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μ -Capture and the n-n Scattering Length

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μ -Capture and the n-n Scattering Length

by

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ABSTRACT

The capture reaction $\mu^- + d \rightarrow n + n + \nu$ is examined with the objective of obtaining a precise measurement of the n-n scattering length. Differential time-of-flight neutron spectra for the two-neutron coincidence experiment are calculated to illustrate the sensitivity of the reaction to a_{nn} . The dominant theoretical uncertainty lies in the unknown short-range character of the n-n scattering wave function as it does for competing reactions such as radiative π -capture.

I. INTRODUCTION

It has been recognized for some time that the nucleon-nucleon (N-N) interaction is not charge independent; i.e., the n-p force is not the same as either the n-n force or the nuclear part of the p-p force. However, the question of charge symmetry--equality of the n-n force and the nuclear part of the p-p force--has not been settled. It is a question of fundamental interest to physics. Because the scattering lengths are large, small differences in the N-N interactions will induce sizeable differences in the respective scattering lengths. Hence a precise measurement of the n-n and p-p scattering lengths is a good method of examining the question of charge asymmetry. The p-p scattering length is well determined, but the uncertainty in the n-n scattering length is of the order of ± 1 f (see Appendix A). Therefore, we wish to examine here the possibility of obtaining a precise value of the n-n scattering length a_{nn} from the μ -capture reaction.

Because the weak interaction is thought to be reasonably well understood, the capture reaction

$$\mu^- + d \rightarrow n + n + \nu$$

should be much more easily interpreted than either of the reactions:

$$\mu^- + d \rightarrow n + n + \gamma$$

$$n + d \rightarrow n + n + p.$$

If indeed the conserved-vector-current theory (CVC) is valid, then half of the matrix elements involved in the general weak interaction, those of the vector current, are exactly the same as those which are measured in electron scattering. Like the electromagnetic force, the weak force is dominated by a single interaction, so that even multiple scattering effects are negligible. Thus, the μ -capture process, being one in which there are only two strongly interacting particles in either the initial or the final state, is one whose theoretical interpretation should be well founded. Several calculations of the μ -capture rate in deuterium have been reported.¹⁻⁶ Results obtained compare favorably with the experimental capture rate⁷ of approximately 350 sec^{-1} . The reasonable agreement of the theoretical and experimental rates adds to our confidence that the calculated neutron spectra will provide realistic estimates as a guide to the experimentalist.

However, it should be noted here that the largest uncertainty in the determination of a_{nn} from the radiative π -capture reaction comes not

from the uncertainty in the initial state but from the unknown short-range character of the final-state n - n wave function. The μ -capture reaction will suffer the same source of uncertainty in the final state, and therefore, it may not offer as much of a theoretical advantage as the above arguments would imply. (See Appendix B.)

II. THEORETICAL SUMMARY

If we denote the weak interaction Hamiltonian by H_W and use first-order perturbation theory, then we can write the probability for μ -capture as

$$dw = (2\pi)^{-5} \delta_4(P_i - P_f) \sum |\langle f | H_W | i \rangle|^2 d^3 p_1 d^3 p_2 d^3 v, \quad (1)$$

where $\vec{p}_1$, $\vec{p}_2$, and $\vec{v}$ are the three momenta of the two neutrons and the neutrino, respectively, ($\vec{P}_f = \vec{p}_1 + \vec{p}_2 + \vec{v}$), and P_i and P_f are the initial and final four momenta. The weak interaction Hamiltonian is assumed to be of the current-current interaction form

$$H_W = - \frac{G}{2} \int j_\lambda^{(+)}(x) j_\lambda^{(-)}(x) d^3 x \quad (2)$$

where the current is the sum of hadronic and leptonic parts:

$$j_\lambda^{(-)} = j_\lambda^{(-)}(\text{hadron}) + j_\lambda^{(-)}(\text{lepton}). \quad (3)$$

The coupling constant G is given by $G = 10^{-5}/m_p^2$, where m_p is the proton mass. The general $V - A$ lepton current can be expressed as

$$j_\lambda^{(-)} = i[\bar{\psi}_e \gamma_\lambda (1 + \gamma_5) \psi_{\nu_e} + \bar{\psi}_\mu \gamma_\lambda (1 + \gamma_5) \psi_{\nu_\mu}] \quad (4)$$

where $\bar{\psi}$ and ψ are the relativistic quantum fields describing the leptons. We have written here the full lepton current in order to emphasize the similarity between the electron and the muon weak currents which lead to β -decay, e -capture, μ -capture, etc.

By restricting our interest to just the deuteron μ -capture reaction, the weak Hamiltonian may be reduced to the following nonrelativistic effective interaction Hamiltonian:

$$H_W = - \frac{G}{2} \sum_i \tau_i^- \{ G_V + [G_P \vec{\sigma}_i \cdot \hat{v} + (F_A/m) \vec{p}_i \cdot \vec{\sigma}_i] \sigma_{\ell} \cdot \hat{v} + [G_A \vec{\sigma}_i + (F_V/m) \vec{p}_i] \cdot \vec{\sigma}_{\ell} \} (1 - \vec{\sigma}_i \cdot \hat{v}), \quad (5)$$

where the subscript i denotes nucleon operators and the subscript ℓ denotes lepton operators. The normalized coupling constants are:

$$\begin{aligned} G_V &= (1 + \frac{v}{2m}) F_V \\ G_A &= F_A - (1 + \mu_p - \mu_n) \frac{v}{2m} F_V \\ G_P &= (F_P - F_A - (1 + \mu_p - \mu_n) F_V) \frac{v}{2m} \\ F_V &= 1. \\ F_A &= -1.24 F_V \\ F_P &= 7 F_A \end{aligned} \quad (6)$$

The subscripts V , A , and P denote the vector, axial vector, and induced pseudoscalar coupling, μ_p and μ_n are the anomalous moments of the proton and the neutron, and τ_i^- is the nucleon isospin lowering operator. (The pseudoscalar coupling F_P is not well determined, but the effect of its uncertainty on the neutron spectra is rather small.)

Using this form of the effective weak Hamiltonian, we can write the appropriate spin-averaged square of the matrix element corresponding to the μ -d atom as

$$\begin{aligned} |\langle f | H_W | i \rangle|^2 &\sim \sum_{m_i} \sum_{m_f} \{ G_V^2 |\langle m_f | \sum_i \tau_i^- e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) | m_i \rangle|^2 \\ &\quad + G_A^2 |\langle m_f | \sum_i \tau_i^- \vec{\sigma}_i e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) | m_i \rangle|^2 \\ &\quad + (G_P^2 - 2G_P G_A) |\langle m_f | \sum_i \tau_i^- \vec{\sigma}_i \cdot \hat{v} e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) | m_i \rangle|^2 \\ &\quad - G_V G_V 2\text{Re}[\langle m_f | \sum_i \tau_i^- e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) | m_i \rangle \\ &\quad \quad \cdot \langle m_f | \sum_i \tau_i^- e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) \partial \cdot \vec{p}_i / m | m_i \rangle] \\ &\quad - (G_A G_A - G_P G_A) 2\text{Re}[\langle m_f | \sum_i \tau_i^- e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) | m_i \rangle \\ &\quad \quad \cdot \langle m_f | \sum_i \tau_i^- e^{i\vec{v} \cdot \vec{r}/2} \varphi_{\mu}(r) \vec{\sigma}_i \cdot \vec{p}_i / m | m_i \rangle] \} \end{aligned} \quad (7)$$

where the latter two terms describe the so-called relativistic corrections to order p_i/m . The matrix element is dominated by the second term, which contains the triplet-singlet transition. This is most easily seen in the S-state approximation for the deuteron, where the result (neglecting the relativistic corrections) reduces to

$$|\langle r | H_W | i \rangle|^2 \sim G_V^2 I_t^2 + G_A^2 (2I_t^2 + I_s^2) + (G_P^2 - 2G_A G_P) \frac{1}{3} (2I_t^2 + I_s^2) \quad (8)$$

The I_s and I_t denote integrals over the triplet and singlet final spin states of the two neutrons. It is I_s that contains the strong final-state s-wave interaction. Because G_P is proportional to v/m it becomes a small correction to the G_A^2 contribution.

Because the contribution of the relativistic corrections to the total capture rates is small, we shall neglect their contributions to the spectra in the curves presented below. The approximation is unimportant for our demonstration purposes, although they would be included in an actual analysis. It should also be pointed out that although G_A and G_P contain a momentum dependence through v , the momentum v is effectively a constant ($v \approx v_{\max}$) over the kinematic region in which we are interested.

The general form of the radial integrals that arise can be represented schematically as

$$I = \int \varphi_\mu(r) \varphi_d(r) e^{i\vec{v} \cdot \vec{r}/2} \varphi_{nn}(p, r) d^3r \quad (9)$$

where φ_d is the deuteron bound state, φ_{nn} is the two-neutron scattering state, $\exp(i\vec{v} \cdot \vec{r}/2)$ represents the neutrino, and φ_μ is the atomic muon wave function. We have used a Reid soft-core deuteron bound state including the D-state. For the n-n scattering state we have used a PRS wave function¹¹ in the singlet s-state, which permits us to vary the short-range character of the final state while holding the scattering length and effective range fixed. We wish to emphasize that the use of zero-range deuteron wave functions and asymptotic forms of the final state such as

$$\varphi_{nn}(p, r) \sim [j_0(pr) \cos \delta - n_0(pr) \sin \delta] \quad (10)$$

can lead to error in extracting a_{nn} from an experimental spectrum. We have assumed throughout that the μ is captured from an s atomic orbit. This should be a good approximation since the μd atom is a one-electron atom, and Stark-effect transitions assure that the predominant capture comes from s-orbitals.¹²

III. NUMERICAL RESULTS

Because measurement of an absolute cross section is difficult, we omit absolute normalizations from our results. Instead we study what can be learned from a measurement of the neutron spectra in a differential time-of-flight experiment. (It is clear that one cannot make a normal two-neutron coincidence measurement, as in the case of radiative π -capture, since one cannot detect the ν as a start pulse.)

The spectra of interest are of the form

$$\frac{d\sigma}{d\Omega_1 d\Omega_2 d(\Delta t)} = \frac{d\sigma}{d\Omega_1 d\Omega_2 dE_1} \frac{dE_1}{d(\Delta t)} \quad (11)$$

$$= \frac{|\langle H_W \rangle|^2 p_1 p_2}{f'} \frac{dE_1}{d(\Delta t)},$$

where p_1, p_2 , are the two neutron momenta, f' is the recoil factor, and $dE_1/d(\Delta t)$ is the transformation Jacobian from E_1 to the differential time of flight Δt . The product of $dE_1/d(\Delta t)$ and $1/f'$ is given by

$$\frac{1}{f'} \frac{dE_1}{d(\Delta t)} \sim \left| p_1^{-3} \left[1 + \frac{m}{v} \left(1 + \frac{p_1}{p_2} \cos \theta \right) \right] + p_2^{-3} \left[1 + \frac{m}{v} \left(1 + \frac{p_2}{p_1} \cos \theta \right) \right] \right|^{-1} \quad (12)$$

and is clearly symmetric in the two neutron momenta.

Because we wish to make the relative momentum $\vec{p} = (\vec{p}_2 - \vec{p}_1)/2$ small to insure that the n-n scattering length dominates the final-state interaction, the opening angle θ_{12} between the two neutrons should be small. We illustrate the expected spectra normalized to unit area for $\theta_{12} = 5^\circ, 10^\circ$, and 20° in Figs. 1-3. It is apparent

Fig. 1. Two-neutron differential time-of-flight spectra normalized to unit area over the symmetric interval $-50 \leq \Delta t \leq +50$ plotted for $\theta_{12} = 5^\circ$ and various values of a_{nn} . Curve (4) corresponds to phase space a_{nn} . Flight path equal 3m.

that differentiating between $a_{nn} = -16.4f$ and either $-18f$ or $-15f$ is quite feasible. However, determining a_{nn} to $\pm 0.3f$ will obviously require large statistics and a good statistical analysis.

For comparison with the sensitivity of the spectrum to a_{nn} we have shown in Fig. 4 the sensitivity of the spectrum to variation in the unknown effective range r_{nn} . Only results for an opening angle of 20° are presented, since differences at 5° and 10° could not be seen on the scale plotted. Although the effects due to variation in r_{nn} are small compared to those due to a comparable percentage variation in a_{nn} , the fact that r_{nn} is not known becomes a non negligible factor for $\theta_{12} > 15^\circ$, such that p becomes non-negligible.

Fig. 2. Two-neutron differential time-of-flight spectra normalized to unit area over the symmetric interval $-50 \leq \Delta t \leq +50$ plotted for $\theta_{12} = 10^\circ$ and various values of a_{nn} . Flight path equal 3m.

Clearly the correct choice of experimental variables is important. The opening angle determines the minimum value of the relative momentum, p_{min} . The time interval Δt determines the range of allowed p and therefore p_{max} . One would like p_{min} to be less than $1/a_{nn}$ or about 12 MeV/c. Thus the opening angles of $5^\circ - 20^\circ$ would be suitable. However, as noted above, for $\theta_{12} > 15^\circ$ the effective range can be a problem. Also, as the opening angle is decreased, the range in p , for a given interval in Δt , increases: $\Delta t = \pm 50$ nsec implies $4.4 \text{ MeV/c} \leq p \leq 7.8 \text{ MeV/c}$ at 10° compared with $2.2 \text{ MeV/c} \leq p \leq 6.8 \text{ MeV/c}$ at 5° . However, one would like to have p_{max} as large as possible in order to make the spectrum sensitive to small variation in a_{nn} . Thus a compromise between the largest reasonable range in p and the largest reasonable value of p_{max} would seem to imply an

Fig. 3. Two-neutron differential time-of-flight spectra normalized to unit area over the symmetric interval $-50 \leq \Delta t \leq +50$ plotted for $\theta_{12} = 20^\circ$ and various values of a_{nn} . Flight path equal 3m.

optimal opening angle in the neighborhood of 10° and an optimal range in Δt of ± 100 nsec.

The more interesting question is: What are the theoretical uncertainties in extracting a_{nn} using the theoretical procedures outlined above? A detailed discussion of the various problems that arise can be found in Appendix B. However, the main uncertainty lies in the sensitivity of the reaction to the unknown short-range properties of the n - n final-state wave function. This is also the source of increased uncertainty that arises in extracting the Coulomb corrected p - p scattering length from the proton-proton scattering data. Thus, the most meaningful analysis in terms of looking for charge asymmetry in the N - N force would involve a simultaneous analysis of the p - p scattering data along with the μ -capture data. In this way the same uncertainty in the short-range character of the

Fig. 4. Two-neutron differential time-of-flight spectra normalized to unit area over the symmetric interval $-50 \leq \Delta t \leq +50$ plotted for $\theta_{12} = 20^\circ$ and various values of r_{nn} . Flight path equal 3m.

scattering state would appear in each analysis.

IV. CONCLUSIONS

Because the weak interaction is thought to be well understood, the μ -capture reaction offers the cleanest theoretical interpretation of the three competing reactions:

However, the largest theoretical uncertainty in the determination of a_{nn} arises from the unknown short-range character of the n - n final-state wave function. This uncertainty is common to all three reactions.

Since the μ -capture experiment entails a differential time-of-flight measurement and the count

rate is much lower than for the analogous radiative π -capture experiment, it would seem that the difficulty associated with μ -capture relative to the difficulty associated with radiative π -capture is large. Because the same source of theoretical un-

certainty dominates our analysis of both of these experiments for determining a_{nn} , it would appear that the π -capture experiment is preferable. However, a comparison of the two reactions would provide a reasonable check on the theoretical assumption concerning the π -d initial state.

APPENDIX A

The low-energy scattering parameters for the singlet neutron-proton and the proton-proton systems are known to high accuracy:¹³⁻¹⁵

$$a_{pp}^C = -7.823 \pm .01 \text{ f} , \quad r_{pp}^C = 2.794 \pm .015 \text{ f} ,$$

$$a_{np}^S = -23.715 \pm .015 \text{ f} , \quad r_{np}^S = 2.73 \pm .13 \text{ f} .$$

The superscript C designates that these are the measured p-p quantities which include Coulomb effects. The uncertainty associated with a_{pp}^C is certainly very small. However, it is the "Coulomb corrected" value of the p-p scattering length that is needed for comparison with the n-n value. The removal of Coulomb effects is a model-dependent process.¹⁶ Thus the uncertainty in the estimate of a_{pp}^S is much larger than the experimental uncertainty in a_{pp}^C . A generally accepted estimate for

the Coulomb corrected scattering length and effective range is^{17,18}

$$a_{pp} = -17.1 \pm .3 \text{ f} , \quad r_{pp} = 2.84 \pm .03 \text{ f} .$$

At present the n-n scattering length is very uncertain. The most recently quoted average value (from the conference on few nucleon problems held at UCLA) is^{18,19}

$$a_{nn} = -16.4 \pm .9 \text{ f} .$$

The uncertainty in r_{nn} is much greater:

$$r_{nn} = 2.8 \pm .5 \text{ f} .$$

The possibility that the N-N interaction is charge symmetric is certainly included within the quoted errors. However, it is quite clear that a precise determination of a_{nn} is lacking.

APPENDIX B

In order to acquire some understanding of the sensitivity of the n-n scattering length to various uncertainties in the theory of the $\mu^- + d \rightarrow n + n + \nu$ reaction, we have analyzed our optimum theoretical model of the n-n differential time-of-flight spectra by modifying the model to account for one or more uncertain aspects; e.g., altering the effective range. The error in the extracted scattering length in such an analysis is then a measure of the theoretical uncertainty characterizing that aspect of the theory being investigated. No attempt has been made to analyze all possible theoretical uncertainties; rather it was attempted to make a thorough analysis of the most important problems.

The model for the deuteron was taken to be the RSC wave function¹⁷ including the D-state component. However, to facilitate the error analysis discussed below, only the renormalized S-state component of the wave function was used; the error in the scattering length due to such an approximation can be seen from Table B-I below to be small. The uncertainty in a_{nn} due to the choice of this model for the ground state was tested by analyzing with the Hulthen form for the deuteron wave function.

The uncertainty in a_{nn} resulting from the unknown value of F_p was determined by varying F_p from its assumed value of $7F_A$ to one of $8F_A$. Such a variation in the coupling constant had only a negligible effect on the extracted scattering

length. This resulted from our restriction of the kinematics to a small region of relative n-n momentum.

The N-N final state was represented by a PRS model¹¹ in which the phase shift was fixed and the wave function was obtained by integrating the Schrödinger equation in over a known local potential (e.g., the RSC singlet S_0 potential) to a radius of one pion Compton wavelength (R), where the wave function and first two derivatives were matched to determine a smooth polynomial fit into the origin ($r\psi = (r\psi)' = (r\psi)'' = 0$ at $r = 0$); a fifth-order polynomial gives a unique fit to the six boundary conditions. To allow for uncertainty in our knowledge of the short-range part of the wave function inside the matching radius, a function of the form $\eta(p)r^3(r-R)^3$ was added to the polynomial; clearly variation in the assumption made about the function $\eta(p)$ will modify the spectra and the value extracted for a_{nn} . The PRS model value corresponds to $\eta(p) = 0$; the model sensitivity was investigated by analyzing the spectra with $\eta(p)$ determined by matching the third derivative at R and by assuming $\eta(p) = \pm 5.0 p$. In addition, the uncertainty in a_{nn}

due to the unknown effective range r_{nn} used to determine the s-wave phase shift

$$p \cot(\delta_0) = \frac{-1}{a_{nn}} + \frac{1}{2} r_{nn} p^2$$

was tested by modifying the effective from its assumed value of 2.8 f.

In the model calculations it was assumed that the only nonzero phase shift of consequence occurred in the $\ell = 0$ partial wave. (All higher multipole contributions were included in the calculation assuming $\delta_\ell = 0$, $\ell \neq 0$.) In order to investigate the uncertainty due to this assumption, a p-wave phase shift of $\delta_1 = 0.1p^2$ was used to analyze the spectra. The results of the calculations showed that although the higher multipoles are important, the uncertainty in the phase shifts in these partial waves is not significant.

In Table B-I are shown the results of our examination of the theoretical uncertainty in the extraction of the n-n scattering length. Note that as discussed in the text, the smaller opening angle provides a larger range in relative momentum p to be investigated for a given time interval. At 30° the

TABLE B-I

THE ERROR (IN f) IN a_{nn} EXTRACTED FROM THE DIFFERENTIAL TIME-OF-FLIGHT SPECTRA DUE TO THE MODEL ERROR NOTED IN THE FIRST COLUMN AS A FUNCTION OF OPENING ANGLE FOR $\Delta t = 50$ NSEC.

	$\theta_{12} =$	5°	10°	20°	30°
RSC with D-state		+ 0.008	+ 0.010	+ 0.020	+ 0.042
Hulthén bound state		+ 0.001	+ 0.001	+ 0.003	+ 0.007
$F_p = 8F_A$		-	-	-	-
$\delta_p = 0.1 p^2$		- 0.008	- 0.007	- 0.006	+ 0.001
$r_{nn} = 2.6 f$		+ 0.079	+ 0.076	+ 0.065	+ 0.037
$\eta(p) \sim (r\psi)'''$		- 0.006	+ 0.002	+ 0.003	+ 0.007
$\eta(p) = 5p$		- 0.031	- 0.045	- 0.063	- 0.098
$\eta(p) = -5p$		+ 0.037	+ 0.040	+ 0.060	+ 0.097
zero range scattering state		- 0.006	- 0.010	- 0.033	- 0.101
Hulthén + zero range scat. state		- 0.015	- 0.017	- 0.041	- 0.117
Hulthén + zero range scat. state ($\ell = 0$ multipole only)		- 0.069	- 0.103	- 0.229	- 0.653

range in p shown here is so small as to make the analysis insensitive to r_{nn} ; however, the sensitivity to a_{nn} is also lost (i.e., the chi-square curve becomes very flat making the uncertainty in a_{nn} very large). Note also that omitting the higher multipoles entirely can produce a large error in the extracted scattering length. It should be pointed out that any momentum dependent correction to the theory (i.e., variation of the phase shift with p due to an uncertainty in r_{nn} as opposed to a constant change from an F_P of $7F_A$ to one of $8F_A$) will produce a much larger effect on the

scattering length than a correction that is not momentum dependent.

From the numerical results in Table B-I one can see that the theoretical uncertainty in a careful analysis of the $\mu^- + d \rightarrow n + n + \nu$ reaction could be made less than some 0.2 f. Since we have not explored all of the possible theoretical problems that might contribute to the uncertainty, a better estimate of its range might be estimated to be in the neighborhood of 0.3 f. However, it should be clear that the theoretical uncertainty in the extraction of a_{nn} is smaller than the present experimental error.

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