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FROM: D. K. Trubey, H. S. Moran, and Alvin M. Weinberg

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Abstract

The slowing-down distribution, to thermal energy, of neutrons from a U^{235} fission source in an infinite water medium up to 160 cm has been determined from experimental data. The Fourier transform of this distribution and the indium resonance distribution, which are the nonleakage probabilities for bare reactors, have also been determined.

Introduction

According to the usual theory of bare homogeneous neutron chain reactors,¹ the nonleakage probability in a finite reactor may be computed from the neutron slowing down distribution in an infinite system, the relation between them being that the nonleakage probability is the Fourier transform of the fission neutron point source slowing down distribution. Thus if $K(B)$ is the nonleakage probability for a bare reactor with geometric buckling B^2 , and $G(r)$ is the point source slowing down kernel, then

$$(1) \quad K(B) = \frac{\int_0^{\infty} G(r) \frac{\sin Br}{Br} 4\pi r^2 dr}{\int_0^{\infty} G(r) 4\pi r^2 dr}$$

where r is the distance from the source.

In the case of moderators which are heavier than deuterium, and to a lesser extent for deuterium itself, continuous slowing down theory is applicable and both $G(r)$ and $K(B)$ are superpositions of Gaussians. In the case of moderation by hydrogen, however, the continuous theory is not at all applicable; the experimentally observed kernel $G(r)$ deviates very strongly from a Gaussian shape. Such formalisms as the Goertzel-Selengut (G-S) method² give fairly good values for the slowing down kernel at small distances -- in fact, rather better values than can be derived from the exact transport equation with presently known cross sections. However, since the G-S method is not rigorous, it can be said that a reliable and consistent calculation of $G(r)$ and $K(B)$ for H_2O which agrees with experiment is at present lacking.

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1. S. Glasstone and M. C. Edlund, The Elements of Nuclear Reactor Theory, Chap. 12, D. Van Nostrand Co., 1952; see also Alvin M. Weinberg and E. P. Wigner, Theory of Chain Reactions, Univ. Chicago Press (to be published).
 2. G. Goertzel, "Criticality of Hydrogen Moderated Reactors," TAB-53 (July 1950); D. Selengut, private communication to G. Goertzel.

On the other hand, there is an abundance of measured data of $G(r)$ for neutrons in H_2O originating from a point source of fissioning U^{235} .³⁻⁵ It was therefore deemed worth while to compute $K(B)$ directly from the observed $G(r)$ for various values of B . The resulting Fourier transform should be useful first to help in the computation of nonleakage probabilities (and therefore critical masses) in bare H_2O reactors (or H_2O -reflected H_2O reactors);⁶ and second, since in most of the theoretical calculations $K(B)$ is computed initially, not $G(r)$, tabulated values of $K(B)$ computed from the experimental values of $G(r)$ according to Eq. 1 should be useful for comparing theoretical computations of the slowing down distribution with experiment. The present paper is a summary of such a computation of $K(B)$ for slowing down of U^{235} fission neutrons in H_2O .

Determination of $G(r)$

Values of $G(r)$ were obtained from two sets of independent experimental data. The first is the well-known slowing down distribution to indium resonance of Hill and Roberts,³ which gives results for distances from the source to about 92 cm. This distribution to indium resonance will be denoted the H-R distribution. The second is the measurement of the distribution of thermal neutrons from a fission source in the water tank of the Lid Tank Shielding Facility (LTSF).⁵ Because the detector was sensitive to thermal neutrons and the intensity of the source was sufficiently large, the distribution in the LTSF experiments could be measured for distances much further than the 92 cm of the H-R experiment; however, the depression of flux near the source was considerably larger than in the H-R experiment, and this makes the LTSF results unusable closer than 15 cm to the source. The difficulty near the source was largely eliminated in the H-R experiment by using a small source, for one series of measurements, located 10 cm from the bottom

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3. J. E. Hill, L. D. Roberts, and T. E. Fitch, J. Appl. Phys. **26**, 1013-1017 (1955).
 4. L. M. Barkov and K. N. Mukhin, J. Nuc. Energy **4**, 91-93 (1957).
 5. W. Zobel et al, Appl. Nuc. Phys. Ann. Prog. Rep. Sept. 1, 1957, paper 9.2, ORNL-2389.
 6. N. C. Francis, H. Hurwitz, Jr., and P. F. Zweifel, Nuc. Sci. Eng. **2**, 253-287 (1957).

of the water tank. Even so, in order to obtain reliable data up to 2 cm from the source it was necessary to extrapolate the data obtained at larger distances back to the source.

Since the LTSF data closer than 10 cm from the source was completely unusable, values of a thermal kernel for this region were based on the H-R indium resonance kernel extrapolated to thermal energy. This extrapolation was actually performed for the entire distance covered by the H-R experiment, that is, out to 92 cm from the source, so that the overlapping regions of the two experiments could be compared.

The extrapolation of the H-R indium kernel to thermal energy was made by assuming that the indium resonance slowing down density and flux are proportional, that the slowing down distribution from that energy to thermal is Gaussian (age theory), and that a diffusion kernel describes the diffusion of thermal neutrons from their point of origin. Thus, if the H-R point kernel at a distance r is denoted $G_{In}(r)$, and the thermal-neutron flux kernel is denoted $G_{th}(r)$,

$$(2) \quad G_{th}(r) \propto \int \int_{\text{all space}} G_{In}(\vec{r}') e^{-\frac{(\vec{r}'' - \vec{r}')^2}{4\tau}} \frac{e^{-\frac{|\vec{r} - \vec{r}''|}{L}}}{|\vec{r} - \vec{r}''|} d\vec{r}' d\vec{r}''$$

where τ is the age from indium resonance to thermal energy and L is the diffusion length of thermal neutrons in water. This is equivalent to

$$(3) \quad G_{th}(r) \propto \int_0^\infty \int_0^\infty G_{In}(r') \frac{r'}{r''} \left[e^{-\frac{(r'' - r')^2}{4\tau}} - e^{-\frac{(r'' + r')^2}{4\tau}} \right] \times \frac{r''}{r} \left[e^{-\frac{|r - r''|}{L}} - e^{-\frac{|r + r''|}{L}} \right] dr' dr''$$

where $G_{th}(r)$ is now the point kernel for thermal neutrons [not for nascent thermal neutrons for which the kernel is denoted $G_{nas}(r)$], and r is the scalar distance from the point source. Equation 3 follows from Eq. 2 since $G_{In}(r')$, the indium resonance flux at a distance r' from a point source, acts as a spherical shell source for the further slowing of the neutrons to thermal and their subsequent diffusion as thermal neutrons.⁷ The expressions in the brackets are the spherical shell kernels for age theory and diffusion theory, respectively. Thus the integration, which was carried out numerically using the Oracle, a high-speed digital computer, was only double rather than sixfold. The resulting thermal-neutron kernel, based on the H-R indium resonance data extrapolated to thermal energy, is shown in Fig. 1 for distances out to 90 cm from the source. The values very near the source (0-2 cm), which are based on data extrapolated back to the source, should be better than values obtained from the actual experimental data since each point is based on two integrations over the entire curve.

Naturally, the constants τ and L are quite important to the calculation. The value of τ was taken to be 1.0 cm^2 , which is the value determined experimentally by Barkov et al.⁸ The value of L was taken to be 2.71 cm which is the weighted average of the experimental results of various investigations summarized by Barkov et al. and agrees within 1% with the experimental value obtained by Barkov et al.

As mentioned above, values of the thermal-neutron flux kernel based on the LTSF⁹ data were calculated for distances that extended out to 160 cm from the source, with the region from 15 to 90 cm overlapping with values based on the H-R data. The source used in this experiment was a 28-in.-dia plate of U^{235} activated by thermal neutrons streaming through a hole in the concrete shield of the ORNL Graphite Reactor (see Fig. 2). The strength of this source is 1.62×10^{11} fissions/sec $\pm 5\%$ as determined by estimating or

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7. S. Glasstone, Principles of Nuclear Reactor Engineering, D. Van Nostrand Co., 592-593, 1955.
 8. L. M. Barkov, V. K. Makarin, and K. N. Mukhin, J. Nuc. Energy 4, 94-102 (1957).
 9. The Lid Tank Shielding Facility has been described in various papers; see, for example, Ref. 7, p. 618.

Fig.1. Thermal-Neutron Flux Kernel for an Infinite Water Medium as a Function of Distance from a Point U^{235} Fission Source.

Fig.2. Lid Tank Source Plate Assembly.

measuring the following:¹⁰ (1) the total number of thermal neutrons captured within the source plate, (2) the rate of neutron production within the source, and (3) the heat production in the source. It has been estimated on the basis of "effective removal cross sections" that because of the intervening structural material between the uranium plate and the water in the tank the effective source strength is 6% smaller, but this correction has not been applied in the normalization of the kernel calculated below as this type of correction is not valid near the source.

The thermal-neutron flux measured in the experiment, designated as the "equivalent $1/v$ " flux, is equivalent to the neutron density times 2200 m/sec and is plotted in Fig. 3. Measurements out to distances of 60 cm from the source plate surface were made with bare and cadmium-covered gold foils; the other measurements taken with fission chambers and BF_3 counters were normalized to the foil measurements. The LTSF data close to the source plate is badly perturbed by the boundary, as was discussed above, and for this reason the data closer than 15 cm was not used in the subsequent analysis.

The transformation which was used to convert the LTSF data from a plane disk source to the point source kernel was one developed by Blizard¹¹ and may be derived as follows:

$$D(z,a) = S \int_0^a G(r) 2\pi\rho \, d\rho$$

where $D(z,a)$ is the flux at a distance z from a plane source of radius a

10. D. R. Otis, Appl. Nuclear Phys. Ann. Prog. Rep. Sept. 10, 1956, ORNL-2081, p. 163.

11. This transformation is reported in Ref. 7, p. 623.

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Fig. 3. Thermal-Neutron Flux in the Water of the LTSF as a Function of Distance from the Fission Source Plate (1.62×10^{11} Fissions per Second). (Reference: ORNL-2389, Paper 9.2).

(see sketch). The source strength per unit area is S and the differential

source area is $2\pi\rho d\rho$. But

$$r^2 = z^2 + \rho^2$$

and

$$2r dr = 2\rho d\rho$$

Therefore,

$$D(z, a) = 2\pi S \int_z^{\sqrt{z^2 + a^2}} G(r) r dr$$

Differentiating with respect to z ,

$$(4) \quad \frac{1}{2\pi S z} \frac{d}{dz} D(z, a) = G(\sqrt{z^2 + a^2}) - G(z)$$

If a function $B(z)$ is defined by

$$B(z) \equiv - \frac{1}{2\pi S z} \frac{d}{dz} D(z, a)$$

then it is found from the recursion formula of Eq. 4 that

$$\begin{aligned}
 (5) \quad G(z) &= B(z) + B(\sqrt{z^2 + a^2}) + B(\sqrt{z^2 + 2a^2}) + \dots \\
 &= \sum_{\nu=0}^{\infty} B(\sqrt{z^2 + \nu a^2})
 \end{aligned}$$

where $G(z)$ is the point kernel at the distance z (or r) from a point source.

Thus from Eq. 5 applied to the LTSF data a thermal-neutron point kernel $G_{th}(r)$ is obtained for r greater than 10 cm. This is plotted in Fig. 1 along with the thermal kernel calculated from the H-R indium kernel, which was normalized to the LTSF kernel at 20 cm. The agreement between the H-R and LTSF kernels is quite good over the region from 15 to 90 cm from the source where the two kernels overlap. It is unfortunate that the LTSF kernel for the region nearer the source is unobtainable since the availability of such data might have served as a basis for explaining the difference between the H-R observed age in H_2O and the various theoretical calculations of the age.¹²

Determination of $K(B)$

Equation 1 was used to calculate the Fourier transform, $K_{th}(B)$, of the distribution $G_{th}(r)$ given in Fig. 1. Similarly, Eq. 1 was used to calculate $K_{In}(B)$ from the distribution reported by Hill and Roberts.² The integrations were carried out numerically using the trapezoidal rule on the Oracle. It was found that the results were slightly greater than those obtained with Simpson's rule and, therefore, were more accurate since the trapezoidal rule integration must be an underestimate. The results for both kernels are shown in Figs. 4, 5, and 6. For purposes of comparison, values of $K_{In}(B)$ obtained by Hurwitz et al.,¹³ both by numerical integration

12. J. E. Wilkins, Jr., R. L. Hellens, and P. E. Zweifel, "Status of Experimental and Theoretical Information on Neutron Slowing Down Distribution in Hydrogenous Media," P/597 PICG V. pp. 62-76 (Aug. 1955).

13. H. Hurwitz, P. Zweifel, and C. D. Petrie, private communication.

Fig. 4. Fourier Transforms of Indium Resonance and Thermal-Neutron Slowing Down Kernels in Water for $B < 0.9$: U^{235} Fission Source.

Fig. 5. Fourier Transforms of Indium-Resonance Slowing-Down Kernel in Water for $B < 0.5$. U^{235} fission source.

Fig. 6. Fourier Transform of Thermal-Neutron Slowing Down Kernel in Water for $B < 0.25$: U^{235} Fission Source.

and by the G-S method, are also plotted in Fig. 5. For large values of B the slope changes radically as shown in Fig. 4, and $K(B)$ even goes negative. This is believed to be due to the shape of the assumed kernel $G(r)$ near 0. In order to verify that the Oracle program was yielding the correct results, a Gaussian and an exponential kernel were computed analytically; the results were identical to those obtained in the Oracle numerical integration.

The transform $K_{\text{nas}}(B)$ of the nascent thermal kernel is also shown in Figs. 4 and 6. Since the thermal neutrons diffuse with a diffusion length L ,

$$(6) \quad K_{\text{nas}}(B) = K_{\text{th}}(B)(1 + L^2 B^2)$$

On the other hand, since the age increment τ from indium resonance to thermal is 1 cm^2 , the following is also very nearly true:

$$(7) \quad K_{\text{nas}}(B) = K_{\text{In}}(B)e^{-B^2}$$

A comparison of the values of $K_{\text{nas}}(B)$ computed by these two methods is shown in Table 1.

In regard to the accuracy of $K_{\text{th}}(B)$, the numerator of Eq. 1 was integrated by hand from 0 to 15 cm, which covered the region where the kernel depends on the indium experiment, and the results are compared with the Oracle computations in Table 2. The table clearly shows that $K(B)$ is almost entirely dependent on the H-R indium experiment for values of B which are of most interest from a practical standpoint. However, it was found that $K(B)$ changed less than 0.5% for $B < 0.2 \text{ cm}^{-1}$ even though the extrapolation of $G(r)$ to $r = 0$ was changed radically. For example, a Gaussian was fitted through the H-R data at $r = 3 \text{ cm}$ and $r = 4 \text{ cm}$. This yielded a $G(0)$ value about 5% higher. A Gaussian was also fitted through LTSF points at $r = 10$ and $r = 20 \text{ cm}$ which yielded a $G(0)$ value nearly 20% lower. For $B > 0.2 \text{ cm}^{-1}$, the uncertainty in $K(B)$ becomes much larger.

Table 1. Ratio of the Fourier Transform $K_{nas}(B)$ Computed from $K_{In}(B)$ to that Computed from $K_{th}(B)$

$B(\text{cm}^{-1})$	$\frac{K_{In}(B)e^{-B^2}}{K_{th}(B)(1 + L^2B^2)}$
0.01	1.0000
0.05	1.0002
0.10	1.0020
0.15	1.0047
0.20	1.0067
0.30	1.0089
0.40	1.0190
0.50	1.0071
0.60	0.9450
0.70	0.9867

Table 2. Ratio of Numerator of Fourier Transform of $G_{th}(r)$ Computed by Hand to that Obtained on the Oracle

$B(\text{cm}^{-1})$	$\frac{\int_0^{15}}{\int_0^{\infty}}$
0.12	1.053
0.15	1.153
0.17	1.236

Since for reactor calculations it is very useful to have a simple multi-group analytic approximation to the experimental Fourier transforms, attempts were made to develop such expressions for the data in Figs. 4, 5, and 6. The following expressions were obtained:

$$K_{In}(B) = \frac{1}{(1 + 26.27B^2)(1 + 2.26B^2)^2}$$

$$K_{th}(B) = \frac{1}{(1 + L^2B^2)(1 + L_1^2B^2)^2(1 + L_2^2B^2)}$$

where

$$L^2 = 7.344 \text{ cm}^2,$$

$$L_1^2 = 2.625 \text{ cm}^2,$$

$$L_2^2 = 26.539 \text{ cm}^2.$$

These simple expressions agree very well with the computed values out to values of $B \approx 0.35 \text{ cm}^{-1}$. The constants, whose sum equals the age or migration area, were chosen simply on the basis of making a good fit over a particular arbitrary region.

Moments of the Kernel

Quantities of interest are the moments, $\overline{r^n}$, of the distribution $G(r)$:

$$(8) \quad \overline{r^n} = \frac{\int_0^{\infty} G(r) r^n 4\pi r^2 dr}{\int_0^{\infty} G(r) 4\pi r^2 dr}$$

The moments $\overline{r_{th}^n}$ determined from the kernel $G_{th}(r)$ are shown in Table 3 for $n = 2, 4, 6, 8$, and 10 . The moments $\overline{r_{th}^n}$ determined from the Hill-Roberts kernel are also presented.

Table 3. Summary of the Moments of Various Slowing Down Distributions

n	$\overline{r_{th}^n}$		$\overline{r_{In}^n}$		From $K_{th}(r)$, cm ⁿ	$\overline{r_1^n}$, cm ⁿ		$\overline{r_2^n}$, cm ⁿ		$\overline{r_{nas}^n}$, cm ⁿ	
	cm ⁿ	f.e.**	From $G_{In}(r)$, cm ⁿ	f.e.**							
2	$(2.348 \pm 0.09) \times 10^2$ *	3.08×10^{-6}	1.847×10^2	1.40×10^{-5}	1.847×10^2	6.0		4.406×10^1		1.907×10^2	
4	$(1.592 \pm 0.13) \times 10^5$ *	1.57×10^{-4}	1.222×10^5	7.95×10^{-4}	1.210×10^5	6.0×10^1		6.472×10^3		1.247×10^5	
6	$(2.769 \pm 0.35) \times 10^8$ *	2.75×10^{-3}	2.27×10^8	1.30×10^{-2}	2.23×10^8	8.4×10^2		1.996×10^6		2.278×10^8	
8	$(1.084 \pm 0.18) \times 10^{12}$ *	2.22×10^{-2}	8.6×10^{11}	8.57×10^{-2}	9.2×10^{11}	1.512×10^4		1.056×10^9		9.376×10^{11}	
10	$(8.230 \pm 1.7) \times 10^{15}$ *	9.59×10^{-2}		3.33×10^{-1}							

*Uncertainty due to estimated maximum possible error in measuring the slope of the kernel. Theory indicates a much larger error (ref. 12).

**Fraction extrapolated.

In order to compute the higher moments, both $G_{th}(r)$ and $G_{In}(r)$ had to be extrapolated by assuming $G(r) \propto e^{-r/\lambda}/r^2$, where $\lambda_{th} = 11.017$ cm for $r > 160$ cm and $\lambda_{In} = 8.786$ cm for $r > 92$ cm. The fractions of both $\overline{r_{th}^n}$ and $\overline{r_{In}^n}$ which depend on extrapolated data are also shown in Table 3. Since the extrapolated fraction of $\overline{r_{In}^n}$ is rather large (as much as 33%), it seems that a more accurate calculation of $\overline{r_{In}^n}$ could be made using $G_{th}(r)$ rather than $G_{In}(r)$. The relationship between the various transforms may be written as follows:

$$(9) \quad K_{th}(B) = K_{In}(B) K_1(B) K_2(B)$$

$$(10) \quad K_{th}(B) = K_{nas}(B) K_2(B)$$

where $K_1(B)$, $K_2(B)$, and $K_{nas}(B)$ are the Fourier transforms of the Gaussian slowing down distribution $e^{-r^2/4\tau}$, the diffusion kernel $e^{-r/L}/r$, and the nascent thermal slowing down distribution. The moments of the Gaussian slowing down distribution and the diffusion kernel are defined as $\overline{r_1^n}$ and $\overline{r_2^n}$, respectively.

If the transform is expressed in terms of the moments

$$(11) \quad K(B) = \sum_{i=0}^{\infty} \frac{(-1)^i}{(2i+1)!} B^{2i} \overline{r^{2i}}$$

where $2i = n$, then by using Eqs. 9 and 10 and equating coefficients of powers of B , the moments of $G_{In}(r)$ and $G_{nas}(r)$ may be computed. This was carried out for $n = 2, 4, 6$, and 8 and the results appear in Table 3. Values for $\overline{r_1^n}$ and $\overline{r_2^n}$ are also given in Table 3.

It may be noted that the estimate for the migration area of U^{235} fission neutrons down to thermal in H_2O is the same whether computed from the thermal or indium second moment. That is, $\overline{r_{th}^2}/6 = 39.13$ cm² and

$\overline{r_{In}^2}/6 + \tau + L^2 = (30.78 + 1.00 + 7.34) \text{ cm}^2 = 39.12 \text{ cm}^2$. The contribution of the Hill-Roberts experiment (0-15 cm from the source) to the integral in computing $\overline{r_{th}^2}$ is only 30%.