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The Effective Resonance Integrals of U^{238} and Th^{232}

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I. Introduction. In calculating the resonance absorption of neutrons in homogeneous mixtures of moderator and resonance absorber, an important quantity is the effective resonance integral,¹ defined by

$$(\sigma_a)_{\text{eff}} = \int \frac{dE}{E} \frac{\sigma_a}{1 + \frac{\sigma_a + \sigma_s}{\sigma_p}} \quad (1)$$

Here, σ_a is the resonance absorption cross section, σ_s the resonance scattering cross section, and σ_p the energy independent scattering cross section, all normalized to one absorbing atom. It is the purpose of this paper to calculate $(\sigma_a)_{\text{eff}}$ for U^{238} and Th^{232} from measured resonance data,² and to compare the calculated values with known experimental values.³

II. Theory. The contribution, $(\sigma_a)_{\text{eff}}$, to Eq. (1) from a single resonance can easily be related to the parameters (energy, widths) of the resonance. We shall confine our attention only to spinless target nuclei, and consider only neutrons of zero orbital angular momentum. Then σ_a and σ_s are given by the Breit-Wigner formula, viz.:

$$\sigma_a = \pi \lambda^2 \frac{\Gamma_n \Gamma_\gamma}{(E - E_0)^2 + (\Gamma/2)^2} = \frac{4\pi \lambda^2 \Gamma_n \Gamma_\gamma}{\Gamma^2} \frac{1}{1 + \left(\frac{E - E_0}{\Gamma/2}\right)^2} \quad (2a)$$

$$\sigma_s = \pi \lambda^2 \frac{\Gamma_n^2}{(E - E_0)^2 + (\Gamma/2)^2} = \frac{4\pi \lambda^2 \Gamma_n^2}{\Gamma^2} \frac{1}{1 + \left(\frac{E - E_0}{\Gamma/2}\right)^2} \quad (2b)$$

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where λ is the neutron wave length divided by 2π , Γ_n is the neutron width, Γ_γ is the radiative width, $\Gamma = \Gamma_\gamma + \Gamma_n$, E_0 is the resonance energy, and E is the neutron energy. Substituting Eqs. (2a) and (2b) in Eq. (1) yields, after some rewriting

$$(\sigma_a)_{\text{eff}} = \frac{\sigma_p \Gamma_\gamma}{E_0} \cdot \frac{\pi/2}{\sqrt{\beta(1+\beta)}} \quad (3)$$

where $\beta = \sigma_p \Gamma / 4\pi \lambda^2 \Gamma_n$. Because the resonances of uranium and thorium are narrow, the energy dependence of λ and Γ_n is slight compared to the resonance denominator, and these quantities have been replaced by their value at resonance in the integration indicated in Eq. (1)

Eqs. (2a, b) and (3) are correct only when the Doppler broadening of resonance lines is ignored. Formulae analogous to Eqs. (2a, b) can be written including the effects of Doppler broadening, viz.:

$$\sigma_a = \frac{4\pi \lambda^2 \Gamma_n \Gamma_\gamma}{\Gamma^2} \phi(\xi, x) \quad (4a)$$

$$\sigma_s = \frac{4\pi \lambda^2 \Gamma_n^2}{\Gamma^2} \phi(\xi, x) \quad (4b)$$

where $\phi(\xi, x)$ is the broadened resonance line shape defined by Born and others⁴ as

$$\phi(\xi, x) = \frac{\xi}{2\sqrt{\pi}} \int_{-\infty}^{+\infty} dy \frac{\exp\left(-\frac{\xi^2}{4}(x-y)^2\right)}{1+y^2} \quad (5)$$

In Eq. (5) x and ξ are defined by

$$x = \left(\frac{E - E_0}{\Gamma/2} \right), \quad \xi = \frac{\Gamma}{\left(\frac{4kTE}{A} \right)^{1/2}} \quad (6)$$

where kT is the temperature of the absorbing medium in energy units, and A is its

mass number. When Eqs. (4a) and (4b) are substituted in Eq. (1) an expression similar to Eq. (3) results:

$$(\sigma_a)_{\text{eff}}^I = \frac{\sigma_p \Gamma_\gamma}{E_0} \cdot J(\xi, \beta) \quad (7)$$

where

$$J(\xi, \beta) = \int_0^\infty \frac{\varphi(\xi, x)}{\varphi(\xi, x) + \beta} dx \quad (8)$$

The function $J(\xi, \beta)$ has been tabulated by the ORACLE for a wide range of values of ξ and β .⁵ Summing Eq. (7) over all resolved resonances gives their cumulative contribution to the effective resonance integral in terms of their resonance parameters.

An equally important contribution to the effective resonance integral comes from resonances at high energies which are unresolved. The effect of these resonances can be estimated in terms of average resonance parameters. Their total contribution, $(\sigma_a)_{\text{eff}}^{\text{II}}$, is given by

$$(\sigma_a)_{\text{eff}}^{\text{II}} = \frac{\sigma_p \Gamma_\gamma}{D} \int_{E_1}^\infty \frac{dE}{E} J(\xi, \beta) = \frac{\sigma_p \Gamma_\gamma}{D} \int_{\beta_1}^\infty J(\xi, \beta) \frac{d\beta}{\beta} \quad (9)$$

where D is the average spacing of resonances, and E_1 is energy of the last resolved resonance.

In carrying out the integral indicated in Eq. (9) Γ_γ was assumed to be energy independent. In evaluating $(\sigma_a)_{\text{eff}}^{\text{II}}$ from Eq. (9) for U^{238} and Th^{232} , ξ was taken fixed and equal to 0.1. The reason for this is as follows: ξ can be written as the sum of two terms, one of which is inversely proportional to square root of the neutron energy and the other of which is energy independent, i.e.,

$$\xi = \frac{\Gamma_\gamma}{\left(\frac{4kTE}{A}\right)^{1/2}} + \frac{\Gamma_n^0}{\left(\frac{4kT}{A}\right)^{1/2}} \quad (10)$$

Here Γ_n^0 is the average neutron width at 1 e.v. In both U^{238} and Th^{232} E_1 is sufficiently large so that the first term in Eq. (10) is at most comparable with the second, and in the limit of high energy vanishes. The second term is about 0.1 in both materials.

Finally, in relating E_1 to β_1 in Eq. (9) it was assumed that $\Gamma_n \gg \Gamma_\gamma$ in the vicinity of energy E_1 , so that $\Gamma_n/\Gamma = 1$. This relation is exact in the limit of infinite E_1 , and for finite E_1 will overestimate $(\sigma_a)_{\text{eff}}''$. If we make the extreme assumption that $\frac{\Gamma_n}{\Gamma} = \frac{1}{2}$ the calculated results described in paragraph III will suffer a decrease of at most 10% for U^{238} and at most 15% for Th^{232} . Hence, $(\sigma_a)_{\text{eff}}$ is not very sensitive to the assumed value of $\frac{\Gamma_n}{\Gamma}$.

The " $\frac{1}{v}$ " part of the absorption cross section below the energy of the first resonance also contributes to the effective resonance integral. Setting $\sigma_a \sim \frac{1}{v}$ and $\sigma_s = 0$ in Eq. (1) we obtain for this contribution

$$(\sigma_a)_{\text{eff}}''' = 2\sigma_p \ln \left(\frac{\sigma_p + \sigma_a(E_0)}{\sigma_p + \sigma_a(E_2)} \right) \quad (11)$$

where E_0 and E_2 are respectively the lower and upper energy limits of the region in which the cross-section is " $\frac{1}{v}$ ". In general, E_0 will be around Cd-cutoff, and E_2 near the energy of the first resonance. In U^{238} and Th^{232} little error is incurred by setting $E_2 = \infty$. Then

$$(\sigma_a)_{\text{eff}}''' = 2\sigma_p \ln \left(1 + \frac{\sigma_a(E_0)}{\sigma_p} \right) \rightarrow 2\sigma_a(E_0) \text{ as } \frac{\sigma_p}{\sigma_a(E_0)} \rightarrow \infty \quad (12)$$

Summing $(\sigma_a)_{\text{eff}}'$ over all resolved resonances, and adding $(\sigma_a)_{\text{eff}}''$ and $(\sigma_a)_{\text{eff}}'''$ yields $(\sigma_a)_{\text{eff}}$, the total effective resonance integral.

III. Results. Plotted in Figure 1 are the results of calculations of the effective resonance integrals of U^{238} and Th^{232} at 300° K based on resonance data reported in reference 2. $(\sigma_a)_{eff}$ has been calculated for both materials for $\sigma_p = 20, 80, 200,$ and 2000 barns. The points have been joined by smooth curves on log-log plots. For both materials these curves have been extrapolated in the pure metal. The points are experimental points taken from the compilation of reference 3. In the case of U^{238} agreement between the curve and the experimental points is excellent. In the case of Th^{232} the calculated curve agrees well with the measured points at high dilution, and for the pure metal. The experimental point at $\sigma_p \approx 21$ barns disagrees with the calculated curve, but this point is considered unreliable.⁶ Hence, we may conclude that agreement between theory and experiment is satisfactory.

Fig. 1. Effective Resonance Capture Integrals of U^{238} and Th^{232} Diluted with Other Materials.

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