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ANALYSIS OF THE CLEAN BUCKLINGS OF 1.3 PER CENT ENRICHED URANIUM-WATER LATTICES

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BROOKHAVEN NATIONAL LABORATORY

M E M O R A N D U M

Date: April 6, 1953

To: I. Kaplan

From: J. Chernick

Subject: Analysis of the Clean Bucklings of 1.3 Per Cent Enriched Uranium-Water Lattices.

Introduction:

The experimental results on the relaxation lengths of the clean, 0.6" diameter, 1.3 per cent U_{235} enriched uranium-water lattices have been completed for the 2:1, 3:1, and 4:1 water to fuel volume ratios. A re-determination of the relaxation lengths of the 1.5:1 lattices as a function of the size of the assembly is presently being carried out. Similar work for the 1:1 water-uranium hexagonal lattices is awaiting the fabrication of the appropriate tube sheets.

On the basis of the experimental results obtained to date, we have investigated some of the questions relating to the accuracy with which the bucklings of these water lattices can be determined.

The standard method of finding the buckling of thermal reactors in the past has involved the fitting of radial flux traverses to determine the lateral reflector savings of the assembly. The method has thus far proved impractical in view of the small size of our enriched uranium-water assemblies. Experimental values are required of the reflector savings correct to about

0.1 cm as a function of loading, azimuth and distance from the source plane. The variation with azimuth is expected because of the irregular loading patterns, and radial flux harmonics will exist near the bottom and top of the assemblies. However, with the recent foil techniques perfected by Kouts and his group, it may prove feasible to use this method of analysis on some of our larger assemblies.

In lieu of direct experimental data, the reflector savings and buckling of the exponential piles have, therefore, been determined by a two parameter fit of the relaxation lengths as a function of loading radius. The method of analysis used for the 0.75" diameter rods assumed that the reflector saving did not vary significantly with loading. The experimental data obtained with the Kidde lattices indicated that this assumption was valid but was not sufficient to settle the point. The data collected to date with the CVR type of lattice is more complete in this respect and is the basis of the present statistical analysis. In addition, a theoretical study of the reflector savings of these lattices as a function of loading has been set up on a two group basis for solution by means of the Univac.

A second point of interest is the question of the statistical weight factors used in the analysis of the experimental data. The convenient least square method which we have selected preferentially weights the relaxation lengths obtained at the larger loadings. In order to judge any bias thus introduced, two alternate weighting systems are studied in the present report.

Method of Analysis:

The experimental data required are the relaxation lengths, L, obtained at each loading. These are listed in Appendix I, and should satisfy the equation

$$L = \left[(2.40483/R+\lambda)^2 - B^2\right]^{-1/2} \qquad (1)$$

where R is the effective loading radius of the assembly, λ the reflector savings

and B^2 the material-buckling of the lattice. The correction factors and statistics involved in the determination of the relaxation lengths and their probable errors are being studied by Kouts for the 2:1 CVR lattices. The residuals obtained in least square fits of equation (1) are given in Appendix II to facilitate future comparison with Kouts' work.

If the reflector savings λ is taken as independent of loading then equation (1) involves a two parameter non-linear fit of the data. If λ_o, B_o^2 are approximations to the best values in the least square sense, a Taylor expansion with $B^2 = B_o^2 + \Delta B^2$, $\lambda = \lambda_o + \Delta\lambda$, $L_o = \left[(2.40483/R+\lambda_o)^2 - B_o^2\right]^{-1/2}$, yields

$$L - L_o = (L_o^3/2)(\Delta B^2) + \left[5.78321\ L_o^3/(R+\lambda_o)^3\right](\Delta\lambda) \qquad (2a)$$

if the higher order terms are neglected. Since equation (2a) is linear it may be fitted by the usual least square technique as it stands. This method makes the sum of the squares of the differences between the observed and calculated values of L a minimum and hence preferentially weights the larger relaxation lengths. An even stronger weighting of the data from the larger assemblies occurs if we fit not L, but L^2 by the above method. The linear formula is then given by

$$L^2 - L_o^2 = L_o^4(\Delta B^2) + \frac{11.56642\ L_o^4}{(R+\lambda_o)^3}(\Delta\lambda) \ . \qquad (2b)$$

Finally, if one divides equation (2a) through by L_o, one obtains the formula

$$\frac{L - L_o}{L_o} = \frac{L_o^2}{2}(\Delta B^2) + \frac{5.78321\ L_o^2}{(R+\lambda_o)^3}(\Delta\lambda) \ . \qquad (2c)$$

A least square fit of equation (2c) gives approximately equal weight to the percentage errors at each loading. All three formulas are convenient to use in least square work, and a comparison of the results indicates how much faith can be placed in the precision estimates derived from the goodness of fit of the data. It is important that the values of B_o^2 and λ_o selected be reasonably close to the exact values. Such a set may be obtained by inverting equation (1)

and solving for B^2 for a few trial values of λ between 6.5 and 7.5 cm. The value of λ which makes B^2 most nearly independent of the loading radius R is then used as a first approximation. Residuals obtained from equation (2) may be compared with exact values based on equation (1) as a check on the linearity of the Taylor expansion. If they disagree, a second run-through of the least square work is required with the improved values of λ_o and B_o^2. Except for 3-parameter fits of the data, we have found that such a repetition of the numerical work is rarely required. The Gaussian method of elimination of the least square equations was used together with check columns to avoid numerical errors. Together with the test for linearity, this method provides a practically complete check on all the calculations.

The results of the least square fits of λ and B^2 and their standard errors are tabulated in Table I below:

Table I

Water to Uranium Volume Ratio = 4:1

Method	λ (cm)	B^2 (10^{-4} cm^{-2})
(2a)	6.673 ± 0.100	50.259 ± 0.366
(2b)	6.800 ± 0.140	49.794 ± 0.499
(2c)	6.644 ± 0.066	50.376 ± 0.265

Water to Uranium Volume Ratio = 3:1

Method	λ (cm)	B^2 (10^{-4} cm^{-2})
(2a)	6.659 ± 0.050	61.201 ± 0.241
(2b)	6.640 ± 0.059	61.291 ± 0.280
(2c)	6.700 ± 0.048	60.989 ± 0.257

Water to Uranium Volume Ratio = 2:1

Method	λ (cm)	B^2 (10^{-4} cm^{-2})
(2a)	7.040 ± 0.070	61.102 ± 0.345
(2c)	7.044 ± 0.060	61.082 ± 0.315

It may be seen that the results are not sensitive to the different methods of weighting the experimental data. The relative error of the buckling is within 1 per cent while the reflector savings is determined to about 0.1 cm on the common basis that the latter does not change with loading.

We next consider the case where the reflector saving is assumed to vary linearly with the loading radius. Equation (1) may be written in the form

$$L = \left[\left(\frac{2.40483}{R+\lambda+c(R-\bar{R})}\right)^2 - B^2\right]^{-1/2} , \qquad (3)$$

where c is a third parameter to be determined and $\bar{R}$ is some mean loading radius in the range of the experimental values. If λ_o, B_o^2, c_o are first approximations and

$$L_o = \left[\left(\frac{2.40483}{R+\lambda_o+c_o(R-\bar{R})}\right)^2 - B_o^2\right]^{-1/2} ,$$

then the linear approximation to equation (3) is given by

$$L-L_o = \frac{L_o^3}{2}(\Delta B^2) + \frac{5.78321\ L_o^3(\Delta\lambda)}{\left[R+\lambda_o+c_o(R-\bar{R})\right]^3} + \frac{5.78321\ L_o^3(R-\bar{R})(\Delta c)}{\left[R+\lambda_o+c_o(R-\bar{R})\right]^3} \qquad (3a)$$

in complete analogy with equation (2a). An initial value of $c_o = 0$ was assumed, together with previous estimates of λ_o and B_o^2. The least square values thus obtained were close to the final values but a second run was generally required to improve the linearity in the expansion of equation (3). The final results are shown in Table II.

Table II

Volume Ratio	λ (cm)	B^2 (10^4 cm^{-2})	c	$\bar{R}$ (cm)
4:1	7.402 ± 0.775	46.460 ± 4.335	0.0930 ± 0.1018	21
3:1	6.303 ± 0.432	63.670 ± 3.063	-0.0455 ± 0.0532	19
2:1	7.065 ± 0.826	60.941 ± 5.399	0.0031 ± 0.1041	19

The striking changes in the results shown by the above table are the standard errors of the bucklings and reflector savings. The relative errors run to several per cent. However, it may be seen that the standard error in the determination of the constant c, which measures the increase in the reflector saving per centimeter increase in loading radius, is greater than the absolute value of c in every case considered. Also the values of c have no apparent pattern, ranging from -0.05 cm/cm to 0.09 cm/cm. Clearly the number of cases examined is still too few to draw definite conclusions, but it again appears likely that the variation of the reflector savings with loading is actually quite small.

Further evidence on this question will be afforded by the values of the reactor bucklings obtained in critical experiments at WAPD and by compiling data of the type shown in Tables I and II for the remaining experimental assemblies. The results of radial flux traverses could, of course, be compared directly with the above fitted values of the reflector savings. It is unfortunate that such a comparison involves a considerable experimental program. Finally, the results of the theoretical calculations which are underway may indicate some of the trends to be expected.

Residuals from the least square fits of the data on the basis of the two parameter fit of equation (2a) and the three parameter fit of equation (3a) are shown in Appendix II. The loading radius R is that of the equivalent cylinder and is obtained from the formula

$$\pi R^2 = NA$$

where N is the number of rods and A the area of a unit cell. From the drawings Kouts finds that:

$R = 1.7431 \sqrt{N}$ 4:1 Lattice

$R = 1.5675 \sqrt{N}$ 3:1 Lattice

$R = 1.3700 \sqrt{N}$ 2:1 Lattice

For 0.6" diameter rods, these values correspond to a gap width (air plus aluminum) of about 33 mils.

An examination of the residuals given in Appendix II shows that the improvement in the fitted values of the relaxation lengths, afforded by a 3-parameter fit of the data, is actually quite trivial, and accounts for the lack of statistical significance in the determination of the constant c. It may also be noted that the residuals tend to be larger for the larger assemblies. The current work of Kouts on the standard errors obtained in fitting L for the 2:1 lattices will show whether this trend is reasonable. If so, it may be preferable in the future to use a least square analysis, such as (2c), which does not preferentially weight the larger assemblies. Finally, the residuals collected in Appendix II should be useful in correlating reactivity effects associated with the relative effectiveness of the various loading patterns, or possible diurnal effects.

Appendix I

Relaxation Lengths of 0.6" dia., 1.3 Per Cent U_{235} Lattices

Volume Ratio	4:1		3:1		2:1	
	N	L (cm)	N	L (cm)	N	L (cm)
	217	44.287	211	42.496	265	40.293
	211	40.515	205	37.255	259	37.526
	205	35.892	199	34.025	253	34.066
	199	34.209	193	31.127	247	32.466
	193	31.410	187	29.026	241	30.071
	187	29.109	181	26.835	235	28.340
	181	27.087	175	25.072	229	26.944
	175	25.958	169	23.387	223	25.657
	169	23.879	163	22.350	217	23.924
	163	23.185	157	21.382	211	23.177
	157	21.796	151	19.955	205	22.044
	151	20.563	145	18.969	199	21.418
	145	19.957	139	18.041	193	20.224
	139	18.910	133	17.342	187	19.685
	133	17.982	127	16.472	181	18.869
	127	17.200	121	15.644	175	18.303
	121	16.500	115	15.116	169	17.208
	115	15.895	109	14.403	163	16.844
	109	15.224	103	14.021	157	16.218
	103	14.539	97	13.418	151	15.702
	97	14.001	91	12.654	145	15.301
	91	13.368	85	12.177	139	14.885
					133	14.308

Appendix II

Residuals from Least Squares Analysis - Volume H_2O:Volume U = 4:1

		(2-Parameter Fit)		(3-Parameter Fit)	
N	$L_{obs.}$	$L_{calc.}$	Res.	$L_{calc.}$	Res.
217	44.29	44.72	-.43	44.60	-.31
211	40.52	40.02	.50	40.00	.52
205	35.89	36.40	-.51	36.43	-.54
199	34.21	33.49	.72	33.54	.67
193	31.41	31.08	.33	31.14	.27
187	29.11	29.02	.09	29.09	.02
181	27.09	27.25	-.16	27.31	-.22
175	25.96	25.68	.28	25.74	.22
169	23.88	24.29	-.41	24.34	-.46
163	23.18	23.03	.15	23.07	.11
157	21.80	21.89	-.09	21.92	-.12
151	20.56	20.84	-.28	20.86	-.30
145	19.96	19.88	.08	19.88	.08
139	18.91	18.98	-.07	18.97	-.06
133	17.98	18.13	-.15	18.12	-.14
127	17.20	17.34	-.14	17.32	-.12
121	16.50	16.60	-.10	16.56	-.06
115	15.90	15.89	.01	15.83	.07
109	15.22	15.21	.01	15.14	.08
103	14.54	14.56	-.02	14.48	.06
97	14.00	13.94	.06	13.84	.16
91	13.37	13.33	.04	13.22	.15

Appendix II (cont.)

Residuals from Least Squares Analysis - Volume H_2O:Volume U = 3:1

		(2-Parameter Fit)		(3-Parameter Fit)	
N	$L_{obs.}$	$L_{calc.}$	Res.	$L_{calc.}$	Res.
211	42.50	42.35	.15	42.39	.11
205	37.26	37.57	-.31	37.57	-.31
199	34.03	33.98	.05	33.96	.07
193	31.13	31.12	.01	31.09	.04
187	29.03	28.79	.24	28.76	.27
181	26.84	26.81	.03	26.78	.06
175	25.07	25.11	-.04	25.08	-.01
169	23.39	23.63	-.24	23.60	-.21
163	22.35	22.31	.04	22.28	.07
157	21.38	21.12	.26	21.11	.27
151	19.96	20.05	-.09	20.04	-.08
145	18.97	19.07	-.10	19.07	-.10
139	18.04	18.17	-.13	18.17	-.13
133	17.34	17.33	.01	17.33	.01
127	16.47	16.54	-.07	16.56	-.09
121	15.64	15.81	-.17	15.82	-.18
115	15.12	15.11	.01	15.14	-.02
109	14.40	14.45	-.05	14.48	-.08
103	14.02	13.82	.20	13.86	.16
97	13.42	13.22	.20	13.26	.16
91	12.65	12.64	.01	12.69	-.04
85	12.18	12.08	.10	12.13	.05

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Appendix II (cont.)

Residuals from Least Squares Analysis - Volume H_2O:Volume U = 2:1

		(2-Parameter Fit)		(3-Parameter Fit)	
N	$L_{obs.}$	$L_{calc.}$	Res.	$L_{calc.}$	Res.
265	40.29	40.59	-.30	40.58	-.29
259	37.53	37.13	.40	37.13	.40
253	34.07	34.36	-.29	34.35	-.28
247	32.47	32.04	.43	32.03	.44
241	30.07	30.06	.01	30.06	.01
235	28.34	28.35	-.01	28.35	-.01
229	26.94	26.84	.10	26.84	.10
223	25.66	25.50	.16	25.50	.16
217	23.92	24.29	-.37	24.29	-.37
211	23.18	23.20	-.02	23.20	-.02
205	22.04	22.20	-.16	22.20	-.16
199	21.42	21.28	.14	21.28	.14
193	20.22	20.42	-.20	20.42	-.20
187	19.68	19.63	.05	19.63	.05
181	18.87	18.89	-.02	18.89	-.02
175	18.30	18.19	.11	18.19	.11
169	17.21	17.53	-.32	17.53	-.32
163	16.84	16.90	-.06	16.90	-.06
157	16.22	16.31	-.09	16.31	-.09
151	15.70	15.74	-.04	15.74	-.04
145	15.30	15.20	.10	15.20	.10
139	14.88	14.68	.20	14.68	.20
133	14.31	14.18	.13	14.17	.14

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