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The Effect of Variations in the Fission Neutron Spectrum at
High Energies on the Age of Fission Neutrons in Water

As has been frequently pointed out, there is a serious discrepancy between the age to indium resonance of fission neutrons in water as observed experimentally (30 cm^2) and as calculated (26.5 cm^2). In a previous report¹ it was shown that no reasonable modification of the fission spectrum below 1 Mev could account for the difference. It has been suggested, however, that uncertainties at the high energy end of the spectrum would have a more powerful effect on the age, because of the large age for very fast neutrons. While accurate calculations of the effect of varying the upper end of the spectrum require detailed machine computations, it is possible to make an estimate of the effect using only the information already available. The results indicate that to account for the discrepancy, the spectrum at high energies would have to be changed by amounts greatly exceeding the known experimental errors in the measurement of the fission spectrum. Of the spectrum variations considered, which can raise the age sufficiently, the one that does the least violence to the accepted spectrum calls for a uniform increase of 60% in all fission neutrons above 2 Mev. Quoted errors in the spectrum from 2 to 6 Mev are 10% or less.²

The calculation leading to this result essentially involves finding the functional derivative of the age with respect to the fission spectrum shape. To keep track of the magnitude of terms neglected it will prove helpful to go back to first principles in obtaining the derivative. The age of fission neutrons to an energy E_0 from a point isotropic source is defined by

$$\tau_f(E_0) = \frac{\int \frac{r^2}{6} \phi_f(r, E_0) dV}{\int \phi_f(r, E_0) dV}, \quad (1)$$

1. H. Goldstein and J. Certaine, Appl. Nuc. Phys. Div. Ann. Prog. Rep. Sept. 1, 1957, Section 8.12, ORNL-2389.
2. L. Cranberg, G. Frye, N. Nereson and L. Rosen, Phys. Rev. 103, 662 (1956).

where $\phi_f(r, E_0)$ is the differential flux at point r and energy E_0 due to a fission source. This flux can also be written as

$$\phi_f(r, E_0) = \int_0^{\infty} \phi(r, E_0, E) N(E) dE, \quad (2)$$

where $N(E)$ is the number spectrum of the source fission neutrons and $\phi(r, E_0, E)$ is the differential flux at r and E_0 from a unit monoenergetic source of energy E . Substituting Eq. (2) in Eq. (1) the age can be written as

$$\tau_f(E_0) = \frac{\int N(E) \int \frac{r^2}{6} \phi(r, E_0, E) dV dE}{\int N(E) \int \phi(r, E_0, E) dV dE}. \quad (3)$$

Introducing the notation

$$\phi_0(E_0, E) = \int \phi(r, E_0, E) dV, \quad (4)$$

Eq. (3) simplifies to

$$\tau_f(E_0) = \frac{\int_0^{\infty} \tau(E_0, E) N(E) \phi_0(E_0, E) dE}{\int_0^{\infty} N(E) \phi_0(E_0, E) dE}. \quad (5)$$

Here $\tau(E_0, E)$ is the age to E_0 from a monoenergetic source at E . In most of the following the variable E_0 will not be displayed explicitly, as none of the operations affect E_0 .

The fission spectrum $N(E)$ will now be broken into two parts:

$$N(E) = N_0(E) + \eta(E),$$

where $N_0(E)$ is the customary normalized spectrum, and $\eta(E)$ is the perturbation whose effect is to be examined. The age with spectrum N can now be written as

$$\tau_f = \frac{\int_0^{\infty} \tau(E) N_0(E) \phi_0(E) dE + \int_0^{\infty} \tau(E) \eta(E) \phi_0(E) dE}{\int_0^{\infty} N_0(E) \phi_0(E) dE + \int_0^{\infty} \eta(E) \phi_0(E) dE} \quad (6)$$

In turn, the integral over $\tau N_0 \phi_0$ in the numerator can be equated to

$\tau_0 \int_0^{\infty} N_0(E) \phi_0(E) dE$, where τ_0 is the age with the unperturbed spectrum

$N_0(E)$. Dividing numerator and denominator by $\int_0^{\infty} N_0(E) \phi_0(E) dE$, the quantity

τ_f becomes

$$\tau_f = \tau_0 \frac{\left[1 + \int_0^{\infty} \frac{\tau(E)}{\tau_0} \eta(E) \phi_0(E) dE \right] / \int_0^{\infty} N_0(E) \phi_0(E) dE}{\left[1 + \int_0^{\infty} \eta(E) \phi_0(E) dE \right] / \int_0^{\infty} N_0(E) \phi_0(E) dE} \quad (7)$$

So far no approximations have been made. It will now be assumed that the ratio of integrals in the denominator is much less than unity. Arguments will be given below to show that ϕ_0 is substantially independent of E . The ratio of integrals in the demoninator then reduces to

$$\int_0^{\infty} \eta(E) dE,$$

which is the fraction of neutrons added to the entire fission source by $\eta(E)$, and it is this fraction which is assumed to be small. Note that it is not required that $\eta(E) \ll N_0(E)$, only that the integral shall be much less than 1. It is this assumption which makes the procedure followed equivalent to finding the functional derivative of the age.

The age now becomes

$$\tau_f = \tau_0 \left\{ 1 + \int_0^{\infty} \left[\frac{\tau(E)}{\tau_0} - 1 \right] \eta(E) \phi_0(E) dE / \int_0^{\infty} N_0(E) \phi_0(E) dE \right\}. \quad (8)$$

At this point the claim is made that neglecting the effect of absorption in H or O, $\phi_0(E)$ is in fact independent of E, so that Eq. (8) becomes

$$\tau_f = \tau_0 \left\{ 1 + \int_0^{\infty} \left[\frac{\tau(E)}{\tau_0} - 1 \right] \eta(E) dE \right\}. \quad (9)$$

The independence of $\phi_0(E)$ on E can be justified in several ways. The final neutron energy E_0 is usually only a few ev or less and in this energy region the oxygen cross section is less than 2% of the hydrogen cross section so that water can be considered here to be the same as hydrogen. Now, in pure hydrogen it is rigorously true that the total slowing down density Q (i.e., the integral over all space) is given by

$$Q(E_0, E) = \sum_t (E_0) \phi_0(E_0, E), \quad (10)$$

where $\Sigma_t(E_0)$ is the total hydrogen macroscopic cross section at E_0 . In the absence of absorption conservation of particles requires that Q must be independent of E_0 and just equal to the source strength, here defined as unity. Hence

$$\phi_0(E) = 1/\Sigma_t,$$

independent of E .

Even if the identity of water with H_2 is not accepted, at low energies E_0 the shape of $\phi_0(E_0, E)$ with respect to E_0 must surely be independent of E . A neutron getting down to the vicinity of E_0 will have suffered many collisions and "lost memory" of its initial energy and ϕ_0 will have some equilibrium shape. Now, the slowing down density at E_0 involves some integral over E' of $\phi_0(E', E)$ times a scattering kernel from E_0 up. The integrand falls off rapidly with increasing E' , and the constancy of the shape of $\phi_0(E', E)$ in this region, coupled with the unity value of Q implies that the magnitude of $\phi_0(E', E)$ is also independent of E .

It has also been pointed out by A. Simon (private communication) that from the start the age might have been defined in terms of the slowing down density $q(r, E)$ instead of the flux $\phi(r, E)$. The effect then would be that ϕ_0 in Eq. (8) would be replaced by Q , which would lead to Eq. (9) immediately since $Q = 1$. Equation (9) thus holds rigorously for the slowing-down age in the absence of absorption, and it is generally accepted that the flux and the slowing-down ages are practically identical. This approach has the virtue of suggesting the direction of the effect of absorption. For low source energies absorption between E and E_0 is quite negligible, but at high energies (> 4 Mev) absorption in oxygen can be appreciable. For such sources $Q(E_0, E) < 1$, and since there are very few fission neutrons at these energies

$$\frac{Q(E_0, E)}{\int_0^\infty N_0(E) Q(E_0, E) dE} < 1.$$

This indicates that Eq. (9) tends to give an overestimate of the change in τ due to the perturbation $\eta(E)$.

By Eq. (9) calculation of the modification of the fission age requires a knowledge of the age from monoenergetic sources as a function of source energy. Accurate information on this function must await more detailed calculations, but enough scattered points are known now to permit drawing a reasonable curve for $\tau(E)$. At 1 Mev the age has been calculated³ by the moments method to be about 14 cm^2 . P. E. Zweifel⁴ has calculated the age for a 2.55-Mev source to be 33.5 cm^2 , and to be 42.5 cm^2 for a 3.47-Mev source. Finally, R. S. Caswell, et al.⁵ have measured the age for a 14.1-Mev source and obtained a value of 150 cm^2 . Admittedly, the gaps in this set of data are large, but a smooth, nearly linear, curve can be passed through these points and the origin. Figure 1 shows the curve of $\frac{\tau(E)}{\tau_f} - 1$ obtained in this manner, with τ_f taken to be the theoretical value of 26 cm^2 .

For calculational purposes Eq. (9) may be written as

$$\frac{\Delta\tau}{\tau_f} = \int_0^{\infty} \left[\frac{\tau(E)}{\tau_f} - 1 \right] \eta(E) dE. \quad (11)$$

The 4 cm^2 difference between the observed and calculated ages corresponds to a $\Delta\tau/\tau_f$ of 15%. To see how this shift might be achieved, three possible forms for $\eta(E)$ were investigated:

- a. $\eta(E) = A\delta(E - E')$;
- b. $\eta(E) = bN_0(E)$, $E > E'$,
 $= 0$ otherwise;
- c. $\eta(E) = cN_0(E)$, $E' < E < E' + \Delta E$,
 $= 0$ otherwise.

3. Quoted in J. E. Wilkins, Jr., R. L. Hellens and P. E. Zweifel, Proceedings of the International Conference at Geneva, 1955, Vol. 5, P. 62.
4. P. E. Zweifel, private communication.
5. R. S. Caswell, R. F. Gabbard, D. W. Padgett and W. F. Doering, Nuc. Sci. Engineering 2, 143 (1957).

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Fig. 1. Age, $\tau(E)$, for Monoenergetic Sources in Water, Plotted in the Form $\frac{\tau(E)}{\tau_f} - 1$.

Each of these variations will be considered as a function of E' .

- a. A monoenergetic source of neutrons is added at E' . The shift in τ is then

$$\frac{\Delta\tau}{\tau_f} = A \int \left(\frac{\tau}{\tau_f} - 1 \right) \delta(E - E') dE = A \left[\frac{\tau(E')}{\tau_f} - 1 \right].$$

The curve of $\left(\frac{\tau}{\tau_f} - 1 \right)$ increases monotonically with E , reaching values around 4 at 14 Mev, so that the 15% shift in τ could thus be accomplished for $E' = 14$ Mev with $A = .04$. However, A should be compared not with unity but rather with the fission spectrum in the neighborhood of E' . To be specific, A will be compared with the fraction of fission neutrons above E' . If $F(E')$ is defined by

$$F(E') = \int_{E'}^{\infty} N_0(E') dE',$$

and the parameter a is given by the ratio

$$a = \frac{A}{F(E')},$$

then the value of a for a given $\Delta\tau/\tau_f$ is to be found from

$$a(E') = \frac{\frac{\Delta\tau}{\tau_f}}{F(E') \left[\frac{\tau(E')}{\tau_f} - 1 \right]}. \quad (12)$$

A curve of $a(E')$ vs E' is shown in Fig. 2. It becomes infinite at $E' = 2$ Mev, since $\tau(E')$ is there equal to τ_f . Obviously the

Fig. 2. Fractional Change in the Fission Spectrum to Increase the Age by 15%, for Three Types of Variation of the Spectrum.

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optimum energy to insert the additional neutrons is in the 3-4 Mev neighborhood, but even there the source strength of these additional neutrons must be more than 1.5 times all the fission neutrons with energies above 4 Mev.

- b. Neutrons are added above E' with the same spectral shape as the fission spectrum. The shift in τ is then

$$\frac{\Delta\tau}{\tau_f} = b \int_{E'}^{\infty} N_o(E) \left[\frac{\tau(E)}{\tau_f} - 1 \right] dE$$

If the integral is denoted by $G(E')$:

$$G(E') = \int_{E'}^{\infty} N_o(E) \left[\frac{\tau(E)}{\tau_f} - 1 \right] dE, \quad (13)$$

then b for given $\Delta\tau/\tau_f$ is given by

$$b(E') = \frac{\Delta\tau/\tau_f}{G(E')} \quad (14)$$

The function $G(E')$ has been evaluated by numerical integration, and the resultant curve of $b(E')$ for 15% shift in τ is plotted in Fig. 2. The curve passes through a minimum for $E' = 2$ Mev, but even then the perturbation required is equivalent to raising the fission spectrum above 2 Mev uniformly by 60%!

- c. Neutrons are added between E' and $E' + \Delta E$ with the same spectral shape as the fission spectrum. Here the parameter $c(E')$ is determined by

$$c(E') = \frac{\Delta\tau/\tau_f}{G(E') - G(E' + \Delta E)}$$

Calculations of $c(E')$ were carried out for $\Delta E = 1$ Mev, i.e., the hump added to the fission spectrum was 1 Mev wide. The curve of $c(E')$ so obtained is also shown in Fig. 2. As might be expected c is everywhere larger than b . The minimum occurs at $E' = 3$ Mev, but even so the required hump in the spectrum must contain twice as many neutrons as the fission spectrum has in the same region!

Of the three modifications considered, the one that requires the least change in the spectrum to produce a given shift of τ is b . But even this case requires a uniform increase of 60% in the fission spectrum from 2 Mev and up. The latest measurement of the fission spectrum² carries errors which are 10% or less up to 6 Mev. The only conclusion that one can draw from these calculations is that reasonable modifications in the fission spectrum can account for a shift in the fission age of at most a few percent.