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DYNAMIC GAMMA ATTENUATION DENSITY MEASUREMENTS

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MASTER

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**DYNAMIC GAMMA ATTENUATION DENSITY
MEASUREMENTS**

by

T. C. Piper

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ABSTRACT

The considerations important in making density measurements, in small response time, by gamma attenuation measurement are enumerated and commented upon. A formula for the gamma source strength versus various parameters is obtained. Errors due to filtering, "streaming", wall adherence, and detector peculiarities are discussed.

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I. INTRODUCTION

In instances where a density measurement is required of a single-phase fluid, at rest or in quasi steady state motion, a variety of measurement methods is available^[1]. But, when the fluid is of more than one phase or in rapid or accelerated motion and when a density measurement is most likely to be needed, few measurement methods are adaptable. This report has been written to show how accurate density measurements can be made by gamma attenuation even under these conditions. The techniques described show how to design a system to given accuracy and response time. An equation is obtained for determining the necessary source strength as a function of the required response time and the gamma energy. In obtaining this equation, discussion is provided to help select the "proper" gamma energy. Since a wide variety of gamma source energies of long half-life and reasonable cost is not available, compromise is usually necessary. The discussion of streaming errors shows how this problem, as reported by others^[2,3,4], can be minimized. The general features of the density measurement system outlined allow it to be readily adapted to such measurements as: two-phase fluid density, or the density of flowing granular solids as in automated materials processing. For purpose of illustration, our discussion can be applied to the measurement of the density of a fluid or mixture flowing in a pipe.

II. THEORY

1. GENERAL

Gamma rays passing close to and through atoms of material are occasionally scattered by interaction with the more tightly bound electrons of the atoms. Quantitative calculations of the attenuation of a beam of gammas passing through assemblies of atoms have been achieved by assigning to each type of atom a total gamma interaction cross section σ . For a given type of atom, σ varies with the gamma energy and has units of cm^2/atom . In a region of density ρ , g/cm^3 , the net interaction cross section per unit volume is then $N\sigma\rho$ where N is the number of atoms of the type in question per gram of the material. Because $N\sigma\rho dx$ represents a scattering cross-sectional area per unit beam area encountered by the beam in distance dx , a gamma beam of intensity I , gammas/ cm^2 sec, will suffer gradual attenuation of gammas or original energy and direction, $dI = -IN\sigma\rho dx$. Replacing I by R/A where A is the cross-sectional beam area at the detector and R is the number of gammas/sec, one has $AdR = -ARN\sigma\rho dx$ or $dR = -RN\sigma\rho dx$. Denoting $N\sigma$ by constant, k , (k being equal to μ/ρ of the literature^[5]) representative of the gamma energy and the material in question,

$$R = R_0 \exp (-k \int \rho dx) . \quad (1)$$

It should be remembered that a gamma-material interaction does not generally result in absorption of the gamma, but only deflection and some loss of energy. Thus, even though the scattered gammas are present, Equation (1) refers only to the rate, R , of gammas passing in the original direction still having the original energy.

If the total attenuation length is a constant, x_0 , the product $k \int \rho dx$ can be replaced by $q \rho_{\text{eff}}$ with $q = kx_0$ resulting in

$$R = R_0 \exp (-q \rho_{\text{eff}}) . \quad (2)$$

Use of the symbol ρ_{eff} rather than ρ calls attention to the fact that the density usually is not constant along the path of integration. Consideration of nonhomogeneity error is given in Section II-4.2.

Hooker and Popper^[2] utilize a less general version of the above equation. Assuming uniform distribution so that $\rho_{\text{eff}} = \rho_{\text{avg}}$ and denoting α as the void fraction, for a two-phase mixture one has:

$$\rho_{\text{eff}} = (1-\alpha) \rho_{\text{liq}} + \alpha \rho_{\text{vap}} = \rho_{\text{liq}} - \alpha(\rho_{\text{liq}} - \rho_{\text{vap}}) . \quad (3)$$

If temperature and pressure, T and p , are fixed at some set of saturation values, ρ_{liq} and ρ_{vap} are each constant. Thus, under these constraints, Equation (2) can be written:

$$R = R_0' \exp \{+q\kappa\alpha\} \quad (4)$$

where $R'_0 = R_0 \exp\{-q\rho_{\text{liq}}\}$ and $\kappa = (\rho_{\text{liq}} - \rho_{\text{vap}})$. With T and p constant, Equation (4) shows that R varies exponentially as a function of the void fraction. Naturally, if T and p vary (i.e., a new saturation temperature is selected) the "constants" R'_0 and κ vary. Thus, Equation (4) is of little usefulness if the saturation temperature and pressure are changing but Equation (2) remains valid.

2. COLLIMATION AND/OR ENERGY ANALYZATION

To use the relationship $R = R_0 \exp\{-q\rho_{\text{eff}}\}$ directly without the need to worry about the intensity due to scattered, secondary gammas, either the energy of each gamma entering the detector must be ascertained before it is counted or secondary (scattered) gammas must be prevented from entering the detector. Since a gamma undergoes a change in direction when it is scattered, collimation of the gamma beam before and after the interaction region will serve to pass nearly all primaries and very few secondaries.

Large source strengths are required to achieve fast, accurate measurements since frequency response is related to beam intensity for a given statistical fluctuation. Because of the high count rates thus encountered and because collimation enhances the ratio of primaries to secondaries reaching the detector, collimation is nearly always necessary even when energy analyzation is used.

In addition, if the material whose density is being measured is itself somewhat radioactive, utilizing collimation on the detector side of the measurement region serves the added purpose of enhancing the ratio of primaries to measurement-region radiation. In fact, it then becomes evident that it is best to place most of the collimation on the detector side of the measurement region.

The actual collimation required is a strong function of the situation. The very least that should be used is containment of the detector such that gammas can only reach it from the general direction of the source. An arrangement that has been found to be more than adequate by the author is to collimate to an extent such that no gammas can directly reach the detector from any portion of the measurement region that is more than 2.0 gamma beam diameters from the beam centerline.

3. MEASUREMENT RESPONSE TIME

The relationship between response time, measurement accuracy, and source strength is the subject of this section. Because the detector output originates from discrete gammas incident upon the detector tube, the calculation of response time attainable versus source size and desired accuracy regresses naturally (even for dc current collection) to counting-statistics calculations.

Since radioactive sources emit gammas in a random manner versus time, Poisson statistics are followed. Thus, for a measurement time τ , and a mean count rate, R , the standard error, σ , in R is $\sqrt{R/\tau}$. A deviation from R of less than $K\sqrt{R/\tau}$ will be encountered in $f(K)\%$ of a series of measurements. Table I lists various values of K and the confidence level, $f(K)$, with the commonly associated statistical title^[6].

Figure 1 illustrates the time variation of a typical dynamic density measurement where R_0 is the detector gamma detection rate for a completely empty measurement region and R_1 is the detector gamma detection rate for a "completely full" measurement region.

TABLE I
VALUES OF $f(K)$ VERSUS K

<u>K</u>	<u>$f(K), \%$</u>	<u>Title</u>
0.67	50.0	Probable Error
1.00	68.3	Standard Error, σ
1.65	90.0	9/10 Error
1.96	95.0	95/100 Error, $\sim 2\sigma$

Fig. 1 Variation of R during typical dynamic density measurement.

Thus, associated with $R(t)$ is a statistical error ΔR ,

$$\Delta R = K\sigma = K \sqrt{R(t)/t} \quad (5)$$

where K is selected from Table I by choosing the desired measurement level of confidence, $f(K)$. Since the value of R varies from R_1 to R_0 , the signal in terms of percent of full scale is:

$$\% \text{ of FS} = 100\% \frac{R(t) - R_1}{R_0 - R_1} \pm 100\% \frac{\Delta R}{R_0 - R_1} \quad (6)$$

The first term on the right hand side of Equation (6) is the mean value of the measurement and the last term is the percent of full scale error associated with the measurement.

By substituting the expression for ΔR from Equation (5) into the last term of Equation (6), an equation relating the percent of full scale error to the gamma detection rate, measurement time, t , and confidence level is obtained.

$$\% \text{ of FS error} = \frac{100 \times K \sqrt{R(t)/t}}{R_0 - R_1} \quad (7)$$

If, for example, a 5% measurement with 0.9 confidence level (i.e., $K = 1.645$) is desired, one has

$$5 = \frac{100 \times 1.645 \sqrt{R(t)/t}}{R_0 - R_1} \quad (8)$$

This expression could then be solved for R_0 to determine the empty-measurement region count rate required to get the desired measurement accuracy for a given t and $q\rho(t)$ product. Instead, the general expression in K and % error (i.e., 5% in the above example) is solved for R_0 in Equation (9) by using $R_1 = R_0 \exp\{-q\rho_{\max}\}$ and $R(t) = R_0 \exp\{-q\rho(t)\}$.

$$R_0 = \frac{\exp\{-q\rho(t)\}}{t} \left\{ \frac{100 K}{(1 - e^{-q\rho_{\max}})(\% \text{ FS error})} \right\}^2 \quad (9)$$

As noted previously, R_0 is the detector gamma detection rate for a completely empty measurement region. To get the actual source strength required one must multiply R_0 by: $\exp\{+q\rho(t)\}$, the measurement region attenuation; $\exp\{+k\rho_{\text{met}}x_{\text{met}}\}$, the wall attenuation; the inverse of the detector efficiency, $1/\eta$; the inverse of the source gamma yield, $1/\gamma$; and the solid angle geometry factor $4h^2/r^2$. The geometry factor originates from $4\pi h^2/\pi r^2$ where the source is assumed to radiate gammas uniformly in all directions, h is the distance from source to detector, and r is the collimation radius at the most restrictive location (usually just in front of the detector as in Figure 2). Thus Equation (9) becomes:

$$\text{Curie Strength} = \frac{4h^2 \exp\{k\rho_{\text{met}}x_{\text{met}}\}}{t \eta \gamma 3.7 \times 10^{10} r^2} \left\{ \frac{100 K}{(1 - e^{-q\rho_{\max}})(\% \text{ FS error})} \right\}^2 \quad (10)$$

Fig. 2 Typical piping gamma attenuation density measurement.

Evans^[7] shows that if a simple RC filter is used rather than counting for time τ , the effective value of τ is $2 RC$. Since $f_{3dB} = 1/2\pi RC$ ^[8], one has $\tau = 1/\pi f_{3dB}$ and the above equation becomes:

$$\text{Curie Strength} = \frac{4h^2 \pi f_{3dB} \exp\{k\rho_{\text{met}} x_{\text{met}}\}}{\eta \gamma 3.7 \times 10^{10} r^2} \left\{ \frac{100 K}{(1-e^{-q\rho_{\text{max}}}) (\% \text{ FS error})} \right\}^2 \quad (11)$$

Because the last term in Equation (11) is squared, variation of any of the terms within requires large variations in the necessary source strength. The term $(1-e^{-q\rho_{\text{max}}})$ is kept from being small by selecting a source with gamma energy such that filling the measurement region with the material whose density is to be measured causes significant decrease in detector output so that a measurement can be made. A value of $e^{-q\rho_{\text{max}}}$ less than 0.05 should be avoided, though, for two reasons. First, the large secondary to primary ratio requires extensive collimation at the detector to eliminate the secondaries. And, second, streaming becomes significant if nonhomogeneities are present (see Section II-4.2). Increasing the frequency response also, naturally, requires a larger source strength to maintain a given value of K and $\%FS$ error. This effect is linear, though, rather than squared. Note, that for a given source strength, 3 dB frequency, and statistical uncertainty, the $\%FS$ error is independent of the value of the measurement region density, $\rho(t)$.

4. ERRORS

4.1 Logging and Data Bias

For Poisson statistics, $\bar{R}$ is located midway between the $\pm K\sqrt{R(t)/\tau}$ statistical error bands. If one utilizes the natural log function to transform Equation (2) to the density domain, the question arises whether or not $\bar{\rho}$ is somewhere near centered within the transformed error bands.

$$\bar{\rho} = -\frac{1}{q} \ln \left(\frac{\bar{R}}{R_o} \right) \quad (12)$$

If for $R = \bar{R} + \Delta R$ the indicated ρ is $\bar{\rho} - \Delta\rho_1$ and for $R = \bar{R} - \Delta R$ the indicated ρ is $\bar{\rho} + \Delta\rho_2$, one can write:

$$\bar{\rho} - \Delta\rho_1 = -\frac{1}{q} \ln \left(\frac{\bar{R} + \Delta R}{R_o} \right) \quad (13)$$

and

$$\bar{\rho} + \Delta\rho_2 = -\frac{1}{q} \ln \left(\frac{\bar{R} - \Delta R}{R_o} \right) \quad (14)$$

Using Equation (12) for $\bar{\rho}$ and $\Delta R = K\sigma$ in each of the above equations gives, after some simple algebra:

$$\Delta\rho_1 = + \frac{1}{q} \ln \left(1 + \frac{K\sigma}{\bar{R}} \right) \quad (15)$$

and

$$\Delta\rho_2 = - \frac{1}{q} \ln \left(1 - \frac{K\sigma}{\bar{R}} \right) . \quad (16)$$

Because, by Equation (7), one can infer that for a statistically meaningful measurement that $\bar{R} \gg K\sigma$, a good approximation of Equations (15) and (16) is $\Delta\rho_1 = \Delta\rho_2 = K\sigma/q\bar{R}$. Thus, in the region of interest where the statistical error is not large, the mean value of density is equal distance from the transformed statistical error bounds. Filtering could then be performed either before or after logging without significant data bias, but is more correctly performed in a stage prior to logging. In a situation where ΔR (due to changing density in the measurement region rather than to statistics) can be oscillatory and significant compared to $\bar{R}$, noticeable error in the indicated versus the actual $\bar{\rho}$ will be experienced if filtering is performed after logging rather than before.

4.2 Gamma "Streaming" or Nonhomogeneity Error

Picture a gamma beam directed along the k axis passing through the measurement region as shown in Figure 3. If there are two phases of the same material in the measurement region, the mixture can be conceptually broken into smaller regions which are either one phase or the other. Summation of portions of the beam passing through each small region then will yield the exiting beam intensity. Equation (17), showing the form of this summation, makes use of the fact that q's for each phase of a material are equal. The sum over k, in the argument of the exponential is simply an approximation to the line integral of Equation (1) for column ij. The double sum over i and j in front of the exponential adds the fluxes from each of the ij columns.

$$R = \frac{R_o}{mn} \sum_{i=1}^m \sum_{j=1}^n \exp \left\{ - \frac{q}{r} \sum_{k=1}^r \rho_{ijk} \right\} \quad (17)$$

Although the space in the near proximity to the beam is usually referred to as the measurement region, density sensing actually only occurs in the small volume "swept out" by the gamma beam. The remainder of this section will be used to show that the transverse extent of the gamma beam should be limited to minimize "streaming". Section II-4.3 shows, though, that a single such measurement, although accurate for the "beam volume", can be in error with respect to the total measurement region density because of nonuniform distribution of the fluid and container geometry.

Fig. 3 Splitting of the gamma beam path into parts.

For a given i and j , the summation $\frac{1}{r} \sum_{k=1}^r \rho_{ijk}$ is the average density, $\rho_{ij\text{avg}}$, of the ij column, so:

$$R_{\text{out}} = \frac{R_o}{mn} \sum_{i=1}^m \sum_{j=1}^n \exp\{-q\rho_{ij\text{avg}}\} \quad (18)$$

Now if every column has the same value, ρ_{avg} , for $\rho_{ij\text{avg}}$, one has:

$$\rho_{\text{avg}} = \frac{1}{mnr} \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^r \rho_{ijk} \quad (19)$$

and

$$R_{\text{avg}} = \frac{R_o}{mn} \sum_{i=1}^m \sum_{j=1}^n \exp\{-q\rho_{ij\text{avg}}\} = R_o \exp\{-q\rho_{\text{avg}}\} \quad (20)$$

In the case of "laminar" flow (two phases separated into laminar planes perpendicular to the gamma beam) and in the case of completely homogeneous flow (the two phases finely and uniformly mixed), $\rho_{ij\text{avg}} = \rho_{\text{avg}}$ for all i and j . Thus, in these two cases, at least, $R = R_o e^{-q\rho_{\text{avg}}}$. We also see that if lamina (thin compared to the beam width) are oriented parallel to the beam, $\frac{1}{r} \sum_{k=1}^r \rho_{ijk} = \rho_{ij\text{avg}}$ has very small value for those ij located in the low density lamina and slightly above ρ_{avg} value for the ij not located in the low density lamina. "Streaming" is then said to occur in this low density lamina because gamma attenuation in the low density lamina region is very low. To show that too large a net output occurs for this material phase configuration we form the ratio $R_{\text{out}}/R_{\text{avg}}$ and find it greater than unity.

$$\frac{R_{\text{out}}}{R_{\text{avg}}} = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n \left(e^{+q[\rho_{\text{avg}} - \rho_{ij\text{avg}}]} \right) \quad (21)$$

Since the quantity $q|\rho_{\text{avg}} - \rho_{ij\text{avg}}|$ is not large we can expand the exponential $e^x = 1 + x + \frac{x^2}{2} + \dots$

Because $\sum_{i=1}^m \sum_{j=1}^n q(\rho_{\text{avg}} - \rho_{ij\text{avg}}) = 0$, one gets to x^2 approximation that:

$$\frac{R_{\text{out}}}{R_{\text{avg}}} = 1 + \frac{1}{2mn} \sum_{i=1}^m \sum_{j=1}^n \left\{ q[\rho_{\text{avg}} - \rho_{ij\text{avg}}] \right\}^2 \quad (22)$$

Thus, since the sum of squared terms must be greater than or equal to zero, R_{out}/R_{avg} is greater than or equal to unity. Because ρ is determined from the output, R_{out} , by $(1/q)\ln(R_o/R_{out})$, ρ_{out} will be less than (or equal) to the actual average density of the material in the gamma beam path. As mentioned just after Equation (11), use of a higher energy gamma has a smaller value of q so the additional term in Equation (22) is reduced in value. Too high an energy gamma, though, leads to insensitivity and to a need for large source strength as indicated by Equation (11) when $e^{-q\rho_{max}}$ is close to unity.

In summary, the streaming error occurs whenever the width of the gamma beam is not small compared to the width of those "fluid" lamina oriented parallel to the beam. Use of a narrow gamma beam and higher energy gammas helps to decrease the streaming. The narrower beam decreases the number of terms in the sum of Equation (22), and the higher energy gamma decreases the value of q .

4.3 Wall Adherence Error (Annular Flow)

When a medium physically separates into two phases, as it often does, the meaning of the attenuated gamma output becomes subject to interpretation. If, for instance, homogeneous mixed flow in a circular pipe of inside diameter, d_o , separates into central flow with diameter d_1 of density ρ_1 and annular flow of density ρ_o , the average density is:

$$\rho_{avg} = \frac{\rho_1 d_1^2 + \rho_o (d_o^2 - d_1^2)}{d_o^2} \quad (23)$$

But the gamma attenuated output is

$$R_{out} = R_o e^{-kd_o} \left[\rho_1 \frac{d_1}{d_o} + \rho_o \left(1 - \frac{d_1}{d_o}\right) \right] \quad (24)$$

whereas the output needed to predict the density of Equation (23) would be

$$R_{avg} = R_o e^{-kd_o} \left\{ \rho_1 \left[\frac{d_1}{d_o}\right]^2 + \rho_o \left(1 - \left[\frac{d_1}{d_o}\right]^2\right) \right\} \quad (25)$$

The ratio R_{out}/R_{avg} is an indication of the magnitude of the output error:

$$\frac{R_{out}}{R_{avg}} = e^{+kd_1(\rho_o - \rho_1)\left(1 - \frac{d_1}{d_o}\right)} \quad (26)$$

Since the density ρ_o of the annular phase is normally larger than ρ_1 , R_{out} is greater than it should be. The predicted density is, therefore, less than it should be. Since d_1 is usually not known, use of a single gamma measurement does not allow accurate evaluation of ρ_{avg} .

Where the flow regime is such that the model of ideal annular flow of density ρ_o with central flow of density ρ_1 is accurate, use of two gamma measurements of distinctly different gamma energies (with attenuation constants k and k') allows measurement of d_1 . Since R_{out} in this case behaves as Equation (24):

$$R_{out} = R_o e^{-kd_o \left[\rho_1 \frac{d_1}{d_o} - \rho_o \left(1 - \frac{d_1}{d_o} \right) \right]} \quad (27)$$

and for gammas of the other energy:

$$R'_{out} = R'_o e^{-k' d_o \left\{ \rho_1 \frac{d_1}{d_o} - \rho_o \left(1 - \frac{d_1}{d_o} \right) \right\}} \quad (28)$$

Solving these for d_1 :

$$d_1 = d_o \left\{ \frac{\rho_o - \frac{\ln \left(\frac{R'_{out}}{R_{out}} \frac{R_o}{R'_o} \right)}{d_o (k-k')}}{\rho_o - \rho_1} \right\} \quad (29)$$

For a true two-phase fluid ρ_o and ρ_1 are the liquid and vapor densities at the saturation pressure for the instant in question. Assuming the primes to be used for the higher energy gamma ($k-k'$) will be positive. Assuming further that $R_o = R'_o$, R'_{out} will be greater than R_{out} . The total log term is then positive. Thus, Equation (29) consists of division of differences between numbers that are of similar size, and care must be taken in measuring R'_{out} and R_{out} to get meaningful answers. Problems such as flow separation due to gravity, etc., may require use of three or more gamma measurements at different energies, angles, etc., to obtain sufficient information to interpret outputs with confidence.

To determine mass flow in a pipe of cross-sectional area A , the problem is more complicated than just measuring density because each of the two separate phases generally have different mean velocities, $\bar{v}_o$ and $\bar{v}_1$. For the present case of separated annular flow:

$$\text{Mass Flow} = (\rho v A)_{avg} = \frac{\pi}{4} \{ \rho_o \bar{v}_o d_o^2 - d_1^2 (\rho_o \bar{v}_o - \rho_1 \bar{v}_1) \} \quad (30)$$

But neither $\bar{v}_o$ nor $\bar{v}_1$ are generally known. If flow is indeed separated as assumed, pitot tubes can be used in each of the two regions to obtain the velocities, but pitot tubes will not yield correct answers in mixed flow.

4.4 Scintillator and Photomultiplier Related Errors

A scintillator coupled to the photocathode of an electron multiplier is one of the most efficient gamma-ray detectors in the 60 keV to 3 MeV gamma-ray energy range of interest. A common scintillator is a crystal of sodium iodide activated by a small percentage of thallium. The sensitivity of this gamma sensing system is influenced by the various factors considered below. Although these factors are not important when counting techniques are used, if the output current of the photomultiplier is used for the value of R_{out} , all of the effects below affect the value of R_{out} (and R_0).

4.41 Scintillator Crystal Temperature Sensitivity. For constant gamma stimulation, the output current from a NaI (Tl) Scintillator electron multiplier tube rises about 0.07%/°F temperature rise when operated near room temperature. Because one eventually logs R_{out} to obtain ρ_{avg} , only the ratio of the current to some reference current R_0 is important. Thus, unless the empty measurement region output, R_0 , can be frequently measured, the drift in R_0 leads to all ratios being formed with an incorrect reference. Containing the photoscintillator in a fixed temperature container will of course eliminate the problem.

4.42 Electron Multiplier Voltage Sensitivity. The electron multiplier^[9] current gain is very sensitive to changes in dynode resistor chain drive voltage. Typical change in current gain per volt change in the total (900 volt nominal) chain voltage is 0.6%/volt. A well-regulated power supply placed in a temperature-stabilized environment is suggested. The effect of dynode drive voltage change is the same as changes in photoscintillator sensitivity discussed above. Temperature stable resistors (derated considerably from manufacturers suggested power rating) are necessary in order that the dynode resistor chain not vary the voltage dynode to dynode versus time or temperature for a fixed total chain voltage.

4.43 Electron Multiplier Space Charge Sensitivity. As the electron current increases at dynodes farther away from the photocathode, the charge in the space about a dynode increases. If the net current in the final stage of the multiplier reaches a value above about one microampere (depending on dynode width, size, drive voltage, etc.), mutual repulsion forces between electrons causes output to increase in a slightly nonlinearly manner with photocathode input rate. Various options are available to prevent this:

If a high count rate is required to minimize statistical fluctuation (see Section II-3), current (or pulses) can be withdrawn from a dynode upstream of the regular collection dynode. When using counting techniques at high counting rate, the upstream dynode collection technique has the additional advantage of decreasing the spread in arrival time of the electrons originating from a given gamma stimulus.

If the high count rate is needed to minimize statistical fluctuations and the space charge effect is only very slightly evident, the dynode resistor chain voltage may

be decreased a few volts. Too low a chain voltage, though, will cause nonconstant current gain and even complete loss of some pulses. This leads to excessive statistical fluctuation.

4.44 Electron Multiplier Dynode Activation Memory Error. With dynode current density at say 10^{-6} ampere/cm², each and every dynode surface atom site is impacted every 100 seconds. With every 100 surface atom sites being stimulated once per second, the dynode suffers a change in its secondary emission "constant" from its value at lower dynode current density. In effect, the microarea remembers that it has recently been impacted and its secondary emission response is slightly greater than if it had not recently been impacted. Thus, if the gamma stimulation rate is suddenly decreased a factor like 10, the photomultiplier output also suddenly decreases a factor of ten but then continues to decrease very slightly for a period of time until the memory effect adjusts to the new dynode current density. For low dynode current density, the electron impingement rate is not sufficient for the memory effect to develop. The solution as in Subsections II-4.42 and -4.43 is to remove the current from the electron multiplier at a stage (before the regular collector dynode) where the dynode current is low enough that the memory effect is not noticeable. An output current of 10^{-8} amperes has been found to be low enough to make the memory effect nearly undetectable.

4.5 Electronics Errors

If the current from the electron multiplier is used as R_{out} and is run into a current-logging amplifier, drift in the logging amplifier ordinarily will change the full scale voltage output change per full scale change in density and both the "full" and "empty" output voltages. Thus, if current logging electronics are used, it must be designed to be as drift free as possible. The heart of a current logging amplifier is usually a transistor in the feedback loop of an operational amplifier. For stable results, a temperature controlled (Peltier cooled) dual logging transistor used with the two transistors in the feedback loops of two operational amplifiers has been found to work well. The two operational amplifiers then drive a differential operational output amplifier. Although temperature compensation techniques might be used rather than absolute temperature control on the logging transistor temperature, the transistor's behavior versus temperature is different at one current than at another. Commercial, temperature-compensated logging circuits and compensation circuits built by the author were all found to be unsatisfactory.

If the current or the raw counts/sec data are linearly recorded and operated on by digital computer to take the log of R to determine ρ , the current-logging amplifier and its drift are eliminated.

III. CONCLUSIONS

A gamma attenuation density measurement using a well-collimated, narrow beam of gammas of proper primary energy allows accurate determination of the total mass of material in the beam path.

The source curie strength necessary to obtain a given response time and percent statistical uncertainty (to a given confidence level) is readily calculated for a given geometry, full-range density change, and gamma energy.

Data bias due to filtering can be prevented by filtering prior to logging of R_{out} .

Gamma streaming is minimized by using a narrow gamma measurement beam of adequate primary energy.

Nonhomogeneous arrangements of two-phase "fluids" in the measurement region force one to use two or more gamma measurements at different angles, and/or gamma energies in order that a meaningful fluid configuration and density value be interpreted from the outputs.

In addition to the above considerations, to obtain accurate results, proper use and understanding of the gamma detection system are important.

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