1

UCRL-3149
~~Chemistry Distribution~~

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

CONCERNING K MIXING IN BOHR-MOTTelson
STATES OF SPHEROIDAL NUCLEI

John O. Rasmussen

September 21, 1955

Printed for the U. S. Atomic Energy Commission

075 1

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

CONCERNING K MIXING IN BOHR-MOTTELSON
STATES OF SPHEROIDAL NUCLEI

John O. Rasmussen
Radiation Laboratory and Department of Chemistry
University of California, Berkeley, California

September 21, 1955

Fundamental to the Bohr-Mottelson¹⁻³ representation of nucleonic wave functions for spheroidal nuclei is the separation of a slow rotational motion of the deformed nucleus from a rapid nucleonic motion in the nuclear frame of reference. The treatment introduces two new approximate quantum numbers, Ω and K , the components along the nuclear symmetry axis of the nucleonic angular momentum and of the total angular momentum, respectively. For the lower-lying states, where no "vibrational" excitation is involved, K and Ω are equal. Thus we shall refer hereafter to K -quantum numbers of states with the implicit understanding that Ω remains equal to it. With the availability of calculated nucleon wave functions for spheroidal potentials⁴ it is appropriate to check the degree to which the separation of rotational and intrinsic nucleonic motion is justified. That is, we may evaluate and include the usually neglected off-diagonal matrix elements of Bohr's U_1 . (Cf. A. Bohr, Ref. 1 pp 29 and 35, also Davidson and Feenberg⁵ and Ford.⁶) These matrix elements connect Bohr states which differ in K (and Ω) by one but have like spins and parities. The matrix elements measure the extent to which the rotational motion decouples the intrinsic nucleonic motion from the symmetry axis.

There is considerable experimental evidence from relative gamma and beta transition probabilities to rotational band members that K is quite a good quantum number in even-even nuclei in the regions of high nuclear deformation.^{3, 7} On the other hand, since no pairing energy need be involved

for intrinsic excitation of an odd nucleon, K mixing in odd-mass nuclei could often be important even in the regions of highest deformation. Bohr and Mottelson⁸ have recently discussed the role of the matrix elements U_1 in connection with rotational moments of inertia.

In attempts⁹ to classify the low-lying nuclear states of Np^{237} the presence of E1 transitions has drawn attention to the even-parity states. Most even-parity states in the 82-126 shell tend in the limit of low deformation toward pure $i_{13/2}$ character, but in the calculations of Nilsson⁴ at large deformation ($\delta = +0.3$) become substantially mixed, principally with $g_{9/2}$ and $d_{5/2}$.

As a numerical example illustrative of the special importance of U_1 in certain cases ($j \gg \Omega$) some of the lowest nuclear energy levels for a single nucleon in the even-parity states of the 82-126 shell were calculated. The wave functions and eigenvalues of Nilsson at $\delta = +0.3$ (large prolate deformation) were used, with assumption of a rotational quantum energy $\left(\frac{\hbar^2}{2\mathfrak{J}}\right)$ of 7 kev, an average for even-even nuclei in the heavy region. The set of levels at the left in Fig. 1 were calculated neglecting off-diagonal contributions of U_1 , and those at the right included them.

There are extensive shifts in the levels and actually a reversal in level order of the lowest two states.

K mixing occurs in the lowest-lying states to an extent that might significantly alter some nuclear properties, such as transition probabilities and magnetic moments. (K mixing cannot, of course, occur in the $I = 1/2$ state.) The spin $5/2$ state has 4.4 percent $K = 3/2$ and 0.03 percent $K = 5/2$, and the spin $9/2$ state, 9 percent $K = 3/2$ and 0.8 percent $K = 5/2$ character.

For a somewhat larger deformation or larger moment of inertia than used in the numerical case of Fig. 1 the $5/2$ state would lie lowest. This

configuration would then be possible for the ground state of Np^{237} . A curious feature of Np^{237} is the anomalously large magnetic moment of its ground state, $+6.0 \pm 2.5$ nuclear magnetons,¹⁰ a value outside the Schmidt limits for spin $5/2$ nuclei.

Magnetic moment calculations with Nilsson's wave functions of the $I = 5/2$, $K = \Omega = 5/2$ state of predominantly $i_{13/2}$ character give about $+3.0$ magnetons, not strongly dependent on deformation.¹¹ Calculations for the $I = 5/2$, $K = \Omega = 1/2$ state without K-mixing give $+3.9$ nuclear magnetons, virtually independent of the degree of prolate deformation.¹² The partial decoupling of the nucleonic angular momentum from the symmetry axis and the consequent K-mixing have an important influence on the magnetic moment. Indeed for the lowest-lying $I = 5/2$ state of the numerical example of Fig. 1 we calculate a magnetic moment of about $+5.2$ nuclear magnetons. The magnetic moment would be larger for smaller deformation, approaching a limit of $+6.1$ nuclear magnetons.

If we interpret the ground state of Np^{237} to be this predominantly $K = 1/2$ state and assume positive intrinsic nuclear deformation, the spectroscopic quadrupole moment, not yet measured experimentally, should be negative. This interpretation also offers the means of explanation for the recent observation by Roberts and Dabbs¹³ that aligned Np^{237} nuclei emit alpha particles preferentially in a direction perpendicular to the nuclear spin axis. The opposite behavior would be expected for $K = 5/2$ nuclear state with prolate surface deformation.^{14, 15}

Difficulty with the above interpretation is encountered in assignment of the 33-kev excited state, and the experimental limits of error on the Np^{237} magnetic moment are still too large to be as restrictive as desirable for nuclear state assignments.

We may generalize from the above calculation that the even-parity nuclear states with an odd nucleon in the 82-126 shell will usually exhibit badly perturbed rotational bands or, at best, bands with very large apparent moments of inertia. The odd-parity states, having generally lower average intrinsic angular momentum (j content), will have smaller matrix elements U_1 and hence be less subject to K-mixing. They should more generally exhibit normal rotational bands.

I am especially indebted to Dr. S. G. Nilsson for sending his calculated wave functions and to Dr. A. Bohr and B. R. Mottelson for other valuable reports in advance of publication.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

Note added in proof: We have just seen a preprint of a paper by Kerman¹⁶ containing a detailed analysis of effects of U_1 , designated RPC in his notation. He finds these matrix elements of importance in the W^{183} spectrum.

REFERENCES

1. A. Bohr, Dan. Mat. Fys. Medd. 26, No. 14 (1952).
2. A. Bohr and B. R. Mottelson, Dan. Mat. Fys. Medd. 27, No. 16 (1953).
3. Alaga, Alder, Bohr, and Mottelson, Dan. Mat. Fys. Medd. 29, No. 9 (1955).
4. S. G. Nilsson, Dan. Mat. Fys. Medd., 29, No. 16 (1955).
5. J. P. Davidson and E. Feenberg, Phys. Rev. 89, 856 (1953).
6. K. W. Ford, Phys. Rev. 90, 29 (1953).
7. Rasmussen, Stephens, Strominger, and Åström, Phys. Rev. 99, 47 (1955).
8. A. Bohr and B. R. Mottelson, Rotations and Vibration of the Nuclear Shell Structure I., Dan. Mat. Fys. Medd., to be published.
9. J. O. Rasmussen, Arkiv för Fysik 7, 185 (1953).
10. Bleaney, Llewellyn, Pryce, and Hall, Phil. Mag. 45, 992 (1954).
11. S. G. Nilsson, private communications (1955).
12. J. O. Rasmussen and D. Strominger, unpublished.
13. L. D. Roberts and J. W. T. Dabbs, private communication (1955).
14. D. L. Hill and J. A. Wheeler, Phys. Rev. 89, 1102 (1953).
15. J. O. Rasmussen, University of California Radiation Laboratory
Unclassified Report No. UCRL-2431 (1953) (unpublished).
16. A. K. Kerman, Dan. Mat. Fys. Medd., to be published.

Fig. 1. Theoretical rotational band spectra for an odd nucleon in orbitals of predominantly $i_{13/2}$ character at large prolate deformation ($\delta = 0.3$).

0757