed that viscous forces, body forces (like gravity), heat conduction, and energy sources are absent. Certain artificial pseudo-viscous forces in connection with shocks will be introduced later, and the possibility for heat conduction will be accomplished by leaving the energy equation in the system rather than eliminating it by use of the constancy of entropy along the world lines of fluid particles.

For spherical symmetry, let r denote the radial coordinate of a small fluid element at time $t = 0$, and let $R = R(r, t)$ be the radial

* This development is found in Ref. 14, Chapter X, and is presented here in brief form to lend continuity to the subject matter.

coordinate of the same fluid element at time t in the motion, so that r and t are independent variables. Each bit of fluid is labeled with a certain value of r , called the Lagrangean coordinate, and retains that label as it moves about. The label r could be chosen in many ways other than the one given above. For example, it could be defined as the radial coordinate at some other fixed time $t = t_0$. Furthermore, if $g(r)$ is any function with a continuous positive derivative, the variable $r' = g(r)$ would also be suitable as a label. In other words, if $h(r) = g'(r)$ is continuous and greater than 0, then

$$r' = \int h(r) dr$$

may be used in place of r as the Lagrangean coordinate. A particular choice, which is convenient and will be used henceforth, is

$$r' = \frac{1}{\rho_0} \int \rho(r) dr,$$

where $\rho(r)$ is the density at $t = 0$, and ρ_0 is a constant having the dimensions of density. The symbol r' can be interpreted as the radial coordinate in a reference configuration of the system in which the various spherical shells of fluid have been moved in or out until they all have common density ρ_0 . This is referred to as the standard reference configuration. Henceforth, r will be written instead of r' .

The Lagrangean form of the differential equations governing a fluid motion with spherical symmetry is

$$\frac{1}{\rho} = \frac{1}{\rho_0} \left[\frac{R(r, t)}{r} \right]^2 \frac{\partial R}{\partial r}, \quad \text{Continuity (7)}$$

$$\frac{\partial u}{\partial t} = - \frac{1}{\rho_0} \left[\frac{R(r, t)}{r} \right]^2 \frac{\partial P}{\partial r}, \quad \text{Motion (8)}$$

$$\frac{\partial E}{\partial t} = \frac{P}{\rho^2} \frac{\partial \rho}{\partial t}, \quad \text{Energy (9)}$$

$$\frac{\partial R}{\partial t} = u, \quad \text{Velocity (10)}$$

and

$$P = f(E, \rho), \quad \text{Equation of State (11)}$$

where u , P , ρ , E , and R represent fluid velocity, pressure, density, internal energy per unit mass, and the Eulerian coordinate, respectively,

and are defined as functions of r and t . Equations 10 and 11 are to be thought of as abbreviations; their substitution into Eqs. 7, 8, and 9 gives three partial differential equations in the three dependent quantities, R , u , and E .

C. The Jump Conditions at a Shock

The motion of an ideal fluid is often characterized by lines or curves in the r, t -plane (more generally surfaces in the $\bar{R}, t$ -space) on which certain of the dependent quantities are discontinuous, but possess one-sided limits on both sides. Even if the flow is smooth in the beginning, discontinuities may develop after a finite time, after which no smooth, single-valued solution of the system (Eqs. 7-11) exists. Examples are an interface, a contact discontinuity (where ρ and E are discontinuous, but P and u are continuous), a shock (where ρ , E , P , and u are all discontinuous), and the head of a rarefaction wave (where only certain derivatives are discontinuous). At such discontinuities, the differential equations (which no longer make sense) must be supplanted by jump conditions, which serve as internal boundary conditions and are needed to make the solution unique.

For an interface, a contact discontinuity, or a rarefaction head, the only jump conditions needed are the continuity of certain variables, and no modification of the finite-difference representations of the system is required, although for an interface, modification may be desirable for increased accuracy. But when shocks are present, the unmodified difference equations do not give even approximately the right answers, except in certain limiting cases.

The Rankine-Hugoniot equations (jump conditions), together with the continuity of $R(r, t)$ across the shock, just provide the needed boundary conditions for the differential equations on each side and determine the motion of the shock front itself. These equations have been used directly in numerical calculations (unpublished work, Los Alamos) in a procedure called "shock fitting." The shock-fitting equations are implicit and must usually be solved by an iterative trial-and-error technique. The equations must be modified when the shock front passes a net point. The shock-fitting technique in general is rather complicated.

To get around these complications, von Neumann and Richtmeyer devised an approximate method of calculation for fluid dynamical problems, in which shocks are taken care of automatically, whenever and wherever they arise. The method is based upon certain physical notions concerning real (and therefore nonideal) fluids rather than on a direct attempt to incorporate the Rankine-Hugoniot conditions into the calculations. Specifically, dissipative mechanisms, like viscosity and heat conduction, have a smoothing effect on a shock, so that the surface of discontinuity is replaced by a thin transition layer in which quantities change rapidly but not discontinuously. The differential equations, with the dissipative mechanism

included, apply in this layer as well as elsewhere, so that no boundary conditions at the shock front are needed. At the same time, the basic conservation laws on which the Rankine-Hugoniot conditions were based are retained, and the jump conditions still hold across the transition layer, in the approximation in which this layer is regarded as thin in comparison with other dimensions occurring in the problem.

D. Effect of Dissipation

Various investigators, including Lord Rayleigh, R. Becker (1922), and L. H. Thomas (1944), have studied the effect of dissipation on shocks. Becker studied the effect of heat conduction and viscosity. He showed that when heat conduction is allowed for, the temperature varies smoothly through the shock layer and that for shocks of strength less than a critical value, the pressure and density vary smoothly also. On the other hand, for stronger shocks, the transition of pressure and density from their initial to their final values is partly by a smooth variation and partly by a discontinuous jump. When viscosity is allowed for, however, all quantities vary smoothly through the shock region for a shock of any strength. In either case, the thickness of the transition zone is proportional to the coefficient of the dissipative mechanism, so that in the limit of no heat conduction and no viscosity, the variations approach the discontinuous ones of the Rankine-Hugoniot theory.

The idea of von Neumann and Richtmeyer was to introduce a purely artificial dissipative mechanism of such form and strength that the shock transition would be a smooth one extending over a small number (say three or four) of intervals Δr of the space variable, and then to construct finite-difference equations with dissipation included, but without the necessity for shock fitting. In a calculation with such equations, the shocks show up automatically as near-discontinuities, across which quantities have very nearly the correct jumps and which travel with very nearly the correct speed through the fluid.

A mechanism like viscosity is more suitable for the dissipative mechanism than one like heat conduction because with viscosity, all quantities vary smoothly. However, Becker showed that with ordinary viscosity, in which the stress is proportional to the rate of shear, and which is, therefore, represented by linear terms in the differential equations, the thickness of the transition layer varies with shock strength, approaching zero for a very strong shock and infinity for a very weak one. But von Neumann and Richtmeyer want the thickness to be about the same--namely, about $3-4 \Delta r$ --for all shocks; they therefore add quadratic terms to the differential equations. This is equivalent to using a small viscosity coefficient for weak shocks and a large one for strong shocks.

By adding these terms, von Neumann and Richtmeyer achieve a thickness of the transition layer that is independent of the shock strength.

Other features of the added terms insure that the thickness is also independent of the pressure and the density of the material into which the shock is running.

The differential equations of the von Neumann-Richtmeyer pseudo-viscous method in Lagrangean formulation for spherical symmetry are

$$V = \frac{1}{\rho_0} \left[\frac{R(r, t)}{r} \right]^2 \frac{\partial R}{\partial r}, \quad \text{Continuity (12)}$$

$$\frac{\partial u}{\partial t} = - \frac{1}{\rho_0} \left[\frac{R(r, t)}{r} \right]^2 \frac{\partial (P + Q)}{\partial r}, \quad \text{Motion (13)}$$

$$\frac{\partial E}{\partial t} = -(P + Q) \frac{\partial V}{\partial t}, \quad \text{Energy (14)}$$

$$\frac{\partial R}{\partial t} = u, \quad \text{Velocity (15)}$$

and

$$P = F(E, V), \quad \text{Equation of State (16)}$$

where $V = 1/\rho$ is the specific volume, and Q , the pseudoviscous pressure, is given by

$$Q = \begin{cases} \frac{(\rho_0 \ell)^2}{V} \left(\frac{\partial V}{\partial t} \right)^2, & \text{if } \frac{\partial V}{\partial t} < 0 \\ 0, & \text{if } \frac{\partial V}{\partial t} \geq 0 \end{cases} \quad (17)$$

where ℓ is a constant having the dimension of length.

One method for the solution of fluid dynamical problems involving shocks is to solve Eqs. 12-17 by a numerical technique.

E. Finite-difference Equations

Finite-difference systems may be obtained in a variety of ways. In the pseudoviscous terms, von Neumann and Richtmeyer replaced ℓ by $a\Delta r(r/R)^2$, where a is a dimensionless constant around 1.5 to 2.0, thus giving the shocks a thickness around 3 to 5 times Δr , i.e., insuring that all shocks cover the same number of net points (namely, about 3-5). If the mesh size is variable, this will give the shocks a greater thickness in the coarse part of the net than in the fine part.

The finite-difference form of Eqs. 12-17 used in this instance is

$$V_{j+\frac{1}{2}}^{n+1} = \frac{1}{\rho_0} \frac{(R_{j+1}^{n+1})^3 - (R_j^{n+1})^3}{r_{j+1}^3 - r_j^3}, \quad \text{Continuity (18)}$$

$$\frac{u_j^{n+1} - u_j^n}{\Delta t} = -\frac{1}{\rho_0} \frac{(P_{j+\frac{1}{2}}^n - P_{j-\frac{1}{2}}^n) + (Q_{j+\frac{1}{2}}^n - Q_{j-\frac{1}{2}}^n)}{\Delta r_j} \left(\frac{R_j^n}{r_j} \right)^2, \quad \text{Motion (19)}$$

$$E_{j+\frac{1}{2}}^{n+1} - E_{j+\frac{1}{2}}^n + \left(\frac{P_{j+\frac{1}{2}}^{n+1} + P_{j+\frac{1}{2}}^n}{2} + Q_{j+\frac{1}{2}}^{n+1} \right) (V_{j+\frac{1}{2}}^{n+1} - V_{j+\frac{1}{2}}^n) = 0, \quad \text{Energy (20)}$$

$$\frac{R_j^{n+1} - R_j^n}{\Delta t} = u_j^{n+1}, \quad \text{Velocity (21)}$$

$$P_{j+\frac{1}{2}}^{n+1} = f(E_{j+\frac{1}{2}}^{n+1}, V_{j+\frac{1}{2}}^{n+1}), \quad \text{Equation of State (22)}$$

and

$$Q_{j+\frac{1}{2}}^n = \begin{cases} \frac{2(a\rho_0\Delta r_{j+\frac{1}{2}})^2}{V_{j+\frac{1}{2}}^n + V_{j+\frac{1}{2}}^{n-1}} \left(\frac{V_{j+\frac{1}{2}}^n - V_{j+\frac{1}{2}}^{n-1}}{\Delta t} \right)^2 \left(\frac{2r_{j+1}}{R_{j+1}^n + R_{j+1}^{n-1}} \right)^4, & \text{if } V_{j+\frac{1}{2}}^n - V_{j+\frac{1}{2}}^{n-1} < 0 \\ 0, & \text{otherwise.} \end{cases}$$

Pseudoviscous Pressure (23)

The system as a whole is correct to $O[(\Delta t)^2] + O[(\Delta r)^2]$. The superscript n represents the time step, and the subscript j represents the net point. The notation r_j denotes the j th net point of the Lagrangean net. The symbols Δr_j and $\Delta r_{j+\frac{1}{2}}$ are defined as

$$\Delta r_j = \frac{r_{j+1} - r_{j-1}}{2},$$

and

$$\Delta r_{j+\frac{1}{2}} = r_{j+1} - r_j.$$

The above set of equations constitutes a system of open-ended, initial-value, finite-difference equations. If all the quantities are known at $t = t^n$ along with the appropriate boundary conditions, the solution can be propagated outward with time.

F. Stability of the Finite-difference Equations

The stability condition that guarantees convergence for the finite-difference equations is

$$\frac{c\Delta t}{R_{j+1}^n - R_j^n} \leq 0.3 \quad (24)$$

and must be satisfied for all n and j , where c is the speed of sound in the medium.

G. Treatment at Interfaces

If an interface between two fluids comes at an index, say $j = J$, the finite-difference equations may be used just as they are, keeping in mind only to use the appropriate equation of state on each side of the interface. However, this may lead to inaccuracies, and it is better to give special treatment to interfaces. The inaccuracies result in several ways: (1) If the densities of the two fluids are very different (as for a gas and a liquid), we may wish to use quite different increments, Δr , on the two sides of the interface with consequent bad centering of the difference equations; and (2) although the pressure gradient, $\partial P / \partial r$, varies continuously across the interface (this is a consequence of the particular definition of the Lagrangean variable adopted here), the derivative of the pressure gradient may not, so that $\partial P / \partial r$ may not be given to the second-order accuracy by the centered difference quotient. To improve the accuracy of the solution at the interface, the expression

$$\begin{aligned} \left. \frac{\partial P}{\partial r} \right|_{r=r_J} \approx & \left\{ 3 \left[\left(P_{j+\frac{1}{2}}^n - P_{j-\frac{1}{2}}^n \right) + \left(Q_{j+\frac{1}{2}}^n - Q_{j-\frac{1}{2}}^n \right) \right] \right. \\ & \left. - \frac{1}{3} \left[\left(P_{j+\frac{3}{2}}^n - P_{j-\frac{3}{2}}^n \right) + \left(Q_{j+\frac{3}{2}}^n - Q_{j-\frac{3}{2}}^n \right) \right] \right\} / (\Delta_1 r + \Delta_2 r), \end{aligned} \quad (25)$$

where $\Delta_1 r$ and $\Delta_2 r$ are the increments of r used on the left and right sides of the interface, respectively, can be used in place of

$$\left. \frac{\partial P}{\partial r} \right|_{r=r_J} \approx \frac{\left(P_{j+\frac{1}{2}}^n - P_{j-\frac{1}{2}}^n \right) + \left(Q_{j+\frac{1}{2}}^n - Q_{j-\frac{1}{2}}^n \right)}{\Delta r_j}$$

in Eq. 19.

Equation 25 can be used even if the two fluids are identical, so that the interface is not a real one but only a point at which the increment Δr is changed.

III. SOLUTION OF A SAMPLE PROBLEM

A. Problem Layout

The idealized reactor system analyzed in this problem simulates a typical sodium-reactor installation. The core is a spherical charge of TNT. The charge radius, a_0 , is 30 cm, and the charge density is 1.5 gm/cm^3 . Thus, the analysis for this problem is based on the destructive component (energy release) of 375 lb of TNT. The liquid coolant in a sodium

reactor would be sodium. However, the properties of water and sodium are very much alike. Since more experimental data are available for water than for sodium, water is used as the coolant in this example. The inert gas is argon, and the biological shield is concrete. The densities of the various media are given in Table I.

TABLE I. Densities of the Various Media

Medium	Density, gm/cm^3	Conditions
TNT	1.51	
Water	0.9982	20°C and 1 atm
Argon	0.00169	15°C and 1 atm
Concrete	2.663	

Solutions for this problem are to be obtained for two different sets of dimensions. The first set (see Fig. 4A) simulates an infinite fluid medium surrounding the charge. The value of the peak pressure at the shock front as it propagates will be determined for this system by the numerical method. Values of the peak pressure at several positions of the shock front will be compared against experimentally measured peak pressures at corresponding positions. This comparison will provide a check on the method used in the numerical solution.

The second set of dimensions (see Fig. 4B) is representative of a typical sodium reactor installation. The effect that the destructive component produces on the concrete containment structure will be determined for this idealized reactor system. The strain rate for the steel strands will then be calculated for this system.

B. Equations of State

The solution of fluid dynamical problems requires the use of an equation

A. SIMULATED INFINITE WATER MEDIA

B. SIMULATED SODIUM REACTOR INSTALLATION

Fig. 4. Example Problem Layouts and Dimensions

of state for the fluid medium of the form $P = f(E, V)$. This problem has three different fluid media and therefore requires an equation of state for each.

1. Products of Detonation

The equation of state for the products of detonation of TNT is taken as that of Jones and Miller.¹⁰ This is a virial equation in powers of the pressure in the form

$$P \frac{v'}{N'} = RT + bP + cP^2 + dP^3, \quad (26)$$

where v' is the volume and N' the number of moles at temperature T and pressure P of the gaseous products of the detonation of one mole of explosive. The symbol R represents the gas constant. At the detonation wave front, for a density of the solid TNT equal to 1.5 gm/cm^3 , the pressure is $15.9 \times 10^{10} \text{ dyne/cm}^2$, the molar volume is 117 cm^3 , the density is 1.95 gm/cm^3 , and the temperature is 3400°K . The values of the virial coefficients are

$$b = 25.4,$$

$$c = -0.104,$$

and

$$d = 2.33 \times 10^{-4},$$

where 1000 kg/cm^2 is taken as the unit of pressure.

The internal energy E in kcal per mole of TNT is represented by the equation*

$$E = 0.0875T - 7.46 + N'E_1, \quad (27)$$

where N' is the total number of moles of the gaseous products (a number that varies from 5.8 at the detonation front to 7.0 when the composition of the gaseous products becomes fixed) per mole of TNT, and E_1 is the energy per mole of gaseous products and is given in kcal by

$$E_1 = -\frac{1}{2} cP^2 - \frac{2}{3} dP^3. \quad (28)$$

If we convert the units and note that v'/N' is specific volume, the resulting equations for the gaseous explosion products used in this treatise are

*This is the equation for internal energy per mole of TNT as given by Jones and Miller¹⁰ and modified by Brode.³

$$PV = RT + 0.6490P - (0.2712 \times 10^{-11}) P^2 + (0.6199 \times 10^{-23}) P^3, \quad (29)$$

and

$$E = (0.1614 \times 10^8) T - 0.1376 \times 10^{10} + (0.1356 \times 10^{-11}) P^2 - (0.4133 \times 10^{-23}) P^3, \quad (30)$$

where P , V , R , T , and E represent the pressure (dyne/cm²), the specific volume (cm³/gm), the gas constant (dyne-cm/gm-°K), temperature (°K), and the internal energy per unit mass of TNT (dyne-cm/gm), respectively. Equations 29-30 combine to give the required function $P = f(E, V)$ for the products of detonation.

2. Water

The equation of state for water is taken as that of Rice and Walsh.¹³ This equation is of the form

$$H(P, V) = H_h(P) + \xi(P)[V - V_h(P)], \quad (31)$$

where H_h and V_h are the specific enthalpy and specific volume for a state on the Hugoniot curve (pressure-volume curve centered at P_0, V_0) and are considered as functions only of pressure.

The values of H_h and V_h are obtained from the Rankine-Hugoniot equations,*

$$V_1/V_0 = (U_s - U_p)/U_s, \quad \text{Mass (32)}$$

$$P_1 - P_0 = U_s U_p / V_0, \quad \text{Momentum (33)}$$

and

$$H_1 - H_0 = \frac{1}{2}(P_1 - P_0)(V_0 + V_1), \quad \text{Enthalpy (34)}$$

and an analytical fit to the experimental Hugoniot curve data for water, initially at 20°C and 1 atm, given by

$$U_s - 1.483 = 25.306 \log_{10} (1 + U_p/5.190), \quad (35)$$

where U_s and U_p are the shock velocity and the particle velocity, respectively, in km/sec.

* These equations are presented here again for continuity and to indicate the form of Eq. 34. Equation 34 is identical to Eq. 6 by the thermodynamic relationship $H = E + PV$.

Equation 31 is based upon the assumption that the thermodynamic variable $(\partial H/\partial V)_P$ may be approximated as a function only of pressure for sufficiently high pressures; i.e.,

$$(\partial H/\partial V)_P \equiv C_P/(\partial V/\partial T)_P \equiv \xi(P),$$

where C_P is the specific heat at constant pressure. This assumption allows $\xi(P)$ to be obtained directly from experimental data¹⁸ for $(\Delta H/\Delta V)_P$. The analytical fit for $\xi(P)$ for the pressure range 25-250 kilobars (kb)* is of the form

$$\log_{10} \xi = 2.17943 + 0.0030338P \quad (36)$$

for ξ and P in kb.

The uncertainty in $(\Delta H/\Delta V)_P$ for a given pressure in the range 100-230 kb is estimated at $\pm 15\%$. However, Walsh and Rice¹⁸ state that this large uncertainty, a consequence of taking differences between nearly equal experimental quantities, does not lead to large errors in the calculated P , V curves since $(\Delta H/\Delta V)_P$ is used only to evaluate small offsets from the measured Hugoniot curve.

On the basis of the above statement, it is assumed that Eq. 31 is applicable for the entire pressure range 0-250 kb without appreciably changing the uncertainty associated with the function $\xi(P)$.

The internal energy of the water is determined from the thermodynamic relationship

$$E = H - PV, \quad (37)$$

where the internal energy and the enthalpy are in cal/gm, the pressure in kb, and the volume in cm^3/gm . Equations 31-37 combine to give the required function $P = f(E, V)$ for the water.

3. Argon

The equation of state of argon is taken as that of Beattie-Bridgman.⁸ This equation is of the form

$$P = \frac{RT \left(1 - \frac{c}{VT^3}\right)}{V^2} \left[V + B_0 \left(1 - \frac{b}{V}\right) \right] - \frac{A_0}{V^2} \left(1 - \frac{a}{V}\right), \quad (38)$$

where a , A_0 , b , B_0 , and c are arbitrary constants evaluated from experimental data. For argon, these constants are

*1 kb = 986.9 atm = 10^9 dynes/cm².

$$a = 0.02328,$$

$$A_0 = 1.2907,$$

$$b = 0,$$

$$B_0 = 0.03931,$$

and

$$c = 5.99 \times 10^{-4}$$

for the pressure in atmospheres, the specific volume in liters per gm-mole, and the temperature in °K. The gas constant $\mathcal{R}$ is 0.08206 atm-liters per gm/mole-°K.

The equation of state reduces to

$$PV^2 = \mathcal{R}T(V+B_0)\left(1 - \frac{c}{VT^3}\right) - A_0\left(1 - \frac{a}{V}\right), \quad (39)$$

and the values of the constants, in the cgs system, are

$$a = 0.58282,$$

$$A_0 = 8.3583 \times 10^5,$$

$$B_0 = 0.98413,$$

$$c = 0.01500,$$

and

$$\mathcal{R} = 2.078 \times 10^6,$$

i.e., for the pressure in dyne/cm², the specific volume in cm³/gm, the temperature in °K, and $\mathcal{R}$ in dyne-cm/gm-°K.

This equation of state is good to within 15% of experimental data⁹ for the pressure range 0-250 atm. An order-of-magnitude check of Eq. 39 reveals the term $c/VT^3 \ll 1$. Thus, the equation of state for argon used in this analysis is

$$PV^2 = \mathcal{R}T(V+B_0) - A_0\left(1 - \frac{a}{V}\right). \quad (40)$$

The relationship for the internal energy of the argon is found by thermodynamics and has the form

$$dE = C_v dt + \frac{A_0}{V}\left(1 - \frac{a}{V}\right) dV, \quad (41)$$

where C_v , the specific heat at constant volume, has the value 3.137×10^6 dyne-cm/gm-°K and is assumed to be constant since the specific heats of monatomic gases are practically independent of temperature.¹² Equations 40-41 combine to give the required function $P = f(E, V)$ for the argon.

C. Finite-difference Net and Equations

The parameters in the differential equations have reference to a particular particle of fluid. However, in this finite-difference approximation, the parameters have reference to a finite zone of fluid. The parameters for pressure, specific volume, specific enthalpy, specific internal energy, speed of sound, temperature, and the pseudoviscous pressure are presumed evaluated at the center of each zone; the value of each of these parameters is considered to be representative of all fluid particles in that zone. The parameters for the Lagrangean coordinate, the Eulerian coordinate, and the velocity are evaluated at the zone interfaces; the value of each of these parameters is considered to be representative of all fluid particles within one-half the adjacent zone thicknesses.

The finite-difference net used in this problem is shown in Fig. 5. Each zone represents a spherical shell of fluid. The first zone of fluid (in the TNT) is designated as zone one, and the center of spherical symmetry is designated as interface one.

Fig. 5. Finite-difference Net

The finite-difference equations (Eqs. 18-23) for this type of net designation become

$$\frac{u(j, t+1) - u(j, t)}{\Delta t} = -V_0 \left[\frac{P(j, t) - P(j-1, t) + Q(j, t) - Q(j-1, t)}{\Delta r(j)_u} \right] \left[\frac{R(j, t)}{r(j)} \right]^2, \quad (42)$$

$$\frac{R(j, t+1) - R(j, t)}{\Delta t} = u(j, t+1), \quad (43)$$

$$V(j, t+1) = V_0 \left[\frac{R(j+1, t+1)^3 - R(j, t+1)^3}{r(j+1)^3 - r(j)^3} \right], \quad (44)$$

$$Q(j, t+1) = \begin{cases} \frac{4[\Delta r(j)_Q]^2}{V_0^2[V(j, t+1) + V(j, t)]} \left[\frac{V(j, t+1) - V(j, t)}{\Delta t} \right]^2 \\ \quad \cdot \left[\frac{2r(j+1)}{R(j+1, t+1) + R(j+1, t)} \right]^4 & \text{if } [V(j, t+1) - V(j, t)] < 0 \\ 0 & \text{otherwise} \end{cases} \quad (45)$$

$$E(j, t+1) - E(j, t) + \left[\frac{P(j, t+1) + P(j, t)}{2} + Q(j, t+1) \right] [V(j, t+1) - V(j, t)] = 0, \quad (46)$$

and

$$P(j, t+1) = f[E(j, t+1), V(j, t+1)], \quad (47)$$

where $V_0 = 1/\rho_0$ and a^2 has been replaced by 2.0. The terms $\Delta r(j)_u$ and $\Delta r(j)_Q$ are defined for proper centering* as

$$\Delta r(j)_u = [r(j+1) + r(j-1)]/2$$

in Eq. 42 and

$$\Delta r(j)_Q = r(j+1) - r(j)$$

in Eq. 45.

*See Appendix G for a pictorial representation for the Δr 's.

At an interface between two different media, the following equation is used in place of Eq. 42:

$$\frac{u(j, t+1) - u(j, t)}{\Delta t} = -V_0 \left\{ \frac{3[P(j, t) - P(j-1, t) + Q(j, t) - Q(j-1, t)]}{\Delta r(j-1) + \Delta r(j+1)} - \frac{\frac{1}{3}[P(j+1, t) - P(j-2, t) + Q(j+1, t) - Q(j-2, t)]}{\Delta r(j-1) + \Delta r(j+1)} \right\} \left[\frac{R(j, t)}{r(j)} \right]^2, \quad (48)$$

where

$$\Delta r(j+1) = [r(j+2) + r(j)]/2$$

and

$$\Delta r(j-1) = [r(j) + r(j-2)]/2.$$

Equations 42-47 or 48 and 43-47, as applicable, are solved in the order presented, and the solution is propagated outward in time. These six equations are solved at each zone or zone interface, as applicable, for time $t+1$ from values known at time t . These newly calculated values are then reassigned as the values for time t , and the process is repeated.

Equations 46 and 47 constitute a system of two simultaneous equations involving two unknowns, $P(j, t+1)$ and $E(j, t+1)$, and are therefore solved by an iterative process until an acceptable solution is obtained. The initial value for $P(j, t+1)$ in this iterative process is either $P(j, t)$ or $[P(j, t) + P(j-1, t+1)]/2$. The former value was used for the solution in the water and the argon, the latter value for the solution in the TNT.

The equations of state for the various fluid media written in finite-difference form are as follows:

1. TNT

$$\begin{aligned} [P(j, t+1)][V(j, t+1)] &= \mathcal{R}[T(j, t+1)] + (0.6490)[P(j, t+1)] \\ &\quad - (0.2712 \times 10^{-11})[P(j, t+1)]^2 \\ &\quad + (0.6199 \times 10^{-23})[P(j, t+1)]^3, \end{aligned} \quad (49)$$

where $\mathcal{R}$ is the gas constant,* and the temperature is obtained from the equation

*See Appendix G for the calculation of the value for $\mathcal{R}$.

$$T(j, t+1) = \{E(j, t+1) + 0.1376 \times 10^{10} - (0.1356 \times 10^{-11})[P(j, t+1)]^2 + (0.4133 \times 10^{-23})[P(j, t+1)]^3\} / (0.1614 \times 10^8). \quad (50)$$

2. Water*

$$H(j, t+1) = H_h(j, t+1) + \xi(j, t+1)[V(j, t+1) - V_h(j, t+1)]23.8892, \quad (51)$$

where the following equations are necessary to obtain a solution:

$$E(j, t+1) = H(j, t+1) - [P(j, t+1)][V(j, t+1)](23.8892), \quad (52)$$

$$U_p = 4.53493 \left(1.0953^{U_s} - 1.1444 \right), \quad (53)$$

$$U_s \left(1.0953^{U_s} - 1.1444 \right) = 0.02209[P(j, t+1)], \quad (54)$$

$$V_h(j, t+1) = 1.0018(1 - U_p/U_s), \quad (55)$$

$$H_h(j, t+1) = 11.9446[1.0018 + V_h(j, t+1)[P(j, t+1)]], \quad (56)$$

and

$$\xi(j, t+1) = 151.16 \left\{ 10^{0.0030338[P(j, t+1)]} \right\}. \quad (57)$$

The units for Eqs. 51-57 are different than the cgs units used for the finite-difference equations (Eqs. 42-47) and therefore must be converted as necessary.

3. Argon

$$[P(j, t+1)][V(j, t+1)]^2 = \mathcal{R}[T(j, t+1)][V(j, t+1) + B_0] - A_0 \left[1 - \frac{a}{V(j, t+1)} \right], \quad (58)$$

where $\mathcal{R}$ is the gas constant, a , A_0 , and B_0 are the empirical constants, and the temperature is obtained from the equation

* The equations presented here are variations of Eqs. 31-37. See Appendix C for the appropriate transformations.

$$C_v[T(j, t+1) - T_0] = [E(j, t+1) - E_0] - \frac{A_0}{V(j, t+1)} \left[1 - \frac{a}{V(j, t+1)} \right] [V(j, t+1) - V_0] \quad (59)$$

D. Assumptions

The assumptions made concerning the solution of the problem are as follows:

1. The equations of state for the products of detonation and the argon are assumed to be valid for the entire range of pressures encountered in the analysis.
2. The detonation of the TNT is assumed to be initiated at the center of the spherical charge.
3. The equation of state for the water [i.e., $P = f(E, V)$], which is given by Eqs. 31-37, is assumed valid for the pressure range 0-250 kb. If in the iterative procedure required for the numerical solution the function $f(E, V)$ is less than zero, the equation of state for the water is taken as $P = 0$ on the following basis: Water can support a small negative pressure (i.e., tensile stress); if the magnitude of the negative pressure applied to the water exceeds the limiting value, separation of the water occurs. Since the magnitude of the limiting negative pressure for the water is small, it is taken as zero for this problem.
4. The concrete containment vessel is assumed to be infinitely rigid until after the major portion of the impulse has been applied. This assumption provides the basis for the second boundary condition necessary for the finite-difference solution.
5. Minute cracks are assumed to be present in the concrete vessel. The basis for this assumption is as follows: The operating temperature of the reactor is relatively high. Shrinkage of the concrete will occur, especially under the influence of this high operating temperature. The minute cracks are presumed to be due to the shrinkage of the concrete. As a result of this assumption, the energy required to cause the concrete to crack into chunks is considered to be negligible.

E. Boundary Conditions

The boundary conditions for this problem are as follows:

1. The particle velocity at the center of the sphere must be zero; i.e., $u(1, t) = 0$ for all t .
2. The particle velocity at the concrete boundary must be zero; i.e., $u(JMAX, t) = 0$ for all t .

F. Initial Conditions

The numerical solution of the differential equations requires that initial values for all the parameters of the corresponding finite-difference equations be known at time t_0 . For this problem, the initial values are prescribed at time $t_0 = 0$, which corresponds to the instant the detonation wave reaches the surface of the spherical charge of TNT. The parameters for which the initial values must be known are Δt , R , u , V , Q , E , P , and r , which represent the time increment, the Eulerian coordinate, the particle velocity, the specific volume, the pseudoviscous pressure, the specific internal energy, the pressure, and the Lagrangean coordinate, respectively. The initial values of these parameters required for the numerical solution of this problem are given in Table II and were obtained as follows:

1. Δt (designated as DELT in Table II).

The allowable value of Δt (DTTEST, Table II) was determined for each zone in the finite-difference net from the stability equation (Eq. 24).* The minimum allowable Δt was used as a basis of establishing the initial value of Δt .

2. R (designated as RR in Table II).

The initial values of the Eulerian coordinate are taken as the initial radii of the zone interfaces defined by the finite-difference net.

3. u , V , E , P (same designation in Table II).

The initial values of these parameters for the products of detonation were obtained from the conditions in the detonation wave prescribed by Taylor as shown in Table XI in Appendix A. The computer was used to calculate the values for E in the detonation products from Eqs. 49 and 50.

The initial values of these parameters for the water and the argon are taken as follows:

<u>Water</u>	<u>Argon</u>
$u_0 = 0$	$u_0 = 0$
$P_0 = 1 \text{ atm} = 1.013274 \times 10^6 \text{ dynes/cm}^2$	$P_0 = 1 \text{ atm} = 1.013274 \times 10^6 \text{ dynes/cm}^2$
$V_0 = 1.0018 \text{ cm}^3/\text{gm}$	$V_0 = 591.5974 \text{ cm}^3/\text{gm}$

*See Appendix G for these calculations.

TABLE II. Initial Values of the Parameters Required for the Finite-difference Solution

	T SEC	DELT SEC	ETUT DYNE-CM	EK DYNE-CM	EINT DYNE-CM	EINT DYNE-CM		
ZONE	0.0000000	0.0000005	5.2770278+015	3.6296949+014	4.9140583+015	5.2797292+015		
J	RR CM	U CM/SEC	V CUB CM/GM	Q DYNES/SQ CM	E DYNE-CM/GM	P DYNES/SQ CM	R CM	DTTEST SEC
1	0.000000+000	0.000000+000	8.110000-001	0.000000+000	1.402390+010	2.190000+010	0.000000+000	5.000000-007
2	3.000000+000	0.000000+000	8.110000-001	0.000000+000	1.402390+010	2.190000+010	3.218704+000	5.000000-007
3	6.000000+000	0.000000+000	8.110000-001	0.000000+000	1.402390+010	2.190000+010	6.437007+000	5.000000-007
4	9.000000+000	0.000000+000	8.110000-001	0.000000+000	1.402390+010	2.190000+010	9.656711+000	5.000000-007
5	1.200000+001	0.000000+000	8.045000-001	0.000000+000	1.482338+010	2.330000+010	1.287561+001	5.000000-007
6	1.500000+001	5.000000+003	7.813000-001	0.000000+000	1.552524+010	2.610000+010	1.611564+001	5.000000-007
7	1.800000+001	1.400000+004	7.519000-001	0.000000+000	1.707184+010	3.090000+010	1.943050+001	5.000000-007
8	2.100000+001	2.520000+004	7.158000-001	0.000000+000	2.098340+010	4.000000+010	2.283341+001	5.000000-007
9	2.400000+001	4.160000+004	6.702000-001	0.000000+000	2.599460+010	5.000000+010	2.634774+001	5.000000-007
10	2.700000+001	6.840000+004	6.270000-001	0.000000+000	3.552874+010	7.200000+010	3.001799+001	5.000000-007
11	2.800000+001	8.280000+004	5.935000-001	0.000000+000	4.714071+010	9.200000+010	3.130528+001	5.000000-007
12	2.900000+001	1.044000+005	5.405000-001	0.000000+000	6.287545+010	1.245000+011	3.264432+001	5.000000-007
13	3.000000+001	1.546500+005	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	3.409285+001	1.000000-005
14	4.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	4.247438+001	1.000000-005
15	5.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	5.162988+001	1.000000-005
16	6.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	6.114709+001	1.000000-005
17	7.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	7.084864+001	1.000000-005
18	8.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	8.065232+001	1.000000-005
19	9.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	9.051065+001	1.000000-005
20	1.000000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.004191+002	1.000000-005
21	1.100000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.103468+002	1.000000-005
22	1.200000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.202916+002	1.000000-005
23	1.300000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.302486+002	1.000000-005
24	1.400000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.402144+002	1.000000-005
25	1.500000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.501068+002	1.000000-005
26	1.600000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.601042+002	1.000000-005
27	1.700000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006	1.701455+002	1.000000-005
28	1.800000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+006	1.013274+006	1.801298+002	1.000000-004
29	1.950000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+006	1.013274+006	1.801573+002	1.000000-004
30	2.100000+002	0.000000+000	0.000000+000	0.000000+000	0.000000+000	0.000000+000	1.801894+002	0.000000+000

<u>Water</u>	<u>Argon</u>
$T_0 = 293^\circ\text{K}$	$T_0 = 288^\circ\text{K}$
$c_0 = 145,000 \text{ cm/sec}$	$c_0 = 33,100 \text{ cm/sec}$
$E_0 = -1.015020 \times 10^6 \text{ dyne-cm/gm}$	$E_0 = 9.034760 \times 10^8 \text{ dyne-cm/gm}$
$H_0 = 0$	$C_V = 3.137 \times 10^6 \text{ dyne-cm/gm-}^\circ\text{K}$

where c_0 is the speed of sound in the medium under these conditions, and E_0 has been calculated from the appropriate relationship.

4. Q (same designation in Table II).

The initial values for the pseudoviscous pressure are taken as zero on the following basis: As the detonation wave propagates through the explosive, it leaves the gaseous products immediately behind the front in a highly compressed state, i.e., a low specific volume. As the front continues outward, the gas away from the front expands rapidly. Because of the ever-increasing radius of the shock front, the specific volume of the gaseous products away from the front at time $t + 1$ is greater than at time t . Therefore, the definition for Q applies everywhere in the gaseous products except at the detonation front. Since the detonation process causes energy to be released in the wave front, it is ambiguous to assume dissipation occurs there. Thus, the initial value of Q everywhere in the gaseous products is taken as zero on the basis of the definition.

The definition of Q also applies to the water and the argon, which, at time $t = 0$, have not been effected by the detonation process.

5. r (designated as R in Table II)

The values of the Lagrangean coordinate represent the locations of the zone interfaces of the finite-difference net designation in the standard reference configuration. The common specific volume used for obtaining the Lagrangean coordinates is taken as that for the water given on page 36, i.e., $V_0 = 1.0018 \text{ cm}^3/\text{gm}$. The computer was used to calculate the values of the Lagrangean coordinates.*

The values given in Table II for ETOT, EK, EINT, and ETNT represent the total energy of the system, the total kinetic energy of the system, the total internal energy of the system, and the total energy of the gaseous explosion products, respectively, at time $t = 0$. These values were calculated by the use of the computer and are given for reference.

*See Appendix C for the derivation of the formula.

G. Flow Charts for Calculations in the Water

The simplified and more detailed flow charts for calculations in the water are shown in Figs. 6 and 7, respectively.

Fig. 6. Simplified Flow Chart for Calculations in the Water

Fig. 7. More Detailed Flow Chart for Calculations in the Water

H. FORTTRAN Names and Their Description

<u>FORTTRAN Name</u>	<u>Description</u>
1. AIPCC	Allowable internal pressure at the inside face of the concrete, kb
2. DELRQ(J)	The value of $\Delta r(j)$ used in the formula to find Q
3. DELRU(J)	The value of $\Delta r(j)$ used in the formula to find u
4. DELT	Time increment, sec
5. DTTEST(J)	The allowable value of Δt resulting from the stability conditions for zone J
6. EINT	Internal energy, dyne-cm
7. E(J, 1)	Specific internal energy of the fluid in zone J at time t, dyne-cm/gm
8. EK	Kinetic energy, dyne-cm
9. EMASS	Mass of the fluid in zone J, gm
10. ENEW	Test value for the accuracy check
11. ENTH	Specific enthalpy of the water, cal/gm
12. ENTHH	Specific enthalpy of the water for the Hugoniot curve, cal/gm
13. EP	Accuracy check value (relative quantity)
14. EPS, EPS1	Accuracy check values, dyne-cm/gm
15. ETNT	Total energy of the TNT products, dyne-cm
16. ETOT	Total energy of the system, dyne-cm
17. IMAX	Maximum number of records written on magnetic tape and maximum number of lines of printout in the final table
18. J	Index for zones and zone interfaces
19. JA	Index of the TNT-water interface
20. JMAX	Index of the argon-concrete interface
21. JW	Index of the water-argon interface
22. KK	Index indicating the zone of maximum pressure outside of the gas bubble

<u>FORTTRAN Name</u>	<u>Description</u>
23. NPLIM	Program control value
24. P(J, 1)	Pressure at the center of zone J at time t, dyne/cm ²
25. PTEST	Auxiliary name used to determine the maximum pressure in the system outside the gas bubble
26. Q(J, 1)	Pseudoviscous pressure term in zone J at time t, dynes/cm ²
27. R(J)	The Lagrangean coordinate of the interface at index J
28. RR(J, 1)	Eulerian coordinate for interface J at time t, cm
29. SOS(J)	Speed of sound in the fluid media in zone J
30. T	Time, sec
31. TEM(J)	Temperature in zone J, °K
32. TEMPE	Temporary value of the specific internal energy used in calculations
33. TEMPP	Temporary value of the pressure used in calculations
34. TMAX	Upper limit on time, sec
35. T1(I)	Time at index I
36. VEL	Average velocity of the fluid particles in zone J
37. VH	Specific volume of the water for the Hugoniot curve, cm ³ /gm
38. V(J, 1)	Specific volume of the fluid in zone J at time t, cm ³ /gm
39. VMAX(J)	Maximum value of the specific volume in zone J that results in a positive pressure in that zone
40. VOL(J), VOL1(J), VOL2(J)	Auxiliary values at index J
41. XI	FORTTRAN name for ξ , kb

<u>FORTRAN Name</u>	<u>Description</u>
42. DELTAT, ARGUE	Auxiliary names used to obtain the maximum allowable Δt
43. K, JAM, JWM, JMAXM, JCUR	Control values of the index
44. LAST, IS3, IFSW, IS2	Program control values
45. N, I, KL, KLL	Indices used as counters
46. T1(I), P1(I), P2(I), P3(I), P4(I), P5(I), P6(I), RR7(I), RR8(I), PMAX(I), KKP(I), ETOTAL	Auxiliary names used to obtain the values of the parameters at critical locations
47. U(J, 2), RR(J, 2), V(J, 2), Q(J, 2), P(J, 2), E(J, 2)	Same description as for U(J, 1),* etc., except applicable for time $t + \Delta t$
48. AAR3, ARG, AR1, AR2, AR3, AR4, AR44, B, CC1, CC2, CC3, DISC, DSQ, EE, E1, PCP, PP, RAD, RAD1, RAD2, RAD3, R1, R2, R3, TEMP, TEMP1, TEMP2, TEMPT, VSQ, VTEST	Auxiliary names

I. List of Data Values Used

	<u>FORTRAN Name</u>	<u>Value (Fig. 4A)</u>	<u>Value (Fig. 4B)</u>
1.	AIPCC	0.0992 kb	0.0992 kb
2.	DELT	0.0000005 sec	0.0000005 sec
3.	EP	0.00005	0.00005
4.	EPS	100,000.0 dyne-cm/gm	100,000.0 dyne-cm/gm
5.	JA	13	13
6.	JW	88	28
7.	JMAX	90	30
8.	TMAX	0.004000 sec	0.022000 sec

*U(J, 1) = velocity in zone J at time 1.

J. FORTTRAN IV Computer Statements

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C      H.C.SORENSEN      EXPLOSION ANALYSIS
C      NUMERICAL ANALYSIS OF A TNT BLAST WAVE IN A LIMITED NON-HOMOGENEOUS
C      FLUID MEDIUM. THE ANALYSIS IS BASED UPON THE VON NEUMANN-RICHTMEYER
C      PSEUDO-VISCOUS METHOD.
C      THIS COMPUTER SOLUTION IS PERFORMED ON THE CDC 3600 COMPUTER.
C      THIS PROGRAM USES FOUR SUBROUTINES WHICH ARE CALLED ROOT3, FCT,
C      ROOT33, AND FCTN.
      COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
      DIMENSION P(50,2),V(50,2),E(50,2),Q(50,2),U(50,2),RR(50,2),TEM(50)
1,R(50),DELRU(50),DELRQ(50),SOS(50),DTTEST(50),VOL(50),VOL1(50),
2VOL2(50),VMAX(50)
      COMMON/A/P1(4000),P2(4000),P3(4000),P4(4000),P5(4000),KKP(4000)
      COMMON/9/P6(4000),RR7(4000),RR8(4000),T1(4000),PMA(4000)
      PRINT 3000
C      READ AND PRINT INPUT DATA.
      READ 1200,JA,JW,JMAX,TMAX,EPS,EP,DELT,AIPCC
      PRINT 1201,JA,JW,JMAX,TMAX,EPS,EP,DELT,AIPCC
      READ 1100,(RR(J,1),U(J,1),P(J,1),V(J,1),E(J,1),Q(J,1),J=1,JMAX)
      PRINT 1101
      PRINT 1102
      PRINT 1103,(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),J=1,JMAX)
      JAM = JA-1
      JWM = JW-1
      JMAXM = JMAX - 1
      R(1) = 0.
      T = 0.
      I = 1
      T1(I) = T
      KK = JA
      KL = 1
      KLL = 1
      TEM(JMAX) = 0.
      DO 130 J=1,JMAXM
130  VMAX(J) = 1000.0
C      CALCULATE THE TEMPERATURE AND THE INTERNAL ENERGY FOR EACH
C      ZONE IN THE TNT PRODUCTS AT TIME T = 0. THE TEMPERATURE, FOR A
C      GIVEN PRESSURE AND VOLUME, IS OBTAINED FROM THE EQUATION OF
C      STATE. THE INTERNAL ENERGY IS A FUNCTION OF THE TEMPERATURE
C      AND PRESSURE.
      DO 150 J=1,JAM
      TEM(J)=(P(J,1)*(V(J,1)-.648986-P(J,1)*(-.271149E+11+.619876E-23*P
1(J,1))))/.521766E7
150  E(J,1)=.161355E8*TEM(J)-.137566E10+P(J,1)**2*(.135575E-11-.413251
1E-23*P(J,1))
      DO 151 J=JA,JWM
151  TEM(J)= 293.
      DO 152 J=JW,JMAXM
152  TEM(J)= 288.
      DO 165 J=1,JMAXM
165  VOL(J) =(RR(J+1,1)-RR(J,1))*(RR(J+1,1)**2+RR(J+1,1)*RR(J,1)+
1RR(J,1)**2)
C      CALCULATE THE TOTAL KINETIC ENERGY OF THE SYSTEM AT TIME T = 0.
C      THIS KINETIC ENERGY IS DUE ONLY TO THE MOTION OF THE EXPLOSION
C      PRODUCTS AS THE OTHER FLUIDS IN THE SYSTEM ARE AT REST.
      EK = 0.
      DO 160 J=1,JAM
      VEL =(U(J+1,1)+U(J,1))**2
160  EK =.5235988*VOL(J)*VEL/V(J,1) + EK
C      CALCULATE THE RADIUS OF EACH ZONE INTERFACE FOR THE STANDARD
C      REFERENCE CONFIGURATION BASED UPON THE DENSITY OF THE WATER
C      UTILIZING THE CONSERVATION OF MASS.

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DO 161 J=2,JMAX
VOL1(J-1)=(RR(J,1)-RR(J-1,1))*(RR(J,1)**2+RR(J,1)*RR(J-1,1)+
1RR(J-1,1)**2)
161 R(J) = (R(J-1)**3+1.0018/V(J-1,1)*VOL1(J-1))**.3333333333
EINT = 0.
C CALCULATE THE INTERNAL ENERGY IN THE TNT PRODUCTS AT TIME T = 0.
DO 162 J=1,JAM
EMASS = 4.1887902*VOL(J)/V(J,1)
162 EINT = EINT + E(J,1)*EMASS
C CALCULATE THE TOTAL ENERGY IN THE TNT PRODUCTS AT TIME T = 0.
ETNT = EINT + EK
C CALCULATE THE TOTAL INTERNAL ENERGY OF THE SYSTEM AT TIME T = 0.
DO 169 J=JA,JMAXM
EMASS = 4.1887902*VOL(J)/V(J,1)
169 EINT = EINT + E(J,1)*EMASS
C CALCULATE THE TOTAL ENERGY IN THE SYSTEM AT TIME T = 0.
ETOT = EINT + EK
DO 168 J=1,JMAXM
168 DELRQ(J) = R(J+1) - R(J)
DO 163 J= 2,JMAXM
163 DELRU(J) = (R(J+1) - R(J-1))/2.
DO 164 J=1,JAM
SOS(J) = 0.
164 DTTEST(J) = DELT
DO 166 J=JA,JWM
SOS(J) = 145000.
166 DTTEST(J) = .000010
DO 167 J=JW,JMAXM
SOS(J) = 33100.
167 DTTEST(J) = .000100
U(1,2) = 0.
U(JMAX,2) = 0.
LAST = JAM
50 PRINT 3000
PRINT 2003
PRINT 2004
PRINT 2005,T,DELT,ETOT,EK,EINT,ETNT
PRINT 2100
PRINT 2101
PRINT 2102,(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),R(J),
1DTTEST(J),J=1,JMAX)
GO TO 51
C CALCULATE THE VELOCITY OF EACH ZONE INTERFACE AT TIME T FROM
C THE FIRST DIFFERENCE EQUATION.
110 DO 200 J=2,JMAXM
IF (J.EQ.JA) GO TO 213
IF (J.EQ.JW) GO TO 213
U(J,2)=U(J,1)-(1.0018*(P(J,1)-P(J-1,1)+Q(J,1)-Q(J-1,1))/DELRU
1(J)*(RR(J,1)/R(J))**2)*DELT
GO TO 200
213 U(J,2)=U(J,1)-DELT*1.0018*((RR(J,1)/R(J))**2)*(3.*(P(J,1)-P(J-1,1)
1+Q(J,1)-Q(J-1,1))-(P(J+1,1)-P(J-2,1)+Q(J+1,1)-Q(J-2,1))/3.)/
2(DELRU(J-1)+DELRU(J+1))
200 CONTINUE
C CALCULATE THE RADIUS OF EACH ZONE INTERFACE AT
C TIME T FROM THE SECOND DIFFERENCE EQUATION.
214 DO 300 J=1,JMAX
300 RR(J,2) = RR(J,1)+U(J,2) *DELT
IF (RR(JMAXM,2).GE.RR(JMAX,2)) RR(JMAXM,2)=RR(JMAXM,1)
IF (RR(JW,2).GE.RR(JMAXM,2)) GO TO 100
210 DO 202 J=1,JMAXM
VOL2(J) = (R(J+1)-R(J))*(R(J+1)**2+R(J+1)*R(J)+R(J)**2)

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202 VOL(J) =(RR(J+1,2)-RR(J,2))*(RR(J+1,2)**2+RR(J+1,2)*RR(J,2)+
1RR(J,2)**2)
C CALCULATE THE SPECIFIC VOLUME FOR EACH ZONE AT TIME T FROM THE
C THIRD DIFFERENCE EQUATION, AND CALCULATE THE PSEUDO-VISCOUS
C PRESSURE TERM FOR EACH ZONE AT TIME T FROM THE FOURTH
C DIFFERENCE EQUATION.
DO 400 J=1,JMAXM
V(J,2) =1.0018*VOL(J)/VOL2(J)
VTEST=V(J,2)-V(J,1)
IF (VTEST) 402,401,401
402 CC1 =3.98563879*(DFLRQ(J))**2/(V(J,2)+V(J,1))
CC2 =(VTEST/DELT)**2
CC3 =(2.*R(J+1)/(RR(J+1,2)+RR(J+1,1)))*4
Q(J,2) =CC1*CC2*CC3
GO TO 400
401 Q(J,2)=0.
400 CONTINUE
C CALCULATE THE TOTAL KINETIC ENERGY OF THE SYSTEM AT TIME T,
EK =0.
DO 201 J =1,JMAXM
VEL =(U(J+1,2) + U(J,2))**2
201 EK =.5235988*VOL(J)*VEL/V(J,2) + EK
ASSIGN 66 TO IS3
IFSW =1
IS2 =1
TEMPP = P(1,1)
C CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN
C THE TNT PRODUCTS AT TIME T BY ITERATION OF THE FIFTH AND SIXTH
C DIFFERENCE EQUATIONS BY THE FOLLOWING PROCEDURE. GUESS P AT
C TIME T. SOLVE THE FIFTH DIFFERENCE EQUATION FOR E AT TIME T.
C THEN USE THIS VALUE OF E TO OBTAIN A CORRECTED VALUE FOR P AT
C TIME T. REPEAT THIS PROCESS UNTIL THE DIFFERENCE BETWEEN TWO
C SUCCESSIVE VALUES FOR P AT TIME T IS LESS THAN A VALUE EPSILON.
DO 699 J=1,JAM
IF (IS2-1)404,403,404
404 TEMPP =(P(J,1)+P(J-1,2))/2.
403 ENEW =0.
AR3 =.648986-V(J,2)
AAR3 =-AR3
AR1 =.619876E-23/AAR3
AR2 =-.271149E-11/AAR3
60 TEMPE = E(J,1)-((TEMPP+P(J,1))/2.+Q(J,2))*(V(J,2)-V(J,1))
GO TO IS3
66 IF(ABS(TEMPE-ENEW)-EPS)68,68,65
65 E1 =TEMPP**2*(.135575E-11-.413251E-23*TEMPP)
AR44 =(TEMPE + .137566E10 - E1)*.32336525
AR4 =AR44/AAR3
N = 25
CALL ROOT3 (TEMPP,EP,G,C,N,M)
R1 = G
C THE EQUATION OF STATE FOR THE TNT PRODUCTS, THE SIXTH DIFF-
CERENCE EQUATION, IS A THIRD ORDER EQUATION IN P. THEREFORE,
C THERE ARE THREE ROOTS TO THIS EQUATION. THE ROOT FINDER SUB-
CROUTINE, CALLED ROOT3, HAS JUST FOUND ONE ROOT TO THIS
C EQUATION. NOW SOLVE FOR THE OTHER TWO ROOTS FROM THE RESULTING
C QUADRATIC EQUATION AND CHOOSE WHICHEVER OF THE THREE ROOTS IS
C CLOSEST TO THE VALUE OF P AT TIME T =T-DELT FOR CONTINUING THE
C ITERATIVE PROCESS.
B =.619876E-23*R1 - .271149E-11
DISC =B**2-2.479504E-23*(AR3+R1 *(.619876E-23*R1-.271149E-11))
IF(DISC)660,670,680
660 TEMPP = R1

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GO TO 190
670 R2 = -B/1.239752E-23
    R3 = R2
    GO TO 96
680 DSQ = SQRTF(DISC)
    R2 = (-B+DSQ)/1.239752E-23
    R3 = (-B-DSQ)/1.239752E-23
96  RAD1 = ABSF(P(J,1)-R1)
    RAD2 = ABSF(P(J,1)-R2)
    RAD3 = ABSF(P(J,1)-R3)
    IF(RAD1-RAD2)91,92,92
91  IF(RAD3-RAD1)93,94,94
93  TEMPP = R3
    GO TO 190
92  IF(RAD3-RAD2)93,95,95
95  TEMPP = R2
    GO TO 190
94  TEMPP = R1
190 ENEW = TEMPE
    GO TO 60
68  P(J,2)=TEMPP
    E(J,2)=TEMPE
    TEM(J) = AR44/.521766E7
    IS2 = 2
C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
    PCP = P(J,2)*1.E-10
    SOS(J) = PCP*(49547.-21.08.*PCP) + 183556.
    DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
699 CONTINUE
    ASSIGN 76 TO IS3
    IFSW = 2
C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C      WATER AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C      EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
    DO 71 J= JA,JWM
    IF (U(J,2))74,82,74
74  TEMPP = P(J,1)
    ENEW = 0.
    GO TO 60
76  IF (TEMPE.GT.1000000.0) GO TO 766
    EPS1 = -EPS/10.
    IF (ABSF(TEMPE-ENEW)-EPS1)78,78,75
766 IF (ABSF(TEMPE-ENEW)-EPS1)78,78,75
75  IF (V(J,2)-VMAX(J)) 756,756,755
756 PP = TEMPP*1.0E-9
    EE = TEMPE*.238892E-7
    N = 25
    CALL ROOT3 (PP,EP,G,C,N,M)
    TEMPP = G*1.0E9
    IF (V(J,2).LE.1.0018) GO TO 759
    IF (TEMPP.LT.0.0) TEMPP=0.0
759 ENEW = TEMPE
    GO TO 60
755 TEMPP = 0.0
    ENEW = TEMPE
    GO TO 60
78  IF(J-LAST)79,79,788
788 IF(TEMPP-1.013274E6)77,77,799
77  TEMPP = 1.013274E6
    TEMPE = -1.015020E6
    ENEW = 0.

```

```

GO TO 79
799 IF (LAST-J) 777,777,79
777 LAST = J
79 IF (VMAX(J).NE.1000.0) GO TO 757
IF (TEMPP.EQ.0.0) VMAX(J) = V(J,2)
757 E(J,2) = TEMPE
P(J,2) = TEMPP
C CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
PCP = P(J,2)*1.E-9
SOS(J) = PCP*(3853.-13.*PCP) + 256906.
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
71 CONTINUE
ASSIGN 86 TO IS3
C CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C ARGON AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
887 DO 80 J=JW,JMAXM
87 IF (LAST - JWM) 82,89,82
89 ENEW = 0.
VSQ = V(J,2)**2
TEMP = 835833.*(1.-.58282/V(J,2))/VSQ
TEMP1 = -.318769E-6*TEMP*(V(J,2)-591.597)
TEMP2 = 2078910.*(V(J,2) + .98413)/VSQ
TEMPP = P(J,1)
GO TO 60
86 IF (ARSF(TEMPE-ENEW)-EPS)88,88,85
85 TEMPT = TEMPE*.318769E-6+TEMP1
TEMPP = TEMPT*TEMP2-TEMP
ENEW = TEMPE
GO TO 60
88 E(J,2) = TEMPE
P(J,2)=TEMPP
TEM(J)=TEMPT
C CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
RAD = 1.66*P(J,2)*V(J,2)
SOS(J) = SQRTF(RAD)
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
80 CONTINUE
GO TO 81
82 JCUR = J
DO 72 J = JCUR,JMAXM
E(J,2) = E(J,1)
72 P(J,2) = P(J,1)
C CALCULATE THE TOTAL INTERNAL ENERGY IN THE SYSTEM AT TIME T.
81 EINT = 0.
DO 500 J=1,JMAXM
EMASS = 4.1887902*VOL(J)/V(J,2)
500 EINT = EINT + E(J,2)*EMASS
C CALCULATE THE TOTAL ENERGY IN THE SYSTEM AT TIME T.
ETOT = EINT + EK
C REASSIGN VALUES.
DO 600 J=1,JMAX
P(J,1)= P(J,2)
RR(J,1)=RR(J,2)
U(J,1)= U(J,2)
Q(J,1)= Q(J,2)
V(J,1)= V(J,2)
600 E(J,1)= E(J,2)
PTEST =P(JA,1)
K =JA + 1

```



```

DO 444 J =K,JMAXM
ARG =P(J,1)
IF (PTEST - ARG) 445,445,444
445 PTEST = ARG
KK = J
444 CONTINUE
IF (T - 0.0000005) 55,54,55
54 PRINT 3000
GO TO 56
55 PRINT 5000
56 PRINT 2000
PRINT 2001
PRINT 2002,T,DELT,ETOT,EK,EINT
PRINT 2100
PRINT 2101
447 PRINT 2102,(J,RR(J,1),U(J,1),V(J,1),O(J,1),E(J,1),P(J,1),R(J),
1DTTEST(J),J=1,JMAX)
C      STORE IN MEMORY THE VALUES OF P AND RR AT THE CRITICAL
C      LOCATIONS.
51 P1(I) = P(1,1)*1.E-9
P2(I) = P(JAM,1)*1.E-9
P3(I) = P(JA,1)*1.E-9
P4(I) = P(JWM,1)*1.E-9
P5(I) = P(JW,1)*1.E-9
P6(I) = P(JMAXM,1)*1.E-9
RR7(I) = RR(JA,1)
RR8(I) = RR(JW,1)
PMAx(I) = P(KK,1)*1.E-9
KKP(I) = KK
ETOTAL = ETOT*.2388915E-13
WRITE(40),T1(I),P1(I),P2(I),P3(I),RR7(I),P4(I),P5(I),RR8(I),P6(I),
1PMAx(I),ETOTAL
C      INCREMENT DELTA T TO THE MAXIMUM VALUE ALLOWED BY THE STABILITY
C      CONDITION FOR THE DIFFERENCE EQUATIONS.
112 DELTAT = DTTEST(1)
DO 44 J=2,JMAXM
ARGUE =DTTEST(J)
IF (DELTAT-ARGUE)44,44,26
26 DELTAT =ARGUE
44 CONTINUE
IF (DELTAT.GE.0.000003) DELT=0.000003
IF (DELTAT.LT.0.000003) DELT=0.000002
IF (DELTAT.LT.0.000002) DELT=0.000001
IF (DELTAT.LT.0.000001) DELT=0.0000005
IF (DELTAT.LT.0.0000005) DELT=0.0000003
IF (DELTAT.LT.0.0000003) DELT=0.0000001
C      INCREMENT T.
T = T + DELT
I = I+1
IF (I.GE.3995) GO TO 100
T1(I) = T
IF(T-TMAX)110,110,100
1100 FORMAT (6E12.6)
1101 FORMAT (///4H J,7X,2HRR,13X,1HU,13X,1HV,13X,1HQ,13X,1HE,13X,1HP)
1102 FORMAT (11X,2HCM,11X,6HCM/SEC,5X,9HCUB CM/GM,4X,11HDYNES/SQ CM,
14X,10HDYNE-CM/GM,4X,11HDYNES/SQ CM/)
1103 FORMAT (14,6E14.6)
1200 FORMAT (3I6,4E14.7,F6.4)
1201 FORMAT (11H INPUT DATA/6H JA,4X,2HJW,3X,4HJMAX,2X,9HTMAX(SEC),
14X,3HEPS,9X,2HEP,5X,9HDELT(SEC),3X,12HA LPCONC(KB)//3I6,F12.7,
2F10.1,F10.5,F12.7,F12.4)
2000 FORMAT (6X,1HT,8X,4HDELT,12X,4HETOT,15X,2HEK,15X,4HEINT)

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2001  FORMAT(5X,3HSEC,8X,3HSFC,11X,7HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM
1/)
2002  FORMAT (2F11.7,3E18.7)
2003  FORMAT(6X,1HT,8X,4HDELT,12X,4HETOT,15X,2HEK,15X,4HEINT,15X,4HETNT)
2004  FORMAT(5X,3HSEC,8X,3HSFC,11X,7HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM
1,11X,7HDYNE-CM/)
2005  FORMAT (2F11.7,4E18.7)
2100  FORMAT (//4H J,7X,2HRR,13X,1HU,13X,1HV,13X,1HQ,13X,1HE,13X,1HP,
113X,1HR,10X,6HDTTEST)
2101  FORMAT (11X,2HCM,11X,6HCM/SEC,5X,9HCUB CM/GM,4X,11HDYNES/SQ CM,
14X,10HDYNE-CM/GM,4X,11HDYNES/SQ CM,7X,2HCM,12X,3HSEC/)
2102  FORMAT (I4,8E14.6)
3000  FORMAT (1H1)
3001  FORMAT (1H1,////////)
5000  FORMAT (///)
5001  FORMAT (/11X,47HNUMBER OF LINES OF PRINT OUT IN THE FINAL TABLE,
13X,3H1 =,I4/11X,33HNUMBER OF RECORDS WRITTEN ON TAPE,3X,3HN =,I4)
8000  FORMAT (/11X,12HPROGRAM OVER)
9695  FORMAT (14X,69HTABULATED VALUES OF THE PRESSURE AND THE RADIAL COO
1RDINATE AT SEVERAL/11X,28HCRITICAL LOCATIONS (PART I).//11X,24HUNI
2TS - TIME, T, SECONDS/19X,21HPRESSURE, P, KILOBARS/19X,38HRADIAL C
3COORDINATE, RR, CENTIMETERS ,24X,14HINSIDE MAX/80X,7HFACE OF
4,3X,8HPRESSURE/80X,8HCONCRETE,1X,10HOUTSIDE OF/34X,19HTNT-WATER IN
5TERFACE,4X,21HWATER-ARGON INTERFACE,3X,6HVESSEL,2X,10HGAS BUBBLE/1
65X,1HT,9X,1HP,8X,1HP,8X,1HP,7X,2HRR,6X,1HP,6X,1HP,7X,2HRR,7X,1HP,
77X,1HP,3X,4HZONE/50X,4HTNT-,18X,6HWATER-/23X,6HCENTER,4X,3HTNT,5X,
85HWATER,3X,5HWATER,3X,5HWATER,2X,5HARGON,3X,5HARGON,4X,5HARGON,4X,
93HMAX,4X,1HJ/)
9696  FORMAT (F20.7,F9.3,F9.3,F8.3,F8.3,F8.3,F7.3,F9.3,F8.3,F8.3,I4)
100   IMAX = I-1
      T = T - DELT
53    NPLIM = 23
      PRINT 3001
      PRINT 9695
      DO 58 I = 1,IMAX
      IF (I - NPLIM) 58,58,59
59    NPLIM = NPLIM + 36
      PRINT 3001
58    PRINT 9696,T1(I),P1(I),P2(I),P3(I),RR7(I),P4(I),P5(I),RR8(I),
1P6(I),PMAX(I),KKP(I)
      PRINT 5001, IMAX,IMAX
      PRINT 8000
      END

```

```

SUBROUTINE ROOT3 (A, EPS, G, C, N, M)
COMMON AR1,AR2,AR3,AR4,IPSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION V(50,2)
C      THIS SUBROUTINE IS A FORTRAN TRANSCRIPTION OF ACM ALGORITHM NO.
C      26(COMM. ACM 4(1960)603). IT COMPUTES A VALUE OF G = Y SATISFYING
C      THE EQUATION Y = FCT(Y). THE ITERATION WILL CONVERGE TO Y PRO-
C      VIDING THAT AT SOME TIME IN THE ITERATION A G IS REACHED SUCH THAT
C      ABS(G-Y)*ABS(D(DFCT/DY)/DY) IS LESS THAN 2*ABS((DF/DY)-1.), WHERE
C      THE DERIVATIVES ARE EVALUATED AT Y.
C      INPUT INCLUDES: A SUBROUTINE, FCT(X,Y), WHICH PLACES THE
C      VALUE OF FCT(X) IN Y; AN INITIAL APPROXIMATION TO THE ROOT, A;
C      A TOLERANCE FOR THE RELATIVE ERROR IN G, EPS; A MAXIMUM NUMBER
C      OF ITERATIONS TO BE PERFORMED, N. THE SUBROUTINE CHANGES N TO
C      THE NUMBER OF ITERATIONS ACTUALLY PERFORMED.
C      OUTPUT INCLUDES: THE NUMBER OF ITERATIONS PERFORMED, N; THE
C      REQUIRED ROOT, G; THE VALUE C = FCT(G) - G; A PARAMETER INDICAT-
C      ING THE SUCCESS OF THE PROCEDURE, M. IF THE TOLERANCE WAS NOT
C      MET, M IS NEGATIVE. THE NUMBER ABS(M)-1 GIVES THE NUMBER OF TIMES
C      THAT THE CORRECTION TO G EXCEEDED THE PRECEDING ONE, IF FCT(Y)-Y
C      HAS THE SAME VALUE FOR TWO SUCCESSIVE APPROXIMATIONS TO G, THEN
C      H = 1, AND WE PRINT CYCLE AND EXIT.
      IF (KL - 2) 11,12,11
12 PRINT 101, J
101 FORMAT (/5X,4HJ1 =,I3,5X,98HIF J1 EQUALS J, THE COMPUTED RESULTS A
1RE ACCEPTABLE. THIS MEANS THAT THE ROOT, G, OBTAINED BY THE/3X,
2113HSUBROUTINE HAS BEEN CORRECTED BY THE PARENT ROUTINE AND THE PA
3RENT ROUTINE HAS CALLED THE SUBROUTINE AGAIN IN ITS/3X,113HATTEMPT
4 TO FIND A SATISFACTORY ROOT. IF J1 DOES NOT EQUAL J, THE ROOT, G
5, OBTAINED BY THE ROOT FINDER SUBROUTINE/3X,68H MAY OR MAY NOT BE A
6CCEPTABLE AND FURTHER INVESTIGATION IS WARRANTED./)
11 KL = 1
   M = 1
   JJ = 0
   C = 0
   CALL FCT (0., F)
   IF (F) 2,1,2
1  G = 0.
   GO TO 50
2  CALL FCT (A, G)
   C = G - A
   D = C
   B = C
   IF (C) 3,50,3
3  DO 40 JJ=1,N
   CALL FCT (G,F)
   C = F - G
   IF (ABS(F)-.0000009)50,50,10
10 IF (ABS(F) - ABSF(G)*EPS)50,50,4
4  H = B/C
   IF (H-1.) 5,6,5
6  PRINT 99, J
   PRINT 100,A,G,F,H
99 FORMAT (/8H CYCLE 3,8X,3HJ =,I4)
100 FORMAT (/6H A =,F15.7,4X,3HG =,E15.7,4X,3HF =,E15.7,4X,3HH =,
1E15.7)
   KL = KL + 1
   GO TO 49
5  IF (H) 7,7,8
8  IF (H-2.) 9,7,7
9  M = M + 1
7  D = D / (H-1.)
   B = C
40  G = G + D
49  M = -M
   JJ = N
50  N = JJ
   RETURN
END

```

```

SUBROUTINE FCT (X,Y)
COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION V(50,2)
IF (IFSW-1)1,1,2
1  Y =(AR1*X+AR2)*X*X+AR4
   GO TO 3
2  NN =50
   PPP =X
   US = 1.75073 + .05486*PPP - .00022*(PPP**2)
   IF (US - 1.483) 6,7,5
6  US = 1.483
   GO TO 7
5  CALL ROOT33 (US,EP,GG,CC,NN,MM,PPP)
   US =GG
7  UP =4.5349294*(1.095275**US - 1.14445)
   VH =1.0018*(1.0-UP/US)
   ENTHH =11.9446*(1.0018+VH)*X
   EXP =.0030338*X
   XI =151.16*10.**EXP
   ENTH =ENTHH+XI*(V(J,2)-VH)*23.8892
   Y = (ENTH-EE)/(V(J,2)*23.8892)
3  RETURN
   END

```

```

      SUBROUTINE ROOT33 (A,EPS,G,C,N,M,PPP)
      COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
      DIMENSION V(50,2)
C      THIS SUBROUTINE IS SIMILAR TO SUBROUTINE ROOT3.
      IF (KLL - 2)11,12,11
12 PRINT 101, J
101 FORMAT (/5X,4HJ1 =.I3,5X,98HIF J1 EQUALS J, THE COMPUTED RESULTS A
1RE ACCEPTABLE. THIS MEANS THAT THE ROOT, G, OBTAINED BY THE/3X,
2113HSUBROUTINE HAS BEEN CORRECTED BY THE PARENT ROUTINE AND THE PA
3RENT ROUTINE HAS CALLED THE SUBROUTINE AGAIN IN ITS/3X,113HATTEMPT
4 TO FIND A SATISFACTORY ROOT. IF J1 DOES NOT EQUAL J, THE ROOT, G
5, OBTAINED BY THE ROOT FINDER SUBROUTINE/3X,68HMAY OR MAY NOT BE A
6CCEPTABLE AND FURTHER INVESTIGATION IS WARRANTED,/)
11 KLL =1
   M =1
   JJ =0
   C = 0
2 CALL FCTN (A,G,PPP)
   C = G - A
   D = C
   B = C
   IF (C) 3,50,3
3 DO 40 JJ=1,N
   CALL FCTN (G,F,PPP)
   C = F - G
   IF (ABSF(C) - ABSF(G) * EPS) 50, 50, 4
4 H = B/C
   IF (H-1.) 5,6,5
6 PRINT 99, J
   PRINT 100,A,G,F,H,PPP
99 FORMAT (/9H CYCLE 33,8X,3HJ =,I4)
100 FORMAT (/6H A =,E15.7,4X,3HG =,E15.7,4X,3HF =,E15.7,4X,3HH =,
1E15.7,4X,5HPPP =,E15.7)
   KLL = KLL + 1
   GO TO 49
5 IF (H) 7,7,8
8 IF (H-2.) 9,7,7
9 M = M + 1
7 D = D / (H-1.)
   B = C
40 G = G + D
49 M = -M
   JJ =N
50 N =JJ
   RETURN
   END

```

```

      SUBROUTINE FCTN (X,Y,PPP)
      COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
      DIMENSION V(50,2)
      Y =(.022090746*PPP+1.14445*X)/(1.095275**X)
      RETURN
      END

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C      H.C.SORENSEN      EXPLOSION ANALYSIS (CONTINUATION)
C      THIS PROGRAM IS THE BASIC PROGRAM ALTERED TO PERMIT THE CONTINUATION
C      OF THE SOLUTION. IT USES THE SAME SUBROUTINES AS THE BASIC PROGRAM.
C      THE INITIAL CONDITIONS FOR THIS PROGRAM ARE THE FINAL CONDITIONS
C      CALCULATED BY THE BASIC PROGRAM.
COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION P(50,2),V(50,2),E(50,2),Q(50,2),U(50,2),RR(50,2),TEM(50)
1,R(50),DELRO(50),DELRO(50),SOS(50),DTTEST(50),VOL(50),VOL1(50),
2VOL2(50),VMAX(50)
COMMON/A/P1(4000),P2(4000),P3(4000),P4(4000),P5(4000),KKP(4000)
COMMON/9/P6(4000),RR7(4000),RR8(4000),T1(4000),PMax(4000)
PRINT 3000
C      READ AND PRINT INPUT DATA.
READ 1200,JA,JW,JMAX,TMAX,EPS,EP,DELT,AIPCC
PRINT 1201,JA,JW,JMAX,TMAX,EPS,EP,DELT,AIPCC
READ 2002,T,DELT,ETOT,EK,EINT
PRINT 2000
PRINT 2001
PRINT 2002,T,DELT,ETOT,EK,EINT
READ 1100,(RR(J,1),U(J,1),V(J,1),Q(J,1),J=1,JMAX)
READ 1100,(E(J,1),P(J,1),R(J),DTTEST(J),J=1,JMAX)
PRINT 2100
PRINT 2101
PRINT 2102,(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),R(J),
1DTTEST(J),J=1,JMAX)
READ 2103,T1(1),P1(1),P2(1),P3(1),RR7(1),P4(1),P5(1),RR8(1),P6(1),
1PMax(1),KKP(1)
PRINT 3000
JAM = JA-1
JWM = JW - 1
JMAXM = JMAX - 1
READ 1000,(VMAX(J),J=JA,JMAXM)
I = 2
T = T + DELT
T1(I) = T
KK = JA
KL = 1
KLL = 1
DO 130 J=1,JAM
130 VMAX(J) = 1000.0
DO 168 J=1,JMAXM
168 DELRO(J) = R(J+1) - R(J)
DO 163 J= 2,JMAXM
163 DELRU(J) = (R(J+1) - R(J-1))/2.
C      CALCULATE THE VELOCITY OF EACH ZONE INTERFACE AT TIME T FROM
C      THE FIRST DIFFERENCE EQUATION.
110 DO 200 J=2,JMAXM
IF (J.EQ.JA) GO TO 213
IF (J.EQ.JW) GO TO 213
U(J,2)=U(J,1)-(1.0018*(P(J,1)-P(J-1,1)+Q(J,1)-Q(J-1,1))/DELRO
1(J)*RR(J,1)/R(J))**2)*DELT
GO TO 200
213 U(J,2)=U(J,1)-DELT*1.0018*((RR(J,1)/R(J))**2)*(3.+(P(J,1)-P(J-1,1)
1+Q(J,1)-Q(J-1,1))-((P(J+1,1)-P(J-2,1)+Q(J+1,1)-Q(J-2,1))/3.))/
2(DELRO(J-1)+DELRO(J+1))
200 CONTINUE
C      CALCULATE THE RADIUS OF EACH ZONE INTERFACE AT
C      TIME T FROM THE SECOND DIFFERENCE EQUATION.
DO 300 J=1,JMAX
300 RR(J,2) = RR(J,1)+U(J,2) *DELT
IF (RR(JMAXM,2).GE.RR(JMAX,2)) RR(JMAXM,2)=RR(JMAXM,1)

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```

      IF (RR(JW,2).GE.RR(JMAXM,2)) GO TO 100
      DO 202 J=1,JMAXM
      VOL2(J) =(R(J+1)-R(J))*(R(J+1)**2+R(J+1)*R(J)+R(J)**2)
202  VOL(J) =(RR(J+1,2)-RR(J,2))*(RR(J+1,2)**2+RR(J+1,2)*RR(J,2)+
      1RR(J,2)**2)
      C      CALCULATE THE SPECIFIC VOLUME FOR EACH ZONE AT TIME T FROM THE
      C      THIRD DIFFERENCE EQUATION, AND CALCULATE THE PSEUDO-VISCOUS
      C      PRESSURE TERM FOR EACH ZONE AT TIME T FROM THE FOURTH
      C      DIFFERENCE EQUATION.
      DO 400 J=1,JMAXM
      V(J,2) =1.0018*VOL(J)/VOL2(J)
      VTEST =V(J,2)-V(J,1)
      IF (VTEST) 402,401,401
402  CC1 =3.98563879*(DELRO(J))**2/(V(J,2)+V(J,1))
      CC2 =(VTEST/DELT)**2
      CC3 =(2.*R(J+1)/(RR(J+1,2)+RR(J+1,1)))*4
      Q(J,2) =CC1*CC2*CC3
      GO TO 400
401  Q(J,2)=0.
400  CONTINUE
      C      CALCULATE THE TOTAL KINETIC ENERGY OF THE SYSTEM AT TIME T.
      EK =0.
      DO 201 J =1,JMAXM
      VEL =(U(J+1,2) + U(J,2))**2
201  EK =.5235988*VOL(J)*VEL/V(J,2) + EK
      ASSIGN 66 TO IS3
      IFSW =1
      IS2 =1
      TEMPP = P(1,1)
      C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN
      C      THE TNT PRODUCTS AT TIME T BY ITERATION OF THE FIFTH AND SIXTH
      C      DIFFERENCE EQUATIONS BY THE FOLLOWING PROCEDURE. GUESS P AT
      C      TIME T. SOLVE THE FIFTH DIFFERENCE EQUATION FOR E AT TIME T.
      C      THEN USE THIS VALUE OF E TO OBTAIN A CORRECTED VALUE FOR P AT
      C      TIME T. REPEAT THIS PROCESS UNTIL THE DIFFERENCE BETWEEN TWO
      C      SUCCESSIVE VALUES FOR P AT TIME T IS LESS THAN A VALUE EPSILON.
      DO 699 J=1,JAM
      IF (IS2-1)404,403,404
404  TEMPP =(P(J,1)+P(J-1,2))/2.
403  ENEW =0.
      AR3 =.648986-V(J,2)
      AAR3 =-AR3
      AR1 =.619876E-23/AAR3
      AR2 =-.271149E-11/AAR3
60  TEMPE = E(J,1)-((TEMPP+P(J,1))/2.+Q(J,2))*(V(J,2)-V(J,1))
      GO TO IS3
66  IF(ABSF(TEMPE-ENEW)-EPS)68,68,65
65  E1 =TEMPP**2*(.135575E-11-.413251E-23*TEMPP)
      AR44 =(TEMPE + .137566F10 - E1)*.32336525
      AR4 =AR44/AAR3
      N = 25
      CALL ROOT3 (TEMPP,FP,G,C,N,M)
      R1 = G
      C      THE EQUATION OF STATE FOR THE TNT PRODUCTS, THE SIXTH DIFF-
      C     ERENCE EQUATION, IS A THIRD ORDER EQUATION IN P. THEREFORE,
      C      THERE ARE THREE ROOTS TO THIS EQUATION. THE ROOT FINDER SUB-
      C      ROUTINE, CALLED ROOT3, HAS JUST FOUND ONE ROOT TO THIS
      C      EQUATION. NOW SOLVE FOR THE OTHER TWO ROOTS FROM THE RESULTING
      C      QUADRATIC EQUATION AND CHOOSE WHICHEVER OF THE THREE ROOTS IS
      C      CLOSEST TO THE VALUE OF P AT TIME T =T-DELT FOR CONTINUING THE
      C      ITERATIVE PROCESS.
      B =.619876E-23*R1 - .271149E-11

```

```

DISC =B*2-2.479504E-23*(AR3+R1  *(.619876E-23*R1-.271149E-11))
IF(DISC)660,670,680
660 TEMPP = R1
GO TO 190
670 R2 = -B/1.239752E-23
R3 = R2
GO TO 96
680 DSQ = SQRTF(DISC)
R2 =(-B+DSQ)/1.239752E-23
R3 =(-B-DSQ)/1.239752E-23
96 RAD1 =ABSF(P(J,1)-R1)
RAD2 =ABSF(P(J,1)-R2)
RAD3 =ABSF(P(J,1)-R3)
IF(RAD1-RAD2)91,92,92
91 IF(RAD3-RAD1)93,94,94
93 TEMPP = R3
GO TO 190
92 IF(RAD3-RAD2)93,95,95
95 TEMPP = R2
GO TO 190
94 TEMPP = R1
190 ENEW = TEMPE
GO TO 60
68 P(J,2)=TEMPP
E(J,2)=TEMPE
TEM(J) =AR44/.521766E7
IS2 =2
C CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
PCP =P(J,2)*1.E-10
SOS(J) =PCP*(49547.-2108.*PCP) + 183556.
DTTEST(J) =.30*(RR(J+1,2)-RR(J,2))/SOS(J)
699 CONTINUE
ASSIGN 76 TO IS3
IFSW =2
C CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C WATER AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
DO 71 J= JA,JWM
74 TEMPP =P(J,1)
ENEW = 0.
GO TO 60
76 IF (TEMPE.GT.1000000.0) GO TO 766
EPS1 = EPS/10.
IF (ABSF(TEMPE-ENEW)-EPS1)78,78,75
766 IF (ABSF(TEMPE-ENEW)-EPS)78,78,75
75 IF (V(J,2)-VMAX(J)) 756,756,755
756 PP = TEMPP*1.0E-9
EE =TEMPE*.238892E-7
N = 25
CALL ROOT3 (PP,EP,G,C,N,M)
TEMPP =G*1.0E9
IF (TEMPP.LT.0.0) TEMPP = 0.0
ENEW = TEMPE
GO TO 60
755 TEMPP = 0.0
ENEW = TEMPE
GO TO 60
78 IF (VMAX(J).NE.1000.0) GO TO 757
IF (TEMPP.EQ.0.0) VMAX(J) = V(J,2)
757 E(J,2) = TEMPE
P(J,2) = TEMPP

```



```

C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
PCP = P(J,2)*1.E-9
SOS(J) = PCP*(3853.-13.*PCP) + 256906.
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
71  CONTINUE
    ASSIGN 86 TO IS3
C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C      ARGON AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C      EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
887  DO 80 J=JW,JMAXM
89    ENEW = 0.
    VSQ = V(J,2)**2
    TEMP = 835833.*(1.-.58282/V(J,2))/VSQ
    TEMP1 = -.318769E-6*TEMP*(V(J,2)-591.597)
    TEMP2 = 2078910.*(V(J,2) + .98413)/VSQ
    TEMPP = P(J,1)
    GO TO 60
86    IF(ABS(TEMPE-ENEW)-EPS)88,88,85
85    TEMPT = TEMPE+.318769E-6*TEMP1
    TEMPP = TEMPT*TEMP2-TEMP
    ENEW = TEMPE
    GO TO 60
88    E(J,2) = TEMPE
    P(J,2) = TEMPP
    TEM(J) = TEMPT
C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
RAD = 1.66*P(J,2)*V(J,2)
SOS(J) = SQRTF(RAD)
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
80  CONTINUE
C      CALCULATE THE TOTAL INTERNAL ENERGY IN THE SYSTEM AT TIME T.
81  EINT = 0.
    DO 500 J=1,JMAXM
    EMASS = 4.1887902*VOL(J)/V(J,2)
500  EINT = EINT + E(J,2)*EMASS
C      CALCULATE THE TOTAL ENERGY IN THE SYSTEM AT TIME T.
    ETOT = EINT + EK
C      REASSIGN VALUES.
    DO 600 J=1,JMAX
    P(J,1) = P(J,2)
    RR(J,1) = RR(J,2)
    U(J,1) = U(J,2)
    Q(J,1) = Q(J,2)
    V(J,1) = V(J,2)
600  E(J,1) = E(J,2)
    PTEST = P(JA,1)
    K = JA + 1
    DO 444 J = K, JMAXM
    ARG = P(J,1)
    IF (PTEST - ARG) 445,445,444
445  PTEST = ARG
    KK = J
444  CONTINUE
55  PRINT 5000
56  PRINT 2000
    PRINT 2001
    PRINT 2002,T,DELT,ETOT,EK,EINT
    PRINT 2100
    PRINT 2101
447  PRINT 2102,(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),R(J),

```

```

1DTTEST(J),J=1,JMAX)
C      STORE IN MEMORY THE VALUES OF P AND RR AT THE CRITICAL
C      LOCATIONS.
51  P1(I) = P(1,1)*1.E-9
    P2(I) = P(JAM,1)*1.E-9
    P3(I) = P(JA,1)*1.E-9
    P4(I) = P(JWM,1)*1.E-9
    P5(I) = P(JW,1)*1.E-9
    P6(I) = P(JMAXM,1)*1.E-9
    RR7(I) = RR(JA,1)
    RR8(I) = RR(JW,1)
    PMAX(I) = P(KK,1)*1.E-9
    KKP(I) = KK
    ETOTAL = ETOT*.2388915E-13
    WRITE(40),T1(I),P1(I),P2(I),P3(I),RR7(I),P4(I),P5(I),RR8(I),P6(I),
1PMAX(I),ETOTAL
C      INCREMENT DELTA T TO THE MAXIMUM VALUE ALLOWED BY THE STABILITY
C      CONDITION FOR THE DIFFERENCE EQUATIONS.
    DELTAT =DTTEST(1)
    DO 44 J=2,JMAXM
    ARGUE =DTTEST(J)
    IF (DELTAT-ARGUE)44,44,26
26  DELTAT =ARGUE
44  CONTINUE
    IF (DELTAT.GE.0.000003) DELT=0.000003
    IF (DELTAT.LT.0.000003) DELT=0.000002
    IF (DELTAT.LT.0.000002) DELT = 0.000001
    IF (DELTAT.LT.0.000001) DELT=0.0000005
    IF (DELTAT.LT.0.0000005) DELT=0.0000003
    IF (DELTAT.LT.0.0000003) DELT=0.0000001
C      INCREMENT T.
    T = T + DELT
    I = I+1
    IF (I.GE.3995) GO TO 100
    T1(I) = T
    IF (T-TMAX)110,110,100
1000 FORMAT (1E14.6)
1100 FORMAT (4E14.6)
1200 FORMAT (3I6,4E14.7,F6.4)
1201 FORMAT (11H INPUT DATA/6H      JA,4X,2HJW,3X,4HJMAX,2X,9HTMAX(SEC),
14X,3HEPS,9X,2HEP,5X,9HDELT(SEC),3X,12HALLPCONC(KB)//3I6,F12.7,
2F10.1,F10.5,F12.7,F12.4///)
2000 FORMAT (6X,1HT,8X,4HDELT,12X,4HETOT,15X,2HEK,15X,4HEINT)
2001 FORMAT (5X,3HSEC,8X,3HSFC,11X,7HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM
1/)
2002 FORMAT (2F11.7,3E18.7)
2100 FORMAT (///4H      J,7X,2HRR,13X,1HU,13X,1HV,13X,1HQ,13X,1HE,13X,1HP,
113X,1HR,10X,6HDTTEST)
2101 FORMAT (11X,2HCM,11X,6HCM/SEC,5X,9HCUB CM/GM,4X,11HDYNES/SQ CM,
14X,10HDYNE-CM/GM,4X,11HDYNES/SQ CM,7X,2HCM,12X,3HSEC/)
2102 FORMAT (I4,8E14.6)
2103 FORMAT (F10.7,3F6.3,F8.3,2F6.3,F8.3,2F6.3,I3)
3000 FORMAT (1H1)
3001 FORMAT (1H1,////)
5000 FORMAT (///)
5001 FORMAT (/11X,47HNUMBER OF LINES OF PRINT OUT IN THE FINAL TABLE,
13X,3HI =,I4/11X,33HNUMBER OF RECORDS WRITTEN ON TAPE,3X,3HN =,I4)
8000 FORMAT (/11X,12HPROGRAM OVER)
9695 FORMAT (14X,69HTABULATED VALUES OF THE PRESSURE AND THE RADIAL COO
1RDINATE AT SEVERAL/11X,28HCRITICAL LOCATIONS (PART I)//11X,24HUNI
2TS - TIME, T, SECONDS/19X,21HPRESSURE, P, KILOBARS/19X,38HRADIAL C
3COORDINATE, RR, CENTIMETERS      ,24X,14HINSIDE      MAX/80X,7HFACE OF

```

4,3X,8HPRESSURE/80X,8HCONCRETE,1X,10HOUTSIDE OF/34X,19HTNT-WATER IN-
 5TERFACE,4X,21HWATER-ARGON INTERFACE,3X,6HVESSEL,2X,10HGAS BUBBLE/1-
 65X,1HT,9X,1HP,8X,1HP,8X,1HP,7X,2HRR,6X,1HP,6X,1HP,7X,2HRR,7X,1HP,
 77X,1HP,3X,4HZONE/50X,4HTNT-,18X,6HWATER-/23X,6HCENTER,4X,3HTNT,5X,
 85HWATER,3X,5HWATER,3X,5HWATER,2X,5HARGON,3X,5HARGON,4X,5HARGON,4X,
 93HMAX,4X,1HJ/)

```

9696. FORMAT (F20.7,F9.3,F9.3,F8.3,F8.3,F8.3,F7.3,F9.3,F8.3,F8.3,I4)
100  IMAX = I-1
      T = T - DELT
53   NPLIM = 23
      PRINT 3001
      PRINT 9695
      DO 58 I = 1,IMAX
      IF (I - NPLIM) 58,58,59
59   NPLIM = NPLIM + 36
      PRINT 3001
58   PRINT 9696,T1(I),P1(I),P2(I),P3(I),RR7(I),P4(I),P5(I),RR8(I),
1P6(I),PMAX(I),KKP(I)
      IMAXM = IMAX - 1
      PRINT 5001,IMAX,IMAXM
      PRINT 8000
      END

```

K. Computer Results

For each increment in time, the finite-difference equations (Eqs. 42-47) were solved for all points in the finite-difference net. Table II represents a typical set of computer output data. A set of data similar to Table II was printed for each time increment. It was impractical to include all the computer output obtained during the solution of this problem.

The appropriate information about the peak pressure in the shock front obtained for the simulated infinite-water-medium layout is presented in tabular form in Table III. The information in columns 1, 2, and 3 of Table III was extracted from the actual computer output for this layout.

For the simulated sodium reactor installation layout, only values of the pressure and the radial coordinate at several critical locations in the finite-difference net are presented. A sample of the tabulated computer results for this layout is given in Table IV.

The order of these computer results is as follows:

1. Tabulated computer results for the simulated infinite water medium (Table III).
2. Graphical representation of the computer results for the simulated infinite water medium (Fig. 8).
3. A sample of the tabulated computer results for the simulated sodium reactor installation (Table IV).
4. Graphical representation of the computer results for the simulated sodium-reactor installation (Figs. 9-18).

TABLE III. Computer Results for Peak-pressure Analysis

Peak Pressure, P_m , kb	Time P_m Occurred, sec	Location of Peak Pressure,* R, cm	a_0/R	Reduced Radius, R/a_0
62.184	0.0000055	31.7605	0.9446	1.059
42.598	0.0000105	33.2910	0.9011	1.110
33.404	0.0000175	34.9995	0.8572	1.167
28.442	0.0000235	36.5528	0.8207	1.218
24.914	0.0000295	38.0820	0.7878	1.269
16.962	0.0000625	47.6551	0.6295	1.589
12.400	0.0001045	57.4176	0.5225	1.914
9.610	0.0001495	67.1947	0.4465	2.240
7.649	0.0001955	76.9436	0.3899	2.565
6.374	0.0002455	86.7915	0.3457	2.893
5.330	0.0002975	96.6564	0.3104	3.222
4.614	0.0003505	106.5312	0.2816	3.551
4.005	0.0004035	116.3906	0.2578	3.880
3.548	0.0004605	126.3269	0.2375	4.211
3.174	0.0005155	136.2160	0.2202	4.541

TABLE III. (Contd.)

Peak Pressure, P_m , kb	Time P_m Occurred, sec	Location of Peak Pressure,* R, cm	a_0/R	Reduced Radius, R/a_0
2.864	0.0005755	146.1831	0.2052	4.873
2.605	0.0006325	156.0993	0.1922	5.203
2.387	0.0006925	166.0547	0.1807	5.535
2.199	0.0007525	176.0063	0.1704	5.867
2.041	0.0008145	185.9781	0.1613	6.199
1.896	0.0008765	195.9477	0.1531	6.532
1.771	0.0009365	205.8960	0.1457	6.863
1.660	0.0010005	215.8830	0.1390	7.196
1.566	0.0010625	225.8498	0.1328	7.528
1.479	0.0011245	235.8166	0.1272	7.861
1.401	0.0011865	245.7838	0.1221	8.193
1.330	0.0012505	255.7668	0.1173	8.526
1.265	0.0013145	265.7490	0.1129	8.858
1.206	0.0013785	275.7307	0.1088	9.191
1.155	0.0014425	285.7119	0.1050	9.524
1.104	0.0015065	295.6916	0.1015	9.856
1.057	0.0015705	305.6733	0.0981	10.189
1.015	0.0016345	315.6548	0.0950	10.522
0.975	0.0016995	325.6423	0.0921	10.855
0.939	0.0017635	335.6240	0.0894	11.187
0.905	0.0018285	345.6110	0.0868	11.520
0.873	0.0018935	355.5989	0.0844	11.853
0.843	0.0019585	365.5863	0.0821	12.186
0.815	0.0020235	375.5738	0.0799	12.519
0.788	0.0020895	385.5663	0.0778	12.852
0.764	0.0021545	395.5539	0.0758	13.185
0.740	0.0022195	405.5416	0.0740	13.518
0.718	0.0022845	415.5340	0.0722	13.851
0.697	0.0023505	425.5220	0.0705	14.184
0.678	0.0024165	435.5144	0.0689	14.517
0.659	0.0024825	445.5067	0.0673	14.850
0.641	0.0025485	455.4991	0.0659	15.183
0.624	0.0026145	465.4915	0.0644	15.516
0.608	0.0026795	475.4802	0.0631	15.849
0.593	0.0027455	485.4728	0.0618	16.182
0.578	0.0028115	495.4654	0.0605	16.516
0.564	0.0028775	505.4581	0.0594	16.849
0.551	0.0029445	515.4544	0.0582	17.182
0.538	0.0030105	525.4472	0.0571	17.515
0.526	0.0030765	535.4401	0.0560	17.848
0.514	0.0031425	545.4331	0.0550	18.181
0.503	0.0032085	555.4261	0.0540	18.514
0.492	0.0032755	565.4225	0.0531	18.847
0.482	0.0033415	575.4157	0.0521	19.181
0.472	0.0034085	585.4120	0.0512	19.514
0.462	0.0034745	595.4053	0.0504	19.847
0.453	0.0035405	605.3988	0.0496	20.180

*R, the location of the peak pressure, is the average of the Eulerian coordinates of the two interfaces for the zone in which the peak pressure occurs and $a_0 = 30$ cm.

TABLE IV. Tabulated Values of the Pressure and the
Radial Coordinate at Several Critical Locations

UNITS - TIME, T, SECONDS

PRESSURE, P, KILOBARS

RADIAL COORDINATE, RR, CENTIMETERS

T	P	TNT-WATER INTERFACE			WATER-ARGON INTERFACE			INSIDE	MAX	
		P	P	RR	P	P	RR	FACE OF	PRESSURE	
	CENTER	TNT	WATER	TNT- WATER	WATER	ARGON	WATER- ARGON	CONCRETE	OUTSIDE OF	
								VESSEL	GAS BURBLE	J
								P	P	
0.0000000	21.900	124.500	0.001	30.000	0.001	0.001	180.000	0.001	0.001	13
0.0000005	21.900	113.998	0.163	30.084	0.001	0.001	180.000	0.001	0.163	13
0.0000010	21.900	102.887	0.348	30.172	0.001	0.001	180.000	0.001	0.348	13
0.0000015	21.900	91.922	0.554	30.264	0.001	0.001	180.000	0.001	0.554	13
0.0000020	21.900	81.603	0.783	30.359	0.001	0.001	180.000	0.001	0.783	13
0.0000025	21.900	72.227	1.034	30.456	0.001	0.001	180.000	0.001	1.034	13
0.0000030	21.900	63.929	1.307	30.555	0.001	0.001	180.000	0.001	1.307	13
0.0000035	21.900	56.731	1.598	30.655	0.001	0.001	180.000	0.001	1.598	13
0.0000040	21.900	50.574	1.917	30.755	0.001	0.001	180.000	0.001	1.917	13
0.0000045	21.900	45.356	2.256	30.856	0.001	0.001	180.000	0.001	2.256	13
0.0000050	21.900	40.955	2.617	30.957	0.001	0.001	180.000	0.001	2.617	13
0.0000055	21.900	37.248	2.999	31.058	0.001	0.001	180.000	0.001	2.999	13
0.0000060	21.900	34.120	3.400	31.158	0.001	0.001	180.000	0.001	3.400	13
0.0000065	21.900	31.470	3.822	31.258	0.001	0.001	180.000	0.001	3.822	13
0.0000070	21.900	29.216	4.264	31.357	0.001	0.001	180.000	0.001	4.264	13
0.0000075	21.900	27.286	4.722	31.455	0.001	0.001	180.000	0.001	4.722	13
0.0000080	21.900	25.624	5.203	31.553	0.001	0.001	180.000	0.001	5.203	13
0.0000085	21.900	24.183	5.701	31.650	0.001	0.001	180.000	0.001	5.701	13
0.0000090	21.900	22.928	6.216	31.746	0.001	0.001	180.000	0.001	6.216	13
0.0000095	21.900	21.828	6.746	31.841	0.001	0.001	180.000	0.001	6.746	13
0.0000105	21.900	20.058	7.827	32.027	0.001	0.001	180.000	0.001	7.827	13
0.0000115	21.901	18.654	8.969	32.210	0.001	0.001	180.000	0.001	8.969	13
0.0000125	21.901	17.522	10.149	32.389	0.001	0.001	180.000	0.001	10.149	13
0.0000135	21.901	16.595	11.361	32.564	0.001	0.001	180.000	0.001	11.361	13
0.0000145	21.902	15.828	12.595	32.736	0.001	0.001	180.000	0.001	12.595	13
0.0000155	21.904	15.186	13.839	32.903	0.001	0.001	180.000	0.001	13.839	13
0.0000165	21.905	14.642	15.078	33.067	0.001	0.001	180.000	0.001	15.078	13
0.0000175	21.907	14.180	16.299	33.227	0.001	0.001	180.000	0.001	16.299	13
0.0000185	21.911	13.785	17.489	33.383	0.001	0.001	180.000	0.001	17.489	13
0.0000205	21.924	13.193	19.572	33.680	0.001	0.001	180.000	0.001	19.572	13
0.0000225	21.945	12.750	21.356	33.962	0.001	0.001	180.000	0.001	21.356	13
0.0000245	21.980	12.415	22.757	34.229	0.001	0.001	180.000	0.001	22.757	13
0.0000265	22.033	12.158	23.707	34.481	0.001	0.001	180.000	0.001	23.707	13
0.0000285	22.108	11.962	24.162	34.718	0.001	0.001	180.000	0.001	24.162	13
0.0000305	22.211	11.812	24.097	34.940	0.001	0.001	180.000	0.001	24.097	13
0.0000325	22.345	11.698	23.517	35.146	0.001	0.001	180.000	0.001	23.517	13
0.0000345	22.503	11.610	22.467	35.339	0.001	0.001	180.000	0.001	22.467	13
0.0000365	22.706	11.532	21.035	35.519	0.001	0.001	180.000	0.001	21.035	13
0.0000385	22.945	11.447	19.337	35.687	0.001	0.001	180.000	0.001	19.337	13
0.0000405	23.216	11.348	17.498	35.845	0.001	0.001	180.000	0.001	17.498	13
0.0000425	23.513	11.230	15.633	35.997	0.001	0.001	180.000	0.001	15.633	13
0.0000445	23.825	11.091	13.834	36.143	0.001	0.001	180.000	0.001	13.834	13
0.0000465	24.143	10.927	12.169	36.287	0.001	0.001	180.000	0.001	12.169	13
0.0000485	24.456	10.735	10.678	36.429	0.001	0.001	180.000	0.001	10.678	14
0.0000505	24.757	10.513	9.378	36.573	0.001	0.001	180.000	0.001	9.378	14
0.0000525	25.040	10.256	8.272	36.719	0.001	0.001	180.000	0.001	8.272	14
0.0000545	25.312	9.966	7.347	36.869	0.001	0.001	180.000	0.001	7.347	14
0.0000565	25.588	9.645	6.601	37.024	0.001	0.001	180.000	0.001	6.601	14
0.0000585	25.896	9.300	6.010	37.184	0.001	0.001	180.000	0.001	6.010	14
0.0000605	26.278	8.934	5.560	37.349	0.001	0.001	180.000	0.001	5.560	14
0.0000615	26.504	8.750	5.371	37.434	0.001	0.001	180.000	0.001	5.371	14
0.0000625	26.767	8.562	5.214	37.519	0.001	0.001	180.000	0.001	5.214	14
0.0000635	27.077	8.372	5.087	37.607	0.001	0.001	180.000	0.001	5.087	14
0.0000645	27.443	8.181	4.989	37.695	0.001	0.001	180.000	0.001	4.989	14
0.0000655	27.870	7.988	4.921	37.786	0.001	0.001	180.000	0.001	4.921	14
0.0000665	28.366	7.796	4.881	37.877	0.001	0.001	180.000	0.001	4.881	14
0.0000675	28.942	7.604	4.868	37.970	0.001	0.001	180.000	0.001	4.868	14
0.0000685	29.591	7.415	4.883	38.065	0.001	0.001	180.000	0.001	4.883	14
0.0000695	30.321	7.228	4.925	38.160	0.001	0.001	180.000	0.001	4.925	14

TABLE IV. (Contd.)

0.0000705	31.131	7.046	4.994	38.257	0.001	n.n01	180.000	0.001	16.137	14
0.0000715	32.017	6.848	5.088	38.355	0.001	n.n01	180.000	0.001	15.833	14
0.0000725	32.971	6.597	5.209	38.454	0.001	n.n01	180.000	0.001	15.495	14
0.0000735	33.984	6.533	5.354	38.554	0.001	n.n01	180.000	0.001	15.126	14
0.0000745	35.042	6.377	5.523	38.654	0.001	n.n01	180.000	0.001	14.732	14
0.0000755	36.126	6.230	5.711	38.754	0.001	n.n01	180.000	0.001	14.315	14
0.0000765	37.217	6.093	5.929	38.855	0.001	n.n01	180.000	0.001	13.882	14
0.0000775	39.302	5.853	6.428	39.056	0.001	n.n01	180.000	0.001	12.958	14
0.0000785	41.113	5.655	6.990	39.255	0.001	n.n01	180.000	0.001	12.020	14
0.0000795	42.443	5.499	7.593	39.451	0.001	n.n01	180.000	0.001	11.007	14
0.0000805	43.103	5.385	8.216	39.642	0.001	n.n01	180.000	0.001	10.212	14
0.0000815	42.910	5.311	8.817	39.827	0.001	n.n01	180.000	0.001	9.381	14
0.0000825	41.811	5.274	9.366	40.005	0.001	n.n01	180.000	0.001	9.669	15
0.0000835	39.707	5.273	9.831	40.176	0.001	n.n01	180.000	0.001	10.238	15
0.0000845	36.720	5.303	10.182	40.339	0.001	n.n01	180.000	0.001	10.742	15
0.0000855	33.193	5.361	10.395	40.494	0.001	n.n01	180.000	0.001	11.230	15
0.0000865	29.150	5.441	10.454	40.641	0.001	n.n01	180.000	0.001	11.632	15
0.0000875	25.218	5.539	10.350	40.780	0.001	n.n01	180.000	0.001	11.957	15
0.0000885	21.563	5.546	10.086	40.911	0.001	n.n01	180.000	0.001	12.107	15
0.0000895	18.340	5.752	9.677	41.035	0.001	n.n01	180.000	0.001	12.346	15
0.0000905	15.603	5.847	9.146	41.154	0.001	n.n01	180.000	0.001	12.400	15
0.0000915	13.331	5.917	8.527	41.267	0.001	n.n01	180.000	0.001	12.358	15
0.0000925	11.469	5.949	7.846	41.376	0.001	n.n01	180.000	0.001	12.223	15
0.0000935	9.946	5.934	7.150	41.481	0.001	n.n01	180.000	0.001	11.908	15
0.0000945	8.699	5.862	6.457	41.585	0.001	n.n01	180.000	0.001	11.601	15
0.0000955	7.671	5.732	5.789	41.689	0.001	n.n01	180.000	0.001	11.313	15
0.0000965	6.817	5.547	5.162	41.792	0.001	n.n01	180.000	0.001	10.876	15
0.0000975	6.103	5.318	4.583	41.897	0.001	n.n01	180.000	0.001	10.393	15
0.0000985	5.499	5.059	4.066	42.003	0.001	n.n01	180.000	0.001	9.876	15
0.0000995	4.984	4.784	3.603	42.111	0.001	n.n01	180.000	0.001	9.340	15
0.0001005	4.347	4.357	3.021	42.279	0.001	n.n01	180.000	0.001	8.512	15
0.0001015	3.990	4.092	2.692	42.393	0.001	n.n01	180.000	0.001	7.974	15
0.0001025	3.678	3.842	2.414	42.511	0.001	n.n01	180.000	0.001	7.449	15
0.0001035	3.403	3.612	2.183	42.631	0.001	n.n01	180.000	0.001	7.712	16
0.0001045	3.160	3.402	1.995	42.754	0.001	n.n01	180.000	0.001	8.087	16
0.0001055	2.944	3.212	1.850	42.881	0.001	n.n01	180.000	0.001	8.429	16
0.0001065	2.750	3.042	1.744	43.010	0.001	n.n01	180.000	0.001	8.738	16
0.0001075	2.576	2.890	1.675	43.142	0.001	n.n01	180.000	0.001	9.007	16
0.0001085	2.419	2.755	1.644	43.277	0.001	n.n01	180.000	0.001	9.232	16
0.0001095	2.276	2.636	1.648	43.414	0.001	n.n01	180.000	0.001	9.408	16
0.0001105	2.147	2.532	1.687	43.554	0.001	n.n01	180.000	0.001	9.531	16
0.0001115	2.029	2.441	1.761	43.695	0.001	n.n01	180.000	0.001	9.599	16
0.0001125	1.920	2.364	1.870	43.839	0.001	n.n01	180.000	0.001	9.610	16
0.0001135	1.821	2.298	2.011	43.983	0.001	n.n01	180.000	0.001	9.565	16
0.0001145	1.774	2.269	2.090	44.055	0.001	n.n01	180.000	0.001	9.529	16
0.0001155	1.729	2.242	2.177	44.128	0.001	n.n01	180.000	0.001	9.480	16
0.0001165	1.686	2.219	2.270	44.200	0.001	n.n01	180.000	0.001	9.417	16
0.0001175	1.644	2.197	2.369	44.272	0.001	n.n01	180.000	0.001	9.341	16
0.0001185	1.604	2.179	2.475	44.344	0.001	n.n01	180.000	0.001	9.252	16
0.0001195	1.529	2.153	2.704	44.487	0.001	n.n01	180.000	0.001	9.028	16
0.0001205	1.458	2.137	2.949	44.628	0.001	n.n01	180.000	0.001	8.743	16
0.0001215	1.393	2.132	3.202	44.767	0.001	n.n01	180.000	0.001	8.442	16
0.0001225	1.332	2.137	3.456	44.903	0.001	n.n01	180.000	0.001	8.133	16
0.0001235	1.275	2.153	3.703	45.036	0.001	n.n01	180.000	0.001	7.777	16
0.0001245	1.222	2.181	3.933	45.165	0.001	n.n01	180.000	0.001	7.413	16
0.0001255	1.172	2.219	4.138	45.290	0.001	n.n01	180.000	0.001	7.040	16
0.0001265	1.125	2.269	4.306	45.411	0.001	n.n01	180.000	0.001	6.665	16
0.0001275	1.081	2.331	4.442	45.528	0.001	n.n01	180.000	0.001	6.293	16
0.0001285	1.039	2.403	4.535	45.641	0.001	n.n01	180.000	0.001	6.210	17
0.0001295	1.000	2.486	4.583	45.751	0.001	n.n01	180.000	0.001	6.473	17
0.0001305	0.962	2.577	4.583	45.857	0.001	n.n01	180.000	0.001	6.718	17
0.0001315	0.910	2.729	4.480	46.009	0.001	n.n01	180.000	0.001	7.036	17
0.0001325	0.862	2.889	4.290	46.155	0.001	n.n01	180.000	0.001	7.296	17
0.0001335	0.818	3.046	4.025	46.297	0.001	n.n01	180.000	0.001	7.489	17
0.0001345	0.777	3.188	3.715	46.436	0.001	n.n01	180.000	0.001	7.598	17
0.0001355	0.739	3.300	3.382	46.574	0.001	n.n01	180.000	0.001	7.649	17
0.0001365	0.703	3.370	3.049	46.712	0.001	n.n01	180.000	0.001	7.610	17
0.0001375	0.670	3.388	2.733	46.852	0.001	n.n01	180.000	0.001	7.494	17
0.0001385	0.639	3.348	2.448	46.996	0.001	n.n01	180.000	0.001	7.304	17
0.0001395	0.610	3.254	2.205	47.144	0.001	n.n01	180.000	0.001	7.051	17
0.0001405	0.583	3.116	2.006	47.297	0.001	n.n01	180.000	0.001	6.744	17
0.0001415	0.558	2.948	1.854	47.456	0.001	n.n01	180.000	0.001	6.395	17

TABLE IV. (Contd.)

0.0002165	0.534	2.768	1.747	47.620	0.001	0.001	180.000	0.001	6.017	17
0.0002195	0.511	2.586	1.684	47.789	0.001	0.001	180.000	0.001	5.622	17
0.0002225	0.490	2.413	1.662	47.963	0.001	0.001	180.000	0.001	5.221	17
0.0002255	0.470	2.254	1.680	48.141	0.001	0.001	180.000	0.001	5.291	18
0.0002275	0.457	2.157	1.708	48.261	0.001	0.001	180.000	0.001	5.422	18
0.0002295	0.445	2.067	1.750	48.382	0.001	0.001	180.000	0.001	5.659	18
0.0002315	0.433	1.986	1.806	48.504	0.001	0.001	180.000	0.001	5.820	18
0.0002335	0.421	1.913	1.874	48.627	0.001	0.001	180.000	0.001	5.944	18
0.0002355	0.410	1.847	1.952	48.749	0.001	0.001	180.000	0.001	6.089	18
0.0002375	0.400	1.789	2.038	48.871	0.001	0.001	180.000	0.001	6.192	18
0.0002395	0.390	1.739	2.129	48.992	0.001	0.001	180.000	0.001	6.274	18
0.0002415	0.380	1.696	2.223	49.113	0.001	0.001	180.000	0.001	6.331	18
0.0002435	0.370	1.659	2.317	49.232	0.001	0.001	180.000	0.001	6.365	18
0.0002455	0.361	1.629	2.408	49.350	0.001	0.001	180.000	0.001	6.374	18
0.0002475	0.352	1.606	2.493	49.465	0.001	0.001	180.000	0.001	6.357	18
0.0002495	0.343	1.589	2.570	49.579	0.001	0.001	180.000	0.001	6.317	18
0.0002515	0.335	1.578	2.635	49.690	0.001	0.001	180.000	0.001	6.252	18
0.0002535	0.327	1.573	2.688	49.799	0.001	0.001	180.000	0.001	6.145	18
0.0002555	0.319	1.573	2.725	49.906	0.001	0.001	180.000	0.001	6.056	18
0.0002575	0.311	1.579	2.746	50.010	0.001	0.001	180.000	0.001	5.928	18
0.0002595	0.304	1.591	2.750	50.111	0.001	0.001	180.000	0.001	5.781	18
0.0002615	0.297	1.608	2.737	50.209	0.001	0.001	180.000	0.001	5.620	18
0.0002635	0.290	1.631	2.709	50.306	0.001	0.001	180.000	0.001	5.445	18
0.0002655	0.283	1.659	2.666	50.400	0.001	0.001	180.000	0.001	5.259	18
0.0002675	0.277	1.691	2.612	50.492	0.001	0.001	180.000	0.001	5.065	18
0.0002695	0.271	1.728	2.549	50.583	0.001	0.001	180.000	0.001	4.845	18
0.0002715	0.264	1.769	2.481	50.672	0.001	0.001	180.000	0.001	4.640	18
0.0002735	0.259	1.812	2.409	50.760	0.001	0.001	180.000	0.001	4.454	18
0.0002755	0.253	1.858	2.337	50.847	0.001	0.001	180.000	0.001	4.433	19
0.0002775	0.247	1.905	2.267	50.934	0.001	0.001	180.000	0.001	4.629	19
0.0002795	0.242	1.952	2.202	51.021	0.001	0.001	180.000	0.001	4.745	19
0.0002825	0.234	2.019	2.115	51.151	0.001	0.001	180.000	0.001	4.942	19
0.0002855	0.226	2.080	2.043	51.283	0.001	0.001	180.000	0.001	5.091	19
0.0002885	0.219	2.131	1.989	51.415	0.001	0.001	180.000	0.001	5.206	19
0.0002915	0.212	2.168	1.951	51.550	0.001	0.001	180.000	0.001	5.286	19
0.0002945	0.206	2.191	1.928	51.686	0.001	0.001	180.000	0.001	5.328	19
0.0002975	0.199	2.195	1.918	51.823	0.001	0.001	180.000	0.001	5.330	19
0.0003005	0.193	2.182	1.917	51.963	0.001	0.001	180.000	0.001	5.293	19
0.0003035	0.187	2.152	1.923	52.104	0.001	0.001	180.000	0.001	5.219	19
0.0003065	0.182	2.106	1.929	52.246	0.001	0.001	180.000	0.001	5.108	19
0.0003095	0.176	2.047	1.931	52.389	0.001	0.001	180.000	0.001	4.965	19
0.0003125	0.171	1.980	1.924	52.533	0.001	0.001	180.000	0.001	4.764	19
0.0003155	0.166	1.907	1.903	52.677	0.001	0.001	180.000	0.001	4.599	19
0.0003185	0.161	1.831	1.865	52.820	0.001	0.001	180.000	0.001	4.385	19
0.0003215	0.157	1.756	1.807	52.963	0.001	0.001	180.000	0.001	4.144	19
0.0003245	0.152	1.684	1.733	53.105	0.001	0.001	180.000	0.001	3.930	19
0.0003275	0.148	1.614	1.644	53.247	0.001	0.001	180.000	0.001	3.882	20
0.0003305	0.144	1.549	1.550	53.388	0.001	0.001	180.000	0.001	4.050	20
0.0003335	0.141	1.508	1.485	53.482	0.001	0.001	180.000	0.001	4.154	20
0.0003365	0.138	1.469	1.420	53.576	0.001	0.001	180.000	0.001	4.250	20
0.0003395	0.136	1.432	1.358	53.670	0.001	0.001	180.000	0.001	4.332	20
0.0003385	0.133	1.396	1.301	53.764	0.001	0.001	180.000	0.001	4.409	20
0.0003405	0.131	1.363	1.251	53.859	0.001	0.001	180.000	0.001	4.474	20
0.0003425	0.128	1.332	1.209	53.954	0.001	0.001	180.000	0.001	4.528	20
0.0003445	0.126	1.303	1.176	54.049	0.001	0.001	180.000	0.001	4.569	20
0.0003465	0.124	1.275	1.155	54.146	0.001	0.001	180.000	0.001	4.597	20
0.0003485	0.122	1.250	1.144	54.242	0.001	0.001	180.000	0.001	4.612	20
0.0003505	0.119	1.226	1.146	54.339	0.001	0.001	180.000	0.001	4.614	20
0.0003525	0.117	1.204	1.160	54.437	0.001	0.001	180.000	0.001	4.603	20
0.0003545	0.115	1.184	1.187	54.535	0.001	0.001	180.000	0.001	4.578	20
0.0003565	0.113	1.165	1.226	54.633	0.001	0.001	180.000	0.001	4.541	20
0.0003585	0.111	1.149	1.275	54.731	0.001	0.001	180.000	0.001	4.491	20
0.0003605	0.109	1.134	1.335	54.830	0.001	0.001	180.000	0.001	4.429	20
0.0003625	0.107	1.122	1.403	54.927	0.001	0.001	180.000	0.001	4.357	20
0.0003645	0.106	1.112	1.477	55.024	0.001	0.001	180.000	0.001	4.279	20
0.0003665	0.104	1.103	1.554	55.121	0.001	0.001	180.000	0.001	4.187	20
0.0003685	0.102	1.097	1.627	55.216	0.001	0.001	180.000	0.001	4.087	20
0.0003705	0.100	1.094	1.702	55.309	0.001	0.001	180.000	0.001	3.979	20
0.0003725	0.099	1.092	1.771	55.401	0.001	0.001	180.000	0.001	3.866	20
0.0003745	0.097	1.093	1.831	55.491	0.001	0.001	180.000	0.001	3.747	20
0.0003765	0.095	1.097	1.878	55.579	0.001	0.001	180.000	0.001	3.624	20
0.0003785	0.094	1.102	1.911	55.665	0.001	0.001	180.000	0.001	3.498	20

TABLE IV. (Contd.)

0.0003865	0.092	1.111	1.926	55.749	0.001	0.001	180.000	0.001	3.379	21
0.0003825	0.091	1.121	1.924	55.831	0.001	0.001	180.000	0.001	3.473	21
0.0003845	0.089	1.135	1.903	55.910	0.001	0.001	180.000	0.001	3.562	21
0.0003865	0.088	1.150	1.864	55.987	0.001	0.001	180.000	0.001	3.644	21
0.0003885	0.086	1.167	1.809	56.063	0.001	0.001	180.000	0.001	3.720	21
0.0003915	0.084	1.196	1.696	56.172	0.001	0.001	180.000	0.001	3.817	21
0.0003945	0.082	1.228	1.566	56.280	0.001	0.001	180.000	0.001	3.895	21
0.0003975	0.080	1.240	1.425	56.385	0.001	0.001	180.000	0.001	3.953	21
0.0004005	0.078	1.291	1.286	56.491	0.001	0.001	180.000	0.001	3.990	21
0.0004035	0.076	1.319	1.157	56.597	0.001	0.001	180.000	0.001	4.005	21
0.0004065	0.075	1.343	1.046	56.705	0.001	0.001	180.000	0.001	3.997	21
0.0004095	0.073	1.359	0.957	56.815	0.001	0.001	180.000	0.001	3.947	21
0.0004125	0.071	1.368	0.893	56.928	0.001	0.001	180.000	0.001	3.915	21
0.0004155	0.069	1.368	0.854	57.045	0.001	0.001	180.000	0.001	3.843	21
0.0004185	0.068	1.359	0.842	57.165	0.001	0.001	180.000	0.001	3.752	21
0.0004215	0.066	1.341	0.854	57.289	0.001	0.001	180.000	0.001	3.644	21
0.0004245	0.065	1.314	0.888	57.417	0.001	0.001	180.000	0.001	3.522	21
0.0004275	0.063	1.281	0.939	57.547	0.001	0.001	180.000	0.001	3.387	21
0.0004305	0.062	1.243	1.002	57.679	0.001	0.001	180.000	0.001	3.244	21
0.0004335	0.061	1.201	1.068	57.813	0.001	0.001	180.000	0.001	3.093	21
0.0004365	0.059	1.159	1.130	57.947	0.001	0.001	180.000	0.001	3.046	22
0.0004395	0.058	1.116	1.181	58.081	0.001	0.001	180.000	0.001	3.157	22
0.0004425	0.057	1.074	1.215	58.215	0.001	0.001	180.000	0.001	3.257	22
0.0004455	0.055	1.035	1.230	58.346	0.001	0.001	180.000	0.001	3.345	22
0.0004485	0.054	0.997	1.224	58.477	0.001	0.001	180.000	0.001	3.418	22
0.0004515	0.053	0.963	1.200	58.606	0.001	0.001	180.000	0.001	3.476	22
0.0004545	0.052	0.931	1.163	58.733	0.001	0.001	180.000	0.001	3.518	22
0.0004575	0.051	0.901	1.118	58.858	0.001	0.001	180.000	0.001	3.542	22
0.0004605	0.050	0.874	1.072	58.983	0.001	0.001	180.000	0.001	3.548	22
0.0004635	0.049	0.849	1.033	59.106	0.001	0.001	180.000	0.001	3.536	22
0.0004665	0.048	0.826	1.004	59.229	0.001	0.001	180.000	0.001	3.507	22
0.0004695	0.047	0.804	0.991	59.351	0.001	0.001	180.000	0.001	3.461	22
0.0004725	0.046	0.785	0.996	59.473	0.001	0.001	180.000	0.001	3.399	22
0.0004755	0.045	0.768	1.018	59.594	0.001	0.001	180.000	0.001	3.322	22
0.0004785	0.044	0.752	1.057	59.714	0.001	0.001	180.000	0.001	3.231	22
0.0004815	0.043	0.738	1.107	59.832	0.001	0.001	180.000	0.001	3.129	22
0.0004845	0.042	0.727	1.163	59.949	0.001	0.001	180.000	0.001	3.017	22
0.0004875	0.042	0.720	1.200	60.025	0.001	0.001	180.000	0.001	2.939	22
0.0004895	0.041	0.714	1.232	60.100	0.001	0.001	180.000	0.001	2.858	22
0.0004905	0.041	0.708	1.257	60.174	0.001	0.001	180.000	0.001	2.775	22
0.0004925	0.040	0.704	1.273	60.246	0.001	0.001	180.000	0.001	2.738	23
0.0004945	0.039	0.701	1.278	60.316	0.001	0.001	180.000	0.001	2.682	23
0.0004965	0.039	0.699	1.269	60.384	0.001	0.001	180.000	0.001	2.643	23
0.0004985	0.038	0.698	1.246	60.451	0.001	0.001	180.000	0.001	2.599	23
0.0005005	0.038	0.697	1.207	60.516	0.001	0.001	180.000	0.001	2.571	23
0.0005035	0.037	0.698	1.117	60.609	0.001	0.001	180.000	0.001	3.037	23
0.0005065	0.036	0.701	1.000	60.699	0.001	0.001	180.000	0.001	3.092	23
0.0005095	0.036	0.706	0.865	60.787	0.001	0.001	180.000	0.001	3.133	23
0.0005125	0.035	0.711	0.723	60.873	0.001	0.001	180.000	0.001	3.161	23
0.0005155	0.034	0.717	0.584	60.960	0.001	0.001	180.000	0.001	3.174	23
0.0005185	0.034	0.723	0.457	61.047	0.001	0.001	180.000	0.001	3.174	23
0.0005215	0.033	0.729	0.348	61.136	0.001	0.001	180.000	0.001	3.158	23
0.0005245	0.033	0.733	0.262	61.229	0.001	0.001	180.000	0.001	3.129	23
0.0005275	0.032	0.737	0.202	61.324	0.002	0.001	180.000	0.001	3.086	23
0.0005305	0.031	0.740	0.170	61.424	0.003	0.001	180.000	0.001	3.030	23
0.0005335	0.031	0.740	0.165	61.528	0.003	0.001	180.000	0.001	2.963	23
0.0005365	0.030	0.740	0.188	61.636	0.004	0.001	179.999	0.001	2.885	23
0.0005395	0.030	0.737	0.239	61.749	0.005	0.001	179.999	0.001	2.797	23
0.0005425	0.029	0.733	0.313	61.865	0.006	0.001	179.999	0.001	2.702	23
0.0005455	0.029	0.728	0.405	61.984	0.007	0.001	179.999	0.001	2.600	23
0.0005485	0.028	0.721	0.507	62.104	0.009	0.001	179.999	0.001	2.493	23
0.0005515	0.028	0.714	0.608	62.226	0.010	0.001	179.999	0.001	2.526	24
0.0005545	0.027	0.707	0.699	62.347	0.012	0.001	179.999	0.001	2.602	24
0.0005575	0.027	0.700	0.770	62.468	0.014	0.001	179.999	0.001	2.670	24
0.0005605	0.027	0.692	0.814	62.587	0.016	0.001	179.999	0.001	2.729	24
0.0005635	0.026	0.685	0.832	62.705	0.019	0.001	179.998	0.001	2.778	24
0.0005665	0.026	0.678	0.826	62.821	0.021	0.001	179.998	0.001	2.817	24
0.0005695	0.026	0.671	0.801	62.936	0.025	0.001	179.998	0.001	2.844	24
0.0005725	0.025	0.664	0.767	63.050	0.028	0.001	179.998	0.001	2.860	24
0.0005755	0.025	0.657	0.733	63.164	0.032	0.001	179.998	0.001	2.864	24
0.0005785	0.025	0.649	0.707	63.277	0.037	0.001	179.998	0.001	2.856	24
0.0005815	0.025	0.641	0.697	63.390	0.042	0.001	179.997	0.001	2.836	24

TABLE IV. (Contd.)

0.0005845	0.024	0.633	0.708	63.504	0.047	0.001	179.997	0.001	2.805	24
0.0005875	0.024	0.624	0.743	63.618	0.053	0.001	179.997	0.001	2.743	24
0.0005905	0.024	0.614	0.800	63.732	0.060	0.001	179.997	0.001	2.711	24
0.0005935	0.024	0.605	0.874	63.846	0.068	0.001	179.997	0.001	2.649	24
0.0005965	0.024	0.595	0.959	63.958	0.076	0.001	179.997	0.001	2.579	24
0.0005995	0.024	0.586	1.043	64.067	0.085	0.001	179.997	0.001	2.501	24
0.0006025	0.024	0.578	1.113	64.173	0.095	0.001	179.997	0.001	2.417	24
0.0006055	0.024	0.570	1.158	64.276	0.106	0.001	179.997	0.001	2.328	24
0.0006085	0.024	0.564	1.168	64.374	0.118	0.001	179.997	0.001	2.282	25
0.0006115	0.024	0.558	1.136	64.467	0.132	0.001	179.998	0.001	2.350	25
0.0006145	0.024	0.552	1.062	64.555	0.146	0.001	179.998	0.001	2.412	25
0.0006175	0.024	0.548	0.952	64.640	0.162	0.001	179.998	0.001	2.446	25
0.0006205	0.025	0.544	0.816	64.721	0.179	0.001	179.999	0.001	2.512	25
0.0006235	0.025	0.540	0.668	64.800	0.197	0.001	180.000	0.001	2.550	25
0.0006265	0.025	0.536	0.521	64.878	0.216	0.001	180.001	0.001	2.579	25
0.0006295	0.026	0.531	0.385	64.957	0.238	0.001	180.003	0.001	2.508	25
0.0006325	0.026	0.526	0.271	65.037	0.260	0.001	180.004	0.001	2.607	25
0.0006355	0.027	0.520	0.182	65.119	0.284	0.001	180.006	0.001	2.607	25
0.0006385	0.028	0.513	0.122	65.204	0.310	0.001	180.009	0.001	2.506	25
0.0006415	0.028	0.506	0.092	65.293	0.338	0.001	180.012	0.001	2.576	25
0.0006445	0.029	0.497	0.090	65.386	0.366	0.001	180.015	0.001	2.546	25
0.0006475	0.030	0.488	0.114	65.482	0.397	0.001	180.019	0.001	2.508	25
0.0006505	0.031	0.479	0.162	65.581	0.429	0.001	180.024	0.001	2.441	25
0.0006535	0.032	0.469	0.228	65.683	0.462	0.001	180.029	0.001	2.406	25
0.0006565	0.033	0.459	0.305	65.787	0.497	0.001	180.035	0.001	2.344	25
0.0006595	0.034	0.450	0.383	65.891	0.532	0.001	180.042	0.001	2.276	25
0.0006625	0.036	0.440	0.453	65.995	0.569	0.001	180.050	0.001	2.203	25
0.0006655	0.037	0.432	0.505	66.099	0.607	0.001	180.059	0.001	2.125	25
0.0006685	0.038	0.423	0.533	66.201	0.646	0.001	180.069	0.001	2.133	26
0.0006715	0.040	0.416	0.531	66.301	0.685	0.001	180.080	0.001	2.189	26
0.0006745	0.041	0.409	0.501	66.400	0.724	0.001	180.093	0.001	2.239	26
0.0006775	0.043	0.402	0.451	66.498	0.763	0.001	180.107	0.001	2.283	26
0.0006805	0.045	0.396	0.390	66.595	0.802	0.001	180.122	0.001	2.349	26
0.0006835	0.047	0.390	0.331	66.691	0.841	0.001	180.139	0.001	2.346	26
0.0006865	0.048	0.385	0.284	66.789	0.878	0.001	180.158	0.001	2.345	26
0.0006895	0.050	0.379	0.258	66.888	0.914	0.001	180.178	0.001	2.375	26
0.0006925	0.052	0.374	0.259	66.988	0.948	0.001	180.200	0.001	2.375	26
0.0006955	0.054	0.369	0.289	67.090	0.980	0.001	180.225	0.001	2.367	26
0.0006985	0.056	0.364	0.350	67.193	1.010	0.001	180.251	0.001	2.348	26
0.0007015	0.058	0.359	0.434	67.297	1.036	0.001	180.279	0.001	2.320	26
0.0007045	0.061	0.355	0.534	67.401	1.059	0.001	180.309	0.001	2.282	26
0.0007075	0.063	0.351	0.636	67.503	1.078	0.001	180.342	0.001	2.236	26
0.0007105	0.065	0.348	0.727	67.602	1.093	0.001	180.377	0.001	2.181	26
0.0007135	0.068	0.345	0.791	67.698	1.103	0.001	180.414	0.001	2.118	26
0.0007165	0.070	0.343	0.818	67.789	1.108	0.001	180.454	0.001	2.048	26
0.0007185	0.072	0.342	0.817	67.848	1.109	0.001	180.481	0.001	1.998	26
0.0007205	0.073	0.342	0.796	67.906	1.107	0.001	180.510	0.001	1.945	26
0.0007225	0.075	0.342	0.755	67.961	1.103	0.001	180.539	0.001	1.890	26
0.0007245	0.077	0.341	0.698	68.014	1.097	0.001	180.570	0.001	1.833	26
0.0007265	0.079	0.342	0.627	68.066	1.087	0.001	180.602	0.001	1.773	26
0.0007285	0.080	0.342	0.547	68.117	1.075	0.001	180.634	0.001	1.712	26
0.0007315	0.083	0.343	0.416	68.191	1.051	0.001	180.686	0.001	1.616	26
0.0007345	0.086	0.344	0.291	68.265	1.021	0.001	180.740	0.001	1.522	26
0.0007375	0.089	0.346	0.183	68.340	0.984	0.001	180.796	0.001	1.421	26
0.0007405	0.092	0.347	0.102	68.416	0.942	0.001	180.855	0.001	1.347	23
0.0007435	0.095	0.348	0.051	68.495	0.895	0.001	180.916	0.001	1.323	23
0.0007465	0.098	0.349	0.034	68.577	0.843	0.001	180.978	0.001	1.282	24
0.0007495	0.101	0.349	0.048	68.663	0.786	0.001	181.043	0.001	1.316	24
0.0007525	0.105	0.349	0.091	68.751	0.725	0.001	181.110	0.001	1.348	24
0.0007555	0.108	0.349	0.156	68.842	0.660	0.001	181.178	0.001	1.376	24
0.0007585	0.111	0.348	0.234	68.934	0.593	0.001	181.248	0.001	1.401	24
0.0007605	0.114	0.348	0.286	68.996	0.547	0.001	181.295	0.001	1.416	24
0.0007625	0.116	0.348	0.336	69.057	0.500	0.001	181.342	0.001	1.430	24
0.0007645	0.118	0.347	0.380	69.119	0.453	0.001	181.391	0.001	1.441	24
0.0007665	0.121	0.347	0.415	69.180	0.405	0.001	181.439	0.001	1.451	24
0.0007685	0.123	0.347	0.440	69.241	0.356	0.001	181.489	0.001	1.458	24
0.0007705	0.126	0.346	0.453	69.301	0.308	0.001	181.538	0.001	1.443	24
0.0007725	0.128	0.346	0.452	69.360	0.259	0.001	181.588	0.001	1.446	24
0.0007745	0.131	0.346	0.438	69.418	0.211	0.001	181.638	0.001	1.448	24
0.0007765	0.133	0.346	0.413	69.476	0.163	0.001	181.688	0.001	1.446	24
0.0007785	0.136	0.346	0.379	69.534	0.115	0.001	181.739	0.001	1.443	24
0.0007805	0.139	0.346	0.340	69.591	0.068	0.001	181.790	0.001	1.457	24

TABLE IV. (Contd.)

0.0007825	0.142	0.346	0.298	69.649	0.021	0.001	181.840	0.001	1.449	24
0.0007845	0.144	0.345	0.258	69.706	0.000	0.001	181.891	0.001	1.439	24
0.0007865	0.147	0.345	0.222	69.764	0.000	0.001	181.942	0.001	1.437	24
0.0007885	0.150	0.345	0.194	69.822	0.000	0.001	181.992	0.001	1.412	24
0.0007905	0.153	0.344	0.176	69.882	0.000	0.001	182.043	0.001	1.395	24
0.0007925	0.156	0.343	0.169	69.942	0.000	0.001	182.094	0.001	1.377	24
0.0007945	0.159	0.342	0.175	70.003	0.000	0.001	182.145	0.001	1.356	24
0.0007965	0.162	0.341	0.194	70.065	0.000	0.001	182.195	0.001	1.334	24
0.0007985	0.165	0.340	0.225	70.128	0.000	0.001	182.246	0.001	1.309	24
0.0008005	0.168	0.338	0.267	70.192	0.000	0.001	182.297	0.001	1.283	24
0.0008025	0.171	0.337	0.319	70.256	0.000	0.001	182.347	0.001	1.256	24
0.0008045	0.174	0.335	0.376	70.320	0.000	0.001	182.398	0.001	1.227	24
0.0008065	0.177	0.333	0.436	70.384	0.000	0.001	182.449	0.001	1.197	24
0.0008085	0.181	0.331	0.494	70.447	0.000	0.001	182.499	0.001	1.165	24
0.0008105	0.184	0.330	0.546	70.510	0.000	0.001	182.550	0.001	1.132	24
0.0008125	0.187	0.328	0.589	70.572	0.000	0.001	182.601	0.001	1.099	24
0.0008145	0.191	0.327	0.618	70.632	0.000	0.001	182.652	0.001	1.067	22
0.0008165	0.194	0.325	0.632	70.692	0.000	0.001	182.702	0.001	1.009	22
0.0008185	0.198	0.324	0.628	70.750	0.000	0.001	182.753	0.001	1.100	22
0.0008205	0.201	0.322	0.608	70.806	0.000	0.001	182.804	0.001	1.099	22
0.0008225	0.205	0.321	0.571	70.861	0.000	0.001	182.854	0.001	1.076	22
0.0008245	0.209	0.320	0.522	70.915	0.000	0.001	182.905	0.001	1.092	22
0.0008265	0.212	0.319	0.462	70.968	0.000	0.001	182.956	0.001	1.086	22
0.0008285	0.216	0.318	0.397	71.020	0.000	0.001	183.006	0.001	1.079	22
0.0008305	0.220	0.317	0.331	71.073	0.000	0.002	183.057	0.001	1.070	22
0.0008325	0.224	0.316	0.268	71.125	0.000	0.002	183.107	0.001	1.040	22
0.0008345	0.227	0.314	0.211	71.177	0.000	0.002	183.158	0.001	1.048	22
0.0008365	0.231	0.313	0.163	71.230	0.000	0.002	183.209	0.001	1.034	22
0.0008385	0.235	0.312	0.127	71.284	0.000	0.002	183.259	0.001	1.020	22
0.0008405	0.239	0.310	0.105	71.339	0.000	0.002	183.310	0.001	1.004	22
0.0008425	0.244	0.308	0.096	71.395	0.000	0.002	183.361	0.001	0.987	22
0.0008445	0.248	0.306	0.101	71.452	0.000	0.002	183.411	0.001	0.949	22
0.0008465	0.252	0.304	0.119	71.510	0.000	0.002	183.462	0.001	0.990	22
0.0008485	0.256	0.302	0.150	71.570	0.000	0.002	183.513	0.001	0.929	22
0.0008505	0.260	0.300	0.190	71.630	0.000	0.002	183.563	0.001	0.908	22
0.0008525	0.265	0.297	0.238	71.690	0.000	0.002	183.614	0.001	0.887	15
0.0008545	0.269	0.295	0.290	71.751	0.000	0.002	183.664	0.001	0.893	15
0.0008565	0.273	0.292	0.342	71.812	0.000	0.002	183.715	0.001	0.891	15
0.0008585	0.278	0.290	0.391	71.872	0.000	0.002	183.766	0.001	0.881	15
0.0008605	0.282	0.288	0.432	71.932	0.000	0.002	183.816	0.001	0.845	15
0.0008625	0.287	0.285	0.464	71.991	0.000	0.002	183.867	0.001	0.840	15
0.0008645	0.291	0.283	0.484	72.049	0.000	0.002	183.918	0.001	0.810	15
0.0008665	0.296	0.281	0.492	72.106	0.000	0.002	183.968	0.001	0.809	23
0.0008685	0.301	0.279	0.486	72.162	0.000	0.002	184.019	0.001	0.815	23
0.0008705	0.305	0.277	0.469	72.217	0.000	0.002	184.069	0.001	0.821	23
0.0008725	0.310	0.275	0.443	72.271	0.000	0.002	184.120	0.001	0.825	23
0.0008745	0.315	0.273	0.410	72.325	0.000	0.002	184.170	0.001	0.829	23
0.0008765	0.320	0.272	0.374	72.378	0.000	0.002	184.221	0.001	0.831	23
0.0008785	0.325	0.270	0.339	72.430	0.000	0.002	184.272	0.001	0.831	23
0.0008805	0.330	0.268	0.307	72.483	0.000	0.002	184.322	0.001	0.831	23
0.0008825	0.335	0.267	0.282	72.535	0.000	0.002	184.373	0.001	0.829	23
0.0008845	0.340	0.265	0.265	72.588	0.000	0.002	184.423	0.001	0.826	23
0.0008865	0.345	0.263	0.258	72.641	0.000	0.002	184.474	0.001	0.821	23
0.0008885	0.350	0.262	0.262	72.694	0.000	0.002	184.525	0.001	0.815	23
0.0008905	0.355	0.260	0.276	72.747	0.000	0.002	184.575	0.001	0.807	23
0.0008925	0.360	0.258	0.299	72.801	0.000	0.002	184.626	0.001	0.798	23
0.0008945	0.365	0.256	0.330	72.855	0.000	0.002	184.676	0.001	0.787	23
0.0008965	0.370	0.255	0.364	72.908	0.000	0.002	184.727	0.001	0.775	23
0.0008985	0.376	0.253	0.400	72.962	0.000	0.002	184.777	0.001	0.741	23
0.0009005	0.381	0.251	0.434	73.014	0.000	0.002	184.828	0.001	0.746	23
0.0009025	0.386	0.250	0.460	73.066	0.000	0.002	184.878	0.001	0.752	21
0.0009045	0.392	0.249	0.477	73.117	0.000	0.002	184.929	0.001	0.758	21
0.0009065	0.397	0.247	0.480	73.166	0.000	0.002	184.980	0.001	0.743	21
0.0009085	0.402	0.246	0.470	73.215	0.000	0.002	185.030	0.001	0.747	21
0.0009105	0.408	0.245	0.444	73.262	0.000	0.002	185.081	0.001	0.770	21
0.0009125	0.413	0.244	0.404	73.308	0.000	0.002	185.131	0.001	0.771	21
0.0009145	0.419	0.243	0.352	73.353	0.000	0.002	185.182	0.001	0.772	21
0.0009165	0.424	0.242	0.291	73.398	0.000	0.002	185.232	0.001	0.771	21
0.0009185	0.429	0.241	0.225	73.442	0.000	0.002	185.283	0.001	0.748	21
0.0009205	0.435	0.240	0.157	73.487	0.000	0.002	185.333	0.001	0.771	16
0.0009225	0.440	0.239	0.092	73.531	0.000	0.002	185.384	0.001	0.779	16
0.0009245	0.446	0.238	0.034	73.576	0.000	0.002	185.434	0.001	0.783	16

TABLE IV. (Contd.)

0.0009245	0.452	0.237	0.000	73.623	0.000	0.002	185.485	0.001	0.742	16
0.0009285	0.457	0.236	0.000	73.670	0.000	0.002	185.535	0.001	0.778	16
0.0009325	0.463	0.235	0.000	73.719	0.000	0.002	185.586	0.001	0.770	16
0.0009365	0.468	0.234	0.000	73.768	0.000	0.002	185.636	0.001	0.758	16
0.0009405	0.474	0.233	0.000	73.819	0.000	0.002	185.687	0.001	0.742	16
0.0009445	0.479	0.232	0.000	73.871	0.000	0.002	185.737	0.001	0.724	16
0.0009485	0.485	0.230	0.000	73.924	0.000	0.002	185.788	0.001	0.702	16
0.0009525	0.490	0.229	0.000	73.979	0.000	0.002	185.838	0.001	0.678	16
0.0009565	0.496	0.227	0.000	74.034	0.000	0.002	185.889	0.001	0.651	16
0.0009605	0.501	0.226	0.050	74.090	0.000	0.002	185.939	0.001	0.629	21
0.0009645	0.507	0.224	0.109	74.147	0.000	0.002	185.990	0.001	0.611	21
0.0009685	0.512	0.223	0.171	74.205	0.000	0.002	186.040	0.001	0.593	21
0.0009725	0.518	0.221	0.232	74.263	0.000	0.002	186.091	0.001	0.574	21
0.0009765	0.523	0.220	0.286	74.320	0.000	0.002	186.141	0.001	0.579	15
0.0009805	0.529	0.218	0.330	74.376	0.000	0.002	186.192	0.001	0.602	15
0.0009845	0.534	0.217	0.359	74.432	0.000	0.002	186.242	0.001	0.622	15
0.0009885	0.540	0.216	0.371	74.487	0.000	0.002	186.293	0.001	0.637	15
0.0009925	0.545	0.214	0.365	74.541	0.000	0.002	186.343	0.001	0.647	15
0.0009965	0.550	0.213	0.342	74.594	0.000	0.002	186.393	0.001	0.652	15
0.0009985	0.556	0.212	0.304	74.646	0.000	0.002	186.444	0.001	0.651	15
0.0009995	0.561	0.211	0.254	74.697	0.000	0.002	186.494	0.001	0.645	15
0.0009995	0.566	0.210	0.197	74.748	0.000	0.002	186.545	0.001	0.633	15
0.0009995	0.571	0.209	0.138	74.799	0.000	0.002	186.595	0.001	0.626	18
0.0009995	0.576	0.208	0.083	74.850	0.000	0.002	186.646	0.001	0.628	18
0.0009995	0.581	0.207	0.034	74.902	0.000	0.002	186.696	0.001	0.627	18
0.0009995	0.587	0.206	0.000	74.954	0.000	0.002	186.747	0.001	0.626	18
0.0009995	0.592	0.205	0.000	75.008	0.000	0.002	186.797	0.001	0.623	18
0.0009995	0.596	0.204	0.000	75.063	0.000	0.002	186.847	0.001	0.618	18
0.0009995	0.601	0.203	0.000	75.119	0.000	0.002	186.898	0.001	0.613	18
0.0009995	0.606	0.202	0.000	75.176	0.000	0.002	186.948	0.001	0.606	18
0.0009995	0.611	0.200	0.010	75.235	0.000	0.002	186.999	0.001	0.598	18
0.0009995	0.616	0.199	0.066	75.295	0.000	0.002	187.049	0.001	0.589	18
0.0009995	0.620	0.198	0.139	75.355	0.000	0.002	187.099	0.001	0.579	18
0.0009995	0.625	0.196	0.225	75.416	0.000	0.002	187.150	0.001	0.571	14
0.0009995	0.629	0.195	0.319	75.477	0.000	0.002	187.200	0.001	0.633	14
0.0009995	0.634	0.194	0.415	75.537	0.000	0.002	187.251	0.001	0.648	14

Fig. 8. Graphical Representation of the Computer Results for the Simulated Infinite Water Medium

Fig. 9
Pressure-Time Curve for the Argon
at the Water-Argon Interface

Fig. 10
Pressure-Time Curve for the TNT
at the TNT-Water Interface

Fig. 11. Pressure-Time Curve for the Inside Face of the Concrete Vessel

Fig. 12. Location-Time Curve for the TNT-Water Interface

Fig. 13. Location-Time Curve for the Water-Argon Interface

Fig. 14. Pressure-Time Curve for the Water at the Water-Argon Interface

Fig. 15. Pressure-Time Curve for the Water at the TNT-Water Interface

Fig. 16. Maximum Pressure in the System Outside of the Gas Bubble

Fig. 17. Pressure-Time Curve for the Center of the Gas Bubble

Fig. 18. Total Energy of the System as a Function of Time

IV. DISCUSSION OF RESULTS

A. Comparison of the Computer Results for the Simulated Infinite Water Medium with Experimental Measurements

To be useful, any new theoretical analysis should be verified experimentally. Experimental measurements of shock-wave pressures have been performed. However, the measurement of shock-wave pressures in the near proximity of the charge is prohibited by the very high pressures there.

One set of experimental data in connection with the Kirkwood-Brinkley theory is presented in Table V. The Kirkwood-Brinkley theory was formulated on the basis of these experimental measurements. Peak pressures in the shock front at distances closer to the charge were then extrapolated by use of their formulation. The values of the peak pressure at various locations, which were calculated by the use of this theory, are given in Table V for reference. The value of the measured pressure given for 25 charge radii appears to be in error. By noticing that the trend of the difference between two successive values of the measured pressure is decreasing, a more appropriate value* for the peak pressure measured at 25 charge radii would be 5,440 psi. Therefore, the peak pressure at this location is not used for comparison.

TABLE V. Peak Pressure in the Spherical Shock Wave from TNT^a

Distance, Charged Radii	Kirkwood-Brinkley Theory ^b Peak Pressure, psi		Penney-Dasgupta Theory Peak Pressure, psi		Pseudoviscous Method Peak Pressure, psi	% Difference for Pseudoviscous Method and Measured Pressure
	Calculated	Measured	Calculated	Calculated		
1	460,000		761,000		(1,512,000)	
2	167,000		141,000		168,000	
3	91,400		67,000		87,130	
5	42,800		30,700		40,030	
7	26,600		19,600		25,020	
10	16,400	16,250	12,600		15,700	3.5
15	9,670	9,640	7,830		9,460	1.9
20	6,720	6,720	5,690		6,640	1.2
25	1,090	1,090	4,470		(5,530)	

^aThe pressures given in this table can be compared on the basis of the similarity principle (see Ref. 5).

^bThese data were obtained from Ref. 5, page 145, Table 4.2.

Values from the Penney-Dasgupta theory are also given in Table V for reference. These values, obtained by the use of the formula

$$P_m(\text{psi}) = 103,000 \left(\frac{a_0}{R} \right) e^{2a_0/R}, \quad (60)$$

*Extrapolated by simple proportion.

agreed closely to the values of the peak pressure in the shock front determined in their step-by-step solution over the range $a_0 < R < 6a_0$.

The values of the peak pressure in the shock front given in Table V for the von Neumann-Richtmeyer pseudoviscous method* were evaluated on the basis of linear interpolation of the computer results given in Table III, except the values of the peak pressure for 1 and 25 charge radii. These two values were extrapolated by the use of the equation

$$P_m = \exp \left[-102.82 \left(\frac{a_0}{R} \right)^6 + 367.79 \left(\frac{a_0}{R} \right)^5 - 504.75 \left(\frac{a_0}{R} \right)^4 + 345.27 \left(\frac{a_0}{R} \right)^3 - 129.58 \left(\frac{a_0}{R} \right)^2 + 30.744 \left(\frac{a_0}{R} \right) - 2.0076 \right], \quad (61)$$

which is an analytic fit** to the data in columns 1 and 4 of Table III.

Another analytic fit was obtained for the data in columns 1 and 2 of Table III of the form

$$P_m = \exp[0.0031079(\ln t)^4 + 0.10858(\ln t)^3 + 1.310(\ln t)^2 + 5.9200(\ln t) + 6.6053]. \quad (62)$$

Equations 61 and 62 are presented graphically in Figs. 8A and 8B, respectively. These two figures may be used together to predict, for a particular distance from the charge, the magnitude of the peak pressure in the shock front and the time at which this peak pressure will occur.

The maximum difference between the measured peak pressures and the peak pressures determined by the pseudoviscous method is 3.5%. There is also very good agreement between the values calculated from the pseudoviscous method and the values calculated from the Kirkwood-Brinkley theory, except in the vicinity of the TNT boundary, where the value calculated by the Penney-Dasgupta theory is closer to the value calculated by the pseudoviscous method. In his book, Underwater Explosions,⁵ Cole states that the Penney-Dasgupta theory seems to be more reliable than the Kirkwood-Brinkley theory for calculations at distances within several charge radii and that the low peak pressures predicted by Penney and Dasgupta are attributed to the low energy release they assumed in the initial conditions (800 cal/gm).†

* 1 kb = 14,505 psi.

** See Appendix B for a description of the computer program.

† A more accepted value is 1100 cal/gm for detonation in a nonoxygen atmosphere. See Appendix F.

Penney* (1943) suggested that calculations of shock waves could profitably be made by using step-by-step calculations out to a few charge radii to provide initial conditions for use of the Kirkwood-Brinkley propagation theory (or some alternative theory) at greater distances. This is precisely what is achieved by the pseudoviscous method alone.

The peak pressure transmitted to the water can only be compared in a qualitative sense. One set of data for comparison is given in Table VI. Since the peak pressure in the transmitted wave is directly related to the peak pressure in the detonation wave, the magnitude of the peak pressure in the transmitted wave obtained from the pseudoviscous method seems to be appropriate by comparison.

TABLE VI. Values of Various Hydrodynamical Parameters for TNT

Parameter	Ref. 17, Page 115	Pseudo- viscous Method	% Difference on Basis of Col. 1
Density of Packed TNT, gm/cm ³	1.60	1.51	5.6
Peak Pressure in Detonation Wave, kb	190	150	21.0
Peak Pressure in Transmitted Wave, kb	133.5	104.2 ^a	22.0

^aObtained from Eq. 61 for $a_0/R = 1$.

On the basis of the above comparisons, the results obtained by the pseudoviscous method for the simulated infinite water medium are acceptable.

B. Computer Results Obtained for the Simulated Sodium Reactor Installation

The computer results for the simulated sodium-reactor installation are presented graphically in Figs. 9-18. These figures show how the values of the pressure and the radial coordinate at several critical locations, which are a result of the destructive component, vary with time. However, the effect that the destructive component produces on the concrete containment structure can be determined completely from Figs. 9 and 11. The destructive component consists of the following three elements: (1) the shock wave, (2) the fluid momentum due to the passage of the shock wave, and (3) the gas bubble at high temperature and pressure pushing on the fluid. These components are described in the following paragraphs.

*See Ref. 5, page 141.

1. The Shock Wave

The shock wave is the fastest element of the destructive component and would, therefore, reach the concrete structure first. However, as the shock wave propagates outward through the water, the magnitude of the peak pressure in the front is decreased by dissipation. The peak pressure in the front, when it passes through the zone in the water adjacent to the water-argon interface, is 1.109 kb and occurs at time $t = 718.5 \mu\text{sec}$. As the shock wave encounters the water-argon interface, a wave of compression is transmitted into the argon, and a rarefaction wave is reflected back into the water. However, when a compressive wave is transmitted from a liquid to a gas, a large attenuation of the peak pressure in the shock front takes place. In this problem, the magnitude of the peak pressure in the shock front of the compressive wave transmitted to the argon is so small that the shock wave is considered to be completely attenuated upon its passage into the argon. This result can be verified in the following manner:

The time interval between the occurrence of the peak pressure in the shock front in two adjacent zones in the water near the water-argon interface is less than $65 \mu\text{sec}$ (see Table III for values near $R = 180$). The speed of sound in water is about four times the speed of sound in argon (both at standard conditions). In this problem, the widths of the argon zones in the finite-difference net are 1.5 times the widths of the water zones. If a shock wave of appreciable magnitude is transmitted into the argon, a noticeable increase in the pressure should result in the zone of argon adjacent to the water-argon interface. This noticeable increase in the pressure should start to occur at approximately $390 \mu\text{sec}$ ($65 \times 4 \times 1.5$) later than when the maximum pressure occurs in the zone of water adjacent to water-argon interface ($718.5 \mu\text{sec}$ as given above), i.e., at time $t = 1.1 \text{ msec}$. Figure 9 shows the pressure-time curve for the zone of argon adjacent to the water-argon interface, and no noticeable pressure increase is indicated at time $t = 1.1 \text{ msec}$. In fact, the first visible increase in the pressure occurs at time $t > 1.5 \text{ msec}$. The pressure peak at approximately 2.0 msec , seen in Fig. 9, is a result of the next element of the destructive component.

Therefore, it is concluded for this particular problem that the shock wave has been completely attenuated during its passage through the water and the argon. Thus, the concrete containment structure is not subjected to a shock wave.

2. The Fluid Momentum due to the Passage of the Shock Wave

The second element of the destructive component to be considered is the fluid momentum due to the passage of the shock wave. As the shock wave propagates outward into the undisturbed water, it imparts

an outward velocity to the water particles behind the shock front. When the shock front reaches the water-argon interface, a rarefaction wave is reflected back into the water, i.e., toward the center of the system. This rarefaction wave gives the water particles behind its front an additional outward velocity.

The passage of the rarefaction wave leaves the water in a very undesirable state as the outer water particles are moving outward faster than those closer to the center of the system. This causes the water particles to separate and, therefore, void spaces to occur in the fluid. (Assumption 5* was used to describe this state of the liquid.) This state continues in the water until the outer particles are slowed down by the pressure increase in the argon. This allows the lagging water particles to "catch up."

The effect of this fluid motion on the concrete containment structure can be interpreted from Fig. 11, the pressure-time curve for the inside of the concrete vessel. The pressure peak that occurs at approximately 2.0 msec is a result of the initial rapid outward motion of the water particles due to the rarefaction wave. The pressure peak at approximately 4.5 msec is a result of the total outward flow of the fluid particles due to the passage of the shock wave and the rarefaction wave. As a result of this initial outward flow of the fluid, an impulse is produced on the concrete containment structure.

3. The Gas Bubble at High Temperature and Pressure Pushing on the Fluid

The slowest element of the destructive component is the gas bubble. In the production of an impulse on the concrete structure, the outward-flowing fluid particles are slowed down. As the pressure in the argon increases, all the fluid particles are given a velocity back toward the center of the sphere. This motion of the fluid particles causes the heretofore expanding sphere of hot gas to decrease in volume. The inward flow of the fluid particles continues until the pressure in the gas bubble increases to a point where it again reverses the direction of the fluid flow. This second outward motion of the fluid particles results in an additional impulse produced on the concrete structure. A third pressure peak, seen in Fig. 11 at approximately 20.0 msec, is a result of this second outward flow of the fluid and, in this case, is caused by the expanding gas bubble. Pulsations of the gas bubble would continue until a quasistatic state of internal pressure would result. The additional impulse on the concrete containment vessel due to each pulsation would decrease. The final effect of the gas bubble on the containment structure would be equivalent to a spherical pressure vessel under the action of a static internal pressure. Because of the cracking

*See page 35.

of the concrete, this idealized quasistatic condition would never be realized. The motion of the gas bubble interface is described in Fig. 12, and the motion of the water-argon interface is described in Fig. 13.

4. Summary

Since the shock wave is attenuated, the only remaining effect of the destructive component that the concrete containment vessel must resist is the impulse. The specific impulse per unit area of concrete at the inside surface is given by

$$I = \int_{t_0}^{t_1} P(t) dt, \quad (63)$$

where $P(t)$ is the pressure at the inside face of the concrete as a function of time. The magnitude of this impulse is related to the area under the pressure-time curve for the inside face of the concrete vessel shown in Fig. 11. Since the magnitude of the impulse can be determined, we can calculate the maximum strain rate to which the high-strength steel strands embedded in the concrete will be subjected. Correlation between the impulse and the strain rate is given in Chapter V.

Figure 18 shows the total energy in the system as a function of time. Since the idealized reactor system analyzed in this study is a closed system, the total energy in the system should remain constant. Constancy of the energy is shown in this figure to the degree of accuracy of the finite-difference system; it indicates that this conservation law is satisfied in the finite-difference solution.

Figures 10 and 14-17 are given for reference to show how the pressures at various critical locations vary with time.

V. STRAIN-RATE CALCULATIONS

The assumption that the concrete containment vessel is infinitely rigid during the action of the impulse appears to be justified because of the short duration over which the major part of the impulse occurs, i.e., 20 msec (see Fig. 11). It does not seem feasible that the huge concrete mass could respond in such a short time. Thus, the maximum strain rate that the steel strands would "see" can be calculated from the impulse obtained from Fig. 11.

The linear impulse is related to the change in the linear momentum by

$$I = \int_{t_0}^{t_1} \Sigma F_r dt = m(v_r - v_0). \quad (64)$$

Since force equals pressure times area, the impulse per unit area can be written

$$I = \int_{t_0}^{t_1} P_r(t) dt = m(v_r - v_0), \quad (65)$$

where the value of the integral is related to the area under the curve shown in Fig. 11. For the concrete mass initially at rest, i.e., $v_0 = 0$,

$$I = mv_r, \quad (66)$$

where I , m , and v_r are the impulse per unit area, mass per unit area, and radial velocity, respectively. The unit area is taken as a unit area on the inside surface of the concrete vessel. By rearrangement of the terms in Eq. 66, the resulting radial velocity of the concrete mass is

$$v_r = \frac{I}{m}. \quad (67)$$

The steel strands embedded in the concrete form circular hoops. The time rate of change of the circumference of any particular strand in terms of the time rate of change of the radius is given by

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}, \quad (68)$$

where C and r are the circumference and radius, respectively.

Since $v_r = dr/dt$, the instantaneous change in the circumference is

$$dC = 2\pi \frac{I}{m} dt. \quad (69)$$

Strain, ϵ , may be defined as the total change in length divided by the length over which the change occurred. In this case,

$$\epsilon = \frac{\Delta C}{C_0}, \quad (70)$$

where C_0 is the original circumference of the steel strand. The instantaneous time rate of change of the strain is given by

$$\frac{d\epsilon}{dt} = \lim_{\Delta t \rightarrow 0} \frac{1}{C_0} \frac{\Delta C}{\Delta t} = \frac{1}{C_0} \frac{dC}{dt}. \quad (71)$$

Substituting Eq. 69 into Eq. 71 and simplifying, the strain rate for any steel strand is given by

$$\dot{\epsilon} = \frac{I}{m \times r_0}, \quad (72)$$

where $\dot{\epsilon}$ represents the time derivative of strain and r_0 is the initial hoop radius of the steel strand.

For this problem, the values for the impulse per unit area* and mass per unit area** are

$$I = 2,607,150 \text{ gm-cm/sec/cm}^2$$

and

$$m = 1007.59 \text{ gm/cm}^2.$$

If r_0 is taken as the smallest possible value, i.e., $r_0 = 210 \text{ cm}$, then, by neglecting the energy absorbed in cracking the concrete, the maximum possible strain rate to which any strand will be subjected is

$$\dot{\epsilon}_{\max} = \frac{2,607,150}{1007.59 \times 210} = 12.32 \text{ sec}^{-1}.$$

*See Appendix B.

**See Appendix E.

Strain rates for metals can be categorized into four classes as shown in Table VII. (The conventional rate of strain for static testing is 10^{-3} sec^{-1} .)

TABLE VII. Strain-rate Classification for Metals^a

Strain Rate, sec^{-1}	Classification
$<10^0$	Low rate of strain
$10^0 - 10^3$	Moderate rate of strain
$10^3 - 10^4$	High rate of strain
$>10^4$	Very high rate of strain

^aThis table and the ensuing discussion is based upon information from Ref. 6, pp. 6.108-6.117.

The maximum strain rate for the strands determined in this analysis is 12.32 sec^{-1} . This is classified in the extreme lower range of moderate rate of strain. To our knowledge, no experimental tests have been performed on high-strength steel strands at this strain rate. However, Klinger¹¹ conducted low strain-rate experiments on the high-strength steel SAE 4340 in the range from the annealed condition (119,000 psi) to a nominal ultimate strength of 220,000 psi. His results indicate

that, in general, the strength properties increase with increasing strain rate. Moreover, the strain-rate effect becomes markedly less as the strength level of the steel is increased. In fact, the 220,000-psi steels (same ultimate stress range as for the steel strands) showed either no change or a slight loss in strength ($<3\%$) with increasing strain rate. No significant effect on the total elongation at any strength level was noted. The range of strain rates for his tests was $10^{-5} \text{ sec}^{-1} < \dot{\epsilon} < 10^1 \text{ sec}^{-1}$.

Thus, the static properties of the steel strands could be used as a good approximation for the design of the concrete containment structure. However, experimental verification of the mechanical properties of the steel strands at the design strain rate would be warranted.

VI. CONCLUSION

In this study, we attempted¹ to determine the effect that the destructive component produces on the concrete containment vessel so that the strain rate for the strands could be calculated, and² to obtain an insight as to how the destructive component changes with time. It is concluded that these objectives have been achieved. The pressure-time curve for the inside face of the concrete vessel, necessary for determining the strain rate to which the steel strands are subjected, was obtained by numerical integration of the differential equations using the von Neumann-Richtmeyer artificial-viscosity technique.

An important factor that influences the results of this problem is the thickness of the layer of inert gas. This gas layer is important because of its role in attenuating the shock wave. It is also effective in controlling the peak pressure at the inside face of the concrete by a buffering action. If the layer of inert gas is very thin, the effectiveness of the layer is reduced. Therefore, a relatively thick layer of gas should be provided because it is essential that the shock wave be eliminated. If a strong shock wave is transmitted to the concrete, serious damage to the structure could occur before the energy-absorbing capability of the strands is developed.

The finite-difference solution was propagated outward in time until the pressure-time curve for the inside face of the concrete vessel (Fig. 11) exhibited the effect of a bubble pulsation. This noticeable effect occurred at about 20 msec. By comparison of the pressure peaks at 4.5 and 20 msec, it was concluded that the magnitudes of the pressure peaks of subsequent pulsations would be small. Therefore, the contribution to the total impulse from all subsequent pulsations was considered negligible. Thus, the computer solution was terminated.

In the process of achieving primary containment, the energy associated with the destructive component is absorbed by the stretching of the steel strands. For the required elongation of the strands to occur, the concrete must be allowed to break up. However, the concrete must not become rubble for its entire thickness as this would deter the effectiveness of the energy-absorbing capability of the strands.

The Rice and Walsh equation of state for water, which was used in the analysis, is relatively simple in form, but requires a rather complicated computer sequence to obtain the results. Except near the charge surface, the magnitude of the peak pressure in the shock front was found to be within the pressure range where the Tait equation of state* is applicable. As the Tait equation of state is simpler than the equation of state by Rice and Walsh, the computer time required for the solution could have been reduced by a combination of these two equations of state.

*See Appendix D for the applicable pressure range.

The pressures in the argon obtained in this analysis were above the range of experimental verification for the equation of state used for the argon. This makes the pressure-time curve for the inside face of the concrete somewhat questionable, as equations of state used for calculations outside the range of experimental verification may not yield reliable results. However, the results obtained indicate the range of pressures for which equations of states are required; i.e., $P > 3500$ atm. The pressures in the argon could have been reduced by using a thicker layer of the gas, but this would have resulted in more computer time required for the solution.

It is readily apparent from this study that the von Neumann-Richtmeyer artificial-viscosity technique offers a very effective method for solving hydrodynamic problems involving shock phenomena. This method offers these features:³

1. No limiting physical assumptions need be imposed on the differential equations to reduce them to ordinary equations or to make them more simply integrable; rather, one deals with the partial differential equations without similarity restrictions.
2. Shock waves are handled automatically, and no special computational technique is required if the problem contains multiple shocks or when shocks impinge on the origin; i.e., there are no floating boundary conditions to be satisfied separately.
3. The complexity of the computations is not seriously increased by allowing for the computation of detailed equations of state from either tabular data or analytic fits.
4. The method is adaptable to large ranges in time and space variables; i.e., expansions around singular points are nowhere necessary.

PART TWO

SOLUTION FOR THE IDEAL REACTOR SYSTEM WITH A REACTOR VESSEL

VII. INTRODUCTION

The problem solved in Part One was for an ideal reactor system that did not include a reactor vessel. However, a reactor vessel is always a component of an integral reactor system.

Often the reactor-vessel material is Type 212 B carbon steel or Type 304 stainless steel (both ductile materials). As toughness (i.e., the material's ability to resist energy load up to fracture) is a function of the ductility of the material, the energy-absorbing capability of a reactor vessel made of either of these two materials is substantial.

The purpose of Part Two is to treat the problem of Part One with the reactor vessel included (see Fig. 19) in order to:

1. Determine the energy-absorbing capability of the reactor vessel.
2. Determine the net effect that the destructive component produces on the concrete containment structure so that the rate of strain for the steel strands can be calculated.

Fig. 19
Idealized Reactor System

The theory in Part One is applicable to Part Two except as noted. The results obtained for this problem will be compared to the results obtained for the problem in Part One.

VIII. SOLUTION OF A SAMPLE PROBLEM

A. Problem Layout

The problem layout for this solution is given in Fig. 20. The reactor-vessel material is Type 304 stainless steel.

Fig. 20. Problem Layout and Dimensions

B. The Theory Associated with the Reactor Vessel*

The reactor vessel is considered to be a thin spherical shell under internal and external pressure and is analyzed by membrane theory. Thus, the stress in the shell is the same at all points, and the stress at any point is the same in all directions.

The energy-absorbing process of the reactor vessel is achieved by large plastic deformation; therefore the elastic behavior of the material is neglected. The material is categorized as a strain-hardening material; i.e., an increase in the stress is required for continued plastic deformation. Several empirical formulas (strain-hardening laws) have been proposed for fitting uniaxial true stress-strain curves. For Type 304 stainless steel, a very close fit for all pertinent portions of the true stress-strain curve is

*See Ref. 6, pp. 6.92-6.195 for more detailed information.

provided by

$$\sigma = \sigma_0(1 + \lambda\epsilon), \quad (73)$$

where σ is the true stress, ϵ is the effective strain, σ_0 is the initial yield stress, and λ is a nondimensional hardening constant.

However, the reactor vessel is in a state of biaxial stress, not uniaxial stress. Therefore, Eq. 73 does not apply directly, but can be converted to a similar expression that applies to the biaxial state of stress by various yield criteria and flow rules. It is common practice to obtain the yield condition for biaxial states of stress from the familiar concept of the yield point under uniaxial stress. This can be accomplished by the use of the von Mises octahedral shear stress yield criterion,

$$\tau_{\text{Oct}} \leq \sigma_0, \quad (74)$$

where σ_0 is the current tensile yield stress. For biaxial states of stress, σ_3 is zero, and the yield criterion reduces to

$$(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_3^2)^{1/2} \leq \sigma_0. \quad (75)$$

For the reactor vessel, where $\sigma_1 = \sigma_2 = \sigma$, the yield criterion reduces to

$$\sigma \leq \sigma_0. \quad (76)$$

Equation 76 gives the result that the yield stress for the spherical shell is identical to the yield stress for simple tension; i.e.,

$$\sigma_{0\text{biaxial}} = \sigma_{0\text{uniaxial}} \quad (77)$$

The effective strain, ϵ , for biaxial states of stress can be obtained for a von Mises material from the relationship

$$\epsilon = \frac{2}{3^{1/2}} (\epsilon_1^2 + \epsilon_1\epsilon_2 + \epsilon_2^2)^{1/2}, \quad (78)$$

which was obtained from the flow rule and where ϵ_1 and ϵ_2 are the strains in the two orthogonal directions. For the reactor vessel, where $\epsilon_1 = \epsilon_2 = \epsilon_c$, the effective strain becomes

$$\epsilon = 2\epsilon_c, \quad (79)$$

where ϵ_c is the circumferential strain in the spherical shell.

Thus, the true stress-strain relationship for the biaxial state of stress for the spherical reactor vessel is obtained by substituting Eqs. 77 and 79 into Eq. 73. This relationship is

$$\sigma = \sigma_0(1 + 2\lambda\epsilon_c), \quad (80)$$

where

σ = true stress in the shell,

σ_0 = yield stress for the material in simple tension,

λ = strain-hardening constant,

and

ϵ_c = circumferential strain in the shell.

The treatment of the reactor vessel in the solution of the problem is somewhat complex. To solve the problem directly by the von Neumann-Richtmeyer technique, an equation of state for the reactor vessel material is needed. An equation of state for the material under biaxial tension is not available. Therefore, the spherical shell is considered as an interface of finite thickness with a velocity obtained from an equation of motion different from Eq. 42. After the reactor vessel attains a certain specified limiting circumferential strain, the vessel is assumed to disintegrate instantaneously. From this point, the solution is propagated with the equations used in Part One.

The equation of motion for a thin spherical shell under an internal pressure where the motion is resisted by the stress in the shell is

$$\frac{du}{dt} = \frac{1}{\rho r h} (Pr - 2\sigma h), \quad (81)$$

where

u = the velocity of the shell,

ρ = density of the shell,

h = thickness of the shell,

r = radius of the shell,

P = internal pressure on the shell,

and

σ = stress in the shell.

As plastic deformation is assumed to produce no volume change of the solid material in the shell, the density of the material remains constant. Consequently, the thickness of the shell can be determined for any position of the shell by the expression

$$h = \left(\frac{r_0}{r} \right)^2 h_0, \quad (82)$$

where

h_0 = initial thickness of the shell,

r_0 = initial radius of the shell,

r = current radius of the shell,

and

h = current thickness of the shell.

A step-by-step numerical solution is to be obtained. Thus, Eq. 81, in incremental form, becomes

$$\frac{\Delta u}{\Delta t} = - \frac{1}{\rho r h} [r(\Delta P) + 2\sigma h], \quad (83)$$

where ΔP is the resultant between the pressure outside the shell, P_0 , and the pressure inside the shell, P_i ; i.e.,

$$\Delta P = P_0 - P_i. \quad (84)$$

The energy associated with the reactor vessel is obtained by the use of the work-energy principle,

$$\Delta U = \Delta E, \quad (85)$$

where ΔU is the net work done on the system by all the external active forces, and ΔE is the total change in the energy. For the thin spherical shell (reactor vessel), the work-energy principle can be written in incremental form as

$$\Delta E = -4\pi r^2(\Delta P)(\Delta r), \quad (86)$$

where r is the radius of the shell and Δr is the incremental change in the radius.

C. Finite-difference Equations

The finite-difference equations associated with the reactor vessel are as follows:

$$\Delta P = P(j,t) - P(j-1,t), \quad (87)$$

$$\epsilon_c = [R(j,t) - R(j,0)]/R(j,0), \quad (88)$$

$$\sigma = \sigma_0(1 + 2\lambda\epsilon_c), \quad (89)$$

$$h = h_0[R(j,0)/R(j,t)]^2, \quad (90)$$

$$u(j,t+1) = u(j,t) - \frac{\Delta t}{\rho h[R(j,t)]} \{\Delta P[R(j,t)] + 2\sigma h\}, \quad (91)$$

and

$$\Delta E = -4\pi[R(j,t)]^2(\Delta P)[R(j,t+1) - R(j,t)]. \quad (92)$$

The indexed terms given in Eqs. 87-92 are referenced to Fig. 5 and are applicable only for the value $j = JW$, which is the index of the reactor vessel (or water-argon interface).

D. Initial Values

The reactor vessel is made of Type 304 stainless steel. The mechanical properties of a material depend upon the strain rate to which the material is subjected. It is required to know the appropriate dynamic tensile properties of the stainless steel. However, since the dynamic properties of Type 304 stainless steel are not adequately known, the static tensile properties are used as a first approximation.

In general,* for austenitic stainless steels, the dynamic ultimate tensile strengths remain the same as static ultimate tensile strengths; the total elongation decreases slightly and the yield stress increases considerably with increasing strain rate. Thus, as toughness is related to the area under the stress-strain curve, the use of the static properties is a conservative approximation.

The static tensile properties for Type 304 stainless steel used in this sample problem are as follows:**

*Ref. 6, page 6.98.

**Ref. 6, page 6.112.

Ultimate true stress = 149,000 psi,

Ultimate true strain = 0.548 in./in.,

Yield stress = 40,400 psi,

and

Hardening constant = 4.9.

The complete list of initial values for the reactor vessel is as follows:

Maximum allowable circumferential strain, $\epsilon_{c\max} = 15\%$;

Yield stress, $\sigma_0 = 2.78 \times 10^9$ dynes/cm²;

Hardening constant, $\lambda = 4.9$;

Initial shell radius, $R(j,0) = 180.0$ cm;

Initial shell thickness, $h_0 = 2.5$ cm;

and

Shell material density, $\rho = 7.85$ gm/cm³.

The input data and the initial conditions are listed in Tables VIII and IX, respectively.

TABLE VIII. Input Data for the Finite-difference Solution

JA	JW	JMAX	TMAX(SEC)	EPS	EPS1	EP	DELTA(SEC)
13	28	30	0.1000000	100000.0	0.00001	0.00005	0.0000005

RRO	LAMRDA	SIGMA0	HZERO	RHO	ECIRCALL
1.800000+002	4.900000+000	2.780000+009	2.500000+000	7.850000+000	1.500000+001

J	RR CM	U CM/SEC	V CM/GM	Q DYNES/SQ CM	E DYNE-CM/GM	P DYNES/SQ CM
1	0.000000+000	0.000000+000	8.110000+001	0.000000+000	0.000000+000	2.190000+010
2	3.000000+000	0.000000+000	8.110000+001	0.000000+000	0.000000+000	2.190000+010
3	6.000000+000	0.000000+000	8.110000+001	0.000000+000	0.000000+000	2.190000+010
4	9.000000+000	0.000000+000	8.110000+001	0.000000+000	0.000000+000	2.190000+010
5	1.200000+001	0.000000+000	8.045000+001	0.000000+000	0.000000+000	2.330000+010
6	1.500000+001	5.000000+003	7.813000+001	0.000000+000	0.000000+000	2.610000+010
7	1.800000+001	1.400000+004	7.519000+001	0.000000+000	0.000000+000	3.090000+010
8	2.100000+001	2.520000+004	7.158000+001	0.000000+000	0.000000+000	4.000000+010
9	2.400000+001	4.160000+004	6.702000+001	0.000000+000	0.000000+000	5.300000+010
10	2.700000+001	6.840000+004	6.270000+001	0.000000+000	0.000000+000	7.200000+010
11	2.800000+001	8.280000+004	5.935000+001	0.000000+000	0.000000+000	9.200000+010
12	2.900000+001	1.044000+005	5.405000+001	0.000000+000	0.000000+000	1.245000+011
13	3.000000+001	1.546500+005	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
14	4.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
15	5.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
16	6.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
17	7.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
18	8.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
19	9.000000+001	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
20	1.000000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
21	1.100000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
22	1.200000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
23	1.300000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
24	1.400000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
25	1.500000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
26	1.600000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
27	1.700000+002	0.000000+000	1.001800+000	0.000000+000	-1.015020+006	1.013274+006
28	1.800000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+008	1.013274+006
29	1.950000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+008	1.013274+006
30	2.100000+002	0.000000+000	0.000000+000	0.000000+000	0.000000+000	0.000000+000

TABLE IX. Initial Values of the Parameters Required for the Finite-difference Solution

T SEC	DELT SEC	ETOT DYNE-CM	EK DYNE-CM	EINT DYNE-CM	ETNT DYNE-CM	ETVES DYNE-CM	ENERGY DYNE-CM
0.0000000	0.0000005	5.2770278+015	3.6296949+014	4.9140583+015	5.2797292+015	0.0000000+000	5.2770278+015

J	RR CM	U CM/SEC	V CUB CM/GM	Q DYNES/SQ CM	E DYNE-CM/GM	P DYNES/SQ CM	R CM	DTTEST SEC
1	0.000000+000	0.000000+000	8.110000+001	0.000000+000	1.402390+010	2.190000+010	0.000000+000	5.000000+007
2	3.000000+000	0.000000+000	8.110000+001	0.000000+000	1.402390+010	2.190000+010	3.218904+000	5.000000+007
3	6.000000+000	0.000000+000	8.110000+001	0.000000+000	1.402390+010	2.190000+010	6.437807+000	5.000000+007
4	9.000000+000	0.000000+000	8.110000+001	0.000000+000	1.402390+010	2.190000+010	9.656711+000	5.000000+007
5	1.200000+001	0.000000+000	8.045000+001	0.000000+000	1.482338+010	2.330000+010	1.287561+001	5.000000+007
6	1.500000+001	5.000000+003	7.813000+001	0.000000+000	1.552524+010	2.610000+010	1.611564+001	5.000000+007
7	1.800000+001	1.400000+004	7.519000+001	0.000000+000	1.707184+010	3.090000+010	1.943050+001	5.000000+007
8	2.100000+001	2.520000+004	7.158000+001	0.000000+000	2.098340+010	4.000000+010	2.283341+001	5.000000+007
9	2.400000+001	4.160000+004	6.702000+001	0.000000+000	2.595460+010	5.300000+010	2.634774+001	5.000000+007
10	2.700000+001	6.840000+004	6.270000+001	0.000000+000	3.552874+010	7.200000+010	3.001799+001	5.000000+007
11	2.800000+001	8.280000+004	5.935000+001	0.000000+000	4.714071+010	9.200000+010	3.130328+001	5.000000+007
12	2.900000+001	1.044000+005	5.405000+001	0.000000+000	6.287545+010	1.245000+011	3.264432+001	5.000000+007
13	3.000000+001	1.546500+005	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	3.409285+001	1.000000+005
14	4.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	4.247438+001	1.000000+005
15	5.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	5.162988+001	1.000000+005
16	6.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	6.114709+001	1.000000+005
17	7.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	7.084864+001	1.000000+005
18	8.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	8.065232+001	1.000000+005
19	9.000000+001	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	9.051665+001	1.000000+005
20	1.000000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.004191+002	1.000000+005
21	1.100000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.103468+002	1.000000+005
22	1.200000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.202916+002	1.000000+005
23	1.300000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.302486+002	1.000000+005
24	1.400000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.402144+002	1.000000+005
25	1.500000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.501868+002	1.000000+005
26	1.600000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.601642+002	1.000000+005
27	1.700000+002	0.000000+000	1.001800+000	0.000000+000	=1.015020+006	1.013274+006	1.701455+002	1.000000+005
28	1.800000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+008	1.013274+006	1.801298+002	1.000000+004
29	1.950000+002	0.000000+000	5.915974+002	0.000000+000	9.034760+008	1.013274+006	1.801573+002	1.000000+004
30	2.100000+002	0.000000+000	0.000000+000	0.000000+000	0.000000+000	0.000000+000	1.801894+002	0.000000+000

E. FORTTRAN Names and Their Description

<u>FORTTRAN Name</u>	<u>Description</u>
1. RRO	Initial radius of reactor vessel
2. XLAMBDA	Hardening constant
3. SIGMAO	Yield stress of reactor vessel
4. HZERO	Initial thickness of reactor vessel
5. RHO	Density of reactor vessel material
6. ECIRCALL	Allowable circumferential strain
7. DELP	Resultant pressure on reactor vessel
8. ECIRC	Circumferential strain
9. SIGMA	True stress in reactor vessel
10. H	Thickness of reactor vessel
11. ETVES, ETVESCAL	Energy associated with reactor vessel
12. ETOT, ETOTAL	Energy associated with fluid
13. ENERGY, ENERCAL	Total energy of system

F. Changes in Computer Program

The computer program used for this solution is the program used in Part One with the following changes:

1. The Tait form of the equation of state* for water has been incorporated.
2. The equations associated with the reactor vessel have been incorporated.
3. The program has been revised to obtain all the output on magnetic tape.
4. The program has been refined to conserve computer time.
5. At the termination of the computer solution, the latest calculated values of all the parameters are punched on cards. These cards then become the input for the continuation program if the use of the continuation program becomes necessary.

*See Appendix D.

G. FORTTRAN IV Computer Statements

```

PROGRAM HCSTNT
COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION P(50,2),V(50,2),E(50,2),Q(50,2),U(50,2),RR(50,2),TEM(50)
1,R(50),DELRU(50),DELRQ(50),SOS(50),DTTEST(50),VOL(50),VOL1(50),
2VOL2(50)
WRITE(39,3000)
C      READ AND PRINT INPUT DATA.
READ1200,JA,JW,JMAX,TMAX,EPS,EP,DELT
WRITE(39,1201),JA,JW,JMAX,TMAX,EPS,EP,DELT
READ 4000,RR0,XLAMBDAS,SIGMA0,HZERO,RHO,ECIRCALL
WRITE(39,4001),RR0,XLAMBDAS,SIGMA0,HZERO,RHO,ECIRCALL
READ 1100,(RR(J,1),U(J,1),P(J,1),V(J,1),E(J,1),Q(J,1),J=1,JMAX)
WRITE(39,1102)
WRITE(39,1103),(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),
1J=1,JMAX)
JAM = JA-1
JWM = JW-1
JMAXM = JMAX - 1
R(1) = 0.
T = 0.
I = 1
ICOUNT=1
I40=1
ASSIGN 740 TO IS6
ASSIGN 510 TO IS7
IS8=1
PLAST=0.
PX=0.
KK = JA
KL = 1
KLL = 1
C      CALCULATE THE TEMPERATURE AND THE INTERNAL ENERGY FOR EACH
C      ZONE IN THE TNT PRODUCTS AT TIME T = 0. THE TEMPERATURE, FOR A
C      GIVEN PRESSURE AND VOLUME, IS OBTAINED FROM THE EQUATION OF
C      STATE. THE INTERNAL ENERGY IS A FUNCTION OF THE TEMPERATURE
C      AND PRESSURE.
DO 150 J=1,JAM
TEM(J)=(P(J,1)*(V(J,1)-.648986-P(J,1)*(-.271149E-11+.619876E-23*P
1(J,1))))/.521766E7
150 E(J,1)=.161355E8*TEM(J)-.137566E10+P(J,1)**2*(.135575E-11-.413251
1E-23*P(J,1))
DO 151 J=JA,JWM
151 TEM(J)= 293.
DO 152 J=JW,JMAXM
152 TEM(J)= 288.
DO 165 J=1,JMAXM
DO 165 J=1,JMAXM
165 VOL(J)=(RR(J+1,1)-RR(J,1))*(RR(J+1,1)**2+RR(J+1,1)*RR(J,1)+
1RR(J,1)**2)
C      CALCULATE THE TOTAL KINETIC ENERGY OF THE SYSTEM AT TIME T = 0.
C      THIS KINETIC ENERGY IS DUE ONLY TO THE MOTION OF THE EXPLOSION
C      PRODUCTS AS THE OTHER FLUIDS IN THE SYSTEM ARE AT REST.
EK = 0.
DO 160 J=1,JAM
160 VEL =(U(J+1,1)+U(J,1))**2

```

```

160 EK = .5235988*VOL(J)*VEL/V(J,1) + EK
C   CALCULATE THE RADIUS OF EACH ZONE INTERFACE FOR THE STANDARD
C   REFERENCE CONFIGURATION BASED UPON THE DENSITY OF THE WATER
C   UTILIZING THE CONSERVATION OF MASS.
DO 161 J=2,JMAX
VOL1(J-1)=(RR(J,1)-RR(J-1,1))*(RR(J,1)**2+RR(J,1)*RR(J-1,1)+
1RR(J-1,1)**2)
161 R(J)=(R(J-1)**3+1.0018/V(J-1,1)*VOL1(J-1))**.3333333333
EINT = 0.
C   CALCULATE THE INTERNAL ENERGY IN THE TNT PRODUCTS AT TIME T = 0.
DO 162 J=1,JAM
EMASS = 4.1887902*VOL(J)/V(J,1)
162 EINT = EINT + E(J,1)*EMASS
C   CALCULATE THE TOTAL ENERGY IN THE TNT PRODUCTS AT TIME T = 0.
ETNT = EINT + EK
C   CALCULATE THE TOTAL INTERNAL ENERGY OF THE SYSTEM AT TIME T = 0.
DO 169 J=JA,JMAXM
EMASS = 4.1887902*VOL(J)/V(J,1)
169 EINT = EINT + E(J,1)*EMASS
C   CALCULATE THE TOTAL ENERGY IN THE SYSTEM AT TIME T = 0.
ETOT = EINT + EK
ETVES = 0.
ENERGY = ETOT + ETVES
DO 168 J=1,JMAXM
168 DELRQ(J) = R(J+1) - R(J)
DO 163 J= 2,JMAXM
163 DELRU(J) = (R(J+1) - R(J-1))/2.
DO 164 J=1,JAM
SOS(J) = 0.
164 DTTEST(J) = DELT
DO 166 J=JA,JWM
SOS(J) = 145000.
166 DTTEST(J) = .000010
DO 167 J=JW,JMAXM
SOS(J) = 33100.
167 DTTEST(J) = .000100
U(1,2) = 0.
U(JMAX,2) = 0.
LAST = JAM
50 WRITE(39,3000)
WRITE(39,2005),T,DELT,ETOT,EK,EINT,ETNT,ETVES,ENERGY
WRITE(39,2101)
WRITE(39,2102),(J,RR(J,1),U(J,1),V(J,1),Q(J,1),E(J,1),P(J,1),
1R(J),DTTEST(J),J=1,JMAX)
WRITE(39,3000)
WRITE(39,9695)
C   TIME ZERO DATA
P1 = P(1,1) *1.E-9
P2 = P(JAM,1) *1.E-9
P3 = P(JA,1) *1.E-9
P4 = P(JWM,1) *1.E-9
P5 = P(JW,1) *1.E-9
P6 = P(JMAXM,1) *1.E-9
PMAX = P(JA,1) *1.E-9
ETOTAL = ETOT*.2388915E-13
ETVESCAL = ETVES*.2388915E-13
ENERCAL = ENERGY*.2388915E-13
WRITE(39,9696),T,P1,P2,P3,RR(JA,1),P4,P5,RR(JW,1),P6,PMAX,KK,
1ETOTAL,ETVESCAL,ENERCAL
WRITE(40),T,P1,P2,P3,RR(JA,1),P4,P5,RR(JW,1),P6,PMAX,ETOTAL,
1ETVESCAL,ENERCAL
T = T + DELT

```

```

      I = I + 1
C      CALCULATE THE VELOCITY OF EACH ZONE INTERFACE AT TIME T FROM
C      THE FIRST DIFFERENCE EQUATION.
110  DO 200 J=2,JMAXM
      IF (J.EQ.JA) GO TO 213
      IF (J.EQ.JW) GO TO 215
      U(J,2)=U(J,1)-(1.0018*(P(J,1)-P(J-1,1)+Q(J,1)-Q(J-1,1))/DELRU
1(J)*(RR(J,1)/R(J))**2)*DELT
      GO TO 200
213  U(J,2)=U(J,1)-DELT*1.0018*((RR(J,1)/R(J))**2)*(3.*(P(J,1)-P(J-1,1)
1+Q(J,1)-Q(J-1,1))-((P(J+1,1)-P(J-2,1)+Q(J+1,1)-Q(J-2,1))/3.))/
2(DELRU(J-1)+DELRU(J+1))
      GO TO 200
215  IF (IS8.EQ.2) GO TO 213
      DELP = P(J,1)-P(J-1,1)
      ECIRC= (RR(J,1)-RRO)/RRO
      SIGMA= SIGMA0*(1.+2.*XLAMBDA*ECIRC)
      H = (RRO/RR(J,1))**2*HZERO
      U(J,2)=U(J,1)-(DELT/(RHO*H*RR(J,1)))*(DELP*RR(J,1)+2.*H*SIGMA)
      IF (U(J,2).LT.0.) U(J,2)=0.
      IF (ECIRC.GE.ECIRCALL) IS8=2
      IF (IS8.EQ.2) WRITE(39,7000)
200  CONTINUE
C      CALCULATE THE RADIUS OF EACH ZONE INTERFACE AT
C      TIME T FROM THE SECOND DIFFERENCE EQUATION.
214  DO 300 J=1,JMAX
300  RR(J,2) = RR(J,1)+U(J,2)*DELT
      IF (RR(JMAX,2).GE.RR(JMAX,1)) RR(JMAX,2)=RR(JMAX,1)
      IF (RR(JW,2).GE.RR(JMAX,2)) GO TO 100
      IF (IS8.EQ.2) GO TO 210
      ETVES = -12.5664*(RR(JW,1)**2)*(P(JW,1)-P(JWM,1))
1*(RR(JW,2)-RR(JW,1)) + ETVES
210  DO 202 J=1,JMAXM
      VOL2(J) =(R(J+1)*R(J))*(R(J+1)**2+R(J+1)*R(J)+R(J)**2)
202  VOL(J) =(RR(J+1,2)-RR(J,2))*(RR(J+1,2)**2+RR(J+1,2)*RR(J,2)+
1RR(J,2)**2)
C      CALCULATE THE SPECIFIC VOLUME FOR EACH ZONE AT TIME T FROM THE
C      THIRD DIFFERENCE EQUATION, AND CALCULATE THE PSEUDO-VISCOUS
C      PRESSURE TERM FOR EACH ZONE AT TIME T FROM THE FOURTH
C      DIFFERENCE EQUATION.
      DO 400 J=1,JMAXM
      V(J,2) =1.0018*VOL(J)/VOL2(J)
      VTEST =V(J,2)-V(J,1)
      IF (VTEST) 402,401,401
402  CC1 =3.98563879*(DELRQ(J))**2/(V(J,2)+V(J,1))
      CC2=(VTEST/DELT)**2
      CC3 =(2.*R(J+1)/(RR(J+1,2)+RR(J+1,1))**4
      Q(J,2) =CC1*CC2*CC3
      GO TO 400
401  Q(J,2)=0.
400  CONTINUE
C      CALCULATE THE TOTAL KINETIC ENERGY OF THE SYSTEM AT TIME T.
      EK =0.
      DO 201 J =1,JMAXM
      VEL =(U(J+1,2) + U(J,2))**2
201  EK =.5235988*VOL(J)*VEL/V(J,2) + EK
      ASSIGN 66 TO IS3
      IFSW =1
      IS2 =1
      TEMPP = P(1,1)
C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN
C      THE TNT PRODUCTS AT TIME T BY ITERATION OF THE FIFTH AND SIXTH

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C      DIFFERENCE EQUATIONS BY THE FOLLOWING PROCEDURE. GUESS P AT
C      TIME T. SOLVE THE FIFTH DIFFERENCE EQUATION FOR E AT TIME T.
C      THEN USE THIS VALUE OF E TO OBTAIN A CORRECTED VALUE FOR P AT
C      TIME T. REPEAT THIS PROCESS UNTIL THE DIFFERENCE BETWEEN TWO
C      SUCCESSIVE VALUES FOR P AT TIME T IS LESS THAN A VALUE EPSILON.
DO 699 J=1,JAM
  IF (IS2-1)404,403,404
404  TEMPP =(P(J,1)+P(J-1,2))/2.
403  ENEW =0.
  AR3   =.648986-V(J,2)
  AAR3  =-AR3
  AR1   =.619876E-23/AAR3
  AR2   =-.271149E-11/AAR3
60  TEMPE = E(J,1)-((TEMPP+P(J,1))/2.+Q(J,2))*(V(J,2)-V(J,1))
  GO TO IS3
66  IF(TEMPE.GT.1.0E6) GO TO 61
  IF(ABSF(TEMPE-ENEW)-EPS)68,68,65
61  IF(ABSF(TEMPE-ENEW)-ABSF(TEMPE)*EPS1)68,68,65
65  E1   =TEMPP**2*(.135575E-11-.413251E-23*TEMPP)
  AR44  =(TEMPE + .137566E10 - E1)*.32336525
  AR4   =AR44/AAR3.
  N     = 25
  CALL ROOT3 (TEMPP,EP,G,C,N,M)
  R1    = G
C      THE EQUATION OF STATE FOR THE TNT PRODUCTS, THE SIXTH DIFF-
C     ERENCE EQUATION, IS A THIRD ORDER EQUATION IN P. THEREFORE,
C      THERE ARE THREE ROOTS TO THIS EQUATION. THE ROOT FINDER SUB-
C      ROUTINE, CALLED ROOT3, HAS JUST FOUND ONE ROOT TO THIS
C      EQUATION. NOW SOLVE FOR THE OTHER TWO ROOTS FROM THE RESULTING
C      QUADRATIC EQUATION AND CHOOSE WHICHEVER OF THE THREE ROOTS IS
C      CLOSEST TO THE VALUE OF P AT TIME T =T-DELT FOR CONTINUING THE
C      ITERATIVE PROCESS.
  B = .619876E-23*R1 - .271149E-11
  DISC =B**2-2.479504E-23*(AR3+R1 - (.619876E-23*R1-.271149E-11))
  IF(DISC)660,670,680
660  TEMPP = R1
  GO TO 190
670  R2 = -B/1.239752E-23
  R3   = R2
  GO TO 96
680  DSQ = SQRTF(DISC)
  R2 =(-B+DSQ)/1.239752E-23
  R3 =(-B-DSQ)/1.239752E-23
96  RAD1 =ABSF(P(J,1)-R1)
  RAD2 =ABSF(P(J,1)-R2)
  RAD3 =ABSF(P(J,1)-R3)
  IF(RAD1-RAD2)91,92,92
91  IF(RAD3-RAD1)93,94,94
93  TEMPP = R3
  GO TO 190
92  IF(RAD3-RAD2)93,95,95
95  TEMPP = R2
  GO TO 190
94  TEMPP = R1
190  ENEW = TEMPE
  GO TO 60
68  P(J,2)=TEMPP
  E(J,2)=TEMPE
  TEM(J) =AR44/.521766E7
  IS2   =2
C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.

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PCP = P(J,2)*1.E-10
SOS(J) = PCP*(49547.-2108.*PCP) + 183556.
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
699 CONTINUE
ASSIGN 76 TO IS3
IFSW = 2
C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C      WATER AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C      EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
DO 71 J= JA,JWM
IF(U(J,2))74,83,74
74 GO TO IS6
740 TEMPP = P(J,1)
ENEW = 0.
GO TO 60
76 IF(TEMPE.GT.1.0E6) GO TO 711
IF(ABSF(TEMPE-ENEW)-EPS)78,78,75
711 IF(ABSF(TEMPE-ENEW)-ABSF(TEMPE)*EPS1)78,78,75
75 PP = TEMPP*1.0E-9
EE = TEMPE*.238892E-7
CALL ROOT3 (PP,EP,G,C,25,M)
TEMPP = G*1.0E9
ENEW = TEMPE
GO TO 60
78 IF(J=LAST)79,79,788
788 IF(TEMPP-1.013274E6)77,77,799
77 TEMPP = 1.013274E6
TEMPE = -1.015020E6
ENEW = 0.
GO TO 79
799 IF (LAST-J) 777,777,79
777 LAST = J
79 P(J,2) = TEMPP
E(J,2) = TEMPE
GO TO 710
73 P(J,2)=316.34E7*((1.0018/V(J,2))**.15-1.)
IF(V(J,2).GT.1.0018)P(J,2)=0.
E(J,2)=E(J,1)-((P(J,2)+P(J,1))/2.+Q(J,2))*(V(J,2)-V(J,1))
IF(J=LAST)710,710,733
733 IF(P(J,2)-1.013274E6)770,770,771
770 P(J,2)=1.013274E6
E(J,2)=-1.01502E6
GO TO 710
771 IF(LAST-J)772,772,710
772 LAST=J
C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
710 PCP = P(J,2)*1.E-9
SOS(J) = PCP*(3853.-13.*PCP) + 256906.
DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
71 CONTINUE
ASSIGN 86 TO IS3
C      CALCULATE THE INTERNAL ENERGY AND PRESSURE FOR EACH ZONE IN THE
C      ARGON AT TIME T BY ITERATION OF THE FIFTH AND SIXTH DIFFERENCE
C      EQUATIONS BY THE SAME PROCEDURE AS FOR THE TNT PRODUCTS.
887 DO 80 J=JW,JMAXM
IF(U(J,2).EQ.0.0) GO TO 83
89 ENEW = 0.
VSQ = V(J,2)**2
TEMP = 835833.+(1.-.58282/V(J,2))/VSQ
TEMP1 = -.318769E-6*TEMP*(V(J,2)-591.597)
TEMP2 = 2078910.*(V(J,2) + .98413)/VSQ

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      TEMPP = P(J,1)
      GO TO 60
86    IF(TEMPE.GT.1.0E6) GO TO 82
      IF(ABSF(TEMPE-ENEW)-EPS)88,88,85
82    IF(ABSF(TEMPE-ENEW)-ABSF(TEMPE)*EPS1)88,88,85
85    TEMPT = TEMPE*.318769E-6+TEMP1
      TEMPP = TEMPT*TEMP2-TEMP
      ENEW = TEMPE
      GO TO 60
88    E(J,2) = TEMPE
      P(J,2) = TEMPP
      TEM(J) = TEMPT
C      CALCULATE THE ALLOWABLE TIME INCREMENT, DELTA T, FROM THE
C      STABILITY CONDITION FOR THE DIFFERENCE EQUATIONS.
      RAD = 1.66*P(J,2)*V(J,2)
      SOS(J) = SQRTF(RAD)
      DTTEST(J) = .30*(RR(J+1,2)-RR(J,2))/SOS(J)
80    CONTINUE
      GO TO 81
83    JCUR = J
      DO 72 J = JCUR, JMAXM
      E(J,2) = E(J,1)
72    P(J,2) = P(J,1)
C      CALCULATE THE TOTAL INTERNAL ENERGY IN THE SYSTEM AT TIME T.
81    EINT = 0.
      DO 500 J = 1, JMAXM
      EMASS = 4.1887902*VOL(J)/V(J,2)
500    EINT = EINT + E(J,2)*EMASS
C      CALCULATE THE TOTAL ENERGY IN THE SYSTEM AT TIME T.
      ETOT = EINT + EK
      ENERGY = ETOT + ETVES
C      REASSIGN VALUES.
      DO 600 J = 1, JMAX
      P(J,1) = P(J,2)
      RR(J,1) = RR(J,2)
      U(J,1) = U(J,2)
      Q(J,1) = Q(J,2)
      V(J,1) = V(J,2)
600    E(J,1) = E(J,2)
      PTEST = P(JA,1)
      K = JA + 1
      DO 444 J = K, JMAXM
      ARG = P(J,1)
      IF (PTEST - ARG) 445,445,444
445    PTEST = ARG
      KK = J
444    CONTINUE
51    P1 = P(1,1)          *1.E-9
      P2 = P(JAM,1)        *1.E-9
      P3 = P(JA,1)         *1.E-9
      P4 = P(JWM,1)        *1.E-9
      P5 = P(JW,1)         *1.E-9
      P6 = P(JMAXM,1)      *1.E-9
      PMAX = P(KK,1)       *1.E-9
      GO TO IS7
513    ASSIGN 515 TO IS7
      ASSIGN 73 TO IS6
      GO TO 515
510    IF(PMAX.GE.PLAST)511,512
512    IF(PX.GE.20.)514,513
511    PX = PMAX
514    PLAST = PMAX

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515  ETOTAL = ETOT*.2388915E-13
      ETVESCAL = ETVES*.2388915E-13
      ENERCAL = ENERGY*.2388915E-13
      WRITE(39,9696),T,P1,P2,P3,RR(JA,1),P4,P5,RR(JW,1),P6,PMAX,KK,
1  ETOTAL,ETVESCAL,ENERCAL
      ICOUNT=ICOUNT+1
      IF(ICOUNT.EQ.6)32,112
32   ICOUNT=1
      WRITE(40),T,P1,P2,P3,RR(JA,1),P4,P5,RR(JW,1),P6,PMAX,ETOTAL,
1  ETVESCAL,ENERCAL
      I40=I40+1
C     INCREMENT DELTA T TO THE MAXIMUM VALUE ALLOWED BY THE STABILITY
C     CONDITION FOR THE DIFFERENCE EQUATIONS.
112  DELTAT = DTTEST(1)
      DO 44 J=2,JMAXM
      ARGUE =DTTEST(J)
      IF (DELTAT-ARGUE)44,44,26
26   DELTAT =ARGUE
44   CONTINUE
      IF (DELTAT.GE.0.000003) DELT=0.000003
      IF (DELTAT.LT.0.000003) DELT=0.000002
      IF (DELTAT.LT.0.000002) DELT=0.000001
      IF (DELTAT.LT.0.000001) DELT=0.0000005
      IF (DELTAT.LT.0.0000005) DELT=0.0000003
      IF (DELTAT.LT.0.0000003) DELT=0.0000001
C     INCREMENT T.
      T = T + DELT
      I = I+1
      IF(TIMELEFT(TO).LE.5000.)GO TO 100
      IF(T=TMAX)110,110,100
1100  FORMAT (6E12.6)
1102  FORMAT (///4H      J,7X,2HRR,13X,1HU,13X,1HV,13X,1HQ,13X,1HE,13X,1HP/
1     11X,2HCM,11X,6HCM/SEC,5X,9HCUB CM/GM,4X,11HDYNES/SQ CM,
14X,10HDYNE-CM/GM,4X,11HDYNES/SQ CM/)
1103  FORMAT(I4,6E14.6)
1110  FORMAT(4E14.6)
1200  FORMAT(3I6,5E12.7)
1201  FORMAT (11H INPUT DATA/6H      JA,4X,2HJW,3X,4HJMAX,2X,9HTMAX(SEC),
14X,3HEPS,8X,4HEPS1,7X,2HEP,5X,9HDELT(SEC)//3I6,F12.7,F10.1,2F10.5,
2F12.7///)
2005  FORMAT(6X,1HT,8X,4HDELT,12X,4HETOT,15X,2HEK,15X,4HEINT,15X,4HETNT,
113X,5HETVES,13X,6HENERGY/5X,3HSEC,8X,3HSEC,11X,7HDYNE-CM,11X,
17HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM,11X,7HDYNE-CM,
1//2F11.7,6E18.7)
2022  FORMAT(4E18.7)
2023  FORMAT(3I5,5E13.4)
2024  FORMAT(2E18.7,3I8)
2101  FORMAT (///4H      J,7X,2HRR,13X,1HU,13X,1HV,13X,1HQ,13X,1HE,13X,1HP,
113X,1HR,10X,6HDTTEST/
1     11X,2HCM,11X,6HCM/SEC,5X,9HCUB CM/GM,4X,11HDYNES/SQ CM,
14X,10HDYNE-CM/GM,4X,11HDYNES/SQ CM,7X,2HCM,12X,3HSEC/)
2102  FORMAT(I4,8E14.6)
2103  FORMAT(F10.7,3F6.3,F8,3,2F6.3,F8,3,2F6.3,13)
3000  FORMAT (1H1)
4000  FORMAT(6E10.4)
4001  FORMAT( 11X,3HRR0,9X,6HLAMBDA,8X,6HSIGMA0,9X,5HHZERO,10X,3HRHO,
18X,8HECIRCALL//E18.6,5E14.6///)
5001  FORMAT (/11X,47HNUMBER OF LINES OF PRINT OUT IN THE FINAL TABLE,
13X,3HI =,17/11X,33HNUMBER OF RECORDS WRITTEN ON TAPE,3X,3HN =,17)
7000  FORMAT(/*      THE STRAIN IN THE REACTOR VESSEL HAS REACHED ITS LIM
9ITING VALUE.*/*      THUS, THE VESSEL IS NO LONGER CONSIDERED AS AN EFF
9ECTIVE PART OF THE SYSTEM.*/)

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8000  FORMAT (/11X,12HPROGRAM OVER)
9695  FORMAT (14X,69HTABULATED VALUES OF THE PRESSURE AND THE RADIAL COO
1RDINATE AT SEVERAL/11X,61HCRTICAL LOCATIONS, AND A RECORD OF THE
1ENERGY- IN THE SYSTEM./ /11X,24HUNI
2TS - TIME, T, SECONDS/19X,21HPRESSURE, P, KILOBARS/19X,38HRADIAL C
3COORDINATE, RR, CENTIMETERS ,24X,14HINSIDE MAX/19X,
323HENERGY, E, MEGACALORIES,38X, 7HFACE OF
4,3X,8HPRESSURE/80X,8HCONCRETE,1X,10HOUTSIDE OF/34X,19HTNT-WATER IN
5TERFACE,4X,21HWATER-ARGON INTERFACE,3X,6HVESSEL,2X,10HGAS BUBBLE/1
65X,1HT,9X,1HP,8X,1HP,8X,1HP,7X,2HRR,6X,1HP,6X,1HP,7X,2HRR,7X,1HP,
77X,1HP,3X,4HZONE,6X,1HE,10X,1HE,9X,1HE/50X,4HTNT-,18X,6HWATER-/
723X,6HCENTER,4X,3HTNT,5X,
85HWATER,3X,5HWATER,3X,5HWATER,2X,5HARGON,3X,5HARGON,4X,5HARGON,4X,
93HMAX,4X,1HJ,5X,5HFLUID,5X,6HVESSEL,5X,5HTOTAL/)
9696  FORMAT(F20.7,2F9.3,3F8.3,F7.3,F9.3,2F8.3,I4,F10.2,2F11.2)
100   IMAX = I-1
      T = T - DELT
      WRITE(39,5001),IMAX,I40
      WRITE(39,8000)
      PUNCH 2023,JA,JW,JMAX,TMAX,EPS,EPS1,EP,DELT
      PUNCH 2022,RRO,XLAMBD, SIGMAO
      PUNCH 2022,HZERO,RHO,ECIRCALL
      PUNCH 2022,T,DELT,ETOT,EK
      PUNCH 2022,EINT,ETVES,ENERGY
      PUNCH1110,(RR(J,1),U(J,1),V(J,1),Q(J,1),J=1,JMAX)
      PUNCH1110,( E(J,1),P(J,1),R(J),DTTEST(J),J=1,JMAX)
      PUNCH 2022,T,P1,P2,P3
      PUNCH 2022,RR(JA,1),P4,P5,RR(JW,1)
      PUNCH2024,P6,PMAX,KK,IS8,LAST
      PUNCH 2022,ETOTAL,ETVESC,ENERCAL
      END

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SUBROUTINE ROOT3 (A, EPS, G, C, N, M)
COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION V(50,2)
C THIS SUBROUTINE IS A FORTRAN TRANSCRIPTION OF ACM ALGORITHM NO.
C 26(COMM. ACM 4(1960)603). IT COMPUTES A VALUE OF G = Y SATISFYING
C THE EQUATION Y = FCT(Y). THE ITERATION WILL CONVERGE TO Y PRO-
C VIDING THAT AT SOME TIME IN THE ITERATION A G IS REACHED SUCH THAT
C  $ABS(G-Y)*ABS(D(DFCT/DY)/DY)$  IS LESS THAN  $2*ABS((DF/DY)-1.)$ , WHERE
C THE DERIVATIVES ARE EVALUATED AT Y.
C INPUT INCLUDES: A SUBROUTINE, FCT(X,Y), WHICH PLACES THE
C VALUE OF FCT(X) IN Y; AN INITIAL APPROXIMATION TO THE ROOT, A;
C A TOLERANCE FOR THE RELATIVE ERROR IN G, EPS; A MAXIMUM NUMBER
C OF ITERATIONS TO BE PERFORMED, N. THE SUBROUTINE CHANGES N TO
C THE NUMBER OF ITERATIONS ACTUALLY PERFORMED.
C OUTPUT INCLUDES: THE NUMBER OF ITERATIONS PERFORMED, N; THE
C REQUIRED ROOT, G; THE VALUE C = FCT(G) - G; A PARAMETER INDICAT-
C ING THE SUCCESS OF THE PROCEDURE, M. IF THE TOLERANCE WAS NOT
C MET, M IS NEGATIVE. THE NUMBER ABS(M)-1 GIVES THE NUMBER OF TIMES
C THAT THE CORRECTION TO G EXCEEDED THE PRECEDING ONE. IF FCT(Y)-Y
C HAS THE SAME VALUE FOR TWO SUCCESSIVE APPROXIMATIONS TO G, THEN
C H = 1, AND WE PRINT CYCLE AND EXIT.
IF (KL - 2) 11,12,11
12 WRITE(39,101),J
101 FORMAT (/5X,4HJ1 =,I3,5X,98HIF J1 EQUALS J, THE COMPUTED RESULTS A
1RE ACCEPTABLE. THIS MEANS THAT THE ROOT, G, OBTAINED BY THE/3X,
2113HSUBROUTINE HAS BEEN CORRECTED BY THE PARENT ROUTINE AND THE PA
3RENT ROUTINE HAS CALLED THE SUBROUTINE AGAIN IN ITS/3X,113HATTEMPT
4 TO FIND A SATISFACTORY ROOT. IF J1 DOES NOT EQUAL J, THE ROOT, G
5, OBTAINED BY THE ROOT FINDER SUBROUTINE/3X,68HMAY OR MAY NOT BE A
6CCEPTABLE AND FURTHER INVESTIGATION IS WARRANTED./)
11 KL = 1
M = 1
JJ = 0
C = 0
CALL FCT (0., F)
IF (F) 2,1,2
1 G = 0.
GO TO 50
2 CALL FCT (A, G)
C = G - A
D = C
B = C
IF (C) 3,50,3
3 DO 40 JJ=1,N
CALL FCT (G,F)
C = F - G
IF (ABSF(C)-.000009)50,50,10
10 IF (ABSF(C) = ABSF(G)*EPS)50,50,4
4 H = B/C
IF (H-1.) 5,6,5
6 WRITE(39,99) J
WRITE(39,100) A,G,F,H
99 FORMAT (/8H CYCLE 3,8X,3HJ =,I4)
100 FORMAT (/6H A =,E15,7,4X,3H G =,E15,7,4X,3H F =,E15,7,4X,3H H =,
1E15,7)
KL = KL + 1
GO TO 49
5 IF (H) 7,7,8
8 IF (H-2.) 9,7,7
9 M = M + 1
7 D = D / (H-1.)
B = C
40 G = G + D
49 M = -M
JJ = N
50 N = JJ
RETURN
END

```

```

SUBROUTINE FCT (X,Y)
COMMON AR1,AR2,AR3,AR4,IFSW,PP,US,UP,VH,ENTHH,XI,ENTH,EE,V,J,EP,KL
1,KLL
DIMENSION V(50,2)
IF (IFSW-1)1,1,2
1  Y = (AR1*X+AR2)*X*X+AR4
   GO TO 3
2  NN =50
   PPP =X
   US = 1.75073 + .05486*PPP - .00022*(PPP**2)
   IF (US - 1.483) 6,7,5
6  US = 1.483
   GO TO 7
5  CALL ROOT33 (US,EP,GG,CC,NN,MM,PPP)
   US =GG
7  UP =4.5349294*(1.095275**US - 1.14445)
   VH =1.0018*(1.0-UP/US)
   ENTHH =11.9446*(1.0018+VH)*X
   EXP =.0030338*X
   XI =151.16*10.**EXP
   ENTH =ENTHH+XI*(V(J,2)-VH)*23.8892
   Y = (ENTH-EE)/(V(J,2)*23.8892)
3  RETURN
   END

```

```

SUBROUTINE ROOT33 (A, EPS, G, C, N, M, PPP)
COMMON AR1, AR2, AR3, AR4, IFSW, PP, US, UP, VH, ENTHH, XI, ENTH, EE, V, J, EP, KL
1, KLL
DIMENSION V(50, 2)
C THIS SUBROUTINE IS SIMILAR TO SUBROUTINE ROOT3.
IF (KLL - 2) 11, 12, 11
12 WRITE(39, 101) J
101 FORMAT (/5X, 4HJ1 =, I3, 5X, 98HIF J1 EQUALS J, THE COMPUTED RESULTS A
1RE ACCEPTABLE. THIS MEANS THAT THE ROOT, G, OBTAINED BY THE/3X,
2113HSUBROUTINE HAS BEEN CORRECTED BY THE PARENT ROUTINE AND THE PA
3RENT ROUTINE HAS CALLED THE SUBROUTINE AGAIN IN ITS/3X, 113HATTEMPT
4 TO FIND A SATISFACTORY ROOT. IF J1 DOES NOT EQUAL J, THE ROOT, G
5, OBTAINED BY THE ROOT FINDER SUBROUTINE/3X, 68HMAY OR MAY NOT BE A
6CCEPTABLE AND FURTHER INVESTIGATION IS WARRANTED./)
11 KLL = 1
M = 1
JJ = 0
C = 0
2 CALL FCTN (A, G, PPP)
C = G - A
D = C
B = C
IF (C) 3, 50, 3
3 DO 40 JJ = 1, N
CALL FCTN (G, F, PPP)
C = F - G
IF (ABSF(C) = ABSF(G) * EPS) 50, 50, 4
4 H = B/C
IF (H-1.) 5, 6, 5
6 WRITE(39, 99) J
WRITE(39, 100) A, G, F, H, PPP
99 FORMAT (/9H CYCLE 33, 8X, 3HJ =, I4)
100 FORMAT (/6H A =, E15.7, 4X, 3HG =, E15.7, 4X, 3HF =, E15.7, 4X, 3HH =,
1E15.7, 4X, 5HPPP =, E15.7)
KLL = KLL + 1
GO TO 49
5 IF (H) 7, 7, 8
8 IF (H-2.) 9, 7, 7
9 M = M + 1
7 D = D / (H-1.)
B = C
40 G = G + D
49 M = M
JJ = N
50 N = JJ
RETURN
END

```

```

SUBROUTINE FCTN (X, Y, PPP)
COMMON AR1, AR2, AR3, AR4, IFSW, PP, US, UP, VH, ENTHH, XI, ENTH, EE, V, J, EP, KL
1, KLL
DIMENSION V(50, 2)
Y = (.022090746 * PPP + 1.14445 * X) / (1.095275 ** X)
RETURN
END

```


H. Computer Results

The computer results for this solution are presented as follows:

1. A sample of the tabulated computer results obtained for various critical parameters (Table X).
2. Graphical representation of the computer results obtained for various critical parameters (Figs. 21-31).

TABLE X. Tabulated Values of the Pressure and Radial Coordinate at Several Critical Locations, and a Record of the Energy in the System

UNITS - TIME, T, SECONDS

PRESSURE, P, KILOBARS

RADIAL COORDINATE, RR, CENTIMETERS

ENERGY, E, MEGACALORIES

T	P	TNT=WATER INTERFACE			WATER=ARGON INTERFACE			INSIDE FACE OF CONCRETE VESSEL	MAX PRESSURE OUTSIDE OF GAS BUBBLE		E	E	E
		P	P	RR TNT=	P	P	RR WATER=	P	P	ZONE			
	CENTER	TNT	WATER	WATER	WATER	ARGON	ARGON	ARGON	MAX	J	FLUID	VESSEL	TOTAL
0.0000000	21.900	124.500	0.001	30.000	0.001	0.001	180.000	0.001	0.001	13	126.06	0.00	126.06
0.0000005	21.900	113.989	0.163	30.084	0.001	0.001	180.000	0.001	0.163	13	137.25	0.00	137.25
0.0000010	21.900	102.885	0.348	30.172	0.001	0.001	180.000	0.001	0.348	13	137.15	0.00	137.15
0.0000015	21.900	91.924	0.554	30.264	0.001	0.001	180.000	0.001	0.554	13	137.11	0.00	137.11
0.0000020	21.900	81.607	0.783	30.359	0.001	0.001	180.000	0.001	0.783	13	137.15	0.00	137.15
0.0000025	21.900	72.226	1.034	30.456	0.001	0.001	180.000	0.001	1.034	13	137.23	0.00	137.23
0.0000030	21.900	63.929	1.307	30.555	0.001	0.001	180.000	0.001	1.307	13	137.36	0.00	137.36
0.0000035	21.900	56.733	1.598	30.655	0.001	0.001	180.000	0.001	1.598	13	137.51	0.00	137.51
0.0000040	21.900	50.576	1.917	30.755	0.001	0.001	180.000	0.001	1.917	13	137.68	0.00	137.68
0.0000045	21.900	45.357	2.256	30.856	0.001	0.001	180.000	0.001	2.256	13	137.84	0.00	137.84
0.0000050	21.900	40.956	2.617	30.957	0.001	0.001	180.000	0.001	2.617	13	138.00	0.00	138.00
0.0000055	21.900	37.249	2.999	31.058	0.001	0.001	180.000	0.001	2.999	13	138.15	0.00	138.15
0.0000060	21.900	34.120	3.400	31.158	0.001	0.001	180.000	0.001	3.400	13	138.29	0.00	138.29
0.0000065	21.900	31.471	3.822	31.258	0.001	0.001	180.000	0.001	3.822	13	138.42	0.00	138.42
0.0000070	21.900	29.217	4.264	31.357	0.001	0.001	180.000	0.001	4.264	13	138.53	0.00	138.53
0.0000075	21.900	27.286	4.722	31.455	0.001	0.001	180.000	0.001	4.722	13	138.63	0.00	138.63
0.0000080	21.900	25.625	5.203	31.553	0.001	0.001	180.000	0.001	5.203	13	138.72	0.00	138.72
0.0000085	21.900	24.185	5.701	31.650	0.001	0.001	180.000	0.001	5.701	13	138.79	0.00	138.79
0.0000090	21.900	22.930	6.216	31.746	0.001	0.001	180.000	0.001	6.216	13	138.85	0.00	138.85
0.0000095	21.900	21.830	6.746	31.841	0.001	0.001	180.000	0.001	6.746	13	138.90	0.00	138.90
0.0000105	21.900	20.061	7.827	32.027	0.001	0.001	180.000	0.001	7.827	13	138.94	0.00	138.94
0.0000115	21.901	18.657	8.969	32.210	0.001	0.001	180.000	0.001	8.969	13	138.96	0.00	138.96
0.0000125	21.901	17.525	10.149	32.389	0.001	0.001	180.000	0.001	10.149	13	138.93	0.00	138.93
0.0000135	21.901	16.598	11.361	32.564	0.001	0.001	180.000	0.001	11.361	13	138.86	0.00	138.86
0.0000145	21.902	15.831	12.596	32.736	0.001	0.001	180.000	0.001	12.596	13	138.74	0.00	138.74
0.0000155	21.904	15.188	13.839	32.903	0.001	0.001	180.000	0.001	13.839	13	138.58	0.00	138.58
0.0000165	21.905	14.644	15.078	33.067	0.001	0.001	180.000	0.001	15.078	13	138.39	0.00	138.39
0.0000175	21.908	14.182	16.300	33.227	0.001	0.001	180.000	0.001	16.300	13	138.17	0.00	138.17
0.0000185	21.911	13.786	17.490	33.383	0.001	0.001	180.000	0.001	17.490	13	137.94	0.00	137.94
0.0000205	21.924	13.194	19.573	33.680	0.001	0.001	180.000	0.001	19.573	13	137.38	0.00	137.38
0.0000225	21.946	12.750	21.357	33.962	0.001	0.001	180.000	0.001	21.357	13	136.84	0.00	136.84
0.0000245	21.980	12.413	22.757	34.229	0.001	0.001	180.000	0.001	22.757	13	136.30	0.00	136.30
0.0000265	22.033	12.156	23.708	34.481	0.001	0.001	180.000	0.001	23.708	13	135.77	0.00	135.77
0.0000285	22.110	11.960	24.163	34.718	0.001	0.001	180.000	0.001	24.163	13	135.23	0.00	135.23
0.0000305	22.214	11.809	24.098	34.940	0.001	0.001	180.000	0.001	24.098	13	134.67	0.00	134.67
0.0000325	22.350	11.696	23.518	35.147	0.001	0.001	180.000	0.001	23.518	13	134.10	0.00	134.10

TABLE X (Contd.)

0.0000345	22.521	11.609	22.467	35.339	0.001	0.001	180.000	0.001	22.467	13	133.55	0.00	133.55
0.0000365	22.713	11.530	21.034	35.519	0.001	0.001	180.000	0.001	21.034	13	133.05	0.00	133.05
0.0000385	22.952	11.446	19.336	35.687	0.001	0.001	180.000	0.001	19.336	13	132.66	0.00	132.66
0.0000405	23.223	11.348	17.496	35.845	0.001	0.001	180.000	0.001	17.496	13	132.40	0.00	132.40
0.0000425	23.518	11.230	15.631	35.997	0.001	0.001	180.000	0.001	15.631	13	132.29	0.00	132.29
0.0000445	23.830	11.091	13.833	36.143	0.001	0.001	180.000	0.001	13.833	13	132.32	0.00	132.32
0.0000465	24.146	10.927	12.168	36.287	0.001	0.001	180.000	0.001	12.168	13	132.47	0.00	132.47
0.0000485	24.458	10.735	10.676	36.429	0.001	0.001	180.000	0.001	12.046	14	132.73	0.00	132.73
0.0000505	24.755	10.513	9.377	36.573	0.001	0.001	180.000	0.001	13.083	14	133.05	0.00	133.05
0.0000525	25.036	10.256	8.271	36.719	0.001	0.001	180.000	0.001	14.053	14	133.44	0.00	133.44
0.0000545	25.305	9.965	7.346	36.869	0.001	0.001	180.000	0.001	14.925	14	133.85	0.00	133.85
0.0000565	25.577	9.644	6.600	37.024	0.001	0.001	180.000	0.001	15.669	14	134.26	0.00	134.26
0.0000585	25.882	9.298	6.009	37.184	0.001	0.001	180.000	0.001	16.262	14	134.67	0.00	134.67
0.0000605	26.261	8.933	5.559	37.349	0.001	0.001	180.000	0.001	16.680	14	135.04	0.00	135.04
0.0000615	26.485	8.748	5.370	37.434	0.001	0.001	180.000	0.001	16.846	14	135.22	0.00	135.22
0.0000625	26.748	8.561	5.213	37.519	0.001	0.001	180.000	0.001	16.962	14	135.38	0.00	135.38
0.0000635	27.058	8.371	5.086	37.607	0.001	0.001	180.000	0.001	17.030	14	135.53	0.00	135.53
0.0000645	27.423	8.179	4.989	37.695	0.001	0.001	180.000	0.001	17.048	14	135.66	0.00	135.66
0.0000655	27.851	7.986	4.919	37.786	0.001	0.001	180.000	0.001	17.015	14	135.77	0.00	135.77
0.0000665	28.348	7.794	3.004	37.877	0.001	0.001	180.000	0.001	16.585	14	135.87	0.00	135.87
0.0000675	28.920	7.597	3.001	37.971	0.001	0.001	180.000	0.001	16.445	14	135.98	0.00	135.98
0.0000685	29.572	7.398	3.022	38.067	0.001	0.001	180.000	0.001	16.253	14	136.07	0.00	136.07
0.0000695	30.310	7.197	3.069	38.164	0.001	0.001	180.000	0.001	16.013	14	136.13	0.00	136.13
0.0000705	31.123	6.998	3.142	38.264	0.001	0.001	180.000	0.001	15.728	14	136.18	0.00	136.18
0.0000715	32.013	6.801	3.240	38.365	0.001	0.001	180.000	0.001	15.401	14	136.20	0.00	136.20
0.0000725	32.971	6.608	3.363	38.468	0.001	0.001	180.000	0.001	15.039	14	136.21	0.00	136.21
0.0000735	33.988	6.422	3.511	38.571	0.001	0.001	180.000	0.001	14.646	14	136.21	0.00	136.21
0.0000745	35.049	6.244	3.684	38.676	0.001	0.001	180.000	0.001	14.227	14	136.19	0.00	136.19
0.0000755	36.137	6.074	3.880	38.782	0.001	0.001	180.000	0.001	13.789	14	136.17	0.00	136.17
0.0000765	37.231	5.914	4.100	38.888	0.001	0.001	180.000	0.001	13.335	14	136.15	0.00	136.15
0.0000785	39.321	5.629	4.619	39.101	0.001	0.001	180.000	0.001	12.377	14	136.11	0.00	136.11
0.0000805	41.133	5.389	5.219	39.314	0.001	0.001	180.000	0.001	11.419	14	136.09	0.00	136.09
0.0000825	42.456	5.195	5.884	39.523	0.001	0.001	180.000	0.001	10.490	14	136.08	0.00	136.08
0.0000845	43.099	5.044	6.592	39.728	0.001	0.001	180.000	0.001	9.609	14	136.09	0.00	136.09
0.0000865	42.917	4.936	7.317	39.927	0.001	0.001	180.000	0.001	8.947	15	136.11	0.00	136.11
0.0000885	41.811	4.867	8.028	40.119	0.001	0.001	180.000	0.001	9.532	15	136.13	0.00	136.13
0.0000905	39.701	4.834	8.690	40.303	0.001	0.001	180.000	0.001	10.088	15	136.15	0.00	136.15
0.0000925	36.709	4.834	9.268	40.479	0.001	0.001	180.000	0.001	10.600	15	136.15	0.00	136.15
0.0000945	33.078	4.863	9.728	40.647	0.001	0.001	180.000	0.001	11.058	15	136.14	0.00	136.14
0.0000965	29.132	4.920	10.040	40.805	0.001	0.001	180.000	0.001	11.448	15	136.10	0.00	136.10
0.0000985	25.200	4.999	10.184	40.956	0.001	0.001	180.000	0.001	11.760	15	136.03	0.00	136.03
0.0001015	19.745	5.158	9.991	41.162	0.001	0.001	180.000	0.001	12.029	15	135.84	0.00	135.84
0.0001045	15.407	5.335	9.419	41.351	0.001	0.001	180.000	0.001	12.084	15	135.58	0.00	135.58
0.0001065	13.148	5.447	8.909	41.471	0.001	0.001	180.000	0.001	12.026	15	135.39	0.00	135.39
0.0001085	11.302	5.540	8.303	41.584	0.001	0.001	180.000	0.001	11.875	15	135.20	0.00	135.20
0.0001105	9.798	5.602	7.635	41.694	0.001	0.001	180.000	0.001	11.637	15	135.00	0.00	135.00
0.0001125	8.569	5.621	6.942	41.800	0.001	0.001	180.000	0.001	11.320	15	134.82	0.00	134.82
0.0001145	7.557	5.588	6.252	41.904	0.001	0.001	180.000	0.001	10.934	15	134.66	0.00	134.66
0.0001165	6.718	5.498	5.589	42.008	0.001	0.001	180.000	0.001	10.494	15	134.53	0.00	134.53
0.0001185	6.016	5.351	4.969	42.111	0.001	0.001	180.000	0.001	10.012	15	134.44	0.00	134.44
0.0001205	5.423	5.154	4.403	42.216	0.001	0.001	180.000	0.001	9.502	15	134.39	0.00	134.39

TABLE X (Contd.)

0.0001225	4.918	4.922	3.895	42.322	0.001	0.001	180.000	0.001	8.977	15	134.39	0.00	134.39
0.0001245	4.484	4.667	3.447	42.431	0.001	0.001	180.000	0.001	8.448	15	134.43	0.00	134.43
0.0001275	3.942	4.261	2.892	42.600	0.001	0.001	180.000	0.001	7.661	15	134.57	0.00	134.57
0.0001305	3.498	3.873	2.457	42.775	0.001	0.001	180.000	0.001	7.345	16	134.79	0.00	134.79
0.0001325	3.244	3.636	2.221	42.895	0.001	0.001	180.000	0.001	7.713	16	134.98	0.00	134.98
0.0001345	3.019	3.419	2.031	43.018	0.001	0.001	180.000	0.001	8.059	16	135.18	0.00	135.18
0.0001365	2.818	3.220	1.884	43.144	0.001	0.001	180.000	0.001	8.375	16	135.40	0.00	135.40
0.0001385	2.637	3.040	1.779	43.273	0.001	0.001	180.000	0.001	8.657	16	135.61	0.00	135.61
0.0001405	2.475	2.879	1.714	43.405	0.001	0.001	180.000	0.001	8.898	16	135.82	0.00	135.82
0.0001425	2.327	2.736	1.686	43.540	0.001	0.001	180.000	0.001	9.094	16	136.01	0.00	136.01
0.0001445	2.194	2.609	1.696	43.677	0.001	0.001	180.000	0.001	9.240	16	136.18	0.00	136.18
0.0001465	2.072	2.497	1.742	43.817	0.001	0.001	180.000	0.001	9.334	16	136.31	0.00	136.31
0.0001485	1.960	2.399	1.824	43.958	0.001	0.001	180.000	0.001	9.374	16	136.42	0.00	136.42
0.0001505	1.858	2.315	1.942	44.101	0.001	0.001	180.000	0.001	9.358	16	136.48	0.00	136.48
0.0001525	1.764	2.243	2.092	44.245	0.001	0.001	180.000	0.001	9.288	16	136.49	0.00	136.49
0.0001545	1.677	2.184	2.272	44.389	0.001	0.001	180.000	0.001	9.165	16	136.47	0.00	136.47
0.0001565	1.597	2.137	2.480	44.533	0.001	0.001	180.000	0.001	8.993	16	136.41	0.00	136.41
0.0001585	1.522	2.101	2.708	44.675	0.001	0.001	180.000	0.001	8.776	16	136.32	0.00	136.32
0.0001605	1.453	2.077	2.953	44.816	0.001	0.001	180.000	0.001	8.519	16	136.20	0.00	136.20
0.0001625	1.388	2.063	3.206	44.954	0.001	0.001	180.000	0.001	8.230	16	136.07	0.00	136.07
0.0001645	1.328	2.060	3.459	45.089	0.001	0.001	180.000	0.001	7.914	16	135.92	0.00	135.92
0.0001665	1.272	2.069	3.703	45.220	0.001	0.001	180.000	0.001	7.578	16	135.77	0.00	135.77
0.0001685	1.219	2.090	3.929	45.348	0.001	0.001	180.000	0.001	7.228	16	135.63	0.00	135.63
0.0001705	1.169	2.122	4.128	45.471	0.001	0.001	180.000	0.001	6.871	16	135.50	0.00	135.50
0.0001725	1.123	2.166	4.293	45.590	0.001	0.001	180.000	0.001	6.512	16	135.39	0.00	135.39
0.0001745	1.079	2.222	4.417	45.705	0.001	0.001	180.000	0.001	6.156	16	135.31	0.00	135.31
0.0001765	1.038	2.291	4.496	45.816	0.001	0.001	180.000	0.001	6.042	17	135.25	0.00	135.25
0.0001785	0.999	2.371	4.529	45.923	0.001	0.001	180.000	0.001	6.296	17	135.23	0.00	135.23
0.0001805	0.962	2.462	4.515	46.026	0.001	0.001	180.000	0.001	6.535	17	135.22	0.00	135.22
0.0001825	0.927	2.562	4.455	46.127	0.001	0.001	180.000	0.001	6.755	17	135.24	0.00	135.24
0.0001845	0.894	2.670	4.353	46.224	0.001	0.001	180.000	0.001	6.952	17	135.28	0.00	135.28
0.0001875	0.847	2.839	4.118	46.365	0.001	0.001	180.000	0.001	7.188	17	135.35	0.00	135.35
0.0001905	0.804	3.007	3.826	46.502	0.001	0.001	180.000	0.001	7.358	17	135.43	0.00	135.43
0.0001935	0.764	3.160	3.504	46.637	0.001	0.001	180.000	0.001	7.454	17	135.52	0.00	135.52
0.0001965	0.727	3.283	3.173	46.773	0.001	0.001	180.000	0.001	7.475	17	135.59	0.00	135.59
0.0001995	0.693	3.364	2.855	46.909	0.001	0.001	180.000	0.001	7.418	17	135.65	0.00	135.65
0.0002025	0.660	3.391	2.563	47.049	0.001	0.001	180.000	0.001	7.287	17	135.67	0.00	135.67
0.0002055	0.630	3.360	2.308	47.193	0.001	0.001	180.000	0.001	7.088	17	135.67	0.00	135.67
0.0002085	0.602	3.271	2.098	47.342	0.001	0.001	180.000	0.001	6.830	17	135.65	0.00	135.65
0.0002115	0.575	3.135	1.936	47.496	0.001	0.001	180.000	0.001	6.523	17	135.61	0.00	135.61
0.0002145	0.550	2.966	1.822	47.657	0.001	0.001	180.000	0.001	6.180	17	135.57	0.00	135.57
0.0002175	0.527	2.783	1.754	47.824	0.001	0.001	180.000	0.001	5.813	17	135.52	0.00	135.52
0.0002205	0.505	2.597	1.730	47.996	0.001	0.001	180.000	0.001	5.432	17	135.49	0.00	135.49
0.0002235	0.484	2.419	1.747	48.173	0.001	0.001	180.000	0.001	5.049	17	135.48	0.00	135.48
0.0002265	0.464	2.255	1.800	48.353	0.001	0.001	180.000	0.001	5.227	18	135.51	0.00	135.51
0.0002285	0.452	2.155	1.851	48.474	0.001	0.001	180.000	0.001	5.407	18	135.54	0.00	135.54
0.0002305	0.440	2.064	1.914	48.595	0.001	0.001	180.000	0.001	5.573	18	135.58	0.00	135.58
0.0002325	0.428	1.981	1.987	48.718	0.001	0.001	180.000	0.001	5.725	18	135.63	0.00	135.63
0.0002345	0.417	1.906	2.068	48.840	0.001	0.001	180.000	0.001	5.860	18	135.69	0.00	135.69
0.0002365	0.406	1.839	2.155	48.961	0.001	0.001	180.000	0.001	5.976	18	135.75	0.00	135.75
0.0002385	0.396	1.780	2.245	49.082	0.001	0.001	180.000	0.001	6.071	18	135.81	0.00	135.81

TABLE X (Contd.)

0.0002405	0.386	1.729	2.335	49.202	0.001	0.001	180.000	0.001	6.145	18	135.87	0.00	135.87
0.0002425	0.376	1.685	2.423	49.320	0.001	0.001	180.000	0.001	6.196	18	135.93	0.00	135.93
0.0002445	0.366	1.648	2.505	49.437	0.001	0.001	180.000	0.001	6.224	18	135.97	0.00	135.97
0.0002465	0.357	1.618	2.578	49.552	0.001	0.001	180.000	0.001	6.228	18	136.01	0.00	136.01
0.0002485	0.349	1.594	2.641	49.664	0.001	0.001	180.000	0.001	6.208	18	136.03	0.00	136.03
0.0002505	0.340	1.576	2.690	49.774	0.001	0.001	180.000	0.001	6.165	18	136.04	0.00	136.04
0.0002525	0.332	1.564	2.724	49.881	0.001	0.001	180.000	0.001	6.099	18	136.03	0.00	136.03
0.0002545	0.324	1.558	2.743	49.986	0.001	0.001	180.000	0.001	6.012	18	136.00	0.00	136.00
0.0002565	0.316	1.557	2.744	50.089	0.001	0.001	180.000	0.001	5.904	18	135.96	0.00	135.96
0.0002585	0.309	1.562	2.729	50.188	0.001	0.001	180.000	0.001	5.779	18	135.91	0.00	135.91
0.0002605	0.301	1.572	2.697	50.285	0.001	0.001	180.000	0.001	5.636	18	135.84	0.00	135.84
0.0002625	0.294	1.587	2.650	50.380	0.001	0.001	180.000	0.001	5.480	18	135.77	0.00	135.77
0.0002645	0.287	1.608	2.591	50.472	0.001	0.001	180.000	0.001	5.311	18	135.70	0.00	135.70
0.0002665	0.281	1.633	2.522	50.562	0.001	0.001	180.000	0.001	5.133	18	135.62	0.00	135.62
0.0002685	0.274	1.662	2.446	50.651	0.001	0.001	180.000	0.001	4.947	18	135.54	0.00	135.54
0.0002705	0.268	1.696	2.366	50.738	0.001	0.001	180.000	0.001	4.755	18	135.48	0.00	135.48
0.0002725	0.262	1.733	2.287	50.825	0.001	0.001	180.000	0.001	4.561	18	135.41	0.00	135.41
0.0002745	0.256	1.773	2.209	50.910	0.001	0.001	180.000	0.001	4.364	18	135.36	0.00	135.36
0.0002765	0.251	1.814	2.136	50.996	0.001	0.001	180.000	0.001	4.433	19	135.32	0.00	135.32
0.0002785	0.245	1.857	2.070	51.081	0.001	0.001	180.000	0.001	4.571	19	135.29	0.00	135.29
0.0002805	0.240	1.900	2.012	51.166	0.001	0.001	180.000	0.001	4.699	19	135.28	0.00	135.28
0.0002825	0.235	1.942	1.962	51.252	0.001	0.001	180.000	0.001	4.817	19	135.28	0.00	135.28
0.0002855	0.227	2.001	1.910	51.382	0.001	0.001	180.000	0.001	4.966	19	135.29	0.00	135.29
0.0002885	0.220	2.052	1.878	51.514	0.001	0.001	180.000	0.001	5.085	19	135.34	0.00	135.34
0.0002915	0.213	2.094	1.867	51.647	0.001	0.001	180.000	0.001	5.169	19	135.40	0.00	135.40
0.0002945	0.206	2.123	1.872	51.782	0.001	0.001	180.000	0.001	5.218	19	135.48	0.00	135.48
0.0002975	0.200	2.139	1.890	51.919	0.001	0.001	180.000	0.001	5.230	19	135.57	0.00	135.57
0.0003005	0.194	2.139	1.916	52.058	0.001	0.001	180.000	0.001	5.204	19	135.66	0.00	135.66
0.0003035	0.188	2.124	1.944	52.198	0.001	0.001	180.000	0.001	5.142	19	135.74	0.00	135.74
0.0003065	0.182	2.095	1.968	52.338	0.001	0.001	180.000	0.001	5.045	19	135.80	0.00	135.80
0.0003095	0.177	2.054	1.983	52.480	0.001	0.001	180.000	0.001	4.916	19	135.85	0.00	135.85
0.0003125	0.171	2.001	1.983	52.621	0.001	0.001	180.000	0.001	4.759	19	135.87	0.00	135.87
0.0003155	0.166	1.942	1.963	52.762	0.001	0.001	180.000	0.001	4.578	19	135.86	0.00	135.86
0.0003185	0.162	1.877	1.920	52.903	0.001	0.001	180.000	0.001	4.379	19	135.83	0.00	135.83
0.0003215	0.157	1.810	1.855	53.042	0.001	0.001	180.000	0.001	4.166	19	135.78	0.00	135.78
0.0003245	0.152	1.742	1.770	53.182	0.001	0.001	180.000	0.001	3.944	19	135.72	0.00	135.72
0.0003275	0.148	1.675	1.670	53.320	0.001	0.001	180.000	0.001	3.784	20	135.66	0.00	135.66
0.0003305	0.144	1.610	1.561	53.459	0.001	0.001	180.000	0.001	3.949	20	135.60	0.00	135.60
0.0003325	0.141	1.569	1.489	53.551	0.001	0.001	180.000	0.001	4.053	20	135.56	0.00	135.56
0.0003345	0.139	1.528	1.419	53.644	0.001	0.001	180.000	0.001	4.149	20	135.54	0.00	135.54
0.0003365	0.136	1.489	1.353	53.736	0.001	0.001	180.000	0.001	4.236	20	135.52	0.00	135.52
0.0003385	0.133	1.451	1.294	53.830	0.001	0.001	180.000	0.001	4.313	20	135.51	0.00	135.51
0.0003405	0.131	1.415	1.242	53.923	0.001	0.001	180.000	0.001	4.380	20	135.50	0.00	135.50
0.0003425	0.129	1.381	1.201	54.018	0.001	0.001	180.000	0.001	4.436	20	135.51	0.00	135.51
0.0003445	0.126	1.348	1.170	54.113	0.001	0.001	180.000	0.001	4.480	20	135.53	0.00	135.53
0.0003465	0.124	1.318	1.152	54.209	0.001	0.001	180.000	0.001	4.512	20	135.55	0.00	135.55
0.0003485	0.122	1.288	1.147	54.305	0.001	0.001	180.000	0.001	4.532	20	135.58	0.00	135.58
0.0003505	0.120	1.261	1.155	54.402	0.001	0.001	180.000	0.001	4.539	20	135.62	0.00	135.62
0.0003525	0.117	1.236	1.177	54.500	0.001	0.001	180.000	0.001	4.533	20	135.65	0.00	135.65
0.0003545	0.115	1.212	1.213	54.598	0.001	0.001	180.000	0.001	4.515	20	135.69	0.00	135.69
0.0003565	0.113	1.191	1.260	54.696	0.001	0.001	180.000	0.001	4.484	20	135.73	0.00	135.73

TABLE X (Contd.)

0.0003585	0.111	1.171	1.319	54.794	0.001	0.001	180.000	0.001	4.442	20	135.77	0.00	135.77
0.0003605	0.109	1.154	1.388	54.892	0.001	0.001	180.000	0.001	4.388	20	135.81	0.00	135.81
0.0003625	0.107	1.139	1.464	54.990	0.001	0.001	180.000	0.001	4.323	20	135.83	0.00	135.83
0.0003645	0.106	1.127	1.544	55.087	0.001	0.001	180.000	0.001	4.247	20	135.86	0.00	135.86
0.0003665	0.104	1.117	1.627	55.182	0.001	0.001	180.000	0.001	4.163	20	135.87	0.00	135.87
0.0003685	0.102	1.109	1.707	55.277	0.001	0.001	180.000	0.001	4.070	20	135.88	0.00	135.88
0.0003705	0.100	1.104	1.783	55.369	0.001	0.001	180.000	0.001	3.970	20	135.87	0.00	135.87
0.0003725	0.099	1.101	1.850	55.460	0.001	0.001	180.000	0.001	3.863	20	135.87	0.00	135.87
0.0003745	0.097	1.101	1.905	55.549	0.001	0.001	180.000	0.001	3.751	20	135.85	0.00	135.85
0.0003765	0.095	1.104	1.945	55.635	0.001	0.001	180.000	0.001	3.635	20	135.82	0.00	135.82
0.0003785	0.094	1.109	1.967	55.719	0.001	0.001	180.000	0.001	3.515	20	135.79	0.00	135.79
0.0003805	0.092	1.117	1.971	55.801	0.001	0.001	180.000	0.001	3.393	20	135.76	0.00	135.76
0.0003825	0.091	1.128	1.955	55.881	0.001	0.001	180.000	0.001	3.400	21	135.72	0.00	135.72
0.0003845	0.089	1.141	1.919	55.958	0.001	0.001	180.000	0.001	3.488	21	135.68	0.00	135.68
0.0003865	0.088	1.156	1.866	56.033	0.001	0.001	180.000	0.001	3.570	21	135.64	0.00	135.64
0.0003885	0.086	1.173	1.796	56.107	0.001	0.001	180.000	0.001	3.646	21	135.60	0.00	135.60
0.0003905	0.085	1.192	1.714	56.178	0.001	0.001	180.000	0.001	3.715	21	135.57	0.00	135.57
0.0003925	0.083	1.223	1.571	56.283	0.001	0.001	180.000	0.001	3.801	21	135.52	0.00	135.52
0.0003945	0.081	1.256	1.419	56.387	0.001	0.001	180.000	0.001	3.868	21	135.48	0.00	135.48
0.0003965	0.079	1.287	1.271	56.490	0.001	0.001	180.000	0.001	3.916	21	135.46	0.00	135.46
0.0004025	0.077	1.316	1.136	56.594	0.001	0.001	180.000	0.001	3.942	21	135.46	0.00	135.46
0.0004055	0.075	1.340	1.020	56.700	0.001	0.001	180.000	0.001	3.948	21	135.47	0.00	135.47
0.0004085	0.073	1.358	0.929	56.809	0.001	0.001	180.000	0.001	3.931	21	135.49	0.00	135.49
0.0004115	0.072	1.369	0.865	56.921	0.001	0.001	180.000	0.001	3.894	21	135.52	0.00	135.52
0.0004145	0.070	1.371	0.828	57.036	0.001	0.001	180.000	0.001	3.836	21	135.57	0.00	135.57
0.0004175	0.068	1.363	0.819	57.156	0.001	0.001	180.000	0.001	3.759	21	135.61	0.00	135.61
0.0004205	0.067	1.347	0.836	57.279	0.001	0.001	180.000	0.001	3.664	21	135.67	0.00	135.67
0.0004235	0.065	1.322	0.875	57.405	0.001	0.001	180.000	0.001	3.555	21	135.72	0.00	135.72
0.0004245	0.064	1.291	0.932	57.534	0.001	0.001	180.000	0.001	3.432	21	135.76	0.00	135.76
0.0004295	0.062	1.254	1.000	57.666	0.001	0.001	180.000	0.001	3.298	21	135.80	0.00	135.80
0.0004325	0.061	1.214	1.071	57.799	0.001	0.001	180.000	0.001	3.157	21	135.82	0.00	135.82
0.0004355	0.060	1.173	1.136	57.933	0.001	0.001	180.000	0.001	3.009	21	135.84	0.00	135.84
0.0004385	0.058	1.131	1.189	58.066	0.001	0.001	180.000	0.001	3.070	22	135.84	0.00	135.84
0.0004415	0.057	1.091	1.224	58.198	0.001	0.001	180.000	0.001	3.173	22	135.83	0.00	135.83
0.0004445	0.056	1.052	1.238	58.330	0.001	0.001	180.000	0.001	3.264	22	135.82	0.00	135.82
0.0004475	0.055	1.015	1.230	58.459	0.001	0.001	180.000	0.001	3.343	22	135.80	0.00	135.80
0.0004505	0.053	0.981	1.205	58.587	0.001	0.001	180.000	0.001	3.407	22	135.77	0.00	135.77
0.0004535	0.052	0.949	1.166	58.714	0.001	0.001	180.000	0.001	3.456	22	135.76	0.00	135.76
0.0004565	0.051	0.919	1.120	58.839	0.001	0.001	180.000	0.001	3.488	22	135.74	0.00	135.74
0.0004595	0.050	0.892	1.075	58.963	0.001	0.001	180.000	0.001	3.504	22	135.74	0.00	135.74
0.0004625	0.049	0.867	1.038	59.087	0.001	0.001	180.000	0.001	3.502	22	135.73	0.00	135.73
0.0004655	0.048	0.843	1.014	59.209	0.001	0.001	180.000	0.001	3.483	22	135.74	0.00	135.74
0.0004685	0.047	0.822	1.007	59.332	0.001	0.001	180.000	0.001	3.448	22	135.74	0.00	135.74
0.0004715	0.046	0.802	1.019	59.453	0.001	0.001	180.000	0.001	3.396	22	135.74	0.00	135.74
0.0004745	0.045	0.784	1.050	59.575	0.001	0.001	180.000	0.001	3.329	22	135.74	0.00	135.74
0.0004775	0.044	0.768	1.098	59.695	0.001	0.001	180.000	0.001	3.249	22	135.73	0.00	135.73
0.0004805	0.043	0.754	1.157	59.813	0.001	0.001	180.000	0.001	3.157	22	135.72	0.00	135.72
0.0004835	0.042	0.742	1.219	59.929	0.001	0.001	180.000	0.001	3.054	22	135.71	0.00	135.71
0.0004855	0.042	0.735	1.258	60.005	0.001	0.001	180.000	0.001	2.981	22	135.70	0.00	135.70
0.0004875	0.041	0.729	1.292	60.080	0.001	0.001	180.000	0.001	2.905	22	135.68	0.00	135.68
0.0004895	0.041	0.724	1.317	60.153	0.001	0.001	180.000	0.001	2.827	22	135.67	0.00	135.67

TABLE X (Contd.)

0.0004915	0.040	0.719	1.332	60.224	0.001	0.001	180.000	0.001	2.746	22	135.65	0.00	135.65
0.0004935	0.040	0.716	1.333	60.293	0.001	0.001	180.000	0.001	2.738	23	135.64	0.00	135.64
0.0004955	0.039	0.714	1.319	60.361	0.001	0.001	180.000	0.001	2.800	23	135.63	0.00	135.63
0.0004975	0.039	0.713	1.289	60.426	0.001	0.001	180.000	0.001	2.857	23	135.62	0.00	135.62
0.0004995	0.038	0.712	1.243	60.490	0.001	0.001	180.000	0.001	2.910	23	135.61	0.00	135.61
0.0005015	0.038	0.713	1.181	60.551	0.001	0.001	180.000	0.001	2.959	23	135.60	0.00	135.60
0.0005045	0.037	0.715	1.058	60.641	0.001	0.001	180.000	0.001	3.021	23	135.59	0.00	135.59
0.0005075	0.036	0.719	0.915	60.727	0.001	0.001	180.000	0.001	3.071	23	135.60	0.00	135.60
0.0005105	0.035	0.724	0.763	60.812	0.001	0.001	180.000	0.001	3.109	23	135.61	0.00	135.61
0.0005135	0.035	0.730	0.613	60.897	0.001	0.001	180.000	0.001	3.133	23	135.63	0.00	135.63
0.0005165	0.034	0.736	0.474	60.982	0.001	0.001	180.000	0.001	3.143	23	135.66	0.00	135.66
0.0005195	0.033	0.741	0.355	61.069	0.002	0.001	180.000	0.001	3.140	23	135.70	0.00	135.70
0.0005225	0.033	0.746	0.261	61.160	0.002	0.001	180.000	0.001	3.122	23	135.73	0.00	135.73
0.0005255	0.032	0.749	0.194	61.254	0.002	0.001	180.000	0.001	3.091	23	135.77	0.00	135.77
0.0005285	0.032	0.751	0.156	61.352	0.003	0.001	180.000	0.001	3.048	23	135.80	0.00	135.80
0.0005315	0.031	0.751	0.148	61.455	0.003	0.001	180.000	0.001	2.991	23	135.82	0.00	135.82
0.0005345	0.030	0.750	0.171	61.562	0.004	0.001	180.000	0.001	2.924	23	135.84	0.00	135.84
0.0005375	0.030	0.746	0.223	61.673	0.005	0.001	180.000	0.001	2.847	23	135.85	0.00	135.85
0.0005405	0.029	0.742	0.301	61.788	0.006	0.001	180.000	0.001	2.760	23	135.85	0.00	135.85
0.0005435	0.029	0.735	0.399	61.906	0.007	0.001	180.000	0.001	2.667	23	135.85	0.00	135.85
0.0005465	0.028	0.728	0.507	62.026	0.008	0.001	180.000	0.001	2.567	23	135.83	0.00	135.83
0.0005495	0.028	0.721	0.615	62.147	0.009	0.001	180.000	0.001	2.462	23	135.80	0.00	135.80
0.0005525	0.027	0.713	0.711	62.268	0.011	0.001	180.000	0.001	2.534	24	135.77	0.00	135.77
0.0005555	0.027	0.706	0.786	62.388	0.012	0.001	180.000	0.001	2.606	24	135.73	0.00	135.73
0.0005585	0.026	0.698	0.831	62.506	0.014	0.001	180.000	0.001	2.670	24	135.70	0.00	135.70
0.0005615	0.026	0.691	0.844	62.623	0.017	0.001	180.000	0.001	2.724	24	135.67	0.00	135.67
0.0005645	0.026	0.684	0.830	62.737	0.019	0.001	180.000	0.001	2.769	24	135.65	0.00	135.65
0.0005675	0.025	0.677	0.795	62.851	0.022	0.001	180.000	0.001	2.803	24	135.64	0.00	135.64
0.0005705	0.025	0.671	0.749	62.964	0.025	0.001	180.000	0.001	2.826	24	135.66	0.00	135.66
0.0005735	0.024	0.663	0.705	63.076	0.029	0.001	180.000	0.001	2.838	24	135.69	0.00	135.69
0.0005765	0.024	0.656	0.671	63.188	0.033	0.001	180.000	0.001	2.839	24	135.74	0.00	135.74
0.0005795	0.024	0.648	0.656	63.301	0.038	0.001	180.000	0.001	2.828	24	135.80	0.00	135.80
0.0005825	0.023	0.639	0.667	63.415	0.044	0.001	180.000	0.001	2.806	24	135.85	0.00	135.85
0.0005855	0.023	0.630	0.706	63.529	0.049	0.001	180.000	0.001	2.773	24	135.90	0.00	135.90
0.0005885	0.023	0.621	0.772	63.644	0.056	0.001	180.000	0.001	2.729	24	135.94	0.00	135.94
0.0005915	0.023	0.611	0.860	63.758	0.064	0.001	180.000	0.001	2.676	24	135.97	0.00	135.97
0.0005945	0.022	0.602	0.960	63.871	0.072	0.001	180.000	0.001	2.613	24	135.97	0.00	135.97
0.0005975	0.022	0.592	1.061	63.981	0.081	0.001	180.000	0.001	2.543	24	135.96	0.00	135.96
0.0006005	0.022	0.584	1.146	64.088	0.092	0.001	180.000	0.001	2.466	24	135.92	0.00	135.92
0.0006035	0.022	0.576	1.204	64.191	0.103	0.001	180.000	0.001	2.383	24	135.87	0.00	135.87
0.0006065	0.022	0.569	1.221	64.289	0.116	0.001	180.000	0.001	2.295	24	135.81	0.00	135.81
0.0006095	0.021	0.564	1.191	64.382	0.130	0.001	180.000	0.001	2.301	25	135.74	0.00	135.74
0.0006125	0.021	0.559	1.112	64.470	0.145	0.001	180.000	0.001	2.365	25	135.67	0.00	135.67
0.0006155	0.021	0.554	0.993	64.554	0.162	0.001	180.000	0.001	2.422	25	135.61	0.00	135.61
0.0006185	0.021	0.551	0.844	64.634	0.181	0.001	180.000	0.001	2.471	25	135.56	0.00	135.56
0.0006215	0.021	0.547	0.683	64.711	0.201	0.001	180.001	0.001	2.513	25	135.53	0.00	135.53
0.0006245	0.021	0.543	0.522	64.788	0.223	0.001	180.001	0.001	2.546	25	135.51	0.00	135.51
0.0006275	0.021	0.539	0.376	64.865	0.247	0.001	180.001	0.001	2.570	25	135.51	0.00	135.52
0.0006305	0.021	0.534	0.254	64.944	0.273	0.001	180.001	0.001	2.585	25	135.53	0.00	135.53
0.0006335	0.021	0.529	0.161	65.025	0.301	0.001	180.002	0.001	2.590	25	135.56	0.00	135.56
0.0006365	0.022	0.522	0.100	65.110	0.332	0.001	180.002	0.001	2.585	25	135.60	0.01	135.60

TABLE X (Contd.)

0.0006395	0.022	0.514	0.072	65.199	0.365	0.001	180.003	0.001	2.571	25	135.64	0.01	135.65
0.0006425	0.022	0.506	0.075	65.291	0.400	0.001	180.004	0.001	2.548	25	135.68	0.01	135.69
0.0006455	0.022	0.497	0.106	65.387	0.438	0.001	180.005	0.001	2.515	25	135.72	0.01	135.73
0.0006485	0.023	0.487	0.163	65.487	0.479	0.001	180.006	0.001	2.474	25	135.75	0.02	135.77
0.0006515	0.023	0.477	0.239	65.589	0.523	0.001	180.007	0.001	2.426	25	135.77	0.03	135.80
0.0006545	0.024	0.467	0.326	65.692	0.570	0.001	180.009	0.001	2.369	25	135.79	0.03	135.82
0.0006575	0.024	0.457	0.412	65.797	0.620	0.001	180.011	0.001	2.307	25	135.79	0.04	135.83
0.0006605	0.025	0.447	0.486	65.901	0.673	0.001	180.013	0.001	2.239	25	135.77	0.05	135.83
0.0006635	0.025	0.438	0.538	66.003	0.729	0.001	180.015	0.001	2.166	25	135.75	0.07	135.82
0.0006665	0.026	0.430	0.562	66.105	0.789	0.001	180.017	0.001	2.113	26	135.71	0.09	135.80
0.0006695	0.027	0.422	0.553	66.204	0.852	0.001	180.020	0.001	2.175	26	135.66	0.11	135.77
0.0006725	0.028	0.415	0.513	66.302	0.918	0.001	180.024	0.001	2.232	26	135.61	0.14	135.75
0.0006755	0.029	0.409	0.449	66.398	0.988	0.001	180.027	0.001	2.283	26	135.55	0.17	135.72
0.0006785	0.030	0.402	0.374	66.493	1.061	0.001	180.031	0.001	2.328	26	135.51	0.21	135.71
0.0006815	0.031	0.397	0.303	66.589	1.137	0.001	180.036	0.001	2.367	26	135.46	0.25	135.72
0.0006845	0.032	0.391	0.247	66.686	1.216	0.001	180.041	0.001	2.398	26	135.43	0.31	135.74
0.0006875	0.034	0.385	0.217	66.784	1.298	0.001	180.046	0.001	2.423	26	135.39	0.37	135.77
0.0006905	0.035	0.380	0.219	66.885	1.383	0.001	180.052	0.001	2.439	26	135.36	0.45	135.81
0.0006935	0.037	0.375	0.256	66.987	1.470	0.001	180.059	0.001	2.449	26	135.32	0.54	135.86
0.0006965	0.038	0.369	0.327	67.091	1.559	0.001	180.066	0.001	2.450	26	135.26	0.64	135.91
0.0006995	0.040	0.365	0.427	67.196	1.650	0.001	180.074	0.001	2.444	26	135.20	0.77	135.96
0.0007015	0.041	0.361	0.502	67.266	1.712	0.001	180.080	0.001	2.437	26	135.14	0.86	136.00
0.0007035	0.042	0.358	0.580	67.335	1.774	0.001	180.086	0.001	2.426	26	135.08	0.96	136.03
0.0007055	0.043	0.356	0.657	67.404	1.836	0.001	180.092	0.001	2.413	26	135.01	1.06	136.07
0.0007075	0.044	0.353	0.728	67.471	1.898	0.001	180.099	0.001	2.396	26	134.92	1.18	136.11
0.0007095	0.045	0.351	0.789	67.536	1.960	0.001	180.106	0.001	2.376	26	134.83	1.31	136.15
0.0007115	0.047	0.350	0.834	67.600	2.022	0.001	180.113	0.001	2.354	26	134.73	1.46	136.19
0.0007135	0.048	0.348	0.861	67.662	2.084	0.001	180.121	0.001	2.329	26	134.63	1.61	136.24
0.0007155	0.049	0.347	0.867	67.722	2.144	0.001	180.129	0.001	2.302	26	134.51	1.78	136.29
0.0007175	0.051	0.346	0.851	67.779	2.204	0.001	180.138	0.001	2.273	26	134.39	1.96	136.35
0.0007195	0.052	0.346	0.813	67.835	2.262	0.001	180.147	0.001	2.262	27	134.26	2.15	136.41
0.0007215	0.053	0.346	0.756	67.888	2.319	0.001	180.157	0.001	2.319	27	134.13	2.36	136.49
0.0007235	0.055	0.346	0.683	67.940	2.375	0.001	180.167	0.001	2.375	27	133.98	2.59	136.57
0.0007255	0.056	0.346	0.598	67.990	2.428	0.001	180.177	0.001	2.428	27	133.84	2.83	136.67
0.0007275	0.058	0.347	0.508	68.039	2.479	0.001	180.188	0.001	2.479	27	133.68	3.09	136.77
0.0007295	0.059	0.348	0.416	68.088	2.528	0.001	180.200	0.001	2.528	27	133.52	3.37	136.89
0.0007315	0.061	0.349	0.327	68.137	2.574	0.001	180.211	0.001	2.574	27	133.35	3.66	137.01
0.0007335	0.063	0.349	0.246	68.186	2.616	0.001	180.224	0.001	2.616	27	133.17	3.97	137.14
0.0007355	0.064	0.350	0.176	68.235	2.656	0.001	180.237	0.001	2.656	27	132.98	4.30	137.28
0.0007375	0.066	0.351	0.118	68.285	2.691	0.001	180.250	0.001	2.691	27	132.77	4.65	137.42
0.0007395	0.068	0.352	0.074	68.336	2.724	0.001	180.264	0.001	2.724	27	132.55	5.02	137.57
0.0007415	0.069	0.352	0.045	68.389	2.752	0.001	180.279	0.001	2.752	27	132.31	5.41	137.72
0.0007435	0.071	0.353	0.032	68.443	2.775	0.001	180.294	0.001	2.775	27	132.05	5.81	137.86
0.0007455	0.073	0.353	0.033	68.499	2.795	0.001	180.310	0.001	2.795	27	131.77	6.24	138.01
0.0007475	0.075	0.353	0.049	68.555	2.809	0.001	180.326	0.001	2.809	27	131.47	6.68	138.15
0.0007495	0.077	0.353	0.078	68.614	2.819	0.001	180.343	0.001	2.819	27	131.15	7.14	138.29
0.0007515	0.079	0.353	0.116	68.673	2.825	0.001	180.360	0.001	2.825	27	130.80	7.62	138.42
0.0007535	0.081	0.353	0.163	68.733	2.825	0.001	180.378	0.001	2.825	27	130.43	8.11	138.54
0.0007555	0.083	0.353	0.214	68.794	2.820	0.001	180.396	0.001	2.820	27	130.04	8.62	138.66
0.0007575	0.085	0.352	0.268	68.856	2.810	0.001	180.415	0.001	2.810	27	129.62	9.14	138.76
0.0007595	0.087	0.352	0.319	68.917	2.795	0.001	180.435	0.001	2.795	27	129.18	9.68	138.86

TABLE X (Contd.)

0.0007615	0.089	0.351	0.366	68.978	2.775	0.001	180.455	0.001	2.775	27	128.72	10.23	138.95
0.0007635	0.092	0.351	0.406	69.039	2.749	0.001	180.476	0.001	2.749	27	128.25	10.79	139.04
0.0007655	0.094	0.350	0.435	69.100	2.719	0.001	180.497	0.001	2.719	27	127.76	11.36	139.12
0.0007675	0.096	0.350	0.453	69.160	2.684	0.001	180.519	0.001	2.684	27	127.25	11.94	139.19
0.0007695	0.099	0.350	0.457	69.219	2.645	0.001	180.541	0.001	2.645	27	126.74	12.53	139.27
0.0007715	0.101	0.350	0.447	69.277	2.601	0.001	180.564	0.001	2.601	27	126.22	13.12	139.33
0.0007735	0.104	0.349	0.425	69.335	2.552	0.001	180.587	0.001	2.552	27	125.69	13.71	139.40
0.0007755	0.106	0.349	0.392	69.393	2.500	0.001	180.611	0.001	2.500	27	125.16	14.31	139.47
0.0007775	0.109	0.349	0.352	69.450	2.443	0.001	180.636	0.001	2.443	27	124.64	14.90	139.54
0.0007795	0.111	0.349	0.307	69.506	2.383	0.001	180.660	0.001	2.383	27	124.12	15.50	139.62
0.0007815	0.114	0.348	0.261	69.563	2.320	0.001	180.686	0.001	2.320	27	123.61	16.09	139.70
0.0007835	0.117	0.348	0.220	69.621	2.253	0.001	180.712	0.001	2.253	27	123.11	16.67	139.78
0.0007855	0.119	0.347	0.185	69.679	2.184	0.001	180.738	0.001	2.184	27	122.62	17.25	139.87
0.0007875	0.122	0.347	0.159	69.738	2.113	0.001	180.764	0.001	2.113	27	122.14	17.82	139.96
0.0007895	0.125	0.346	0.146	69.797	2.039	0.001	180.791	0.001	2.039	27	121.67	18.39	140.06
0.0007915	0.128	0.345	0.146	69.858	1.963	0.001	180.819	0.001	1.963	27	121.22	18.93	140.15
0.0007935	0.131	0.343	0.161	69.920	1.886	0.001	180.847	0.001	1.886	27	120.78	19.47	140.25
0.0007955	0.134	0.342	0.190	69.983	1.807	0.001	180.875	0.001	1.807	27	120.35	19.99	140.34
0.0007975	0.137	0.340	0.231	70.047	1.727	0.001	180.904	0.001	1.727	27	119.94	20.50	140.44
0.0007995	0.140	0.339	0.285	70.111	1.647	0.001	180.933	0.001	1.647	27	119.53	20.99	140.53
0.0008015	0.143	0.337	0.346	70.176	1.566	0.001	180.962	0.001	1.566	27	119.14	21.46	140.61
0.0008035	0.146	0.335	0.412	70.241	1.485	0.001	180.991	0.001	1.485	27	118.77	21.92	140.69
0.0008055	0.150	0.333	0.478	70.305	1.404	0.001	181.021	0.001	1.454	25	118.40	22.35	140.76
0.0008075	0.153	0.331	0.540	70.369	1.323	0.001	181.051	0.001	1.487	25	118.05	22.77	140.82
0.0008095	0.157	0.329	0.593	70.431	1.242	0.001	181.082	0.001	1.520	25	117.72	23.16	140.89
0.0008115	0.160	0.327	0.633	70.493	1.162	0.001	181.112	0.001	1.552	25	117.40	23.54	140.94
0.0008135	0.164	0.326	0.657	70.553	1.083	0.001	181.143	0.001	1.583	25	117.10	23.89	140.99
0.0008155	0.167	0.324	0.663	70.611	1.005	0.001	181.174	0.001	1.613	25	116.82	24.22	141.05
0.0008175	0.171	0.323	0.649	70.668	0.928	0.001	181.205	0.001	1.642	25	116.56	24.53	141.09
0.0008195	0.174	0.322	0.618	70.724	0.852	0.001	181.237	0.001	1.669	25	116.33	24.82	141.14
0.0008215	0.178	0.320	0.571	70.778	0.778	0.001	181.268	0.001	1.695	25	116.11	25.08	141.19
0.0008235	0.182	0.319	0.511	70.831	0.705	0.001	181.300	0.001	1.719	25	115.92	25.33	141.25
0.0008255	0.186	0.318	0.444	70.883	0.634	0.001	181.332	0.001	1.742	25	115.76	25.55	141.31
0.0008275	0.190	0.317	0.373	70.935	0.564	0.001	181.364	0.001	1.762	25	115.62	25.75	141.37
0.0008295	0.194	0.316	0.303	70.986	0.496	0.001	181.396	0.001	1.781	25	115.51	25.93	141.43
0.0008315	0.198	0.315	0.239	71.038	0.430	0.001	181.428	0.001	1.797	25	115.42	26.08	141.50
0.0008335	0.202	0.313	0.184	71.090	0.366	0.001	181.460	0.001	1.811	25	115.36	26.22	141.58
0.0008355	0.206	0.312	0.141	71.143	0.304	0.001	181.493	0.001	1.823	25	115.32	26.34	141.66
0.0008375	0.210	0.310	0.111	71.197	0.244	0.001	181.525	0.001	1.833	25	115.30	26.43	141.73
0.0008395	0.215	0.309	0.096	71.252	0.185	0.001	181.557	0.001	1.840	25	115.30	26.51	141.81
0.0008415	0.219	0.307	0.095	71.308	0.129	0.001	181.590	0.001	1.844	25	115.32	26.57	141.89
0.0008435	0.223	0.305	0.109	71.365	0.074	0.001	181.622	0.001	1.846	25	115.35	26.61	141.96
0.0008455	0.228	0.303	0.137	71.423	0.022	0.001	181.654	0.001	1.846	25	115.40	26.63	142.03
0.0008475	0.232	0.300	0.175	71.483	0.000	0.001	181.687	0.001	1.843	25	115.45	26.64	142.09
0.0008495	0.237	0.298	0.223	71.542	0.000	0.001	181.719	0.001	1.837	25	115.51	26.64	142.15
0.0008515	0.242	0.296	0.275	71.602	0.000	0.001	181.751	0.001	1.829	25	115.55	26.64	142.19
0.0008535	0.246	0.293	0.328	71.663	0.000	0.001	181.784	0.001	1.818	25	115.59	26.64	142.23

NUMBER OF LINES OF PRINT OUT IN THE FINAL TABLE I = 389

NUMBER OF RECORDS WRITTEN ON TAPE N = 78

PROGRAM OV--

Fig. 21. Pressure-Time Curve for the Inside Face of the Concrete Vessel

Fig. 22. Location-Time Curve for the TNT-Water Interface

Fig. 23. Location-Time Curve for the Reactor Vessel (Water-Argon Interface)

Fig. 24. Maximum Pressure in the System Outside of the Gas Bubble

Fig. 25. Pressure-Time Curve for the Water at the Reactor Vessel (Water-Argon)

Fig. 26. Pressure-Time Curve for the Water at the TNT-Water Interface

Fig. 27. Pressure-Time Curve for the Center of the Gas Bubble

Fig. 28. Pressure-Time Curve for the TNT at the TNT-Water Interface

Fig. 29. Total Energy of the System as a Function of Time

Fig. 30. Energy Associated with the Fluid as a Function of Time

Fig. 31. Energy Associated with the Reactor Vessel as a Function of Time

IX. DISCUSSION OF RESULTS

A. Comparison of Results

Computer results for the simulated sodium reactor installation without the reactor vessel are compared to those results obtained with the reactor vessel by use of the following figures:

1. The pressure-time curve for the center of the gas bubble (Figs. 17 and 27).

The pressure at the center of the gas bubble is virtually unaffected by the addition of the reactor vessel to the system.

2. The pressure-time curve for the TNT at the TNT-water interface (Figs. 10 and 28).

The pressure in the TNT at the TNT-water interface is virtually unaffected by the addition of the reactor vessel to the system.

3. The pressure-time curve for the water at the TNT-water interface (Figs. 15 and 26).

The pressure in the water at the TNT-water interface is virtually unaffected by the addition of the reactor vessel to the system.

4. The location-time curve for the TNT-water interface (Figs. 12 and 22).

In a minor way, the motion of the TNT-water interface is affected by the addition of the reactor vessel to the system. The period of the second bubble pulsation is increased because of the retarding action caused by the resistance of the reactor vessel. The maximum size of the gas bubble is virtually unaffected by the addition of the reactor vessel.

5. The pressure-time curve for the water at the reactor vessel (water-argon interface) (Figs. 14 and 25).

The pressure history for the water at the reactor vessel or water-argon interface is changed considerably with the addition of the reactor vessel to the system. Because of the resistance of the reactor vessel, the first pressure peak ($t = 0.75$ msec), which is due to the shock wave, increases from 1^+ to 3^- kb. Also, the peak pressures due to the fluid momentum are displaced from 4.5 and 20 msec to 5 and 23 msec, respectively. The magnitudes of these two peak pressures are virtually unchanged.

6. The location-time curve for the reactor vessel (water-argon interface) (Figs. 13 and 23).

The motion of the boundary separating the water and the argon is greatly affected by the addition of the reactor vessel to the system. The boundary is slowed down because of the resistance of the reactor vessel. This results in a longer period for the second pulsation.

7. The pressure-time curve for the inside face of the concrete vessel (Figs. 11 and 21).

The addition of the reactor vessel to the system has resulted in three noticeable differences between the two pressure-time curves for the inside face of the concrete vessel: (a) The pressure peak at 2.0 msec (Fig. 11) has been eliminated by the reactor vessel as a result of the resistance of the reactor vessel, (b) the maximum pressure has been decreased, and (c) the period of the second pressure peak has been increased.

8. The maximum pressure in the system outside of the gas bubble (Figs. 16 and 24).

These graphs are presented for information only.

9. The total energy in the system as a function of time (Figs. 18 and 29).

The system analyzed in this problem constitutes a closed energy system. Therefore, the total energy in the system should remain constant. These two graphs depict energy conservation to within the accuracy of the finite-difference solution.

Figures 30 and 31 are presented to show the way in which the various energy components vary.

B. General Discussion of Results

Figure 31 shows that the reactor vessel removes a substantial amount of energy from the fluid. The maximum reactor vessel energy is 33.35 Mcal. The total energy in the system is 136.60 Mcal. Thus, the energy of the reactor vessel represents 24.4% of the total energy. The energy remaining in the fluid is thereby decreased by this amount, and thus the effect of the destructive component on the concrete containment vessel is diminished.

The impulse on the concrete containment vessel is 2,066,292 gm-cm/sec/sq cm. The resulting maximum strain rate is 9.77 sec^{-1} . This corresponds to a 20.7% decrease compared to the analysis made without the reactor vessel.

X. CONCLUSION

It can be concluded from the results obtained from this solution that the reactor vessel is a very good energy-absorbing device and should be included as part of the primary containment system. The addition of the reactor vessel to the system reduces the effect that the destructive component produces on the concrete structure as compared to the effect produced without a reactor vessel. Thus, the use of the static properties of the steel strands for design becomes a better approximation.

The incorporation of the Tait form of the equation of state for the water into the computer program decreases the computer time required for the solution with virtually no change in the accuracy of the solution.

It is known that the stress in a strain-hardening steel like Type 304 stainless is a function of strain ϵ and the strain rate $\dot{\epsilon}$; i.e.,

$$\sigma = \sigma(\epsilon, \dot{\epsilon}). \quad (93)$$

In this problem, the static properties of the material were used as a first approximation. To refine the solution, it is necessary to have a function relating the strain rate and the dynamic yield stress similar to that given for mild steel,*

$$\sigma_d = \sigma_y \left[\left(\frac{\dot{\epsilon}}{40.4} \right)^{0.2} + 1 \right], \quad (94)$$

where

σ_d = dynamic yield stress, psi;

σ_y = static yield stress, psi;

and

$\dot{\epsilon}$ = strain rate, sec^{-1} .

A function similar to Eq. 94, together with a linear strain-hardening function, could be used to obtain a more exact true stress-strain curve for the conditions to which the reactor vessel is subjected. The strain rate can be obtained at any given time from the relationship

$$\dot{\epsilon}_c = \frac{u}{r_0}, \quad (95)$$

*See Ref. 23, page 38.

where u and r_0 are the velocity at time t and the initial radius of the reactor vessel, respectively. The effective strain rate, $\dot{\epsilon}$, obtained from an expression similar to Eq. 78 is given as

$$\dot{\epsilon} = 2\dot{\epsilon}_c. \quad (96)$$

A very important factor in the analysis is the thickness of the argon layer in comparison to the allowable increase in the radius of the reactor vessel before rupture. The reactor vessel must not be allowed to impinge against the concrete vessel while retaining its integrity, as this would cause adverse effects. Thus, the thickness of the argon layer must be correlated to the plastic deformation of the vessel.

Further work in the nuclear-reactor containment field on this concept should include (1) a more reliable equation of state for the argon, (2) optimization of the computer program as to the zone widths used in the solution, and (3) extension of the analysis to a two-dimensional system, which resembles more closely the actual reactor system. The fracture criteria for the reactor vessel should be based upon the strain rate as well as the total strain. It is also necessary to obtain a new solution for distribution of the pressure, particle velocity, etc., for TNT by the use of high-speed computers and present explosive technological data.

APPENDIX A

Tables of Experimental Data
Extracted from References

TABLE XI. Spherical Detonation Wave in TNT^a

r/R	ξ	ψ	c/U	f	u/U	$p \times 10^{-10}$	ρ
1.0	0.2424	0.320	0.7575	2.60	0.2424	15.00	2.000
0.99904	0.230	0.3084	0.742	2.06	0.230	14.45	1.972
0.99632	0.220	0.2986	0.733	1.80	0.219	13.90	1.942
0.99271	0.210	0.2874	0.724	1.20	0.208	13.34	1.928
0.9885	0.20	0.2748	0.719	1.00	0.1977	12.68	1.880
0.9821	0.19	0.2612	0.714	0.70	0.1866	12.11	1.850
0.9744	0.18	0.2469	0.710	0.60	0.1754	11.09	1.797
0.9656	0.17	0.2323	0.708	0.60	0.16415	10.59	1.773
0.9557	0.16	0.2177	0.702	0.60	0.1529	9.44	1.709
0.9448	0.15	0.2028	0.700	0.62	0.14172	8.91	1.683
0.9320	0.14	0.1879	0.694	0.80	0.1305	7.94	1.639
0.9178	0.13	0.1737	0.688	1.45	0.1193	7.08	1.595
0.8997	0.12	0.1601	0.675	1.97	0.1080	6.46	1.562
0.8804	0.11	0.1468	0.659	2.52	0.09684	5.875	1.531
0.8577	0.10	0.1337	0.641	3.00	0.08577	5.43 ₃	1.506
0.8275	0.09	0.1205	0.618	3.40	0.07448	4.84	1.468
0.7960	0.08	0.1071	0.596	3.88	0.06368	4.41 ₆	1.440
0.7584	0.07	0.09350	0.568	4.22	0.05309	3.98	1.407
0.7148	0.06	0.07966	0.538	4.50	0.04289	3.56 ₅	1.372
0.6668	0.05	0.06514	0.512	4.65	0.03334	3.23 ₆	1.342
0.6166	0.04	0.05084	0.487	4.72	0.02466	2.95 ₁	1.316
0.5672	0.03	0.03673	0.461	4.74	0.01702	2.66	1.285
0.5186	0.02	0.02327	0.446	4.70	0.01037	2.48	1.265
0.4950	0.015	0.01695	0.441	4.70	0.00741	2.43	1.259
0.4723	0.010	0.01096	0.430	4.66	0.00472	2.34	1.249
0.4474	0.005	0.00527	0.425	4.64	0.00224	2.29	1.244
0.418	0	0	0.418	4.60	0	2.19	1.233

^aExtracted from Ref. 15. The figures in columns 4, 7, and 8 are taken from an unpublished report by H. Jones. They are not identical with the revised figures published by Jones and Miller (Ref. 10, Table 3, p. 497), but Taylor decided the difference was not sufficiently great to warrant the recalculation of the spherical detonation wave.

The detonation velocity, U , has the value 638,000 cm/sec based on a density, ρ , of the packed TNT of 1.51 gm/cm³; the unit of pressure, p , is dynes/cm².

TABLE XII. Shock-wave and Thermodynamic Data for the Water Hugoniot
Centered at 20°C and 1 atm^a

P, kb	V, cm ³ /gm	T, °C	U _s , km/sec	U _p , km/sec	C, km/sec	ΔS, cal/gm/deg	ΔE, cal/gm	ΔH, cal/gm
0	1.0018	20	1.483	0	1.483	0	0	0
5	0.8773	36	2.008	0.250	2.350	0.005	7.4	112
10	0.8204	54	2.352	0.426	2.800	0.018	21.7	218
15	0.7845	74	2.632	0.571	3.130	0.038	38.9	320
20	0.7583	96	2.871	0.698	3.390	0.062	58.2	420
25	0.7378	121	3.083	0.812	3.611	0.087	78.8	519
30	0.7207	145	3.273	0.918	3.810	0.114	100.7	617
40	0.6940	196	3.611	1.110	4.114	0.169	147.0	810
50	0.6735	251	3.910	1.281	4.330	0.224	196.0	1000
60	0.6561	310	4.173	1.440	4.513	0.276	247.7	1188
70	0.6414	372	4.415	1.588	4.666	0.326	301.2	1373
80	0.6287	436	4.639	1.728	4.795	0.374	356.4	1558
90	0.6175	502	4.848	1.860	4.907	0.420	413.0	1740
100	0.6075	570	5.045	1.986	5.015	0.463	470.8	1922
110	0.5980	638	5.228	2.108	5.113	0.504	530.4	2101
120	0.5893	710	5.404	2.225	5.214	0.543	591.1	2280
130	0.5813	781	5.570	2.338	5.295	0.580	652.7	2457
140	0.5740	854	5.731	2.447	5.380	0.615	715.2	2634
150	0.6558	927	5.883	2.554	5.462	0.649	779.1	2810
170	0.5538	1074	6.160	2.760	5.626	0.712	909.4	3158
190	0.5420	1223	6.428	2.956	5.781	0.771	1043.2	3503
200	0.5363	1297	6.567	3.051	5.856	0.798	1111.7	3673
210	0.5310	1371	6.679	3.144	5.928	0.825	1180.6	3844
230	0.5209	1520	6.916	3.326	6.072	0.876	1320.7	4182
250	0.5112	1667	7.151	3.502	6.211	0.923	1464.5	4517
300	0.4897	2037	7.668	3.920	-	1.031	1834.5	5343
350	0.4705	2407	8.131	4.312	-	1.127	2220.5	6153
400	0.4530	2777	8.552	4.685	-	1.212	2621.3	6949
450	0.4371	3147	8.943	5.041	-	1.290	3034.3	7732

^aExtracted from Ref. 13.

APPENDIX B

Description of the Miscellaneous Computer Programs1. Least-squares Curve-fitting Program

The analytic fits to experimental data were all obtained by a least-squares curve-fitting program adapted to the CDC-160-A computer. This computer program produced a least-squares polynomial fit to a set of discrete data by the use of orthogonal polynomials.* This program performed the least-squares operations for four types of polynomial forms:

- a. Polynomial: $Y = A + BX + CX^2 + \dots$
- b. Exponential: $Y = \exp(A + BX + CX^2 + \dots)$.
- c. Power: $Y = \exp[A + B(\ln X) + C(\ln X)^2 + \dots]$.
- d. Logarithmic: $Y = A + B(\ln X) + C(\ln X)^2 + \dots$

The form that resulted in the best fit to each set of data was used.

2. Program for CalComp 580 Plotter

The graphical representation of the computer results was obtained by the use of a plotting subroutine** adapted to the CDC-3600 computer. The required output from the main computer program was saved on a separate magnetic tape. The information on this tape was transformed to appropriate instructions for a CalComp 580 plotter.

All the graphs resulting from this analysis were drawn in original form by a CalComp 580 plotter. These graphs were then traced by a draftsman and reduced photographically to the form presented in this report.

3. Program for the Impulse

The area under the pressure-time curve was obtained by the use of a program written for the CDC-3600 computer. This program was based on the trapezoid rule for performing the numerical integration. The area was computed from t_1 , the time at which the pressure on the concrete first exceeded P_0 , to t_2 , the time at which the computer solution was ended.

The magnitude of the impulse obtained by the use of this computer program is

$$\text{Impulse} = 2,607,150 \text{ gm-cm/sec/cm}^2.$$

*The method was taken from a paper by Forsythe, G. E., Generation and Use of Orthogonal Polynomials for Data Fitting with a Digital Computer, J. Soc. Indust. Appl. Math. 6 (1957).

**Technical Memorandum No. 70, ANL, Off-Line Plotting Techniques Using CalComp Magnetic Tape Plotting System 580, LeVee, C. G., and Ohde, J. A. (April 20, 1964).

APPENDIX C

Equations1. Equations for the Speed of Sounda. TNT

The speed of sound in the TNT explosion products is determined by an analytic fit to the data contained in columns 4 and 7 of Table XI of the form

$$c_{\text{TNT}} = 183556.0 + 49547.0 P - 2108.0 P^2, \quad (97)$$

where P and c are in dynes/cm² and cm/sec, respectively.

b. Water

The speed of sound in the water is determined by an analytic fit to the data contained in columns 1 and 6 of Table XII of the form

$$c_{\text{water}} = 256906.0 + 3853.0 P - 13.0 P^2, \quad (98)$$

where P and c are in kb and cm/sec, respectively.

c. Argon

The speed of sound in the argon is determined by the following equation:*

$$c_{\text{argon}} = (\gamma PV)^{1/2}, \quad (99)$$

where P and V are in dynes/cm² and cm³/gm, respectively, and γ , the ratio of the specific heats, has the value 1.66.

2. Equation for Determining the Shock Speed, U_s , in the Water

In an iterative process involving finite-difference equations, the number of iterations can be made relatively small by making a good initial guess for the parameter involved. In the determination of the shock speed in the water, the initial value was obtained by an analytic fit to the data contained in columns 1 and 4 of Table XII of the form

$$U_s = 1.75073 + 0.05486 P - 0.00022 P^2, \quad (100)$$

where P and U_s are in kb and km/sec, respectively.

* This equation can be found in any elementary physics book. See for example, Hausmann, E., and Slack, E. P., Physics, Third Edition (1948).

3. Equation for the Internal Energy of the Argon

The equation of state for the argon is of the form

$$P = \frac{RT}{V^2} (V + B_0) - \frac{A_0}{V^2} \left(1 - \frac{a}{V}\right). \quad (101)$$

From thermodynamics, the change in specific internal energy can be expressed in terms of temperature and volume changes as

$$dE = \left(\frac{\partial E}{\partial T}\right)_V dT + \left(\frac{\partial E}{\partial V}\right)_T dV. \quad (102)$$

Also from thermodynamics,

$$\left(\frac{\partial E}{\partial T}\right)_V = C_v$$

and

$$\left(\frac{\partial E}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P.$$

Therefore, by substitution, Eq. 102 becomes

$$dE = C_v dT + \left[T \left(\frac{\partial P}{\partial T}\right)_V - P \right] dV. \quad (103)$$

By differentiating Eq. 101, one obtains

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{R}{V^2} (V + B_0). \quad (104)$$

Substitution of Eqs. 101 and 104 into Eq. 103 gives the required equation for the internal energy of the argon:

$$dE = C_v dT + \left[\frac{A_0}{V} \left(1 - \frac{a}{V}\right) \right] dV. \quad (105)$$

4. Transformation of the Equations for the Water

The following equations have been obtained from pages 28-30:

$$H(P, V) = H_h(P) + \xi(P)[V - V_h(P)], \quad (106)$$

$$V_1/V_0 = (U_s - U_p)/U_s, \quad (107)$$

$$P_1 - P_0 = U_s U_p / V_0, \quad (108)$$

$$H_1 - H_0 = \frac{1}{2} (P_1 - P_0) (V_0 + V_1), \quad (109)$$

$$U_s - 1.483 = 25.306 \log_{10} (1 + U_p / 5.190), \quad (110)$$

$$\log_{10} \xi = 2.17943 + 0.0030338 P, \quad (111)$$

and

$$E = H - PV. \quad (112)$$

The units used for the parameters in Eqs. 106-112 are as follows:

<u>Symbol</u>	<u>Unit</u>
P, ξ	kb
H, E	cal/gm
V	cm ³ /gm
U	km/sec

By considering the units, Eqs. 106 and 112 become

$$H = H_h + \xi (V - V_h) 23.8892 \quad (113)$$

$$E = H - (PV) 23.8892. \quad (114)$$

By taking antilogs and simplifying, Eq. 110 becomes

$$U_p = 4.53493 (1.0953^{U_s} - 1.1444). \quad (115)$$

By considering the units and substituting for U_p (Eq. 115), Eq. 108 becomes

$$U_s (1.0953^{U_s} - 1.1444) = 0.02209 P, \quad (116)$$

where P_0 and V_0 have been replaced by the values zero and 1.0018, respectively.

For the Hugoniot curve, $V_1 = V_h$ and $H_1 = H_h$, so that Eq. 107 becomes

$$V_h = 1.0018 (1 - U_p / U_s), \quad (117)$$

and, by considering the units and choosing $H_0 = 0$, Eq. 109 becomes

$$H_h = 11.9446 P(1.0018 + V_h). \quad (118)$$

By taking antilogs and simplifying, Eq. 111 becomes

$$\xi = 151.16 \left(10^{0.0030338P} \right). \quad (119)$$

Equations 51-57 represent Eqs. 113-119, respectively.

5. The Equation for the Lagrangean Coordinates

For a spherical shell with an inside radius and an outside radius designated r_1 and r_2 , respectively, and a density ρ_0 , the mass of the shell is given by

$$m = \frac{4}{3} \pi \rho_0 (r_2^3 - r_1^3). \quad (120)$$

Similarly, for a spherical shell of radii R_1 and R_2 and density ρ , the mass of the shell is given by

$$M = \frac{4}{3} \pi \rho (R_2^3 - R_1^3). \quad (121)$$

If the masses of the two spherical shells are equal, then

$$\rho_0 (r_2^3 - r_1^3) = \rho (R_2^3 - R_1^3). \quad (122)$$

If the terms on the right side of Eq. 122 are associated with the actual physical system, i.e., the finite-difference net, at time $t = 0$, then the terms on the left side are associated with the standard reference configuration of density ρ_0 .

The Lagrangean coordinates are calculated by Eq. 122. The first zone of fluid, which is at the center of the system, is a small sphere. For this zone, $r_1 = R_1 = 0$, and therefore the Lagrangean coordinate of interface 2 is

$$r_2 = \frac{\rho}{\rho_0} R_2. \quad (123)$$

The Lagrangean coordinate, r_2 , becomes the inside radius in the standard reference configuration for the next outer zone of fluid, which is a spherical shell. Hence, the Lagrangean coordinate of interface 3 is

$$r_3 = \left[r_2^3 + \frac{\rho}{\rho_0} (R_3^3 - R_2^3) \right]^{1/3}. \quad (124)$$

The process is repeated until all the Lagrangean coordinates have been calculated, i.e., until the entire physical system is transformed to the standard reference configuration.

The finite-difference form of Eq. 122 used for the calculation of the Lagrangean coordinates is

$$R(J) = \left\{ R(J-1)^3 + \frac{V_0}{V(J-1)} [RR(J, 1)^3 - RR(J-1, 1)^3] \right\}^{1/3}. \quad (125)$$

where $R(J-1)$ and $R(J)$ are the Lagrangean coordinates representing the radii of the zone interfaces in the standard reference configuration, and $RR(J-1, 1)$ and $RR(J, 1)$ are the Eulerian coordinates representing the radii of corresponding zone interfaces in the actual physical system at time $t = 0$.

APPENDIX D

Comparison of Computer Results with Those
Given by the Tait Equation

The adiabatic Tait form of the equation of state for water⁴ is

$$P = 3.047 \left[\left(\frac{1.0018}{V} \right)^{7.15} - 1 \right] \quad (126)$$

for the pressure range $0 < P < 25$ kb.

Various values for the specific volume and the associated values for the pressure have been extracted from the computer results given in Table III and are shown in Table XIII. The values for the volumes were used in Eq. 126 to obtain the corresponding pressures. These values of volume and pressure are also given in Table XIII.

TABLE XIII. Pressure-Volume Data for Water

Computer Solution		Equation 126		% Diff. in Pressure
Spec. Vol, cm ³ /gm	Pressure, kb	Spec. Vol, cm ³ /gm	Pressure, kb	
0.7676	24.162	0.7676	17.404	38.8
0.7753	16.962	0.7753	15.991	6.06
0.8019	12.358	0.8019	11.914	3.72
0.8427	7.649	0.8427	7.447	2.70
0.9079	3.174	0.9079	3.111	2.02
0.9614	1.057	0.9614	1.042	1.44
0.9808	0.503	0.9808	0.500	0.60

For pressures below 16 kb, there is good agreement between the computer results and the values given by Eq. 126. Therefore, the adiabatic Tait form of the equation of state for water (Eq. 126) could have been used in the computer solution when the peak pressure in the shock front dropped below 16 kb.

APPENDIX E

Calculations for the Concrete Vessel1. Mass per Unit Area of the Inside Surface

The mass per unit area, m , is given by

$$m = \frac{\text{Total Mass of Concrete}}{\text{Area of Inside Surface}}$$

For $r_i = 210$ cm, $r_o = 390$ cm, and $\rho = 2.663$ gm/cm³, one obtains:

- The area of the inside surface, $A = 554,178$ cm²;
- The volume of concrete; $V = 209,682,950$ cm³;
- The total mass of concrete, $M = 558,385,695$ gm.

Hence, the mass per unit area is $m = 1007.59$ gm/cm².

2. Calculation for the Allowable Internal Pressure

The tangential stress in the concrete, based upon the equation from elasticity* for a spherical container under internal pressure, is given by

$$\sigma_t = \frac{P_i a^3}{2R^3} \frac{2R^3 + b^3}{b^3 - a^3}, \quad (127)$$

where σ_t is the stress in the tangential direction, P_i is the internal pressure, a and b are the inside and outside radii, respectively, and R is the radial coordinate of the point of stress.

The allowable internal pressure, taken as that internal pressure for which the tangential stress at the outside surface equals the allowable stress, is given by

$$P_i \Big|_{R=b} = \frac{2}{3} (\sigma_t) \left[\left(\frac{b}{a} \right)^3 - 1 \right]. \quad (128)$$

For $\sigma_t = 400$ psi, the allowable internal pressure is

$$P = 0.0992 \text{ kb.}$$

*See Timoshenko, S., and Goodier, J., Theory of Elasticity, Second Edition, 1951, p. 359.

APPENDIX F

Energy Calculations

1. Energy Released by the Detonation of TNT

The energy released by the detonation of solid TNT can be determined if the appropriate information about the gaseous explosion products is known. The required information for the gaseous products of detonation for TNT of density 1.5 gm/cm^3 is given in Table XIV. In addition, for the solid TNT, the heat of formation is 13.0 kcal/mole , and the molecular weight is 227 gm . Thus, by converting the units, the heat of formation of solid TNT is 57.30 cal/gm .

TABLE XIV. Information about the Products of Detonation of TNT^a

Product	Quantity, moles/kg	Heat of Formation, kcal/mole	Energy Released, cal/gm
CO	8.7	+26.4	229.68
CO ₂	6.5	+94.05	611.32
H ₂	1.6	0	0
H ₂ O	4.6	+57.80	265.88
N ₂	6.6	0	0
CH ₄	2.4	+17.89	42.94
C	13.1	0	0

^aThe information in this table is extracted from Table 12.18a (page 310) and Table ii.5 (pages 404-405) of Cook, M. A., The Science of High Explosives, Reinhold Publishing Corporation, New York (1958).

The energy released by detonation of TNT is given as

$$U = \sum U_{\text{Products}} - U_{\text{Solid TNT}} = 1092.52 \text{ cal/gm.}$$

In this problem, $a_0 = 30 \text{ cm}$, so that the mass of explosive is given as

$$M_{\text{TNT}} = 169,645 \text{ gm.}$$

Therefore, the total energy released by the explosive charge is

$$U = 1092.52 \times 169,645 = 185,124,000 \text{ cal.}$$

2. Initial Kinetic Energy of the Concrete Mass

The initial kinetic energy of the concrete mass, as a result of the impulse, is given by

$$\text{K.E.} = \frac{1}{2} M v_T^2. \quad (129)$$

The initial radial velocity, as given by Eq. 67, is

$$v_r = \frac{I}{m} = 2590 \text{ cm/sec.}$$

For the total mass $M = 558,385,695$ gm, the initial kinetic energy is given as

$$\text{K.E.} = 18.36445 \times 10^{14} \text{ dyne-cm} = 43,871,100 \text{ cal.}$$

3. Comparison of Energies

The energies that have been determined are as follows:

- a. The total energy released by the TNT: 185,124,000 cal.
- b. The total energy of the detonation products* at time $t = 0$: 126,128,500 cal.
- c. The initial kinetic energy of the concrete mass: 43,871,100 cal.

The initial kinetic energy of the concrete mass represents 23.7% of the total energy released by the detonation process and 34.8% of the energy in the detonation products at time $t = 0$.

*Obtained from Table II as $\text{ETNT} = 5.2797292 \times 10^{15}$ dyne-cm.

APPENDIX G

Miscellaneous Considerations1. Calculation of the Gas Constant, $\mathcal{R}$, for the Detonation Products

The equation of state for the detonation products is given by Eq. 29 as

$$PV = \mathcal{R}T + 0.6490 P - (0.2712 \times 10^{-11}) P^2 + (0.6199 \times 10^{-23}) P^3.$$

The conditions immediately behind the detonation front, as given by Taylor, are

$$P = 15.00 \times 10^{10} \text{ dynes/cm}^2,$$

$$V = 0.500 \text{ cm}^3/\text{gm},$$

and

$$T = 3400^\circ\text{K}.$$

Thus, the value of the gas constant, $\mathcal{R}$, for the detonation products is given as

$$\mathcal{R} = 5,217,660 \text{ dyne-cm/gm-}^\circ\text{K}.$$

2. Calculations for the Allowable Δt 's at $t = 0$

The minimum allowable time increment for each fluid media is as follows:

a. TNT

For $\Delta R = 1.0 \text{ cm}$, $P = 12.45 \times 10^{10} \text{ dynes/cm}^2$, and $c = 473,670 \text{ cm/sec}$, the time increment is given as

$$\Delta t \leq 0.0000006 \text{ sec.}$$

Therefore, DTTEST for the TNT products is taken as 0.0000005 sec.

b. Water

For $\Delta R = 10.0 \text{ cm}$, $P = 1.013274 \times 10^6 \text{ dynes/cm}^2$, and $c = 145,000 \text{ cm/sec}$,

$$\Delta t \leq 0.0000206.$$

Therefore, DTTEST for the water is taken as 0.0000100 sec.

c. Argon

For $\Delta R = 15.0$ cm, $P = 1.013274 \times 10^6$ dynes/cm², and
 $c = 33,100$ cm/sec,

$$\Delta t \leq 0.0001360.$$

Therefore, DTTEST for the argon is taken as 0.00010000 sec.

The allowable time increment for the finite-difference calculations is governed by that for the TNT products and is taken as

$$\text{DELT} = 0.0000005 \text{ sec.}$$

3. Pictorial Representation for the Δr 's

The pictorial representation for the Δr 's is shown in Fig. 32:

Fig. 32

Pictorial Representation for Δr 's

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*Copies of Refs. 19-22 could not be located by the authors. However, the necessary information from each is found in Ref. 4.