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NEUTRON LEAKAGE FROM THE 30
MEGAWATT SGR - P4 REACTOR

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NEUTRON LEAKAGE FROM THE 30 MEGAWATT SGR - P4 REACTOR

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ABSTRACT

The fast and thermal neutron leakage from the 30 megawatt SGR-P4 reactor has been studied by three independent methods. The appropriate reactor constants were calculated in another report¹ and are summarized in Table I. Results are tabulated for comparison in Table II. The values obtained are for the radial leakage only; axial leakage can be analyzed by the same procedures. However, axial leakage is complicated by neutron streaming through the process tube channels.

Method I. studies the fast neutron leakage and consists of the application of Fermi Age theory to fission neutrons in a one-dimensional infinite geometry with the source taken as constant in the multiplying region where $x \leq 0$ and attenuated exponentially in the non-multiplying region where $x \geq 0$. An energy spectrum is obtained for the leakage neutrons from the reflector and is shown in Fig. 4. The energy spectrum is integrated to give the total fast leakage current, and the results are modified by a correction factor to convert to cylindrical geometry. The total fast leakage current is also computed by the method given in Reference 8, and the result shown in Table II. for comparison. The leakage through an 8 inch iron thermal shield is studied by a similar age theory procedure.

Method II. obtains the thermal leakage and integrated fast

leakage by evaluating the average diffusion coefficients and measuring the gradient of the flux in a two-group, two-region flux plot taken from Reference 2 which is shown in Fig. 14.

Method III. is based on the leakage probability for the equivalent bare reactor; i.e., a bare critical reactor having the same core constants as the reactor being studied. Corrections are then applied to the bare reactor fast and thermal leakage to take into account the effect of the reflector and thermal shield.

Good accuracy is not obtainable by these approximate methods, but a glance at Table II shows that they yield results which are consistent to within a factor of about 5. It is difficult to make a reliable estimate of the error involved in any of the methods, but certain features of the calculations are examined at various points in the text which indicate that none of the results should be considered accurate to better than a factor of about two. The best estimate of the accuracy is probably supplied by comparing the results of Table II.

The question of nuclear instrument response is considered for instruments located outside the thermal shield. Since results correct to a factor of 5 are adequate for an evaluation of this question, it is believed that the conclusions reached are sound. Response to fast neutrons is taken proportional to $1/v$, and the energy spectrum of leakage neutrons so weighted is integrated to give the ratio of the response to fast and thermal neutrons. It was found that for an 8 inch shield the fast neutrons give about 3 times as much signal as the thermal neutrons, but that the signal due to the combined effect of thermal and fast neutrons is too low by about a factor of 50 to operate the Westinghouse compensated ionization chambers at their rated efficiency. An iron shield about 5 inches thick would permit adequate thermal leakage to operate these chambers. The Westinghouse fission chamber, to be used for reactor startup instrument, should be able to yield a satisfactory counting rate at least 10 decades below full power. The flux available outside an 8 inch iron shield is too low by a factor of 10^5 , which shows the necessity of inserting the startup instrument into the reflector.

METHOD I. AGE EQUATION

A reasonably good approximation to the fast neutron leakage current from the reflector of a reactor can be obtained by solving the age equation with the boundary conditions which are imposed by the reactor configuration and evaluating dq/dx at the reflector boundary, provided a satisfactory fast neutron source term is used. However, the exact solution of this problem for a specific case presents several annoying problems because (1) the age in the core and reflector are not usually the same, (2) the virgin fast neutron distribution (a virgin fast neutron will be defined as a fission neutron which has not experienced a collision) is not accurately known and is a rather disagreeable function of position for a lattice in any event, (3) the energy of fission neutrons is not unique so we must either integrate over the fission spectrum which is tedious or decide on some suitable way of averaging the constants involved, and (4) the form of the solutions for the particular geometry involved may be discouragingly complicated. Moreover, the age theory is known to give a leakage which is too low for neutrons which have made few collisions, and it is these exceptional neutrons which give the main contribution to the high energy neutron leakage. We are, therefore, persuaded to make some simplifying approximations, several of which come to mind immediately. The manner in which any one approximation is applied to the problem is not unique, and ease of computation from the final formulas was a strong influencing factor. Several of the approximations adopted in this analysis were suggested by Dr. E. R. Cohen, to whom the author wishes to express his appreciation.

A. Fast Neutron Leakage from the Reflector

The simplified model which will be examined* consists of two semi-infinite, one-dimensional regions. It is assumed that the age is the same for these two regions, and that the virgin neutron source distribution is given by

$$q(x,0) = \begin{cases} q_0 = \text{constant}, & x \leq 0 \\ ce^{-x/\lambda}, & x > 0, \quad c = \text{constant} \end{cases} \quad \dots (1)$$

* Another model which could be used is an infinite cylinder core of

λ is the mean free path in the reflector and depends on energy. The age equation is then solved for this geometry and the current obtained by evaluating dq/dx at a point $x = T$, the reflector thickness. This result is then applied to the particular reactor configuration under consideration by making the proper choice of T , q_0 , τ and λ . We claim that the energy distribution of the leakage neutrons will be sufficiently well represented by Eq. 4, and that the total fast leakage current from the center of a face of the shield is given approximately by integrating $J(F,T)$ from F_{th} to ∞ and introducing certain geometry dependent correction factors to be discussed presently which modify the above results so that they are more descriptive of the actual reactor configuration. q_0 can be taken as proportional to the reactor power and is given by:

$$(q_0)_{av.} = \frac{P(\text{watts}) \times 3.1 \times 10^{10} (\text{fission/watt}) \times \gamma (\text{neutron/fission})}{\text{volume of core}}$$

* radius R surrounded by an infinite hollow cylinder of thickness T for the reflector. If the age in core and reflector are the same, the solution of the age equation satisfying the boundary condition that $q = 0$ on the extrapolated boundary $A = R + T + 0.71 \lambda_{tr}$ is

$$q(r, \tau) = \sum_{n=1}^{\infty} C_n e^{-w_n^2 \tau} J_0(w_n r)$$

where

$$C_n = \frac{2}{A^2 J_1^2(w_n A)} \int_0^A r f(r) J_0(w_n r) dr$$

and w_n is given from the roots $x_1, x_2, x_3, \dots$ of $J_0(x)$ by the relation $w_n = x_n/A$. $f(r)$ is the virgin fast neutron source term, which for a lattice will be complicated and depend on the azimuthal angle as well as r . Except for the simplest case where $f(r)$ is taken to be constant, the evaluation of C_n presents a disagreeable problem. For this reason, and also because several other approximations have been made, it was decided to use the simpler approximation outlined in the text.

With $\nu = 2.5$ neutrons/fission, $P = 3 \times 10^7$ watts and $V = 9.98 \times 10^6 \text{ cm}^3$ (corresponding to $R = 93.2$ cm and 26.8 cells loaded), this gives
 $(q_0) = 2.33 \times 10^{11}$ neutrons/cm³

The $e^{-x/\lambda}$ dependence of $q(x,0)$ for $x > 0$ provides a simple approximate way to improve the results of age theory calculation by considering a fission neutron which enters the reflector to be a virgin fission neutron until it experiences its first collision. It is in error, however, since it holds for the case where all of the virgin neutrons travelling from the core into the reflector cross the boundary at normal incidence. If the expression $e^{-x/(\lambda \cos \theta)}$ is integrated over a solid angle of 2π , we obtain for the probability that a virgin neutron which crosses the core-reflector boundary will make its first collision at a distance x from the boundary:

$$\frac{1}{2\pi} \int_0^\pi e^{-\frac{x}{\lambda \cos \theta}} \sin \theta d\theta = \int_0^1 e^{-x/\lambda \mu} d\mu = e^{-x/\lambda} + \frac{x}{\lambda} E_1\left(-\frac{x}{\lambda}\right)$$

where $E_1(-t)$ is the exponential integral $-\int_t^\infty \frac{e^{-t}}{t} dt$.

If this is normalized to unity, the results for the source function

$$q(x,0) = \frac{2}{\lambda} \left[e^{-x/\lambda} + \frac{x}{\lambda} E_1\left(-\frac{x}{\lambda}\right) \right] ; \quad x > 0 \quad (2)$$

Fig. 1 shows a plot of this function and of the function $\frac{1}{\lambda} e^{-x/\lambda}$ for comparison. In the interests of simplification, the $e^{-x/\lambda}$ dependence is adopted for the source function $q(x,0)$.

The constant c is determined by calculating the leakage from the half-space $x < 0$ due to a constant source q_0 in that region.

$$\int_0^{2\pi} d\phi \int_0^{\pi/2} \sin \theta d\theta \int_0^\infty dr \quad q_0 r^2 \frac{\cos \theta}{4\pi r^2} e^{-r/\lambda} = \frac{\lambda}{4} q_0$$

This must equal the area under the curve $c e^{-x/\lambda}$ from 0 to ∞ , which is λc . Therefore, $c = q_0/4$. In view of the neglecting of the $E_1(-x/\lambda)$ term, no attempt is made to correct q when $x < 0$ for the neutrons which escape from the negative half-space before experiencing their first collision.

The required solution of the age Eq. is given by:

$$q(x, \tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} q(\alpha, 0) e^{-\frac{(\alpha-x)^2}{4\tau}} d\alpha$$

where $q(x,0)$ is given by Eq. 1 with $c = q_0/4$. When the integration is performed, there results

$$q(x, \tau) = \frac{q_0}{2} \left\{ \left[1 - \operatorname{erf} \left(\frac{x}{2\sqrt{\tau}} \right) \right] + \frac{\frac{\tau}{\lambda^2} - \frac{x}{\lambda}}{4} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] \right\} \quad (3)$$

where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. The flux per unit energy is related

to the slowing down density by the expression

$$q(x, \tau) = \xi E \sum_g(E) \varphi(x, E)$$

where τ corresponds to the energy reduction from a fission energy E_0 to the lower energy E . The current at point x in the semi-infinite one-dimensional reflector may therefore be written as

$$\begin{aligned} J(x, E) &= - \frac{\lambda \operatorname{tr}(E)}{3} \operatorname{grad} \varphi(x, E) \\ &= - \frac{\lambda \operatorname{tr}(E)}{3 \xi E \sum_g(E)} \frac{dq(x, \tau)}{dx} \\ J(x, E) &= \frac{\lambda \operatorname{tr}(E)}{3 \xi E \sum_g(E)} \frac{q_0}{8} \left\{ \frac{3}{\sqrt{\pi \tau}} e^{-x^2/4\tau} + \frac{\tau/\lambda^2 - x/\lambda}{\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] \right\} \quad (4) \end{aligned}$$

In Eq. 4, λ and τ depend on the energy E_0 of the fission neutrons, so Eq. (4) should be averaged over the fission spectrum. However, we will avoid this tedious computation and take average values for λ and τ . For

λ_{av} we take

$$\lambda_{av} = \frac{\int_{E_{th}}^{\infty} \lambda(E) N(E) dE}{\int_{F_{th}}^{\infty} N(E) dE} = 5.78 \text{ cm.} \quad (5)$$

where $(5) N(E) = (\text{constant}) e^{-E} \sinh \sqrt{2E}$ and $\lambda(E) = 1/\sum_{tot}(E)$ was evaluated

for graphite of density 1.60 gms/cm^3 by taking the total cross section for graphite given in reference 3.

For $\tau(F)_{av}$ we take

$$\tau(F)_{av} = \left[\tau(1.44) - \tau(E, 1.44) \right] \frac{\int_E^{\infty} N(E) dE}{\int_E^{\infty} N(E) dE} \quad (6)$$

where $\tau(1.44) = 311 \pm 3 \text{ cm}^2$ which is the experimental value for the age of fission neutrons to the indium resonance in graphite of density 1.60 gms/cm^3 given in reference 4 and

$$\tau(E, 1.44) = \frac{1}{3 \xi \left(1 - \frac{2}{3A}\right) (N_0 \rho/A)^2} \int_{1.44}^E \frac{1}{\sigma_s(E)^2} \frac{dE}{E}$$

The two integrals on the right of Eq. (6) are included because a fission neutron must have its energy greater than E in order that it may be able to age to energy E . Consequently since $\tau(1.44)$ refers to the entire fission spectrum, the quantity in the square bracket on the right will give too small a value for $\tau(F)_{ave}$ if F is a few tens of kilovolts or larger.

$\sigma_s(E)$ was taken equal to $\sigma_{tot}(E)$, as given in reference 3. A curve of $\tau(E)_{av}$ vs F is given in Fig. 3.

The current $J(F)$ has been computed from Eq. (4) for values of $x = 61 \text{ cm}$ and 84 cm . Curves of $J(F)/q_0$ vs E are given in Fig. 4. A numerical integration of each curve was performed to obtain the total fast neutron leakage. A curve of the form $J(x) = J_0 e^{-x/a}$ where $a = 8.74$ was fitted to these two values and is shown in Fig. 5.

It will be assumed that the energy spectrum of the leakage neutrons from a cylindrical reactor is sufficiently well represented by the curves of Fig. 4. However, the total integrated leakage will be in error because of the geometrical approximations. One-group diffusion theory will be used to investigate the size of this error.

To compare the current in a slab of thickness T to that in a semi-infinite region, we must examine the ratio of the gradients of the flux for these two geometries. If the currents are normalized to unity at $x = 0$, there results

$$\frac{J_{finite}}{J_{\infty}} = \left[\frac{w \cosh w(T-x)/\cosh wT}{w e^{-wx}} \right]_{x=T} = \frac{2 e^{wT}}{e^{wT} + e^{-wT}} \approx 2 \quad (7)$$

With $w = 1/\sqrt{\tau} = 0.0557 \text{ cm}^{-1}$, the negative exponential can be neglected

and the resulting error be less than 1 per cent if $T > 41.5$ cm.

To compare the current in the plane slab of thickness T to that in a cylindrical slab of the same thickness and inner radius R , we again normalize the fluxes so that the same current enters in both cases. Letting $A = R + T$, there results

$$\phi_{\text{slab}} = \frac{\sinh w(T - x)}{\cosh wT}$$

$$\phi_{\text{cyl}} = \frac{I_0(wR) K_0(wA) - I_0(wA) K_0(wR)}{I_1(wR) K_0(wA) + I_0(wA) K_1(wR)}$$

For the ratio of the currents at $x = T$ and $r = A$ we have

$$\frac{J_{\text{cyl}}}{J_{\text{slab}}} = \cosh(wT) \frac{I_1(wA) K_0(wA) + I_0(wA) K_1(wA)}{I_1(wR) K_0(wA) + I_0(wA) K_1(wR)} = 0.671 \quad (8)$$

The cylinder has a considerably lower leakage than the slab, so the application of the infinite geometry formula to reflectors which are approximated by cylinders will not introduce an excessive error. Since the reflector being considered is neither infinite slab nor cylinder, the above calculation only serves to indicate the magnitude of the error. A correction of $J = 1.5 J_\infty$ will be arbitrarily adopted, where J_∞ is given by Eq. (4).

The value of $q_0 = 2.33 \times 10^{11}$ neutrons/cm³ is the average over the core. If we want the leakage in the midplane of the reactor (where it will be peak), this value should be increased by approximately the vertical peak-to-average ratio, which is about 1.33 for a chopped cosine vertical thermal flux distribution in this reactor. The resulting q_0 may give a J which is a little too high due to the radial variation of q . We will adopt a correction of 1.3 for q_0 which is probably not far from the best value, and gives $q_0 = 3.03 \times 10^{11}$. Taking values of $1/q_0 J_\infty$ from Fig. 4, we then obtain for the peak fast leakage:

$$J_f(61) = 6.50 \times 10^{10} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_f(84) = 4.68 \times 10^9 \text{ neutrons/cm}^2 - \text{sec.}$$

B. Fast Neutron Leakage From the Reflector by the Empirical Method of Reference 8.

Experimental work done at Harwell⁽⁸⁾ on the fast neutron leakage from graphite moderated Uranium reactors indicate that the attenuation of the integrated fast flux from a small source of fission neutrons by thick slabs of graphite can be described by the empirical relation

$$\phi_f = \frac{1.9S}{r} e^{-\frac{r}{13.7}}$$

While diffusion theory calls for a $\frac{1}{r^2}$ dependance in front of the exponential factor, the $\frac{1}{r}$ dependence gave a good fit to their data and was chosen because it leads to a convenient expression when an integration is made along a line source. S is the fission neutron source per unit length of a fuel channel. For the peak leakage due to a single fuel channel they obtain the expression

$$\Delta \phi_f = 3.8 S \frac{\text{Sec } i(\alpha)}{x} ; \alpha = \frac{x}{13.7}$$

where x is the perpendicular distance from the fuel channel to the point on the reflector surface where the leakage is being computed and S is evaluated at the foot of this perpendicular. If this expression is summed over all the fuel channels the total fast leakage is obtained. A curve of $\frac{\text{Sec } i(\alpha)}{x}$ vs x is given in reference 8.

When this calculation was carried out for the SCR-P4 reactor, there was obtained

$$J_f(61) = 5.7 \times 10^{10} \text{ neutrons/cm}^2 - \text{sec}$$

$$J_f(84) = 7.7 \times 10^9 \text{ neutrons/cm}^2 - \text{sec}$$

Such excellent agreement of these results with the earlier ones is without doubt accidental and should not be taken to mean that either method yields highly accurate results. There is a considerable uncertainty in determining S, and as is pointed out in reference 8, the leakage as calculated in this manner differs considerably from that measured in an actual reactor.

C. The Peak-To Average Ratio For Leakage Neutrons

The question of the value for the peak-to-average leakage ratio will now be considered. One way to obtain a value for this is to assume a full cosine distribution for the leakage current vertically and horizontally over the face of the reflector. (We are now thinking of the actual graphite stack rather than the cylindrical approximation.) If this is done, there results

$$J_{\text{peak}} = (\pi/2)^2 \quad J_{\text{ave}} = 2.47 J_{\text{ave}} \quad (9)$$

This may be a suitable approximation for the thermal neutron leakage, but there is reason to believe that it is inappropriate for the fast neutron leakage. The attenuation of the thermal flux and current in an infinite medium is given by $e^{-\lambda x}$ where $\lambda = 1/L$. Since for fast neutrons $\sqrt{T}$ is the quantity corresponding to L for thermal neutrons, we expect the fast flux and current to be attenuated by $e^{-x/\sqrt{T}}$. (The results of reference 8 indicate that for graphite thicknesses of more than 60 cm the value of $\sqrt{T} = 13.7$ cm should be used rather than $\sqrt{322}$.) Hence, an increase in the distance in the moderator will reduce the fast current faster than the thermal current, because $\sqrt{T} \ll L$. Further justification for this is seen in Fig. 4, which shows that the fast current is reduced by about a factor of 10 by the addition of 23 cm of reflector. This means that as one moves from the center of a face of the stack towards an edge, the fast leakage will at first drop slowly and then quite rapidly as one comes opposite the edge of the core.

Referring to Fig. 6, if the thermal leakage is represented by the full cosine curve between $\pm b$, the fast leakage will be given by some sort of bell shaped curve. To obtain an upper limit on the fast leakage peak-to-average ratio, let us take the leakage to be given by a cosine between $\pm a$ and zero between a and b . (dotted curve) The average value for this curve is

$$y_{\text{av}} = \frac{1}{b} \int_0^a \cos \frac{\pi x}{2a} dx = \frac{a}{b} \frac{2}{\pi}$$

For the graphite stack of the SGR-P4 reactor, we have in the vertical direction, $a = 6$ ft and $b = 8$ ft, while in the horizontal direction $a = 3$ ft and $b = 5$ ft, so we would have for the fast leakage

$$J_{\text{peak}} = \left(\frac{8}{6}\right) \left(\frac{5}{3}\right) \left(\frac{\pi}{2}\right)^2 J_{\text{av}} = 2.2 \left(\frac{\pi}{2}\right)^2 J_{\text{av}} \quad (10)$$

or about twice the peak-to-average ratio as for the thermal leakage. Values for the peak-to-average ratio of 2.5 for thermal leakage and 5 for fast leakage will probably not be greatly in error.

D. Fast Neutron Leakage From the Thermal Shield

The attenuation of the fast neutron leakage by the thermal shield will be obtained by solving the age equation for a semi-infinite one-dimensional region with an exponentially attenuated source function, $q(x,0) = 1/\lambda e^{-x/\lambda}$. λ and τ depend on the energy E_1 of the source neutrons. For the age in iron we will take (6)

$$\tau(E_1, E) = 1/\xi \int_E^{E_1} \frac{\lambda_{\text{tr}}(E)}{\xi \sigma_s + \gamma \sigma_a} \frac{dE}{E}, \quad E_1 > E \quad (11)$$

where $\xi = 0.0354$ and $\gamma = 0.0237$ for iron. Values were computed for iron of density 7.7 grams/cm³, taking $\sigma_a(E) = (2.43/1.128) \sqrt{0.025/E}$, and σ_s as equal to σ_t in reference 3. An average value of σ_s was adopted for each energy decade and this used in Eq. 11 for the computation of $\tau(E_1, E)$. The results are plotted in Figs. 7 and 8. Capture in the iron was taken into account by multiplying $\text{grad } q(x, \tau)$ by the capture escape probability (6)

$$p(E_1, E) = \exp \left(- \int_E^{E_1} \frac{\sigma_a}{\xi \sigma_s + \gamma \sigma_a} \frac{dE}{E} \right) \quad (12)$$

Eq. (12) is plotted in Fig. 9 for various values of E_1 and E . σ_s and σ_a are taken to be the same as in the age calculation.

Two different boundary conditions for the age equation are considered. The first is $q(0, \tau) = 0$ and $q(\infty, \tau) = 0$, which gives the solution

$$q(x, \tau) = \frac{1}{2\sqrt{\pi\tau}} \int_0^\infty q(\alpha, 0) \left[e^{-\frac{(\alpha-x)^2}{4\tau}} - e^{-\frac{(\alpha+x)^2}{4\tau}} \right] d\alpha.$$

When the integration is carried out, there results

$$q(x, \tau) = \frac{e}{2\lambda} \left\{ e^{-x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] - e^{x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} + \frac{x}{2\sqrt{\tau}} \right) \right] \right\}$$

$$\frac{dq}{dx} = \frac{e}{\lambda\sqrt{\pi\tau}} - \frac{e}{2\lambda^2} \left\{ e^{-x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] + e^{x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} + \frac{x}{2\sqrt{\tau}} \right) \right] \right\} \quad (13)$$

Eq. (13) is plotted in Fig. 10 for various values of λ and τ with x taken to be 20 cm. This boundary condition will lead to too small a leakage from the shield because it permits too great a leakage back into the reflector. In fact if q vanished at $x = -.71 \lambda_{tr}$ instead of at $x = 0$, the back leakage would correspond to that where a vacuum replaces the reflector.

As seen from Fig. 10, Eq. (13) gives a negative (backward) current at $x = 20$ cm if $\tau > 190 \text{ cm}^2$. For a finite shield the leakage is always positive, so it is evident that results based on Eq. (13) will not be reliable unless $\tau > 190 \text{ cm}^2$. In the computations, $\frac{dq}{dx}$ was taken to be zero for $\tau > 190 \text{ cm}^2$. The error due to this effect will not be extreme because not many neutrons of this age are involved. Other approximations probably contribute a much greater uncertainty.

The second boundary condition is $\frac{dq(0, \tau)}{dx} = 0$ and $q(\infty, \tau) = 0$, which gives the solution

$$q(x, \tau) = \frac{1}{2\sqrt{\pi\tau}} \int_0^\infty q(\alpha, 0) \left[e^{-\frac{(\alpha-x)^2}{4\tau}} + e^{-\frac{(\alpha+x)^2}{4\tau}} \right] d\alpha$$

When the integration is carried out, there results

$$q(x, \tau) = \frac{e}{2\lambda} \left\{ e^{-x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] + e^{x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} + \frac{x}{2\sqrt{\tau}} \right) \right] \right\}$$

$$\frac{dq}{dx} = -\frac{e}{2\lambda^2} \left\{ e^{-x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} - \frac{x}{2\sqrt{\tau}} \right) \right] - e^{x/\lambda} \left[1 - \operatorname{erf} \left(\frac{\sqrt{\tau}}{\lambda} + \frac{x}{2\sqrt{\tau}} \right) \right] \right\} \quad (14)$$

Eq. (14) is plotted in Fig. 11, with x taken to be 20 cm. The result is not very sensitive to changes in λ . For comparison the curve of Fig. 10 with $\lambda = 1.36$ cm is repeated on the same scale. This boundary condition will lead to too high a leakage from the shield because it corresponds to the case where the boundary between the reflector and shield is a perfect reflector for neutrons which are trying to diffuse from the shield back into the reflector.

The current at point x in the semi-infinite medium due to unit incident current of energy E_1 from the reflector is

$$J(x, E_1, E) = - \frac{\lambda_{tr}(E_1) P(E_1, E)}{3 \xi E \sum_s (E)} \text{grad } q(x, \tau) \quad (15)$$

Equation (15) is then multiplied by the source strength $J(E_1)$, which is obtained from Eq. (4), and integrated over E_1 to obtain the total current of energy E at point x . This is too small a value for the leakage from a shield of thickness x because of the use of semi-infinite geometry in the solution of the age equation. On the basis of the earlier discussion of this point, we will double the results obtained from Eq. (15) and adopt this as the leakage from a shield of thickness x .

The integration of Eq. (15) was done numerically as follows: The energy was divided into 7 intervals of one decade each and $J(E_1)$ obtained at the geometric mean (which is 3.16×10^n electron volts) of each interval from Fig. 4. These $J(E_1)$ were then used in Eq. (15) to obtain the current at certain values of $E < E_1$ which is due to the source $J(E_1)$ for each of the 7 decades. The values of E which were chosen are 0.075, 0.1, 0.316, 1.0, 3.16, 10, 100 and 1000 electron volts. The various quantities which appear in Eq. (15) were taken from the curves in the text for each pair of values (E_1, E) involved. These results were then summed over E_1 giving $1/q_0 J(20 \text{ cm}, E)$. The results with dq/dx given by Eq. (13) are plotted in Fig. 12. A separate calculation was carried out for reflector thickness of 61 cm and 84 cm. To investigate the effect of changing the source term $J(E_1)$, the curves of Fig. 4 were read at 4.2×10^n electron volts for each decade instead of at the geometric mean and the calculation repeated. The result is plotted as curve c in Fig. 12. To investigate the effect of the boundary condition on $q(x, \tau)$, the calculation was repeated for an 84 cm reflector taking dq/dx from Eq. (14), and the result plotted in Fig. 13. The corresponding curve with dq/dx taken from Eq. (13) is also plotted in Fig. 13 for comparison. It is evident that the second

boundary condition on $q(x, \tau)$ gives rise to about 4 times the leakage that is obtained from the first boundary condition.

Curves a and b of Fig. 12 were integrated to give the total fast leakage from the shield. The results are

$$\frac{1}{q_0} J_f = 7.37 \times 10^{-4} \text{ for the 61 cm reflector (curve a).}$$

$$\frac{1}{q_0} J_f = 3.17 \times 10^{-5} \text{ for the 84 cm reflector (curve b).}$$

$$\frac{1}{q_1} J_f = 6.30 \times 10^{-4} \text{ (curve c).}$$

These values do not have a correction for finite geometry or the boundary condition on $q(x, \tau)$ incorporated. It was decided to multiply the above values by 8 which should give an upper limit on the fast leakage. The results are

$$J_f \text{ (61 cm refl., 20 cm sh.)} = 1.79 \times 10^9 \text{ neutrons/cm}^2 \text{ - sec.}$$

$$J_f \text{ (84 cm refl., 20 cm sh.)} = 7.68 \times 10^7 \text{ neutrons/cm}^2 \text{ - sec.}$$

METHOD II. TWO GROUP FLUX EQUATIONS

Neutron leakage probability for a bare reactor is determined by the buckling, so if an infinite cylinder and an infinite square prism have the same buckling, they will have the same total leakage. The leakage current from two such configurations averaged around their perimeters will be inversely proportional to their perimeters. The radius R of a cylinder having the same buckling as a rectangular prism whose base is a square of side A is $R = (2.405/\pi\sqrt{2})A$. The ratio of perimeter of the cylinder to that of the prism is thus 0.850, so the average radial leakage from the cylinder should be multiplied by 0.850 to obtain the average leakage from the sides of the prism.

A. Reflector Leakage by Evaluating $d\phi/dr$ at the Edge of the Reflector of a Cylindrical Model.

The leakage from the cylinder is $J = -\lambda_{tr}/3 \text{ grad } \phi$, where $\text{grad } \phi$ is the flux gradient at the edge of the reflector and is obtained from the two-group flux plot for cylindrical geometry shown in Fig. 14. This figure was taken from reference 2. For graphite $1/3 \lambda_{tr} = 0.935$ centimeters, and the

same value for both thermal and fast neutrons is adopted since the scattering cross section is constant up to about 0.10 Mev and few collisions occur above this energy. The gradients are evaluated at a point $0.71 \lambda_{tr} = 2.00$ centimeters from the extrapolated boundary. From reference 2, we have for the flux in the core

$$\begin{aligned}\phi_{1c} &= \phi_0 [1.0245 J_0(\alpha r) - 0.0004986 I_0(\beta r)] \\ \phi_{2c} &= \phi_0 [J_0(\alpha r) + 0.0005177 I_0(\beta r)]\end{aligned}\quad (16)$$

where $\alpha = 0.016930$ and $\beta = 0.081196$.

$$\begin{aligned}\phi_{1r} &= -2.583 \times 10^{-6} [I_0(\eta_{10} r) - 4.543 \times 10^7 K_0(\eta_{10} r)] \phi_0 \\ \phi_{2r} &= 3.595 \times 10^{-6} [I_0(\eta_{10} r) - 4.543 \times 10^7 K_0(\eta_{10} r)] \phi_0 - 0.05968 \\ &\quad [I_0(\eta_{20} r) - 104.5 K_0(\eta_{20} r)] \phi_0\end{aligned}\quad (17)$$

where $\eta_{10} = 0.056136$ and $\eta_{20} = 0.017061$.

ϕ_0 is the value of the thermal flux at the center, which will be related to reactor power. Since there are k/p_v fissions per thermal neutron absorbed

$$\text{Power} = \frac{k}{p_v} \frac{10^{-10}}{3.1} \int_{\text{core}} \phi_{2c} \Sigma_{2c} dv \text{ watts} \quad (18)$$

Taking the vertical flux to be given by a chopped cosine distribution and the radial distribution by Eq. (16), there results

$$P = \phi_0 \frac{k \Sigma_{2c}}{p_v} \frac{10^{-10}}{3.1} 4H \sin\left(\frac{\pi}{2} \frac{h}{H}\right) \left[\frac{R}{2} J_1(\alpha R) + .00051768 \frac{R}{\beta} I_1(\beta R) \right] \quad (19)$$

whence $\phi_0 = 1.066 \times 10^{14}$.

(The average flux is $\bar{\phi} = 6.12 \times 10^{13}$.)

Then from equation (17) we find at $r = 167 - 1/3 \lambda_{tr} = 165$ cm

$$\frac{d\phi_{1r}}{dr} = -4.90 \times 10^{10}$$

$$\frac{d\phi_{2r}}{dr} = -8.64 \times 10^{11}$$

For the average leakage around the central plane of the core there results

$$J_f = .850 \sum_{1r} \tau \frac{d\phi_{1r}}{dr} = 4.65 \times 10^{10}$$

$$J_{th} = .850 \sum_{2r} \angle_r^2 \frac{d\phi_{2r}}{dr} = 6.40 \times 10^{11}$$

Peak values will be obtained by multiplying these by appropriate peak-to-average ratios, which must here consider the horizontal distribution only, because the values of $d\phi/dr$ given above are at the center plane of the reactor. For the thermal flux, assuming full cosine distributions, this is $\pi/2$. For the fast flux it is less than $b/a(\pi/2) = 5/3(\pi/2)$ but greater than $\pi/2$. The value of the fast peak to average ratio will be taken equal to $\sqrt{5}$. (See page 11)

$$J_f \text{ peak} = 1.04 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} \text{ peak} = 1.005 \times 10^{12} \text{ neutrons/cm}^2 - \text{sec.}$$

B. Reflector Leakage by Evaluating $d\phi/dr$ at 61 cm from the Core-Reflector Boundary.

The leakage from the reflector will now be obtained by an alternate method to that given in part A of this section. If it is assumed that the fast and thermal fluxes in directions normal to a face of the graphite stock are represented reasonably well by the curves of figure 14, a lower bound on the peak leakage from a face of the reflector can be obtained by using the value of $d\phi/dr$ at $r = 93.2 + 61 = 154.2$ cm. and omitting the peak-to-average and prism-to-cylinder corrections. Justification for this assumption is that the distance from the center of the core to a face is 3 feet = 91.4 cm, which is very close to the cylindrical core radius of 93.2 cm. A lower bound is obtained because the flux in this direction would extrapolate to zero at $91.4 + 61 + 2 = 154.2$ cm, so the actual gradient would be steeper than that obtained by the above method.

The gradients have the value

$$\frac{d\phi_{1r}}{dr} = -6.58 \times 10^{10}$$

$$\frac{d\phi_{2r}}{dr} = -9.29 \times 10^{11}$$

The peak leakage currents are then found to be

$$J_f = -\tau \sum_{1r} \frac{d\phi_{1r}}{dr} = 7.35 \times 10^{10} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = -L^2 \sum_{2r} \frac{d\phi_{2r}}{dr} = 8.10 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

C. Leakage from the Thermal Shield.

The leakage from the thermal shield will be estimated by multiplying the results obtained in parts A and B by $e^{-x/L}$ and $e^{-x/\sqrt{\tau}}$, respectively. For

iron at 400° C we take $\sum_a = 0.117 \text{ cm}^{-1}$, $\sum_s = 0.906 \text{ cm}^{-1}$, $\sum_{tr} = 0.887 \text{ cm}^{-1}$ and

$\sum_{tot} = 1.023 \text{ cm}^{-1}$. Then $\eta^2 = 3 \sum_{tr} \sum_a (1 - \frac{4}{5} \frac{\sum_a}{\sum_{tot}}) = 0.282 \text{ cm}^{-2}$, so

$L = 1/\eta = 1.89 \text{ cm}$. For the age we have $\tau = 60 \text{ cm}^2$ when averaged over the leakage spectrum for a 61 centimeter reflector and $\tau = 36 \text{ cm}^2$ when averaged over the leakage spectrum for an 34 cm reflector. When this modification is made to the results of part A there results for the peak leakage from a thermal shield of 20 centimeter thickness

$$J_f(61) = 1.04 \times 10^{11} \times e^{-2.582} = 7.89 \times 10^9 \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = 1.005 \times 10^{12} \times e^{-10.585} = 2.55 \times 10^7 \text{ neutrons/cm}^2 - \text{sec.}$$

Part B gives

$$J_f = 5.56 \times 10^9 \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = 2.06 \times 10^7 \text{ neutrons/cm}^2 - \text{sec.}$$

METHOD III. LEAKAGE PROBABILITY

The values of the leakage probabilities of the equivalent bare reactor are $1 - e^{-B^2 \tau}$ for fast neutrons and $L^2 B^2 / (1 + L^2 B^2)$ for thermal neutrons. Using

the core constants given in Table I we find B^2 for the equivalent bare reactor by solving for B^2 the equation

$$ke^{-B^2\tau} = 1 + L^2B^2$$

This gives $B^2 = 320.31 \times 10^{-6} \text{ cm}^{-2}$. Subtracting the vertical buckling = $46.76 \times 10^{-6} \text{ cm}^{-2}$, we have for the radial buckling of a bare cylinder $B_r^2 = 273.55 \times 10^{-6}$. The leakage probabilities are then found to be

$$P_f = 1 - e^{-B_r^2\tau} = 0.0877$$

$$P_{th} = L^2B_r^2/(1 + L^2B_r^2) = 0.0784 \quad (19)$$

The number of fast neutrons produced per second in a reactor operating at power p is if $p = 30$ megowatts

$$\begin{aligned} N_f &= p(\text{watts}) \times 3.1 \times 10^{10} \text{ (fissions/watt)} \times \nu \text{ (neutrons/fission)} \\ &= 3 \times 10^7 \times 3.1 \times 10^{10} \times 2.5 = 2.32 \times 10^{18} \text{ neutrons/sec.} \end{aligned}$$

The number of thermal neutrons produced per second is for a bare reactor:

$$N_{th} = N_f P_{th} = 2.32 \times 10^{18} \times 0.912 = 1.954 \times 10^{18} \text{ neutrons/sec.}$$

The average radial leakage is then found to be

$$J_f = \frac{N_f P_f}{S} = 4.84 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = \frac{N_{th} P_{th}}{S} = 3.65 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.} \quad (20)$$

where

$$S = 2\pi RH = 4.20 \times 10^5 \text{ cm}^2$$

$$R = \frac{2.405}{B_r} = 145.5 \text{ cm}$$

These values are to be multiplied by $(\pi/2)^2$ to obtain the peak radial leakage of the bare core, which are

$$J_f = 11.9 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = 9.00 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

We now obtain the peak leakage from the reflector by considering the effect of a 61 centimeter reflector on the peak leakage values of the core. A way of considering the effect of the reflector is to attenuate the thermal and fast leakage by $e^{-T/L}$ and $e^{-T/\sqrt{t}}$, respectively, where $T = 61$ cm is the reflector thickness. Taking the attenuation constant $\sqrt{t} = 13.7$ cm from reference 8, the result of this calculation gives for peak values of the leakage

$$J_f = 11.92 \times 10^{11} \times 0.0334 = 1.39 \times 10^{10} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} = 9.00 \times 10^{11} \times 0.385 = 3.46 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

The leakage from the thermal shield is obtained as in Method II and is for a 61 cm reflector

$$J_f \text{ peak} = 3.98 \times 10^{10} \times 0.0757 = 1.05 \times 10^9 \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} \text{ Peak} = 3.46 \times 10^{11} \times 2.54 \times 10^{-5} = 8.79 \times 10^6 \text{ neutrons/cm}^2 - \text{sec.}$$

For the 84 cm reflector the corresponding currents are:

$$J_f \text{ (ref)} = 2.60 \times 10^9 \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} \text{ (ref)} = 2.42 \times 10^{11} \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_f \text{ (sh)} = 9.48 \times 10^7 \text{ neutrons/cm}^2 - \text{sec.}$$

$$J_{th} \text{ (sh)} = 6.14 \times 10^6 \text{ neutrons/cm}^2 - \text{sec.}$$

The above values of J_{th} neglect the contribution due to fast neutrons which age to thermal in the shield and then leak out. This will not be very great because of the large thermal absorption cross section of iron.

INSTRUMENT RESPONSE

It is desired to evaluate the response of ionization chambers located outside the thermal shield. Since the chambers are boron coated, we assume that their response to neutrons of energy E is proportional to $1/E$. The response to thermal neutrons is $J_{th} \Sigma_a(th)$, where $\Sigma_a(th)$ is the macroscopic absorption cross section of the instrument for thermal neutrons. The response to fast neutrons is

$$\int_{E_{th}}^{\infty} J(E) \Sigma_a(E) dE$$

The ratio of the response to fast neutrons to the response to thermal neutrons is

$$R = \frac{\int_{E_{th}}^{\infty} J(E) \sum_a (th) \sqrt{E_{th}/E} dE}{J_{th} \sum_a (th)} \quad (21)$$

$$= \frac{\sqrt{E_{th}}}{J_{th}} q_0 \left\{ 1/q_0 \int_{E_{th}}^{\infty} \frac{J(E)}{\sqrt{E}} dE \right\}$$

The quantity in the curly bracket was obtained by numerical integration of the curves of Fig. 12 and for a 61 centimeter reflector is 1.38×10^{-4} . A factor of 8 times this to give an upper limit is too high because of the ineffectiveness of the high energy leakage in instrument response due to the $1/\sqrt{E}$ dependence of $\sum_a$. A factor of 4 is probably about right, and will be used in the following.

With $E_{th} = 0.075$ ev, $q_0 = 3 \times 10^{11}$ and $J_{th} = 1.5 \times 10^7$, this gives $R = 3.0$.

The Westinghouse compensated ionization chambers should be in a thermal flux of about 2×10^9 when the reactor is at full power. The thermal flux outside the thermal shield is too low by about a factor of 200, and the fast leakage contributes a response only about 3 times greater than the thermal leakage, so the combined responses are low by about a factor of 50. In order to increase the thermal leakage fiftyfold, the thermal shield would have to be reduced to a thickness x given by

$$e^{-\frac{x}{1.89}} = 2.54 \times 10^{-5} \times 50 = 0.00127$$

$$x = 12.6 \text{ cm} = 4.96 \text{ inches.}$$

This could be accomplished by removing iron from the thermal shield in the vicinity of the ion chambers, and in order maintain the γ -ray shielding, replacing it by some material having low thermal absorption cross section such as zirconium. Neutron absorber can then be placed behind the counter to prevent an excess of thermal neutrons from diffusing into the biological shield.

The Westinghouse fission chamber gives a response of 0.7 counts/sec in a flux of 1 neutron/cm² - sec. The statistical probable error in N counts is

$$(0.6745/\sqrt{N}) \times 10^{-2} \text{ percent}$$

Thus 1,145 counts are required in order to reduce the statistical error to 2 percent. If 10 seconds are allowed to obtain this many counts, the flux required is $\phi = 164$ neutrons/cm² -sec. If a statistical error of 10 percent is allowed 45.8 counts are required and a flux of 65 neutrons/cm² - sec. required if 1 second is allowed to obtain these counts. So a flux of about 100 is a practical minimum to give reliable readings on a counting rate meter although lower fluxes will give a signal which can by use of a stop watch and a count register be related to flux level. Since the flux outside the thermal shield at full power is about 10^7 and the startup instrument should be able to give reliable results at least 10 decades below full power, (12 decades are required if no extraneous source is used for startup) a factor of 10^5 must be gained. This can only be done by inserting the startup instrument into the reflector.

SUMMARY OF RESULTS

The first three columns of Table II summarize the results of the neutron leakage calculations by the three methods described in this paper. In the fourth column are listed peak values of the several leakage currents which the author considers to be the best values to adopt for the SGR-P4 reactor. They were obtained by inspection of the calculated values in the preceeding columns and giving greater weight to the value which the author considered the most reliable. It is believed that the adopted values are reliable to a factor of two.

The following points are noteworthy:

1. Methods II and III give reasonably good agreement (factor of 3.0) with regard to the thermal leakage but rather poor agreement (factor of 7.5) with regard to the fast leakage.
2. The attenuation by the thermal shield is the same in Methods II and III by definition, and is 2.54×10^{-5} for thermal neutrons, 7.57×10^{-2} for fast neutrons with a 61 cm reflector, and 3.65×10^{-2} for fast neutrons with an 84 cm reflector.
3. The attenuation by the thermal shield from Method IA is 2.75×10^{-2} with a 61 cm reflector and 1.18×10^{-2} with an 84 inch reflector.

It is believed that the shield attenuation factors in Method I are more reliable than the others, because

some of the detail of the aging process and energy distribution of the fast neutrons was taken into account. The good agreement of Methods IA and IB lends support to this belief.

4. The attenuation of fast neutrons by an additional 23 cm of graphite is 0.072 by Method IA, 0.13 by Method IB, and 0.187 by Method III which is based on the empirical formula of reference 8.
5. It is believed that Method III leads to too small a value for the leakage from the reflector because the neutron density in the core of a reflected reactor must, because of its smaller volume, be greater than that in an unreflected reactor operating at the same power level. Moreover the first flight of fast neutrons is ignored in applying the $e = x/\sqrt{\tau}$ formula and this would lead to a higher fast leakage.

DISCUSSION

One of the objectives of this memorandum is to point out some of the major difficulties in an accurate calculation of neutron leakage from a reactor. Methods II and III are of questionable accuracy for the fast leakage, but are simple and easy to apply. However, they do not give information as to the energy distribution of the fast leakage neutrons. In fact, it is not possible to treat the fast leakage through the thermal shield adequately without referring to the results of Method I, because the nuclear properties of the shield for fast neutrons must be averaged over the energy spectrum of the fast neutrons. While Method I does give the energy distribution, it is lengthy and tedious to apply in practice even with the aid of the approximations which have been adopted, which in turn limit its accuracy. The shape of the distribution is probably reliable, but the accuracy of the integrated value of the leakage is debatable. Improvement in accuracy could be obtained at the expense of greatly increased computational effort by solving the age equation in cylindrical geometry using a more accurate source function (e.g., a chopped Bessel function in the core and the more accurate first collision function in the reflector given by Eq. (2)) and averaging the solution over the fission spectrum. However, even this geometry is only an approximation to the problem being considered. A still better calculation would involve solving the age equation for a rectangular prism having suitably attenuated line sources located at the cell centers in the core.

The fast leakage through the shield could be more accurately obtained by solving the age equation with the source given by Eq. (2) for two finite slabs whose dimensions and ages correspond to the stack and shield respectively. The thermal leakage through the shield should be augmented by the contribution of those fast neutrons which become thermal in the shield and then leak out. This could be obtained by solving the thermal diffusion equation in the finite shield with the source term taken to be $q(x, \tau_{th})$.

Numerical methods would have to be used with each step, and it would also be necessary to divide the energy intervals more finely in order to accurately take into account the variation of cross sections with energy. In view of the additional labor involved it is questionable whether a more sophisticated calculation is justifiable. It is probably more desirable to attempt to measure the leakage experimentally, a suggested procedure for which is being described in reference 7. Even more desirable would be a criticality experiment wherein the actual geometry of the reactor is closely approximated. Flux and cadmium ratio measurements taken at many points throughout the structure would then yield the desired information.

A fourth method of obtaining information about the leakage currents should also be mentioned here. This involves the solution of the multigroup diffusion equations in three or more regions. The two-group, three-region case is a practical one which might give somewhat more accurate results than the ones obtained herein. However, its accuracy is also limited by geometrical considerations as well as the determination of the proper average values for the reactor constants in the shield. These must be averaged over the neutron energy spectrum in the shield which we do not know accurately, and to make matters worse, depends upon position in the shield because of the strong $1/v$ absorption of the shield which greatly perturbs the distribution near thermal energies where most of the neutrons lie. (The curves of Fig. 4 apply to an infinite non-absorbing graphite medium.) So good values for the reactor constants in the shield are difficult to calculate. If a two group three region solution were carried out in cylindrical geometry for the 165 megawatt SGR-P12 reactor, which being octagonal in shape is reasonably well represented by a cylinder, leakage from the shield correct to a factor of 2 might be obtained.

TABLE I.

DATA FOR A TYPICAL 30 MEGAWATT SGR - P4 REACTOR

(Taken from NAA-SR-Memo-563)

I. General Description:

Graphite stack dimensions, 10 ft. x 10 ft. x 16 ft.

Height of core = $h = 12 \text{ ft.} = 365.7 \text{ cm.}$

Cell size = 12-9/16 inches square

Cell area = 1018 cm^2

Core size at criticality with the data given below = 26.8 cells

Reflector thickness = 61 cm. = 2 ft.

Average graphite temperature = 400°C.

Average fuel temperature = 400°C.

Average coolant temperature = 350°C.

II. Equivalent Cylinder Dimensions:

$R = \text{core radius at criticality} = 93.2 \text{ cm.}$

$A = R + T = \text{stack radius} = 165 \text{ cm.}$

$T = \text{reflector thickness} = 71.8 \text{ cm.}$

Radius of equivalent bare reactor of height H is $R_b = 145.5 \text{ cm.}$

III. Core Constants:

$k = 1.22346$

$P_c = 0.92288$

$L_c^2 = 310.85 \text{ cm}^2$

$\sum 2_c = 0.0028732 \text{ cm}^{-1}$

$\tau_c = 333.32 \text{ cm}^2$

$\sum 1_c = 0.0033494 \text{ cm}^{-1}$

$B^2 = 320.31 \times 10^{-6} \text{ cm}^{-2}$

Vertical buckling = $B_z^2 = 46.76 \times 10^{-6} \text{ cm}^{-2}$

Unreflected height = $H = 459.8 \text{ cm.}$

IV. Reflector Constants:

$P_r = 1.0$

$L_r^2 = 4092.8 \text{ cm}^2$

$\lambda_r^2 = 1/L = 0.01563 \text{ cm}^{-1}$

$\sum 2_r = 0.00021286 \text{ cm}^{-1}$

TABLE I. (continued)

$$\begin{aligned}\tau_n &= 322.12 \text{ cm}^2 \\ \mu_n &= 1/\tau_n = 0.05570 \text{ cm}^{-1} \\ \Sigma_{1n} &= 0.0034674 \text{ cm}^{-1}\end{aligned}$$

V. Miscellaneous Constants:

$$V(T = 400^\circ \text{ C}) = 22.2 \times 10^5 \quad \sqrt{T/293} = 3.33 \times 10^5$$

$$\tau_0 = \frac{1}{V \Sigma_a} = 1.045 \times 10^{-3} \text{ sec.}$$

$$\ell = \frac{\tau_0}{1 + L^2 B^2} = 0.954 \times 10^{-3} \text{ sec.}$$

$$\text{Slowing down time in graphite} = 1.5 \times 10^{-4} \text{ sec.}$$

$$\text{Generation time} = 1.10 \times 10^{-3} \text{ sec.}$$

$$3.1 \times 10^{10} \text{ fission per watt.}$$

$$\nu = 2.5 \pm .1 \text{ neutrons per fission.}$$

TABLE II

COMPARISON OF PEAK VALUES OF NEUTRON LEAKAGE (NEUTRONS/CM²-SEC) AS CALCULATED
BY DIFFERENT METHODS FOR SGR-P4

<u>LEAKAGE FROM THE REFLECTOR</u>				
<u>Neutrons/cm²-sec</u>	<u>Method I</u>	<u>Method II</u>	<u>Method III</u>	<u>Adopted Values</u>
J_f (61 cm refl.)	A. 6.50×10^{10}	A. 10.4×10^{10}	1.39×10^{10}	6×10^{10}
	B. 5.7×10^{10}	B. 7.35×10^{10}		
J_f (84 cm refl.)	A. 4.68×10^9	---	2.60×10^9	7×10^9
	B. 7.7×10^9			
<hr/>				
J_{th} (61 cm refl.)	---	A. 10.05×10^{11}	3.46×10^{11}	5×10^{11}
		B. 8.10×10^{11}		
J_{th} (84 cm refl.)	---	---	2.42×10^{11}	---
<hr/>				
<u>LEAKAGE FROM THE THERMAL SHIELD</u>				
J_f (20 cm shield) (61 cm refl.)	1.79×10^9 (upper lim.)	A. 7.89×10^9 B. 5.56×10^9	1.05×10^9	2×10^9
J_f (20 cm shield) (84 cm refl.)	7.68×10^7 (upper lim.)	---	9.48×10^7	1×10^8
J_{th} (20 cm shield) (61 cm refl.)	---	A. 2.55×10^7 B. 2.06×10^7	0.879×10^7	1.5×10^7
J_{th} (20 cm shield) (84 cm refl.)	---	---	6.14×10^6	---
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FIGURE CAPTIONS

- I. Curves Representing the Virgin Fast Neutron Distribution in the Reflector.
- II. Plot of $\frac{1}{q_0} \frac{dq}{dx}$ vs τ , as obtained from Equation 3.
- III. $\tau(E)$ vs E for fission neutrons in graphite of density 1.60 gms/cm^3 .
- IV. Normalized Leakage Current from Graphite Reflector as a Function of Energy for 61 cm. and 84 cm. Reflectors.
- IVa. Relative Fast Leakage per Decade from Reflector.
- V. Integrated Fast Leakage per Fast Neutron Produced in the Core vs Reflector Thickness.
- VI. Assumed Distribution of Neutron Leakage over a Face of the Graphite Stack.
- VII. Age of Neutrons in Iron.
- VIII. Age of Neutrons in Iron. (Cross plot of Figure VII)
- IX. Capture Escape Probability for Neutrons Which are Slowing Down in Iron.
- X. Plot of $\frac{dq}{dx}$ from equation (13) for unit source at 20 cm in a semi-infinite region.
- XI. Plot of $\frac{dq}{dx}$ from equation (14) for unit source at 20 cm. in a semi-infinite region.
- XII. Fast Neutron Current vs Energy at $x = 20 \text{ cm.}$ in a semi-infinite iron region, with boundary condition on $q(x, \tau)$ as $q(0, \tau) = 0$
- XIII. Comparison of Fast Neutron Current vs energy at $x = 20 \text{ cm.}$ in a semi-infinite iron region for the two boundary conditions on q .
- XIV. Fast and Thermal Flux Plot (Taken from Memo-733.).

Fig. 1

Fig. 4

Fig. 4A

INTEGRATED FAST LEAKAGE/PER NEUTRON PRODUCED →

0.15
0.10
0.05
0

60

70

80

90

REFLECTOR THICKNESS (CM) →

Fig. 5

Fig. 6

36

37

38

40

41

Fig. 9

E ev

41

Fig. 10

Fig. 11