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INFORMATION ON THE NEUTRONS PRODUCED
IN THE $H^3(d,n)He^4$ REACTION

By
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January 1954

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ABSTRACT

This compilation of tables and graphs of the more important kinematical relations describing the D-T reaction is intended for the reference of experimenters working with the Cockroft-Walton Accelerator.

Livermore, California

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I. KINEMATICS

Consider the following two-body reaction. Initially nucleon 1 is incident on nucleon 2 which is at rest in the lab system; after the interaction only nucleons 3 and 4 emerge.

The general expression for the energy of nucleon 3 in the lab system may be written in the form

$$E_L(m_3, \phi) = \left(\frac{m_1}{m_1 + m_2} \right)^2 \frac{m_3}{m_1} E_0 \cos 2\phi \frac{m_4}{m_3 + m_4} \left(\frac{m_2}{m_1 + m_2} E_0 + Q \right) \quad (1)$$

$$\pm \frac{2 \cos \phi}{m_1 + m_2} \sqrt{m_1 m_3 \frac{m_4}{m_3 + m_4} E_0 \left(\frac{m_2}{m_1 + m_2} E_0 + Q \right)} \cdot \sqrt{1 - \frac{m_1 m_3}{m_1 + m_2} \frac{E_0 \sin^2 \phi}{m_4 \left(\frac{m_2}{m_1 + m_2} E_0 + Q \right)}}$$

where E_0 is the energy of nucleon 1 in the lab system, ϕ is the lab angle made by the velocity vector of nucleon 3 with the beam direction, the m_j 's are the respective nucleon masses, and the Q of the reaction is defined customarily as $Q = m_3 + m_4 - m_1 - m_2$.

For the case of the reaction $d + T \rightarrow n + He^4$ Eq. (1) yields for the neutron energy

$$E_L(E_0, \phi_n) = 0.08 E_0 \cos 2\phi_n + 0.8 (.6E_0 + 17.6) + 0.8 \cos \phi_n \sqrt{0.4E_0 (.6E_0 + 17.6)} \sqrt{1 - \frac{E_0 \sin^2 \phi_n}{10(.6E_0 + 17.6)}} \quad (2)$$

$Q = 17.6$ Mev was determined from the mass values appearing in the Segré chart.

In Table I, Eq. (2) has been evaluated as a function of ϕ_n for six values of the parameter $\bar{E}_0$. Fig. 1 is a system of curves representing the data of Table I. Since the tritium target is "thick" (relative to the range of the incident deuterons) deuterons of all energies from the bombarding energy to zero will yield neutrons. The energy spread of neutrons (see section IV "Line Shape") to be found at any angle ϕ_n may be found from Fig. 1 by measuring the energy spread between the curves for E_0 = bombarding energy and $E_0 = 0$. It is clear that one will find his neutron source to be most monoenergetic if he looks at about 98° to the beam direction.

TABLE I

NEUTRON ENERGY VS LAB ANGLE FOR DEUTERON BOMBARDING ENERGY E_0

$\phi_n(\text{lab})$	$E_0=0$	$E_0=0.1$	$E_0=0.2$	$E_0=0.3$	$E_0=0.4$	$E_0=0.5$
0	14.080	14.808	15.144	15.417	15.655	15.874
10	14.080	14.797	15.129	15.398	15.632	15.849
20	14.080	14.766	15.083	15.340	15.566	15.774
30	14.080	14.714	15.009	15.248	15.458	15.650
40	14.080	14.644	14.908	15.123	15.312	15.486
50	14.080	14.546	14.785	14.971	15.134	15.286
60	14.080	14.460	14.644	14.795	14.931	15.056
70	14.080	14.352	14.490	14.605	14.710	14.806
80	14.080	14.238	14.326	14.404	14.485	14.544
90	14.080	14.120	14.166	14.200	14.240	14.280
100	14.080	14.004	13.996	13.998	13.999	14.020
110	14.080	13.892	13.838	13.807	13.786	13.772
120	14.080	13.788	13.692	13.629	13.581	13.544
130	14.080	13.695	13.561	13.445	13.398	13.340
140	14.080	13.614	13.450	13.333	13.244	13.168
150	14.080	13.550	13.359	13.224	13.118	13.030
160	14.080	13.502	13.293	13.144	13.026	12.974
170	14.080	13.473	13.253	13.096	12.972	12.867
180	14.080	13.464	13.243	13.079	12.953	12.819

Fig. 1 Neutron energy from the reaction $T(D,n)He^4$ as a function of observation angle in the lab system. E_0 is the energy in Mev of the incident deuteron.

It may be of interest to know where, in the lab frame of reference, the neutron and associated α -particle from a given D-T encounter are to be found. Eq. (3) relates ϕ_α , the angle made by the α -particle with the beam direction, to ϕ_n , that made by the neutron and the beam direction.

$$\tan \phi_\alpha = \frac{\frac{1}{2} \sin 2\phi_n + \sin \phi_n \sqrt{\frac{1}{2} - \sin^2 \phi_n}}{-\sin^2 \phi_n + \cos \phi_n \sqrt{\frac{1}{2} - \sin^2 \phi_n} - \frac{m_\alpha}{m_n}} \quad (3)$$

Here, $\gamma = \frac{V_{c.m.}}{v_n}$ where $V_{c.m.}$ is the velocity of the center of mass in the lab system and v_n is the neutron velocity in the center of mass system.

In Table II and Fig. 2, Eq. (3) has been evaluated for several values of the deuteron bombarding energy, E_0 .

TABLE II												
ANGLE BETWEEN ALPHA PARTICLE AND INCIDENT BEAM FOR DEUTERON BOMBARDING ENERGY E_0												
$\phi_n(\text{lab})$	0.1			0.2			0.3			0.4		
	0.5											
0	180°	00'	00"	180°	00'	00"	180°	00'	00"	180°	00'	00"
10	168	41	38	168	04	28	167	36	40	167	06	23
20	157	26	28	156	14	16	154	58	03	154	22	23
30	146	17	26	144	34	11	143	10	16	141	56	15
40	135	17	00	133	08	10	131	24	34	129	54	00
50	124	27	13	121	58	48	120	01	05	118	19	28
60	113	49	23	111	07	55	109	01	30	107	13	41
70	103	24	22	100	36	03	98	26	03	96	36	19
80	93	12	14	90	22	54	88	13	46	86	25	51
90	83	12	46	80	27	34	78	23	03	76	39	57
100	73	25	23	70	48	44	68	51	57	67	16	00
110	63	49	01	61	24	51	59	58	15	58	11	12
120	54	22	52	52	14	07	50	39	42	49	23	00
130	45	05	20	43	14	40	41	53	50	40	48	37
140	35	55	28	34	24	30	33	18	37	32	25	35
150	26	51	15	25	41	48	24	51	42	24	11	29
160	17	51	56	17	04	45	16	31	02	16	04	01
170	8	55	14	8	31	40	8	12	26	8	00	56
180	0	00	00	0	00	00	0	00	00	0	00	00

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600
009

$\phi_n(\text{LAB})$

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Fig. 2 Relation between the angles made by the neutron and associated α -particle with incident deuteron beam line. E_0 is the energy in Mev of the incident deuteron.

To determine the ratio of the solid angles, in the c.m. system primed quantities and the lab system, into which the neutron is scattered one writes

$$dw'/dw^* = \sin\theta d\theta / \sin\phi d\phi \quad . \quad (4)$$

The relation between the c.m. angle θ and the lab angle ϕ is

$$\cos\theta = -r\sin^2\phi \pm \cos\phi\sqrt{1-r^2\sin^2\phi} \quad . \quad (5)$$

Differentiating,

$$\sin\theta d\theta = \left(2r\sin\phi\cos\phi + \sin\phi\sqrt{1-r^2\sin^2\phi} + \frac{r^2\cos^2\phi\sin\phi}{\sqrt{1-r^2\sin^2\phi}} \right) d\phi \quad (6)$$

where only the plus sign in Eq. (5) has been retained. By inserting Eq. (6) in Eq. (4) and rearranging,

$$(dw'/dw)_n = \frac{r(\cos\phi + \sqrt{\frac{1}{r^2} - \sin^2\phi})}{\sqrt{\frac{1}{r^2} - \sin^2\phi}} \quad . \quad (7)$$

Again, by definition

$$\frac{1}{r^2} = \frac{v_n'}{V_{c.m.}} = \frac{m_\alpha}{m_n} \frac{(m_1+m_2)}{m_1} \left[\frac{m_2}{m_1+m_2} + \frac{Q}{E_0} \right]$$

where m_1 = mass of incident particle

m_2 = mass of target particle

m_n = mass of neutron

m_α = mass of alpha

E_0 = incident energy

To obtain $(dw'/dw)_\alpha$, one could just use Eq. (7) employing for $\frac{1}{r^2}$ the expression above after interchanging m_α and m_n . Eq. (7) has been evaluated as a function of ϕ_n and E_0 . These values appear in Table III and in Fig. 3.

* Throughout this report dw'/dw represents dw'/dw .

TABLE III

RATIO $\left(\frac{dw'}{dw}\right)$ OF NEUTRON SOLID ANGLES FOR DEUTERON
BOMBARDING ENERGY E_0

$\theta_n(\text{lab})$	0.1	0.2	0.3	0.4	0.5
0	1.0483	1.0684	1.0837	1.0969	1.1085
10	1.0476	1.0674	1.0823	1.0954	1.1068
20	1.0454	1.0642	1.0784	1.0908	1.1016
30	1.0417	1.0590	1.0720	1.0834	1.0933
40	1.0368	1.0520	1.0634	1.0734	1.0820
50	1.0308	1.0435	1.0528	1.0611	1.0683
60	1.0239	1.0336	1.0407	1.0470	1.0525
70	1.0163	1.0227	1.0274	1.0316	1.0352
80	1.0082	1.0113	1.0133	1.0154	1.0171
90	0.9999	0.9996	0.9990	0.9988	0.9986
100	0.9917	0.9880	0.9848	0.9825	0.9804
110	0.9837	0.9768	0.9712	0.9669	0.9629
120	0.9763	0.9664	0.9586	0.9523	0.9468
130	0.9696	0.9571	0.9472	0.9394	0.9324
140	0.9639	0.9491	0.9376	0.9283	0.9201
150	0.9593	0.9426	0.9298	0.9194	0.9102
160	0.9559	0.9379	0.9240	0.9128	0.9030
170	0.9538	0.9350	0.9206	0.9088	0.8986
180	0.9531	0.9340	0.9194	0.9075	0.8971

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$\phi_n(\text{LAB})$

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Fig. 3 The ratio of the solid angles in the center of mass (primed quantities) and lab systems into which the neutron is produced as a function of the neutron lab angle.

II. D+T YIELD

The differential cross section for the production of neutrons from the D+T reaction has been measured by Conner et al.¹ in the energy region 15 to 530 kev, by Arnold et al.² from 10 to 120 kev, and by Argo et al.³ from 80 to 1200 kev tritons. In the overlapping regions the agreement is excellent as seen in Fig. 4. The angular dependence of the differential cross section has been found to be isotropic to 200 kev by Allan and Poole⁴ and to 570 kev by Argo et al.³ Thus, the total thick target yield of neutrons per incident deuteron of energy E is given by

$$Y = \int_0^E \frac{n_t \, d\sigma/d\omega' \, 4\pi}{d\epsilon/dx} \, d\epsilon \quad (8)$$

where n_t is the number of tritium nuclei/cm³

$d\sigma/d\omega'$ is the c.m. differential cross section for the D(T,n)He⁴ reaction

and $d\epsilon/dx$ is the rate of energy loss of deuterons of energy ϵ in the Ti-T targets.

To determine n_t we assume a loading factor of one tritium per titanium atom. (A survey of reports on this point shows that this is a reasonable average number). Then since the density of Ti is 4.54 gm/cm³ and since the volume expansion of the hydride has been reported to be some 15 percent, the density of loaded titanium, ρ , is

$$\rho_{\text{Ti-T}} = \frac{48+3}{48} \times 4.54 \times .85 = 4.1 \quad (\pm 5\%)$$

244 014

This does not agree well with the measured value of TiH_2 (3.76 gm/cm^3). The discrepancy, however, is in the right direction since two hydrogens probably take more room in the lattice than one tritium.

Thus, the number of tritium atoms per cm^3 is

$$n_t = \frac{\rho_{\text{Ti-T}} 6.02 \times 10^{23}}{A_{\text{Ti-T}}} = \frac{6.02 \times 10^{23} \times 4.1}{51} = 4.8 \times 10^{22}.$$

The rate of energy loss of deuterons in Ti-T is given by

$$d\epsilon/dx = \frac{48}{48 + 3N} \left(\frac{dE}{dx} \right)_{\text{Ti}} + \frac{3N}{48 + 3N} \left(\frac{dE}{dx} \right)_{\text{T}} \quad (9)$$

if all rates of energy loss are measured in mg/cm^2 . $(dE/dx)_{\text{Ti}}$ represents the rate of energy loss of deuterons in normal titanium, $(dE/dx)_{\text{T}}$ that in tritium, and N represents the number of tritium atoms per titanium (it is assumed $N=1$).

Determination of $(dE/dx)_{\text{Ti}}$:

S. D. Warshaw⁵ has made some good measurements of the rate of energy loss of protons in Al, Cu, Ag, and Au in the energy range 50 to 350 kev. Although no specific measurements of dE/dx in Ti are known of, it may be inferred from this data by interpolating between Al and Cu with respect to $A^{-1/2}$. This interpolation presupposes no anomalous behaviour of low-velocity protons in Ti. Below 50 kev a reasonably smooth extrapolation is guessed at. Warshaw's data with the interpolated Ti curve appears in Fig. 5.

Finally, since we are interested in the rate of energy loss of deuterons in Ti, we employ the proton data to derive a deuteron curve (Fig. 6) by assuming that the rate of energy loss is a function only of the velocity of the particle; i.e.

$$(dE/dx)_p(\epsilon) = (dE/dx)_d(2\epsilon) \quad (10)$$

Fig. 5 The rate of energy loss of protons in various elements.

Fig. 6 The rate of energy loss of deuterons in titanium - derived from the curve of Fig. 5.

Determination of $(dE/dx)_T$:

Recent data from Calif. Inst. of Tech.⁶ on the rate of energy loss of protons in hydrogen has been used to derive the rate of energy loss of deuterons in tritium in the deuteron energy range 60 to 700 kev. At 120 kev the rate of energy loss is about 2 percent higher than that reported by Phillips⁷ at Los Alamos, and 10 percent higher at 160 kev, the highest energy reported by the Los Alamos group. It has been suggested by Phillips⁸ that the discrepancy might be due to the fact that Calif. Inst. of Tech. used higher target gas pressures. This increased scattering would make one tend to underestimate the gas target thickness and hence overestimate (dE/dx) . It appears that, in view of this explanation, the Calif. Inst. of Tech. data would be more applicable to our own case.

There is a rather large discrepancy between the Calif. Inst. of Tech. data and that of C. M. Crenshaw⁹. Crenshaw's data yields dE/dx values which are about 150 kev/mg/cm² less over the whole energy range from 100 to 600 kev. It is not easy to explain away this discrepancy; however, in view of its age and the fact that Crenshaw failed to specify his target gas temperature, Crenshaw's data has been rejected.

Combining the data of Figs. 6 and 7 according to Eq. 10, we obtain Fig. 8, the rate of energy loss of deuterons in the tritium-titanium target.

With the information thus developed, one may calculate by numerical integration of Eq. (8) the thick target yield of neutrons per incident deuteron of energy E . The results of this calculation appears in Table IV and Fig. 9.

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TABLE IV

THICK TARGET YIELD OF NEUTRONS FROM $T(d,n)He^4$ REACTION

$$Y = \int_0^E n_t 4\pi \frac{d\sigma/dw'}{d\epsilon/dx} d\epsilon = 4\pi n_t \sum_E \frac{d\sigma/dw'}{d\epsilon/dx} \Delta\epsilon$$

Energy interval (kev)	$\frac{d\sigma/dw'}{d\epsilon/dx} \times 10^4$	$\sum \frac{d\sigma/dw'}{d\epsilon/dx} \times 10^4$	Y neutrons/deuteron
0-25	.32	.32	11.85×10^{-8}
25-50	2.3	2.62	97.0
50-75	6.8	9.43	349.
75-100	11.4	20.82	770
100-125	12.3	33.12	1225
125-150	10.8	43.92	1625
150-175	8.4	52.32	1925
175-200	6.8	59.12	2180
200-225	5.65	64.77	2395
225-250	4.65	69.42	2550
250-275	3.95	73.37	2710
275-300	3.45	76.82	2840
300-325	3.05	79.87	2950
325-350	2.75	82.62	3050
350-375	2.50	85.12	3150
375-400	2.35	87.47	3225
400-425	2.15	89.62	3315
425-450	2.0	91.62	3390
450-475	1.9	93.52	3460
475-500	1.8	95.32	3525

This yields: 10^{10} neutrons /45.4 μ coul. deuterons at 500 kev

10^{10} neutrons /49.6 μ coul. deuterons at 400 kev

10^{10} neutrons /56.3 μ coul. deuterons at 300 kev

10^{10} neutrons /73.4 μ coul. deuterons at 200 kev

Fig. 7 The rate of energy loss of deuterons in tritium (0°C - 760 mm). ● Data from California Institute of Technology; ⁶ x Phillips' Data; ⁷ ○ Crenshaw's Data. ⁹

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Fig. 8 The rate of energy loss of deuterons in a titanium-tritide target - derived from the data of Figs. 6 and 7.

Fig. 9 Theoretical thick target yield of neutrons from $T(d,n)He^4$ reaction produced in titanium-tritide target.

III. NEUTRON MONITOR

One of the methods used to monitor the production of 14 Mev neutrons from the reaction $T(d,n)He^4$ is to count the associated α -particles with a detector subtending a very small solid angle at the target.

To determine the relation between the number of α 's counted and the number of neutrons intercepted by the target, the following considerations are made.

In the $T(d,n)He^4$ reaction, let $N'_\alpha(w'_\alpha, \epsilon)d\epsilon$ be the number of α 's per unit solid angle scattered into $w'_\alpha(c.m.)$ as a result of a reaction by deuterons of energy between ϵ and $\epsilon+d\epsilon$; similarly $N'_n(w'_n, \epsilon)d\epsilon$ is defined for neutrons. Converting to the lab system

$$N_\alpha(w_\alpha, \epsilon)dw_\alpha = N'_\alpha(w'_\alpha, \epsilon)dw'_\alpha,$$

and

$$N_\alpha(w_\alpha, \epsilon)\Delta\Omega_\alpha = N'_\alpha(w'_\alpha, \epsilon) \left(\frac{dw'_\alpha}{dw_\alpha}\right)_{\alpha, \epsilon} \Delta\Omega_\alpha \quad (11)$$

is the number of α 's detected in the solid angle $\Delta\Omega_\alpha$ in the lab system as a result of interactions by deuterons of energy ϵ . No integration was performed because our physical setup employs such a small $\Delta\Omega_\alpha$ that $(dw'_\alpha/dw_\alpha)_\alpha$ is sensibly constant in the interval. For the neutrons

$$N_n(w_n, \epsilon)dw_n = N'_n(w'_n, \epsilon)dw'_n$$

and if the target subtends, at the neutron source, a solid angle $\Delta\Omega_n$, the number of neutrons intercepted by the target is

$$N_n(w_n, \epsilon)\Delta\Omega_n = \int_{\Omega} N'_n(w'_n, \epsilon) \left(\frac{dw'_n}{dw_n}\right)_n dw_n. \quad (12)$$

Since the D-T reaction yields neutrons isotropically in the c.m. system up to deuteron bombarding energies of 600 kev,

$$N'_n(w'_n \epsilon) = N'_\alpha(w'_\alpha \epsilon) = K(\epsilon) ,$$

i.e., the yield is a function of energy alone.

Further, the number of reactions per unit solid angle due to deuterons of energy between ϵ and $\epsilon + d\epsilon$ is

$$K(\epsilon)d\epsilon = n_d(\epsilon) n_t(\epsilon) dx \frac{d\sigma}{dw}(\epsilon) ,$$

or

$$K(\epsilon)d\epsilon = n_d(\epsilon) n_t(\epsilon) \frac{d\epsilon}{(d\epsilon/dx)} \frac{d\sigma}{dw}(\epsilon) .$$

where $n_d(\epsilon)d\epsilon$ is the number of deuterons per second of energy incident on target nuclei. Since the cross section is of the order of a few barns, we shall assume that the depletion of deuterons of energy ϵ as they enter the tritium-titanium target is insignificant. Thus $n_d(\epsilon) \approx f(\epsilon)$ and is equal to the number of deuterons per second of the incident beam. $n_t(\epsilon)$ = number of tritons per unit volume of the titanium target. The ϵ -dependence has been indicated to account for an uneven distribution of tritium in the titanium. However, since it would be difficult to know this distribution, we shall assume it to be constant.

$$(d\sigma/dw')(\epsilon) = \text{known differential cross section}$$

$$(d\epsilon/dx)(\epsilon) = \text{rate of energy loss of deuterons in tritium loaded target.}$$

Hence

$$K(\epsilon) = \frac{n_d n_t (d\sigma/dw')(\epsilon)}{(d\epsilon/dx)(\epsilon)} \quad (13)$$

Substituting this into Eq. (11) and (12) and forming the ratio, one finds for the number of neutrons scattered into the solid angle $\Delta\Omega_n$ per α -particle scattered into $\Delta\Omega_\alpha$

$$\frac{N_n \Delta \Omega_n}{N_\alpha \Delta \Omega_\alpha} = \frac{\int_E \frac{(d\sigma/dw')(\epsilon)}{(d\epsilon/dx)(\epsilon)} \Delta \Omega_n (dw'/dw)_n (\epsilon) dw_n d\epsilon}{\Delta \Omega_\alpha \int_E \frac{(d\sigma/dw')(\epsilon)}{(d\epsilon/dx)(\epsilon)} \cdot (dw'/dw)_\alpha (\epsilon) d\epsilon} \quad (14)$$

For the special case of total neutron production per detected α -particle, Eq. (14) becomes

$$N = \frac{4\pi \int_0^E \frac{d\sigma/dw'}{d\epsilon/dx} d\epsilon}{\int_0^E \frac{(d\sigma/dw')(\epsilon)}{(d\epsilon/dx)(\epsilon)} \cdot (dw'/dw)_\alpha d\epsilon} \quad (15)$$

$$= \frac{4\pi}{\Delta \Omega_\alpha} R$$

The ratio of the two integrals, R , represents the correction to be applied to a computation of N which assumes the reaction to be isotropic in the lab system. The value of R as a function of incident deuteron energy appears in Table V and Fig. 10.

TABLE V

TOTAL NEUTRON PRODUCTION PER DETECTED α -PARTICLE

E (kev)	$\frac{d\sigma}{dw} \frac{dE}{dx}$	$\left(\frac{dw'}{dw}\right)_{\text{at } 135^\circ \text{lab}}$	$\frac{d\sigma}{dw} \frac{dw'}{dw} \left(\frac{dw'}{dw}\right)$	$\sum \frac{d}{dx}$	$\sum \frac{d}{dx} \frac{dw'}{dw}$	$R = \frac{\sum}{\sum}$
0-25	0.2	.964	0.193	0.2	0.193	1.035
25-50	2.3	.924	2.125	2.5	2.318	1.080
50-75	6.8	.897	6.10	9.3	8.42	1.103
75-100	11.3	.876	9.89	20.60	18.31	1.124
100-125	12.3	.860	10.58	32.90	28.89	1.130
125-150	10.8	.846	9.14	43.70	38.03	1.150
150-175	8.4	.833	7.00	52.10	45.03	1.156
175-200	6.8	.822	5.59	58.9	50.62	1.163
200-225	5.6	.810	4.53	64.5	55.15	1.17
225-250	4.7	.800	3.76	69.2	58.91	1.175
250-275	3.9	.790	3.08	73.1	61.99	1.180
275-300	3.4	.780	2.67	76.5	64.66	1.183
300-325	3.1	.771	2.39	79.6	67.05	1.186
325-350	2.8	.762	2.135	82.4	69.18	1.191
350-375	2.5	.753	1.883	84.9	71.07	1.194
375-400	2.3	.745	1.714	87.2	72.78	1.198
400-425	2.2	.737	1.62	89.4	74.40	1.201
425-450	2.0	.730	1.46	91.4	75.86	1.204
450-475	1.9	.723	1.372	93.3	77.23	1.209
475-500	1.8	.716	1.29	95.1	78.52	1.211

Fig. 10 Total number of neutrons produced per α -particle detected in proportional-counter monitor, $N = \frac{4\pi}{\Delta\Omega} R$

IV. LINE SHAPE

Probably even more interesting than the total yield is the neutron line shape* at any particular lab angle, since this shows for a specific experimental geometry the energy spread of the neutrons striking the target.

The ordinate of the line shape is given by an expression proportional to the neutron yield in the lab system of deuterons of energy ϵ .

$$L(\phi_{\text{lab}}) \approx \frac{\frac{d\sigma}{dw'}(\epsilon)}{\frac{d\epsilon}{dx}(\epsilon)} \frac{dw'}{dw}(\epsilon) \quad (16)$$

The abscissa is the neutron energy resulting from the reaction of a deuteron of energy ϵ .

$$E_n(\epsilon, \phi) = 0.08\epsilon \cos 2\phi + 0.8(0.6\epsilon + 17.6) \\ + 0.8 \cos \phi \sqrt{0.4\epsilon(0.6\epsilon + 17.6)} \sqrt{1 - \frac{\epsilon \sin^2 \phi}{10(0.6\epsilon + 17.6)}} \quad (\text{Same as Eq. (2)})$$

Neglected in this calculation of line shape has been the scattering of the incident deuteron in the target. If one were to compare the predicted line shape with that observed in a neutron detector, one must also consider the finite angle subtended by the detector.

Neutron line shapes calculated in this way for several angles are presented in Fig. 11a, b, and c.

* Energy distribution of the neutrons.

Fig. 11a Neutron Line Shape. The energy distribution of neutrons coming off at an angle ϕ_n (lab) with respect to the incident deuteron beam. ($\phi_n = 0^\circ, 30^\circ$ and 60°).

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Fig. 11b Neutron Line Shape. ($\phi_n = 98^\circ$).

Fig. 11c Neutron Line Shape. ($\phi_n = 120^\circ$ and 150°).

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