POSSIBLE MEASUREMENT OF THE VACUUM POLARIZATION
IN HEAVY ION SCATTERING⁺

Johann Rafelski and Abraham Klein
Department of Physics, University of Pennsylvania
Philadelphia, Pennsylvania 19174

UPR-0028N-

11/15/73

ABSTRACT

A measurement of vacuum polarization effects in nonrelativistic elastic heavy ion scattering is suggested. We find that in a typical subcoulomb barrier event enough momentum is transferred from the projectile to the target nucleus to make the high q^2 behavior of the photon propagator observable. The elastic cross section is increased in a typical case by 2.5% above the Rutherford value. Only relative measurements are needed.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

⁺Supported in part by the U.S. Atomic Energy Commission.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The only high precision experiment in which vacuum polarization effects give a dominant additional contribution, is the measurement of certain transition energies in muonic atoms, such as $5g_{7/2} - 4f_{5/2}$ in muonic lead.^(1,2) In all other precision tests of quantum electrodynamics such as Lambshift, g-factor, etc. vacuum polarization is a small, albeit well-established, part of the effect observed. Furthermore, unlike the case of heavy muonic atoms, essentially only low momentum transfer contributions are tested ($q^2/m_e^2 \approx 1$). In high energy scattering of e^+ , e^- , etc., which is of course a high momentum transfer test and therefore essential for detecting possible new interactions or deviations at short distances,⁽³⁾ one tests in practice only Born diagrams at high energies. The effects of the vacuum polarization has also been taken into account to obtain an improved value for the scattering length^(4,5,6) in low energy proton-proton scattering. This experiment of course does not measure vacuum polarization effect by itself as strong interactions give the main unknown contribution to the calculations. Hence one has to believe the theoretical predictions for vacuum polarization at $q^2 \gg m_e^2$.

Simple considerations show, that in nonrelativistic heavy ion scattering we can achieve a rather direct test of the structure of the spectral function for the photon propagator at large

momentum transfer $|\vec{q}|^2$, though the relative precision presently available is in no sense comparable to that of muonic atoms. For illustration, we choose the scattering of ^{12}C on ^{208}Pb . To bring both nuclei to a separation of 10 fm (at which part the nuclear densities begin to overlap), we need at least a kinetic energy, E , of 66 MeV for a ^{12}C ion. We choose an $E_p = 40$ MeV ^{12}C -beam (which most tandem laboratories can achieve). For the relative momentum we find $p^2 \sim 0.9 \text{ GeV}^2/c^2$. The momentum transfer is $|\vec{q}|^2 = |\vec{p}|^2 4 \sin^2 \theta/2$, where θ is the scattering angle. We see, that with the chosen conditions, we easily reach a momentum transfer $|\vec{q}| \approx 1 \text{ GeV}/c$ at a subcoulomb energy. With ^{208}Pb as target, the available energy resolution of better than 1% allows discrimination against coulomb excitations of the target.

As we calculate below, we find an increase of the elastic scattering cross section due to vacuum polarization effects by 1% at $|\vec{q}| = 30 \text{ MeV}$, at $|\vec{q}| = 100 \text{ MeV}$ by 1.4%, at $|\vec{q}| = 1 \text{ GeV}$ by 2.1% and at 3 GeV by 2.4%. No absolute measurements of the cross section are necessary as we may view $|\vec{q}|$ as function of the scattering angle. As the precision of the relative measurements now can be as high as .5%⁽⁷⁾ we believe that the necessary experiments can be carried out in the near future.

We will justify below that it is sufficient to consider

first order Born approximation to calculate the effects outlined above. In the nonrelativistic approximation, the elastic scattering of two charged point-like particles is described completely by the photon propagator $D_{\mu\nu}(|\vec{q}|^2)$ ($q_0 = 0$). In first order Born approximation we have

$$\frac{d\sigma}{d\Omega} \propto (D_{00}(|\vec{q}|^2))^2 \quad (1)$$

We also know, that $D_{00}(|\vec{q}|^2)$ may be represented using the spectral function $\sigma(t)$ (8)

$$D_{00}(|\vec{q}|^2) = |\vec{q}|^2 + \int_1^\infty \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt \quad (2)$$

m_e is the electron mass. In first order in α , the spectral function is given by (9)

$$\sigma(t) = 2\alpha/3\pi \left(1 + \frac{1}{2t^2}\right) (t^2 - 1)^{1/2} t^{-2} \quad (3)$$

Combining Eqs. (1), (2) we obtain

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_R = 1 + 2|\vec{q}|^2 \int_1^\infty \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt \quad (4)$$

$R = \text{Rutherford.}$

As all quantities on the right hand side of Eq. (4) are positive definite, we see that the actual cross section should be larger than the Rutherford cross section.

The right hand side is easily evaluated for different values of $|\vec{q}|^2$. We find that already for $|\vec{q}|^2 \sim 5 \times (2m_e)^2$ the spectral

function reaches the asymptotic form (see Uehling⁽⁹⁾)

$$\int_1^{\infty} \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt = \alpha/3\pi |\vec{q}|^{-2} [\ln(|\vec{q}|^2 (2m_e)^{-2}) - 5/3] \quad (5)$$

Noting that $|\vec{q}|^2 = 8\sin^2 \theta/2 \cdot E_p \mu_p$, where $E_p = p^2/2\mu_p$ is the relative kinetic energy and μ_p the reduced mass, we obtain

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right) / \left(\frac{d\sigma}{d\Omega} \right)_R &= 1 + 2\alpha/3\pi [\ln(|\vec{q}|^2 (2m_e)^{-2}) - 5/3] \\ &= 1 + 2\alpha/3\pi [\ln(2\sin^2(\theta/2) E_p \mu_p / m_e^2) - 5/3] \end{aligned} \quad (6)$$

This expression yields the numerical results quoted above.

The above consideration in momentum space can be easily understood in the configuration space. As it is well known, the electron vacuum polarization potential has the same sign as the inducing potential and has a range of about $\lambda_e/2 \sim 190$ fm, where λ_e is the Compton wave length of the electron. Due to this interaction we have in addition to the Coulomb force an additional "long range" force (when comparing with the nuclear sizes). Thus the probability for scattering must increase.

We will now justify the use of first order Born approximation to derive Eq. (4). We have so far considered the contributions shown in Fig. 1 to the scattering amplitude. As the coupling constant is $\alpha' = Z_1 \cdot Z_2 \cdot \alpha > 1$ we cannot use the standard argument $\alpha' < 1$. The characteristic parameter for Coulomb scattering

is $\eta = Z_1 \cdot Z_2 \cdot \alpha \cdot c / v_\infty$ where v_∞ is the asymptotic velocity of the projectile. For the case considered we have $\eta = 11$. This indicates that no expansion in η is permissible. However, there is a very small parameter, $v = (2m_e c)^2 / p^2$. We will now consider the scattering amplitude in all orders in η and approximate the result using $\eta \cdot v^{1/2}$ as a small parameter. In fact, $\eta v^{1/2} = Z_1 \cdot Z_2 \alpha m_e c^2 / E_p$ which even in case of a very heavy ion collision is a reasonably small quantity. Setting E_p proportional to the Coulomb energy at the turning point, we obtain (Z dependence cancels out) $\eta v^{1/2} = R_{ca} / \lambda_e$, here R_{ca} is the distance of closest approach in the center of mass system and $\lambda_e = 386$ fm, the electron's Compton wavelength. In Fig. 2 we outline our approach. We use exact Schroedinger Coulomb functions for the scattered heavy ion to calculate the vacuum polarization scattering amplitude. We rely heavily on the work done in proton-proton scattering.⁽⁶⁾ As seen in Fig. (2) we do not consider graphs with two or more vacuum polarization loops, as this would imply computing to next order in the vacuum polarization coupling constant $\alpha/3\pi$. This contribution should be considerably smaller in the case considered hereto. Using the expression given by Durand⁽⁶⁾ for the vacuum polarization scattering amplitude $M_{vp}(q^2)$ calculated with Schrodinger Coulomb functions we obtain

$$M(|\vec{q}|^2) = M_c(|\vec{q}|^2) + M_c(|\vec{q}|^2) \propto (\sin^2 \theta/2)^{-i\eta}. \quad (7)$$

$$\cdot \left\{ |\vec{q}|^{-2} + \int_0^\infty \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} \cdot [(1 + vt^2/2\sin^2 Q/2)/(1+vt^2/2)]^{-i\eta} \right. \\ \left. \cdot \exp[-2\eta \tan^{-1}((v/2)^{1/2}t)] \cdot |\Gamma(1+i\eta)|^2 \cdot {}_2F_1(-i\eta, 1+i\eta, 1; (1+t^2 v/2\sin^2 Q/2)^{-1}) \right\}$$

Note that the ${}_2F_1$ function is singular for $v \rightarrow 0$. Therefore we consider

$$|\Gamma(1+i\eta)|^2 {}_2F_1(-i\eta, 1+i\eta, 1; (1+\epsilon)^{-1}) = -i\eta \ln [(1+\epsilon)/\epsilon]. \quad (8)$$

$$\cdot {}_2F_1(-i\eta, 1+i\eta, 1; \epsilon/(1+\epsilon)) - i\eta \Gamma(-i\eta)^{-1} \Gamma(1+i\eta)^{-1} \sum_{n=0}^{\infty} \Gamma(n-i\eta).$$

$$\Gamma(n+1+i\eta)/(n!)^2 (\epsilon/(1+\epsilon))^n \cdot [(n-i\eta)^{-1} \sum_{m=n+1}^{\infty} 2 \eta^2 (m(m^2+\eta^2))^{-1}]$$

The sum over n may be truncated after $n = 0$, as the expansion parameter is $\eta^2 \cdot v$. We thus obtain

$$(8) = 1 - i\eta \ln[(1+\epsilon)/\epsilon] {}_2F_1(-i\eta, 1+i\eta, \epsilon/(1+\epsilon)) + i\eta 2\text{Re}(\psi(1+i\eta) + \gamma) \quad (9)$$

where $\psi(z) = d(\ln \Gamma(z))/dz$ and $\gamma = 0.5772$. Even for $\eta \sim 10$ the asymptotic expansion $\text{Re}(\psi(1+i\eta)) = \ln(1+\eta)$ is very good. Inserting Eq. (8) into Eq. (7) we see that in the domain of integration over t in which $vt^2 < 1$ we can neglect all additional factors in Eq. (7) but

$$1 - i\eta [\ln (1+(vt^2/2\sin^2 Q/2)^{-1}) + 2 \ln (1+\eta) + 2\gamma].$$

For $vt^2 > 1$ however the integrand in Eq. (7) does not contribute

significantly. Thus

$$M(|\vec{q}|^2) \propto (|\vec{q}|^{-2} + \int \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt + i\eta \int [\ln(1+\eta) - \ln(1+(\nu t^2/2\sin^2 Q/2)^{-1})] \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt \quad (10)$$

We emphasize that this result is valid for any value of η .

As the additional term in Eq. (10) is purely imaginary, no destructive interference between Coulomb and vacuum polarization parts in the amplitude occurs. Proceeding as before we obtain

$$\frac{d\sigma}{d\Omega} \frac{d\sigma}{d\Omega}_R = 1 + 2|\vec{q}|^2 \int_1^\infty \sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1} dt - \eta^2 (A \cdot |\vec{q}|^2)^2 \quad (11)$$

where $A \cdot |\vec{q}|^2 = |\vec{q}|^2 \int [2\ln(1+\eta) + 2\gamma - \ln[1+(\nu t^2/2\sin^2 Q/2)^{-1}]]$

$$\sigma(t) [|\vec{q}|^2 + 4m_e^2 t^2]^{-1}$$

As long as $\eta\alpha < 1$ the contribution $(A \cdot |\vec{q}|^2)^2$ can be neglected. When considering possible future Pb-Pb experiments this term should be carefully evaluated, however. Then other diagrams of the order $(Z_1\alpha)(Z_2\alpha)$ should also be included.

Let us finally discuss a number of effects which may possibly enter into the discussion of Coulomb elastic cross section. The most obvious effect is the electron screening of the target nucleus (we assume that the light projectile is completely stripped). The s-electrons have essentially constant charge distribution in the nucleus - this will cause no effect as a function of the

scattering angle or energy - the effects due to the variation of electronic charge in the target atom can be estimated using a model charge distribution and is found to be of the order of 0.1% and less of the total cross section in the region of interest.

As the collision is slow compared with the electron motion, molecular orbitals⁽¹⁰⁾ must be used to describe the electron during the collision states. This will give rise to effective molecular potentials similar to those found in chemical binding, as it has been outlined in Reference (11). The precise calculation of those effects, which are very important in very heavy ion - atom collisions are in progress. However, if we choose light projectiles with $Z < 20$ and heavy target with $Z \sim 80 - 90$ this effect is also negligible.⁽⁷⁾ All radiative effects like self energy, vertex corrections, Bremsstrahlung are negligible due to the large mass of the projectile and its non-relativistic motion. In the non-relativistic case the possible spin-spin and spin-orbital coupling of nuclear spins are also almost negligible - they are smaller by a factor $(\frac{v}{c})^2$ than the Coulomb scattering, for $\frac{v}{c} \sim \frac{1}{15}$ we have a possible 0.5% effect. This effect should be evaluated more carefully, when higher precision scattering data becomes available. We conclude this discussion observing that in scattering of very heavy ions, such as lead on lead at sub-Coulomb barrier energy, we can reach $|\vec{p}| = 134 \text{ GeV}^2$ which may

lead to $|\vec{q}|^2 \geq 500 \text{ GeV}^2$ (center of mass) under favorable conditions. Such experiments will be possible as soon as heavy ion accelerators in Darmstadt and Berkeley - Super - Hilac are working with Pb and heavier beams. With these accelerators we will have unique tools to test vacuum polarization at large momentum transfer.

The idea that vacuum polarization may play an important role in heavy ion scattering arose in discussions with Berndt Muller and Walter Greiner in Frankfurt. The work did not proceed until a discussion with Chris Davies, who brought this problem again to our attention.

Figure Captions

- 1) First order Born diagrams for Coulomb + vacuum polarization scattering.
- 2) The complete Born series to all orders in $Z_1 Z_2 \alpha$ and to first order in $Z_1 Z_2 \alpha \alpha$.

References

+ Supported in part by the U. S. Atomic Energy Commission.

1. The experiments are partially contradictory with respect to the sign of the discrepancy, which may be as large as 3.5 standard deviations compared to the best available theory, which also includes higher order effects of vacuum polarization, see G. Backenstoss et al., Phys. Letters 31B, 233 (1970), 43B, 539 (1973); M. S. Dixit, H. L. Anderson et al., Phys. Rev. Lett. 27, 878 (1971) and H. K. Walter, J. M. Vuilleumier et al., Phys. Lett. 40B, 197 (1972).
2. For theoretical calculation see M. K. Sundaresan and P. J. S. Watson, Phys. Rev. Lett. 29, 15, 1122 (1972); J. Blomquist, Nucl. Phys. B48, 95 (1972); T. L. Bell, Phys. Rev. A7, 1480 (1973); P. Vogel, Phys. Rev. A7, 63 (1973). For larger summary of the present status see J. Rafelski, B. Muller, G. Soff and W. Greiner "Critical Discussion of the Vacuum Polarization Measurements with Muonic Atoms" Frankfurt, September, 1973, to be published. In a recent preprint G. A. Rinker, Jr. and L. Wilets claim additional contribution to higher order vacuum polarization calculations. This contribution enlarges the discrepancy in lead by one standard deviation.

3. S. J. Brodsky, S. D. Drell, Ann. Rev. Nuc. Sci. 20, 147 (1970).
4. For review see E. M. Henley in "Isospin in Nuclear Physics", edited by D. H. Wilkinson, North-Holland (1969).
5. L. Durand, III, Phys. Rev. 108, 1597 (1957).
6. L. Heller, Phys. Rev. 120, 627 (1960).
7. T. Fortune, private communication: We thank Dr. Fortune for supplying us with the experimental information needed.
8. We refer here to standard books on Field Theory, such as J. D. Bjorken and S. D. Drell or S. Schweber.
9. J. Schwinger, Phys. Rev. 75, 651 (1949), see also E. A. Uehling, Phys. Rev. 48, 55 (1935).
10. B. Muller, J. Rafelski and W. Geiner, to appear in Physics Letters; B. Muller, Ph.D. Thesis, Frankfurt 1973.
11. J. Rafelski, B. Muller and W. Geiner, Lett. Nuovo Cim. 4, 469 (1972).
12. In this case the molecular K-orbitals are already created when both nuclei are separated by app. 150 fm. The energy of the K-shell does not change considerably when the ions further approach each other, thus no appreciable effect on the scattering cross section is found in the range of $|\vec{q}|^2$ considered.

Fig 1 Rapelski - Klein

Fig 2 Rapelshu - Klein