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BDX-613-1079

APPLICATION OF STATISTICAL
METHODS TO MANUFACTURING
OPERATIONS

Technological Spinoff Report

Prepared by:
C. Latimer and
J. H. Carder

Published March 1974

MASTER

Prepared for the
United States Atomic Energy Commission
Under Contract Number AT(29-1)-613 USAEC

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APPLICATION OF STATISTICAL METHODS TO MANUFACTURING OPERATIONS

BDX-613-1079, UNCLASSIFIED Technological Spinoff Report,
Published March 1974

Prepared by C. Latimer and J. H. Carder, D/141

A wide variety of statistical techniques have application to manufacturing operations. This paper describes the application of some of these techniques to a number of specific cases in a large multi-product manufacturing organization. The cases chosen involve plastic molding and functional testing. The statistical techniques used include analysis of variance, discriminant analysis, non-linear regression, response surfaces, non-parametric statistics, bivariate normal distribution, and Grubbs' method for estimating components of variability. The emphasis is on relating the statistical techniques to the manufacturing situation under study.

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KANSAS CITY DIVISION

KANSAS CITY, MISSOURI

A prime contractor for the
Atomic Energy Commission

Contract Number AT(29-1)-613 USAEC

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INTRODUCTION

Variation is inherent in all manufacturing processes. "Statistics" is a body of knowledge that has application in the study of such variation. Statistics can be used to estimate the influence of variables in a manufacturing process and to isolate specific causes from random fluctuations. The body of knowledge that has accumulated in the statistical field has resulted in a large number of techniques, each suitable for study of a certain class of situations.

The major categories of application include identification of sources of variation, determination of process and/or equipment capability, and optimizing manufacturing processes. In any manufacturing operation where variability of end product parameters is greater than desired, reduction in variability requires identification of its source. Variation in product parameters results from variations in raw materials or process parameters, or from combinations of the two. Statistical techniques can be used to identify such relationships. Once identified, variations in materials and processes can be controlled in many cases to achieve acceptable levels of variation in product parameters.

In many cases, product features can be generated by more than one process or type of equipment. If tolerances are tight relative to available manufacturing capabilities, it may be important to determine which of the available process/equipment choices will best achieve the required product results. Capability studies involving statistical techniques can be used to determine the capabilities of the various available process/equipment choices for specific product needs. If the best available choice is still poor, at least the knowledge of what can be expected permits informed planning. For example, design trade-offs can be made to increase the tolerance in one place and reduce it in another to get a better combined yield, or plans can be made to produce a quantity that yields acceptable parts in sufficient quantities to meet schedule requirements.

While identification of sources of variation is useful in its own right, in some situations the further application of statistical techniques can determine an optimum combination of process settings (i.e., times, temperatures, pressures, etc.) to achieve a maximum yield of in-tolerance parts. This approach usually involves a designed experiment in which the process variables previously determined to affect the critical product parameters are varied incrementally in various combinations. Statistical techniques can then be applied to the results to establish a "recipe" for the optimum process.

Variation in a manufacturing process becomes evident during the inspection operation. Sampling inspection frequently is appropriate, and the level of sampling inspection is an economic balance between cost and consequence of accepting defective product, which is usually determined prior to the beginning of production. Once production has passed the "pilot stage," the quality of the product can be determined and the amount of sampling adjusted accordingly.

To illustrate the practical application of statistical techniques to manufacturing operations, a number of specific cases related to the various types of applications will be discussed.

These specific applications and the various categories to which they pertain are as follows.

- Identifying Sources of Variation
 1. Solder Bond Strength
 2. Transmitter Tube Arcing
- Determination of Process and/or Equipment Capability
 1. Valve Pressure Differential Test Failures
 2. Moisture Pick-Up of Molded Desiccant
 3. Peak Current Variation in an Electrical Assembly
- Optimizing Manufacturing Processes
 1. Injection Molding of a Nylon Cover
 2. Rise-Rate Testing of Urethane
- Optimizing Trade-Offs Between Inspection Effort and Protection
 1. Inspection of a Cellular Silicone Product

Statistical terms in this paper are italicized and are defined in the glossary.

IDENTIFYING SOURCES OF VARIATION

Situations where identifying sources of variation is applicable include those where the percentage of unacceptable product is high, volume is large, schedules are tight, and parts are relatively expensive, or where it has not been possible to make any parts acceptable on all parameters.

An ideal method of identifying sources of variation that have not previously been investigated is to first establish a tentative hypothesis that a particular source is a contributing cause and then to plan and conduct an experiment designed to either support or refute the hypothesis, followed by appropriate analysis and interpretation of the results. An alternate method is to analyze data previously collected on production log sheets, inspection records prepared for acceptance purposes, or other records. Usually, this method is not as satisfactory as the planned approach.

An example involving solder bond strength will serve to illustrate the planned investigation approach.

Solder Bond Strength

Solder bond failures were found in parts consisting of a cylindrical plunger soldered in a sleeve. There were some areas of the bond where voids were observed; thus, voids were considered as possibly contributing to the failures. A designed experiment studying four main effects and utilizing *analysis of variance*¹ techniques was planned to investigate variations in the soldering process, with void area and ultimate strength being the responses used to evaluate the process. The variable factors studied were two levels of temperature of the hot plate used in soldering, three fluxing methods, and the use or nonuse of post soldering treatment (induction heating with vibration).

Other potential variables such as thickness of bond area, cleanliness, flux batch, and time between soldering and testing were, to the extent practical, held constant during the course of this experiment.

Tables 1 and 2 list the averages of the responses measured at each of the levels discussed above. The significant results in the analysis that are common to both the strength and void area (summarized in Tables 3 and 4) were an effect due to operators and the interaction between operators and the vibration/no-vibration treatment. The data illustrating these effects is shown in Table 5. Operator One obtained better results for both strength and void area by vibrating, while Operator Two obtained better results by not vibrating.

Because of the skill required of the operator, this factor tends to override the other factors under consideration. Thus, the operator effect was the most influential variable in the system, while the temperature effect was somewhat inconsistent. A temperature was subsequently chosen commensurate with technical knowledge of the soldering process.

Table 1. Solder Bond Strengths (Pounds)*

Flux	Operator 1				Operator 2			
	550°F		950°F		550°F		950°F	
	Vibrate	Not Vibrate	Vibrate	Not Vibrate	Vibrate	Not Vibrate	Vibrate	Not Vibrate
A	9303	8800	9092	8970	8733	8720	8781	8880
B	8870	8785	9197	9150	8586	8764	8857	9050
C	9235	9084	9233	9100	8664	8726	8489	8417
*Each entry is the mean of ten observations.								

Table 2. Solder Bond Void Areas (Square Inches)*

Flux	Operator 1				Operator 2			
	550°F		950°F		550°F		950°F	
	Vibrate	Not Vibrate	Vibrate	Not Vibrate	Vibrate	Not Vibrate	Vibrate	Not Vibrate
A	0.0035	0.0104	0.0086	0.0012	0.0102	0.0001	0.0258	0.0006
B	0.0058	0.0162	0.0039	0.0055	0.0150	0.0091	0.0286	0.0009
C	0.0036	0.0026	0.0057	0.0030	0.0077	0.0025	0.0515	0.0194
*Each entry is the mean of ten observations.								

Table 3. Strength Results

Source of Variation	Degrees of Freedom	Mean Square	F Ratio
O, Operator	1	7,182,960.0	135.479**
T, Temperature	1	372,881.7	7.033**
F, Flux	2	43,058.8	0.812
V, Vibration	1	147,015.0	2.773
OT	1	61,440.0	1.159
OF	2	914,996.2	17.258**
OV	1	922,560.0	17.401**
TF	2	944,052.9	17.806**
TV	1	77,041.7	1.453
FV	2	197,791.2	3.731*
OTF	2	174,423.8	3.290*
OTV	1	82,140.0	1.549
OFV	2	59,723.8	1.126
TFV	2	123,550.4	2.330
OTFV	2	18,926.2	0.357
Residual	216	53,019.0	
Total	239		
*Denotes significance at the 5% level. **Denotes significance at the 1% level.			

Table 4. Void Area Results

Source of Variation	Degrees of Freedom	Mean Square $\times 10^{10}$	F Ratio
O, Operator	1	42,951,420	4.975*
T, Temperature	1	19,181,760	2.222
F, Flux	2	4,104,507	0.475
V, Vibration	1	40,352,200	4.674*
OT	1	38,680,511	4.480*
OF	2	10,124,512	1.173
OV	1	54,254,551	6.284*
TF	2	16,457,262	1.906
TV	1	32,612,253	3.777
FV	2	1,276,602	0.148
OTF	2	6,351,950	0.736
OTV	1	6,353,761	0.736
OFV	2	537,162	0.062
TFV	2	12,560	0.001
OTFV	2	1,839,212	0.213
Residual	216	8,633,679	
Total	239		
*Denotes significance at the 5% level.			

Table 5. Averages of Solder Test Results

Strength Averages (Pounds)			
	Operator 1	Operator 2	Average
Vibration	9155	8685	8920
No Vibration	8982	8760	8871
Average	9068	8722	8895
Void Averages (Square Inches)			
	Operator 1	Operator 2	Average
Vibration	0.0052	0.0231	0.0142
No Vibration	0.0065	0.0054	0.0060
Average	0.0058	0.0143	0.0101

Additional analyses were performed, but they did not alter the conclusions. This experiment was typical in that selected variables were varied systematically and attempts were made to control others that influence the product. Experiments of this type are frequently useful in illustrating the variables in the experiment as conducted, and provide documentation so that further work may proceed based on fact. Because of this experiment, attention was directed to the operator techniques.

Transmitter Tube Arcing

Arcing of a transmitter tube had been a problem in its testing as well as the testing of the next assembly.

A technique was needed to screen out arcers prior to the next assembly. *Discriminant analysis*² is a statistical technique used to assign units to categories on the basis of values of variable properties of the units. Discriminant analysis was used to classify a tube into one of two groups: arcers and non-arcers.

Fifteen variables representing properties of the tube were used to determine the function for discriminating between the two groups. The variables data analyzed consisted of 137 assemblies found to contain 121 non-arcing tubes and 16 arcing tubes.

The form of discriminant function was $Y = a_0 + a_1X_1 + a_2X_2 + \dots + a_{15}X_{15}$, where the a 's are constants derived from the data available and the X 's are the values of the variables. If the value of the equation was positive for the tube being classified, it was predicted to be an arcer.

The table below shows the observed classification and the classification using the discriminant function for the tube used in the analysis.

Classification		Predicted Arcers		Predicted Non-Arcers		Total	
Observed	Arcers	12	8.8%	4	2.9%	16	11.7%
Observed	Non-Arcers	25	18.2%	96	70.1%	121	88.3%
	Total	37	27.0%	100	73.0%	137	100.0%

The table shows 11.7 percent of the tubes analyzed actually arced. If only the 100 tubes predicted as non-arcers by the discriminant function were used in the next level of build, the expected percent of arcers would be $4/100 = 4$ percent as compared with 11.7 percent. This reduction would be at the expense

of discarding 27.0 percent of the tubes in all, erroneously discarding 18.2 percent of the tubes. In this case, the discarding of so much good product to improve the percent defective from 11.7 percent to 4 percent was not justified. However, in some situations it might be economical to take advantage of this degree of discrimination. For the arcing tube situation, attention was directed to another avenue of approach, resulting ultimately in elimination of the problem.

Discriminant analysis is useful in a situation where a final test results in distinct classifications and the outcome of the test is dependent upon variable data obtainable earlier in the production sequence.

DETERMINATION OF PROCESS AND/OR EQUIPMENT CAPABILITY

It is frequently worthwhile to investigate the capability of a process independently of any specifications that are imposed on the product. The process spread may be estimated for the entire process or for some portion of it. Also, when specifications are imposed on the product, it is worthwhile to estimate limits for the percent out-of-tolerance expected to be generated by the process. Statistical techniques are useful to make such estimates, especially when there is not much data available. In addition, when a particular step in the process is under consideration, indirect means through statistical techniques may be employed to obtain process capability estimates.

In a process capability investigation, the measurement system frequently must be incorporated into the study. A calibration control program requires the periodic investigation of measuring equipment. Equipment is investigated and judged to be in or out of calibration according to specific tolerances. The equipment might have developed such a systematic error that measurement of a standard exhibits an excessive bias or such an imprecision that measurement of the standard exhibits excessive variation. Adjustments or repairs would then be made to bring the equipment back within the tolerances.

With the presence of systematic error and imprecision in the measuring equipment, errors of misclassification of product may exist (i.e., good called bad and bad called good). These errors depend not only on the systematic error and imprecision of the measuring equipment, but also on the product distribution of the characteristic that is measured. An objective of a calibration control program is to ensure that these errors remain relatively small.

Measuring instruments that are judged out of calibration may or may not raise chances of misclassification of product to an

intolerable amount. The probability of accepting a bad unit and the probability of rejecting a good unit can be estimated given the estimates of the tester bias, tester standard deviation, product standard deviation, and the product percent defective. The preliminary estimates can be obtained by drawing a random sample of product, measuring the product characteristic under consideration several times, and applying the analysis of variance techniques. *Bivariate normal analysis*³ is then applied to estimate the probabilities of misclassification. The size of these risks permits informed planning with respect to measuring equipment.

Tests on Pressure Regulating Valve

Failures of outlet pressure on a pressure regulating valve under specified input conditions were resulting in an excessive failure rate of about two percent. A study was undertaken to evaluate the process standard deviation and the measurement error standard deviation. Also, a plan was established to study the influence of these measures of variation on the observed percent defective and on the misclassification of parts as defective or non-defective. Table 6 shows the results obtained from five repeated measurements on each of 15 parts.

Through the use of bivariate normal distribution analysis, estimates were made of percent of defective parts called non-defective and non-defective parts called defective by the measurement system. Further, estimates were made of these percentages of the observed non-defective parts. These results are shown in Table 7.

This table provides the following estimates. Of a typical 100 parts produced, 97.70 are accepted. If these are retested by a comparable measuring system, 96.58 are accepted. There are 0.08 parts actually defective among those accepted on subsequent testing. There are 2.30 parts called defective on original test and an additional 1.12 parts called defective on subsequent testing. Assuming that the 0.08 parts ultimately defective among those passing both tests are tolerated, the problem becomes one of increasing the yield.

An investigation was made of reducing the process standard deviation by 1.0 psi, and of reducing the tester standard deviation by 1.0 psi. The process improvement option led to an estimate of 98.93 parts accepted out of 100 produced on first inspection, and 98.30 parts accepted on subsequent inspection. The tester improvement option led to an estimate of 98.06 parts accepted on first inspection and 97.40 accepted on second inspection. The number of parts actually defective among those accepted on second inspection was estimated to be 0.04 for the process improvement option and 0.03 for the tester improvement option.

Table 6. Summary of Inspection Results on a Valve

Outlet Pressure Requirement (psi)	150-190
Average (psi)	169.74
Process Standard Deviation*	8.49
Measurement Error Standard Deviation (psi)**	2.31
Percent Defective	2.30
<p>*Isolated from measurement error by statistical procedure. **Estimated by repeated measurements on a typical valve.</p>	

Table 7. Misclassification of Good and Bad Valves by First and Second Measurements

Calculation of Actual Percent	First Inspection Observed Percent			Second Inspection Observed Percent		
	Defective	Non-Defective	Total	Defective	Non-Defective	Total
Defective	1.47	0.38	1.85	0.30	0.08	0.38
Non-Defective	0.83	97.32	98.15	0.82	96.50	97.32
Total	2.30	97.70	100.00	1.12	96.58	97.70

The number of parts called defective on subsequent inspection is 0.63 for the process improvement option and 0.60 for the tester improvement option. Since the yield would be improved by about one part per 100 produced under the process improvement option as compared with the tester option, attention was directed to the process.

Moisture Pick-Up of Molded Desiccant

There are situations where product parameters are related to process variables in an indirect manner. Statistical techniques frequently are useful in studying process capabilities in such situations.

A problem with the shelf life of mixed material used to make molded desiccant blocks led to a study of moisture absorption of completed parts in relation to the age of the premixed material. Molded desiccant blocks were made from material stored 1, 2, 3, and 5 days. The blocks were all then subjected to the same humidity environment. An equation of the form

$$Y = A - Ae^{Bt^C}$$

was fitted to each category. This "stacked index" model was fitted by a *non-linear regression*⁴ technique. The constants in the fitted equation provided estimates of the rate of moisture pick-up and the asymptotic level of saturation without actually waiting for saturation to occur. This technique has supplied information for controlling variables in the molded desiccant process in order to achieve desired results for rate of moisture absorption and saturation level. These results were then used to establish a permissible shelf life. It has been found that this type of equation fits many phenomena, including some stress-strain curves and loss of volatiles, as well as moisture absorption and loss. This empirical equation is simpler to deal with than some series expressions derived from differential equation assumptions. The exponent applied to *t* (time) permits rates of change that vary with time. Constants in the formula provide means of comparing different process conditions, and also provide means of assessing process capability.

Peak Current Variation in an Electrical Assembly

Excessive variation in peak current on an electrical part was causing a high percentage of rejects. Since both tester precision and product stability were possible contributors to the excessive variation in readings, a study was undertaken to determine the relative contribution of each. An experimental plan was devised to test two units simultaneously on the same tester.

Twenty such pairs of observations were taken. The standard deviation of the readings obtained on one unit was 0.05232 and on another unit was 0.05808, and the *covariance*⁵ was found to be 0.002468. From this information, the standard deviations representing the instability of each unit and the tester error were computed by the technique known as *Grubbs' method*.⁵

Standard Deviation

Tester	0.0497 amperes
Unit A	0.0164 amperes
Unit B	0.0301 amperes

Since the tester standard deviation was larger than the product standard deviation, an improvement was incorporated in the tester. With this change, the number of units exhibiting out-of-tolerance readings was significantly reduced.

OPTIMIZING MANUFACTURING PROCESSES

Often an important goal is to optimize a particular product parameter, such as the strength, cost, or yield, by determining its relationship to controllable process variables. When one product parameter is optimized, some other parameter is apt to be far from its optimum level, and, further, it may be impossible to optimize two or more product parameters simultaneously. On the other hand, it is usually difficult, if not impossible, to quantify and in turn specify a complete list of product parameters that are optimum from an overall standpoint. Nevertheless, using the best knowledge and judgement at his disposal, the designer does in fact make such specifications. If the process ultimately falls short with respect to one product parameter, a study of the process with respect to levels of process variables is in order. Such a situation is a common occurrence, and statistical methods for coping with it are well developed. Trade-offs between competing product parameters require some knowledge of optimum levels of process variables for each parameter separately. Processes with well defined, adjustable, independent variables are more readily adaptable to this approach.

Optimization through *response surface*⁶ techniques first requires a presumption that the product parameter, Y , may be regarded as a function of the process parameters, $X_1, X_2, \dots, X_n$. In many situations, the function possesses some curvature in the region of interest. The true functional relationship may not be known, but it can be assumed to be approximated by a second degree equation. The technique is best described by an example.

Injection Molding of a Nylon Cover

A nylon cover was exhibiting excessive warping. A study was planned to determine levels of five process variables which would yield a part with minimum warp. The parts are cast in a center-gated mold in an injection molding press.

Five process variables were selected for study.

- X_1 , Mold Temperature: Temperature in degrees F of the mold cavity surface when the machine is on cycle.
- X_2 , Melt Temperature: Temperature in degrees F of the material at the time of a shot when the machine is on cycle. The material is discharged into a cup and immediately measured with a needle pyrometer.
- X_3 , Temperature Difference: ΔT of the cavity and punch side of the mold. $\Delta T = \text{Cavity Temperature} - \text{Punch Temperature}$. The ΔT is to allow for any discrepancy in removing heat from the two sides of the part when cast. An uneven cooling of the part would be expected to induce warp.
- X_4 , Injection Pressure: A setting of the hydraulic pressure applied to the injection cylinder to force the molten material into the mold cavity.
- X_5 , Press Closed Time: The time the press is forward and closed. Approximately one second is used to fill the mold cavity; for the remaining time, the part is held restrained in the mold cavity.

The levels of these process variables are shown in Table 8.

The following variables were held constant:

- Speed Setting: 0 (maximum).
- Percent Cushion: 5 percent (at some conditions, the cushion was lost because the parts tend to flash excessively, allowing the cylinder to bottom out).
- Back Pressure: 150 psi.
- Injection Pressure Application Time: 5 seconds.
- Cooling Time: 5 seconds less than press closed time.
- Press Open Time: 10 seconds.
- Clamp Pressure: 5 turns.

Table 8. Levels of Variables in Nylon Cover Experiment

Level	Mold Temperature (°F) X_1	Melt Temperature (°F) X_2	Cavity- Punch, ΔT (°F) X_3	Inject Pressure (psi) X_4	Press Closed Time (sec) X_5
-2	100	550	-10	550	25
-1	120	575	- 5	700	40
0	140	600	0	850	55
+1	160	625	5	1000	70
+2	180	650	10	1150	85

The experimental plan, or design, was typical of those recommended in the statistical literature for investigations of this type. Table 9 shows the design matrix consisting of 32 lines and the experimental results. Each of the 32 lines had ten parts molded per line. A stepwise *regression*⁴ procedure was used to determine the quadratic response surface equation of best fit with highest multiple correlation coefficient (R) and small standard error (s.e.).

Four characteristics were observed for each line of parts.

- Characteristic No. 1 is the warp and is measured as the maximum deviation from a flat surface. A positive measure is concave; negative is convex. The desired condition is zero warp.
- Characteristic No. 2 is the range of warp found in the group of 10 parts. A group may have a satisfactory mean dimension while the spread or range may be large, which is undesirable.
- Characteristic No. 3 is the fit of a pattern of holes in the part over pins in the gage used for inspection. The scale used is an arbitrary one, determined by feel. The following scale was used.

- +2 - Oversize, tight fit on pins.
- +1 - Oversize, slight drag on pins.
- 0 - Good Fit, pins do not touch sides of holes, part loose on gage.
- 1 - Undersize, slight drag on pins.
- 2 - Undersize, tight fit on pins.

The desired condition is zero, a loose fit, where the pins do not touch the sides of holes.

- Characteristic No. 4 is surface condition. The material has a tendency to produce an uneven, wavy surface which would not be desirable and would contribute to runout in thickness. The parts were observed by reflected light and visually judged by the following scale.

- +2 - Mirror Surface, no visible sinks or aberrations.
- +1 - Good Surface, slight runout, some runout at corners.
- 0 - Sinks (low spots on surface visible by reflected light), not sufficient to affect thickness of part.
- 1 - Objectionable Waviness, may affect thickness of part.
- 2 - Obvious Waviness, imparts a bad appearance to the part.

A good process must consistently produce parts with minimum warp, a good size, and a mirror surface.

Table 9. Nylon Cover Experiment

Line No.	Matrix of Experiment					Response			
	Mold Temp. X_1	Melt Temp. X_2	ΔT^* X_3	Injection Pressure X_4	Press Closed Time X_5	Warp $\times 10^{-3}$	Range $\times 10^{-3}$	Size	Surface Aberrations
1	-1	-1	-1	-1	1	9.0	8.0	2	2
2	1	-1	-1	-1	-1	26.4	10.0	-1	1
3	-1	1	-1	-1	-1	34.6	8.0	0	0
4	1	1	-1	-1	1	22.1	6.0	0	0
5	-1	-1	1	-1	-1	13.1	8.0	0	1
6	1	-1	1	-1	1	18.6	5.0	0	1
7	-1	1	1	-1	1	11.6	6.0	0	-1
8	1	1	1	-1	-1	35.2	8.0	-2	-1
9	-1	-1	-1	1	-1	30.4	4.0	0	0.5
10	1	-1	-1	1	1	27.0	4.0	-1	1
11	-1	1	-1	1	1	21.4	6.0	0	-1
12	1	1	-1	1	-1	34.2	14.0	-2	-1
13	-1	-1	1	1	1	10.4	4.0	0	0.5
14	1	-1	1	1	-1	25.8	8.0	-1	0.5
15	-1	1	1	1	-1	36.8	10.0	-0.5	-2
16	1	1	1	1	1	18.6	6.0	-1	-1
17	0	0	0	0	0	30.4	4.0	-1	-1
18	0	0	0	0	0	30.0	4.0	-1	-1
19	0	0	0	0	0	30.8	4.0	-1	-1
20	0	0	0	0	0	30.5	8.0	-1	-1
21	0	0	0	0	0	29.1	4.0	-1	-1
22	0	0	0	0	0	28.4	4.0	0	-1
23	-2	0	0	0	0	21.2	6.0	2	-1
24	2	0	0	0	0	25.4	6.0	-2	-1
25	0	-2	0	0	0	-2.0	12.0	0	0
26	0	2	0	0	0	40.6	12.0	-1	-1
27	0	0	-2	0	0	30.0	4.0	-1	-1
28	0	0	2	0	0	32.0	4.0	-1	-2
29	0	0	0	-2	0	24.4	6.0	0	1
30	0	0	0	2	0	23.8	6.0	0	0
31	0	0	0	0	-2	36.2	4.0	-1	-1
32	0	0	0	0	2	17.2	4.0	0	-2

* ΔT = Cavity Temperature Minus Punch Temperature

The quadratic equations of best fit have the terms arranged in descending order of significance. The level of significance of the coefficients is indicated by the number of underlines (that is, triple underlined terms are highly significant, double underlined terms are significant at the 99.95 percent level, single underlined terms are significant at the 99.50 percent level, and all other terms shown are significant at the 99.00 percent level).

Characteristic No. 1, warp, or runout from flat over the width of part, equals $Y_1 \times 10^{-3}$ inches.

$$\hat{Y}_1^* = 29.29 + \underline{\underline{5.79}}X_2 - \underline{\underline{5.66}}X_5 - \underline{2.59}X_2^2 + 2.95X_4 - 3.54X_2X_5 - 2.54X_1X_4.$$

$$R = 0.888 \text{ (21 d.f.)}.$$

$$\text{s.e.} = 4.369.$$

The melt temperature, X_2 , is the most significant variable with minimum warp at X_2 equals -1.12 (572°F). The press closed time is second in significance. The warp is reduced with increased press closed time, giving a decrease of 3.75×10^{-3} inches for each 10 second increase. The inject pressure causes increased warp with increased pressure, giving 2.0×10^{-3} inches for each 100 psi increase. The negative interaction of melt temperature with press closed time, X_2X_5 , indicates increased warp with high melt temperature and short press closed time, resulting in insufficient time for the part to cool in the mold. The ΔT variable, X_3 , is not a significant variable over the range of values used. The optimum levels for minimum warp are as follows:

- X_1 - Mold Temperature: -1.0 , 120°F
- X_2 - Melt Temperature: -1.8 , 555°F
- X_3 - ΔT (Cavity Temperature - Punch Temperature): 0 (not significant)
- X_4 - Inject Pressure: -1 (700 psi)
- X_5 - Press Closed Time: $+1$ (70 seconds)

Characteristic No. 2, the range in warp of 10 parts cast, equals $Y_2 \times 10^{-3}$.

* $\hat{Y}$ denotes the predicted value of the response Y .

$$\hat{Y}_2 = 4.45 - \underline{\underline{1.95X_2^2}} - \underline{\underline{1.04X_5}} + 0.875X_2 + 0.85X_1X_4 + 0.95X_2X_4.$$

$$R = 0.811 \text{ (21 d.f.)}.$$

$$\text{s.e.} = 1.61 \times 10^{-3}.$$

The most significant variable for range in warp within group of parts is the melt temperature, with minimum range at $X_2 = -0.22$ (595°F). The range in warp also decreases as the press closed time, X_5 , is increased. The range in warp is important as a group of parts such as Line 25 (Table 9) may have a small mean value for the warp while the range of values may be large. The most desirable condition is minimum warp and minimum range within the group of parts.

Characteristic No. 3 is size. The gage for part size is a functional gage. Therefore, the response Y is a relative number where zero is a good fit, a negative value is undersize, and a positive value is oversize.

$$\hat{Y}_3 = 0.875 - \underline{\underline{0.79X_1}} + \underline{\underline{0.33X_4^2}} - \underline{\underline{0.27X_2}} + 0.21X_1^2 + 0.22X_5 \\ - 0.41X_4X_5 - 0.30X_3X_5.$$

$$R = 0.916 \text{ (19 d.f.)}.$$

$$\text{s.e.} = 0.38.$$

The mold temperature, X_1 , has the greatest effect on size of part and should be used to control part size. High mold temperature gives reduced size; low mold temperature gives increased size. The low level of the melt temperature causes an increase in part size. The part size was not consistent with injection pressure. (There was considerable flash, indicating the cavity pressure was not holding, and the injection cylinder would bottom, indicating that no cushion was left in cylinder.) The clamp pressure was not sufficient to prevent flash on other than low mold temperature, low melt temperature, and low injection pressure, such as lines 1 and 5. Fortunately, these are the levels for least warp and should also give a part with minimum flash.

Characteristic No. 4 is surface flatness. The scale used is relative, where a +2 is desirable (no visible aberrations). A surface rated less than zero would be objectionable.

$$\hat{Y}_4 = -1.13 - \underline{\underline{0.64X_2}} - \underline{\underline{0.51X_4}} - \underline{\underline{0.44X_3}} - \underline{\underline{0.37X_4^2}} + 0.24X_2^2.$$

$$R = 0.857 \text{ (24 d.f.)}.$$

$$\text{s.e.} = 0.525.$$

The melt temperature is the most significant variable and should be low, near the -2 level (550-560°F) for the best surface. The temperature difference, (X_3), between cavity and punch appears as a significant variable. Better surfaces occurred at low injection pressure levels. This is probably due to losing the cavity pressure when the mold flashed. A higher clamp pressure, eliminating the flash, should improve surface contour. The press closed time, (X_5), has no effect on the surface. The levels of the five variables giving the best surface were the following:

- X_1 - Mold temperature: 120 - 130°F.
- X_2 - Melt temperature: 555 - 565°F.
- X_3 - ΔT : 0 - 5°F.
- X_4 - Inject pressure: 750 - 800 psi.
- X_5 - Press closed time: 65 - 75 seconds.

After the evaluation of 32 lines of parts of the designed experiment, two sets of conditions which appeared to be close to optimal point were used. Forty parts were run at each of the following conditions.

<u>Process Parameters</u>	<u>Group A</u>	<u>Group B</u>
Mold Temperature	125°F	140°F
Melt Temperature	560°F	560°F
ΔT	0	0
Inject Pressure	800 psi	800 psi
Press Closed Time	65 sec	70 sec
<u>Part Characteristics</u>	<u>Group A</u>	<u>Group B</u>
Warp	0.004 or less	0.004 or less
Flash	Minimum about outer edge and hole	Minimum
Surface	Good (+1 or better)	Good (+1 or better)
Size	Outermost pin touches, slightly oversize	Good, 0, loose on gage

Both groups should have a high percentage of acceptance. The increase in mold temperature for Group B was effective in reducing the size. Group B conditions were recommended and adopted.

Rise-Rate Testing of Urethane Foams

Optimizing process conditions begins at an early stage of the planning. Statistical methods are useful in laboratory and pilot plant studies to serve as a basis for determining near optimum conditions when actual production begins.

It is necessary to establish the working time available to mix and pour a foam into a mold during production. The foam rise time and gel time are responses that limit the working time. These flow properties were measured using a Rise-Rate Machine and Pressure-Gel Rate Machine which measure the flow properties of the foams with respect to time. A designed experiment was planned to study the effect that several independent variables had on the flow properties. Response surface techniques were used to illustrate these results. These plots permit one to readily visualize the minimum (or maximum) and the neighboring surface.

The designed experiment (see Table 10) was a 2^4 factorial design with a star and center points. Three repeat measurements were taken at each line, these responses were averaged, and the response equations were fitted to these averages. An equation of the form

$$\begin{aligned}\hat{Y} = & a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + a_{11}X_1^2 + a_{22}X_2^2 + a_{33}X_3^2 \\ & + a_{44}X_4^2 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{14}X_1X_4 + a_{23}X_2X_3 + a_{24}X_2X_4 \\ & + a_{34}X_3X_4\end{aligned}$$

was used to explain the responses.

The following independent variables were varied for the experiment.

- X_1 - Catalyst Concentration (parts per hundred).
- X_2 - Temperature of the R-Component (degrees F).
- X_3 - Temperature of the T-Component (degrees F).
- X_4 - Viscosity of the T-Component (centipoise).
- X_5 - Calculated Temperature of the Mixture (degrees F).

Table 10. Averages for Responses

	Line No.	Design Matrix					Responses	
		X ₁	X ₂	X ₃	X ₄	X ₅	Y ₁	Y ₂
Center Points	1	1	1	1	1	1.0	180.0	134.0
	2	-1	1	1	1	1.0	314.7	257.5
	3	1	-1	1	1	0.5	188.3	138.3
	4	-1	-1	1	1	0.5	335.0	265.3
	5	1	1	-1	1	-0.5	196.7	150.0
	6	-1	1	-1	1	-0.5	353.0	286.0
	7	1	-1	-1	1	-1.0	216.7	170.0
	8	-1	-1	-1	1	-1.0	436.7	343.3
	9	1	1	1	-1	1.0	167.3	133.0
	10	-1	1	1	-1	1.0	330.0	255.7
	11	1	-1	1	-1	0.5	181.7	139.3
	12	-1	-1	1	-1	0.5	355.0	282.3
	13	1	1	-1	-1	-0.5	195.0	148.0
	14	-1	1	-1	-1	-0.5	381.7	291.7
	15	1	-1	-1	-1	-1.0	219.0	182.7
	16	-1	-1	-1	-1	-1.0	420.0	331.5
Star	17	0	0	0	0	0.0	255.7	197.3
	18	0	0	0	0	0.0	279.4	199.7
	19	0	0	0	0	0.0	239.2	202.0
	20	0	0	0	0	0.0	247.5	-----
	21	2	0	0	0	0.0	161.7	123.0
	22	-2	0	0	0	0.0	655.0	499.0
	23	0	2	0	0	0.5	232.0	173.3
	24	0	-2	0	0	-0.5	271.7	213.7
	25	0	0	2	0	1.5	183.3	139.7
	26	0	0	-2	0	-1.5	286.7	223.7
	27	0	0	0	2	0.0	247.7	190.3
	28	0	0	0	-2	0.0	241.0	193.7

The independent variables and their coded levels are shown in Table 11. Variable X_5 , the calculated temperature for the mixture, was generated by weighting the temperature of the R and T components according to the amount of each in the mixture. This is the mixture temperature that would be obtained if both of the components were at the same temperature when mixed and was introduced into the equation to replace variables X_2 and X_3 . It was desirable that variable X_5 produce an equation of about the same reliability as that using X_2 and X_3 . This would make the response equation easier to evaluate and the response surface easier to represent (i.e., three variables as opposed to four). Thus, two equations were predicted for each response. The first is the result of the designed experiment using the independent variables X_1 , X_2 , X_3 , and X_4 to predict the responses. The second set uses the calculated temperature for the mixture in place of the temperatures of the R and T components (i.e., variables X_1 , X_4 , and X_5 to predict the responses). The first set of equations are identified by "A" in the test and the second set are identified by "B."

The equations selected were those that had the smallest standard error with all of the terms significant. The significance of each term is indicated by the number of lines under each term in the equation. Listed with each equation are the standard error of the response (s.e.), the multiple correlation coefficient (R), and the coefficient of determination, (R^2), in percent (this represents the percent of the variation in the response that is explained by the equation).

The responses obtained from the Rise-Rate and Pressure-Gel Rate machines were the following.

- Y_1 -Rise Time: Time interval in seconds from the start of mixing to the time when 95 percent of the maximum foam height occurs.
- Y_2 -Gel Time: Time interval in seconds from the start of mixing to the time when the foam gels.

The average experimental results for rise time (Y_1) and gel time (Y_2) for each line of the experiment are listed in Table 10. Two sets of predictive equations for rise time and gel time were developed from these experimental results. The first set of equations (designated A) includes separate terms (X_2 and X_3) for the temperatures of the R and T components. The second set of equations (designated B) includes one term (X_5) for the calculated temperature of the mixture which replaces the X_2 and X_3 terms. Since viscosity of the T-component, X_4 , had no significant effect on either of the responses, this term does not appear in any of these equations. The B equations provide

Table 11. Levels of the Variables

Coded Level	Catalyst Concentration X_1 (Parts per Hundred)	*R Temperature X_2 ($^{\circ}$ F)	*T Temperature X_3 ($^{\circ}$ F)	T Viscosity X_4 (Centipoise)
-2	0.0	80	60	3640
-1	0.6	90	70	4800
0	1.2	100	80	5420
+1	1.8	110	90	6300
+2	2.4	120	100	7580

Coded Levels and Calculated Temperatures for Variable X_5							
Coded Level	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
Calculated Temp. for Mixture, X_5	70	75	80	85	90	95	100

*Variable X_5 , the calculated temperature for the mixture, was generated by weighting the temperatures of the R and T components according to the weight of each in the mixture.

a simpler model without any significant loss in accuracy, and are presented in the form of response surfaces in Figures 1 and 2.

Rise Time (Y_1): Equation A

$$\hat{Y}_1 = 252.05 - \underline{\underline{87.85X_1}} - \underline{\underline{13.06X_2}} - \underline{\underline{23.90X_3}} + \underline{\underline{24.36X_1^2}} \\ + 6.29X_1X_2 + \underline{\underline{9.16X_1X_3}}.$$

$$s.e. = 14.02.$$

$$R = 0.988, R^2 = 97.5\%.$$

The terms X_1 , X_2 , X_3 , X_1^2 , X_1X_2 , and X_1X_3 are significant and explain 97.5 percent of the variation in the response. The rise time increases as the catalyst concentration and the temperatures of the R and T components decrease.

Equation B

$$\hat{Y}_1 = 252.05 - \underline{\underline{87.85X_1}} - \underline{\underline{33.90X_5}} + \underline{\underline{24.36X_1^2}} + \underline{\underline{13.51X_1X_5}}.$$

$$s.e. = 14.52.$$

$$R = 0.985, R^2 = 97.1\%.$$

The terms X_1 , X_5 , and X_1^2 , and X_1X_5 are significant and explain 97.1 percent of the variation in the response. The rise time increases as the catalyst concentration and calculated temperature decrease (see Figure 1).

Gel Time (Y_2): Equation A

$$\hat{Y}_2 = 196.63 - \underline{\underline{71.23X_1}} - \underline{\underline{11.55X_2}} - \underline{\underline{19.40X_3}} + \underline{\underline{19.94X_1^2}} + \underline{\underline{5.37X_1X_3}} \\ + \underline{\underline{6.69X_2X_3}}.$$

$$s.e. = 8.668.$$

$$R = 0.993, R^2 = 98.6\%.$$

Y_1 = Rise Time
 X_1 = Catalyst Concentration
 X_5 = Calculated Temperature

$$Y_1 = 252.05 - 87.85 X_1 - 33.90 X_5 + 13.51 X_1 X_5 + 24.36 X_1^2$$

Figure 1. Rise Time Response Surface

Y_2 = Gel Time
 X_1 = Catalyst Concentration
 X_5 = Calculated Temperature

$$Y_2 = 196.63 - 71.23X_1 - 27.90X_5 + 8.11X_1 X_5 + 19.94X_1^2$$

Figure 2. Gel Time Response Surface

The terms X_1 , X_2 , X_3 , X_1^2 , X_1X_3 , and X_2X_3 are significant and explain 98.6 percent of the variation in the response. The gel time will increase as the catalyst concentration and temperatures of the R and T components decrease.

Equation B

$$\hat{Y}_2 = 196.63 - \underline{\underline{71.23X_1}} - \underline{\underline{27.90X_5}} + \underline{\underline{19.94X_1^2}} + \underline{\underline{8.11X_1X_5}}.$$

s.e. = 10.84.

$R = 0.987$, $R^2 = 97.5\%$.

The terms X_1 , X_5 , X_1^2 , and X_1X_5 are significant and explain 97.5 percent of the variation in the response. The gel time will increase as the catalyst concentration and calculated temperature decrease (see Figure 2).

After observing the response surfaces in Figures 1 and 2, increases and decreases are easily noted. The amounts of these response equations with the remaining variables set to zero level are also important. These results were calculated for the significant independent variables in the region between +1 and -1, and their effects on the responses are shown in Table 12.

OPTIMIZING TRADE-OFF BETWEEN INSPECTION EFFORT AND PROTECTION

The statistical literature contains much information on sampling inspection. A large portion of the literature deals with the action to be taken after risks are specified. The problem of coping with the trade-offs involved in specifying risks remains a difficult problem, in spite of all that has been written on the subject. In most instances, after a system has operated for a time data will suggest changes in the inspection level to achieve the desired level of protection with a minimum of inspection effort.

Inspection of a Cellular Silicone Product

A cellular silicone part is made by compression molding of preformed silicone gum. The inspection procedure for the compression/deflection characteristics of this product required inspection on all units submitted to the inspection department. The compression value was obtained at four positions, defined by specification on the part. The compression characteristics are measured at two specified levels of compression.

Table 12. Urethane Foam Response Summary

Dependent Variables			Independent Variables				
			Catalyst Concentration ↓X ₁		T Viscosity X ₄	Calculated Temperature ↓X ₅	
Response	Coefficient of Determination	Change in Response	+1 to 0	0 to -1		+1 to 0	0 to -1
Rise Time	97.1%	↑Actual ↑Percent	↑63.5 ↑24.1%	↑112.2 ↑ 42.6%	Not Significant	↑33.9 ↑12.9%	↑33.9 ↑12.9%
Gel Time	97.5%	↑Actual ↑Percent	↑51.3 ↑24.9%	↑ 91.2 ↑ 44.2%	Not Significant	↑27.9 ↑13.5%	↑27.9 ↑13.5%
<p>↑ means that the variable is increasing ↓ means that the variable is decreasing</p> <p>For example: A decrease in X₁ from +1 to 0 results in the response Rise Time increasing 63.5 seconds, or 24.1 percent.</p>							

Because of the low rejection rate and high testing workload on this part, a statistical evaluation of the load deflection on the part was made to determine the feasibility of reducing the number of test locations from four to two and of the initiation of sampling inspection.

The data was grouped so that each of the six combinations of two out of four test positions was investigated. The percent beyond specification and the variability for each combination was studied. Variation for each combination of two positions was compared with that for four positions by a standard statistical method, *Bartlett's test*.⁷ Also, the estimates of the percentage out of specification were compared. Variation between positions within parts was also studied. A common procedure for studying such variation requires a prior assumption of a normal distribution. In this case, the normal distribution did not appear to be appropriate, and a *non-parametric*⁸ approach was used which requires no assumptions about the distribution. These analyses led to the conclusion that two of the four positions would be adequate for detecting shifts toward an unacceptable situation.

Further, the stability of the process was such that the 100-percent inspection requirement could be reduced to a sampling basis. For product of this type, it was decided that a 2.0 percent maximum defectiveness could be risked if defective lots were screened. On this basis, it was estimated that 94 percent of the lots would pass the corresponding criteria without screening. Further, the percent of product misclassified by the two positions recommended, as compared to the four positions originally specified, was estimated. Negligible misclassification was found. Accordingly, the number of test positions for the part and the number of parts inspected per lot were reduced, resulting in substantial savings.

CONCLUSION

Variation is inherent in all manufacturing and measuring processes. Knowledge of the sources and amounts of variation in such processes can provide a sound basis for choosing among various alternatives, for determining optimum combinations of multiple controllable variables, and for better understanding of the factors influencing process behavior. Statistical techniques provide an organized means of dealing quantitatively and objectively with these inherent variations.

In recent years, these techniques have been rather extensively applied to the inspection function, and a broad spectrum of sampling plans to meet almost any conceivable need has been made available. However, the potential benefits of applying statistical techniques to manufacturing processes have not been widely

recognized. The specific cases treated in this paper are illustrative of these benefits and hopefully may encourage similar application by others.

GLOSSARY

Analysis of Variance -- A technique that subdivides the total variance of a set of data into meaningful component parts associated with specific sources of variation.

Bartlett's Test -- A test, provided by M. S. Bartlett, for the homogeneity of a set of variances from a number of independent normal samples.

Bivariate Normal Analysis -- An analysis using the bivariate normal distribution, a model showing frequencies in a population whose individuals possess two measurable characteristics.

Coefficient of Determination -- A measure of the proportion of the variance for random variable Y, attributed to another random variable X, when Y is expressed as a linear regression on X.

Covariance -- A measure of the joint variation of two or more random variables.

Discriminant Analysis -- A technique for studying the extent that different populations overlap or diverge from one another.

F-Ratio -- A test based on the ratio of two independent statistics, each of which is distributed as the variance in samples from normal populations with the same variance.

Grubbs' Method -- Method of determining the standard deviation of the product distribution and standard deviation of the tester error.

Multiple Correlation Coefficient -- A measure of the degree of association between the dependent variable and the function of the independent variables represented by the regression equation.

Non-Linear Regression -- A trend of one random variable in relation to one or more variables that are not linear.

Non-Parametric -- A method of testing a hypothesis that does not depend upon the distribution.

Regression -- The method used to describe the relationship of one random variable with one or more other variables.

Response Surface Methodology -- A method used to establish how one random variable responds to the change in one or more other variables.

Standard Error of Estimate -- An expression for the standard deviation of the observed values about a regression line.

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