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FLUCTUATION EFFECTS ON THE CONDUCTIVITY
OF Nb AND NbN FILMS

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ABSTRACT

Standard four-terminal resistance measurements were done on sputtered Nb and NbN films to study the effect of fluctuations on the electrical conductivity. For one of the NbN samples, data were also taken in perpendicular magnetic fields up to 60 kG. Agreement of the data with the theory was only qualitative. The zero field data seemed to indicate a change in sample dimensionality - a behavior also observed in the magnetic field data. However, the change took place at values of the magnetic field higher than predicted by theory. One way of reconciling this apparent discrepancy is to use in the theoretical expressions a smaller film thickness than that measured. This suggests that the granular structure of the superconductor affected the superconducting transition of the samples. This interpretation is supported by the high activation energy of the normal resistance as determined from the perpendicular field data. Even with this adjustment, however, difficulties remained. The behavior of the refractory films used here appears to be more complex than the description provided by the theory for homogeneous superconductors.

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I. INTRODUCTION

The superconducting transition can be described in terms of an order parameter Ψ , where $|\Psi|^2$ gives the density of the superconducting electrons. Ψ has a finite value below the critical temperature T_c , going to zero as T approaches T_c . Above the critical temperature the average value of the order parameter is zero but thermodynamic fluctuations create temporary regions of finite superconducting density which become manifested in observable effects. One of these is the rounding of the resistance curve at the transition to the superconducting state. Other manifestations occur in fluctuation-induced diamagnetic susceptibility, Hall effect, Josephson current, specific heat, and tunneling density of states.

An early estimate (1) gave a very narrow temperature range for the size of the critical region. By using materials with short mean free paths, the temperature range can be increased considerably (2). A further extension is obtained by using one- or two-dimensional samples, where the dimensionality gets reduced by one when the temperature dependent coherence length becomes larger than a linear dimension of the sample. The coherence length measures the range of the coherent fluctuations in the order parameter.

Although the critical region was very narrow,

which suggested that fluctuation effects would be impossible to probe experimentally, observations indicated otherwise. The problem was resolved with the understanding that the criterion for critical behavior was not the same as the condition governing the observation of fluctuation effects.

Theories based either on a microscopic approach (3), or the time-dependent Ginzburg-Landau theory (4)-(6) have been put forward to account for fluctuation effects on the electrical conductivity. Equations have been derived which give specific results that can be tested experimentally. The verification of the theory can take two approaches. One is to take the predicted functional dependence on temperature (or magnetic field) and compare the parameters calculated from the data with those provided by the theory. Another approach would be to test qualitative features of the theory. For instance, according to the theory, a two-dimensional sample (thin film) behaves like a three-dimensional sample when the temperature dependent coherence length becomes shorter than the sample thickness. This change in the dimensionality occurs also in a magnetic field, and is in fact predicted also for specific heat measurements (7).

Resistance measurements on thin films (8)-(17) and one-dimensional systems (18)-(22) give good qualitative agreement with theory. However, attempts to make a

quantitative comparison are plagued with the difficulty of having to vary a number of parameters in order to give a good fit. Assuming that the normal resistance can be measured independently, Testardi et. al. (11) proposed a scheme which enables a test of the temperature dependence of the data without the use of fitted parameters. They get the predicted temperature dependence for their lead films but the width parameters differed (being 2-14 times smaller) from the theoretical values which are calculated from fundamental constants and the normal sheet resistances $R_n \frac{w}{l}$, where w and l are the width and length of the film respectively. A discrepancy also obtained in the results of Glover (8) on amorphous bismuth films, Masker and Parks (17) on aluminum films, and Smith et. al. (9) on lead films. Strong coupling corrections (23) reduce the width parameter whereas depairing mechanisms (24) and sample inhomogeneities (25) increase it.

In addition to pair-type fluctuations, normal electron fluctuations contribute to the electrical conductivity (26)-(32). Early calculations of the latter contribution, called the Maki term, gave a singularity for one and two dimensional samples. Thompson (27) introduced a pair-breaking mechanism to get rid of the divergence, but Patton (31) showed that the Maki term was well-behaved in two or fewer dimensions without the use of a pair-breaking mechanism.

Aside from the difficulty with the width parameter, the experiments (8)-(10), (16) also failed to show the theoretically predicted transition to three dimensional behavior.

Using an analysis with no fitted parameters, Schaer (12) found that a transition from the three to the two dimensional regime occurred in NbN(Ti) films. A limitation of the analysis was the method used to determine the normal state resistance R_n . For samples with low critical fields H_c , resistance measurements in fields above H_c give R_n directly. NbN has an inaccessible high critical field so R_n , which happened to be temperature dependent, was assumed to be given by a linear extrapolation of the high temperature data to the temperature range of interest.

A refinement in the determination of R_n can be made following the technique used by Kajimura and Mikoshiba (13). The method is based on the theoretical model for the excess conductivity in a magnetic field. It entails taking resistance measurements in a finite field, keeping the temperature constant, and making a comparison with the extension of the theory to the static field case. The development of the theory again starts from a microscopic (33) or a phenomenological (34)-(39) approach.

Much progress has been made in the theoretical

framework. Difficulties, like the narrow temperature range of the critical region or the divergence of the Maki term for one- or two-dimensional superconductors, have been resolved. The progress in the theoretical field was accompanied by a flurry of experimental activity. The hope was that the experimental results would confirm the validity of the models used, or in case of disagreement, provide clues as to how the models should be modified to give a closer representation of the process being described. The experiments are also useful even if they only serve to define the limits of applicability of the theoretical models.

Most of the experiments on fluctuation effects on the electrical conductivity have been done on Type I superconductors. Comparison with the theory have been met with varying degrees of success. In this study, samples are made of type II superconductors (coherence length $\leq$ penetration depth) prepared by sputtering. The study focuses on dirty superconductors, for which the mean free path is even shorter than the coherence length. There are two reasons for this: a) the use of short mean free paths increases the temperature range for which fluctuation effects become observable, and 2) thicknesses can be chosen so that, in theory at least, the samples exhibit bulk as well as two-dimensional behavior in this temperature range.

II. THEORY

Although the effect of a magnetic field on the fluctuation contribution to the electrical conductivity was considered in the early Aslamazov-Larkin paper (3) the correct and more rigorous derivations appeared later. The treatment here will deviate from the historical pattern for simplicity. Any proper extension of the theory must reduce to the correct limiting forms, so the expressions for the zero-field case can be obtained from the magnetic field results by letting H go to zero.

Quantum mechanics gives a prescription for calculating the current from the order parameter, which requires the solution of the equation of motion of Ψ . Using the Ginzburg-Landau expression for the free energy, the thermal averages of the fluctuations (and the corresponding currents) can be calculated. The fluctuation-dissipation theorem then relates these to the electrical conductivity. The derivation in this section will be valid for finite fields only, since the free energy will be assumed to be given by the Ginzburg-Landau theory. In addition, the treatment will be confined to the case when the magnetic field is perpendicular to the sample.

From the time-dependent Ginzburg-Landau theory, the equation of motion of the order parameter Ψ in

a magnetic field with vector potential A is (4), (26)

$$\left[\frac{\partial}{\partial t} + v \left\{ \alpha + \frac{1}{2m} (-i\nabla - zeA)^2 \right\} \right] \Psi(r, t) = 0 \quad (1)$$

with $\hbar = k = c = 1$. In the dirty limit,

$$v = \frac{4}{3} m v_F l$$

$$\alpha = \frac{12 T_c}{\pi m v_F l} \cdot \frac{T - T_c}{T_c}$$

where v_F is the Fermi velocity and l the mean free path.

Let the magnetic field be in the z -direction, and perpendicular to the sample. Choosing the gauge

$$A = (0, xH, 0)$$

$\Psi(r, t)$ can be expanded in terms of the eigenfunctions of the operator

$$\frac{1}{2m} (-i\nabla - zeA)^2$$

Hence

$$\Psi(r, t) = \sum_{n, k, q} \Psi_{n, k, q}(t) \varphi_{n, k, q}(r) \quad (2)$$

For bulk material

$$\varphi_{n, k, q}(r) = e^{-i(ky + qz)} \chi_n(x - \frac{k}{zeH}) \quad (3)$$

where χ_n is the normalized harmonic oscillator eigenfunction

$$\frac{1}{2m} \left\{ -\frac{\partial^2}{\partial x^2} + 4e^2 x^2 H^2 \right\} \chi_n(x) = \varepsilon_n \chi_n(x) \quad (4)$$

Applying periodic boundary conditions in the y and z

directions,

$$k = \frac{2\pi}{L} n_k$$

$$n_k = 0, \pm 1, \pm 2,$$

$$q = \frac{2\pi}{L} n_q$$

$$n_q = 0, \pm 1, \pm 2, \dots$$

By letting L go to infinity the summation over k and q in equation (2) can be replaced by an integration.

For a film of thickness d oriented $\perp$ to H ,

$$q = \frac{2\pi}{d} n_q$$

$$n_q = 0, \pm 1, \pm 2$$

The eigenvalue corresponding to $\rho_{n,k,q}$ is

$$\varepsilon_{n,q} = \frac{q^2}{2m} + (n + 1/2) \omega_H; \quad \omega_H = \frac{2eH}{m} \quad (6)$$

Substituting (6) into the equation of motion,

$$\left(\frac{\partial}{\partial t} + \Gamma_{nq} \right) \psi_{n,k,q}(t) = 0 \quad (7)$$

where

$$\Gamma_{nq} = \gamma \left\{ \alpha + (n + 1/2) \omega_H + \frac{q^2}{2m} \right\}$$

Define the Fourier transformations

$$\psi(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_{\omega} e^{i\omega t} d\omega$$

$$\psi_{\omega} = \int_{-\infty}^{\infty} \psi(t) e^{-i\omega t} dt$$

so that the Fourier transform of (7) is

$$(i\omega + \Gamma_{nq}) \psi_{n,k,q}(\omega) = 0,$$

from which we get the frequency spectrum of the fluctuations of the order parameter (40, Chapter XII)

$$\langle |\psi_{n,k,q}^{(\omega)}|^2 \rangle = \langle |\psi_{n,k,q}^{(t)}|^2 \rangle \frac{2\Gamma_{nq}}{\Gamma_{nq}^2 + \omega^2} \quad (8)$$

To calculate thermal averages, each fluctuation is weighted by the free energy associated with it, i.e.

$$\langle |\psi_{n,k,q}^{(t)}|^2 \rangle = \frac{\int e^{-\beta F} |\psi_{n,k,q}^{(t)}|^2 d\{\psi^*, \psi\}}{\int e^{-\beta F} d\{\psi^*, \psi\}} \quad (9)$$

In the Ginzburg-Landau theory,

$$F = F_n + \int d^3x \left\{ \alpha |\psi|^2 + \frac{1}{2m} |(-i\nabla - 2eA)\psi|^2 + \frac{H^2}{8\pi} \right\}$$

where F_n is the free energy in the normal state. Substituting into (9) and retaining only terms of order $|\psi|^2$, we find (41, p. 174)

$$\langle |\psi_{n,k,q}^{(t)}|^2 \rangle = \frac{\nu T}{\Gamma_{nq}}$$

The current is obtained from

$$\mathbf{j} = \frac{ie}{m} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{2e}{m} A \psi^* \psi$$

Using (2) and (3), we get the Fourier coefficients (Appendix A)

$$\begin{aligned} j_{\omega}^x(x, k, p) &= \frac{ie}{m} \sum_{n_1, n_2} \int \frac{dk' dp'}{(2\pi)^3} \psi_{n_1, k'-k/2, p'-p/2, \omega'-\omega/2}^* \\ &\quad \cdot \psi_{n_2, k'+k/2, p'+p/2, \omega'+\omega/2} \\ &= F_{n_1 n_2}^x(x, k'+k/2, k'-k/2) \end{aligned}$$

where

$$F_{n,n_2}(x, k_1, k_2) = \psi_{n_1}\left(x - \frac{k_1}{2eH}\right) \psi'_{n_2}\left(x - \frac{k_2}{2eH}\right) \\ - \psi_{n_2}\left(x - \frac{k_2}{2eH}\right) \psi'_{n_1}\left(x - \frac{k_1}{2eH}\right)$$

From the fluctuation-dissipation theorem,

$$\sigma(\omega) = \frac{1}{\omega} \tanh \frac{\hbar\omega}{2T} \int_{-\infty}^{\infty} dx \langle J_{\omega}\left(\frac{x}{2}, 0, 0\right) J_{-\omega}\left(-\frac{x}{2}, 0, 0\right) \rangle$$

For the current correlation function,

$$\langle J_{\omega}^x\left(\frac{x}{2}, 0, 0\right) J_{-\omega}^x\left(-\frac{x}{2}, 0, 0\right) \rangle = -\frac{e^2}{m} \sum_{n,m} \int \frac{dk'}{2\pi} \frac{dP'}{2\pi} \frac{d\omega'}{2\pi} \\ \times (2\pi T)^2 \frac{1}{(\omega' - \omega/2)^2 + \Gamma_{n,p'}^2} \frac{1}{(\omega' - \omega/2)^2 + \Gamma_{n,p'}^2} \\ \times F_{mn}\left(\frac{x}{2}, k', k'\right) F_{nm}\left(-\frac{x}{2}, k', k'\right)$$

Consider the static conductivity. Using properties of the harmonic oscillator eigenfunctions, Tilley and Parkinson (39) evaluate the integral over x and k' as

$$\int F_{mn}\left(\frac{x}{2}, k', k'\right) F_{nm}\left(-\frac{x}{2}, k', k'\right) dx dk' = \\ -2(2eH)^2 \{ (n+1) \delta_{n,m-1} + n \delta_{n,m+1} \}$$

Hence

$$\sigma^{xx}(0) = \frac{e^2}{2T} (2\pi T)^2 2\omega_H^2 \mathcal{G}$$

where

$$\mathcal{G} = \frac{2}{(2\pi)^3} \sum_n (n+1) \int dP' d\omega' \left(\frac{1}{\omega'^2 + \Gamma_{n+1,p'}^2} + \frac{1}{\omega'^2 + \Gamma_{n,p'}^2} \right)$$

Performing the w' integration,

$$J = \left(\frac{1}{2\pi}\right)^2 \sum_n \int dP' \left(\frac{A}{\Gamma_{n+1, P'}} + \frac{B}{\Gamma_{n, P'}} \right) \quad (10)$$

with

$$A = -B = - \frac{n+1}{2\gamma\omega_H (\Gamma_{n, P'} + \frac{1}{2}\gamma\omega_H)}$$

To go to the two-dimensional limit, replace $1/(2\pi) dP'$ by $\frac{1}{d} \sum_{P'}$. If the film is thin enough, it suffices to take only the $P' = 0$ term. Hence

$$J = \frac{1}{2\gamma^3\omega_H^3} \cdot \frac{1}{2\pi d} \sum_{n, P} \frac{n+1}{(n+1/2+\gamma+\delta_P)(n+1+\gamma+\delta_P)(n+3/2+\gamma+\delta_P)} \quad (11)$$

for finite films, and

$$J_{2D} = \frac{1}{4\pi d} \frac{1}{\gamma^3\omega_H^3} \sum_n \frac{n+1}{(n+1/2+\gamma)(n+1+\gamma)(n+3/2+\gamma)} \quad (11')$$

$$\gamma = \frac{d}{\omega_H} = \frac{4T_c}{2\pi v\ell} \frac{E}{H}; \quad \delta_P = \frac{P^2}{2m\omega_H}$$

for two-dimensional samples. We may write

$$\sigma'_{2D} = \sigma'_{2D}(E) G_{2D}(\ell/2h) \quad (12)$$

where

$$\sigma'_{2D}(E) = \frac{e^2}{4d} \quad (13)$$

$$h = 2c \xi^2(0) H = \frac{\pi v\ell}{12T_c} H$$

$$G_{2D}(\gamma) = 2\gamma \sum_{n=0}^{\infty} (n+1) \left\{ \frac{1}{(n+1/2+\gamma)(n+1+\gamma)(n+3/2+\gamma)} \right\}$$

In the limit $H \rightarrow 0$, the summation can be replaced by an integration and we recover (Appendix B) the

Aslamazov-Larkin result (13), with the identification $T_c(H=0) = T_{c0}$. For finite H but small T , $G(T) \propto T$ and

$$\sigma'_{3D} \propto T^{1/2} \quad (14)$$

Going back to (10) and performing the P' integration,

$$\sigma'_{3D} = \sigma'_{3D}(T) G_{3D}(T/2H) \quad (15)$$

for bulk material, where

$$\sigma'_{3D}(T) = \frac{e^2}{32\pi^2} \frac{1}{T^{1/2}} \quad (16)$$

$$G_{3D}(T) = 4T^{1/2} \sum_{n=0}^{\infty} (n+1) \left\{ \frac{1}{\sqrt{n+1/2+1}} + \frac{1}{\sqrt{n+3/2+1}} - \frac{2}{\sqrt{n+1+1}} \right\}$$

Again, in the limit $H \rightarrow 0$, replacing the summation by an integration and expanding the arguments (Appendix B) yields (16).

By an ingenious algebraic manipulation, Aoi (private communication to Prof. Goldman) transforms (11) to

$$\sigma' = \frac{3e^2 T}{4\pi v_F l} \left\{ \frac{1}{2ed\Delta H} + \frac{1}{\sqrt{2e\Delta H}} \coth(d\sqrt{2e\Delta H}) \right\} \quad (17)$$

$$\Delta H = H - H_{c2}(T)$$

where H_{c2} is obtained by setting $\Gamma_{00} = 0$ in (7'), i.e.

$$\alpha + \frac{4\pi^2 c_2}{2} = 0 \quad (18)$$

We note the striking resemblance of (17) to the expres-

sion derived by Schmidt for the zero field case. Depending on the magnitude of ΔH , (17) goes in the limit as $1/\epsilon$ (2-dimensional) or $1/\sqrt{\epsilon}$ (3-dimensional behavior) where H is incorporated into ϵ (equations (12) and (15) are derived by Usadel via a more general approach and the field dependence of ϵ enters through T_c).

Later we will have occasion to use and compare a few versions, valid in the Ginzburg-Landau region, derived by various authors. The results are:

Usadel

3-dimensional case

$$\sigma' = \frac{e^2}{4\pi} \sqrt{\frac{6T}{\pi v l}} \frac{1}{\sqrt{\epsilon}}$$

2-dimensional case

$$\sigma' = e^2 / 4\pi d \epsilon$$

where

$$\epsilon = \ln T / T_c$$

and T_c is defined from

$$\ln(T_c/T_0) + \psi\left(\frac{1}{2} + \frac{v l c H_{c2}(T_c)}{6\pi T_c}\right) - \psi\left(\frac{1}{2}\right) = 0$$

With the definition

$$H_{c2}(T_c) / T_c = H / T$$

and for finite H , the expression for ϵ simplifies to

$$\epsilon = \ln T / T_0 + P/2; \quad P = \frac{\pi v l c H}{6T}$$

Mikeska-Schmidt

Depending upon how the parameter $\rho (= \frac{DeH}{2T_c})$ com-

pares with ε , the expressions are

3-dimensional case

$$\sigma' = \frac{e^2}{4} \sqrt{\frac{2T_c}{\pi D}} \frac{1}{\sqrt{\varepsilon}} \quad p \ll \varepsilon \ll 1$$

$$= \frac{e^2}{4} \sqrt{\frac{2T_c}{\pi D}} \frac{1}{\sqrt{\varepsilon}} \quad \varepsilon \ll p \ll 1$$

2-dimensional case

$$\sigma' = \frac{e^2}{16d\varepsilon} \quad p \ll \varepsilon \ll 1$$

$$= \frac{e^2}{4d\varepsilon} \quad \varepsilon \ll p \ll 1$$

where

$$p = \frac{DeH}{2\pi T_c} = \frac{v_F L_H}{6\pi T_c}$$

and T_c is defined from

$$\ln(T_c/T_{c0}) + \psi\left(\frac{1}{2} + \frac{2DeH}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right) = 0$$

Aoi

3-dimensional case

$$\sigma' = \frac{e^2 T}{D\pi} \frac{1}{\sqrt{2e\Delta H}} \quad d\sqrt{2e\Delta H} \gg 1$$

2-dimensional case

$$\sigma' = \frac{eT}{D\pi} \frac{1}{d\Delta H} \quad d\sqrt{2e\Delta H} \ll 1$$

where

$$\Delta H = H - H_{c2}(T)$$

and $H_{c2}(T)$ is obtained from $\alpha + w_{c2} = 0$.

The derivation given previously takes a phenomenological approach and hence does not contain the so-called Maki term. This term, which arises in the micro-

scopic calculation, is in the zero field limit (using Thompson's results):

$$\sigma'_{MT} = \frac{e^2}{8d} \left[\ln(\epsilon/\delta) / (\epsilon - \delta) \right] \times$$

$$\left[\frac{1}{2} + \ln \left\{ \frac{\xi(0)}{d\delta^{1/2}} \operatorname{sech} \left(\frac{d\epsilon^{1/2}}{\xi(0)} \right) \right\} / \ln(\epsilon/\delta) \right]$$

$$\delta = \text{pair-breaking parameter} = T_{co, bulk} - T_{co}$$

and

$$\sigma' = \sigma'_{AL} + \sigma'_{MT}$$

where

$$\sigma'_{AL} = \frac{e^2}{4d\epsilon} \frac{1}{2} \left[\frac{1}{2} + \frac{d}{\xi} \operatorname{cath} \frac{d}{\xi} \right]$$

Note that from eq. (14) the excess conductivity is approximately linear with the inverse of applied field for a two-dimensional sample close to T_{co} . Inclusion of the Maki-Thompson term preserves the linearity in $1/H$, but with a bigger slope (46).

III. EXPERIMENTAL PROCEDURE

The experiments conducted in this work are an extension of Mr. Schaer's investigations (12), using essentially the same apparatus and procedure. Samples were prepared by Mr. Somasundaram of the Metallurgy Department of the University of Minnesota, using a sputtering technique similar to that used by Mr. Shy in making the samples for Mr. Schaer's experiments. A razor blade was used to scrape off the film edges and eliminate edge effects.

To minimize errors due to lead resistances, both the sample and thermometer resistances were measured with a four terminal technique. Measuring currents were supplied by lockin amplifiers (PAR model HR-8) which also detected error signals from the bridge. The bridge output was coupled to the detector by a 10 megohm input resistance preamplifier, which required the use of high frequencies for the bridge circuits.

The temperature of the sample block was regulated by feeding the error signal from either the sample or thermometer bridge into a temperature-regulating power supply designed and built by Mr. John T. Anderson. The circuit consists of two operational amplifiers in a buffer and summing configuration.

The sample block was inside an exchange gas can which was separated from the helium bath by a vacuum.

The exchange gas can also served as a heat shield, and its temperature was regulated by a Wheatstone bridge with a lockin phase sensitive detector. Its output was fed into two operational amplifiers in a summing configuration. The Wheatstone bridge was operated at 10^3 hertz.

Resistance measurements in a finite field were done with the 60 kG persistent current magnet acquired during the later stages of the study. The experiments were conducted in a 6" I.D. stainless steel dewar. The magnet was supported by three stainless steel tubings which were attached to the brass plate adaptor at the top of the dewar. The cryostat, which was the same one used by Mr. Schaer for his high field data, was fastened to this adaptor, and the length of the stainless steel supports was calculated so that upon assembly the sample would be at the center of the magnet. Four plates of 1/16" thick copper at various levels provided radiation shielding. Electrical connections to the magnet came out through the side dewar pump-out flange.

To assemble the apparatus, the cryostat was put together and the magnet attached to the dewar-cryostat adaptor facility. The cryostat was then slipped into the magnet and the whole assembly lowered into the dewar.

The NbTi magnet had a 2" bore and a .1% field

homogeneity at the center throughout a 1 cm. diameter spherical volume. Its rated field at the center was 60 kG, with a current of 49 amps. A 1 Ω -20 watt resistor was connected across the terminals to dissipate the power should the magnet suddenly go normal.

A precalibrated CryoCal thermometer (unit #852) was used to measure sample temperatures. For the finite field data a thermistor (National Carbon Co, type 100 K-HE) was used for regulating the temperature, having a much smaller magnetoresistance. The CryoCal was used as a transfer standard in zero field for the magnetic field measurements.

An early paper (42) showed that thermistors were suitable thermometers in the presence of a magnetic field, giving at most a 0.2% change in the resistance at 4.2°K in a field of 19 kG. The later paper by the same authors (42) showed that the type of thermistor used in these experiments caused a temperature shift of approximately -28°mK in a field of 60 kG when used as the control system sensor. This was at a temperature of 8.9°K, the highest temperature presented. The shift at higher temperatures would be bigger. However, this does not invalidate our data, as the measured resistance values at the high field end tended to converge (this will be explained in more detail in the next section).

IV. DATA AND ANALYSIS

An important parameter in the theory is the coherence length. To determine it experimentally, we use the relationship

$$H_{c2}(T) = \frac{\phi_0}{2\pi f \lambda_0} \left[\frac{T_c - T}{T_c} \right], \quad \phi_0 = 2 \times 10^{-7} \text{ g-cm}^2$$

assuming that at $R = \frac{1}{2}R_n$, $T_c(H) = T_{c0}$. Two other conditions are assumed to be satisfied. One is that (47)

$$H_{c1}(T) = H_{c2}(T)$$

where $H_{c1}(T)$ is the surface normal critical field and $H_{c2}(T)$ is, as before, the bulk critical field of type II superconductors. This condition is satisfied if T is close to T_c or if the electronic mean free path is short. The second is related to the polycrystalline nature of the samples. The equation

$$\xi(T) = 0.15 \left[\xi_0 \lambda_{eff} T_c / (T_c - T) \right]^{1/2} = \xi(0) \left[T_c / (T_c - T) \right]^{1/2}$$

(for dirty metals) relies on coherence between quasiparticles extending over many grains ($\xi(T) \gg a$, where a is the grain size); this is satisfied if the transmissivity of the grain boundaries is sufficiently large (48).

Field and temperature measurements at which sample resistance was half the maximum value are shown in Figs. 1-4. The value of $|dH/dT|$ for the different samples as well as their geometric properties are summarized in Table 1.

Zero field resistance measurements are presented

FIGURES 1-4
MAGNETIC FIELD VERSUS TEMPERATURE
AT WHICH SAMPLE RESISTANCE IS HALF
ITS MAXIMUM VALUE

in Figs. 5-10. Measuring currents were 100 uA or less for the Nb samples, 40 uA for the NbN. Using smaller currents gave no shift in the resistance curves, and this was taken as an indication that non-ohmic behavior was not taking place. For the CryoCal and thermistor thermometers, the currents were chosen so as not to exceed the maximum power recommended by the manufacturers. With a 40 uA current, the best accuracy of the resistance measurements was 5 parts in 10^5 . Temperature resolution was better than .2 mK.

For the Nb samples, R_n was taken to be the flat portion of the R vs. T curve. Application of a perpendicular field showed no increase in the measured resistance. The resistance curves for the NbN samples had the same behavior as the Nb samples (see Fig. 11). There is a very small negative dR/dT for the NbN between 20° K and 30° K (of about .002 ohms/ $^\circ$ K). This increase in resistance with decreasing temperature is so small that it is not visible from the graph.

The magnetic field necessary to quench the superconductivity of the NbN was inaccessibly high. Using the technique of Kajimura and Mikoshiba (13), R_n can be determined from the data in an applied perpendicular field. According to the theory, in the 2-dimensional limit when

$$e/2c \xi^2(0)H \ll 1$$

(eq. (14)) a plot of $1/R$ vs. $1/H$ keeping T constant

FIG. 5. The Resistive Transition of Nb Sample 1

Fig.7. The Resistive Transition of Nb Sample 3

Fig. 8. The Resistive Transition of NbN 40a

Fig.9. The Resistive Transition of NbN 40b

Fig. 10. The Resistive Transition of NbN 40c

Fig.11. A Typical Nb Resistance Curve

gives a straight line with intercept at $1/R_n$.

Resistance measurements in a perpendicular field were taken for NbN 40b, and the results plotted in Figs. 12-16. The calculated values of R_n using the intercepts in the graphs are compared with the calculated values from a linear extrapolation of the measured temperature dependence of R above 20°K (in zero field) in Table 4. We notice that the Kajimura-Mikoshiba method gives a much bigger temperature dependence of R_n .

To go back to the question of thermistor magnetoresistance, Schaer (12) tested a thermistor from the same batch as the one used here, and found that at 4.2°K a field of 10 kG introduced a temperature error of less than 10°uK . However, the later paper by Schlosser and Munnings (42) indicated that the temperature shift at higher fields was much larger than previously believed. Using the same type of thermistor as we had for the control system sensor, their data showed that fields of up to 16 kG caused a negligible temperature shift at 8.9°K . As the field was increased, the shift ranged from $\approx -5^\circ\text{mK}$ at 30 kG to $\approx -23^\circ\text{mK}$ at 50 kG.

In the finite field data, the temperature was kept constant as the applied field was varied. Since the thermistor was used to control the temperature, any appreciable magnetoresistance would lead to serious errors. Fortunately in this case, at field values corresponding

Fig.12. $1/R$ versus $1/H$ for NbN 40b

Fig.13. $1/R$ versus $1/H$ for NbN 40b

Fig.14. $1/R$ versus $1/H$ for NbN 40b

Fig.15. $1/R$ versus $1/H$ for NbN 40b

Fig.16. $1/R$ versus $1/H$ for NbN 40b

to non-negligible temperature shifts, the resistance curve was also relatively flat in the temperature range under consideration. In Figs. 12-16, the straight line portions are virtually unaffected by thermistor magneto-resistance. As we move to the curved portions, we can compare the data for a given field at different temperatures and see that corrections are negligible. For instance, at 40 kG where the magnetoresistive temperature shift is $\approx 12^\circ\text{mK}$, the R values at 12.320°K and 12.392°K differ by .14%. As the field is increased, the measured resistances at different temperatures tend to converge.

The zero field data can be analyzed without the use of adjustable parameters by using the scheme of Testardi et. al. However, R_n must be determined independently, and for the NbN samples a definitive way of assigning correct values to R_n is difficult. (see Table 4).

Disregarding this problem for the moment, let us see what results from the analysis. By differentiating both sides of (13) and (16) with respect to T , a log-log plot of

$$\frac{1}{R^2} \frac{dR}{dT} - \frac{1}{R_n^2} \frac{dR_n}{dT}$$

versus

$$(R_n - R)/R$$

will give straight lines. The slope depends on the dimensionality of the sample, so a change in sample dimen-

sionality shows up as a change in the slope of the "log-log" plot.

The results are presented in Figs. 19-24. For R_n we used the flat portion of the resistive transition curve. In the NbN samples then, the temperature dependence of R_n is neglected. If the temperature dependence is assumed to be given by a linear extrapolation of the high temperature behavior of the resistive curve, the corrected "log-log" plot would be shifted to the right, but the change in the slope would be negligible.

The graphs exhibit straight line portions, but the slopes are different from the theoretical values of 2 or 3. Qualitatively the graphs look similar to those presented by Schaer (12).

An interesting result is the change in slope at a temperature which we denote by T_b . Note that (13) and (16) become equal when

$$d = 2\xi(T). \quad (20)$$

Calculating the coherence length corresponding to T_b , we find that $\xi(T_b)$ is approximately half of the film thickness for Nb samples 1 and 2 (Table 5); for the NbN samples, however, $\xi(T_b)$ is much less than the film thickness. It seems reasonable to infer that the grain size (rather than the film thickness) is the relevant parameter in the change in sample dimensionality. The magnetic field data for NbN40b supports this interpretation,

Fig.17. Log-Log Plot for Nb1

Fig.18. Log-Log Plot for Nb2

Fig.19. Log-Log Plot for Nb3

Fig.20a. Log-Log Plot for NbN 40a

Fig.20b. Log-Log Plot for NbN 40a

Fig.21a. Log-Log Plot for NbN 40b

Fig.21b. Log-Log Plot for NbN 40b

Fig.22a. Log-Log Plot for NbN 40c

Fig.22b. Log-Log Plot for NbN 40c

which will be discussed further later on. It seems that the effect of the granular structure is the reduction of the diffusional mean free path (45). It is also possible that the samples used here have high grain boundary resistances so that the assumption of having homogeneous superconductors becomes tenuous (49).

The comparison of the zero-field data with either the Aslamazov-Larkin or the Aslamazov-Larkin plus Maki-Thompson theories is shown in Tables 2 & 3. The data are fitted to the theories treating the transition temperature, width parameter, and pair-breaking parameter (for the Maki-Thompson model) as adjustable. Agreement between the measured and calculated values of R is better than 1%. For the Nb samples, the fitted T_c decreases monotonically with increasing R_n^{sq} ; for the NbN samples T_c increases monotonically with increasing R_n^{sq} . There seems to be no correlation between the pair-breaking parameter and R_n^{sq} .

The fitted width parameter using the A-L theory alone is larger than the calculated value. Including the M_T term improves the agreement. However, the fitted pair-breaking parameter is much smaller than the expected $(T_{co} - T_c)/T_{co}$, where T_{co} is the zero field transition temperature for the bulk specimen. The lack of one-to-one correspondence stems from the presence of two competing mechanisms (46): 1) decrease in T_c due to

pair breaking from the imaginary part of the electron-phonon interaction, and 2) increase in T_c from the real part of the interaction.

Another explanation for the shift in the transition temperature (from the bulk value) is the effect of an activated conduction mechanism (43). If the temperature dependent R_n is expressed as

$$R_n = R_0 \exp(A/T) \quad (21)$$

then the activation energy $k_B A$ is related to the shift in T_{co} by

$$T_{co}' - T_{co} \approx \frac{1}{2} A$$

provided that the dominant influence is the normal state activation conduction mechanism. On the other hand,

$$2\pi T_{co}' \approx A$$

if the activation energy were due to size quantization in small particles. No temperature dependence in R_n was observed for the Nb samples. For the NbN, the transition curve gives only a small T dependence ($\sim 0.003 \Omega/K$ between 20 and 30°K). Applying the Mikoshiba-Kajimura technique, the calculated values of R_n for NbN40b can be fitted to (21) with $R_0 \approx 2.0$ ohms and $A \approx 54^\circ K$. This activation energy is close to T_{co}' for bulk NbN, since $T_{co}' = 17^\circ K$ for the bulk sample (50).

The data in a magnetic field again show a change in the dimensionality of the sample (Figs. 12-16).

Fig.23. Experimental Excess Conductivity

-52-

Fig.24. Experimental Excess Conductivity -53-

For the departure from straight $1/H$ behavior to occur at the experimentally observed fields, a smaller film thickness (than the experimentally measured value) has to be used in the theoretical equations. This smaller film thickness is approximately equal to the effective thickness determined from the zero field data.

The excess conductivity obtained by subtracting out R_n (determined either from a linear extrapolation of the high temperature behavior or using the Kajimura-Mikoshiba model) is shown in Figs. 23 & 24. In Fig. 24 only the temperature dependence of the zero-field data is used. A simple linear extrapolation would give values of R_n smaller than the maximum measured resistances in the presence of finite fields. For comparison purposes, the graphs from the theory are presented in Figs. 25-26. No attempt is made for a quantitative fit, as thermometer magnetoresistance would have to be corrected for. In the Aoi model, only the curves corresponding to a temperature of 12.320°K (below the assumed critical temperature of 12.342°K) are presented. It gives the same result as the Usadel model, as expected. To use the Aoi equations above T_c would require a redefinition of H_{c2} for the excess conductivity to vary with temperature (keeping H fixed) in accordance with what was experimentally observed.

Note that if the experimental value of film thickness ($1611 \text{ \AA}$) is used in the theoretical equations, the

Fig.25. Excess Conductivity, Aoi Model -55-

Fig.26. Excess Conductivity, Usadel Model

3-dimensional behavior already predominates when the applied field is as small as 2 kG. This can be seen from Fig. 25, as well as by calculating

$$d\sqrt{2e\Delta H} \quad (22)$$

and remembering that (22) must be small compared to 1 for 2-dimensional behavior to be observed.

The Kajimura-Mikoshiba method for determining R_n yields excess conductivity curves that do not converge at high fields, unlike the behavior of the theoretical curves.

V. CONCLUSION

At best the agreement of the data with the theory is qualitative. Other sputtered samples of Nb, NbN, and NbC were tried. In all cases, attempts to make a quantitative fit were unsuccessful. Two recent articles (49), (50) may provide clues as to how the choices of samples could be improved. One crucial assumption in the analysis was that as long as $\xi(T) \gg a$, where a is some average grain size, the theory as developed for homogeneous superconductors would apply to polycrystalline samples. In this study, preference was given to samples with high resistivity because interest was primarily on observing the change in dimensionality. The higher the resistivity, the shorter the electronic mean free path and the wider the temperature range over which fluctuation effects become manifest.

A detailed study of the effects of granular structure on the superconducting transition (49) indicates that films of lower resistivity would have been more suitable here. The differences in the transition temperatures observed for several granular aluminum films were attributed not to wider dispersion of grain sizes in some of the films (i.e. greater inhomogeneity), but rather to the building up of insulating barriers between grains and a concomitant decrease in T_c and broadening of the transition.

The second paper (50) deals with the influence of Pauli paramagnetism on the critical field H_{c2} for NbN films prepared by sputtering. The authors showed that H_{c2} depended not so much on critical temperature and film thickness as on resistivity ρ_n . If ρ_n was less than 200 $\mu\Omega\text{cm}$, then the effect of Pauli paramagnetism on H_{c2} was not appreciable. Higher resistivity led to a decrease in H_{c2} ; in these films the transmissivity of the grain boundaries was low.

With the above limitations in mind, it is interesting to see the deviation from straight $1/H$ behavior at high fields. The curvature seemed to indicate that a transition to a three dimensional regime had taken place. However, substituting the measured sample parameters into the equations gave a much different value of the magnetic field at which the transition should occur.

Another result that emerged from the study was the difficulty of assigning a definitive value for R_n for the NbN samples. A simple extrapolation of the high temperature zero field data yielded values of R_n that were smaller than the maximum values recorded in a perpendicular magnetic field. Using the magnetic field data gave a strongly temperature dependent R_n which could be interpreted as being due to an activated conduction mechanism. However, subtracting out the normal resistance determined in this manner yielded curves for the excess conductivity

that differed from the theoretical ones, which tended to converge at high fields.

The theory of the critical behavior of small, isolated and coupled superconducting grains has been developed (52),(53). What is needed is a description of granular samples considered as aggregates of weakly coupled grains.

With a better choice of samples (i.e., so the theory for homogeneous superconductors would be a close approximation) further experiments could investigate the angular dependence on magnetic field, similar to what has been done for Al films (51). Higher fields could be tried, and temperature controllers devised that do not have the magnetoresistance limitation imposed by the thermistor used here.

TABLE 1
SAMPLE CHARACTERISTICS

Sample	Width (in.)	Length (in.)	Thickness (A)	dH/dT ($\times 10^3 g/^{\circ}K$)
Nb1	.100	.217	10,000	3.25
Nb2	.054	.167	10,000	3.02
Nb3	.082	.233	5,000	3.57
NbN40a	.048	.164	2,000	10.6
NbN40b	.076	.113	1,611	13.9
NbN40c	.077	.127	2,000	12.0

TABLE 2
NB SAMPLE PARAMETERS

	Nb1	Nb2	Nb3
R_n (ohms)	.351	.750	1.574
R_n^{sq}	.162	.243	.553
τ_o (calc)	2.46E-6	3.69E-6	8.40E-6
fitted parameters (A-L theory)			
T_c (°K)	8.584	8.558	8.303
$\xi(0)^*$	107 Å	111	108
τ_o	1.99E-5	4.02E-5	3.33E-5
fitted parameters (A-L & M-T theory)			
T_c (°K)	8.585	8.554	8.303
$\xi(0)^*$	107 Å	111	108
τ_o	2.798E-6	5.761E-6	3.232E-6
δ	6.498E-6	3.825E-8	8.661E-7

*calculated using $\xi^2(0) = \frac{\phi_o}{2\pi T_c} \cdot \frac{1}{dH/dT}$

TABLE 3
NbN SAMPLE PARAMETERS

	NbN40a	NbN40b	NbN40c
R_n (ohms)	28.620	22.632	20.740
R_n^{sq}	8.43	15.2	12.6
ζ_o (calc)	1.28E-4	2.32E-4	1.92E-4
fitted parameters (A-L theory)			
T_c (°K)	10.566	12.342	11.622
$\xi(0)^*$	53.5 Å	43.2	47.9
ζ_o	1.971E-4	2.701E-4	2.685E-4
fitted parameters (A-L & M-T theory)			
T_c (°K)	10.549	12.321	11.593
$\xi(0)^*$	53.5 Å	43.2	48.0
ζ_o	1.325E-5	3.045E-5	1.326E-5
δ	7.428E-15	3.957E-15	1.406E-15

*calculated using $\xi^2(0) = \frac{\phi_o}{2\pi T_c} \cdot \frac{1}{dH/dT}$

TABLE 4
COMPARISON OF CALCULATED R_n VALUES FOR NbN40b

T (°K)	R_n (Kaj-Mik) (ohms)	R_n (Ext) (ohms)
12.320	24.213	22.6639
12.354	23.941	22.6638
12.392	23.770	22.6637
12.411	23.474-	22.6636
12.479	23.084	22.6635

Kaj-Mik - using Kajimura-Mikoshiba model of taking
intercept of perpendicular field data

Ext - using linear extrapolation of high T data

TABLE 5
COHERENCE LENGTH WHEN SLOPE OF LOG-LOG PLOT CHANGES

Sample	T_c (°K)	T_B (°K)	$\xi(0)$ (Å)	$\xi(T_B)$ (Å)	d (Å)
Nb1	8.584	8.588	107	4,954	10,000
Nb2	8.558	8.562	111	5,139	10,000
Nb3	8.303	8.303	108	4,018	5,000
NbN40a	10.566	10.984	53.5	269	2,000
NbN40b	12.342	12.864	43.2	221	1,611
NbN40c	11.622	12.397	47.9	186	2,000

Appendix A.

In view of the different definitions used for the Fourier transform, we illustrate the calculation of the Fourier component of the current. Consider only one variable y .

Define

$$\psi(y) = \int_{-\infty}^{\infty} \psi_k e^{-iky} dk$$

$$\psi_k^* = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(y) e^{iky} dy$$

The Fourier component of the current

$$J_y = \frac{ie}{m} \left\{ \psi^*(y) \frac{\partial}{\partial y} \psi(y) - \psi(y) \frac{\partial}{\partial y} \psi^*(y) \right\}$$

is therefore

$$J_{k''} = \frac{e}{2\pi m} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_k^* \psi_{k'} (k' - k) \\ \cdot e^{i(k - k' + k'')y} dk' dk dy$$

$$= \frac{e}{m} \int_{-\infty}^{\infty} k'' \psi_{k' - k''}^* \psi_{k'} \frac{dk'}{2\pi}$$

$$J_k = \frac{e}{m} \int_{-\infty}^{\infty} k \psi_{k - k/2}^* \psi_{k' + k/2} \frac{dk'}{2\pi}$$

Appendix B.

We note that the eigenvalues

$$E_{n,q} = \frac{q^2}{2m} + (n + \frac{1}{2})w_H$$

become a continuous function of n in the limit as H goes to zero. Hence the summation over n can be replaced by an integration.

For the thin film case, resolve the integrand into partial fractions

$$\frac{(n+1)}{(n+\frac{1}{2}+a)(n+\frac{3}{2}+a)(n+1+a)} = \frac{1-2a}{n+\frac{1}{2}+a} + \frac{4a}{n+1+a} - \frac{1+2a}{n+\frac{3}{2}+a}$$

Integration with respect to n yields

$$G_{2D}(a) = 2a \left\{ (1-2a) \ln(n+\frac{1}{2}+a) - (1+2a) \ln(n+\frac{3}{2}+a) + 4a \ln(n+1+a) \right\}$$

The upper limit ($n \rightarrow \infty$) gives zero. At the lower limit, expand the logarithms to get

$$\begin{aligned} G_{2D}(a) &= 2a \left\{ (1-2a) \left(\frac{n+\frac{1}{2}}{a} - \frac{(n+\frac{1}{2})^2}{2a^2} + \dots \right) \right. \\ &\quad - (1+2a) \left(\frac{n+\frac{3}{2}}{a} - \frac{(n+\frac{3}{2})^2}{2a^2} + \dots \right) \\ &\quad \left. + 4a \left(\frac{n+1}{a} - \frac{(n+1)^2}{2a^2} + \dots \right) \right\} \\ &= 1, \end{aligned}$$

taking the lower limit. For finite H but small a ,

taking terms to second order in a gives

$$\begin{aligned} G_{2D}(a) &= -2a \{ \ln(1+2a) - \ln(3+2a) - 2a \ln 3 + 4a \ln 2 \} \\ &= 2a \{ \ln 3 - a \left(\frac{11}{6} + 2 \ln 3 - 4 \ln 2 \right) \} \end{aligned}$$

For the three-dimensional case, integration gives

$$G_{3D}(a) = 4a^{\frac{1}{2}} \left\{ 2 \left[\frac{1}{3}(a+n+\frac{1}{2}) - (a+\frac{1}{2}) \right] \sqrt{a+n+\frac{1}{2}} \right. \\ + 2 \left[\frac{1}{3}(a+n+\frac{3}{2}) - (a+\frac{3}{2}) \right] \sqrt{a+n+\frac{3}{2}} \\ - 4 \left[\frac{1}{3}(a+n+1) - (a+1) \right] \sqrt{a+n+1} \\ + 2\sqrt{a+n+\frac{1}{2}} + 2\sqrt{a+n+\frac{3}{2}} \\ \left. - 4\sqrt{a+n+1} \right\}$$

The upper limit gives zero. At the lower limit, the last three terms expanded in a add up to zero:

$$(a+n)^{\frac{1}{2}} \left\{ 1 + \frac{1}{2} \cdot \frac{\frac{1}{2}}{n+a} - \frac{1}{8} \left(\frac{\frac{1}{2}}{n+a} \right)^2 + \dots + 1 + \right. \\ \left. \frac{1}{2} \cdot \frac{3/2}{n+a} - \frac{1}{8} \left(\frac{3/2}{n+a} \right)^2 + \dots - 2 - \frac{1}{n+a} \right. \\ \left. \frac{1}{8} \left(\frac{1}{n+a} \right)^2 + \dots \right\}$$

We're left with

$$G_{3D}(a) = 4a^{\frac{1}{2}} \left\{ 2 \left(\frac{n}{3} - \frac{2a}{3} - \frac{1}{3} \right) \sqrt{n+a} \left[1 + \frac{1}{2} \cdot \frac{\frac{1}{2}}{n+a} \right. \right. \\ \left. \left. - \frac{1}{8} \left(\frac{\frac{1}{2}}{n+a} \right)^2 + \dots \right] + 2 \left(\frac{n}{3} - \frac{2a}{3} - 1 \right) \cdot \right. \\ \left. \sqrt{n+a} \left[1 + \frac{1}{2} \cdot \frac{3/2}{n+a} - \frac{1}{8} \left(\frac{3/2}{n+a} \right)^2 + \dots \right] \right. \\ \left. - 4 \left(\frac{n}{3} - \frac{2a}{3} - \frac{2}{3} \right) \sqrt{n+a} \left[1 + \frac{1}{2} \cdot \frac{1}{n+a} \right. \right. \\ \left. \left. - \frac{1}{8} \left(\frac{1}{n+a} \right)^2 + \dots \right] \right\} \\ = 1$$

when evaluated at the lower limit.

For finite H but small a :

$$G_{3D} = 4a^{\frac{1}{2}} \left\{ 2 \left(-\frac{2a}{3} + \frac{2}{3} \right) \sqrt{a+n+\frac{1}{2}} \right.$$

$$\begin{aligned}
 & + 2\left(-\frac{2a}{3}\right)\left(a+n+\frac{3}{2}\right)^{\frac{1}{2}} \\
 & - 4\left(-\frac{2a}{3}+\frac{1}{3}\right)\left(a+n+1\right)^{\frac{1}{2}} \Big\} \\
 = & 4a^{\frac{1}{2}} \Big\{ 2^{\frac{1}{2}}\left(-\frac{2a}{3}+\frac{2}{3}\right)\left(1+a-\frac{a^2}{2}+\dots\right) \\
 & + 2^{\frac{1}{2}}\left(-\frac{2a}{3}\right)\left(3^{\frac{1}{2}}+\dots\right) \\
 & + 2\left(-\frac{4a}{4}-\frac{2}{3}\right)\left(1+\frac{a}{2}+\dots\right) \Big\} \\
 \approx & \frac{16}{3\sqrt{2}} \left(2^{\frac{1}{2}}-1\right)a^{\frac{1}{2}}
 \end{aligned}$$

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REFERENCES

1. V.L. Ginzburg, Sov. Phys. - Solid State 2,1824(1960).
2. R.A. Ferrell and H. Schmidt, Phys. Letters 25A,544 (1967).
3. L.G. Aslamazov and A.I. Larkin, Sov. Phys. - Solid State 10,875; Phys. Letters 26A,238(1968).
4. H. Schmidt, Z. Physik 216,336(1968).
5. E. Abrahams and J.F. Woo, Phys. Letters 27A,117 (1968).
6. S. Marcelja, Ph.D. thesis, The University of Rochester (1970)(unpublished).
7. Patrick A. Lee and Subodh R. Shenoy, Phys. Rev. Lett. 28,1025(1972).
8. R.E. Glover, Phys. Letters 25A,542(1967).
9. R.O. Smith, B. Serin, and E. Abrahams, Phys. Letters 28A,224(1968).
10. M. Strongin, O.F. Kammerer, J. Crow, R.S. Thompson, and H.L. Fine, Phys. Rev. Lett. 20,922(1968).
11. L.R. Testardi, W.A. Reed, P.C. Hohenberg, W.H. Haemmerle, and G.F. Brennert, Phys. Rev. 181,800 (1969).
12. F.M. Schaer, Ph.D. thesis, The University of Minnesota (1971); A.M. Goldman, F.M. Schaer, and L. Toth, Solid State Commun. 2,557(1971); A.M. Goldman, F.M. Schaer, L.E. Toth, and J. Zbasnik, Physica 55,492(1971)(Proc. Int. Conf. on the Science of Superconductivity, Stanford, 1969).
13. K. Kajimura and N. Mikoshiba, J. Low Temp. Phys. 4,331(1971).
14. T. Aomine and L. Rinderer, J. Low Temp. Phys. 6,323 (1972).
15. B. Serin, R.O. Smith, and T. Mizusaki, Physica 55, 224(1971)(Proc. Int. Conf. on the Science of Superconductivity, Stanford, 1969).
16. N.H. Sze and G.T. Meaden, Phys. Can. 27,43(1971) (abstract #65690, Phys. Abs. 74,3701(1971)).

17. W.E. Masker and R.D. Parks, Phs. Rev. B 1,2164 (1970).
18. W.W. Webb and R.J. Warburton, Phys. Rev. Lett. 20, 461(1968).
19. J.W. Cook, Jr., M.J. Skove, E.P. Stillwell, R.S. Thompson, Phys. Letters 32A,445(1970).
20. R.S. Newbower, M.R. Beasley, and M. Tinkham, Phys. Rev. B 5,864(1972).
21. A.F. Mayadas and R.B. Laibowitz, Phys. Rev. Lett. 28,156(1972).
22. G.A. Thomas and R.D. Parks, Phys. Rev. Lett. 27, 1276(1971).
23. P. Fulde and K. Maki, Phys. Kondens. Materie 8, 371(1969).
24. H. Schmidt, Phys. Letters 27A,658(1968).
25. H.L. Testardi, Phys. Letters 35A,33(1971).
26. K. Maki, Progr. Theor. Phys. 40,193(1968).
27. R.S. Thompson, Phys. Rev. B 1,327(1970).
28. A. Schmid, Z. Physik 215,210(1968); Z. Physik 243,346(1971).
29. J. Keller and V. Korenman, Phys. Rev. Lett. 27, 1270(1971); Phys. Rev. B 5,4367(1972).
30. H. Winter, Phys. Letters 35A,303(1971).
31. B.R. Patton, Phys. Rev. Lett. 27,1270(1971).
32. H. Takayama and K. Maki, Progr. Theoret. Phys. 46,42(1971).
33. K.D. Usadel, Z. Physik 227,260(1969).
34. R.S. Thompson, Physica 55,296(1971)(Proc. Int. Conf. on the Science of Superconductivity, Stanford, 1969).
35. G.E. Clarke and D.R. Tilley, J. Phys. C. 3,2448(1971).
36. H.H. Mikeska and H. Schmidt, Z. Physik 230,239(1970).

37. P. Fulde and K. Maki, Z. Physik 238,233(1970).
38. E. Abrahams, R.E. Prange, and M.J. Stephen, Physica 55,230(1971)(Proc. Int. Conf. on the Science of Superconductivity, Stanford, 1969).
39. D.R. Tilley and J.B. Parkinson, J. of Physics C (Solid State Physics) 2,2175(1969).
40. L.D. Landau and E.M. Lifshitz, Statistical Physics (Oxford: Pergamon, 1959).
41. P.G. de Gennes, Superconductivity of Metals and Alloys (New York: W.A. Benjamin, Inc., 1966).
42. W.F. Schlosser and R.H. Munnings, Rev. Sci. Instr. 40,1359(1969); paper presented at Fifth Temperature Symposium (1971).
43. Myron Strongin, R.S. Thompson, O.F. Kammerer, and J.E. Crow, Phys. Rev. B 1,1078(1970).
44. R. Koepke and G. Bergmann, Z. Physik 242,31(1972).
45. J.H.P. Watson, (abstract #37086, Phys. Abs. (1971)).
46. R.A. Craven, G.A. Thomas, and R.D. Parks, Phys. Rev. B 7,157(1973).
47. G. Luders and K.-D. Usadel, The Method of the Correlation Function in Superconductivity Theory, Vol. 56 of Springer Tracts in Modern Physics, G. Hohler, ed. (Berlin: Springer-Verlag, 1971).
48. B. Abeles, Roger W. Cohen, and W.R. Stowell, Phys. Rev. Lett. 18,902(1967).
49. G. Deutscher, H. Fenichel, M. Gershenson, E. Grunbaum and Z. Ovadyahu, J. Low Temp. Phys. 10,231(1973).
50. Tsutomu Yamashita, Shogo Kitahara, and Yutaka Onodera, J. Appl. Phys. 43,4749(1972).
51. T. Aomine and L. Rinderer, J. Low Temp. Phys. 10,527 (1973).
52. J.P. Hurault, K. Maki and M.T. Beal-Monod, Phys. Rev. B 3,762(1971).
53. G. Deutscher, Phys. Lett. 35A,28(1971).