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FRAGMENT EFFECTS ON STRUCTURAL ELEMENTS

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IMPACT EFFECT OF FRAGMENTS STRIKING STRUCTURAL ELEMENTS

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ABOUT THE AUTHORS OF THIS PAPER

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INTRODUCTION

In the design of modern industrial plants, engineers are quite often faced with certain hazards inherent in process systems operating under high pressure or with explosive mixtures. In general, the basic consideration is in the containing of this pressure or possible detonation within the system of piping and pressure vessels used. If an explosion should occur which is not contained, the skin of the building is almost universally designed to blow out to prevent damage to the main structural framework. Similarly, if high velocity fragments are produced as a result of such a detonation, or some component of the system is suddenly ejected because of the high pressure, it is very rare that any attempt is made to design against the possible impact on any part of the building.

With the advent of nuclear process systems, it has become of prime importance to reconsider this engineering philosophy. Certain types of such plants have systems utilizing high pressures, others result in the formation of explosive mixtures, and some have both. As a result, the ability of fragments or missiles resulting from conceivable accidents to penetrate the outer shell or vapor container of the structure must be evaluated. That this must be prevented for those accidents believed feasible is obvious, otherwise excessive radiation may escape to contaminate the surrounding areas. This paper is aimed directly at the effect of those fragments resulting from an explosion within a pressure vessel of a nuclear power reactor; however, the equations derived are applicable to any missile of known mass travelling at a specific velocity.

BASIC DESIGN CONSIDERATIONS

When the pressure generated by an explosion inside a pressure vessel exceeds the yield point of its metal, and cannot be contained, the shell bursts. If the rate of the explosion is rapid, fragments or projectiles of irregular shape will be thrown at moderate to high velocities. The effect of a fragment on the adjacent walls of the room or vapor container depends on its shape, mass, initial velocity, the distance from point of explosion, the angle at which the fragment strikes, and the contact areas. The items of major importance to the designer are mass, initial velocity and the contact area. The action of a fragment striking a wall may result in partial or complete penetration, in damage by the load of impact, or both. This paper concerns itself essentially with the effect of impact, and assumes that all the required parameters are known. However, means for obtaining the values for the parameters will be discussed.

INITIAL VELOCITY

The initial velocity of a fragment is nearly independent of its mass and shape, and is approximately equal to the velocity of expansion of the exploding pressure vessel (1). Unfortunately for this discussion, much definite source material available for predicting this value is in classified reports developed as a part of the weapons program (1, 2). Those with access may utilize these reports by converting the explosive power available in the system under consideration into an equivalent quantity of TNT. Once this is known, the velocity can be expressed as a function of the ratio of the charge weight

to the weight of the container. Of the two, it is believed that reference 2 will provide the more realistic value. For the purpose of this report it is assumed the initial velocity is known.

MASS AND SHAPE

The impact loading and penetration of fragments depend on their shape, weight and contact area. In general, fragments will vary considerably in size. Unfortunately, insufficient evidence appears to be available to permit the prediction of the size and shape of the fragments resulting from an explosion. The fact that the size of these fragments can be quite large is illustrated by the results of the Whiting explosion (3). A figure of 10 percent of the weight of the pressure vessel is recommended as the maximum size of a fragment to be considered.

PENETRATION

The penetration of a fragment into a resisting mass is governed by the velocity at impact, the physical properties of the missile, and the material characteristics of the resisting mass. These factors have been combined in various forms by a number of authorities in ballistics to obtain empirical expressions for penetration. Here, too, the sources of information known to the writers for calculating the depth of penetration are classified reports (1, 2) which resulted from the weapons program.

The phenomenon of penetration can be described in terms of several degrees of damage. When a fragment of relatively slow velocity strikes a wall of a finite thickness, it will penetrate slightly causing a crater around the point of contact, provided the contact is made within a relatively small area. As the velocity is increased, the penetration will become greater and scabbing will take place in back of the wall, which is usually wider and shallower than the front spall crater. As the velocity is increased still further, progressively deeper craters will be formed until the wall is completely perforated and the fragment passes through.

Formulas have been derived by both Army and Navy agencies to determine the extent of penetration. These formulas have been derived on the basis of round missiles in connection with specific charges of TNT or other military explosives. In order to utilize the equations, therefore, they must be converted for use with the rectangular shape to be expected with the fragments resulting from a vessel explosion. It

is suggested that such conversion be based on equivalent cross-sectional areas. The modified equations can then be used to determine the extent of penetration to a degree of accuracy which is sufficient for the impact analysis.

EFFECT OF IMPACT

The usual damage discussed in reference to projected fragments is that of penetration. However, a fragment can also cause extensive damage solely from its force of impact. The following analysis derives a method to evaluate the impact effect of a high-velocity fragment striking a structural element. The loading on any structure being struck is in the nature of an impulse or dynamic load. The overall effect is also dependent on the extent of penetration of the fragment. The analysis when penetration takes place is different than that where none occurs, and both cases are presented.

CASE WITH PENETRATION

Under a dynamic load, a structure will deform under the load and return completely or partially to its initial position. The amount of maximum and permanent deformation is a function of the type of impulse loading and material characteristics of the structure. The impulse loading on the structure is a function of the mass and velocity of the fragment as well as the depth of penetration. The relation between load and deformation or between resistance and deflection of a structural element may take any number of forms. It is convenient, however, to define the structural resistance in the same terms and units as the external loading. The procedure proposed develops a means of calculating a static load which will permit the designer to establish the required yield strength of the structure to resist the impulse loading of the fragment.

The basic assumption made for this case is that the force of impact, F , and its velocity, v , both reduce linearly to zero as the fragment penetrates the beam. The total kinetic energy of the fragment is expended while the fragment travels a distance, X , equal to the penetration obtained by either of the formulas contained in reference 1 or 2. The deflection of the structural element is assumed negligible when compared to the penetration. Under these assumptions, the work done by the fragment is equal to its kinetic energy, thus

$$\frac{F_i X}{2} = \frac{wv^2}{2g}$$

$$F_i = \frac{wv^2}{gX} \quad (1)$$

Furthermore, because the velocity is assumed to reduce linearly from an initial value of V , the time of the impulse or the duration of the impact force is determined by the equation.

$$t_i = \frac{2X}{v} \quad (2)$$

Equations (1) and (2) provide the two factors required to establish the limits of the impulse load, namely F_i and t_i . The capability of a structure to resist dynamic loads is a function of its natural period of vibration, T , the time of the impulse, t_i , the yield strength of the material and its ductility. For this analysis, the ductility shall be described by the symbol μ , and is defined as the ratio of the deflection of a structure at failure to its deflection at yield. Its value can be readily established from the stresses-strain curve, and is a measure of the capability of a structure to absorb energy in the plastic range.

The value of F_i obtained by equation (1) is generally too large and too conservative to be used for practical design, therefore, an equivalent static load should be determined. The method proposed here for obtaining this load has been developed by Newmark (4) in studies he has made in connection with blast loading. He has established equations based on triangular impulse loadings, which are consistent with the basic assumptions mentioned previously. The factor by which the value of F_i can be reduced is expressed by the equation

$$\frac{F_i}{F} = K \quad (3)$$

then

$$K = \frac{(2\mu - 1)^{0.5} T}{\pi t_i} + \frac{1 - \frac{0.5}{\mu}}{1 + \frac{0.7T}{t_i}} \quad (4)$$

Figure 1 has been developed for values of $\mu = 1, 20$ and 50 . For a completely brittle structure, $\mu = 1.0$; for moderately brittle structures,

$\mu = 3$ to 5 ; for moderately ductile structures, $\mu = 10$ to 30 ; and for very ductile structures, μ may vary from 40 to 100 . The use of equation (3) with equation (4) or Figure 3 will allow the designer to determine a static load which, if it stresses the material exactly to its yield point, will absorb the energy of the impulse while deflecting the beam through a distance of μ times the yield deflection. Therefore, the designer should utilize the value of F obtained in this manner together with a suitable factor of safety on the dynamic yield stress of the material. In other words the great ability of any material to absorb energy in the plastic stress range is being utilized, inferring of course that permanent deformation is permissible for this hazard.

The natural period of vibration, T , which appears in equation (3), must be determined for the structural element being struck. However, to utilize the formula developed by Newmark, it is theoretically necessary to use the period corresponding to the stiffness of the elasto-plastic state, as the structure is assumed to enter the plastic range while deflecting. This refinement is not believed necessary in this case. Standard procedures are available for the computation of the period of the structure in its initial elastic state, either analytically or by numerical means, and are believed entirely adequate for this analysis.

CASE WITH NO PENETRATION

For the case where a mass may strike flat, resulting in little or no penetration, the preceding method cannot be used. A search into the literature reveals that a fair amount of work has been undertaken on this subject, but none could be found for the condition where the structural element receiving the impact is stressed beyond the elastic limit. All work known to the writers has been conducted within the yield point for both elastic and inelastic impact. The basic concept behind the following analysis is that a fragment striking a wall must be contained within the structure, but by allowing permanent deformation to take place. This allows the designer to utilize the great capacity of any material to absorb impact or impulse loads in the plastic range. The following, therefore, is an attempt to evaluate the effect of a high velocity fragment striking a beam at midspan under inelastic impact. An expression is derived for the required yield resistance of the beam in terms of the mass and velocity of the frag-

IMPACT

FIG. 2. BEAM AND EQUIVALENT MASS - SPRING SYSTEM

FIG. 3. MOTION OF MASS -
SPRING SYSTEM

FIG. 4. LOAD - DEFLECTION CURVE
FOR SPRING

ment, and the ductility factor and period of vibration of the beam. A second expression is derived for the maximum deflection of the beam in terms of the mass and velocity of the fragment, the yield resistance of the beam, and its "effective" mass.

The derivation parallels that developed by Timoshenko (5), and is limited by the same assumptions. To maintain the same concepts used by Newmark (4), the derivation is carried out on a single degree of freedom mass-spring system replacing the actual beam. (See Figure 2). This system is equivalent to the beam if the spring constant, k , is made equal to the static load-deflection ratio of the beam and if the period of vibration of the system is made equal to that of the beam by using an "effective" mass, M_e , which is less than the beam mass, M .

From Figure 2, if U = kinetic energy of the mass, m , just prior to impact, then

$$U = \frac{mv^2}{2} = mgh \quad (5)$$

At the instant of impact, local inelastic deformation is assumed to occur, after which both masses are assumed to move together and spring compression takes place. From momentum considerations the common velocity, V , of the two masses at the instant local inelastic deformation is complete is given by

$$V = v \left(\frac{1}{1 + \frac{M_e}{m}} \right) \quad (6)$$

Setting

$$c = \frac{1}{1 + \frac{M_e}{m}} \quad (7)$$

then

$$V = vc \quad (8)$$

If U^1 = total kinetic energy of both bodies at the time local inelastic deformation is complete

$$U^1 = \left(\frac{m + M_e}{2} \right) V^2 \quad (9)$$

Substituting equations (7) and (8) and simplifying

$$U^1 = \frac{cmv^2}{2} \quad (10)$$

or

$$U^1 = mghc \quad (11)$$

The total work done in compressing the spring is equal to the total kinetic energy available at the time local inelastic deformation is complete (U^1) plus the potential energy available resulting from the distance the spring will compress. From Figures 2 and 3, this value is expressed by the terms

$$mghc + mgx_m$$

The total strain energy stored in the spring must equal the total work done. From Figures 3 and 4, the value of the strain energy is

$$q_y x_m = q_y \frac{x_y}{2}$$

Equating

$$q_y x_m = q_y \frac{x_y}{2} = mghc + mgx_m \quad (12)$$

Substituting

$$x_y = \frac{q_y}{k} \text{ and } x_m = \mu x_y = \frac{\mu}{k} q_y$$

into Equation 12

$$\frac{\mu}{k} q_y^2 = \frac{q_y^2}{2k} = mghc + \frac{mg\mu}{k} q_y$$

Simplifying

$$q_y^2 - \left(\frac{2mg\mu}{2\mu - 1} \right) q_y = \frac{2kmghc}{2\mu - 1}$$

Completing the square and solving for q_y

$$q_y = \frac{mg\mu}{2\mu - 1} \left[1 + \sqrt{2 + \left(\frac{2\mu - 1}{\mu^2} \right) \left(\frac{2hck}{mg} \right)} \right]$$

Substituting

$$h = \frac{v^2}{2g}; c = \frac{m + M_e}{m}; k \approx \frac{4\pi^2}{T^2} (M_e + m)$$

$$q_y = \left(\frac{\mu}{2\mu - 1} \right) mg$$

$$\left[1 + \sqrt{1 + \left(\frac{2\mu - 1}{\mu^2} \right) \left(\frac{2\pi v}{gT} \right)^2} \right] \quad (13)$$

When the last term under the radical is large when compared to unity, equation (13) reduces to

$$q_y = \frac{2 \pi m v}{T} \sqrt{\frac{1}{2\mu-1}} \quad (14)$$

Equations (13) and (14) are expressions for the static load, which, if it is applied as a concentrated load, will give the designer a basis of design to provide the necessary yield strength to resist the fragment impact in the plastic range. The value of q_y is equivalent to the value of F given in equation (3) for the condition where the fragment penetrates the element under consideration.

In order to obtain a rough concept of the degree of the maximum dynamic deflection, the following derivations may be of interest

Substituting $q_y = mg \frac{x_y}{x_s}$ into Equation (12)

$$\frac{x_y}{x_s} x_m - \frac{x_y^2}{2x_s} = ch + x_m$$

Substituting $x_y = \frac{x_m}{\mu}$

$$\frac{x_m^2}{\mu x_s} + \frac{x_m^2}{2\mu^2 x_s} = ch + x_m$$

Simplifying

$$x_m^2 - \left(\frac{2\mu^2}{2\mu-1}\right) x_m x_s = \left(\frac{2\mu^2}{2\mu-1}\right) ch x_s$$

Completing the square and solving for x_m

$$x_m = \left(\frac{\mu^2}{2\mu-1}\right)$$

$$x_s \left[1 + \sqrt{1 + \left(\frac{2\mu-1}{\mu^2}\right) \left(-\frac{2h}{x_s}\right) \left(\frac{1}{1 + \frac{M_e}{m}}\right)} \right] \quad (15)$$

When $\mu = 1$, Equation (15) coincides with Equation 192 of Reference (5). If $h^2 = v^2/2g$ is substituted into Equation 15.

$$x_m = \left(\frac{\mu^2}{2\mu-1}\right)$$

$$x_s \left[1 + \sqrt{1 + \left(\frac{2\mu-1}{\mu^2} \frac{v^2}{g x_s} \right) \left(\frac{1}{1 + \frac{M_e}{m}}\right)} \right] \quad (16)$$

When the last term under the radical is large when compared to unit, Equation (16) becomes

$$x_m = x_s \mu v \sqrt{\frac{1}{(2\mu-1) x_s g \left(1 + \frac{M_e}{m}\right)}} \quad (17)$$

An expression for x_m in terms of q_y is useful when the value of q_y is known. If the proper substitutions are made in Equation (12), the following expression can be obtained

$$x_m = - \frac{mv^2}{q_y \left(\frac{2\mu-1}{\mu}\right) - 2mg} \left(\frac{1}{1 + \frac{M_e}{m}}\right) \quad (18)$$

The various equations for deflectors necessitate a means of establishing a value of the "effective" mass (or weight). From an article written by Whitney, Anderson and Cohen (6), the following values can be used.

BEAM TYPE	W_e	
	Elastic Range	Plastic Range
Simple span, center concentrated load	.494 W	.333 W
Fixed span, center concentrated load	.383 W	.333 W
Cantilever span, concentrated end load	.243 W	.333 W

The value of W_e to be used is dependent on whether the design is to be for loading within the elastic range ($\mu = 1.0$) or in the plastic range ($\mu > 1.0$). It is further dependent on the type of restraint conditions of the beam or structural element under consideration.

ILLUSTRATIVE EXAMPLES

In order to illustrate the use of the preceding equations, and also compare the relative effect of penetration on the equivalent static design load, the following examples are provided.

CASE 1 — PENETRATION:

Fragment Conditions:	$v = 500$ ft/sec
	$w = 1000$ lbs
Impact Conditions	$X = 3.0$ ft
	$T = 0.1$ seconds
	$\mu = 20$
From equation (1),	$F_i = 2,600,000$ lbs
From equation (2),	$t_i = .012$ seconds
	$\frac{T}{t_i} = \frac{0.1}{.012} = 8.3$
From Figure 1,	$K = 23$
From equation (3),	$F = 113,500$ lbs

CASE 2 — PENETRATION:

Fragment Conditions:	$v = 500$ ft/sec
	$w = 1000$ lbs
Impact Conditions:	$X = 1.0$ ft
	$T = 0.1$ seconds
	$\mu = 20$
From equation (1),	$F_i = 7,830,000$ lbs
From equation (2),	$t_i = .004$ seconds
	$\frac{T}{t_i} = 24.9$
From Figure 1,	$K = 66$
From equation (3),	$F = 118,800$ lbs

CASE 3 — NO PENETRATION:

Fragment Conditions:	$v = 500$ ft/sec
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	$w = 1000$ lbs
Impact Conditions:	$X = 0$
	$T = 0.1$ seconds
	$\mu = 20$
From equation (14),	$q_v = 156,000$ lbs

COMMENTS

From the above examples, it is seen that the effective impact load of a high velocity fragment is very high, many times that of the fragment weight. In general, the effect of penetration is to decrease the damage due to impact alone. This can be partially explained on the basis of absorbing some of the kinetic energy by penetration which will also soften the impact force. The reduction in the impact force, F_i , with increased penetration is apparent from equation 1. However, as the penetration increases, so does the time of application of the load (Equation (2)). In the field of structural dynamics, the length of time an impulse loading acts has a great effect on the equivalent static load. Therefore, the increase in the equivalent static load with decreased penetration is not as marked as one may expect. For the three examples shown, the difference between Case 1, 3 feet penetration and Case 3, zero penetration is not great. It is quite possible that under conditions of greater penetration the value of F may actually increase. However, for the purpose of most analysis, the limiting condition for impact effect can be assumed to be that of zero penetration.

SUMMARY

The preceding analysis has developed equations to permit a designer to evaluate the impact effect of missile striking a structural element at high velocity. Although the degree of accuracy cannot in a practical approach be as that of normal static loading considerations, the degree of accuracy is sufficient to permit the engineer to cope with the problem with a reasonable degree of confidence. The computed equivalent static load may be off by as much as a factor of two from the true value, but considering the complexity of this type of problem, it is felt that this is reasonably close.

In application to nuclear power plants where structural containment is extremely important, it is hoped this paper may be of some help. The

problem of evaluating various hazards has included the effect of high velocity missiles in many reactors. It is particularly to aid the engineer in his valuations of the effects of a fragment, other than physical penetration, that this paper has been written.

NOTATIONS

F = equivalent static load (lb)
 F_i = force of impact (lb)
 g = acceleration of gravity (ft/sec²)
 k = spring constant (lb/inch)
 M = mass of beam (lb-sec²/ft)
 m = mass of fragment (lb-sec²/ft)
 P = concentrated load (lb)
 q = resistance of beam (lb)
 T = natural period of vibration (seconds)
 t_1 = duration of impulse or impact force (seconds)

U = kinetic energy (ft-lb)
 V = velocity of fragment and point of contact after impact (ft/sec)
 v = velocity of fragment just before impact (ft/sec)
 W = weight of beam (lb)
 w = weight of fragment (lb)
 X = penetration (ft)
 x = deflection of beam (ft)
 μ = ductility factor

SUBSCRIPTS

e = effective
 m = maximum
 s = static
 y = yield

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