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Informal Report

EFFECT OF SYNCHROTRON MOTION ON THE
CLOSED ORBIT DISTORTION IN CROSSING BEAMS

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Abstract

When beams collide with non-zero crossing angles, there is a dipole component to the beam-beam force, thus introducing a closed orbit distortion. If the force experienced by a particle is antisymmetric about the collision center, then the distorted orbit still passes through the collision center but with a changed slope, resulting in a shift from the nominal crossing angle. This antisymmetry in the force occurs for coasting beams, while for bunched beams, it is the case for particles at the center of the bunches, assuming that the bunches overlap exactly around the collision center. The presence of synchrotron motion means that the beam-beam force experienced by a particle modulates as the particle moves back and forth within the bunch. Specifically, a symmetric component to the force is introduced. If the synchrotron period is much longer than the betatron period, the modulation of the force is adiabatic, and so the effect is a modulation of the closed orbit. The symmetric component of the dipole force causes a shift in the closed orbit at the collision center. This indicates a tendency for the beams to separate, which in turn changes the nature of the force between the two beams. The closed orbit shift is calculated. Some comments on self-consistency are made. It is indicated how under certain circumstances, a self-consistent solution may not exist. The case of SPEAR is considered. It is shown that for small crossing angles, the orbit shift at the collision center is larger than the beam size, indicating a major effect. Furthermore, it is argued that a self-consistent

solution cannot exist. For coasting beams, small angle crossings can be designed so that only the antisymmetric component of the dipole force is present. Thus, the predominant effect is a shift in the nominal crossing angle.

Introduction

When beams collide with non-zero crossing angles, there is a dipole component to the beam-beam force, thus introducing a closed orbit distortion.¹ If the force experienced by a particle is antisymmetric about the collision center, then the distorted orbit still passes through the collision center but with a changed slope, resulting in a shift from the nominal crossing angle. This antisymmetry in the force occurs for coasting beams, while for bunched beams, it is the case for particles at the center of the bunches, assuming that the bunches overlap exactly around the collision center. The presence of synchrotron motion means that the beam-beam force experienced by a particle modulates as the particle moves back and forth within the bunch. Specifically, a symmetric component to the force is introduced. If the synchrotron period is much longer than the betatron period, the modulation of the force is adiabatic, and so the effect is a modulation of the closed orbit.

The symmetric component of the dipole force causes a shift in the closed orbit at the collision center. This indicates a tendency for the beams to separate, which in turn changes the nature of the force between the two beams.

In Section 2, we calculate the closed orbit shift and discuss the question of self-consistency. We will indicate how, under certain circumstances, a self-consistent solution for the closed orbit may not exist. In Section 3, the case of SPEAR is considered. We estimate that the modulation of the closed orbit due to the synchrotron motion in the bunches is large, relative to the transverse dimensions of the beam. This large orbit shift indicates a major effect, and in fact, we argue that for SPEAR, a self-consistent solution for the equilibrium orbit in the case of small crossing angles cannot exist.

The absence of synchrotron motion in coasting beams allows the design of small angle crossings where only the antisymmetric component of the dipole force is present. Thus, we expect that for coasting beams, the predominant influence of the dipole force would be a shift from the nominal crossing angle.

2. The Beam-Beam Closed Orbit Distortion

If beams cross vertically at an angle α , the dipole component of the force between them (that is, the force of the strong beam on the weak beam) can be described by the field, E_y , given by²

$$\frac{1}{e} E_y = \frac{2\lambda(s)}{s\alpha} \left[1 - e^{-s^2\alpha^2/2\sigma^2} \right], \quad (2.1)$$

where

$$\sigma = \sigma(s) = \sqrt{\epsilon\beta(s)} \quad (2.2)$$

is the rms 1/2-width of the strong beam and we have assumed a circular beam; ϵ is the rms beam emittance, $\beta(s)$ is the β -function,³ and s is the azimuthal distance measured from a center of symmetry, so that in the gradient free region around the symmetry point,

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}, \quad (2.3)$$

with β^* the value of $\beta(s)$ at the symmetry center, which is just the nominal collision center. The quantity $\lambda(s)$ is the strong beam density per unit length and for Gaussian beams, the density seen by the weak beam is

$$\lambda(s) = N \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_L} e^{-2(s-s_{ch})^2/\sigma_L^2}, \quad (2.4)$$

where σ_L is the rms bunch 1/2-length, N is the number of particles per bunch, and we have taken account of the apparent strong beam contraction. The synchrotron motion of the particles in the weak beam is taken account of by including a shift, s_{ch} , in the center of the density seen by the weak beam. It can be shown that this shift is given roughly by

$$s_{ch} = \sigma_L \sin(2\pi t/T_s). \quad (2.5)$$

Since the synchrotron period, T_s , is much longer than the betatron period,

the time dependence in s_{ch} can be treated as an external parameter. That is, s_{ch} can be written

$$s_{ch} = \sigma_L \delta , \quad (2.6)$$

where δ is an adiabatic parameter which fluctuates between -1 and +1.

The closed orbit distortion can be found from the equation

$$\eta'' + \nu^2 \eta = \nu^2 g(\phi) , \quad (2.7)$$

where ν is the tune, η is the normal coordinate, related to the displacement, y , by

$$\eta = y / \sqrt{\beta(s)} , \quad (2.8)$$

differentiation is with respect to

$$\phi = \frac{1}{\nu} \int_0^s \frac{ds}{\beta(s)} , \quad (2.9)$$

and $g(\phi)$ is given by

$$g(\phi) = \frac{4r_0 \beta^{3/2} \lambda(s)}{\gamma s \alpha} \left[1 - e^{-s^2 \alpha^2 / 2\sigma^2} \right] , \quad (2.10)$$

with γ the energy in units of particle mass, and r_0 the classical particle radius.

Expressing the solution to (2.7) in a Fourier series in ϕ , we find for the periodic solution, i.e. the closed orbit,³

$$\eta_{co}(s) = \sum_k \frac{\nu^2 g_k e^{ik\phi}}{\nu^2 - k^2} , \quad (2.11)$$

where

$$g_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} g(\phi) e^{-ik\phi} d\phi . \quad (2.12)$$

If $v \approx K$, we can take only the $k = \pm K$ terms in the sum, obtaining for the vertical closed orbit distortion,

$$y(s) = \frac{v \sqrt{\beta(s)}}{v-K} \operatorname{Re} \left\{ g_K e^{iK\phi} \right\} . \quad (2.13)$$

Since $g(\phi)$ is only non-zero in the crossing regions, the rapid fall-off of $\lambda(s)$ with s allows us to approximate g_K , for the appropriate values of K , by

$$g_K = \frac{n_c}{2\pi v} \int_{-\infty}^{\infty} \frac{1}{\beta(s)} g(\phi) e^{-iK\phi} ds , \quad (2.14)$$

where n_c is the number of identical crossings, and the field free expressions for $\beta(s)$ and $\phi(s)$ can be used: that is, $\beta(s)$ given by (2.3) and $\phi(s)$ by

$$\phi(s) = \frac{1}{v} \tan^{-1} \left(\frac{s}{\beta^*} \right) . \quad (2.15)$$

For two identical crossings, we have

$$g_K = \frac{1}{\pi v} \int_{-\infty}^{\infty} \frac{1}{\beta(s)} g(\phi) e^{-iK\phi} ds, \quad K \text{ even} \quad (2.16)$$

$$g_K = 0, \quad K \text{ odd} .$$

Introducing the quantities,

$$t = s/\beta^* , \quad (2.17)$$

$$\eta = \frac{\alpha\beta^*}{\sigma^*} \quad (\sigma^* = \sigma \text{ at the crossing point}), \quad (2.18)$$

$$\text{and} \quad v = \beta^*/\sigma_\ell , \quad (2.19)$$

(2.13) can be put in the form

$$y(s) = A_K \sqrt{\beta^*\beta(s)} \left\{ Q_s(\eta, v, \delta) \sin K\phi + Q_c(\eta, v, \delta) \cos K\phi \right\} , \quad (2.20)$$

where,

$$A_K = \frac{4r_o \beta^* N}{\sqrt{2} \pi^{3/2} \gamma \sigma_t \sigma^* (v-K)} \quad , \quad (2.21)$$

$$Q_s (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} \frac{(1+t^2)^{1/2}}{t} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2 / (1+t^2)} \right] \sin\left(\frac{K}{v} \tan^{-1} t\right) dt \quad , \quad (2.22)$$

and

$$Q_c (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} \frac{(1+t^2)^{1/2}}{t} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2 / (1+t^2)} \right] \cos\left(\frac{K}{v} \tan^{-1} t\right) dt \quad , \quad (2.23)$$

For $K \sim v$, we have

$$Q_s (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2 / (1+t^2)} \right] dt \quad , \quad (2.24)$$

and

$$Q_c (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2 / (1+t^2)} \right] \frac{1}{t} dt \quad . \quad (2.25)$$

To obtain approximate analytical expressions for Q_s and Q_c , we neglect t^2 with respect to 1 in the exponential. Thus,

$$Q_s (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2} \right] dt \quad , \quad (2.26)$$

and

$$Q_c (\eta, v, \delta) = \frac{2}{\eta} \int_{-\infty}^{\infty} e^{-2(vt-\delta)^2} \left[1 - e^{-\frac{1}{2}\eta^2 t^2} \right] \frac{1}{t} dt \quad , \quad (2.27)$$

Performing the Q_s integral, we have

$$Q_s(\eta, v, \delta) = \frac{\sqrt{2\pi}}{v\eta} \left[1 - \frac{1}{\sqrt{1+\eta^2/4v^2}} e^{-2\delta^2 f} \right], \quad (2.28)$$

where

$$f = \frac{\eta^2}{\eta^2 + 4v^2}; \quad (2.29)$$

while Q_c can be put in the form

$$Q_c(\eta, v, \delta) = \frac{2\delta\sqrt{2\pi}}{\eta} \int_0^f \frac{e^{-2\delta^2 y}}{\sqrt{1-y}} dy. \quad (2.30)$$

Since $f < 1$, we can expand the square root factor in powers of y . Separating off the lowest order term,

$$Q_c(\eta, v, \delta) = \frac{2\delta\sqrt{2\pi}}{\eta(2\delta^2)} \left[1 - e^{-2\delta^2 f} \right] + \Delta Q_c, \quad (2.31)$$

with

$$\Delta Q_c(\eta, v, \delta) = \frac{2\delta\sqrt{2\pi}}{\eta} \sum_{n=1}^{\infty} \frac{1.3 \dots (2n-1)}{2^n n!} f^{n+1} H_n, \quad (2.32)$$

and

$$H_n = \int_0^1 e^{-2\delta^2 f y} y^n dy. \quad (2.33)$$

The quantity H_n can be obtained from the recurrence relation,

$$H_n = \frac{1}{2\delta^2 f} \left[-e^{-2\delta^2 f} + n H_{n-1} \right]. \quad (2.34)$$

The antisymmetric term (i.e. the term involving $\sin K\phi$) is responsible for a shift in the orbit slope at the collision center. By relating this slope to the actual crossing angle, one can find a self-consistent result for α . Thus, the only effect of the antisymmetric term is a shift in the nominal crossing angle. In other words, without synchrotron motion, i.e. with coasting beams, since the crossing can be arranged so that only the antisymmetric term contributes, a stable crossing configuration will result. Of course, for "large" crossing angles, where the dipole effect is falling off, this is evident. Our result, however, is perfectly general and applies to small crossing angles. We will henceforth neglect the antisymmetric term, which has already been treated in Ref.(1).

With synchrotron motion, the symmetric $\cos K\phi$ term appears, inducing a shift in the orbit position at the collision center. Using (2.20) and (2.30), we obtain an approximate expression for this shift,

$$y^* = \frac{\sqrt{2\pi} A_K \beta^*}{\delta \eta} \left[1 - e^{-2\delta^2 f} \right] . \quad (2.35)$$

There is a peak in y^* as a function of crossing angle, or η . It can be found from the equation

$$e^{-2\delta^2 f} = \frac{1}{1 + 4\delta^2 f(1-f)} . \quad (2.36)$$

For small δ , that is, for particles at the center of the bunch, the peak occurs at $f = 1/2$, or

$$\eta = 2v , \quad (2.37)$$

corresponding to a crossing angle

$$\alpha = 2 \frac{\sigma^*}{\sigma_L} ; \quad (2.38)$$

while for $\delta^2 = 1$, that is, for particles at the ends of the bunch, $f = .31$, $\eta = 1.3 v$, or

$$\alpha = 1.3 \frac{\sigma^*}{\sigma_L} . \quad (2.39)$$

Combining (2.18), (2.21) and (2.35), the orbit shift at the collision center becomes

$$y^* = \frac{4r_o \beta^* N}{\pi \gamma \sigma_\ell (v-K) \alpha \delta} \left[1 - e^{-2\delta^2 f} \right], \quad (2.40)$$

where

$$f = 1 / \left(1 + \frac{4\sigma^{*2}}{\alpha^2 \sigma_\ell^2} \right). \quad (2.41)$$

From (2.20) and (2.28), we can calculate the slope at the collision center:

$$y'^* = \frac{4r_o N}{\pi \gamma \beta^* \alpha (v-K)} \left[1 - \left(1 + \frac{\alpha^2 \sigma_\ell^2}{4\sigma^{*2}} \right)^{\frac{1}{2}} e^{-2\delta^2 f} \right]. \quad (2.42)$$

Consider now the question of self-consistency. Along part of its synchrotron path, the closed orbit distortion will lie completely to one side of the strong beam (i.e. above or below). This follows from the expressions (2.41) and (2.42) for y^* and y'^* respectively. For example, if $K > v$ and $\delta > 0$, the weak beam orbit lies below the strong beam at the collision center (i.e. $y^* < 0$), while the slope of the orbit is negative (i.e. $y'^* < 0$). Now since $\delta > 0$, and given our assumed sign convention, the strong beam is situated in the region $s > 0$ and $y > 0$. This means there is a tendency not to cross. Thus, to find a self-consistent solution, we look for a closed orbit which is a fixed distance from the strong beam along the entire bunch length. This fixed distance is, of course, the orbit itself. Let us neglect the β -function variation, and let y^* be the beam separation; y^* is also the orbit distortion at $s = 0$. The field, E_y is

$$\frac{1}{e} E_y = \frac{2 \lambda(s)}{y^*} \left[1 - e^{-y^{*2}/2\sigma^{*2}} \right]. \quad (2.43)$$

Thus, we obtain for the closed orbit at $s = 0$,

$$y^* = \frac{4r_o \beta^* N}{\pi \gamma (v-K)} \frac{1}{y^*} \left[1 - e^{-\frac{1}{2} (y^*/\sigma^*)^2} \right]. \quad (2.44)$$

If we set

$$x = y^*/\sigma^* \quad , \quad (2.45)$$

this equation for x can be written

$$x^2 = \frac{4r_o \beta^* N}{\pi \gamma \sigma^{*2} (\nu - K)} \left[1 - e^{-\frac{1}{2} x^2} \right] . \quad (2.46)$$

This means that if $K > \nu$, i.e. that the dominant harmonic is above the tune, no solution exists. It is interesting that if the beams have the same sign of the charge, these arguments do not apply. In fact, a non-zero solution for x could exist.

3. Application to SPEAR

The SPEAR parameters are:⁴

$$\sigma^* = \sqrt{\sigma_V^* \sigma_H^*} = 0.62 \times 10^{-2} \text{ cm} \begin{cases} \sigma_V^* = 10^{-3} \text{ cm} \\ \sigma_H^* = 3.8 \times 10^{-2} \text{ cm} \end{cases}$$

$$\sigma_\ell = 10 \text{ cm}$$

$$\beta^* = 10 \text{ cm}$$

$$\gamma = 3000$$

$$N = 5 \times 10^{10}$$

$$\nu = 5.25$$

$$K = 6$$

$$r_o = 2.8 \times 10^{-13} \text{ cm} .$$

Since the synchrotron period, $T_s = 0.19 \text{ msec}$, and the betatron period, $T_\beta = 0.14 \text{ } \mu\text{sec}$, we have $T_s \gg T_\beta$ and the adiabatic approximation is valid. From (2.39), we obtain the crossing angle for the maximal orbit distortion,

$$\alpha = 0.8 \text{ mrad, assuming } \delta^2 = 1 .$$

At this crossing angle, we obtain an orbit distortion and slope,

$$y^* = -4.4 \times 10^{-3} \text{ cm}, y'^* = -0.53 \text{ mrad} \quad .$$

With such a large distortion (relative to transverse beam size), we can apply our model of self-consistency given in the previous section. Since SPEAR has a symmetry of 2, the closest even harmonic is $K = 6$, above the tune. Thus, from (2.46), no self-consistent solution exists. This could perhaps explain the inability of SPEAR to operate at small crossing angles.

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