APEX
448

MASTER

THE ACTIVITY OF THE FISSION PRODUCTS OF U²³⁵

Submitted to
General Electric Company
Aircraft Nuclear Propulsion Department
Cincinnati 15, Ohio

Submitted by
Internuclear Company
7 N. Brentwood Blvd.
Clayton 5, Missouri

OCTOBER 31, 1958

GENERAL ELECTRIC
ATOMIC PRODUCTS DIVISION
AIRCRAFT NUCLEAR PROPULSION DEPARTMENT

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

115-001

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor the Air Force, nor any person acting on behalf of the Commission or the Air Force:

- A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above "person acting on behalf of the Commission or Air Force" includes any employee or contractor of the Commission or Air Force to the extent that such employee or contractor prepares, handles, or distributes, or provides access to, any information pursuant to his employment or contract with the Commission or Air Force.

Printed in USA. Price \$2.25. Available from the

Office of Technical Services
U.S. Department of Commerce
Washington 25, D.C.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

APEX-448

UC-34 Physics and
Mathematics
TID-4500 (14th Edition)

**THE ACTIVITY OF THE
FISSION PRODUCTS OF U^{235}**

SUBCONTRACT AT-118

**W. E. Knabe
G. E. Putnam
INTERNUCLEAR COMPANY**

October 31, 1958

United States Air Force

Contract No. AF 33(600)-38062

United States Atomic Energy Commission

Contract No. AT (11-1)-171

GENERAL ELECTRIC

**AIRCRAFT NUCLEAR PROPULSION DEPARTMENT
ATOMIC PRODUCTS DIVISION**

Cincinnati 15, Ohio

**Published by
Technical Publications Sub-Section
January 1959**

.

.

.

44

TABLE OF CONTENTS (continued)

	<u>Title</u>	<u>Page</u>
5.0	Average Gamma Energy Group Cross Sections for Photoneutron Production with D and Be . . .	64
6.0	Effect of Neutron Flux Magnitude on the Decay Rate	67
6.1	Introduction	67
6.2	The Effect of Neutron Absorption in the Unstable Nuclides	68
6.3	The Effect of Neutron Absorption in the Stable Nuclides	70
7.0	Effects of Neutron Energy	73
7.1	Introduction	73
7.2	Mass Yield Distribution.	74
7.3	Charge Distribution.	75
8.0	Fission Product Decay Rates Following a Reactor Shutdown.	77
9.0	Comparison of Gamma Energy Decay Rates According to Various Sources.	79

APPENDIX

	Abstracts from the Literature Search.	82
--	---	----

LIST OF TABLES

	<u>Title</u>	<u>Page</u>
2.1	The Energy Group Definitions	11
2.3.1	The Values of E_g and E_{g-1} (in Mev)	17
2.3.2	Photons/fiss-sec Versus Time After an Instantaneous Fission	22-26
3.2	Beta Decay from an Instantaneous Event of Fissions	42-43
3.3.a	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 1$ hr)	46
3.3.b	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10$ hrs)	47
3.3.c	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10^2$ hrs)	48
3.3.d	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10^3$ hrs)	49
3.3.e	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10^4$ hrs)	50
3.3.f	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10^5$ hrs)	51
3.3.g	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = 10^7$ hrs)	52
3.3.h	Beta Energy Decay Rate Versus Time After Shutdown ($\tau = \infty$ hrs)	53
5.0	The Average $\gamma_{,n}$ Cross Section of D and Be for Each Gamma Energy Group	65

LIST OF FIGURES

	<u>Title</u>	<u>Page</u>
2.A	Photons/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $1 - 10^4$ sec)(Groups 1, 2, 3, 4, 5, 6, VII, T)	27
2.B	Photons/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $10^4 - 10^8$ sec)(Groups 1, 2, 3, 4, 5, 6, VII, T).	28
2.C	Photons/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $1 - 10^4$ sec)(Groups 7, 8, 9, 10, 11, 12).	29
2.D	Photons/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $10^4 - 10^8$ sec)(Groups 7, 8, 9) .	30
2.E1	Gamma Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $1 - 10^4$ sec)(Groups 2, 4, 6, T). .	31
2.E2	Gamma Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $1 - 10^4$ sec)(Groups 1, 3, 5, VII).	32
2.F	Gamma Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $10^4 - 10^8$ sec)(Groups 1, 2, 3, 4, 5, 6, VII, T).	33
2.G	Gamma Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $1 - 10^4$ sec)(Groups 7, 8, 9, 10, 11, 12).	34
2.H	Gamma Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission (Time Range $10^4 - 10^8$ sec)(Groups 7, 8, 9) .	35

LIST OF FIGURES (continued)

	<u>Title</u>	<u>Page</u>
2.I	Average Energy per Photon Versus Time After an Instantaneous Fission (Time Range 1 - 10^7 sec)(Groups 1, 2, 3, 4, 5, 6, VII, T)	36
2.J	Average Energy per Photon Versus Time After an Instantaneous Fission (Time Range 1 - 10^7 sec)(Groups 7, 8, 9, 10, 11, 12)	37
3.A	Beta Energy and Beta Particle Decay Rate Following a Delta Function of Fissions at $t = 0$ (Time Range 1 - 10^4 sec)	54
3.B	Beta Energy and Beta Particle Decay Rate Following a Delta Function of Fissions at $t = 0$ (Time Range 10^4 - 10^8 sec)	55
3.C	Beta Energy per Beta Particle Versus Time After Fission (Time Range 1 - 10^7 sec)	56
4.A	Fraction of Beta Energy Converted into Bremsstrahlung Versus Atomic Number Z.	62
4.B	Fraction of Beta Energy Converted into Bremsstrahlung Versus Beta Particle Energy for $Z = 92$	63
5.A	γ, n Cross Section of D and Be Versus Photon Energy	66
9.A	Comparison of Gamma Energy Decay Rates	81

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the kind assistance rendered by: J. F. Perkins in providing supplementary data from the very thorough fission product analyses conducted by him and R. W. King; by R. W. Peelle in providing reports on the experimental results obtained by him, F. C. Maienshein, W. Zobel, and T. A. Love; and by John Krase and his group at General Electric Company, Aircraft Nuclear Propulsion Department in providing numerous helpful suggestions and references.

1.0 INTRODUCTION

Recent data from both experimental and analytical sources enable the prediction of beta and gamma fission product activity with much greater accuracy than previously possible for time ranges of one second to a few hours after a given history of fissions in a nuclear reactor. While there have existed, for some time, several complete and adequate treatments of beta and gamma decay activity for times over an hour after fission or reactor shutdown, it is only recently that data have been obtained enabling treatment of activities for shorter periods of time. The older data are quite sufficient for estimations of gamma shielding requirements, activities in nuclear fuel which is to be processed, power generation capabilities of gross fission products, and the amounts of specific long-lived fission product nuclides produced in a reactor. Activities at shorter times (up to a few hours after shutdown) must be known, also, in order to allow estimation of such effects as the heat generation in the fuel, moderator, structural material, etc. of a reactor during reactor operation and immediately following the start of a reactor shutdown or scram.

With regard to the general problem of heat generation due to fission products in components of either operating or shutdown reactors it is usually most important to know, as a function of time:

- a) The energy distribution of the fission product gammas
- b) The energy release rate of the fission product gammas
- c) The energy release rate of the fission product betas

Each of these can be easily computed (provided that the time function of reactor fission rate, the total reactor operating time, and the time after shutdown are known) if the beta energy release rate and the energy and photon number release rates of a sufficient number of gamma energy groups are known, as a function of time, for an instantaneous fission event. This report presents these release rate data in both tabular and graphical form, and the methods for computing (a), (b), and (c), above, are outlined in detail.

The data given for beta and gamma decay rates pertain directly to the thermal fission of U^{235} . The possible effects of neutron energy spectra, other than thermal, and neutron flux level upon the fission product decay rates were examined and the results are reported.

The importance of bremsstrahlung caused by beta particles from fission products and the effect of photoneutron production due to the photonuclear reactions of the fission product gammas with D or Be were studied. Methods of evaluating these effects in the reactor are briefly described.

2.0 GAMMA ENERGY RELEASE

2.1 Introduction

The gamma energy release from fission products between 1 and 10^8 seconds after an instantaneous fission event is considered. Curves and tables are presented for an instantaneous fission event that show the energy and photon number release rates per fission as a function of time. Also included are plots of the time behavior of the average energy per photon and the average energy per photon in each of the assumed individual energy groups. The calculation of the fission product gamma energy release for a reactor operation period τ , from the basic data herein presented for an instantaneous event of fissions, can be easily performed as outlined in the last chapter of this section.

Two recent sources of information are used and the data from these sources are combined. For short times after fission (1-1550 sec), experimental results of Oak Ridge National Laboratory* are available, whereas, for long period decay, computed decay curves obtained by Perkins and King are adopted from their publication**. The two sets of data are in good agreement so that they can be easily combined: Beyond 1550 sec, the experimental decay curves are extrapolated in such a way as to join smoothly the calculated curves at 10^4 sec from which point the calculated data are used directly. The fact that the two sets of data disagree to some extent in the region where the two time ranges overlap, i.e., between 100 and 1550 sec, is due to the incomplete data available for computing the activity of short-lived fission products. As a result of this, the computed curve lies below the experimental one.

The photons are classified, first, into seven energy groups covering the energy range between .1 and 5.5 Mev. The group limits are the same as given by Perkins and King. In addition, Group VII of the Perkins and King data is split up into six individual sub-groups so that actually twelve energy groups are considered. The subdivision of Group VII provides a detailed picture of the decay characteristics in the upper half of the energy range of interest. The group limits are listed in the following table.

* See references #17a, 17b, and 26

** See reference #25

Table 2.1
The Energy Group Definitions

12-Group Subdivision	Perkins & King Groups	Energy Range (in Mev)	Average Energy (in Mev)
1	I	0.1 - 0.4	0.3
2	II	0.4 - 0.9	0.63
3	III	0.9 - 1.35	1.10
4	IV	1.35 - 1.8	1.55
5	V	1.8 - 2.2	1.99
6	VI	2.2 - 2.6	2.38
7-12	VII	2.6 - 5.5	Varies between 3.5 and about 3.1
	(Group VII Subgroups)		
7	VII, a	2.6 - 3.0	2.75
8	VII, b	3.0 - 3.5	3.25
9	VII, c	3.5 - 4.0	3.7
10	VII, d	4.0 - 4.5	4.22
11	VII, e	4.5 - 5.0	4.70
12	VII, f	5.0 - 5.5	5.25

2.2 Discussion of the Curves and Results

2.2.1 Photon Number Release Rates

Figure 2.A shows, for an instantaneous event of fissions, the photon number release rate per fission as a function of time. For times between 1.75 and 1550 sec after fission, the curves are obtained by numerical integration of the ten energy spectrum curves (one time point per spectrum integral) representing the experimental results of Oak Ridge National Laboratory. Since no information about the decay characteristics of Group 1 is available for this time range, it is assumed that the number release rate for this group is about half of that of Group 2 throughout this interval. It is found that, by this assumption, the corresponding curve joins smoothly the curve obtained from Perkins and King's results at 10^4 sec. This indicates that no severe discrepancies are expected to result from this choice.

The curves from Figure 2.A are continued in Figure 2.B for times after fission between 10^4 and 10^8 sec. They are calculated, for this time range, directly from Perkins and King's results.

In order to obtain the photon number decay rates from the calculated energy release rates, it is necessary to know the time behavior of the average energy per photon for each of the individual groups. It is found that the average energies of the twelve groups are effectively constant in time and the results are listed in Table 2.1. Dividing, for each group, the energy release rate by the average energy as given in Table 2.1, one obtains the number release rate. For times after fission from 10^4 to 10^8 sec the energy release rates of Perkins & King are used directly.

The curve labeled with T in Figures 2.A and 2.B is the sum of Groups 1 to 12 and represents the total photon number release rate per fission. The area below this curve gives the total number of photons per fission emitted between 1 and 10^8 sec; its value is found to be 6.4 photons/fission. Group VII (the sum of Groups 7 through 12) is also shown in Figures 2.A and 2.B.

Figures 2.C and 2.D show the photon release rate for Groups 7 to 12 (the six subgroups of Group VII). In the time interval 1-1550 sec, the curves are obtained as outlined above from the Oak Ridge data. The method of extrapolation beyond 1550 sec is discussed in Section 2.2.2. It is seen from Figure 2.C that Groups 10, 11, and 12 can be completely neglected from about 10^4 sec on. Therefore, only three curves (Groups 7, 8, and 9) are plotted in Figure 2.D.

2.2.2 Gamma Energy Release Rates

Figures 2.E and 2.F show the energy release rates for Groups 1-6 as well as the total energy release rate, marked "T", and the sum of Groups 7-12 (Perkins & King Group VII).

Except for Group 1, the curves are obtained for short period decay from the Oak Ridge spectrum curves by numerical integration of

$$\int_{E_g^l}^{E_g^u} n(E,t) E dE \quad \left(\frac{\text{Mev}}{\text{fiss. sec}} \right),$$

where E_g^l and E_g^u are the lower and upper limits, respectively, of the g^{th} group, and $n(E,t)$ is the number of photons/fiss-sec per unit energy at time t after the instantaneous fission event. In Group 1, the average energy is assumed to be 0.3 Mev, by which assumption the curve for Group 1 in Figure 2.E2 is gained directly (by multiplication) from

the corresponding curve for the number release rate in Figure 2.A. As is done for the number release rates, the energy release rate curves are fitted by eye to the ten calculated points in the range $1 \leq t \leq 1550$ sec, and are made to join the corresponding curves of Perkins and King at 10^4 sec. For $10^4 \leq t \leq 10^8$ sec, Perkins and King's curves are simply replotted (Figure 2.F).

To serve as a check, the curve marked "T", representing the total gamma energy release rate per fission, has been integrated numerically between 1 and 10^8 sec. The result, $E_T = 6.6$ Mev/fission, may be compared with Table I in reference 26. Therein, E_T is given as $5.9(\pm 0.8)$ Mev/fission for the (same) time range 1- 10^8 sec and the energy range 0.28-5.0 Mev. The two results are based on the same sources of data so that they can be compared directly and they should be equal for the same energy range. If the energy intervals 0.1-0.28 Mev and 5-5.5 Mev are included, as is done in this report, a comparison with other data listed in the above-quoted table indicates that the value of about 5.9 Mev/fission should be raised by an amount which would bring the result close to the value herein obtained by numerical integration, 6.6 Mev/fission. Deviations from this number due to different handling of the original data in the time interval between 1550 and about 10^4 sec, can be neglected.

This is, of course, only an internal check, mainly with respect to the assumptions upon Group 1, and cannot be considered as a proof of the validity of the result. Such a validity, however, is indicated by the good agreement between the two independent sets of experimental and computed data with one another. The combined result checks with the 6 Mev/fission, a value estimated and commonly accepted by many authors, within the error limits of such an estimate.

In Figures 2.G and 2.H the energy release rates for the six groups 7 -12 (Group VII) are plotted.

For short period decay, the curves are determined in the same way as previously outlined, namely, by numerical integration over the appropriate energy ranges of the energy spectra.

For times $t > 1550$ sec the following extrapolation method is employed. The ratios among the ordinate values of Groups 7, 8, and 9 are assumed to remain unchanged from 1550 sec upwards. On the other hand, the sum of Groups 7, 8, and 9 must equal the value for Group VII in Figures 2.E1, 2.E2, and 2.F at each time point (with a small correction for Groups 10, 11, and 12 which are seen to fall off very

steeply so that they need no longer be considered singly and can be entirely dropped beyond 10^4 sec). Thus, the energy release rate curves for Groups 7, 8, and 9 are determined.

On the grounds of the above assumption, Groups 8 and 9 are probably overestimates of the corresponding energy release rates beyond 10^4 or 10^5 sec; consequently Group 7 is probably underestimated by an equivalent amount, but even if the extrapolated parts of the curves for Groups 7, 8, and 9 deviate very highly from the actual behavior of the corresponding subgroups, the validity of their total is conserved. Moreover, in the neighborhood of 10^4 - 10^5 sec after fission, the energy release rate of Group VII declines to values which are at least one order of magnitude smaller than those of any of the other groups, so that the knowledge of its detailed structure is no longer of much importance.

By dividing the energy release rate by the respective average energy in each individual group--7, 8, or 9--one obtains the corresponding values for the photon number release rate curves used for the extrapolation of the latter in Figures 2.C and 2.D. The average energies of Groups 7, 8, and 9 are assumed to remain constant in the considered time range, as is indicated by a straight-line extrapolation beyond 1550 sec in the plot of the average energy versus time (Figure 2.J).

The superposition of these photon number release rate curves of the groups gives the extrapolation of Curve VII in Figures 2.A and 2.B.

2.2.3 Average Energy per Photon

In Figure 2.I the average energy per photon of each principal group is plotted versus the time after fission. The ten time point values for each group between 1.75 and 1550 sec are calculated from the Oak Ridge spectrum curves according to

$$\overline{E(t_k)} = \frac{\int_{E_g^l}^{E_g^u} E_g^n(E, t_k) E dE}{\int_{E_g^l}^{E_g^u} E_g^n(E, t_k) dE} \quad \left(\frac{\text{Mev}}{\text{photon}} \right)$$

The average energy per photon for a given group is observed to be constant in the time range, 1-1550 sec, except for Group VII. Beyond 1550 sec, the same average energies as for shorter times are assumed for Groups 2 through 6. This is justified only because the changes of the average energy within the relatively narrow energy intervals (average group width is about 0.4 Mev) are probably not very appreciable. For Group 1, for which no information is available from the Oak Ridge spectrum curves, the average energy is assumed to be 0.3 Mev.

The average energy for Group VII, which is not constant in time, is analyzed in more detail. Figure 2.J shows the average energies for the six subgroups of Group VII. As seen from the diagram, these values are constant in time up to 1550 sec and they are assumed to remain unchanged throughout the whole time range of interest.

If one divides the energy release rates for Groups 7, 8, and 9 in the extrapolation range by the respective average energies, one obtains the number release rates of the subgroups which, in turn, total up to the photon number release rate of Group VII in the range of extrapolation. With this information, the extrapolated curve for the average energy of Group VII is obtained as shown in Figure 2.I. For long period decay, the average energy of Group VII is probably overestimated as a consequence of the nature of the assumption for extrapolating Groups 7, 8, and 9. After about 3×10^3 sec, the extrapolated curve remains constant in time with a value of about 3.1 Mev/photon. The error is not likely to exceed +10%.

The ratio of the total energy to the total number release rates, at any time t after fission, gives the average energy per photon at that time. This ratio is represented by the curve marked "T" in Figure 2.I. At long times after fission it is not very sensitive to the accuracy with which the subgroups in Group VII are treated in particular. This is because of the rapid decay of this group and, consequently, its small contribution to the total at long times after fission.

2.3 Applications of the Basic Data

2.3.1 Definitions

The basic data are represented by two functions: (a) the function of the photon number release rate per fission versus time after fission for each of the twelve energy

groups, and (b) the function of the gamma energy release rate per fission versus time after fission for each of the twelve energy groups. Each of these functions corresponds to an instantaneous event of fissions at time 0.

Definitions:

$f_g(x)$ $\equiv$ Photon number release rate per fission for the g^{th} energy group (measured in photons/fission-sec).

$h_g(x)$ $\equiv$ The gamma energy release rate per fission for the g^{th} energy group (measured in Mev/fission-sec).

x $\equiv$ The time following an instantaneous event of fissions at $x = 0$ (sec).

$R(t')$ $\equiv$ The fission rate at time t' sec (fissions/sec).

t' $\equiv$ The time following the start of fissions in a reactor (sec).

τ $\equiv$ The maximum value of t' , i.e., the total operating time of the reactor (sec).

t $\equiv$ The time measured from the shutdown of the reactor, i.e., elapsed time following the point at which $R(t')$ becomes effectively zero (sec).

In addition to the basic data, it is implied that $R(t')$ is a known function of t' , and that τ and t are given.

From these data, one can derive:

$F_g(\tau, t)$ $\equiv$ The photon number release rate for the g^{th} energy group following a reactor operating time, τ sec, and a shutdown time, t sec, (photons/sec).

$U_g(\tau) \equiv F_g(\tau, 0)$ $\equiv$ The photon number release rate for the g^{th} energy group during reactor operation but after an operating period of τ sec (photons/sec).

$H_g(\tau, t)$ $\equiv$ The gamma energy release rate for the g^{th} energy group following a reactor operating time, τ sec, and a shutdown time, t sec (Mev/sec).

$V_g(\tau) \equiv H_g(\tau, 0) \equiv$ The gamma energy release rate for the g^{th} energy group during reactor operation but after an operating period of τ sec (Mev/sec).

$\bar{E}_g \equiv H_g(\tau, t) / F_g(\tau, t) \equiv$ The average energy of a photon emitted in the g^{th} group (Mev/photons).

$$\mathcal{J}(E, \tau, t) \equiv \sum_{m=1}^{m=g-1} F_m(\tau, t) + \frac{F_g(\tau, t)}{\Delta E_g} (E - E_g) \equiv$$

The cumulative energy distribution of the photon number release rate following an operating time, τ sec, and a shutdown time, t sec, where $E_{g-1} \leq E < E_g$ and $1 \leq g \leq 12$. $\mathcal{J}(E, \tau, t)$ is the number of photons released per second which have energies less than or equal to E . $\Delta E_g = E_g - E_{g-1}$ by definition, where E_g is the upper bound of the g^{th} energy group, and E_{g-1} is the lower bound. (E_{g-1} for the first group is $E_0 = 0.1$ Mev.)

The form of the basic data imposes the restriction that $\mathcal{J}(E_0, \tau, t) = 0$.

The normalized cumulative energy distribution, $\mathcal{J}_{\text{norm}}(E, \tau, t)$, is obtained by simply dividing $\mathcal{J}(E, \tau, t)$ by $\sum_{m=1}^{m=12} F_m(\tau, t)$. Hence,

$$\mathcal{J}_{\text{norm}}(E, \tau, t) \equiv \frac{\mathcal{J}(E, \tau, t)}{\sum_{m=1}^{12} F_m(\tau, t)}$$

The energy intervals into which the basic data are grouped are listed in Table 2.1. E_{g-1} and E_g for the twelve energy groups are listed in Table 2.3.1 in consistency with Table 2.1.

Table 2.3.1

The Values of E_g and E_{g-1} (in Mev)

g	1	2	3	4	5	6	7	8	9	10	11	12
E_g	.4	.9	1.35	1.8	2.2	2.6	3.0	3.5	4.0	4.5	5.0	5.5
E_{g-1}	.1	.4	.9	1.35	1.8	2.2	2.6	3.0	3.5	4.0	4.5	5.0

The basic data are presented in the form of curves for $f^g(x)$ versus x , in Figures 2.A through 2.D. $f^g(x)$ is tabulated in Table 2.3.2, where for each group 25 values of $f^g(x)$ are listed versus the corresponding values of x . In Table 2.3.2, the sum of Groups 7 through 12 is listed for comparison to what is defined as Group VII.

For $h^g(x)$, the basic data are presented only in curve form on Figures 2.E1 through 2.H.

$U_g(\tau)$, $V_g(\tau)$, $\bar{E}_g$, $\mathcal{J}(E, \tau, t)$, and $\mathcal{J}_{\text{norm}}(E, \tau, t)$ are only special forms or simple functions of $F_g(\tau, t)$ and $H_g(\tau, t)$. $H_g(\tau, t)$ is calculated completely analogously to $F_g(\tau, t)$, such that whatever is the operation on $f^g(x)$ to produce $F_g(\tau, t)$ the same operation on $h^g(x)$ produces $H_g(\tau, t)$. For this reason, only the method of computing $F_g(\tau, t)$ from the tabular values of $f^g(x)$ versus x is outlined herein, $H_g(\tau, t)$ may be obtained analogously, and all other values may be obtained by means of the given definitions.

2.3.2 Calculation of $F_g(\tau, t)$

In Table 2.3.2 the values of $f^g(x)$ are listed for values of x ranging from 1 sec to 10^8 sec (3.17 years). There is a maximum of 25 values of $f^g(x)$ given for each of the energy groups. These values of $f^g(x)$ and the corresponding values of x are indexed by n as shown, such that $1 \leq n \leq 25$. The partitioning of the x dimension is such that

$$x_{n+1} = 2.154434689x_n \quad \text{and} \quad (1)$$

$$\ln\left(\frac{x_{n+1}}{x_n}\right) = 0.7675283643 \quad \text{for any value of } n.$$

This partitioning is chosen so that the loss of accuracy is small compared to the accuracy of the basic data if a straight line interpolation is assumed on a log-log plot of $f^g(x)$ versus x . This is equivalent to the statement that there is no significant error in the assumption that

$$f^g(x) = A_n^g x^{a_n^g} \quad \text{for } x_n \leq x < x_{n+1}, \quad \text{where } A_n^g \text{ and } a_n^g \quad (2)$$

are constants

Now, $F_g(\tau, t)$, as defined, is determined by the value of an integral as follows:

$$F_g(\tau, t) = \int_0^{\tau} R(t') f^g(\tau + t - t') dt' . \quad (3)$$

If one chooses $x = \tau + t - t'$, then

$$F_g(\tau, t) = \int_t^{t+\tau} R(\tau + t - x) f^g(x) dx . * \quad (4)$$

It is now assumed that, on a basis similar to the assumption of Equation (2),

$$R(\tau + t - x) f^g(x) = A_n^g x^{a_n^g} \quad \text{for } x_n \leq x < x_{n+1} \quad (5)$$

On this basis, Equation (4) becomes, after integrating piece-wise,

$$F_g(\tau, t) = \frac{A_1^g (x_{i+1}^{a_{i+1}^g} - t^{a_{i+1}^g})}{a_{i+1}^g} \\ + \sum_{n=i+1}^{n=j-1} \frac{A_n^g (x_{n+1}^{a_{n+1}^g} - x_n^{a_{n+1}^g})}{a_{n+1}^g} + \frac{A_j^g ((t+\tau)^{a_{j+1}^g} - x_j^{a_{j+1}^g})}{a_{j+1}^g} , \quad (6)$$

where $x_i \leq t < x_{i+1}$, $x_j \leq t+\tau < x_{j+1}$, $1 \leq t, *$ and $j \geq i+1$ (The summation term in Equation (8) is omitted if $j \leq i+1$.) If $j = i$, $t+\tau$ replaces x_{i+1} in the first term of Equation (6) and the other terms are omitted.

* Note: If $t < 1$ (as for $U_g(\tau) = F_g(\tau, 0)$) use $t = 1$ in Equation (4) for the lower limit of the interval of integration. The result is correct because the basic data are derived on the assumption that the proper number of photons per fission is obtained from the integral,

$$\int_1^{\infty} f(x) dx, \text{ instead of from the integral, } \int_0^{\infty} f(x) dx.$$

Moreover, as shown for beta emission in Section 3.0, $U_g(\tau) = F_g(\tau, 1)$ for $\tau \gg 1$. The error is insignificant for $\tau = 10$ seconds. Similarly, use $V_g(\tau) = H_g(\tau, 1)$ for $\tau \geq 10$ seconds.

For any $1 \leq n \leq 25$,

$$a_n^g = \left[1 / \ln \left(\frac{x_{n+1}}{x_n} \right) \right] \ln \left[\frac{R(\tau+t-x_{n+1}) f^g(x_{n+1})}{R(\tau+t-x_n) f^g(x_n)} \right], \text{ or} \quad (7)$$

$$a_n^g = 1.302883446 \ln \left[\frac{R(\tau+t-x_{n+1}) f^g(x_{n+1})}{R(\tau+t-x_n) f^g(x_n)} \right], \text{ and}$$

$$A_n^g = R(\tau+t-x_n) f^g(x_n) / x_n a_n^g.$$

For numerical calculation on a digital computer, it is more efficient and convenient to express $F_g(\tau, t)$ as follows:

$$\begin{aligned} F_g(\tau, t) = & \quad (8) \\ & \left[R(\tau+t-x_{i+1}) f^g(x_{i+1}) (x_{i+1}) - R(\tau+t-x_i) f^g(x_i) (x_i) \left(\frac{t}{x_i} \right)^{a_{i+1}^g} \right] / (a_{i+1}^g) \\ & + \sum_{n=i+1}^{n=j-1} \left[R(\tau+t-x_{n+1}) f^g(x_{n+1}) (x_{n+1}) - R(\tau+t-x_n) f^g(x_n) (x_n) \right] / (a_{n+1}^g) \\ & + \left[R(\tau+t-x_{j+1}) f^g(x_{j+1}) (x_{j+1}) \left(\frac{\tau+t}{x_{j+1}} \right)^{a_{j+1}^g} - R(\tau+t-x_j) f^g(x_j) (x_j) \right] / (a_{j+1}^g), \\ \text{where } a_n^g = & 1.302883446 \ln \left[\frac{R(\tau+t-x_{n+1}) f^g(x_{n+1})}{R(\tau+t-x_n) f^g(x_n)} \right] \text{ for } 1 \leq n \leq 24. \end{aligned}$$

The index, i , is chosen such that $x_i \leq t \leq x_{i+1}$, and the index, j , is chosen such that $x_j \leq \tau + t \leq x_{j+1}$. It is restricted that $i \leq 1$, $x_1 = 1$, $j \leq 24$, $x_{25} = 10^8$, $t \geq 1$ (sec), and $t + \tau \leq 10^8$. Equation (8) applies for $j \geq i+1$, with the summation term neglected when $j \leq i+1$. When $j = i$, the proper expression for $F_g(\tau, t)$ is

$$\begin{aligned} F_g(\tau, t) = & \left[R(\tau+t-x_{i+1}) f^g(x_{i+1}) (x_{i+1}) \left(\frac{\tau+t}{x_{i+1}} \right)^{a_{i+1}^g} \right. \\ & \left. - R(\tau+t-x_i) f^g(x_i) (x_i) \left(\frac{t}{x_i} \right)^{a_{i+1}^g} \right] / (a_{i+1}^g) \end{aligned} \quad (9)$$

One of the properties of $f^g(x)$ is that $\int_{x_n}^{x_{n+1}} f^g(x) dx$

is of about the same order of magnitude for every value of n ; the integral shows a tendency to decrease as n increases, for the partitioning adopted herein. This property guarantees, for example, that if $R(\tau+t-x)$ is zero over a time range small compared to the interval x_n to x_{n+1} in which R is zero then the error is very small if the decrease in R is neglected entirely. It follows that the numerical integration represented by Equation (8) for $F_g(\tau, t)$ is justified in most practical cases, and that, in unusual cases in which $R(\tau+t-x)$ varies rapidly and strongly, suitable averages of the R function can be easily constructed.

Table 2.3.2

γ Photons/fiss-sec Versus Time
After An Instantaneous Fission

(Time Range 1 - 10^8 sec)

1	time, * x_1 , sec	**Group 1	Group 2	Group 3
1	1.0 (0)	[†] 5.90 (-2)	1.2 (-1)	7.60 (-2)
2	2.15 + (0)	3.78 (-2)	7.7 (-2)	3.82 (-2)
3	4.64 + (0)	2.15 (-2)	4.43 (-2)	1.91 (-2)
4	1.0 (1)	1.12 (-2)	2.39 (-2)	9.60 (-3)
5	2.15 + (1)	5.40 (-3)	1.12 (-2)	4.80 (-3)
6	4.64 + (1)	2.46 (-3)	5.10 (-3)	2.40 (-3)
7	1.0 (2)	1.12 (-3)	2.29 (-3)	1.20 (-3)
8	2.15 + (2)	5.05 (-4)	1.02 (-3)	6.10 (-4)
9	4.64 + (2)	2.28 (-4)	4.44 (-4)	2.95 (-4)
10	1.0 (3)	1.00 (-4)	2.00 (-4)	1.27 (-4)
11	2.15 + (3)	4.28 (-5)	8.70 (-5)	4.50 (-5)
12	4.64 + (3)	1.78 (-5)	3.80 (-5)	1.35 (-5)
13	1.0 (4)	7.20 (-6)	1.65 (-5)	3.58 (-6)
14	2.15 + (4)	3.02 (-6)	6.90 (-6)	1.23 (-6)
15	4.64 + (4)	1.28 (-6)	2.90 (-6)	4.20 (-7)
16	1.0 (5)	5.40 (-7)	1.20 (-6)	1.42 (-7)
17	2.15 + (5)	2.27 (-7)	4.50 (-7)	4.70 (-8)
18	4.64 + (5)	9.30 (-8)	1.56 (-7)	1.70 (-8)
19	1.0 (6)	3.80 (-8)	5.45 (-8)	9.00 (-9)
20	2.15 + (6)	1.45 (-8)	2.12 (-8)	5.30 (-9)
21	4.64 + (6)	4.80 (-9)	9.50 (-9)	3.00 (-9)
22	1.0 (7)	1.20 (-9)	4.49 (-9)	1.53 (-9)
23	2.15 + (7)	2.23 (-10)	1.38 (-9)	3.74 (-10)
24	4.64 + (7)	2.45 (-11)	1.80 (-10)	2.55 (-11)
25	1.0 (8)	1.20 (-12)	1.18 (-11)	

* x_1 is given to only 3 significant digits.

** All γ group data is in units of photons/fiss-sec.

[†] The numbers in parentheses are the exponent of 10 which multiplies the given number; e.g., 5.90 (-2) = 5.90×10^{-2}

Table 2.3.2 (Continued)

γ Photons/fiss-sec Versus Time
After An Instantaneous Fission

(Time Range 1 - 10^8 sec)

1	time, x_1 , sec	Group 4	Group 5	Group 6
1	1.0 (0)	3.43 (-2)	2.39 (-2)	8.30 (-3)
2	2.15 + (0)	1.13 (-2)	1.39 (-2)	7.55 (-3)
3	4.64 + (0)	1.20 (-2)	7.51 (-3)	5.39 (-3)
4	1.0 (1)	6.50 (-3)	3.80 (-3)	2.00 (-3)
5	2.15 + (1)	3.34 (-3)	1.89 (-3)	1.49 (-3)
6	4.64 + (1)	1.60 (-3)	8.70 (-4)	6.85 (-4)
7	1.0 (2)	7.49 (-4)	3.88 (-4)	3.03 (-4)
8	2.15 + (2)	3.39 (-4)	1.69 (-4)	1.31 (-4)
9	4.64 + (2)	1.47 (-4)	7.00 (-5)	5.48 (-5)
10	1.0 (3)	6.15 (-5)	2.80 (-5)	2.31 (-5)
11	2.15 + (3)	2.53 (-5)	1.11 (-5)	9.25 (-6)
12	4.64 + (3)	1.02 (-5)	4.41 (-6)	3.51 (-6)
13	1.0 (4)	4.10 (-6)	1.75 (-6)	1.05 (-6)
14	2.15 + (4)	1.22 (-6)	8.20 (-7)	2.83 (-7)
15	4.64 + (4)	3.20 (-7)	3.80 (-7)	4.20 (-8)
16	1.0 (5)	6.20 (-8)	8.00 (-8)	5.80 (-9)
17	2.15 + (5)	3.60 (-8)	8.00 (-9)	3.05 (-9)
18	4.64 + (5)	2.88 (-8)	1.28 (-9)	2.19 (-9)
19	1.0 (6)	2.29 (-8)	4.64 (-10)	1.50 (-9)
20	2.15 + (6)	1.12 (-8)	1.03 (-10)	6.90 (-10)
21	4.64 + (6)	2.11 (-9)	9.50 (-12)	1.20 (-11)
22	1.0 (7)	9.00 (-11)		
23	2.15 + (7)			
24	4.64 + (7)			
25	1.0 (8)			

Table 2.3.2 (Continued)

γ Photons/fiss-sec Versus Time
After An Instantaneous Fission

(Time Range 1 - 10^8 sec)

1	time, x_1 , sec	Group 7	Group 8	Group 9
1	1.0 (0)	7.30 (-3)	6.25 (-3)	4.60 (-3)
2	2.15 + (0)	5.58 (-3)	4.68 (-3)	3.45 (-3)
3	4.64 + (0)	3.65 (-3)	3.00 (-3)	2.20 (-3)
4	1.0 (1)	2.11 (-3)	1.69 (-3)	1.23 (-3)
5	2.15 + (1)	1.10 (-3)	8.25 (-4)	6.18 (-4)
6	4.64 + (1)	5.00 (-4)	3.30 (-4)	2.68 (-4)
7	1.0 (2)	2.00 (-4)	1.40 (-4)	1.01 (-4)
8	2.15 + (2)	7.60 (-5)	5.10 (-5)	3.80 (-5)
9	4.64 + (2)	2.69 (-5)	1.70 (-5)	1.30 (-5)
10	1.0 (3)	8.60 (-6)	5.42 (-6)	4.00 (-6)
11	2.15 + (3)	2.60 (-6)	1.56 (-6)	1.12 (-6)
12	4.64 + (3)	6.00 (-7)	3.41 (-7)	2.40 (-7)
13	1.0 (4)	8.85 (-8)	5.20 (-8)	3.80 (-8)
14	2.15 + (4)	1.48 (-8)	8.30 (-9)	6.00 (-9)
15	4.64 + (4)	1.74 (-9)	9.7 (-10)	7.00 (-10)
16	1.0 (5)	7.80 (-11)	4.40 (-11)	3.30 (-11)
17	2.15 + (5)	3.25 (-11)	1.84 (-11)	1.38 (-11)
18	4.64 + (5)	2.41 (-11)	1.41 (-11)	1.01 (-11)
19	1.0 (6)	2.25 (-11)	1.33 (-11)	9.50 (-12)
20	2.15 + (6)	1.51 (-11)	8.40 (-12)	6.3 (-12)
21	4.64 + (6)	1.40 (-12)	7.80 (-13)	5.60 (-13)
22	1.0 (7)			
23	2.15 + (7)			
24	4.64 + (7)			
25	1.0 (8)			

Table 2.3.2 (Continued)

γ Photons/fiss-sec Versus Time
After An Instantaneous Fission

(Time Range 1 - 10^8 sec)

1	time, x_1 , sec	Group 10	Group 11	Group 12
1	1.0 (0)	3.70 (-3)	2.03 (-3)	9.20 (-4)
2	2.15 + (0)	2.00 (-3)	1.12 (-3)	5.30 (-4)
3	4.64 + (0)	1.07 (-3)	6.00 (-4)	2.94 (-4)
4	1.0 (1)	5.60 (-4)	3.05 (-4)	1.51 (-4)
5	2.15 + (1)	2.80 (-4)	1.48 (-4)	7.40 (-5)
6	4.64 + (1)	1.32 (-4)	6.35 (-5)	3.20 (-5)
7	1.0 (2)	5.75 (-5)	2.45 (-5)	1.18 (-5)
8	2.15 + (2)	2.15 (-5)	8.20 (-6)	3.65 (-6)
9	4.64 + (2)	6.25 (-6)	2.15 (-6)	8.25 (-7)
10	1.0 (3)	1.20 (-6)	3.80 (-7)	1.18 (-7)
11	2.15 + (3)	1.59 (-7)	4.90 (-8)	7.90 (-9)
12	4.64 + (3)	2.70 (-9)	5.00 (-10)	
13	1.0 (4)			
14	2.15 + (4)			
15	4.64 + (4)			
16	1.0 (5)			
17	2.15 + (5)			
18	4.64 + (5)			
19	1.0 (6)			
20	2.15 + (6)			
21	4.64 + (6)			
22	1.0 (7)			
23	2.15 + (7)			
24	4.64 + (7)			
25	1.0 (8)			

Table 2.3.2 (Continued)

γ Photons/fiss-sec Versus Time
After An Instantaneous Fission

(Time Range 1 - 10^8 sec)

1	time, x_1 , sec	Group VII (7 - 12)	Total
1	1.0 (0)	2.48 (-2)	3.40 (-1)
2	2.15 + (0)	1.74 (-2)	2.03 (-1)
3	4.64 + (0)	1.08 (-2)	1.21 (-1)
4	1.0 (1)	6.05 (-3)	6.31 (-2)
5	2.15 + (1)	3.02 (-3)	3.12 (-2)
6	4.64 + (1)	1.33 (-3)	1.44 (-2)
7	1.0 (2)	5.35 (-4)	6.59 (-3)
8	2.15 + (2)	1.98 (-4)	2.97 (-3)
9	4.64 + (2)	6.61 (-5)	1.31 (-3)
10	1.0 (3)	1.99 (-5)	5.59 (-4)
11	2.15 + (3)	5.50 (-6)	2.26 (-4)
12	4.64 + (3)	1.18 (-6)	8.86 (-5)
13	1.0 (4)	1.79 (-7)	3.40 (-5)
14	2.15 + (4)	2.91 (-8)	1.35 (-5)
15	4.64 + (4)	3.41 (-9)	5.35 (-6)
16	1.0 (5)	1.56 (-10)	2.03 (-6)
17	2.15 + (5)	6.47 (-11)	7.71 (-7)
18	4.64 + (5)	4.86 (-11)	2.98 (-7)
19	1.0 (6)	4.53 (-11)	1.26 (-7)
20	2.15 + (6)	2.98 (-11)	5.30 (-8)
21	4.64 + (6)	2.74 (-12)	1.99 (-8)
22	1.0 (7)		7.32 (-9)
23	2.15 + (7)		2.00 (-9)
24	4.64 + (7)		2.30 (-10)
25	1.0 (8)		1.30 (-11)

Fig. 2.H γ Mev/fiss-sec for an Instantaneous Fission Event Versus Time After Fission
(Time Range $10^4 - 10^8$ sec)

Figure 2.1
Average Energy per Photon Versus Time After an Instantaneous Fission
(Time Range 1 - 10^7 sec)

MODEL

DATE

Figure 2.J
Average Energy per Photon Versus Time After an Instantaneous Fission
(Time Range 1 - 10^7 sec)

3.0 BETA ENERGY RELEASE

3.1 Results

The rate at which beta particle energy is released after an instantaneous fission event is represented by the function,

$$M(t) = 1.9t^{-1.2} - 1.15t^{-1.957} - 10.9(t+10^5)^{-1.366}, \quad (1)$$

which is probably correct within $\pm 20\%$ in the interval $2 \leq t \leq 3 \times 10^7$ sec. $M(t)$ is given in mev/fission-sec and t in seconds.

By this formula, the total beta energy emitted between 1 and 10^8 sec after fission is 7.65 mev. Of this total, 6.78 mev are emitted between 1 and 10^4 sec. The total energy emitted after 1 sec is 7.86 mev, by the formula, which overestimates the energy decay rate beyond 3×10^7 sec.

This formula may be used to obtain the beta energy decay rate at a time $t \geq 1$, following any history of fissions in a time interval, 0 to τ . Let $R(t')$ be the fission rate (fissions/sec at time t'), then the beta energy decay rate (mev/sec) after a time t , is

$$N(\tau, t) = \int_0^{\tau} R(t') M(\tau + t - t') dt'. \quad (2)$$

This integral is expected to be of accuracy comparable to that of $M(t)$ for $t + \tau \leq 3 \times 10^7$ sec.

The rate at which the beta particles are emitted following an instantaneous fission event is represented by the formula,

$$p = t^{-1.115} - 0.7t^{-1.561} \quad (\beta \text{ particles/fission-sec}). \quad (3)$$

The error is estimated to be less than about $\pm 25\%$ for $2 \leq t \leq 10^7$. The number of beta particles emitted between 1 and 10^8 sec after the fission event by this formula is 6.4 β particles per fission. The probably correct value for the total number of β particles per fission is 6.1,

whereas the value obtained by formula (3) is 7.45 particles per fission. The difference is almost entirely accountable to the fact that formula (3) for P begins to greatly over-estimate the beta particle decay rate after 10^7 seconds. The error at 10^8 sec is about 170%.

3.2 The Method Used to Determine the Beta Decay Formulae

The formulae derived for $M(t)$ and $P(t)$ are reasonably good approximations to data obtained from:

- a) Energy Release from the Decay of Fission Products, J. F. Perkins and R. W. King, Nuclear Science and Engineering: Vol 3, #6, pp.726-746 (1958).
- b) Beta or Antineutrino Energy per 0.5 Mev per Fission per Second (Figure obtained from J. F. Perkins by private communication; data were obtained by J. F. Perkins and R. W. King for times after fission of 10^2 , 10^3 , 10^4 , and 10^5 seconds).
- c) Inverse Beta Decay and the Two Component Neutrino, R. W. King and J. F. Perkins, UCRL-8337, June, 1958.
- d) Gamma Decay of Fission Products from the Slow Neutron Fission of U^{235} , Carl F. Miller, USNRDL-TR-187, July 11, 1957.

The beta energy decay at times after fission from 5×10^3 sec to 10^8 sec is taken from reference (a) above. Reference (b) data are used as a check on the beta energy decay rate of reference (a) at 10^4 and 10^5 sec after fission and to obtain the energy per beta particle (mev/ β particle) at 10^2 , 10^3 , 10^4 , and 10^5 sec after fission.

Reference (a) and reference (c) provide estimates of the total beta energy emitted per fission. According to these references, it can be assumed that the total beta energy per fission is about 7.4 to 7.8 mev. Hence, the value of the integral of $M(t)$ from 0 to ∞ should be within this range. It is reasonable to assume that the beta decay energy emitted below about 1 sec is small with respect to the total beta decay energy and that the energy emitted after 10^8 sec is also small. (The beta energy emitted after 10^8 sec is probably less than 0.2 mev.)

The total number of beta particles emitted after fission is obtained from reference (c). The number is 6.1 β 's/fission.

Reference (d) is used to obtain the number of beta particles per disintegration as a function of time after fission in the time range 2.75×10^3 – 8×10^7 sec. Reference (a) provides the data for the disintegration rate (beta decays plus isomeric transitions) from 5×10^3 sec to 10^8 sec. Multiplication of the reference (d) data (the betas per disintegration) and the disintegration rate data from reference (a) provides the data for the beta particle decay rate per fission in the time range 2×10^3 – 10^8 sec. (Reference (a) disintegration rates are taken only because the rates are slightly higher than those of reference (d). The agreement of the two sets of data is very good: within 3% with few exceptions.)

In deriving the beta energy decay rate function for $1 \leq t \leq 10^4$, the time range of greatest interest herein, it is assumed that: the reference (a) data are exact from about 2×10^3 to 10^4 sec after fission; that the total beta energy emitted from 1 to 10^4 sec is about 6.8 mev; and that the decay rate tends to be roughly level at times after fission up to the neighborhood of 1 sec. The integral of beta energy decay rate from 10^4 to 10^8 sec (from the reference (a) data, directly) is about 0.87 mev. This number subtracted from an assumed total of 7.4 to 7.8 implies that about 6.5 to 6.9 mev are emitted prior to 10^4 sec after fission. On a log-log plot of beta energy decay rate versus time, a straight line through the reference (a) data between 2×10^3 and 10^4 sec, gives a total of about 8.0 mev for $1 \leq t \leq 10^4$. This strongly implies that the leveling off effect at about 1 sec should account for about 1.2 mev, such that, when this amount is subtracted from the straight line extrapolation, a total of 6.8 mev remains. By trial and error methods it is found that the function, $1.15t^{-1.957}$, when subtracted from the extrapolated log-log line, leaves a total of 6.8 mev and provides the desired leveling off effect at 1 sec after fission.

If it can be assumed that little beta energy is emitted between 0 and 1 second after fission, then the expected total of 6.8 mev between 1 and 10^4 seconds is only 18% overestimated by the integral (8.0 mev) of the single extrapolated line from the reference (a) data. Moreover, to effect the expected leveling off at about 1 sec, about half of the 1.2 mev correction is accounted for in the interval from 1 to 2 seconds. Nine-tenths of the 1.2 mev overestimate is accounted for in the interval from 1 to about 10 seconds. Therefore, there is reason to believe

that the error of the extrapolated line from the reference (a) data is of roughly the same order of magnitude as the error in the data (from reference (a)) for the β mev/fission-sec near 10^4 sec after fission and the error in the number used for the total beta energy emitted per fission between 1 and 10^4 sec. The equation of the single extrapolated line is $1.9(t)-1.2$, hence the beta energy decay curve for the interval, $1 \leq t \leq 10^4$ sec is approximated by the formula $1.9(t)-1.2-1.15(t)-1.957$.

Again, by trial and error, it is found that if an additional term, $10.9(t+10^5)-1.366$, is subtracted from the formula derived for the time range, $1-10^4$ sec, the data of reference (a) for the time range, $10^4-3 \times 10^7$ sec, are also approximated quite well. The final formula obtained is given above as $M(t)$.

In the time range $10^4-3 \times 10^7$ sec, $M(t)$ differs from the reference (a) data by an amount usually less than $\pm 10\%$ except in the range, 10^5-10^6 sec, where the $M(t)$ function gives an overestimate of up to 23%. It is in this time range that the experimental data of Day and Cannon, Borst, and Brady and Turkevich (see reference (a)) also fall above the reference (a) data, so that $M(t)$ represents a compromise between the experimental and calculated results. The values given by $M(t)$ become increasingly higher than the reference (a) data beyond 3×10^7 sec such that the error is about 50% at 10^8 seconds.

The function, $P(t)$, is derived to represent the beta particle decay rate in the time range $1-10^4$ seconds. Data obtained from reference (a) and reference (c) are extrapolated and leveled off at 1 sec (as with the beta energy decay rate function) in order to obtain a total integral of 4.43 beta particles emitted in the time range $1-10^4$ seconds. There are 1.67 beta particles emitted in the time range 10^4-10^8 sec (by numerical integration of the data from references (a) and (c)), such that the total for the time range $1-10^8$ is 6.1 beta particles per fission. Again, the number emitted outside this time range is considered to be negligible. The beta particle decay rate, $P(t)$, approximates the data obtained from references (a) and (c), as well as the data of reference (b), to within $\pm 10\%$ in the time range 10^3-10^5 seconds. Even beyond 10^5 sec (up to 10^7 sec) the formula overestimates by no more than about 25%.

As a final check, the ratio, M/P , is computed and found to vary by less than about $\pm 15\%$ from the values for the beta energy per particle obtained from references (a), (b), and (c) in the time range 10^2-10^7 seconds. (The deviation is less than about $\pm 5\%$ in the range $2 \times 10^2-2 \times 10^4$ seconds.)

Table 3.2

Beta Decay From An Instantaneous Event Of Fissions

t(sec)	Dis/fiss-sec †P & K	β/Dis ‡Miller	β/fiss-sec	β/fiss-sec P(t)
1 X 10 ⁰				3.00 X 10 ⁻¹
2 X 10 ⁰				2.24 X 10 ⁻¹
5 X 10 ⁰				1.09 X 10 ⁻¹
1 X 10 ¹				5.75 X 10 ⁻²
2 X 10 ¹				2.89 X 10 ⁻²
5 X 10 ¹				1.12 X 10 ⁻²
1 X 10 ²				5.36 X 10 ⁻³
2 X 10 ²				2.54 X 10 ⁻³
5 X 10 ²				9.36 X 10 ⁻⁴
1 X 10 ³				4.37 X 10 ⁻⁴
2 X 10 ³	2.13 X 10 ⁻⁴	0.967	2.06 X 10 ⁻⁴	2.04 X 10 ⁻⁴
5 X 10 ³	8.00 X 10 ⁻⁵	0.933	7.46 X 10 ⁻⁵	7.39 X 10 ⁻⁵
1 X 10 ⁴	3.75 X 10 ⁻⁵	0.912	3.42 X 10 ⁻⁵	3.43 X 10 ⁻⁵
2 X 10 ⁴	1.80 X 10 ⁻⁵	0.895	1.61 X 10 ⁻⁵	1.59 X 10 ⁻⁵
5 X 10 ⁴	6.30 X 10 ⁻⁶	0.871	5.49 X 10 ⁻⁶	5.73 X 10 ⁻⁶
1 X 10 ⁵	2.80 X 10 ⁻⁶	0.864	2.42 X 10 ⁻⁶	2.65 X 10 ⁻⁶
2 X 10 ⁵	1.15 X 10 ⁻⁶	0.900	1.04 X 10 ⁻⁶	1.22 X 10 ⁻⁶
5 X 10 ⁵	3.70 X 10 ⁻⁷	0.953	3.53 X 10 ⁻⁷	4.41 X 10 ⁻⁷
1 X 10 ⁶	1.95 X 10 ⁻⁷	0.956	1.86 X 10 ⁻⁷	2.04 X 10 ⁻⁷
2 X 10 ⁶	9.50 X 10 ⁻⁸	0.942	8.95 X 10 ⁻⁸	9.42 X 10 ⁻⁸
5 X 10 ⁶	3.50 X 10 ⁻⁸	0.917	3.21 X 10 ⁻⁸	3.39 X 10 ⁻⁸
1 X 10 ⁷	1.55 X 10 ⁻⁸	0.979	1.52 X 10 ⁻⁸	1.57 X 10 ⁻⁸
2 X 10 ⁷	5.50 X 10 ⁻⁹	0.964	5.30 X 10 ⁻⁹	7.23 X 10 ⁻⁹
5 X 10 ⁷	1.20 X 10 ⁻⁹	0.973	1.17 X 10 ⁻⁹	2.60 X 10 ⁻⁹
1 X 10 ⁸	4.70 X 10 ⁻¹⁰	0.933	4.39 X 10 ⁻¹⁰	1.20 X 10 ⁻⁹

† Data from Figure 2 of Energy Release From The Decay of Fission Products, J. F. Perkins & R. W. King, N S and E: 3,726-7 & 6 (1958)

‡ Data derived from Table 1, Gamma Decay of Fission Products from the Slow Neutron Fission of U-235, Carl F. Miller, USNRDL-TR-187, July 11, 1957

* Data derived from a figure obtained from J. F. Perkins, Beta or Antineutrino Energy per 0.5 Mev per Fission per Second, by J. F. Perkins and R. W. King, (1958)

Table 3.2 (Continued)

Beta Decay From An Instantaneous Event Of Fissions

t(sec)	β Mev/fiss-sec	β Mev/fiss-sec M(t)	β Mev/ β $\dagger$ P & K $\dagger$ and Miller	particle $\dagger$ $\frac{M(t)}{P(t)}$
1 X 10 ⁰		7.50 X 10 ⁻¹		2.500
2 X 10 ⁰		5.31 X 10 ⁻¹		2.365
5 X 10 ⁰		2.26 X 10 ⁻¹		2.066
1 X 10 ¹		1.07 X 10 ⁻¹		1.864
2 X 10 ¹		4.89 X 10 ⁻²		1.692
5 X 10 ¹		1.68 X 10 ⁻²		1.504
1 X 10 ²		7.42 X 10 ⁻³	1.577*	1.385
2 X 10 ²		3.25 X 10 ⁻³		1.282
5 X 10 ²		1.09 X 10 ⁻³		1.163
1 X 10 ³		4.74 X 10 ⁻⁴	1.029*	1.084
2 X 10 ³	2.03 X 10 ⁻⁴	2.06 X 10 ⁻⁴	0.985	1.010
5 X 10 ³	6.80 X 10 ⁻⁵	6.76 X 10 ⁻⁵	0.912	0.915
1 X 10 ⁴	2.95 X 10 ⁻⁵	2.87 X 10 ⁻⁵	0.862*	0.837
2 X 10 ⁴	1.24 X 10 ⁻⁵	1.18 X 10 ⁻⁵	0.770	0.746
5 X 10 ⁴	3.80 X 10 ⁻⁶	3.44 X 10 ⁻⁶	0.692	0.600
1 X 10 ⁵	1.28 X 10 ⁻⁶	1.27 X 10 ⁻⁶	0.529*	0.481
2 X 10 ⁵	4.00 X 10 ⁻⁷	4.67 X 10 ⁻⁷	0.385	0.382
5 X 10 ⁵	1.10 X 10 ⁻⁷	1.36 X 10 ⁻⁷	0.312	0.308
1 X 10 ⁶	5.30 X 10 ⁻⁸	5.89 X 10 ⁻⁸	0.285	0.289
2 X 10 ⁶	2.75 X 10 ⁻⁸	2.70 X 10 ⁻⁸	0.307	0.287
5 X 10 ⁶	1.03 X 10 ⁻⁸	9.88 X 10 ⁻⁹	0.321	0.291
1 X 10 ⁷	4.70 X 10 ⁻⁹	4.61 X 10 ⁻⁹	0.309	0.295
2 X 10 ⁷	2.05 X 10 ⁻⁹	2.14 X 10 ⁻⁹	0.387	0.296
5 X 10 ⁷	5.80 X 10 ⁻¹⁰	7.66 X 10 ⁻¹⁰	0.496	0.294
1 X 10 ⁸	2.10 X 10 ⁻¹⁰	3.49 X 10 ⁻¹⁰	0.478	0.290

$\dagger$ Data from Figure 2 of Energy Release From The Decay of Fission Products, J. F. Perkins & R. W. King, N S and E: 3,726-7 & 6 (1958)

$\dagger$ Data derived from Table 1, Gamma Decay of Fission Products from the Slow Neutron Fission of U-235, Carl F. Miller, USNRDL-TR-187, July 11, 1957

* Data derived from a figure obtained from J. F. Perkins, Beta or Antineutrino Energy per 0.5 Mev per Fission per Second, by J. F. Perkins and R. W. King, (1958).

The values of $P(t)$, $M(t)$, and $M(t)/P(t)$ at times from 1 to 10^8 sec after fission are listed (versus the comparable data derived from references (a), (b), (c), and (d)) in Table 3.2. These data are plotted on Figures 3.A, 3.B, and 3.C.

3.3 Formulae for the Decay Rate of Beta Energy Following Shutdown

Formula (2) gives the rate of beta energy release at a time after shutdown, t sec, following a period of operation time, τ sec, during which fissions take place according to some fission rate function, $R(t')$.

If $R(t')$ is constant, then $N(\tau, t)$ is given by

$$N(\tau, t) = \quad (4)$$

$$R9.5 \left[t^{-0.2} - (t+\tau)^{-0.2} \right] - 1.202 \left[t^{-0.957} - (t+\tau)^{-0.957} \right]$$

$$- 29.78 \left[(t+10^5)^{-0.366} - (t+\tau+10^5)^{-0.366} \right] \frac{\text{mev}}{\text{sec}}.$$

where R is the number of fissions per second taking place during τ . The error in $N(\tau, t)$ is probably within $\pm 20\%$ for $2 \leq t \leq 3 \times 10^7$ sec and $0 \leq \tau \leq \infty$.

If $R(t')$ is not constant, the integral of formula (2) must be evaluated.

A special case is that in which the beta energy release rate is desired for times during the operating period. In this case let τ sec be the time after start of operation. The variable, t , in formula (2) now has no meaning, and the beta energy release rate at τ sec after start of operation is

$$Q(\tau) = \int_0^{\tau-1} R(t') M(\tau-t') dt' \frac{\text{mev}}{\text{sec}}. \quad (5)$$

$R(t')$ is given in units of fissions/sec.

It should be noted that the upper limit of this integration interval is $\tau-1$ sec instead of τ . This is the correct upper limit because $M(t)$ is derived with the assumption that an insignificant amount of beta energy is released within 1 sec after an instantaneous fission event and that the proper value of the total beta energy per fission ($\pm 5\%$ error) is obtained from the formula:

$$Q(\infty) = \lim_{\tau \rightarrow \infty} \int_0^{\tau-1} M(\tau-t') dt' = \int_1^{\infty} M(x) dx, \text{ i.e.,} \quad (6)$$

$$Q(\infty) \doteq N(\infty, 1) = 7.86 \text{ mev/fission.}$$

$Q(\tau)$ is almost exactly equal to $N(\tau, 1)$, numerically, when $\tau \gg 1$; so that if $\tau \approx 10$ sec, for example, one may use values of $N(\tau, 1)$ in place of $Q(\tau)$ with a maximum error of about 3% at $\tau = 10$.

Tables 3.3 a, b, c, d, e, f, g, and h list values of $N(\tau, t)$ in units of β mev/sec per fission/sec and β mev/watt-sec at times between 1 and 10^8 sec after shutdown. The values correspond to a unit fission rate or a unit reactor wattage for operation times, τ , of lengths 1 hour, 10 hours, 10^2 hours, 10^3 hours, 10^4 hours, 10^5 hours, 10^7 hours, and ∞ .

Table 3.3a

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 1$ hr)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	6.45 (0)	2.09 (11)	1.0 (5)	4.47 (-3)	1.45 (8)
1.5 (0)	6.09 (0)	1.98 (11)	1.5 (5)	2.50 (-3)	8.13 (7)
2.0 (0)	5.80 (0)	1.88 (11)	2.0 (5)	1.66 (-3)	5.40 (7)
3.0 (0)	5.35 (0)	1.74 (11)	3.0 (5)	9.49 (-4)	3.08 (7)
5.0 (0)	4.78 (0)	1.55 (11)	5.0 (5)	4.87 (-4)	1.58 (7)
7.0 (0)	4.40 (0)	1.43 (11)	7.0 (5)	3.22 (-4)	1.05 (7)
1.0 (1)	4.01 (0)	1.30 (11)	1.0 (6)	2.12 (-4)	6.88 (6)
1.5 (1)	3.59 (0)	1.17 (11)	1.5 (6)	1.34 (-4)	4.34 (6)
2.0 (1)	3.30 (0)	1.07 (11)	2.0 (6)	9.71 (-5)	3.16 (6)
3.0 (1)	2.92 (0)	9.48 (10)	3.0 (6)	6.21 (-5)	2.02 (6)
5.0 (1)	2.47 (0)	8.02 (10)	5.0 (6)	3.55 (-5)	1.16 (6)
7.0 (1)	2.20 (0)	7.14 (10)	7.0 (6)	2.46 (-5)	8.00 (5)
1.0 (2)	1.93 (0)	6.26 (10)	1.0 (7)	1.66 (-5)	5.41 (5)
1.5 (2)	1.64 (0)	5.33 (10)	1.5 (7)	1.06 (-5)	3.43 (5)
2.0 (2)	1.45 (0)	4.72 (10)	2.0 (7)	7.69 (-6)	2.50 (5)
3.0 (2)	1.21 (0)	3.93 (10)	3.0 (7)	4.88 (-6)	1.59 (5)
5.0 (2)	9.33 (-1)	3.03 (10)	5.0 (7)	2.78 (-6)	9.04 (4)
7.0 (2)	7.73 (-1)	2.51 (10)	7.0 (7)	1.92 (-6)	6.23 (4)
1.0 (3)	6.21 (-1)	2.02 (10)	1.0 (8)	1.24 (-6)	4.03 (4)
1.5 (3)	4.71 (-1)	1.53 (10)			
2.0 (3)	3.81 (-1)	1.24 (10)			
3.0 (3)	2.74 (-1)	8.90 (9)			
5.0 (3)	1.72 (-1)	5.60 (9)			
7.0 (3)	1.24 (-1)	4.01 (9)			
1.0 (4)	8.48 (-2)	2.76 (9)			
1.5 (4)	5.37 (-2)	1.75 (9)			
2.0 (4)	3.82 (-2)	1.24 (9)			
3.0 (4)	2.31 (-2)	7.51 (8)			
5.0 (4)	1.18 (-2)	3.83 (8)			
7.0 (4)	7.41 (-3)	2.41 (8)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3b

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.09 (0)	2.30 (11)	1.0 (5)	3.66 (-2)	1.19 (9)
1.5 (0)	6.73 (0)	2.19 (11)	1.5 (5)	2.18 (-2)	7.07 (8)
2.0 (0)	6.44 (0)	2.09 (11)	2.0 (5)	1.50 (-2)	4.86 (8)
3.0 (0)	5.99 (0)	1.95 (11)	3.0 (5)	8.86 (-3)	2.88 (8)
5.0 (0)	5.42 (0)	1.76 (11)	5.0 (5)	4.68 (-3)	1.52 (8)
7.0 (0)	5.04 (0)	1.64 (11)	7.0 (5)	3.14 (-3)	1.02 (8)
1.0 (1)	4.65 (0)	1.51 (11)	1.0 (6)	2.08 (-3)	6.76 (7)
1.5 (1)	4.23 (0)	1.37 (11)	1.5 (6)	1.32 (-3)	4.29 (7)
2.0 (1)	3.94 (0)	1.28 (11)	2.0 (6)	9.62 (-4)	3.13 (7)
3.0 (1)	3.55 (0)	1.15 (11)	3.0 (6)	6.18 (-4)	2.01 (7)
5.0 (1)	3.10 (0)	1.01 (11)	5.0 (6)	3.54 (-4)	1.15 (7)
7.0 (1)	2.83 (0)	9.20 (10)	7.0 (6)	2.45 (-4)	7.97 (6)
1.0 (2)	2.56 (0)	8.31 (10)	1.0 (7)	1.66 (-4)	5.39 (6)
1.5 (2)	2.27 (0)	7.37 (10)	1.5 (7)	1.06 (-4)	3.44 (6)
2.0 (2)	2.07 (0)	6.74 (10)	2.0 (7)	7.70 (-5)	2.50 (6)
3.0 (2)	1.82 (0)	5.92 (10)	3.0 (7)	4.89 (-5)	1.59 (6)
5.0 (2)	1.53 (0)	4.97 (10)	5.0 (7)	2.75 (-5)	8.95 (5)
7.0 (2)	1.35 (0)	4.40 (10)	7.0 (7)	1.89 (-5)	6.13 (5)
1.0 (3)	1.18 (0)	3.83 (10)	1.0 (8)	1.26 (-5)	4.09 (5)
1.5 (3)	9.97 (-1)	3.24 (10)			
2.0 (3)	8.78 (-1)	2.85 (10)			
3.0 (3)	7.23 (-1)	2.35 (10)			
5.0 (3)	5.50 (-1)	1.79 (10)			
7.0 (3)	4.49 (-1)	1.46 (10)			
1.0 (4)	3.54 (-1)	1.15 (10)			
1.5 (4)	2.62 (-1)	8.50 (9)			
2.0 (4)	2.06 (-1)	6.70 (9)			
3.0 (4)	1.42 (-1)	4.62 (9)			
5.0 (4)	8.34 (-2)	2.71 (9)			
7.0 (4)	5.66 (-2)	1.84 (9)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3c

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10^2$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.37 (0)	2.40 (11)	1.0 (5)	1.43 (-1)	4.63 (9)
1.5 (0)	7.02 (0)	2.28 (11)	1.5 (5)	1.02 (-1)	3.33 (9)
2.0 (0)	6.73 (0)	2.19 (11)	2.0 (5)	8.00 (-2)	2.60 (9)
3.0 (0)	6.28 (0)	2.04 (11)	3.0 (5)	5.57 (-2)	1.81 (9)
5.0 (0)	5.70 (0)	1.85 (11)	5.0 (5)	3.46 (-2)	1.13 (9)
7.0 (0)	5.33 (0)	1.73 (11)	7.0 (5)	2.51 (-2)	8.16 (8)
1.0 (1)	4.94 (0)	1.60 (11)	1.0 (6)	1.77 (-2)	5.76 (8)
1.5 (1)	4.51 (0)	1.47 (11)	1.5 (6)	1.18 (-2)	3.85 (8)
2.0 (1)	4.23 (0)	1.37 (11)	2.0 (6)	8.86 (-3)	2.88 (8)
3.0 (1)	3.84 (0)	1.25 (11)	3.0 (6)	5.84 (-3)	1.90 (8)
5.0 (1)	3.39 (0)	1.10 (11)	5.0 (6)	3.42 (-3)	1.11 (8)
7.0 (1)	3.12 (0)	1.01 (11)	7.0 (6)	2.39 (-3)	7.78 (7)
1.0 (2)	2.84 (0)	9.24 (10)	1.0 (7)	1.63 (-3)	5.30 (7)
1.5 (2)	2.55 (0)	8.30 (10)	1.5 (7)	1.05 (-3)	3.40 (7)
2.0 (2)	2.36 (0)	7.67 (10)	2.0 (7)	7.63 (-4)	2.48 (7)
3.0 (2)	2.11 (0)	6.85 (10)	3.0 (7)	4.87 (-4)	1.58 (7)
5.0 (2)	1.82 (0)	5.90 (10)	5.0 (7)	2.75 (-4)	8.92 (6)
7.0 (2)	1.64 (0)	5.32 (10)	7.0 (7)	1.88 (-4)	6.10 (6)
1.0 (3)	1.46 (0)	4.75 (10)	1.0 (8)	1.25 (-4)	4.07 (6)
1.5 (3)	1.28 (0)	4.15 (10)			
2.0 (3)	1.16 (0)	3.76 (10)			
3.0 (3)	9.97 (-1)	3.24 (10)			
5.0 (3)	8.14 (-1)	2.65 (10)			
7.0 (3)	7.05 (-1)	2.29 (10)			
1.0 (4)	5.99 (-1)	1.95 (10)			
1.5 (4)	4.89 (-1)	1.59 (10)			
2.0 (4)	4.19 (-1)	1.36 (10)			
3.0 (4)	3.31 (-1)	1.08 (10)			
5.0 (4)	2.38 (-1)	7.73 (9)			
7.0 (4)	1.87 (-1)	6.08 (9)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3d

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10^3$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0) *	7.51 (0)	2.44 (11)	1.0 (5)	2.63 (-1)	8.55 (9)
1.5 (0)	7.16 (0)	2.33 (11)	1.5 (5)	2.17 (-1)	7.04 (9)
2.0 (0)	6.86 (0)	2.23 (11)	2.0 (5)	1.89 (-1)	6.13 (9)
3.0 (0)	6.42 (0)	2.09 (11)	3.0 (5)	1.55 (-1)	5.04 (9)
5.0 (0)	5.84 (0)	1.90 (11)	5.0 (5)	1.20 (-1)	3.90 (9)
7.0 (0)	5.46 (0)	1.78 (11)	7.0 (5)	1.00 (-1)	3.27 (9)
1.0 (1)	5.07 (0)	1.65 (11)	1.0 (6)	8.22 (-2)	2.67 (9)
1.5 (1)	4.65 (0)	1.51 (11)	1.5 (6)	6.41 (-2)	2.08 (9)
2.0 (1)	4.36 (0)	1.42 (11)	2.0 (6)	5.29 (-2)	1.72 (9)
3.0 (1)	3.98 (0)	1.29 (11)	3.0 (6)	3.94 (-2)	1.28 (9)
5.0 (1)	3.53 (0)	1.15 (11)	5.0 (6)	2.61 (-2)	8.49 (8)
7.0 (1)	3.25 (0)	1.06 (11)	7.0 (6)	1.94 (-2)	6.32 (8)
1.0 (2)	2.98 (0)	9.69 (10)	1.0 (7)	1.40 (-2)	4.54 (8)
1.5 (2)	2.69 (0)	8.75 (10)	1.5 (7)	9.40 (-3)	3.06 (8)
2.0 (2)	2.50 (0)	8.12 (10)	2.0 (7)	7.02 (-3)	2.28 (8)
3.0 (2)	2.24 (0)	7.29 (10)	3.0 (7)	4.59 (-3)	1.49 (8)
5.0 (2)	1.95 (0)	6.34 (10)	5.0 (7)	2.65 (-3)	8.61 (7)
7.0 (2)	1.77 (0)	5.77 (10)	7.0 (7)	1.83 (-3)	5.95 (7)
1.0 (3)	1.60 (0)	5.20 (10)	1.0 (8)	1.23 (-3)	4.00 (7)
1.5 (3)	1.41 (0)	4.60 (10)			
2.0 (3)	1.29 (0)	4.20 (10)			
3.0 (3)	1.13 (0)	3.68 (10)			
5.0 (3)	9.50 (-1)	3.09 (10)			
7.0 (3)	8.41 (-1)	2.73 (10)			
1.0 (4)	7.34 (-1)	2.38 (10)			
1.5 (4)	6.23 (-1)	2.03 (10)			
2.0 (4)	5.52 (-1)	1.80 (10)			
3.0 (4)	4.62 (-1)	1.50 (10)			
5.0 (4)	3.66 (-1)	1.19 (10)			
7.0 (4)	3.12 (-1)	1.01 (10)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3e

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10^4$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.62 (0)	2.48 (11)	1.0 (5)	3.67 (-1)	1.19 (10)
1.5 (0)	7.26 (0)	2.36 (11)	1.5 (5)	3.19 (-1)	1.04 (10)
2.0 (0)	6.97 (0)	2.26 (11)	2.0 (5)	2.91 (-1)	9.45 (9)
3.0 (0)	6.52 (0)	2.12 (11)	3.0 (5)	2.56 (-1)	8.32 (9)
5.0 (0)	5.95 (0)	1.93 (11)	5.0 (5)	2.19 (-1)	7.11 (9)
7.0 (0)	5.57 (0)	1.81 (11)	7.0 (5)	1.97 (-1)	6.40 (9)
1.0 (1)	5.18 (0)	1.68 (11)	1.0 (6)	1.76 (-1)	5.71 (9)
1.5 (1)	4.76 (0)	1.55 (11)	1.5 (6)	1.53 (-1)	4.97 (9)
2.0 (1)	4.47 (0)	1.45 (11)	2.0 (6)	1.38 (-1)	4.48 (9)
3.0 (1)	4.08 (0)	1.33 (11)	3.0 (6)	1.17 (-1)	3.81 (9)
5.0 (1)	3.63 (0)	1.18 (11)	5.0 (6)	9.34 (-2)	3.04 (9)
7.0 (1)	3.36 (0)	1.09 (11)	7.0 (6)	7.89 (-2)	2.57 (9)
1.0 (2)	3.09 (0)	1.00 (11)	1.0 (7)	6.48 (-2)	2.11 (9)
1.5 (2)	2.80 (0)	9.09 (10)	1.5 (7)	5.04 (-2)	1.64 (9)
2.0 (2)	2.60 (0)	8.46 (10)	2.0 (7)	4.15 (-2)	1.35 (9)
3.0 (2)	2.35 (0)	7.63 (10)	3.0 (7)	3.06 (-2)	9.96 (8)
5.0 (2)	2.06 (0)	6.68 (10)	5.0 (7)	2.00 (-2)	6.51 (8)
7.0 (2)	1.88 (0)	6.11 (10)	7.0 (7)	1.48 (-2)	4.80 (8)
1.0 (3)	1.70 (0)	5.54 (10)	1.0 (8)	1.05 (-2)	3.41 (8)
1.5 (3)	1.52 (0)	4.94 (10)			
2.0 (3)	1.40 (0)	4.54 (10)			
3.0 (3)	1.24 (0)	4.02 (10)			
5.0 (3)	1.05 (0)	3.43 (10)			
7.0 (3)	9.45 (-1)	3.07 (10)			
1.0 (4)	8.38 (-1)	2.72 (10)			
1.5 (4)	7.28 (-1)	2.37 (10)			
2.0 (4)	6.57 (-1)	2.14 (10)			
3.0 (4)	5.67 (-1)	1.84 (10)			
5.0 (4)	4.70 (-1)	1.53 (10)			
7.0 (4)	4.16 (-1)	1.35 (10)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3f

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10^5$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.69 (0)	2.50 (11)	1.0 (5)	4.46 (-1)	1.45 (10)
1.5 (0)	7.34 (0)	2.39 (11)	1.5 (5)	3.98 (-1)	1.29 (10)
2.0 (0)	7.05 (0)	2.29 (11)	2.0 (5)	3.70 (-1)	1.20 (10)
3.0 (0)	6.60 (0)	2.15 (11)	3.0 (5)	3.35 (-1)	1.09 (10)
5.0 (0)	6.02 (0)	1.96 (11)	5.0 (5)	2.97 (-1)	9.66 (9)
7.0 (0)	5.65 (0)	1.84 (11)	7.0 (5)	2.75 (-1)	8.95 (9)
1.0 (1)	5.26 (0)	1.71 (11)	1.0 (6)	2.54 (-1)	8.24 (9)
1.5 (1)	4.83 (0)	1.57 (11)	1.5 (6)	2.30 (-1)	7.49 (9)
2.0 (1)	4.55 (0)	1.48 (11)	2.0 (6)	2.15 (-1)	6.98 (9)
3.0 (1)	4.16 (0)	1.35 (11)	3.0 (6)	1.93 (-1)	6.28 (9)
5.0 (1)	3.71 (0)	1.21 (11)	5.0 (6)	1.68 (-1)	5.45 (9)
7.0 (1)	3.44 (0)	1.12 (11)	7.0 (6)	1.51 (-1)	4.92 (9)
1.0 (2)	3.16 (0)	1.03 (11)	1.0 (7)	1.35 (-1)	4.39 (9)
1.5 (2)	2.87 (0)	9.34 (10)	1.5 (7)	1.17 (-1)	3.80 (9)
2.0 (2)	2.68 (0)	8.72 (10)	2.0 (7)	1.05 (-1)	3.41 (9)
3.0 (2)	2.43 (0)	7.89 (10)	3.0 (7)	8.86 (-2)	2.88 (9)
5.0 (2)	2.14 (0)	6.94 (10)	5.0 (7)	6.99 (-2)	2.27 (9)
7.0 (2)	1.96 (0)	6.36 (10)	7.0 (7)	5.86 (-2)	1.90 (9)
1.0 (3)	1.78 (0)	5.80 (10)	1.0 (8)	4.77 (-2)	1.55 (9)
1.5 (3)	1.60 (0)	5.20 (10)			
2.0 (3)	1.48 (0)	4.80 (10)			
3.0 (3)	1.32 (0)	4.28 (10)			
5.0 (3)	1.13 (0)	3.68 (10)			
7.0 (3)	1.02 (0)	3.33 (10)			
1.0 (4)	9.17 (-1)	2.98 (10)			
1.5 (4)	8.07 (-1)	2.62 (10)			
2.0 (4)	7.36 (-1)	2.39 (10)			
3.0 (4)	6.46 (-1)	2.10 (10)			
5.0 (4)	5.49 (-1)	1.78 (10)			
7.0 (4)	4.95 (-1)	1.61 (10)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3g

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = 10^7$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.79 (0)	2.53 (11)	1.0 (5)	5.39 (-1)	1.75 (10)
1.5 (0)	7.43 (0)	2.42 (11)	1.5 (5)	4.92 (-1)	1.60 (10)
2.0 (0)	7.14 (0)	2.32 (11)	2.0 (5)	4.63 (-1)	1.50 (10)
3.0 (0)	6.70 (0)	2.18 (11)	3.0 (5)	4.28 (-1)	1.39 (10)
5.0 (0)	6.12 (0)	1.99 (11)	5.0 (5)	3.90 (-1)	1.27 (10)
7.0 (0)	5.74 (0)	1.87 (11)	7.0 (5)	3.68 (-1)	1.20 (10)
1.0 (1)	5.35 (0)	1.74 (11)	1.0 (6)	3.47 (-1)	1.13 (10)
1.5 (1)	4.93 (0)	1.60 (11)	1.5 (6)	3.24 (-1)	1.05 (10)
2.0 (1)	4.64 (0)	1.51 (11)	2.0 (6)	3.08 (-1)	1.00 (10)
3.0 (1)	4.26 (0)	1.38 (11)	3.0 (6)	2.86 (-1)	9.31 (9)
5.0 (1)	3.81 (0)	1.24 (11)	5.0 (6)	2.61 (-1)	8.47 (9)
7.0 (1)	3.53 (0)	1.15 (11)	7.0 (6)	2.44 (-1)	7.94 (9)
1.0 (2)	3.26 (0)	1.06 (11)	1.0 (7)	2.27 (-1)	7.39 (9)
1.5 (2)	2.97 (0)	9.65 (10)	1.5 (7)	2.09 (-1)	6.80 (9)
2.0 (2)	2.78 (0)	9.02 (10)	2.0 (7)	1.97 (-1)	6.39 (9)
3.0 (2)	2.52 (0)	8.19 (10)	3.0 (7)	1.80 (-1)	5.84 (9)
5.0 (2)	2.23 (0)	7.24 (10)	5.0 (7)	1.59 (-1)	5.18 (9)
7.0 (2)	2.05 (0)	6.67 (10)	7.0 (7)	1.47 (-1)	4.77 (9)
1.0 (3)	1.88 (0)	6.10 (10)	1.0 (8)	1.34 (-1)	4.36 (9)
1.5 (3)	1.69 (0)	5.50 (10)			
2.0 (3)	1.57 (0)	5.10 (10)			
3.0 (3)	1.41 (0)	4.58 (10)			
5.0 (3)	1.23 (0)	3.99 (10)			
7.0 (3)	1.12 (0)	3.63 (10)			
1.0 (4)	1.01 (0)	3.28 (10)			
1.5 (4)	9.00 (-1)	2.93 (10)			
2.0 (4)	8.29 (-1)	2.69 (10)			
3.0 (4)	7.39 (-1)	2.40 (10)			
5.0 (4)	6.42 (-1)	2.09 (10)			
7.0 (4)	5.88 (-1)	1.91 (10)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

Table 3.3h

 β Energy Decay Rate Versus Time After Shutdown(Time of Operation at Unit Fission Rate or Power: $\tau = \infty$ hrs)

t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec	t, time after shutdown	β Mev/sec fiss/sec	β Mev watt-sec
1.0 (0)*	7.86 (0)	2.55 (11)	1.0 (5)	6.08 (-1)	1.98 (10)
1.5 (0)	7.50 (0)	2.44 (11)	1.5 (5)	5.61 (-1)	1.82 (10)
2.0 (0)	7.21 (0)	2.34 (11)	2.0 (5)	5.32 (-1)	1.73 (10)
3.0 (0)	6.77 (0)	2.20 (11)	3.0 (5)	4.97 (-1)	1.62 (10)
5.0 (0)	6.19 (0)	2.01 (11)	5.0 (5)	4.60 (-1)	1.49 (10)
7.0 (0)	5.81 (0)	1.89 (11)	7.0 (5)	4.38 (-1)	1.42 (10)
1.0 (1)	5.42 (0)	1.76 (11)	1.0 (6)	4.16 (-1)	1.35 (10)
1.5 (1)	5.00 (0)	1.62 (11)	1.5 (6)	3.93 (-1)	1.28 (10)
2.0 (1)	4.71 (0)	1.53 (11)	2.0 (6)	3.77 (-1)	1.23 (10)
3.0 (1)	4.32 (0)	1.41 (11)	3.0 (6)	3.56 (-1)	1.16 (10)
5.0 (1)	3.88 (0)	1.26 (11)	5.0 (6)	3.30 (-1)	1.07 (10)
7.0 (1)	3.60 (0)	1.17 (11)	7.0 (6)	3.14 (-1)	1.02 (10)
1.0 (2)	3.33 (0)	1.08 (11)	1.0 (7)	2.97 (-1)	9.65 (9)
1.5 (2)	3.04 (0)	9.87 (10)	1.5 (7)	2.79 (-1)	9.05 (9)
2.0 (2)	2.84 (0)	9.25 (10)	2.0 (7)	2.66 (-1)	8.65 (9)
3.0 (2)	2.59 (0)	8.42 (10)	3.0 (7)	2.49 (-1)	8.09 (9)
5.0 (2)	2.30 (0)	7.47 (10)	5.0 (7)	2.29 (-1)	7.44 (9)
7.0 (2)	2.12 (0)	6.89 (10)	7.0 (7)	2.16 (-1)	7.03 (9)
1.0 (3)	1.95 (0)	6.32 (10)	1.0 (8)	2.03 (-1)	6.61 (9)
1.5 (3)	1.76 (0)	5.72 (10)			
2.0 (3)	1.64 (0)	5.33 (10)			
3.0 (3)	1.48 (0)	4.81 (10)			
5.0 (3)	1.30 (0)	4.21 (10)			
7.0 (3)	1.19 (0)	3.86 (10)			
1.0 (4)	1.08 (0)	3.51 (10)			
1.5 (4)	9.70 (-1)	3.15 (10)			
2.0 (4)	8.99 (-1)	2.92 (10)			
3.0 (4)	8.08 (-1)	2.63 (10)			
5.0 (4)	7.11 (-1)	2.31 (10)			
7.0 (4)	6.57 (-1)	2.14 (10)			

* Numbers in parentheses indicate the power of ten multiplying the given decimal number.

$$M = \frac{\beta \text{ Mev}}{\text{fiss-sec}}$$

$$P = \frac{\beta \text{ Particles}}{\text{fiss-sec}}$$

4.0 BREMSSTRAHLUNG IMPORTANCE

4.1 Introduction

The beta rays from the radioactive decay of fission products in the fuel region of a reactor are accompanied by four different components of gamma radiation:

- 1) Nuclear de-excitation radiation of the daughter nuclei from beta decay (discussed in Section 2.0);
- 2) External bremsstrahlung, which is a secondary radiation and results from the stopping of the beta particles in the material adjacent to the point of beta emission;
- 3) Inner bremsstrahlung, which occurs during the beta disintegration process itself; and
- 4) Characteristic x-rays which are of such low energy that they can be usually neglected. (This kind of radiation arises from transitions in the atomic shells of the target material due to the ejection of shell electrons during ionizing collisions with beta particles.)

A comparison of the total energy release per fission in each of these components shows that the nuclear de-excitation radiation is highly predominant, so that all of the other components can be neglected for most of the practical applications.

In fact, the total energy released through inner bremsstrahlung per fission is at least two to three orders of magnitude smaller than the energy carried away by the beta particles per fission*. The beta energy per fission, in turn, is comparable with the energy release per fission through nuclear de-excitation gammas. Thus, inner bremsstrahlung can be neglected in all practical cases.

Although it can be expected that the influence of external bremsstrahlung is not appreciable, it is worthwhile to estimate an upper limit for the amount of energy

* Compare: B. T. Price, C. C. Morton, and K. T. Spinney; Radiation Shielding (1957), p. 81

converted into gamma radiation by this effect. Usually this component of radiation was neglected, for the gross beta spectrum from fission products was not known until recently. In 1957 and in the current year, however, several authors have published experimental and calculated results for the shape of the beta spectrum from fission products*. A comparison of the findings by the various authors indicates that the curve obtained from Levy's investigation represents the true spectrum very well. Levy's spectrum pertains to the beta energy spectrum which is attained in equilibrium concentrations of fission products following a very long irradiation in a reactor. Therefore, this curve is picked as the basis of the following calculations for energy conversion into bremsstrahlung.

4.2 Maximum Effect of External Bremsstrahlung

Based on the theory of Bethe and Heitler, the average fraction of energy converted into bremsstrahlung during the stopping of an electron with initial kinetic energy, E mev, is, in the energy range of interest, given by

$$w(E,Z) = \frac{1.98 \times 10^{-4} (1.96 E + 2)}{(1 + 0.35 \log_{10}(\frac{82}{Z}))} Z^{**} \quad (1)$$

where Z is the atomic number of the target material. This formula holds for thick targets, i.e., targets in which the beta particles are completely stopped. In Figure 4.A, $w_k = w(E_k, Z)$ is plotted versus Z , with $E = E_k$ as parameter ($E_k = 1, 2, 3, \dots, 7$ mev). If the target material consists of a mixture of elements with atomic numbers Z_i , an effective atomic number $\bar{Z}$ can be calculated according to

$$\bar{Z} = \frac{\sum_i a_i Z_i^2}{\sum_i a_i Z_i} \quad (2)$$

* See #27 in the Appendix; R. W. King and J. F. Perkins, Inverse Beta Decay and the Two Component Neutrino; also references given therein.

** See: B. T. Price, C. C. Morton, and K. T. Spinney, Radiation Shielding (1957), p. 76

where a_i denotes the fractional number of atoms with the atomic number $Z = Z_i$.

Since bremsstrahlung production varies with the composition of the target material (in a heterogeneous reactor, for example, the fuel plates) an upper limit will be given herein.

The amount of energy radiated as bremsstrahlung increases with the atomic number of the material in which the electrons are brought to rest so that an upper limit for bremsstrahlung production in a reactor is obtained by assuming that the beta particles are completely stopped by uranium. Thus, with $Z = 92$, Equation (1) becomes

$$w(E,92) = .03634E + .03708.$$

This straight line is plotted in Figure 4.B. The total amount of beta energy converted into bremsstrahlung per fission is given by

$$E_{Br} = \int_0^{6.75 \text{ meV}} n(E)w(E,92)EdE \quad (3)$$

where $n(E)$ is the number of beta particles per fission per unit energy (see Appendix, 27, Levy's curve); the spectrum can be neglected beyond 6.75 meV. If Equation (3) is integrated numerically, the result is $E_{Br} = .877$ meV per fission. (The corresponding value for Al is only .068 meV.)

It should be noted that the computed value of the upper limit, $E_{Br} = .877$ meV/fission, is subject to some error: Equation (1) of this section, by which the result is obtained, is based on the original theory of Bethe and Heitler which, using Born's approximation, is not accurate for high atomic numbers. However, it has been found that the deviation of the theoretical predictions from experimental results are not very appreciable in this case even at very high atomic numbers*. It is estimated that the calculated value of $E_{Br} = .877$ meV/fission might be too high by about 12-15%. Thus, the upper limit for external bremsstrahlung in a reactor is about $E_{Br} = .75$ meV/fission.

* Compare, for instance, Heisenberg, Kosmische Strahlung, (1953) p. 326 and 354

4.3 Energy Spectrum of External Bremsstrahlung

The energy distribution of bremsstrahlung energy radiated along the whole track of the electron, i.e., resulting from the complete stopping, is given approximately by the expression*

$$I(h\nu, E) = C \left\{ 4 \left(1 - \frac{h\nu}{E} \right) - 3 \frac{h\nu}{E} \ln \left(\frac{E}{h\nu} \right) \right\} \quad (4)$$

(gamma energy per unit energy range),

where $h\nu$ is the gamma energy and the parameter, E , the initial energy of the beta particle. This formula, which has been derived from Bethe and Heitler's theory is in good agreement with experimental results.

The constant, C , in Equation (4) may be determined for any beta emission spectrum from the fact that the total bremsstrahlung energy, given by

$$\int_0^{6.75 \text{ mev}} n(E) \left[\int_0^E I(h\nu, E) d(h\nu) \right] dE,$$

must be equal to E_{Br} , given by Equation (3). Therefore,

$$C = \frac{\int_0^{6.75 \text{ mev}} n(E) w(E, \bar{Z}) dE}{\int_0^{6.75 \text{ mev}} n(E) \left[\int_0^E \left\{ 4 \left(1 - \frac{h\nu}{E} \right) - 3 \frac{h\nu}{E} \ln \left(\frac{E}{h\nu} \right) \right\} d(h\nu) dE \right]} \quad (5)$$

where $\bar{Z}$ is computed by Equation (2). The energy spectrum of bremsstrahlung energy per fission is then given by

* See S. J. Wyard, Proc. Phys. Soc., A65, p.377 (1952), or, B. T. Price, C. C. Morton, and K. T. Spinney; Radiation Shielding (1957) p. 78

$$\phi_e(h\nu) = \int_0^{6.75 \text{ mev}} I(h\nu', E) n(E) dE \quad (6)$$

(gamma energy per unit energy range),

and the energy spectrum for the number of γ quanta is given by

$$\phi_p(h\nu) = \int_0^{6.75 \text{ mev}} \frac{I(h\nu', E)}{h\nu'} n(E) dE \quad (7)$$

(photons per unit gamma energy range per fission).

Equations (6) and (7) can be used to obtain the bremsstrahlung energy and photon emission rates per fission in any energy interval by simply changing the energy interval of the integrals to correspond to the energy interval of interest.

4.4 Conclusions

If it is assumed that the bremsstrahlung production occurs in material which is 100% uranium, the upper limit of the contribution of bremsstrahlung energy is less than 15% of the total de-excitation gamma energy released from fission products per fission. This percentage falls rapidly to about 1% if the beta stopping material has an effective Z of 27 (aluminum).

The influence of external bremsstrahlung lies well within the error limits (about $\pm 20\%$) both of experimental and calculated results. The effect is clearly negligible if the effective Z of the beta stopping material is small.

Figure 4.A

Fraction of Beta energy converted into
Bremstrahlung versus Atomic Number Z .
($E \equiv$ Energy of Beta particle)

Figure 4.B

Fraction of Beta Energy Converted Into
Bremstrahlung Versus Beta Particle Energy
For $Z = 92$.

5.0 AVERAGE GAMMA ENERGY GROUP CROSS SECTIONS FOR PHOTONEUTRON PRODUCTION WITH D AND Be

High-energy gamma rays from fission products produce photoneutrons in a reactor if deuterium and/or beryllium are present.

The thresholds for the two (γ, n) -reactions, with D and Be, are 2.23 mev and 1.63 mev, respectively.

The group-averaged cross-sections for these two reactions have been determined for a typical γ -spectrum (one existing in a reactor after 8×10^3 sec and corresponding to the Oak Ridge continuous fuel belt experiment) according to

$$(\bar{\sigma}_{\gamma, n}) = \frac{\int_{E_g^l}^{E_g^u} \sigma_{\gamma, n}(E) n(E) dE}{\int_{E_g^l}^{E_g^u} n(E) dE},$$

where $n(E)$ denotes the γ -spectrum from the fuel belt as given in reference 26 of the Appendix. The results are listed in Table 5.0*.

The curves of $\sigma_{\gamma, n}(E)$ for deuterium and beryllium are taken from Weinberg and Wigner, The Physical Theory of Neutron Chain Reactors, 1958, p. 30. For Be, the curve has been modified such as to fit the experimental point values instead of presenting the theoretical results. The two curves are shown in Figure 5.A; they are essentially identical with those presented in Figure 2.1.2 of the Reactor Handbook (Physics)**. For Be, $\sigma_{\gamma, n}$ in the groups above 3.0 mev is not given in Table 5.0 because of a lack of data in that range.

* AECD - 3645, Reactor Handbook, Vol I, 1955

** Ibid.

Table 5.0
THE AVERAGE (γ, n) CROSS SECTION OF D AND Be
FOR EACH GAMMA ENERGY GROUP

Group No.	Energy Range (in Mev)	$(\sigma_{\gamma, n})_g$ in Millibarns	
		Be	D
1-3	.1 -1.35	0	0
4	1.35-1.8	0.12	0
5	1.8 -2.2	0.42	0
6	2.2 -2.6	0.38	0.76
7	2.6 -3.0	0.68	1.50
8	3.0 -3.5		1.77
9	3.5 -4.0		1.93
10	4.0 -4.5		2.07
11	4.5 -5.0		2.19
12	5.0 -5.5		2.29

If it is assumed that the energy spectrum within each gamma energy group does not change appreciably with time, then the number of photoneutrons produced per cm^3 per sec for each group, g , is proportional to

$$N \times (\overline{\sigma_{\gamma, n}})_g \times F_g(\tau, t),$$

where N denotes the number of D or Be nuclei, respectively, per cm^3 , $(\overline{\sigma_{\gamma, n}})_g$ is the average (γ, n) cross section of the g^{th} group, and $F_g(\tau, t)$ is the number of photons per sec released in the g^{th} group.

Note: Other methods exist for obtaining the photoneutron effects in a reactor. See Section 8.0.

6.0 EFFECT OF NEUTRON FLUX MAGNITUDE

ON THE DECAY RATE

6.1 Introduction

The effect of the neutron flux magnitude on fission product decay rate is felt primarily through changes in the fission rate during operation. R , the fission rate, is directly proportional to the neutron flux so that the decay rate of gammas or betas is directly proportional to the neutron flux level maintained during the operating period.

Secondary effects of the neutron flux magnitude are felt in that unstable nuclides in the decay chains, which determine the rate of fission product decay, are altered or added by the absorption of neutrons in the various fission product nuclides. There are changes in the concentrations of unstable nuclides due to neutron absorption, and also the stable nuclides can absorb neutrons and, in some cases, become unstable nuclides. The first effect can alter the decay rate by changing the concentrations of decaying nuclides. The second effect increases the decay rate by increasing the effective length of existing decay chains or by producing new decay chains.

It can be shown that in fully enriched U^{235} reactors practical values of ϕ and $\phi\tau$ (neutron flux and neutron flux times the operating time, respectively) assure that these secondary effects due to neutron absorption in the fission products are small if one is interested in decay rates at less than a few hours after shutdown.

Let it be assumed that in a fully enriched U^{235} reactor practical limitations require that

$$a. \quad \sigma_{a25}\phi\tau \leq .693 \quad (1)$$

(this corresponds to 50% U^{235} burnout),

$$b. \quad \phi \leq 10^{15} \text{ neutrons/cm}^2\text{-sec,}$$

where, ϕ = neutron flux during operation,
 τ = effective operating time, and
 σ_{a25} = effective absorption cross section of U^{235} .

If all absorption cross sections are referred to point values at 2200 m/sec neutron velocity, one may take $\sigma_{a25} = 690$ barns, so that limitation (1) a. becomes

$$\phi\tau = 10^{21} \text{ neutrons/cm}^2. \quad (2)$$

6.2 The Effect of Neutron Absorption in the Unstable Nuclides

The absorption of a neutron in an unstable nuclide has, first, the effect of reducing the corresponding fraction of the decay rate due to this nuclide. The nuclide produced by the absorption can be stable or it can be unstable; and in the latter case the decay rate will be increased by the activity of the nuclide produced. A survey of the unstable nuclides listed by Perkins and King (Nuclear Science and Engineering, Vol 3, #6, June, 1958) which were used to compute fission product decay after shutdown indicates that only about 20% of the nuclides are transformed to stable nuclides upon absorption of a neutron.

The unstable nuclides produced by absorption of neutrons in the original unstable nuclides can be expected, as a group, to have roughly the same decay properties as the original group. (On a purely statistical basis, the new group should contain, however, a somewhat greater number of shorter lifetime nuclides.)

If one considers an unstable nuclide of decay constant $\lambda \text{ sec}^{-1}$, cumulative yield y , and absorption cross section $\sigma \text{ cm}^2$, and assumes that the predecessors of the nuclide are of short lifetimes, then the number of these nuclides formed during an operating period, τ , is

$$\frac{YR}{(\lambda + \sigma\phi)} \left[1 - e^{-(\lambda + \sigma\phi)\tau} \right], \quad (3)$$

where R is the fission rate in fissions/second. The ratio of this number to the number of nuclides which would result under the same conditions if σ were zero is

$$F \equiv \frac{\lambda}{(\lambda + \sigma\phi)} \left[\frac{1 - e^{-(\lambda + \sigma\phi)\tau}}{1 - e^{-\lambda\tau}} \right]. \quad (4)$$

It is clear from expression (4) that the nuclides most affected are those of large lifetime (lifetime, $\ell = 1/\lambda$) and large cross section.

For any given value of $\sigma\phi\tau$, the smallest value F can assume (as $\lambda\tau \rightarrow 0$) is

$$F_{\min} = \frac{(1 - e^{-\sigma\phi\tau})}{\sigma\phi\tau} ; \quad (5)$$

and in no case is F less than $\lambda/\lambda + \sigma\phi$. For example, if $\sigma\phi\tau \leq .1$ and $\sigma\phi/\lambda \leq .05$, then F is greater than 0.95; i.e., the fraction of the decay rate due to such a nuclide is not reduced by more than 5%.

It is reasonable to assume that, if an unstable nuclide absorbs a neutron, the maximum effect upon the decay rate after shutdown is achieved if the nuclide formed is stable (or very long-lived) and that the activity after shutdown due to this influence is therefore reduced. If the nuclide formed is again unstable it can be assumed that the increase in the total decay rate at times less than a few hours after shutdown is a negligible percentage, for there is significant neutron absorption only in the longer-lived nuclides for which $\sigma\phi > \lambda$ and the sum of the yields per fission of such nuclides is a small percentage of the sum of the yields of the shorter-lived nuclides.

It is also noted that if the operation time, τ , is increased from, say, τ_1 to τ_2 , the corresponding increase in the decay rate at any time following shutdown is due to the increase in the number of nuclides which are of life-time greater than about $\tau_2/2$, for all others are effectively saturated. (If $\ell = \tau/2$ for a nuclide, the nuclide attains a concentration greater than 85% of the saturated value at τ .)

It follows from these considerations that if the operation time is increased from τ_1 to τ_2 and there are no nuclides with $\ell \geq \tau_1/2$, $\sigma \leq 1/10\phi\tau_2$, and $\sigma \leq 1/20\phi\ell$, then the increase in decay rate (due to the increase in operation time) at any time after shutdown can be affected by no more than -5% by neutron absorption in unstable nuclides. It is noted also that if $\sigma \leq 1/\phi\tau_2$, F is still greater than 0.68.

On this basis, it is found that if the operating time is ten hours ($\tau = 3.6 \times 10^4$ sec) and $\phi \leq 10^{15}$ neutrons/cm²-sec, a nuclide must have a $\sigma \leq 2,800$ barns if the fraction of the decay rate after shutdown due to the nuclide is to be affected by more than -5%. The only two unstable nuclides in the Perkins and King listing for which this occurs are Xe¹³⁵ and Sm¹⁵¹. Perkins and King (in the Nuclear Science and Engineering article)* examined the effect of Xe¹³⁵ and

* See Appendix, 25

found that for a flux range of 10^{12} to 10^{15} n/sec-cm² the decay rate of energy Group 1 was changed by only 25% in the neighborhood of 10^5 sec after shutdown. The effect at times less than a few hours after shutdown may, therefore, be assumed small even in Group 1. Sm^{151} has a low yield (.450%) and emits only very soft gammas (.02 mev) and soft betas (average energy $\pm$.02 mev). Therefore, the effect of this nuclide also may be neglected.

According to the General Electric Chart of the Nuclides and A/CONF. 15/P/2483, USA, 12 August, 1958 by Donald J. Hughes, none of the other unstable nuclides listed by Perkins and King have reported values of over 1000 barns, and only one nuclide, Pr^{143} , is of lifetime greater than 10^4 sec and of cross section over 100 barns. (σ_a for Pr^{143} is 124 barns, according to the latter reference.)

Now, if, by restriction (2), $\phi\tau \leq 10^{21}$, then σ must be greater than 100 barns if the fraction of the decay rate due to a given nuclide is reduced by more than 5% and σ must be greater than 1000 barns if the rate is reduced by more than 32%. A rough survey of the Chart of Nuclides indicates that about 20% of all the reported fission product cross sections are more than 100 barns. It is reasonable to assume that no more than 20% of the unstable nuclides have cross sections greater 100 barns, and that of these 20% insignificantly few might have a cross section over 1000 barns.

After ten hours of operation, the decay activity at about an hour after shutdown is already half of the value corresponding to an infinite operation time ($\tau_2 \rightarrow \infty$). Hence, the upper limit of the effect on the decay activity at times less than or equal to an hour is $(.5)(.32)(.20) = 3.2\%$, if it is assumed that the entire 20% of the nuclides of lifetime more than five hours ($\tau_1/2$) have 1000 barn cross sections. It can be safely concluded, therefore, that neutron absorption in unstable nuclides produces a very small effect on the decay rate at times after shutdown less than a few hours if $\phi \leq 10^{15}$ and $\phi\tau \leq 10^{21}$.

6.3 The Effect of Neutron Absorption in Stable Nuclides

The upper limit of the effect on shutdown activity of neutron absorption in stable nuclides can be estimated in terms of the conditions previously determined for a fully enriched reactor in which $\phi\tau \leq 10^{21}$. Consider a nuclide which absorbs a neutron to become an unstable nuclide of lifetime, $\ell = 1/\lambda$. The activity at shutdown due to the unstable nuclide is given by

$$YR \left[(1 - e^{-\lambda\tau}) - [\lambda/(\lambda - \sigma\phi)](e^{-\sigma\phi\tau} - e^{-\lambda\tau}) \right], \quad (6)$$

where σ is the cross section of the stable nuclide, Y the effective yield (the cumulative yield if the lifetimes of predecessors in a decay chain are short compared to $1/\sigma\phi$), and R is the fission rate during operation of the reactor. It is apparent that, if $\sigma\phi$ is small, the activity of the produced unstable nuclide becomes small too. If this activity were due to a normally occurring unstable nuclide of direct fission yield, the activity at shutdown would be

$$YR(1 - e^{-\lambda\tau}). \quad (7)$$

The ratio of expression (6) to expression (7) represents a measure of the disadvantage an unstable nuclide suffers in being formed by neutron absorption instead of by direct fissioning or short-lived decay of fission product predecessors. One may define the effective yield, $Y_e = GY$ for the unstable nuclide which is formed by neutron absorption. The ratio, G , is

$$G = \left[1 - \left(\frac{\lambda}{\lambda - \sigma\phi} \right) (e^{-\sigma\phi\tau} - e^{-\lambda\tau}) / (1 - e^{-\lambda\tau}) \right]. \quad (8)$$

This ratio is always less than unity and less than $\sigma\phi\tau$. Sm^{152} , Eu^{153} , and Eu^{151} are the only stable nuclides which have cross sections over 50 barns and are transformed into unstable nuclides upon absorption of a neutron, according to the General Electric Chart of the Nuclides, 1956. The sum of the total yields of these nuclides is less than 1% and the unstable nuclides formed have lifetimes of greater than 10 hours. Therefore, these nuclides can have only a very negligible effect upon the decay rate until, perhaps, a few years after shutdown.

It follows that, if $\phi\tau$ is limited to 10^{21} , $\sigma\phi\tau$ cannot be greater than .05 for all the other stable nuclides which can form unstable ones. Only about ten of these nuclides have a cross section of more than 10 barns, corresponding to $\sigma\phi\tau \approx .01$.

Therefore, only about ten nuclides can produce unstable nuclides with effective yields greater than .01 Y . If Y were 6% for each of the ten nuclei, and each nuclide were of cross section 50 barns, the total effective yield would

be only 3%. It is clearly indicated that at times up to a few hours after shutdown, there can be no significant effect on the decay rate due to the small numbers of unstable nuclides produced by neutron absorption in stable nuclides, so long as $\phi\tau \leq 10^{21}$.

At long times after shutdown, the decay rate could be greatly affected, however, even for $\phi\tau \leq 10^{21}$. At about a year after shutdown there are so few nuclides contributing significantly to the total decay rate that a small production of some long-lived nuclides could easily produce an order of magnitude change in the decay rate. Also, as $\phi\tau$ increases (as it might in a breeding or converting reactor), the effective yield of an unstable nuclide produced by neutron absorption will approach the yield of the stable parent nuclide, even if the stable nuclide is of small cross section.

23 07

7.0 EFFECTS OF NEUTRON ENERGY

7.1 Introduction

The energy of the neutrons causing fission is of influence on the gross decay characteristics of the fission products due to two effects:

- 1) Modification of the mass distribution;
- 2) Change of the distribution in Z , the atomic number, along the decay chain for a fixed mass number, A .

For thermal neutron fission, the distributions in mass and charge of the fission products have been extensively studied by many investigators. Data are also available for higher neutron energies, however, this information does not yet reveal the detailed dependence of the distribution curves on the neutron energy.

A compilation of recent data along with a brief survey was published by S. Katcoff in Nucleonics, April, 1958*. For thermal fission of U^{235} , in particular, Perkins and King have compiled, partially based on Katcoff's list, a table** which also includes the half-lives and decay schemes of the individual nuclides. This table directly provides the input for the computer program employed by these investigators. The calculations can, of course, be easily extended to the case of fission at higher energies if the corresponding fission yields are known.

Herein it is attempted to estimate only whether or not the deviation from the gross fission product decay at thermal fission is very appreciable as the neutron energy changes from thermal up to about 1 mev. For this purpose, the two previously mentioned effects are considered in more detail.

* See Appendix, 18

** Table I in Appendix 25 and private communication

7.2 Mass Yield Distribution

The first question arising is: does the mass distribution curve change considerably?

The mass distribution curve of the fission products from thermal U^{235} fission is characterized by its peaks in the two mass regions between 90 and 101 and between 133 and 144, as well as by the deep trough between the two humps. The ratio of the peak values to the value of the lowest point in the valley of the curve is about 600 in this case, manifesting the high predominance of asymmetric fissions. The peaks exhibit "fine-structure" which is ascribed to shell effects.

In U^{235} fission by 14 mev neutrons, symmetric fission is much more probable. There the ratio of the yields at the peaks and the valley is only about 6 and the fine-structure at the peaks is absent. The area below the fission yield curve totals 200% independently of the particular shape of the curve, so that a rise of the symmetric fission yield causes a corresponding drop in the peak values.

This behavior is qualitatively understood and can be explained in terms of the excitation energy of the compound nucleus upon absorption of the neutron. Whereas at low energy fission the probability for asymmetric fission modes is much higher than for symmetric modes, the latter become energetically less unfavorable with increasing excitation energy. The peaks of the yield curve will, therefore, become less pronounced with increasing neutron energy, as seen in the case of 14 mev neutrons. Of course, the peaks will drop only slightly as long as the bottom of the trough is not raised to values comparable to the peak values. For fission by fission spectrum neutrons, for instance, where the minimum of the yield curve is still more than by a factor of hundred below the peaks, the height of the peaks is practically the same as at thermal fission. This does not seem very surprising if the excitation energies for the two cases, thermal and fast fission, are compared with one another. For 14 mev neutrons, however, the change in excitation energy is significant, for, in this case, the contribution of the kinetic energy of the neutron to the excitation energy of the ($U^{235}+n$)-nucleus is about twice as high as the amount contributed by the binding energy of the neutron.

The values quoted in the previous paragraphs may be compared to a curve given by Hill and Wheeler*. The authors plotted the fission yield ratio $\text{Ag}^{111}/\text{Ba}^{140}$ versus the excitation energy (in mev). This diagram shows that this ratio, which is a measure of the asymmetry of fission, varies uniformly with the excitation energy. The ratio remains below 1/100 up to an excitation energy of about 8 mev.

At resonance energies, where a different distribution in the relative probabilities of mass division might be expected, Nasuhoglu, et. al.** have measured the fission yields from neutron fission between 1 and 10 ev. The results indicate no significant deviation from those established for thermal fission.

From the preceding considerations it can be concluded that the deviations from the gross fission product decay characteristics of thermal fission are not very appreciable, due to the considered effect, for neutron energies of interest herein, i.e., up to about 1 mev. Eventual changes in the fine-structure of the peaks themselves could influence the behavior in decay but it is not believed that this effect alone, if present, would be so severe as to modify the decay curves appreciably, especially in the time region up to a few hours after fission (or shutdown) in which many nuclides are contributing to the gross decay rate.

7.3 Charge Distribution

The second point to be considered is the shift of the charge distribution curve for a given mass number, A , with the neutron energy.

The independent fission yields of the individual members (elements) in the decay chain of a given mass, A , have been found to be approximately distributed according to a Gaussian curve about a most probable value, Z_p , of charge. By using Pappas' modification*** of Glendenin's postulate of equal chain lengths one can easily find Z_p as a function of A ****.

* Phys. Rev. 89, 1102, (1953), (Fig. 53, p. 1145)

** Phys. Rev. 108, 1522, (1957)

*** See, for instance, Geneva Papers, P/881

**** See also: Physics of Nuclear Fission (International Series of Monographs on Nuclear Energy), 1958; Chapter 3, A. N. Murin, p. 37

The shape of the distribution curve (half-width) is roughly independent of the mass number, so that its "universal" character enables one to estimate easily the independent fission yields of those chain members (in long decay chains) which are not accessible to direct measurement. Since it is very difficult, in many cases, to find the independent yield experimentally, predictions on the basis of a general distribution law are very helpful, and have, indeed, been widely used in establishing the systematics of independent fission yields.

For 14 mev neutron fission, as compared to thermal neutron fission, it is found that the charge distribution curve is shifted toward the beta-stable end of the decay chains*. This effect can possibly be explained by the fact that more neutrons are "boiled off" in high energy fission than in thermal fission. The half-width of the distribution curve is assumed to remain unchanged by the shift. Wahl** studied the charge distribution shift in the mass region $A = 131-135$, for 14 mev neutrons and found a displacement in Z_p of 0.7 charge units toward beta-stability from the position for thermal neutron fission.

This shift, in effect, shortens the decay chain length and thereby influences the gross fission product decay pattern. By arguments concerning the excitation energy (similar to those used in connection with the mass yield distribution), one can conclude, however, that the shifting of the charge distribution curve is still negligible at neutron energies of primary interest in reactors.

On the grounds of the assessed smallness of the two effects, it can be expected that the gross decay pattern of the fission products is not appreciably influenced by variations in the mean neutron energy causing fission in the range up to about 1 mev.

* G. P. Ford, AECD-3597 (1953)

** Phys. Rev. 99, 730 (1955)

8.0 FISSION PRODUCT DECAY RATES

FOLLOWING A REACTOR SHUTDOWN

The formulae for the beta and gamma decay rate due to fission products in a reactor apply, by definition, to a reactor of operating period, τ sec, during which the reactor power or fission rate is not zero. Reactor "shutdown" refers, then, to the instant at which the fission rate, R , becomes zero. In many cases, the fission rate is reduced to so small a value with respect to the previous level within a few seconds after a fairly large amount of negative reactivity is introduced that R may be safely considered to be zero and the "time after shutdown", t , may be considered to be known with an accuracy of a few seconds. However, it is often the case that the reactivity in the reactor is reduced slowly or the total amount of negative reactivity ($K-1/K$) introduced is not very large so that the effective sources (delayed neutrons and photoneutrons, primarily) cause a much more gradual reduction in the fission rate. In these cases, the reactor power due to fissions exceeds the fission product decay energy release rate for a significant length of time following the start of power decline. When this occurs, terminating the operating time, τ , at the point where shutdown only begins to occur must obviously be avoided in order to be able to evaluate the fission product decay rates after start of shutdown. The time after the introduction of negative reactivity must be regarded as part of the operating time, τ , until a time is reached at which the reactor power is really negligible.

If one is interested in the decay rate of the fission products during the power decline of the shutdown process one must make use of the given formulae for $U_g(\tau)$ and $V_g(\tau)$ (Section 2.0) and/or $Q(\tau)$ (Section 3.0) which apply to an operating reactor. The integrals in these formulae can always be evaluated numerically so long as the fission rate $R(\tau)$ is known. $R(\tau)$ can be calculated from the set of equations consisting of the reactor kinetic equation and the six delayed neutron equations if no photoneutrons are present. If deuterium is present in the reactor, eight other equations must be added to the set to account for the production of photoneutrons from the absorption of the high energy gammas from eight resolved groups of known half-life and photoneutron yield*. For beryllium, also, half-lives and photoneutron

* See Phys. Rev. 71, 573, (1947), S. Bernstein, et.al.

yields have been resolved*, so that equations can be added to the original set of reactor kinetics equations in order to solve for $R(\tau)$ after the beginning of shutdown. The sets of first order differential equations so formed are easily solved by a number of simple numerical methods for this time region during which the reactor is subcritical.

* See A/CONF. 15/P/2146, USSR, 7 July, 1958, Physical Characteristics of Beryllium Moderated Reactors, by A. K. Krasin, et.al.

9.0 COMPARISON OF GAMMA ENERGY DECAY RATES

ACCORDING TO VARIOUS SOURCES

The J. F. Perkins and R. W. King paper in Nuclear Science and Engineering, Vol 3, No. 6, June, 1958, shows a comparison of their results with other commonly used data for fission product decay following an instantaneous fission event. In the Nuclear Science and Engineering paper, the data are given for a time range of 10^2 to 10^8 sec after fission, and the decay rates versus time are compared for gamma energy release, total energy (gamma + beta + anti-neutrino) release, gamma plus beta energy release, and total disintegration.

Figure 9.A is a comparison of the results obtained in Section 2.0 for the total gamma energy release rate to the gamma energy rates shown in the Perkins and King paper. The time scale, however, is extended down to 1 sec and terminated at 10^6 sec after fission.

The data of Section 2.0 correspond well to the experimental results of Borst for times after shutdown of over an hour. The Way-Wigner rule-of-thumb expression ($1.26t^{-1.2}$) is also satisfactory at times above about twenty hours, but below these times it is impossible for the single straight line on the log-log graph to follow the shape of the gamma energy decay rate.

A report by S. Untermyer and J. F. Weills, Heat Generation in Irradiated Uranium, AECD-3454, February 25, 1952, presents total gamma and beta energy decay rates after shutdown for various times of reactor operation. The data in the latter report for 1 sec of reactor operation are shown only up to 10^2 sec after shutdown. If the gamma and beta energy decay rates of Sections 2.0 and 3.0 are added to obtain the total it is found that there is good agreement (for an instantaneous fission event) with the (1 sec operation) data of AECD-3454. The data of the latter report are consistently low compared to the data of Sections 2.0 and 3.0 up to 10^2 sec after fission, although there is less than about 5% difference at 10^2 sec. The percentage of the difference increases rapidly for shorter times after fission to about 40% at 10 sec after fission. This difference at short times after fission may be understood at least partly in terms of the fact that the value for the fission product heat production (in AECD-3454) attains a maximum of about 5.7% of the total fission heat production after long irradiation. If the total fission

energy is 190 mev, for example, this percentage corresponds to a total gamma and beta energy release of 10.8 mev. A more probable value is 13.9 ± 0.5 mev, so that it is to be expected that the 20% difference in total energy release is caused primarily by an underestimation of the decay rate below about 10^2 sec by the AECD-3454 data.

Figure 9.A

APPENDIXABSTRACTS FROM THE LITERATURE SURVEY

This compilation is the result of a literature search performed in connection with work on the beta and gamma decay of U^{235} fission products with primary interest on short periods after reactor shutdown. Papers, published or prepared for publication, and unclassified reports on fission product decay are listed in chronological sequence and are briefly abstracted. Included is literature of more general or different character as far as it is found helpful in investigating the problem. The literature search is essentially restricted to work performed in the U. S.

Efforts are made to present a compilation valuable for direct study of the more recent results in this field with the particular problem in mind rather than to enumerate all existing publications in the related fields. As to information not directly connected with the subject, the reader will find useful references in the extensive bibliography which is attached to some of the reports herein quoted. Such reports are marked by an asterisk.

In particular, there are five recent works which are deemed to represent the best sources of information at the present time. Three of them are experimental contributions of Oak Ridge National Laboratory* related to the gamma decay at short times after shutdown; the remaining two works** give the gross beta and gamma energy release rates calculated from the individual nuclides which represent the fission products. The experimental results cover the time range between 1.7 and 1550 sec. The calculated values are given between 10^2 and 10^8 sec. The methods to be used in investigating the problem, experimentally and analytically, are clearly illustrated and thoroughly carried through in these works. The results are directly used in this report.

* See numbers 17a, 17b, and 26 of this compilation

** See numbers 25 and 27 of this compilation

Abstracts

1. C. D. Coryell and N. Sugerman, Radiochemical Studies: The Fission Products, Book I, Part IV, 1951* Papers 34-43: Studies are reported of the beta and gamma rays emitted from gross fission products (paper 34). Papers 35 to 37 deal with the intensity, decay characteristics, and average energy of the beta and gamma radiations of unseparated fission products produced in uranium during short irradiations (10 sec to 5 min). The radiation from fission products during pile operation and after shutdown is estimated. The decay of gross fission products is discussed, on the basis of experimental results, in comparison with the decay of the sum of the separated fission products (papers 38 to 40). A direct calorimetric study of the decay of combined beta and gamma energy is described in paper 41; the results are compared with calculated values by another author (paper 42). Paper 43 is the well-known "Way-Wigner" work which presents a theoretical treatment of the gross fission product decay in a statistical approach.
2. J. E. Brolley, Jr. and M. S. Livingston, Short Period Gamma Rays from U²³⁵ Fission Products, LA-1188 (Jan. 1951): In the introduction of this report, earlier results of investigations of short-period activities are briefly summarized and discussed. It is concluded from the quoted works that there seems to be no evidence for the existence of decay periods in the m-sec range. The authors (and cooperators) have performed experiments to detect gamma ray activities with period 1-100 m-sec from thermal neutron fission of U²³⁵ which gave negative results. The shortest period observed was 0.43 seconds.
3. W. K. Ergen, Hard Gamma-Emitters Among Fission Fragments, ANP 59 (May, 1951): Hard gammas are investigated with respect to their (γ ,n) reactions with D and Be (photoneutrons).
4. J. Howlett, et.al., Tables of Fission Product Activities After Irradiation and Decay, AERE T/R-589: Tables with an introduction and an outline of the calculation method are given.
5. J. M. West, Calculations On U²³⁵ Fission Product Decay Chains, ANL 4807 (May, 1952): Calculations on the U²³⁵ fission product decay schemes are performed. Results for a typical example are tabulated and plotted. The method of calculation is briefly discussed.

6. J. Moteff, APEX - 134 (1953) (See Number 12 of this compilation.):
(The excerpt published in Nucleonics rather than the original work has been abstracted, because it includes modified data.)
7. A. L. Gluckstern, M. H. Hull, Jr., G. Breit; Photon Spectrum from Bremsstrahlung, YALE(LA)-3 (Aug., 1953):
The photon spectrum of bremsstrahlung is calculated. (As to refinements of the original Bethe-Heitler theory a short remark is given on page 6.)
8. E. D. Arnold, Activities of Fission Products from Core Solution of K-23 Reactor, CF-54-4-62 (April, 1954):
The individual activities of the major medium and long-life ($>2\frac{1}{2}$ days) fission products and their gross activity are calculated. For shorter times, the Borst-Wheeler data are used to approximate the total activity. Tables and graphs are included. An empirical equation describing buildup and decay of the major fission products has been determined.
9. F. H. Clark, Decay of Fission Product Gammas,* NDA 27-39 (Dec., 1954):
This report presents an extension of Moteff's work APEX - 134 (See Number 6) and it includes experimental information particularly with respect to decays at times shorter than treated by Moteff. Seven energy groups for the gammas are used, as introduced by Moteff. The author attempts to include all known gammas contributing significantly to dose between 30 minutes and 1000 days after shutdown. General comments to related problems are given, e.g., the influence of the power level on Xe¹³⁵ removal. The major part of the report consists of tables and curves.
10. P. T. Wagner, Calculating Half-Life for Low Activities of Short-Lived Nuclides, Nucleonics, Vol 13, No. 4 (April, 1955):
A method is described of calculating half-life, with direct application to radionuclides of short half-life and low activities.
11. F. E. Obenshain and A. Foderaro, Energy from Fission Product Decay, WAPD-P-652 (May, 1955):
This report is a compilation of data and equations describing the rate of energy release from fission products after a period, T_0 , of constant fissioning. The decay curve $f(t)$ has been fitted in two ways: by 1) a piece-wise continuous curve, and 2) a continuous curve which is a sum of exponentials.

A short note on the influence of delayed neutrons is made with reference to two papers wherein this problem is considered.

12. J. Moteff, Energy Spectrum of Gammas from Gross Fission Products, Nucleonics, Vol 13, No. 5, Page 28, (May, 1955): (The article is a condensation of APEX - 134 (1953)---compare with Number 6 of this compilation---and includes modified data.) The results of energy spectrum calculations are plotted showing the spectral lines at various times after shutdown, for the gammas from thermal-fission products of U²³⁵. Not included is bremsstrahlung caused by fission-product betas. Curves are shown of the energy decay rates as a function of time after shutdown for seven energy groups. The influence of operation time is discussed and illustrated by graphs. The data are compared with results of other authors. A table is presented of nuclides with their characteristic data as they are used in the calculations. The criteria applied in selecting the nuclides are quoted in brief and it is mentioned that some nuclides with high gamma energies were omitted because of their short half-life. The energy decay rates given in the table include a factor of about 1.04 to correct the fission yield to a modified standard of 6.37%. The method of calculation is outlined and the basic formula are presented. Although the numerical values are out-dated, this work must be considered, on methodical grounds, as the basis for the more recent calculations (Clark, and Perkins and King).
13. S. J. Wyard, Radiative-Source Corrections for Bremsstrahlung and Scatter, Nucleonics, Vol 13, No. 7, Page 44 (July, 1955): Correction terms are given for bremsstrahlung from beta rays and internal conversion electrons as well as for scattering. Intensity and spectral distributions are discussed.
14. D. H. Peirson, The Gamma Ray Spectrum of Fission Products from Slow Neutron Irradiation of U²³⁵, British Journal of Applied Physics, Vol 6, Page 444 (Dec., 1955): Reported are measurements of the gross fission product decay gamma-spectrum by means of a scintillation spectrometer. The experimental array as well as the evaluation of the measurements are described in detail. Indicated, also, is the scintillation spectrometer's use as an analytical tool in other applications. (See also Nature 173, Page 990.) The results are plotted and discussed.
15. E. D. Arnold, Fission Product Activities in Irradiated Natural Uranium, Enriched Uranium, and Thorium, ORNL-2039 (Dec.)(March, 1956): The total activities of the fission products of uranium and thorium are presented for neutron fluxes of 10^{12} - 10^{15} n/cm²/sec.

16. M. H. Kalos, A Monte Carlo Calculation of the Transport of Gamma Rays, NDA 56-7 (July, 1956):
The method, which uses two devices to speed the Monte Carlo calculation, is described in detail, and the program is discussed. A sample is given (Appendix C).
- 17a. R. W. Peelle, W. Zobel, T. A. Love, Measurement of the Spectrum of Short-Lived Fission Product Decay Gamma Rays Emitted from a Rotating Fuel Belt,* ORNL-2081 (Applied Nuclear Physics Division Annual Report, Sept. 10, 1956):
The authors point out that the major part of radiation from fission products is emitted at times smaller than those that allow radiochemical analysis.

Two methods of approach to determine short half-lived fission product gammas are suggested: Measurements (1) with rotating fuel belts and (2) with samples which are exposed for very short times to a flux of thermals and are then quickly removed to a gamma ray spectrometer. Both methods are used by the Oak Ridge group (see also Number 17b of this compilation).

Method 1: The rotating belt consists of fuel plate sections irradiated by thermal neutrons leaking from the reflector of the ORNL Graphite Reactor. The gamma ray scintillation spectrometer employed is of the multiple-crystal type developed for use at BSF (See ORNL-1142 and ORNL-1929). (It has a favorable characteristic: straight-forward relationship between voltage spectrum of pulses obtained from the spectrometer and the photon energy spectrum. Such a unique response is required for complicated spectra.) The results are plotted and discussed (accuracy is estimated). The photon energy spectrum is integrated between 0.36 and 5.8 mev to obtain an average of 4.8 mev/fission for this energy range.

- 17b. W. Zobel and T. A. Love, Time and Energy Spectra of Fission Product Gamma Rays Measured at Short Times after Uranium Sample Irradiations, ORNL-2081 (Sept. 10, 1956) - See 17a:

Method 2: The experiment consists of two parts: Study of a) the detailed time dependence of relatively large photon energy groups, and b) the detailed energy spectra measured at different times after fission. A description of the experimental equipment is presented, which is supplemented by paper #50 on Page 208 of the same progress report. Results are plotted: decay rate versus time and photon intensity versus photon energy. Estimated error: 20 to 25%.

18. New Nuclear Data, Nuclear Science Abstracts, Vol 11, No. 12B, (June 30, 1957):
This is a compilation of recent nuclear data.
19. C. F. Miller, Gamma Decay of Fission Products from the Slow-Neutron Fission of U²³⁵, Report USNRDL-TR-187:
Photon energy rates and emission rates from fission products produced by slow-neutron bombardment of U²³⁵ are computed. Determined, as a function of time after fission, are the number of photons and the mean photon energy per disintegration, the average energy per photon and the number of beta rays per photon. Shown are also the relative contributions of individual fission products. Tables and graphs are presented. Time range considered: 45 min - 300 years. (Due to the lack of decay scheme information for many short-lived nuclides, the calculations have not been performed for times shorter than quoted.) Recent data are used, including internal conversion data. The photon emission rates include all photons with energy greater than 20 kev.
20. G. de Saussure, A Calculation of the Gamma Ray Spectrum of the Bulk Shielding Reactor, ORNL 5707-105 (July, 1957):
Four sources of primary gamma rays are considered (during reactor operation): 1) prompt fission gammas, 2) capture gamma rays from Al, 3) capture gammas from U, and 4) fission product decay gamma rays. The high-energy portion of the gamma ray flux at the surface of the uranium-aluminum BSF reactor is determined. Four energy groups of the gammas are assumed.
21. J. O. Blomeke and M. F. Todd, Uranium 235 Fission Products Production as a Function of Thermal Neutron Flux, Irradiation Time, and Decay Time, Part I:
Levels of fission products resulting from thermal fission of U²³⁵ in reactor fuels are computed for a wide range of reactor operating conditions and decay times. Values for approximately 300 fission products are presented in graphical form together with gross totals of their activities, radiation powers, and thermal neutron poisoning. The gamma spectrum is further broken into four groups of specified energy ranges, which are suitable for use in shielding design. Tabulations of these properties are arranged as to chain or mass number, element, and as to the rare-gas and rare-earth groups. The calculations assume constant replenishment of U²³⁵, constant reactor power operation, and no fission-product separations during irradiation.

22. R. Nasuhoglu, et.al., Fission Yields from Epithermal Neutron Fission, Phys. Rev. 108, 1522 (1957):
Radiochemical analyses of samples of U^{235} irradiated with monoenergetic neutrons at 1.1, 3.1 and 9.0 ev. indicate that the fission yield of Ag^{111} , Cd^{115} , and Sb^{127} relative to the yield of Sr^{89} does not change from that produced by thermal neutrons.

23. D. Strominger, J. M. Hollander, and G. T. Seaborg, Table of Isotopes, Rev. Mod. Phys. Vol 30, No. 2, Part II (April, 1958):
The table represents a complete list of all the radioactive and stable isotopes of the elements, together with a number of their salient features, as recorded in the literature or by private communication by approximately February, 1958.

24. S. Katcoff, Fission Product Yields from U, Th and Pu,* Nucleonics, Vol 16, No. 4, Page 78 (April, 1958):
This article presents a compilation of recent data accompanied by a very clear and informative survey. Included are graphs and tables of re-normalized fission yields and decay chain schemes.

25. J. F. Perkins, and R. W. King, Energy Release from the Decay of Fission Products, Nuclear Science and Engineering, Vol 3, No 6, (June, 1958):
The total disintegration rates, rates of beta and gamma energy release, and gamma ray energy spectrum, are calculated for fission products due to thermal neutron fission of U^{235} . Information on decay schemes was largely obtained from the compilations of the Nuclear Data Group of the National Research Council to July, 1957. Total fission yields are from Katcoff and from Steinberg and Glendenin. Nuclear charge distributions are taken from Pappas' work, which includes the effect of closed shells. Reactor operating times of 1, 10, 100, and 1000 hours are treated, and the results for instantaneous operation are compared to other calculations and measurements. The present results fall below Way and Wigner's predictions of both disintegration rate and total energy release over the entire range of decay times, though they agree satisfactorily with the Way-Wigner rule-of-thumb expressions. The present results are in very good agreement with experimental measurements. The gamma spectrum is found to vary considerably with decay time but to be only a weak function of reactor operating time. The total beta and antineutrino energies per fission are found to be 7.6 ± 0.5 and 10.0 ± 0.7 , respectively.

115-089

26. R. W. Peelle, W. Zobel, T. A. Love, and F. C. Maienschein, Recent Results for the Energy Spectra of Fission - Associated Radiation,* (Unpublished): Current experimental results for the gamma ray energy spectra associated with thermal-neutron fission of U^{235} are discussed and compared with results obtained by other workers. The work is divided into three sections according to the length of the time interval between fission and the observation of the gamma rays; (1) a revised absolute time and energy dependence is shown for gross fission-product gamma rays emitted between 1 sec and 1600 sec after fission, (2) pulse-height spectra are given to illustrate the time dependence of gamma rays emitted between 5×10^{-8} sec and 10^{-6} sec after fission and (3) a new absolute energy spectrum of the prompt gamma rays from fission is presented for the energy range from 0.3 to 10 mev. The first has been made available by an improved analysis of previously reported experiments, while the other sections incorporate new data. Error estimates are included. The importance of these spectra to shielding is discussed.
27. R. W. King and J. F. Perkins, Inverse Beta Decay and the Two Component Neutrino,* UCRL-8337 (June, 1958): This work includes very valuable information with respect to the beta decay spectrum from U^{235} fission products. Discussed is the systematics of beta decay end points and the shape of the beta energy spectrum. (The average cross section is calculated per anti-neutrino from U^{235} fission products, for the inverse beta decay. The predictions of the two-component neutrino theory are found to be in accord with the experimental results.)