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AN EVALUATION OF HIGH FLUX
NUCLEAR SYSTEMS FOR RESEARCH
WITHIN THE LIMITATIONS OF THE
ATOMS-FOR-PEACE PROGRAM

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August 1955

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OAK RIDGE SCHOOL OF REACTOR TECHNOLOGY

F. C. VonderLage, Director

"AN EVALUATION OF HIGH FLUX NUCLEAR SYSTEMS FOR RESEARCH
WITHIN THE LIMITATIONS OF THE ATOMS-FOR-PEACE PROGRAM"

Prepared by:

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August, 1955

PREFACE

In September, 1954, a group of men representing various scientific and engineering fields embarked on the twelve months of study which culminated in this report. For nine of these months, formal classroom and student laboratory work occupied their time. At the end of that period, these six students were presented with a problem in reactor design.

This is a summary report of the study, the research, the problems and the solutions which developed during the final ten-weeks period of the school term. It must be realized that, in so short a time, a study of this scope can not be guaranteed complete or free of error. This "thesis" is not offered as a polished engineering report but rather as a record of the work done by the group under the leadership of the group leader. It is reproduced for use by those persons competent to assess the uncertainties inherent in the results obtained in terms of the preciseness of the technical data and analytical methods employed in the study. In the opinion of the students and faculty of ORSORT, the problem has served the pedagogical purpose for which it was intended.

As a matter of historical fact and pride we point out that similar investigations by student groups of previous ORSORT classes have led to sufficiently encouraging results to warrant more exhaustive studies; in at least one instance, a reactor first investigated by a student group is soon to become a physical reality. There is also recorded an instance in which calculations contained in a similar report were uncritically abstracted and applied to a study for which they were never intended. It is to avoid the recurrence of the latter experience that we have taken some pains to acquaint the reader with the character of this report.

The faculty wishes to join the authors in an expression of appreciation for the assistance which various members of the Oak Ridge National Laboratory have so generously contributed. In particular, the guidance of the group consultant, James A. Lane, is gratefully acknowledged.

F. C. VonderLage

for

The Faculty of ORSORT

ACKNOWLEDGMENT

The members of this group wish to express their sincere gratitude to their advisors, J. A. Lane and R. B. Briggs, without whose patient and wise counsel this report could not have been written.

Our thanks go in equal measure to T. Fowler, who gave us much invaluable help in applying his ORACLE code to our calculations. We appreciate particularly his willingness to come to the ORACLE after hours to help us operate the calculator.

To J. H. Erwin we are indebted for much vital information in connection with the metallurgical aspects of this work.

We wish to thank Harriet Madden for her unfailingly cheerful help and advice on innumerable occasions.

We are grateful also to M. Tobias, J. P. Sanders, W. E. Kinney, and the operators of the ORACLE for their helpfulness to us.

ABSTRACT

This report presents a study of possible reactor systems within the limitations of the Atoms-for-Peace Program, i.e. using no more than 6 kilograms of U-235 in enrichment of 20%. The basic investigation is by means of a two group - two region ORACLE calculation, but burnup, poison effects, and operating times are also taken into account, and engineering and cost estimates for the most interesting cases are included.

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1. INTRODUCTION

1.A. Postulates, Goals, and Limitations

Under the Atoms-for-Peace program a quantity of 20% enriched uranium has been set aside for the purpose of building nuclear research reactors in foreign countries. A limit of six kilograms of U-235 has been established for any one country. The purpose of this study is to determine the materials, dimensions, and required fuel masses for feasible reactors with an average thermal neutron flux of 10^{14} neutrons per cm^2 per second to be built within these limitations.

The scope of this study was confined to an investigation of feasible core materials, moderator materials and reflector materials, and their most useful combinations, and some of the initial engineering considerations for plate type fuel elements. This limitation was imposed since a detailed design of a specific reactor would depend on the desires and special needs of the user. A single optimum configuration is not specified, because the selection of materials, core size, and fuel loading is dependent upon the operating cycle, material costs, and the purposes of the reactor. This report provides information which may be used by the designer as a means of selecting the system most suited to his needs as the basis for a detailed design.

Due to the large number of permutations possible among the parameters, it was necessary to make a study of previous reactor designs of this type, and to make a series of exploratory calculations to determine the useful range or parameters.

For engineering reasons the core was considered to contain aluminum plate fuel elements similar to those of the "Swimming Pool Reactor" and the

Materials Testing Reactor. The coolant-moderator materials considered were:

1. Light water (H_2O)
2. A mixture of 50% light water and 50% heavy water (HDO)
3. A mixture of 10% light water and 90% heavy water (10-90)
4. Heavy water (D_2O)

The reflector materials selected were:

1. Light water
2. A mixture of 50% light water and 50% heavy water
3. A mixture of 10% light water and 90% heavy water
4. Beryllium
5. Graphite

With these materials a total of twenty-four core-reflector combinations is possible. Other variables considered were the dimensions of both the reflector and the core and the metal-to-water ratio. If five values of each of these three variables were considered for each of the twenty-four reactors, a total of 3000 combinations would need to be calculated. Fortunately, the change in critical mass with variation of the metal-to-water ratio is nearly linear and sufficiently small that the behavior can be shown by calculations at only two or three values of this ratio. To reduce the calculations further only two values of reflector thickness and three or four values of core radius were used. The larger of the reflector thicknesses was chosen to allow observation of the maximum fuel savings realized with an essentially "infinite" reflector.

It was also observed that some of the possible combinations of reflectors and cores would not be practical from economic or engineering considerations. Thus, the original list was reduced to approximately 200 reactor combinations.

In this study no attempt was made to account for the differences caused by the non-sphericity and non-homogeneity of the actual reactors. However, an attempt was made to have all assumptions conservative so that the required fuel loading would be overestimated in all cases.

The engineering calculations on element temperatures and cooling were limited to the core itself. They included computations on the maximum expected metal surface temperatures of the element plates with different core parameters as well as pressure drops expected across the core for different element designs and water velocities. The results are all presented in graphical form.

The economic evaluation presents in tabular form the value of the uranium beryllium, graphite, and D₂O in the systems under consideration, so that a momentary comparison can be made by the reader, but these costs are limited to the reactor and do not reflect the total cost of the reactor system.

1.B. Summary of Results

From the point of view of expense and simplicity it would be most desirable to build the reactor system with a water core and a water reflector permitting use of a pool modified to assure adequate flow through the core. Unfortunately, it is impossible to operate such a system at an average thermal flux of 10^{14} neutrons/cm²/sec with the limits imposed on construction. Under the most favorable conditions, the required mass is well above the six kilogram limit and the fuel capacity of the core.

Even so, there is considerable interest in water-moderated cores with better reflectors. However, an H₂O reflector does not bring the mass down to the required level. A 10-90 reflector gives an operating mass just under six kilograms in the most favorable case, i.e., a thick reflector and

low metal-to-water ratio. But, under these circumstances, the fuel capacity is so small that no reactor is physically possible. With 75 cm D_2O reflectors the reactors are feasible only for high metal-to-water ratios. With a core radius of 30 cm and M/W of 1.3 systems of 4.5 kg U-235 are possible with up to eighteen plates per element. When the reflector thickness is increased to 120 cm, there is a large range of possibilities. Reactors with design loadings of only a little more than 3 kg are possible in every case at high metal-to-water ratios. Even at the M/W of 0.7, there are possible systems, but these have large cores with somewhat higher masses.

Considerable savings in required fuel can be realized by using a graphite or beryllium reflector. Beryllium reflectors 15 cm thick give results comparable to thick (120 cm) D_2O reflectors but beryllium reflectors 40 cm thick yield requires masses as low as 2.5 to 3.0 kg. Graphite, while not as effective as beryllium, leads to systems with required masses between 3.0 and 4.0 kg. From all considerations, it would appear that such systems provide the cheapest and most satisfactory approach to the problem. It is also of benefit that the economically feasible thick reflectors (50 to 100 cm) give a much larger volume in which the flux is fairly high for experimental purposes. It should be emphasized, however, that the calculations were based on pure graphite. Water coolant in the graphite reflector may greatly lower its effectiveness.

When the core moderator is enriched to 50% D_2O , the required fuel concentration is reduced sufficiently that even at low metal-to-water ratios a system with only a 15 cm beryllium reflector becomes physically possible with operational masses in the order of three kilograms. The use of H $2O$ for the moderator has the additional advantage that the reflector coolant can

also be H₂O resulting in less poisoning of the reflector by its coolant. If it is desired to save fuel at the expense of greatly increasing the amount of beryllium, a system can be built using a H₂O core coolant, a 40 cm thick beryllium reflector and a M/W of 0.7. The required loading for this system is only 2.5 kg. Graphite reflected, H₂O cooled systems with critical masses of three or three-and-a-half kilograms are entirely possible at a M/W of 0.7. It does not appear very feasible to construct a system using H₂O in both core and reflector even though such a system would be quite simple mechanically. The operational mass of such a system is just below the limit (about 5.8 kg) and can be accommodated by pushing the element fuel capacity to the limit and using an eight-plate fuel element.

Results were also obtained for cores in which the moderator is enriched to 90% D₂O. It appears that no large fuel saving can be realized by going to this type core. The thermal utilization is so much higher for these systems that although the clean-cold masses may be lowered the operational loadings will actually be more than for the corresponding H₂O system. For a 35 cm core radius, a reactor with a 10-90 reflector having 5.5 kg operational loading is entirely feasible and there is a wide choice of number of plates per element.

The systems using D₂O cores require large quantities of D₂O. The cores which require the smallest masses determined have radii in the vicinity of 40 cm making the core and reflector volumes very large. Indications are that by using thick graphite or beryllium reflectors, operational loadings of three kilograms or less can be achieved. A system with a 35 cm core radius, a 100 cm graphite reflector, and a 0.5 M/W has an operational loading of 3.4 kg. The reactors considered in this report all have a total heat output of 10

megawatts or more, with power densities ranging from about 25 to 650 watts per cubic centimeter. To properly cool the type of elements chosen for these systems under these conditions it will, in practically all cases, be necessary to use a forced circulation system in order to avoid boiling within the core. Generally speaking, this condition of keeping the metal surface temperature below the boiling point is the criterion for the minimum cooling in these systems, rather than any other temperature considerations which might apply elsewhere. From the graphical presentations in section 4, it is possible to determine the pressure drop across the core and the amount of water required to cool the reactors considered in this report.

An economic evaluation has also been made of representative systems. In these systems the value of uranium, beryllium, graphite, and D_2O has been tabulated for core, reflector and external coolant. This evaluation shows a considerable economic advantage of the graphite reflectors over any other; with every coolant, the graphite reflector provides by far the cheapest system. In systems using other than H_2O for coolant, the external coolant values are often as much as the total core value, clearly showing the economic advantage of H_2O cooling.

2. FUEL ACCOMMODATION CALCULATIONS AND RESULTS

The type of fuel elements has been fixed to be the same as that of the elements in the Materials Testing Reactor and the Bulk Shielding (Swimming Pool) Reactor, i.e., are to be aluminum clad plates of aluminum-uranium alloy assembled with side plates to make an element 3 x 3 inches. The reasons for this choice are several. For one, the technology of making these elements is well established so much of the cost and time of development can be avoided.

A second reason is that the chemical technology for processing these elements is well in hand, thus again saving cost and time in development. The third, and possible most cogent reason, arises out of the fact that most of the reactors will be built in countries having little or no chemical processing facilities. Substantial savings can, therefore, be realized by making elements which can be processed in the United States in existing handling, dissolver, and processing equipment.

For simplicity of calculation, this study is based upon a modified version of the MTR elements using flat plates and nominal dimensions. The results of the calculations of fuel accommodation, pressure drops, and heat transfer should be good approximations to values for the actual elements.

Two types of fuel elements were considered in this study. One is a conservative design which is well within present technological limits but has a relatively low fuel capacity. The second design is more optimistic in fuel accommodability, shaving clad and side-plate thicknesses somewhat. Both have outside dimensions of 3 x 3 inches and an active region 24 inches long. The former element will be referred to as the "low capacity element"; the latter, the "high capacity element".

The first, low-capacity, element has 0.150 inch thick side plates and 0.020 inch aluminum cladding on each side of the fuel bearing weight uranium. Elements of this type were considered with eight to eighteen plates. Fig. 2.A.1 shows a cross section of such an element having ten plates, and a metal-to-water ratio of 1.0.

The high capacity element was considered when it became apparent that many reactors which have masses well within the limits set by the program were nevertheless unaccommodable in the low capacity elements. The high

FIGURE 2.A: CROSS SECTION OF A TYPICAL FUEL ELEMENT

capacity element has side plates 0.125" thick and 0.015" aluminum cladding. The weight per cent of uranium in the fuel-bearing alloy was increased to 27.5% by weight. Here, too, elements with eight to eighteen plates were considered.

The mass of U-235 that can be accommodated in a fuel element is calculated as follows:

$$\begin{aligned}\text{Metal-to-water ratio} &= M/W = (\text{volume of metal}/\text{volume of water}) \\ &= \frac{(\text{volume of fuel plates}) + (\text{volume of side plates})}{\text{volume of water}}\end{aligned}$$

Volume of fuel plates = $(n)(x)(a)(L)$ cubic inches

Volume of side plates = $6(b)(L)$ cubic inches

Volume of water = $(n)(y)(a)(L)$ cubic inches

where: n = No. of fuel plates in the element

x = fuel plate thickness (in inches)

a = width of fuel plates (between side plates; in inches)

b = thickness of side plates (in inches)

L = active length of fuel plates

y = water channel thickness (in inches)

$$\text{also: } n(x + y) = 3 \quad (1)$$

$$\text{then } M/W = \frac{(n)(x)(a) + 6(b)}{(n)(y)(a)} \quad (2)$$

Solving these simultaneously:

$$x = \frac{3(M/W)(a) - 6(b)}{((M/W) + 1)(a)(n)}$$

$$y = \frac{3(a) + 6(b)}{((M/W) + 1)(a)(n)}$$

The volume of the active portion of the element, V_m (in cubic inches), is given by

$$V_m = (n) [x - 2(c)] (d) (L) \quad (3)$$

where: c = clad thickness (in inches)

d = active width of plates (in inches)

The mass of U-235 in the element is given by:

$$M_{25} = v_{25} V_m P_U \quad (4)$$

where: v_{25} = volume fraction of U-235

P_U = density of uranium

$$\text{also: } v_{25} = \frac{0.2 (m_U/P_U)}{(m_U/P_U) + (m_{Al}/P_{Al})} \quad (5)$$

where: m_U = mass fraction of uranium in the alloy

m_{Al} = mass fraction of aluminum in the alloy

P_U = density of metallic uranium

P_{Al} = density of metallic aluminum

Consider, as an example, a ten-plate element of the high capacity type whose metal-to-water ratio is 0.7. In this element:

$$a = 2.75 \text{ "}$$

$$b = 0.125 \text{ "}$$

$$L = 21.0 \text{ "}$$

$$n = 10$$

$$c = 0.015 \text{ "}$$

$$d = 2.55 \text{ "}$$

$$\text{then } x = \frac{3(.7)(2.75) - 6(0.125)}{1.7 (2.75) (10)} = 0.108 \text{ "}$$

$$y = \frac{3(2.75) + 6(0.125)}{1.7 (2.75) (10)} = 0.192 \text{ "}$$

Then from equation (3) we have:

$$V_m = (10)(0.108 - 2(0.015)) (2.55) (24) \\ = 47.7 \text{ cubic inches}$$

This element is to have an alloy with 27.5% by weight uranium so, from equation (4):

$$v_{25} = \frac{(0.2) (0.275/18.7)}{(0.275/18.7) + (.725/2.7)} = 0.0104$$

and then from equation (5):

$$M_{25} = (0.0104)(57.7) \text{ in}^3 (18.7) \text{ gm/cm}^3 (2.54) \text{ cm}^3/\text{in}^3 \\ = 152 \text{ grams of U-235}$$

The results of these calculations are embodied in Fig. 2.A.2 and Fig. 2.A.3, which show the maximum amount of U-235 which can be put in each of the two types of fuel elements with varying numbers of plates per element.

The mass of U-235 which can be accommodated was taken to be the volume of the spherical core divided by the volume of the active region of an element times the maximum mass per element. Figure 2.A.4 shows the number of fuel elements of the above specified dimensions (3"x3"x24") which occupy volumes equal to spheres of various radii. Figure 2.A.5 to Figure 2.A.9 give the masses of U-235 which can be accommodated in volumes equal to the spheres of various radii using fuel elements of both specified types. As the mass needed to make the spherical reactor critical has been multiplied by a factor 1.15 to allow for the fact that the actual reactor is not spherical, but the extra volume so gained has not been allowed for in the maximum accommodable mass calculations, the assessment of the feasibility of a given configuration is conservative. That is, to say, a given reactor can contain a little more fuel than was allowed for, so that a margin of safety in estimates of the limits of possibility exists.

FIGURE 2.A.2 U²³⁵ MASS PER ELEMENT

FIGURE 2.A.3: U^{235} MASS PER ELEMENT

FIGURE 2.A.4: NUMBER OF FUEL ELEMENTS PER REACTOR

FIGURE 2.A.5 U^{235} CAPACITY OF REACTOR

FIGURE 2.A.6

FIGURE 2.A.7 U^{235} CAPACITY OF REACTOR

FIGURE 2.A.8 U^{235} CAPACITY OF REACTOR

FIGURE 2.A.9 U^{235} CAPACITY OF REACTOR

3. NUCLEAR CALCULATIONS

3.A. The Oracle Calculations

The basic calculations on which the results are based were made on the ORACLE and were two-group, two-region calculations. (2G2R calculations) These were carried out for spherical systems, and adjusted for non-spherical configurations.

The input data to the code are the parameters of the core and of the moderator, the following being used:

Fermi age

Fast diffusion coefficient

Thermal diffusion coefficient

Resonance escape probability

Total thermal macroscopic absorption cross section (in the reflector only)

Extrapolation distance at the outer reflector boundary

η , the number of fission neutrons produced per absorption in fuel.

The resonance escape probability in the reflector is set at 1.0 for all reactors considered in this report. The input also includes the dimensions of the core and reflector. Details on the choice of the constants used are to be found in Appendix I, Section 5.A.

The ORACLE takes this input data and, after converting it into machine notation, computes the quantities entering the determinant given in "Glasstone and Edlund" (ref. 3-1). It then calculates the value of this determinant, adjusts the input and iterates the calculation until the value of the determinant is less than some pre-assigned number, which is generally taken to be 5×10^{-4} to guarantee five figure accuracy in the results. The machine then punches out the results. These results include, at the option

of the individual, both fast and thermal fluxes at four points in the core, at the interface between the core and the reflector, and at four points in the reflector. In any case, one gets the thermal absorption cross section of the fuel, the total thermal absorption cross section in the core, fast and thermal leakage, and a number of other quantities of less immediate interest to the purposes of this report.

The mass needed is then computed from the cross section of the fuel, as detailed elsewhere in this report.

Certain difficulties can and do occur in the use of this code. Where the reactor computed has a core size so small as not to be at least several diffusion lengths, and in cases where the loading comes out to be very small, the ORACLE may not calculate. This takes place if a negative square root occurs in the calculation. This has occurred several times in the process of this investigation. Another type of failure in this calculation is possible. The machine may continue to iterate and never bring the value of the determinant to a low enough value.

Both these effects are most likely in reactors having large diffusion lengths, and small cores. Due to this, it has not always been possible to extend the calculation to radii as small as might otherwise appear desirable.

It should be noted, however, that the cases that will not run because the core is too small are not very meaningful anyway, since the method of calculation assumes diffusion theory to hold, and diffusion theory does not hold in cores of the order of magnitude of only a few diffusion lengths. For such cases more powerful techniques, such as the Feynman-Welton Method are needed.

This study was restricted to the use of the 2G2R code, without

attempt to extend the curves to areas where the method breaks down. This does not, in most cases, restrict the usefulness of the curves, for the following reason: the amount of fuel needed for operation at the design conditions is strongly dependent on the thermal utilization, f , and rises sharply as f does. The smaller cores have higher leakage, therefore higher f factors, and therefore higher masses. In fact, the masses often rise quite sharply with decreasing radii.

When this is combined with the fact that the amount of fuel that can be accommodated in a given core drops with the core radius, it will be perceived that the smaller cases are not often feasible from the fuel accommodation consideration. An examination of the graphs of loading and accommodation, figures 3.D.2 to 3.D.63 will serve to clarify this point.

3.B. Reduction of Data and Required-Loading Calculations

One of the results from the 2G2R code is the macroscopic thermal absorption cross section of the fuel in the core. This data is first converted to the useful form of the (critical mass) of U^{235} in kilograms in the following fashion:

We observe that $N(25) \sigma_a(25) + N(28) \sigma_a(28) = \Sigma_a^F$

or $\Sigma_a^F = N(25) \left[\sigma_a(25) + \frac{N(28)}{N(25)} \sigma_a(28) \right]$

Now $\frac{N(28)}{N(25)} = 4 \times \frac{238}{235} = 3.95$

so $N(25) = \frac{\Sigma_a^F}{\sigma_a(25) + 3.95 \sigma_a(28)} = \frac{\Sigma_a^F \times 10^{24}}{687 + 3.95 \times 2.74}$

or $N(25) = \frac{10^{24}}{698} \Sigma_a^F$

Therefore $W(25) = Vol (cm^3) \times N(25) \frac{235}{6 \times 10^{24}} = \frac{V \times \Sigma_a^F \times 235 \times 10^{24}}{698 \times .6 \times 10^{24}}$

$= .56 \times Vol \times \Sigma_a^F (cm^{-1})$

This equation, yields the cold clean critical loading of the spherical system from the 2G2R calculation. The loading of the actual, non-spherical cold, clean system is then computed by multiplying this spherical critical loading by a factor of 1.15. This factor was determined with the following considerations in mind. A simple calculation shows that the ratio of the mass of a bare cubical reactor to that of a bare spherical reactor is 1.22, if both are made of homogeneous material of the same density and composition. The ratio of volumes is, therefore, the same. The same ratio, when the right cylinder is compared with the sphere, is 1.145. The reactors considered here will be made to have a shape as close to that of a right cylinder as the shape of the individual fuel elements will permit. From this consideration alone one would take the factor to be close to the lower value of 1.145. However, all the reactors we are considering are reflected, and one expects that this fact would reduce the difference in the masses. Nonetheless, the factor has been chosen as 1.15 in order to be sure that the estimate of required mass is conservative, that is to say, on the high side. With similar intent, this additional 15% in volume has not been added to the estimate of the fuel which can be accommodated in a given core in order to make this estimate on the low side, that is to say, again conservative. The resulting mass figure is not yet that of an operative reactor since insertion of any poison, burnup of fuel, or Xenon buildup during running would shut the reactor down immediately. To allow for those a factor "A" has been computed by which the cold clean non-spherical critical loading must be multiplied to obtain the required loading for the operating reactor. Of course, the excess reactivity thus produced must be poisoned out initially by means of rods and, perhaps, burnable poison. To obtain this "A" factor, the following effects

must be taken into consideration: fuel burnup, xenon override, experiments, and contingencies. Each of these will be discussed in turn.

Burnup: If the flux in a reactor is specified, the burnup rate depends on the amount of fuel present. Here, the fuel is defined as Uranium 235. Therefore, on a unit volume basis, $\frac{dN}{dt} = -\Sigma_z(25) \phi_0 = N \sigma_a \phi_0$

where $\sigma_a = 678$ barns, $\phi_0 = 10^{14}$, $t =$ seconds
 $\frac{N}{N_0} = e^{-\sigma_a \phi_0 t} = e^{-2.4 \times 10^{-3} T}$, $T =$ hours

Thus, when $T = 440$ hours (18 days) $\frac{N}{N_0} = .90$

$T = 290$ hours (12 days) $\frac{N}{N_0} = .933$

This is the fraction of fuel which is left in the reactor after the times specified.

For ready combination with the other factors it is convenient to convert this fraction of fuel remaining into the form of a $\Delta k/k$.

Let Σ_u = the macroscopic absorption cross section of the uranium

Σ_m = the macroscopic absorption cross section of all that is not U in the cold clean reactor

Σ_b = the macroscopic absorption cross section of fuel that is burned out.

Then, the original thermal utilization, $f = \frac{\Sigma_u}{\Sigma_u + \Sigma_m}$

and the "burned out" thermal utilization $f' = \frac{\Sigma_u - \Sigma_b}{\Sigma_u + \Sigma_b - \Sigma_b}$

dividing each of these fractions on the right hand side by Σ_u , and defining

$$B = \frac{\Sigma_b}{\Sigma_u} = 1 - \frac{N}{N_0}$$

$$\text{and } y = \frac{\Sigma_m}{\Sigma_u}$$

$$f = 1/(1+y)$$

$$\text{or } y = (1-f)/f$$

and $f' = (1 - B)/(1 + y - B)$.

$$\text{So, } (f' - f)/f' = k/k = 1 - f/f' = \frac{(f - 1) B}{1 - B}$$

Thus for 440 hours $B = 1 - .9 = .10$,

$$\text{or } \Delta k/k = -(1 - f) = .111$$

and for 290 hours $B = 1 - .933 = .067$

$$\text{or } \Delta k/k = (f - 1) = .072 = -.072(1 - f)$$

Experiments: An allowance of either four or five per cent in $\Delta k/k$ has been made for experiments to be inserted in the reactor. Of course, one must in practice provide safeguards which assure that experimental assemblies or materials involving even a fraction of this reactivity are never removed while the reactor is in operation.

Contingencies: An additional allowance of two per cent in $\Delta k/k$ has been made for temperature effect, samarium and other low poison buildups, and for unknown effects. The samarium steady state poisoning gives values for $\Delta k/k$ of anywhere from 0.7% to 0.9% (the poisoning is a function of thermal utilization). This is attained in about 11 days at a flux of 10^{14} . The temperature effect is judged to be about 0.8%, leaving about 0.6% for unknown effects. In MTR experience 5% $\Delta k/k$ for experiments was found to be very ample or even excessive, so that this allowance may complement the 0.6% for unknown effects.

Xenon Override: From the equation (equation 11.66.3 in Elements of Nuclear Reactor Theory by Glasstone and Edlund)

$$\text{where } \phi_0 = 10^{14} \text{ neutrons/cm}^2 \cdot \text{sec}$$

$$\lambda_1 = \text{Iodine decay constant } 2.9 \times 10^{-5} \text{ sec}^{-1}$$

$$\lambda_2 = \text{Xenon decay constant } 2.1 \times 10^{-5} \text{ sec}^{-1}$$

$$\gamma_1 = \text{Fractional yield of Iodine } .056$$

γ_2 = Fractional yield of Xenon .003

σ_2 = Microscopic neutron capture cross section of Xenon
 $3.5 \times 10^{-18} \text{ cm}^2$

$\frac{\Sigma_f}{\Sigma_n} = .83$ (for 20% enriched U)

t = Time after shutdown in seconds.

The following results are obtained:

1. Steady state ($t=0$) $P = 0.046$
2. 1/2 an hour after shutdown $P = 0.0751$
3. 1/4 hour after shutdown $P = 0.0586$

By defining f as the unpoisoned thermal utilization

$$f = \frac{\Sigma_u}{\Sigma_u + \Sigma_o}$$

$$f' = \frac{\Sigma_u}{\Sigma_u + \Sigma_o + \Sigma_p}$$

Where $y = \frac{\Sigma_o}{\Sigma_u}$ and $P = \frac{\Sigma_p}{\Sigma_u}$

Σ_u = macroscopic fuel absorption cross section

Σ_o = macroscopic non-fuel absorption cross section

Σ_p = macroscopic xenon cross section

The following relations can be derived:

$$f = \frac{1}{1 + y} \quad f' = \frac{1}{1 + y + P}$$

also $(f' - f)/f' = (k'_{\text{eff}} - k_{\text{eff}})/k'_{\text{eff}} = \Delta k/k = P/(1 + y) = fP$

the $\Delta k/k$ needed to override the xenon effect in terms of the thermal utilization of the clean reactor. Thus:

steady state operation $\Delta k/k = 0.046$

1/2 hour override $\Delta k/k = 0.0751$

1/4 hour override $\Delta k/k = 0.0596$

Allowing 5.0% for experiments, 2.0% for contingencies, 18 days of operation and 1/2 hour xenon override we find:

$$-\Delta k/k = .07 + 0.111(1 - f) + 0.0751 f = 0.181 - 0.0395 f; \quad (1)$$

5.0% for experiments, 2.0% for contingencies, and 1/4 hour xenon override and 18 days operation we find:

$$-\Delta k/k = 0.07 + 0.072(1 - f) + 0.0751 f = 0.142 + 0.003 f; \quad (2)$$

5.0% for experiments, 2.0% for contingencies, and 1/2 hour xenon override for 12 days operation we find:

$$-\Delta k/k = 0.07 + 0.072(1 - f) + 0.0751 f = 0.142 + 0.0031 f. \quad (3)$$

It is now necessary to translate these figures in $\Delta k/k$ into the form of "A", the factor by which to multiply the cold clean cylindrical mass to give it a k/k equal to that which is taken away by the above effects. Now, instead of the original amount of uranium present, an amount is added to change Σ_u into $A\Sigma_u$.

$$\text{Defining } f'' = \frac{A\Sigma_u}{A\Sigma_u + \Sigma_m} \qquad f = \frac{\Sigma_u}{\Sigma_u + \Sigma_m}$$

similar to the above

$$\Delta k/k = (1 - f/f'')$$

$$\text{now } \frac{\Sigma_m}{\Sigma_m + \Sigma_u} = 1 - \frac{\Sigma_u}{\Sigma_u + \Sigma_m} = 1 - f$$

$$\text{So } \frac{\Delta k}{\Delta k} = (A - 1)(1 - f)/A \quad \text{or} \quad A = \frac{1 - f}{\Delta k/k}$$

$$\text{Thus, from case (1) above } A \text{ becomes : } A = \frac{1 - f}{0.181 - 0.0395 f}$$

Figure 3.B.1 shows the value of 'A' as a function of f in several cases of interest. Figures 3.B.2 and 3.B.3 show the effect on the operating loading in two reactors for several choices of operating design conditions.

FIGURE 3.B.1: OPERATING MASS FACTOR

FIGURE 3.B.2: EFFECT OF DESIGN REQUIREMENTS

FIGURE 3.B.3: EFFECT OF DESIGN REQUIREMENTS

An 18 day (440 hour) core life has been chosen to give a three week cycle with time for exchange of fuel elements and making experimental adjustments. The figure shows that the amount of fuel to be inserted can vary considerably depending upon the conditions one imposes. The following criterion has been chosen: there be available at all times an excess of 5.0% in $\Delta k/k$ for experiments, and a 2.0% allowance in $\Delta k/k$ for temperature effects. In addition, the system will have 1/2 hour of xenon override for the first 12 days of its core life (290 hours) and at least 1/4 hour xenon override for the remaining six days. It is also of interest to see how long one can operate after the nominal 18 day period if one does not need to shut down. It is clear that the permissible shut-down time after the end of the 18 days progressively shortens. Eventually, even a very short accidental scram allows the xenon level to rise beyond the ability of the system to overcome. If, however, one can continue to operate without shut-down, one can operate for a considerably longer time than the design limit of 18 days. This extra time has been calculated as follows:

The steady state poisoning due to xenon at a flux of 10^{14} is 0.046 so that the $\Delta k/k$ is 0.046 f.

Previously derived was the equation

$$\frac{N}{N_0} = e^{-0.24 \times 10^{-3} X}$$

where X is the running time in hours. The running time with only steady state xenon override is set equal to the case for the 1/4 hour - 18 day running time. Since the loading initially is the same, the $\Delta k/k$ is the same,

$$0.111 (1-f) + 0.07 + 0.0751 f = (N_0 - N)(1 - f)/N + 0.07 + 0.046 f$$

One can solve this for N/N_0 and thus find X .

For the case where mass loading is determined by the 1/2 hour - 12 day condition the equation becomes:

$$0.072(1 - f) + 0.07 + 0.0751 f = (N_0 - N)(1 - f)/N + 0.07 + 0.046 f$$

The result of these two conditions is given in Fig. 3.B.4. As may be expected, the cases with high f which need much extra mass compensate by yielding a grace period ranging over 200 hours.

3.C. Flux Plots

As has been stated earlier in this report, the ORACLE code used in this investigation (T. Fowler's 2G2R code) calculates both fast and thermal fluxes as well as fuel cross section and other information, and will punch out fluxes at the choice of the individual. If fluxes are punched out, they are given at the following points:

1. at the center of the core
2. at $a/4$ where a is the core radius
3. at $a/2$
4. at $3a/4$
5. at a , i.e., at the core-reflector interface
6. at $a + T/4$ where T is the reflector thickness
7. at $a + T/2$
8. at $a + 3T/4$
9. at the extrapolated boundary of the reflector.

The last item ought to be zero in all cases, and often is, but sometimes it comes out to be a very small number instead, due to rounding off of the values entering the calculation.

FIGURE 3.B.4: EXTENDED OPERATING TIME

In the results of the calculation one also gets a number, A_{2c}^T , which is the integral over the core of the product of thermal flux and thermal absorption cross section divided by 4π . Therefore, one has:

$$\phi_p(r) = \frac{\phi(r)}{4\pi} \times \frac{P_c}{\Sigma_f(\text{fuel})} \times \frac{k \Sigma_{th}(T)}{A_{2c}^T}$$

where $\phi(r)$ = the flux at r as normalized by the ORACLE

$\phi_p(r)$ = the flux at r at operating power

$\Delta k = 3.38 \times 10^{16}$ fissions per Mw per second

$\Sigma_{th}(T)$ = total thermal absorption cross section in the core

P_c = power in the core

Σ_f = macroscopic fission cross section

but, in our case $\bar{\Phi}_{th}$ is specified as 10^{14} neutrons per cm^2 per second.

So
$$P_c = \frac{\bar{\Phi}_{th} \times \Sigma_f(\text{fuel}) \times \text{volume}}{k}$$

So
$$\phi_p(r) = \frac{\phi(r)}{4\pi} \times \bar{\Phi} \times \frac{\Sigma_2(T)}{A_{2c}^T} \times \text{volume}$$

using the factor $\frac{10^{14}}{4\pi} \times \frac{\Sigma_2(T)}{A_2(T)} \times \text{volume}$ the fluxes were normalized to

$$\bar{\Phi}_{th} = 1 \times 10^{14} \text{ neutrons/cm}^2/\text{sec}$$

Figures 3.C.1 through 3.C.8 show some examples of the resulting fluxes which are chosen to be representative, and of interest. Figure 3.C.1 shows the fluxes in two reactors which are the same except that the M/W is different. It will be noted that in the upper curve there is a distinct rise in the thermal flux at the core-reflector interface, which is suppressed in the lower curve, which give the fluxes where $M/W = 0.7$, whereas the upper one corresponds to $M/W = 1.3$. This is due to the fact that at the high M/W there is a much larger leakage of fast neutrons from the core into the

FIGURE 3.C.1: FLUX PLOTS-SPHERICAL SYSTEMS

FIGURE 3.C.2: FLUX PLOTS-SPHERICAL SYSTEMS

FIGURE 3.C.3 FLUX PLOTS - SPHERICAL SYSTEMS

FIGURE 3.C.4: FLUX PLOTS - SPHERICAL SYSTEMS

FIGURE 3.C.5: FLUX PLOTS-SPHERICAL SYSTEMS

FIGURE 3.C.6: FLUX PLOTS-SPHERICAL SYSTEMS

FIGURE 3.C.7: FLUX PLOTS-SPHERICAL SYSTEMS

FIGURE 3.C.8: FLUX PLOTS-SPHERICAL SYSTEMS

reflector where they are then thermalized, and thus yield the large thermal flux observed. It will be noted that the fast flux is indeed higher in the high M/W case, since there the conversion from fast to thermals is less effective, and so there must be more fast neutrons to yield the same thermal average flux. Similar behavior is to be noted in the other similar pairs of curves, except that the effect is in some cases less pronounced. It is especially to be noted that the graphite reflector is very effective in maintaining the thermal flux over a large volume, and it thus offers advantages for experiments and irradiations.

3.D. Results and Discussion

A discussion of the results obtained in this study can well commence with some general observations concerning the effect of the parameters on the required masses. The mass of U-235 which makes a cold clean reactor critical depends on the leakage from the core, on the core volume, on the absorption in the core, and on the moderating properties of the materials.

It can be seen that, as core size increases, the fuel density approaches a constant which is that for the infinite system. Since the volume increases as the cube of the radius, one would expect the mass curve to approach in slope a cubic curve as the radius gets very large. As a core diminishes in size, the leakage from the core becomes relatively more dominant so that one expects a rise in the mass curve or at least a flattening of the aforementioned cubic curve. This behavior is indeed observed. One can reason further that the position of the minimum is dependent on the leakage, so that as the leakage increases, the minimum is displaced toward larger radii. This is also observed. For example, H₂O moderated core with 15 cm H₂O reflector

shows a minimum at about 25 cm core radius for the metal-to-water ratio of 0.7, but at a radius of 28.0 cm where the metal-to-water ratio is 1.3.

Another effect on the clean cold critical curves which is readily observed is that, all other things remaining constant, the thicker reflector yields the lower mass.

More interesting are the operating loadings which have been obtained from the information in Table 5.B.1 and Figure 3.B.1. The operational mass depends strongly on the thermal utilization of the core. Smaller cores have larger fuel concentrations because the leakage is higher. They have higher thermal utilizations and, consequently, higher multiplying factors. This results in accentuating the leakage effect by raising the required mass at these smaller radii. Thus, the minima of the operating mass curves are always displaced toward larger radii from those of the corresponding cold, clean mass curves. A few examples of this will suffice. (Note that the location of the minimum as to radius is referred to here. Of course, the hot-dirty curves are always higher than the cold-clean ones.) Consider the following case: H_2O core, 15 cm beryllium reflector, $M/W = 1.3$; cold clean minimum is at 23.5 cm radius, hot dirty minimum at 25.5 cm. Or another case: HDO core, 15 cm HDO reflector, $M/W = 0.7$; cold clean minimum is at 24 cm. Hot dirty at 30.8 cm.

Since the operating mass curves reflect the leakage effect, or at least are covariant with it, one would also look for a shift in the location of the minimum with change in reflector thickness. Again this occurs. For example, in the HDO core, $M/W = 1.3$ reactor, the 50 cm graphite reflector yields a minimum at 27.2 cm, whereas the 100 cm reflector yields the minimum at 25.8 cm.

KEY TO FIGURES

FIGURE 3.D.1

Or, if the leakage is increased by keeping the reflector constant but increasing the metal-to-water ratio, then, again, the location of the minimum in the operating mass curve shifts to larger radii. This effect, it should be noted, is most apparent in the water and H₂O cores. Here the migration length is much larger in aluminum than in the liquid so that putting in more aluminum strongly increases the probability that a neutron will leak out. This effect is demonstrated, for example, in the H₂O core systems with 50 cm graphite reflectors. The $M/W = 1.3$ curve of hot, dirty loading has its minimum at 27.5 cm, whereas the corresponding curve with $M/W = 0.7$ has its minimum at 25.8 cm.

Consider now the masses. The methods and calculations from which the curves are obtained have been discussed before, and will not be reported here. This discussion will be restricted to pointing out the pertinent features of the results and some especially advantageous systems. It must be borne in mind that any calculated masses for design operation which are larger than 9 kg will not appear on the graph. The Table 5.B.1 in Appendix II lists all masses calculated, even those above 9 kg.

It should be pointed out here how the results are presented. Fig. 3.D.1 is a sample sheet which shows what each of the curves is. It will be observed that there are three heavy curves in the graph. They are, from the bottom up, the cold, clean critical mass in the spherical system, the cold clean mass adjusted to the non-spherical reactor, and on top, the mass required for operation under the design conditions. There appear also a number of lighter curves (usually 6) some with dashed and some with solid lines. These present information as to the maximum amount of U-235 that can be put into a core of

the given radius under two sets of conditions. The dotted curves give the amount for the fuel elements having 20 mil thick cladding, an active core of U-Al alloy which is 20% by weight uranium, and 150 mil thick side plates. The solid curves are similarly constructed for elements having 15 mil thick cladding, 125 mil side plates and an active section which is 27.5% by weight uranium. The numbers appearing on top of each of these curves are the numbers of plates per element. The effect of various choices of these numbers is presented elsewhere in this report. Figure 3.D.2 shows the required mass for simplest 'swimming pool' type, of system, which is merely a set of elements in water with an effectively infinite water reflector. It is obvious at a glance that, whereas one could make a critical assembly of this type, one cannot make an operating reactor in this way. The masses needed are far above what can be put in even if the fuel were available. The metal-to-water ratio of 0.7 is the only one at which this system was investigated but higher M/W ratios would yield even higher masses. The next four graphs, Figs. 3.D.3 - 3.D.6, show the effect of enriching the reflector to 50% in D₂O. As may be seen, a reflector which is 'thin' to neutrons (7.5 cm thick) is not even as good as an infinite water reflector. The masses for the M/W = 0.7 case are higher than the ones for the thick water reflector with the same M/W, and for the M/W = 1.3 case they are higher still. The migration length in H₂O is 8.0 cm, so the 15 cm H₂O reflector considered here is thick⁺. In this case the reflector is, as one would expect, just a little better than infinite water, but not nearly enough so to be useful. The lowest mass is still above 6 kg and beyond the range of accommodability. The next possible improvement is to enrich the reflector still more and, since the effect of the water is dominant at most enrichments, as seen in Fig. 5.A.1

⁺ Glasstone and Edlund state in Elements of Nuclear Reactor Theory that a reflector which is 1.5 migration lengths thick is practically infinite, insofar as mass is concerned (8.30, p. 236).

FIGURE 3.D.2: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.3: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.4: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.5: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.6: U^{235} LOADING FOR DESIGN OPERATION

the mixture chosen was one which was 90% D₂O and 10% H₂O. This mixture has a migration length of 18.2 cm so 15 cm and 30 cm were chosen as representative reflectors for this case. The 'thin' reflector yields masses which are lower than the masses for the 'thin' H₂O reflector, as one would expect. Thus, comparing Fig. 3.D.3 with 3.D.7, it is seen that there is a drop from 8.8 to 6.6 kg in the minimum operating loading in going from a thin H₂O to a thin 10-90 reflector, at a M/W of 0.7. Making the same comparison between Fig. 3.D.5 and Fig. 3.D.9., it is seen that a like saving from 6.1 down to 5.3 kg is effected by enriching the 'thick' reflector. Here, in Fig. 3.D.9 one sees the first system for which the operating mass is below the 6 kg limit. However, at the radii where this low mass is attained, one cannot get the required fuel in, so the system is not possible. If one attempts to get more fuel in by increasing the M/W in this case, one finds (Fig. 3.D.10) that the leakage has increased sufficiently to push the mass above 6 kg again, so that again the system is not feasible. If one takes the final logical step in attempting to improve this system to make it work one can make the reflector fully enriched or pure D₂O Figs. 3.D.11 and 3.D.14 show the effect of doing so. The migration length of pure D₂O is 162 cm, but from motives of economy reflectors of 75 and 120 cm thickness are considered. This changes the situation considerably. Fig. 3.D.11 shows that even a 75 cm reflector of D₂O suffices to give loadings which are as low as 3 kg and thus come well under the limit. The figure shows also that above 3.4 kg one comes into the range where the metal may be accommodated and thus, the reactor presents itself as a practical possibility. It will be seen also that an increase in the number of plates per element requires going to larger masses, and that this effect is pronounced due to the steep slope of the curve. A 14-plate element of the 'optimistic' type requires 4.4 kg, and

FIGURE 3.D.7: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.8: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.9 U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.10: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.II: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.12: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.13: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.14: U^{235} LOADING FOR DESIGN OPERATION

an 18 plate element of this type is entirely out of reach. The other types of elements, which will henceforth be designated as "low-capacity" elements, are out of this picture entirely. In this case an advantage is to be gained by going to a higher M/W. Fig. 3.D.12 shows the same case as the previously discussed one, except that it has $M/W = 1.3$. Here the lowest possible mass is 3.75 kg as against about 3.0 kg in the $M/W = 0.7$ case, but the masses are still well below the limit, and the range of possible choices as to plate numbers and element types is far wider. At about 30 cm radius one has a required loading of 4.4 kg, and here one may choose from all the possible types of elements up to 18-plate elements of the low-capacity type. If one goes to a 120 cm thick D₂O reflector, one realizes further savings in the masses, but these are probably too small to offset the increased cost of the D₂O. Comparing Fig. 3.D.11 and 3.D.13 one sees that the maximum saving in loading is only about 0.25 kg of U-235, and it is less over most of the range. This makes a thick reflector definitely unattractive. Making the same comparison for the $M/W = 1.3$ case, (Figs. 3.D.12 and 3.D.14) one sees that here the differences are somewhat larger, as much as 0.5 kg of U-235, in fact; but over most of the range, here also, the saving is much smaller, and it does not appear that the extra 45 cm thickness of D₂O is justified.

Still better systems are possible if aqueous reflectors are abandoned altogether in favor of beryllium and graphite, which are the best choices among other materials.* Figs. 3.D.15 to 3.D.20 show what can be done with beryllium reflectors on water systems. For these cases, three M/W values have been considered in order to have a better picture of the effect of this variable. In beryllium the migration length is 22.6 cm. Therefore, 15 and 40 cm have been chosen as being respectively thin and thick for this material. Consider first

* It should be noted, though, that in practice considerable difficulty would be encountered in surrounding a cylindrical reactor with a reflector on all sides, since room must be left for coolant headers and control elements.

FIGURE 3.D.15: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.16 : U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.17: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.18: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.19 : U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.20: U^{235} LOADING FOR DESIGN OPERATION

the thin reflector. It can be seen that above 27.5 cm radius, the reduction in mass to be gained by going to lower M/W is small, not more than 1.3 of a kg at most in going from M/W = 1.3 curve to rise sharply, as the leakage effect becomes dominant, so that at these radii the system becomes impossible. In the other two lower M/W cases this effect does not appear down to the lowest radii of interest, so these are still feasible there. Fig. 3.D.16, in particular, shows that a system requiring less than 4.5 kg of U-235 is possible and also feasible from the fuel accommodation viewpoint, even with only this thin beryllium reflector. Figs. 3.D.19 through 3.D.20 indicate that thick beryllium reflectors offer truly fine possibilities. The leakage is still dominant in the M/W = 1.3 case so that the characteristic rise at low radii is observed, but the minimum has come down from 4.2 to 3.2 kg, and thus, at about 25 cm radius we have a system which is flexible, indeed, as to fuel plate configuration and has a low enough mass to be very advantageous. It may be emphasized here that the advantages of low mass are more than the reduced cost of the U-235 inventory. In a reactor of the type considered here, where the design parameter fixes the flux and not the power, the power is proportional to the loading, so that one saves also in heat dissipation and external systems costs by reducing the mass. Fig. 3.D.18 shows that by taking an 8-plate high-capacity type of element one can have a reactor with a design-operation loading of 2.35 kg, which is very low income. However, this is in a core of only 23 cm radius, which makes it difficult to make any sort of approximation to a square cylinder with 3 by 3 inch cross section elements, and the volume available for experiments and control rods is also very small. It has been felt that 25 cm represents the lower limit of convenience in this respect. Note that Fig. 3.D.19 shows that a 25 cm

core, 2.6 kg reactor is very easily feasible, and appears to be sound in every way, except that 40 cm of beryllium at about \$16,000 per cubic foot represents a considerable value. To reduce this, one can replace the beryllium by graphite which costs only a small fraction of the former material. Figs. 3.D.21 through 3.D.24 present this information. In graphite the migration length is 54.3 cm, so 50 cm, and 100 cm have been chosen as the thicknesses to be used. Comparison will show that graphite is only very little worse than beryllium, so that from this viewpoint, graphite seems more desirable due to the economy involved. However, it must be emphasized that the reflector will have to be cooled. This means that the reflector will not be pure graphite but must contain other material as coolant. The graphs showing the masses of cores with D₂O reflectors, when compared with similar ones with 10-90 reflectors clearly shows that even so small an admixture of light water as 10% by volume, will severely 'poison' the reflector, and thus cause major increases in the loading. Beryllium, being a better conductor of heat than graphite will thus have the advantage of needing a smaller volume fraction of coolant in it than graphite would. The graphite could, of course, be cooled by D₂O, which presents engineering difficulties, or with gas, in which case this effect would not enter.

An advantage which such a coolant in the graphite or beryllium confers is that the migration length in graphite or beryllium is considerably reduced by the addition of water coolant to the reflector, which thus considerably diminishes the thickness required, though not, of course, being as good as the pure substance.

There is considerable advantage to be gained in enriching the moderator-coolant in the core. This advantage comes about for the following reasons:

- (1) The longer mean free path and larger age spreads the system over a larger

FIGURE 3.D.21: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.22: ^{235}U CORE RADIUS (cm) LOADING FOR DESIGN OPERATION

FIGURE 3.D.23 U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.24: U^{235} LOADING FOR DESIGN OPERATION

volume, so that lower M/W ratios should be feasible for getting in the required metal; (2) The deuterium atoms will also moderate, though not quite as well as light hydrogen ones, and they have a far lower absorption cross section so that one gains in moderation as against absorption; (3) The larger core will give more room for experiments and irradiations.

And, accordingly, core moderators of HDO, 10% H₂O - 90% D₂O, and pure D₂O have been investigated.

Figs. 3.D.25 through 3.D.39 represent the results attained with cores moderated with HDO. There is again the possibility of having both core and reflector made of the same material, with its attendant advantages and simplicity. Therefore, Figs. 3.D.25 to 3.D.28 present results of this type of arrangement. It is observed, as before, that HDO is not a very good reflector. Thin reflectors of this material, as in the case of the water cores, do not give low enough masses to be of interest. The HDO core does improve things somewhat. If Fig. 3.D.3, which is a water moderated core, is compared with 3.D.25, which is a HDO moderated core, both having $M/W = 0.7$, and both having a 7.5 cm HDO reflector, it is seen that the enrichment of the core reduces the minimum in the mass curve from 8.4 kg down to 7.6 kg. And, as expected, the minimum for the HDO core occurs for a radius which is 4 cm higher than in the H₂O case. A thick HDO reflector is, of course, better than a thin one, but again, the improvement is not sufficient to put the systems in the useful range. The only possible exception is the HDO core, HDO reflected (15 cm) system which has a minimum in the curve at 5.8 kg, just below the limit, and one can just get enough fuel in by using an eight plate element of the high capacity type. The similar system with $M/W = 1.3$ has requirements of the order of 8 to 9 kg, but it is possible to get that much fuel in quite readily.

FIGURE 3.D.25: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.26: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.27: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.28: U^{235} LOADING FOR DESIGN OPERATION

It was felt that it would not be of interest to have two different liquids in one system, so the possibilities of systems which have H₂O cores and 10-90 or D₂O reflectors were not investigated. It can be inferred, though, from the data on water cores, that such systems could be made, especially with D₂O reflectors.

There is information, however, on systems with beryllium or graphite reflectors. These could be cooled with H₂O, which is present in the core and thus can be cooled in one external system, with attendant savings. Using H₂O as coolant one can permit twice as much liquid in the reflector as would be the case were water used. It is unfortunate that in many cases the ORACLE results are incomplete, so that there is only one point known for a given system, but useful inferences can nonetheless be drawn. Fig. 3.D.29 is an attempt to see what effect a very thin beryllium reflector would have. Only one point is known, and it is so high, that it is safe to assume it lies on the rising part of the mass curve. This is not surprising, because the combination of H₂O in the core and a very thin reflector would make the leakage so high as to shift the minimum in the curve considerably to the right, that is, to large values of radius. When Figs. 3.D.30, 3.D.31, and 3.D.32 are examined, it appears that a 15 cm beryllium reflector, although thin, shows very real possibilities. The expected shift in the location of the minimum both toward higher mass and larger radius as M/W increases progressively is again observed. Here, however, the M/W = .7 system requires so low a mass that the metal can be accommodated at this ratio. A system here with a radius of 27.5 cm has a required mass of 3.0 kg of U-235, and a fairly good choice of fuel element types is available. Fig. 3.D.31 shows that going to M/W = 1.9 causes a small rise in the required mass,

FIGURE 3.D.29: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.30: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.31: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.32 ^{235}U LOADING FOR DESIGN OPERATION

but yields a wider choice of fuel elements in compensation. The largest M/W (Fig. 3.D.32) carries this one step further, and shows that if one wishes to accept a 3.6 kg mass one can make the elements in any of the configurations considered. Figs. 3.D.33 through 3.D.35 consider the effect of thick reflectors of beryllium on the H₂O systems. In these cases no points were obtained for radii below 30 cm but the results that are presented show that these are very low mass systems. One which seems very good is at 30 cm core radius with M/W = 0.7; it requires only 2.6 kg of U-235 and can be made with almost any of the fuel element types. It does not seem that the increase in mass occasioned by going to higher M/W ratios is justifiable here, small though it be, since even the lowest M/W lets one put virtually any of the systems to use. However, should it be desirable to make an 18-plate element of the low-capacity type, one can do so by using the M/W = 1.0 reactor at 30 cm which requires only 2.75 kg of U-235. The figures indicate that still smaller radii, of the order of 25 cm might make a reactor possible with only about 2.4 kg and M/W = 1.0. The graphite reflected reactor configurations again show a small but significant increase in operating mass over the corresponding beryllium system. The graphite is not quite as good a reflector, so one finds that at higher M/W ratios, where the leakage to the reflector is larger, the disadvantage of graphite over beryllium is more pronounced. The low M/W reactor case (Fig. 3.D.36) requires a minimum of 3.9 kg, as compared to 3.5 kg with beryllium (Fig. 3.D.32). In general with these systems, the needed fuel can be inserted easily and their slightly higher mass is probably more than compensated for by their lower cost. However, again it might well be pointed out that the cooling of a graphite reflector without putting hydrogen in it presents difficulties.

Still higher enrichment of the core moderator and coolant was

FIGURE 3.D.33: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.34: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.35: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.36: U^{235} LOADING FOR DESIGN OPERATION

investigated. Systems were taken with moderator-coolant 90% enriched in D_2O (Figs. 3.D.40 - 3.D.50). Looking first at Figs. 3.D.40 to 3.D.43 one finds the systems which have the same enrichment in the core as in the reflector, and are for that reason desirable. It appears, however, that there is no feasible system among them, and in fact, they require masses which are quite a bit higher than similar systems using H_2O in both core and reflector. The reason for this is that the increase in the mean free path in the core causes increased leakage from the core, and the reflector property improvement in these cases is not sufficient to offset this disadvantage. Of more interest are the beryllium and graphite reflected systems. Consider Figs. 3.D.30 and 3.D.44; these two cases are alike in their reflectors and M/W ratios and differ in only that in the one case the core contains H_2O and in the other, 10-90. The beryllium reflector here is thin, so leakage is high. In the 10-90 system, it is so much higher that the minimum in the loading curve occurs at a much larger radius than in the H_2O case. This results in a higher mass using 10-90 when the cores are small (below 33 cm radius). This is because of the aforementioned leakage effect. At larger core radii, where one would expect the leakage to be relatively less important one finds that the 10-90 systems yield lower masses. Unfortunately, only one point is known on the $M/W = 1.3$ curve, but it is about what one should expect, with the higher leakage giving somewhat higher required masses. These highly enriched systems are of interest chiefly if it is desired to build systems of larger volume without larger investment of mass. Of course, the expense of the moderator rises accordingly. The same holds for the beryllium reflected systems. For example, the one point that was found with a 40 cm beryllium reflector and $M/W = 0.7$ yields a mass of only 4.4 kg with a 45 cm core radius. Of course, the expense is formidable. In general, it is not observed that systems

FIGURE 3.D.37: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.38: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.39 U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.40: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.41: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.42: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.43: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3D.44: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.45: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.46: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.47: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.48: U^{235} LOADING FOR DESIGN OPERATION

of this enrichment offer unique advantages, compensating for their high cost. An exception to this, if large core systems are desirable, is the 100 cm graphite reflected, $M/W = 0.7$, 10-90 (Fig. 3.D.49). This system makes a reactor at 35 cm possible with 3.4 kg, or one at 40 cm with 4.05 kg. In either case fuel accommodation is no problem. And, since 90 per cent enriched fluid is present in any case, it ought to be easy to cool the graphite without seriously damaging its reflecting properties. Twenty per cent by volume of fluid, which should be more than ample to cool, would include 2.0% of H_2O in the reflector which should not be serious.

The last group of systems presented here has fully enriched D_2O as moderator-coolant (Figs. 3.D.51 to 3.D.53). In these reactors one must take into account the very long neutron mean free paths. This results in large reactors, so that consideration of systems with radii less than 35 cm was not attempted, and often results were obtained only at 45 or 55 cm. Another factor, which should be borne in mind when these systems are considered, is the question of maximum loadings. In these large systems, it is not necessarily most advantageous to fill the entire core volume with fuel elements as is done in smaller cores. This is particularly true where the metal-to-water ratio is very low. One sees this most easily by considering the case of a system with a M/W so low that the plate thickness reduces to twice the clad thickness. In that case, no fuel can be put in at all. If in this case only half as many elements are used, and they are spaced further apart, then the M/W in the elements is made higher to keep the average at the original value, and then one can put fuel in. It is clear, therefore, that such large, low M/W reactors might more advantageously be build with few, spaced elements. However, if this is done, the system quickly loses its homogeneity, and the simple calculations done here lose meaning.

FIGURE 3.D.49: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.50: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.51: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.52: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.53: U^{235} LOADING FOR DESIGN OPERATION.

Therefore, in this report all curves for mass accommodation are calculated on the assumption that the elements are close-packed, and accordingly for $M/W = 0.3$ the available mass becomes very low. It should be remembered, however, that by suitably spacing the elements one could materially increase the accommodation.

Figures 3.D.52 through 3.D.63 represent the D_2O moderated reactors. Reflecting D_2O cores with D_2O again offers simplicity of operation and construction, but in this case at the expense of truly large volumes of expensive D_2O . Because of its large migration length and attendant large core sizes and reflector thickness, one must use volumes equivalent to spheres of 100 to 120 cm radius of D_2O and this is an expensive item, indeed. However, this does promise low-mass systems of large volume. For example, Fig. 3.D.51 shows that a 40 cm core reactor with $M/W = 0.3$ can be made having only 2.5 kg, which is probably the best that can be done as to mass for so large a system, and has the simplicity of using the same reflector and moderator besides. In these reactors beryllium is not as advantageous as in smaller ones. Comparing Figs. 3.D.51 and 3.D.57 we see that the masses required with only 60 cm of D_2O (which is 'thin') are almost exactly the same as those required with 40 cm of beryllium (which is thick). As D_2O is cheaper on a volume basis than beryllium, by a factor of ten, and has the additional advantage of simplicity of construction, it is clear that there is no advantage to be gained by using beryllium in these reactors.

Graphite reflection still merits attention because of its low cost. Figs. 3.D.60 to 3.D.63 show these configurations. It may be observed that thick graphite is not quite as good as thick beryllium, but with the cooling being simpler if D_2O is used, graphite provides a very attractive possibility in this

FIGURE 3.D.54: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.55: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.56: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.57: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.58: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.59: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.60: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.61: ^{235}U LOADING FOR DESIGN OPERATION

range. For example, Fig. 3.D.62 shows that at 35 to 40 cm, with $M/W = 0.5$, a reactor reflected with 100 cm of graphite needs only about 3.5 kg of U-235, and this is not so large a core as to be prohibitive.

It has been assumed throughout that the variation in the required mass with changes in M/W , all else being constant, is regular enough, and small enough to be obtained for any desired M/W from a knowledge of only two or three points. This is borne out by the results shown in Figs. 3.D.64 and 3.D.65. These graphs represent the variation in cold, clean critical mass with changes in M/W , and for easier comparison they have been normalized to 1.0 at $M/W = 0.7$. It will be seen that these curves are, indeed, smooth and regular enough so that no difficulty should be encountered in extrapolating intermediate values. It will also be observed that good reflectors cause small variations in mass with changing M/W .

4. INTRODUCTION- ~~ENGINEERING~~

4.A. Fluid Flow, Pressure Drop, and Temperature Considerations

These calculations have been limited to the core and have not taken into account either the cooling of the reflector and shield or any consideration of the external cooling system. These items were omitted as no specification of the experimental facilities or location of the reactor are given and each change of these requirements would involve a quite different system.

In general, the information in this section was limited to water flow considerations. The results which are based on MTR type fuel elements are presented in four sets of curves. Given the power density, the heat flux for elements with different numbers of plates can be obtained from Fig. 4.A.1.

Fig. 4.A.2 assumes an inlet coolant temperature of 100°F and for various maximum

FIGURE 3.D.62: ^{235}U LOADING FOR DESIGN OPERATION

FIGURE 3.D.63: U^{235} LOADING FOR DESIGN OPERATION

FIGURE 3.D.64: EFFECT OF METAL TO WATER RATIO

FIGURE 3.D.65: EFFECT OF METAL TO WATER RATIO

FIGURE 4.A.1: HEAT FLUX IN CORE

FIGURE 4A.2: METAL SURFACE TEMPERATURE

heat fluxes relates the water flow velocity to the maximum metal surface temperature. The total pressure drop across the elements for a given velocity between the plates can be determined from Figs. 4.A.3 and 4.A.4. The total water flow rate per element is obtainable for each metal-to-water ratio from Fig. 4.A.5.

Calculation of Pressure Loss:

The primary reference for the pressure drop calculations is, "Heat Transfer and Pressure Loss in Proposed ORR Fuel Assemblies", by J. P. Sanders (4.1). The formulas and methods used in this report were only modified where necessary because of the slightly different requirements of this application.

Description of Assembly:

Except for the fuel section, the fuel element assembly considered here is identical to that used in the above cited report and described in ORNL Dwg. D-15898. A typical cross section of the fuel plate portion of an element is shown in Fig. 2.A.1 where, for simplification, the plates are shown to be flat instead of curved. Another change to drawing D-15898 is the enlargement of the exit straight sections from 2.00 inches inside diameter to 2.250 inches. This was also done by Sanders. Fig. 2.A.1 shows fuel plates with 20 mil cladding and 150 mil side plates although elements with 15 mil cladding and 125 mil side plates were also considered. In plotting the results of calculations the plate spacing rather than the number of plates is used as the variable when this parameter is involved in the calculation. However, this plate spacing can be obtained for all considered numbers of plates and metal-to-water ratios from Fig. 4.A.6.

FIGURE 4.A.3: PRESSURE DROP

FIGURE 4.A.4. PRESSURE DROP

FIGURE 4.A.5: COOLANT FLOW BETWEEN PLATES

FIGURE 4.A.6: WATER CHANNEL THICKNESS

FIGURE 4.A.7 PLATE THICKNESS

Empirical Relationships Used to Determine Pressure Loss:

The losses due to sudden contraction were calculated according to equation (31) in Badger and McCabe (4.2). Changing this relationship from head loss to pressure loss in pounds per square inch, it becomes:

$$\Delta P = \frac{K_c U^2}{2(144) g_c}$$

The constant, K_c , is evaluated according to Fig. 15 of Badger and McCabe (4.3). The significance of the remaining terms is found in Section 5.C. The losses due to sudden expansion were calculated from Eq. (30) in Badger and McCabe (4.5), which is identical with equation (102) in Vennard (4.6). Again changing to pressure loss in pounds per square inch, the equation is:

$$\Delta P = \frac{(U_H - U_L)^2}{2(144) g_c}$$

In the case of a more gradual expansion the formula is modified according to Eq. (103) in Vennard (4.7), giving

$$\Delta P = \frac{K_e (U_H - U_L)^2}{2(144) g_c}$$

The constant K_e was evaluated according to Fig. 84 of this reference (4.8).

The losses due to friction in straight lengths of constant cross sectional area were calculated according to the Fanning friction equation shown in McAdams (4.9). The integrated form of Eq. (8a) of this test converted to pressure loss in pounds per square inch is

$$\Delta P = \frac{2f U^2 L}{(144) g_c D_e}$$

The Fanning friction factor, f , is found by computing the Reynolds number and applying the relationship expressed graphically in Fig. 51 of McAdams

(4.10). A line drawn one-fourth the distance from line A to G on the plot was found to best relate the friction loss due to flow between the plates.

In applying the friction equation and in determining the Reynolds number for flow in channels other than circular, the equivalent hydraulic diameter was calculated according to the suggestion in Badger and McCabe (4.11). The equivalent diameter is given as four times the cross sectional area of the channel divided by the wetted perimeter of the channel.

Method of Calculation:

In calculating the pressure loss through the fuel assembly, the velocity between the fuel plates was assigned a value, V . By calculating the relative flow area throughout the assembly, the velocity at the other points could be determined in terms of this velocity. The pressure loss over each section was then calculated in terms of this velocity. In order that the pressure loss calculated might compare with the pressure loss which might be determined in an experimental apparatus, the effect of changes in velocity head was calculated. The net change in velocity head over the entire assembly was zero since it was assumed that velocities were the same above and below the assembly support.

Calculations were made for two plate spacings (0.075 inch and 0.250 inch) and two metal-to-water ratios (0.7 and 1.3). These calculations were made assuming that the average water temperature is 100°F. The viscosity of water at this temperature was determined from the nomograph, in Badger and McCabe (4.12). The density of water at this temperature was determined as the reciprocal of the specific volume given in Keenan and Keys (4.13)

An example of these calculations is shown in Appendix 5D of this report.

Result of Pressure Loss Calculations:

The relationship giving the pressure loss through the assembly as a function of velocity through this fuel plate assembly is given by the following expressions.

For a metal-to-water ratio of .7

$$\text{Upper end box } \Delta p = .00639 v^2 + .00095 v^{1.79}$$

$$\text{Lower end box } \Delta p = .00894 v^2 + .0017 v^{1.79}$$

$$\text{Total of end boxes } \Delta p = .0153 v^2 + .00265 v^{1.79}$$

$$\text{Fuel plates with .250" spacing } \Delta p = .00238 v^2 + .012 v^{1.79}$$

$$\text{Fuel plates with .075" spacing } \Delta p = .00238 v^2 + .050 v^{1.79}$$

$$\text{Total assembly (.250" spacing) } \Delta p = .0177 v^2 + .01465 v^{1.79}$$

$$\text{Total assembly (.075" spacing) } \Delta p = .0177 v^2 + .05265 v^{1.79}$$

For a metal-to-water ration of 1.3

$$\text{Upper end box } \Delta p = .00411 v^2 + .00051 v^{1.79}$$

$$\text{Lower end box } \Delta p = .00410 v^2 + .0010 v^{1.79}$$

$$\text{Total end boxes } \Delta p = .00821 v^2 + .00151 v^{1.79}$$

$$\text{Fuel plates with .250" spacing } \Delta p = .00385 v^2 + .012 v^{1.79}$$

$$\text{Fuel plates with 0.75" spacing } \Delta p = .00385 v^2 + .050 v^{1.79}$$

$$\text{Total assembly (.250" spacing) } \Delta p = .01206 v^2 + .0135 v^{1.79}$$

$$\text{Total assembly (.075" spacing) } \Delta p = .01206 v^2 + .0515 v^{1.79}$$

Calculation of Maximum Surface Temperature:

Development of Equations:

Again these calculations follow those of J. P. Sanders (4.14) except where changes were made to allow for variation in conditions.

In calculating the maximum surface temperatures, the assumption was

made that the neutron flux, and consequently the heat flux, was distributed, as in the MTR according to a cosine curve which dropped to 0.4 times its maximum at the end of the plates. Thus, if Q_0 represents the maximum heat flux at the center of the length of the fuel plate, $0.4 Q_0$ represents the flux at the end of the fuel plate. In addition, it was assumed that no heat was conducted linearly along the fuel plate.

The heat flux in the fuel plate can now be expressed as

$$Q = Q_0 \sin \pi x/L \quad (a)$$

where L is the extrapolated length of the fuel plate that has zero flux at the ends. L is related to the actual length of the plate, a , by

$$\begin{aligned} L &= a / \left[1 - (2) (0.4416)/\pi \right] \\ &= a/0.7380 \end{aligned} \quad (b)$$

Designating the temperatures of the water as T_w and that of the surface as T_m , the water flow rate in pounds per foot of width of plate per second as G a heat balance from the entrance (designated sub 1) and any point, x , along the plate is

$$3600 G C_p(T_w - T_{w1}) = 2 \int_0^x Q_0 \sin \pi x/L dx \quad (c)$$

$$\frac{0.4116L}{\pi}$$

Integrating this expression

$$T_w = T_{w1} + \frac{2 Q_0 L}{3600 \pi C_p} G (0.9165 - \cos \pi x/L) \quad (d)$$

Since it is assumed that all the heat generated in the plate is transferred directly into the water, the following relationship is valid at any point x

$$h(T_m - T_w) = Q_o \sin \pi x/L \quad (e)$$

$$T_m = T_w + (Q_o/h) (\sin \pi x/L) \quad (f)$$

Substituting the value of T_w in equation (f) the surface temperature may be expressed

$$T_m = T_{w1} + \frac{2 Q_o L}{3600 \pi C_p G} (0.9165 - \cos \frac{\pi x}{2}) + \frac{Q_o}{h} \sin \frac{\pi x}{L} \quad (g)$$

In order to find the point at which the maximum value of T_m occurs

$$\frac{dT_m}{dx} = \frac{2Q_o}{3600 \pi C_p G} \sin \frac{\pi x}{L} + \frac{Q_o \pi}{h L} \cos \frac{\pi x}{L} = 0 \quad (h)$$

$$\text{or } \tan \frac{\pi x}{L} = - \frac{3600 \pi C_p G}{2 h L} \quad (i)$$

The value of the heat transfer coefficient, h can be obtained from an empirical equation given in McAdams (4.15). The equation is

$$h = 5.6 (1 + 0.011 t) (G')^{0.8} / (D')^{0.2} \quad (j)$$

where t is the bulk temperature of the fluid in degrees Fahrenheit, G' is the pounds of fluid flowing per second-square foot of cross section and D' is the hydraulic diameter in inches. The diameter is taken as twice the gap width which is approximately equal to the value obtained by dividing four times the area by the perimeter.

The equation (j) above was modified for these calculations, to allow for the varying gap spacing, to read as follows:

$$h = \frac{35.6 (1 + 0.011 t) G'^{0.8}}{d'/2} \quad (k)$$

In order to evaluate this heat transfer coefficient and also the heat capacity, and density, it is necessary to know the average bulk temperature of the liquid. This is taken as the arithmetic average between the inlet and exit water temperatures.

The total heat, H, removed from a foot width of plate in one second is

$$H = \frac{2}{3600} \int_{0.4116 L/\pi}^{2.730 L/\pi} Q_0 \sin \frac{\pi x}{L} dx \quad (1)$$

Integrating

$$H = 3.24 \times 10^{-4} Q_0 L \text{ BTU/ft. sec} \quad (m)$$

The temperature rise of the water in flowing by the plate is found to be

$$\Delta T = H/G \text{ cp} \quad (n)$$

As a first estimate the value of Cp may be taken as 1.00 BTU per pound-degree Fahrenheit; L is 2.781 feet. The temperature rise in terms of the maximum heat flux and flow rate is

$$\Delta T = 9.02 \times 10^{-4} Q_0/G \quad (o)$$

The average bulk water temperature is then found to be

$$t = \frac{T_{w1} \times (T_{w1} + \Delta T)}{2} \quad (p)$$

Method of Calculation:

Three values of maximum heat flux, Q_0 , were selected and in each case for a series of values of mass flow rate, G, in pounds per foot of width-second, the rise in water temperature, ΔT , was calculated from (o), next, for an entrance water temperature of 100°F the average bulk water temperature, t, was calculated from equation (p).

For the known gap widths it is now possible to calculate the mass flow rate, G' , and the linear velocity, V , for each assumed value of G . Sufficient values are now known to calculate the film heat transfer coefficient, h , from equation (k).

In order to locate the point of maximum surface temperature the values of $\tan \pi x/L$ are calculated from equation (i) and the corresponding values of $\sin \frac{\pi x}{L}$ and $\cos \pi x/L$ are read from tables. Since these are only values needed in determining the maximum surface temperature, it is unnecessary to solve for the quantity x .

The values obtained are substituted in Equation (g) to obtain the maximum surface temperature.

An example of these calculations is shown in Appendix 5-D.

Results of Calculations:

The results of the calculations outlined in this section are shown in Fig. 4.A.2. It is interesting to note that there is very little variation in the maximum metal surface temperature for a given flow velocity and heat flux for the two extremes of plate spacing considered in these core systems (.075" and .250").

In calculating these values of maximum surface temperature, the stipulation was made that boiling does not occur in the water film. As this is the desired operating condition in a research reactor of the types considered in the report these results are applicable.

4.B. Economic Evaluation

Heretofore, in this report, all cores have been discussed from operational mass criteria. In this section the volumes and corresponding

value for some of the systems are tabulated for an economic comparison. The materials considered here are uranium, beryllium, graphite, and D₂O; their value in no way reflects the total cost of the reactor system. No effort is made to estimate the physical plant cost, i.e., buildings, handling equipment, control systems, and cooling systems.

The volumes and masses of material were calculated on the following basis:

1. The total core volume was taken as 115% of that of the spherical reactor. (As explained in the introduction, the factor 1.15 changes the spherical core to the nuclear-equivalent cylindrical one).
2. The uranium investment was taken as that necessary for design operation.
3. The coolant volume in the core was calculated from cylindrical core volume and metal-to-water ratio.
4. The reflector volume was taken to be the volume of a sleeve of the designated thickness surrounding the 24 inch long cylindrical core. (Fuel element processing requires a 24 inch long active lattice.)

Fig. 4.B.1 shows these volumes as a function of spherical reactor radius.

The prices used for uranium and D₂O are those quoted by the United States at the 1955 Atoms-for-Peace Conference held in Geneva:

20% enriched uranium - \$25/gm of U²³⁵.

D₂O - \$28/pound (\$0.0678/cm³).

The price for fabricated beryllium is in the order of \$150/pound (\$0.569/cm³); for fabricated graphite, \$1.00/pound (\$0.00373/cm³).

By combining the volumes from Fig. 4.B.1 with the above prices,

FIGURE 4.B.1: CORE and REFLECTOR VOLUMES

the economic value of the core can be obtained. In the case of the 50% H₂O - 50% D₂O and 10% H₂O - 90% D₂O systems, the D₂O volume was taken to be 0.50 and 0.90 respectively of the liquid volume.

Table 4.B.1 gives the estimated economic value of representative systems. Systems using other than H₂O for cooling will, of course, require much more D₂O than that contained in the core; the amount depending upon the actual engineering design. Table 4.B.1 included this external volume and its value based upon an estimate of 8 kw per liter for the cooling capacity of the external system. In addition to this added investment, these systems require a completely closed and leak-tight cooling system which is inherently expensive compared to the simpler systems applicable to H₂O cooling.

This tabulation clearly shows the economic advantage of the graphite systems. The cost of these systems is about one half that of the 15 cm beryllium reflected systems. The 50-50 coolant systems are not extremely expensive, the total cost being approximately the same as that of water systems. Both the 10-90 and D₂O coolant systems are quite expensive.

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TABLE 4.B.1
APPROXIMATE VALUE OF REPRESENTATIVE SYSTEMS

CORE			REFLECTOR		DESIGNED OPERATIONAL LOADING	COOLANT IN CORE		REFLECTOR		FUEL	TOTAL CORE VALUE	GENERATED POWER	COOLANT IN EXTERNAL SYSTEM	
Coolant	Size	M/W	Material	Thick- ness		Volume	Value	Volume	Value	Value	VALUE		Volume	Value
	cm.			cm.	kg.	liters	10 ⁶ \$	liters	10 ⁶ \$	10 ⁶ \$	10 ⁶ \$	Mw	liters	10 ⁶ \$
H ₂ O	25	0.7	Be	15	3.55	44.3	----	155	0.092	0.089	0.181	16.9	2100	----
H ₂ O	25	0.7	C	100	3.46	44.3	----	2677	0.010	0.086	0.096	16.7	2090	----
H ₂ O	30	0.7	10 - 90	30	5.79	76.6	----	474	0.029	0.144	0.173	27.5	3440	----
H ₂ O	30	0.7	D ₂ O	75	4.21	76.6	----	1840	0.124	0.105	0.229	20.0	2500	----
H ₂ O	35	0.7	Be	15	5.90	124	----	232	0.138	0.148	0.286	28.0	3500	----
50 - 50	30	0.7	50 - 50	15	5.82	76.6	0.002	194	0.007	0.146	0.155	27.6	3450	0.117
50 - 50	30	0.7	Be	15	3.35	76.6	0.002	194	0.115	0.084	0.201	15.9	1980	0.067
50 - 50	30	0.7	C	100	3.16	76.6	0.002	2920	0.011	0.079	0.092	15.0	1870	0.063
50 - 50	30	1.3	Be	15	3.90	56.5	0.002	194	0.115	0.098	0.215	18.5	2310	0.078
10 - 90	30	0.7	Be	15	3.88	76.6	0.005	194	0.115	0.097	0.217	18.4	2300	0.140
10 - 90	30	0.7	C	100	3.43	76.6	0.005	2920	0.011	0.086	0.101	16.3	2030	0.124
10 - 90	35	0.7	10 - 90	30	5.52	124	0.008	552	0.034	0.138	0.180	26.2	3270	0.200
D ₂ O	35	0.5	C	100	3.42	138	0.009	3180	0.012	0.086	0.106	16.3	2030	0.137
D ₂ O	45	0.5	D ₂ O	60	3.72	289	0.020	1780	0.120	0.093	0.233	17.7	2200	0.150

5. APPENDICES

5.A. Appendix 1. Nuclear Constants

The 2G2R ORACLE code requires as input data certain quantities which serve to specify the system. These include the extrapolation distance in the reflector, the Fermi age in the reflector and the core, fast and thermal group diffusion coefficients in both core and reflector, the resonance escape probability in the core and reflector, the thermal macroscopic absorption cross section of the non-fuel components of the core, and the total thermal absorption cross section in the reflector. The value of η , the number of fission neutrons produced per neutron absorbed in the fuel is also required.

It is assumed in each case that the reflector is made of the pure material making no allowance for canning materials, coolants, or channels. The core is assumed to consist of aluminum and of the moderator-coolant. The presence of the fuel in the core is not allowed for in calculating the age and the diffusion constants in the core for its effect on these quantities is small. The pure material constants from which the data are calculated are listed in Table 5.A.1. These were selected from several sources with the intent of finding the best values possible in each case, although it would be difficult to give a rigorous justification for the choices. The subscripts 1 and 2 refer to fast and thermal group values respectively. Where the constants are not directly available from the literature, they are computed from the following equations and the values so calculated are the ones quoted for the pure material constants in Table 5.A.1:

I. Macroscopic cross sections

$$\Sigma = N \sigma$$

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Figure 5.A.1: AGE FOR $\text{H}_2\text{O} - \text{D}_2\text{O}$ MIXTURES

II. Microscopic transport cross sections

$$\sigma_{tr} = \sigma_s \left[1 - \frac{2}{3A} \right]$$

III. Diffusion coefficients

$$D = \frac{1}{3\Sigma_{tr}}$$

The migration and diffusion lengths quoted in Table 5.A.1 are used only as criterion for choosing reflector thicknesses. They are found by the relations:

$$L = \sqrt{D_2/\Sigma_{a2}}$$

and

$$M = \sqrt{L^2 + \tau}$$

IV. Resonance escape probabilities.

These have been chosen to be 1.0 in the reflectors since the materials picked as reflectors have very low fast absorption cross sections. In the core, the situation is not so simple. The resonance escape probability is required as input datum. However, it depends on the fuel concentration, and therefore, on the outcome of the calculation. Moreover, even if the fuel concentration were known, it would be difficult to calculate the resonance escape probability. This number is strongly affected by the 'lumping' of fuel, and thus, depends upon the plate thicknesses and the plate numbers in the fuel elements. Some doubt has also been cast on the validity of the available methods for the computation of this number by the fact that previous calculations for a reactor of this type were not substantiated by experimental evidence. For these reasons a constant value of 0.97 was used for all calculations. This

value is in accord with that used for other related reactors from experimental results. It is somewhat higher than calculated values, but this is accounted for by the fact that fast fission is not taken into account in the ORACLE code, but does occur to some extent since a large amount of U-238 is present in these reactors. For any specific reactor, the value may be adjusted by choosing the proper number of plates per element, and therefore, the plate thicknesses.

It should be pointed out that the fast microscopic scattering cross section of aluminum was determined by a $1/E$ average of the total microscopic cross section as plotted in AEC-2040 over the energy range from 0.025 ev to 1.0 Mev.

The fast group macroscopic transport cross sections for H_2O and D_2O were found from the fast microscopic cross sections for scattering of $^1H^1$, $^1D^2$, and ^{16}O in the following fashion:

$$\Sigma_{tr1}^{H_2O} = \frac{2}{18} \sigma_{s1}^{1H^1} \frac{.602 \times 10^{24}}{1} (1 - 2/3) + \frac{16}{18} \sigma_{s1}^{^{16}O} \frac{.602 \times 10^{24}}{16} (1 - \frac{2}{3 \times 16})$$

and

$$\Sigma_{tr1}^{D_2O} = \frac{2}{20} \sigma_{s1}^{1D^2} \frac{.602 \times 10^{24}}{2} (1 - \frac{2}{3 \times 2}) + \frac{16}{20} \sigma_{s1}^{^{16}O} \frac{.602 \times 10^{24}}{16} (1 - \frac{2}{3 \times 16})$$

where: $\sigma_{s1}^{1H^1} = 20$ barns

$\sigma_{s1}^{1D^2} = 10$ barns

$\sigma_{s1}^{^{16}O} = 3.8$ barns

The following constants for U-235 are not included in Table 5.A.1

$\sigma_{f2} = 580$ barns

$\checkmark = 2.46$ neutrons per fission

The following symbols are used:

N is the atomic or molecular density of the pure material in atoms/cc.

σ_s is the microscopic cross section for scattering in barns per atom or molecule.

σ_{tr} is the microscopic cross section for transport in barns per atom or molecule.

σ_a is the microscopic cross section for absorption in barns per atom or molecule.

Σ_{tr} is the macroscopic cross section for transport in cm^{-1} .

Σ_a is the macroscopic cross section for absorption in cm^{-1} .

D is the diffusion coefficient in cm.

τ is the Fermi age in cm^2 .

L is the diffusion length in cm.

M is the migration length in cm.

The following relationships were used to obtain the required input data from these basic values.

1. Extrapolation distance:

$$\tau = \frac{.71}{\Sigma_{tr2}} \quad \text{where } \Sigma_{tr2} \text{ is the property of the reflector.}$$

2. Fermi age: The Fermi age of light water - heavy water mixtures was computed from data obtained by private communication and is shown in Fig. 5.A.1. The ages of mixtures other than this, i.e., of the various coolants with aluminum, were computed by the equations

$$\frac{1}{\sqrt{\tau_{\text{mix}}}} = \frac{V_a}{\sqrt{\tau_a}} + \frac{V_b}{\sqrt{\tau_b}}$$

where v_i is the volume fraction of the i 'th component of the mixture and τ_i is the age of the i 'th component of the mixture.

3. Diffusion Coefficients: The diffusion coefficient of a mixture was found by the equation:

$$D_{\text{mix}} = \frac{(D_a) (D_b)}{(v_a) (D_b) + (v_b) (D_a)}$$

Where v_i is the volume of the i 'th component of the mixture and D_i is the diffusion coefficient of the i 'th component.

4. Resonance Escape Probability: The resonance escape probability was taken to be 0.97 in the core and 1.0 in the reflector in all cases.

5. Macroscopic Cross Sections: Macroscopic cross sections of mixtures were computed by the formula:

$$Z_{\text{mix}} = v_a \Sigma_a + v_b \Sigma_b$$

Where: v_i is the volume fraction of the i 'th component of the mixture

Σ_i is the macroscopic cross section of the i 'th component

6. η : η was computed to be 2.05 for the 20% enriched mixture.

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TABLE 5.A.1 NUCLEAR CONSTANTS

	N_{cm}^{-3}	$\sigma_{s2}(b)$	$\sigma_{s1}(b)$	$\sigma_{tr2}(b)$	$\sigma_{tr1}(b)$	$\sigma_{a2}(b)$	$\Sigma_{a2}^{(cm)^{-1}}$	$\Sigma_{tr2}^{(cm)^{-1}}$	$\Sigma_{tr1}^{(cm)^{-1}}$	D_{2cm}	D_{1cm}	$\tau(cm^2)$	$L(cm)$	$M(cm)$
H ₂ O	3.34×10^{22}					.660 ⁽³⁾	.0221	2.34 ⁽⁴⁾	.564 ⁽⁵⁾	.142	.590	33.8 ⁽⁶⁾	2.54	6.35
D ₂ O	3.31×10^{22}					.00093 ⁽³⁾	3.06×10^{-5}	.418 ⁽⁴⁾	.303 ⁽⁵⁾	.80	1.10	120.0 ⁽⁶⁾	162.0	162
Be	12.3×10^{22}	6.2 ⁽¹⁾	6.0 ⁽¹⁾	5.74	5.52	.009 ⁽¹⁾	.00111	.71	.68	.47	.49	89.0 ⁽⁷⁾	20.6	22.6
Al	6.02×10^{22}	1.6 ⁽¹⁾	1.9 ⁽²⁾	1.56	1.87	.215	.0129	.094	.112	3.55	2.97	308.0 ⁽⁷⁾	16.7	24.2
U ²³⁵	4.79×10^{22}					687.								
U ²³⁸	4.73×10^{22}					2.74								
HDO							.0110	1.38	.434	.242	.767	42.3 ⁽⁶⁾	4.67	8.0
H _{0.2} D _{1.8} O							.00223	.610	.329	.548	1.011	84.5 ⁽⁶⁾	15.7	18.2
C	8.12					.00435	.00353	.365 ⁽³⁾	.368	.912	.905	350 ⁽⁴⁾	50.8	54.3
<div> <div> (1. A.E.C.U. - 2040 (2. Computed from A.E.C.U. - 2040 (3. Hughes <u>Pile Neutron Research</u> (4. Stevenson <u>Introduction to Nuclear Engineering</u> </div> <div> (5. From σ_{s1} of $^1H^1$, $^1D^2$, ^{16}O, in A.E.C.U. - 2040 (6. Computed from ORNL-CF-54-1-45 (7. Personal Communication, T. Fowler </div> </div>														

5.B. APPENDIX II.

Table of Results

The following table summarizes our results in tabular form.

The reactor configurations are arranged in the following order:

1. Core moderator-coolant:

H_2O ; HDO ; $10\%H_2O - 90\%D_2O$; D_2O

2. Core radius

3. Moderator composition:

H_2O ; HDO ; $10\%H_2O - 90\%D_2O$; D_2O ; Beryllium; Graphite

4. Reflector thickness

5. Metal-to-water ratio

The columns contain the following information:

1. Core moderator material

2. Core radius in cm.

3. Reflector material

4. Reflector thickness in cm.

5. Metal-to-water ration in the core

6. Thermal utilization (f)

7. Adjusted cold clean critical loading in Kg. (U-235)

8. Loading for design operation in Kg. (U-235)

9. Power in Megawatts

10. Power density in Watts /cm³

11. Macroscopic thermal absorption crossection of the Uranium
in the core in cm⁻¹

5.B.1 Table of Results

1	2	3	4	5	6	7	8	9	10	11
H ₂ O	22.5	H ₂ O	10	0.7	.838	2.84	-	-	-	.0925
"	25	"	"	"	.795	2.97	9.80	42.6	650	.0704
"	30	"	"	"	.733	3.62	7.85	37.3	330	.0496
"	35	"	"	"	.686	4.58	8.52	40.5	225	.0398
"	25	HDO	7.5	0.7	.810	3.30	-	-	-	.0782
"	30	"	"	"	.740	3.78	8.47	40.2	355	.0518
"	35	"	"	"	.689	4.67	8.72	41.5	224	.0404
"	25	"	"	1.3	.870	4.68	-	-	-	.111
"	30	"	"	"	.791	4.63	14.8	70.4	624	.0636
"	35	"	"	"	.733	5.53	12.0	57.0	308	.0463
"	25	"	15	0.7	.777	2.67	7.64	36.3	55	.0632
"	30	"	"	"	.716	3.34	6.70	31.8	281	.0458
"	35	"	"	"	.673	4.31	7.75	36.8	199	.0373
"	25	"	"	1.3	.820	3.26	-	-	-	.07731
"	30	"	"	"	.755	3.80	8.00	38.0	336	.0522
"	35	"	"	"	.706	4.86	8.31	39.5	214	.0407
"	25	.1 H ₂ O .9D ₂ O	"	0.7	.783	2.58	7.70	36.6	559	.0645
"	30	"	"	"	.720	3.37	6.94	33.0	292	.0464
"	35	"	"	"	.673	4.34	7.81	37.2	201	.0375
"	25	"	"	1.3	.828	3.42	-	-	-	.0810
"	30	"	"	"	.760	3.84	9.65	45.9	406	.0530
"	35	"	"	"	.709	4.73	9.40	44.6	241	.0410
"	25	"	30	0.7	.748	2.28	5.34	25.4	388	.0540
"	30	"	"	"	.700	2.99	5.79	25.5	226	.0410
"	35	"	"	"	.654	3.98	6.91	32.8	177	.0344

1	2	3	4	5	6	7	8	9	10	11
H ₂ O	25	1 H ₂ O	30	1.3	.784	2.58	7.8	37.0	565	.0606
"	30	.9 D ₂ O	"	"	.724	3.10	6.50	30.9	273	.0442
"	35	"	"	"	.683	4.19	7.74	36.8	198	.0361
"	25	D ₂ O	75	0.7	.696	1.75	3.32	15.75	240	.0415
"	30	"	"	"	.649	2.45	4.21	20.0	177	.0337
"	35	"	"	"	.619	3.40	5.61	26.8	149	.0294
"	25	"	"	1.3	.720	1.8	3.77	17.9	273	.0428
"	30	"	"	"	.669	2.47	4.41	20.9	185	.0339
"	35	"	"	"	.635	3.38	5.66	27.4	152	.0292
"	25	"	120	0.7	.683	1.66	3.06	14.5	221	.0415
"	30	"	"	"	.640	2.35	3.98	18.9	167	.0337
"	35	"	"	"	.606	3.27	5.25	25.0	139	.0294
"	25	"	"	1.3	.709	1.68	3.32	15.9	243	.0398
"	30	"	"	"	.656	2.33	4.05	19.2	169	.0321
"	35	"	"	"	.625	3.22	5.32	25.2	140	.0279
"	22.5	Be	15	0.7	.731	1.53	3.30	15.7	330	.0499
"	25	"	"	"	.703	1.82	3.55	16.8	257	.0430
"	30	"	"	"	.658	2.54	4.43	21.0	168	.0350
"	35	"	"	"	.629	3.65	6.06	28.8	155	.0306
"	25	"	15	1.0	.719	1.89	3.90	18.5	282	.0448
"	30	"	"	"	.6275	2.61	4.70	22.3	197	.0359
"	35	"	"	"	.641	3.60	6.09	28.9	160	.0310
"	22.5	"	"	1.3	.789	1.94	6.15	29.2	614	.0628
"	25	"	"	"	.732	1.94	4.20	19.9	304	.0458
"	30	"	"	"	.682	2.64	4.86	23.1	204	.0362
"	35	"	"	"	.650	3.70	6.36	30.2	163	.0310

1	2	3	4	5	6	7	8	9	10	11
H ₂ O	22.5	Be	40	0.7	.690	1.24	2.33	11.1	233	.0404
"	25	"	"	"	.665	1.52	2.69	12.8	269	.0361
"	30	"	"	"	.629	2.23	3.70	17.6	269	.0307
"	35	"	"	"	.603	3.29	5.26	25.0	221	.0276
"	25	"	"	1.0	.659	1.535	2.68	12.7	194	.0365
"	30	"	"	"	.636	2.23	3.74	17.7	156	.0307
"	35	"	"	"	.613	3.19	5.16	24.5	136	.0275
"	22.5	"	"	1.3	.764	1.67	4.31	20.4	430	.0544
"	30	"	"	"	.646	2.23	3.80	18.0	159	.0306
"	35	"	"	"	.619	3.24	5.28	25.1	135	.0272
"	22.5	C	50	0.7	.750	1.66	5.86	27.8	584	.0541
"	25	"	"	"	.714	1.90	3.84	18.2	274	.0451
"	30	"	"	"	.663	2.58	4.55	21.6	191	.0354
"	35	"	"	"	.628	3.63	6.02	28.6	154	.0305
"	22.5	"	"	1.3	.782	1.85	5.47	26.0	545	.0604
"	25	"	"	"	.744	2.06	4.74	22.5	344	.0488
"	30	"	"	"	.686	2.68	4.95	23.5	208	.0369
"	35	"	"	"	.648	3.59	6.15	29.2	162	.0310
"	22.5	"	100	0.7	.736	1.54	5.58	26.5	555	.0501
"	25	"	"	"	.701	1.78	3.46	16.4	250	.0423
"	30	"	"	"	.651	2.46	4.25	20.2	179	.0336
"	35	"	"	"	.619	3.52	5.74	27.3	148	.0296
"	22.5	"	"	1.3	.764	1.67	4.31	20.3	431	.0544
"	25	"	"	"	.726	1.89	3.99	18.9	289	.0448
"	30	"	"	"	.674	2.52	4.65	21.6	191	.0347
"	35	"	"	"	.637	3.42	5.75	27.3	151	.0295

1	2	3	4	5	6	7	8	9	10	11
HDO	25	HDO	7.5	0.7	.845	2.72	-	-	-	.0642
"	30	"	"	"	.777	2.96	8.50	40.4	356	.0407
"	35	"	"	"	.723	3.66	7.62	36.2	196	.0307
"	25	"	"	1.3	.887	3.94	-	-	-	.0932
"	30	"	"	"	.814	3.84	-	-	-	.0527
"	35	"	"	"	.759	4.50	11.2	53.2	287	.0377
"	25	"	15	0.7	.805	2.08	-	-	-	.0485
"	30	"	"	"	.747	2.50	5.82	27.6	244	.0344
"	35	"	"	"	.699	3.26	6.29	29.9	166	.0274
"	25	"	"	1.3	.835	2.56	-	-	-	.0607
"	30	"	"	"	.775	3.00	8.40	39.8	352	.0412
"	35	"	"	"	.730	3.82	8.18	38.6	209	.0320
"	25	Be	7.5	0.7	.795	1.90	5.45	25.9	395	.0450
"	22.5	"	15	"	.761	1.14	2.89	13.7	287	.0370
"	25	"	"	"	.725	1.32	2.77	13.2	202	.0314
"	30	"	"	"	.683	1.82	3.35	16.0	142	.0251
"	35	"	"	"	.648	2.49	4.25	20.2	112	.0216
"	22.5	"	"	1.0	.770	1.21	3.26	15.5	326	.0395
"	25	"	"	"	.738	1.41	3.13	14.9	228	.0334
"	30	"	"	"	.690	1.92	3.60	17.1	151	.0264
"	35	"	"	"	.658	2.61	4.57	21.7	117	.0226
"	25	"	"	1.3	.748	1.50	3.48	16.5	252	.0355
"	30	"	"	"	.699	2.02	3.90	18.5	164	.0278
"	35	"	"	"	.666	2.74	4.86	23.1	129	.0237
"	30	"	40	0.7	.645	1.55	2.65	12.6	111	.0213

1	2	3	4	5	6	7	8	9	10	11
HDO	35	Be	40	0.7	.619	2.20	3.60	17.1	95.2	.0190
"	30	"	"	1.0	.651	1.61	2.77	13.1	116	.0221
"	35	"	"	"	.623	2.28	3.75	17.8	99	.0196
"	35	"	"	1.3	.630	2.33	3.82	18.1	101	.0202
"	25	C	50	0.7	.738	1.39	2.22	10.6	162	.0333
"	30	"	"	"	.686	1.81	3.35	15.9	141	.0256
"	35	"	"	"	.646	2.49	4.26	20.2	112	.0216
"	25	"	"	1.3	.760	1.60	4.02	19.1	292	.0379
"	30	"	"	"	.704	2.05	4.02	19.1	169	.0283
"	35	"	"	"	.662	2.72	4.78	22.7	126	.0235
"	25	"	100	0.7	.725	1.30	2.70	12.8	195	.0308
"	30	"	"	"	.672	1.76	3.16	15.0	133	.0241
"	35	"	"	"	.637	2.38	3.96	18.8	106	.0206
"	25	"	"	1.3	.740	1.45	3.24	15.4	236	.0343
"	30	"	"	"	.686	1.91	3.54	16.8	146	.0263
"	35	"	"	"	.650	2.57	4.17	19.2	106	.0222
.1H ₂ O .9D ₂ O	25	.1 H ₂ O .9 D ₂ O	15	0.7	.843	1.49	-	-	-	.0351
"	35	"	"	"	.790	2.98	9.52	45.1	244	.0250
"	30	"	"	1.3	.857	3.28	-	-	-	.0450
"	35	"	"	"	.797	3.81	-	-	-	.0319
"	30	"	30	0.7	.797	1.85	-	-	-	.0255
"	35	"	"	"	.749	2.35	5.52	26.2	146	.0197
"	40	"	"	"	.713	2.83	5.69	27.0	104	.0164
"	30	"	"	1.3	.793	2.26	7.26	34.5	305	.0311
"	35	"	"	"	.750	2.90	6.84	32.5	175	.0243
"	30	Be	15	0.7	.763	1.52	3.88	18.4	163	.0209

1	2	3	4	5	6	7	8	9	10	11
.1 H ₂ O	35	Be	15	0.7	.720	1.95	4.06	19.3	107	.0168
.9 D ₂ O	45	"	"	"	.665	3.15	5.52	26.2	68.6	.0129
"	35	"	"	1.3	.718	2.38	4.89	23.2	129	.0206
"	45	"	40	0.7	.628	2.69	4.38	20.8	54.5	.0110
"	30	C	50	"	.765	1.55	4.02	19.1	169	.0123
"	35	"	"	"	.718	1.93	3.98	18.9	105	.0167
"	45	"	"	"	.656	3.07	5.35	25.4	66.5	.0125
"	35	"	"	1.3	.740	2.35	5.28	25.1	139	.0203
"	45	"	"	"	.652	3.76	6.40	30.4	79.5	.0153
"	35	"	100	0.7	.700	1.77	3.43	16.2	89.5	.0153
"	45	"	"	"	.643	2.87	4.87	23.1	60.4	.0117
"	45	"	"	1.3	.641	3.52	5.79	20.4	53.4	.0143
D ₂ O	35	D ₂ O	60	0.3	.758	1.06	2.61	12.4	69.0	.00919
"	45	"	"	"	.682	1.62	2.98	14.2	37.2	.00661
"	55	"	"	"	.648	2.43	4.16	19.8	28.2	.00542
"	45	"	"	.5	.666	2.12	3.72	17.7	46.4	.00861
"	55	"	"	"	.630	3.24	5.30	25.6	36.3	.00723
"	55	"	"	0.7	.625	3.86	6.27	29.8	42.9	.00868
"	35	Be	15	0.3	.796	1.32	-	-	-	.0114
"	45	"	"	"	.730	1.98	4.21	20.3	53.2	.00797
"	55	"	"	"	.683	2.85	5.26	25.0	23.4	.00636
"	35	"	"	0.5	.770	1.65	4.44	21.1	117	.0143
"	45	"	"	"	.707	2.52	4.98	23.7	62.1	.01027
"	55	"	"	"	.664	3.77	6.67	31.7	45.6	.0084
"	35	"	"	0.7	.758	1.90	4.70	22.3	124	.0164
"	45	"	"	"	.698	2.96	5.68	27.0	70.7	.01203

1	2	3	4	5	6	7	8	9	10	11
D ₂ O	55	Be	15	0.7	.642	4.46	7.59	36.0	52.0	.00995
"	45	"	40	0.3	.685	1.59	2.95	14.0	36.7	.00645
"	55	"	"	"	.648	2.42	4.14	19.5	28.1	.00542
"	45	"	"	0.5	.664	2.06	3.65	17.3	40.3	.00836
"	55	"	"	"	.629	3.22	5.35	25.4	32.5	.00718
"	55	"	"	0.7	.616	3.84	6.13	29.1	42.0	.00852
"	35	C	50	0.5	.769	1.63	4.36	20.7	115	.0141
"	45	"	"	"	.698	2.42	4.62	21.9	57.2	.00983
"	55	"	"	"	.651	3.58	6.16	29.3	42.0	.00798
"	35	"	"	0.7	.757	1.86	4.59	21.8	121	.01615
"	45	"	"	"	.687	2.83	5.16	24.5	64.3	.0115
"	55	"	"	"	.644	4.23	7.18	34.1	48.9	.00945
"	35	"	100	0.5	.749	1.46	3.42	16.2	90.0	.0126
"	45	"	"	"	.680	2.23	4.02	19.1	50.0	.00907
"	55	"	"	"	.637	3.36	5.65	26.8	38.4	.00743
"	45	"	"	0.7	.669	2.55	4.58	21.8	57.0	.0106
"	55	"	"	"	.626	3.98	6.56	29.3	42.1	.00883

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TABLE 5.B.2 RESULTS CORRELATED WITH REFLECTORS

Reflector H ₂ O - 100 cm												
$\frac{M}{W} = 0.7$	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50H ₂ O - 50D ₂ O												
10H ₂ O - 90D ₂ O												
D ₂ O												

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Reflector 7.5 cm 50 H ₂ O - 50 D ₂ O												
M/W = .7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		3.30	3.78	4.67				--	8.47	8.73		
50 H ₂ O - 50 D ₂ O		2.72	2.96	3.66				--	8.50	7.67		
10 H ₂ O - 90 D ₂ O												
D ₂ O												

Reflector 7.5 cm - 50 H ₂ O - 50 D ₂ O												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		4.68	4.63	5.39				--	14.8	11.7		
50 H ₂ O - 50 D ₂ O		3.94	3.84	4.50				--	--	11.2		
10 H ₂ O - 90 D ₂ O												
D ₂ O												

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Reflector 15 cm 10 H ₂ O - 90 D ₂ O												
M/W = .7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		2.58	3.37	4.34				7.70	6.94	7.81		
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O		1.29		2.59					19			
D ₂ O												

Reflector 15 cm 10 H ₂ O - 90 D ₂ O												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		3.42	3.84	4.73					9.65	9.4		
50 H ₂ O - 50 H ₂ O												
10 H ₂ O - 90 D ₂ O			3.28	3.81								
D ₂ O												

Reflector 30 cm 10 H₂O - 90 D₂O

Reflector 30 cm 10 H ₂ O = 90 D ₂ O												
M/W = .7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		2.28	2.99	3.98				5.34	5.79	6.91		
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O			1.85	2.35					--	5.52		
D ₂ O												

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Reflector 30 cm 10 H₂O - 90 D₂O

Reflector 30 cm 10 H ₂ O - 90 D ₂ O												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		2.58	3.10	4.19				7.80	6.50	7.74		
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O			2.26	2.90					7.26	6.84		
D ₂ O												

Reflector 60 cm D ₂ O												
M/W = 0.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O				1.06	1.62	2.43				2.61	2.98	4.16

Reflector 60 cm D ₂ O												
M/W = 0.5	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O					2.12	3.24					3.72	5.30
10 H ₂ O - 90 D ₂ O												
D ₂ O												

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Reflector 60 cm D ₂ O												
M/W	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O						3.86						6.27

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Reflector D ₂ O - 75 cm												
M/W = .7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		1.75	2.45	3.40				3.32	4.21	5.61		
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O												

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Reflector D ₂ O - 75 cm												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O		1.81	2.47	3.38				3.77	4.41	5.66		

Reflector 120 cm D ₂ O												
M/W = 0.7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		1.66	2.35	3.27				3.06	3.98	5.25		
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - .90 D ₂ O												
D ₂ O												

[illegible]

Reflector 7.5 cm Be

Reflector 7.5 cm Be												
M/W = 0.7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	40	45	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O		1.90						6.26				
10 H ₂ O - 90 D ₂ O												
D ₂ O												

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[illegible]

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Reflector 15 cm Be												
M/W = 0.5	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O				1.65	2.52	3.77				4.44	4.98	6.67

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Reflector 15 cm Be												
M/W = 0.7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		1.82	2.54	3.55				3.55	4.43	5.90		
50 H ₂ O - 50 D ₂ O	1.14	1.32	1.82	2.49			2.89	2.77	3.35	4.25		
10 H ₂ O - 90 D ₂ O			1.52	1.95	3.15				3.88	4.06	5.52	
D ₂ O				1.90	2.96	4.46				4.70	5.68	7.59

Reflector 15 cm Be

[illegible]

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Reflector 15 cm Be

[illegible]

[illegible][illegible]

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Reflector 40 cm Be												
M/W = 0.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O					1.59	2.42					2.95	4.14

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Reflector 40 cm Be												
M/W = 0.5	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O					2.06	3.22					3.65	5.35

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Reflector 40 cm Be												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O			2.33	3.15					3.80	5.14		
50 H ₂ O - 50 D ₂ O				2.33						3.02		
10 H ₂ O - 90 D ₂ O												
D ₂ O												

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Reflector Graphite 50 cm												
M/W = 0.5	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O				1.63	2.42	3.58				4.36	4.62	6.16

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Reflector Graphite 50 cm												
M/W = 0.7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		1.90	2.58	3.54				3.84	4.55	5.86		
50 H ₂ O - 50 D ₂ O		1.39	1.81	2.495				2.22	3.35	4.26		
10 H ₂ O - 90 D ₂ O			1.55		1.93	2.07			4.02		3.98	5.35
D ₂ O				1.86	2.83	4.23				4.59	5.16	7.18

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Reflector Graphite 50 cm												
M/W = 1.3	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		2.06	2.68	3.59				4.74	4.95	6.15		
50 H ₂ O - 50 D ₂ O		1.60	2.05	2.72				4.02	4.02	4.78		
10 H ₂ O - 90 D ₂ O				2.35	3.76					5.28	6.40	
D ₂ O												

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Reflector Graphite 100 cm												
M/W = 0.5	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O												
50 H ₂ O - 50 D ₂ O												
10 H ₂ O - 90 D ₂ O												
D ₂ O				1.46	2.23	3.36				3.42	4.09	5.65

Reflector Graphite . 100 cm

Reflector Graphite . 100 cm												
M/W = .7	Adjusted Cold Clean Mass						Loading for Designed Operation					
Core	22.5	25	30	35	45	55	22.5	25	30	35	45	55
H ₂ O		1.78	2.46	3.42				3.46	4.25	5.58		
50 H ₂ O - 50 D ₂ O		1.30	1.755	2.38				2.70	3.16	3.96		
10 H ₂ O - 90 D ₂ O			1.77		2.87				3.43		4.87	
D ₂ O					2.55	3.98					4.58	6.56

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Reflector Graphite 100 cm

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5.C. Appendix III. Table of Engineering Nomenclature

A	Length of fuel plate, feet
C _p	Specific heat, BTU/lb °F
D ^l	Hydraulic diameter, inches
D _e	Equivalent hydraulic diameter, feet
f	Forming friction factor
g _c	Gravity constant, 32.2 feet/sec ²
G	Water flow rate, pounds/foot of width-second
G ^l	Water flow rate, pounds/foot ² -second
h	Film heat transfer coefficient BTU/foot ² -hour-°F
H	Total heat generated, BTU/foot of width-second
K _c	Constant relating head losses due to sudden contraction to flow rate
K _e	Constant relating head loss due to expansion to flow rate
L	Length, feet
Δ _p	Pressure loss, pounds/inch ²
Q	Heat flux, BTU/foot ² hour
s	Specific volume foot ³ /pound
t	Average bulk water temperature, °F
T _m	Surface temperature, °F
ΔT	Temperature change in water flowing past fuel plate, °F
T _w	Water temperature, °F
u	Flow rate, feet/second
u _H	Velocity in smaller cross section, feet/second
u _L	Velocity in larger cross section, feet/second
V	Velocity between fuel plates, feet/second
ρ	Density, pounds/foot ²
μ	Viscosity, pounds/foot-second

5.D. Appendix IV. Engineering Calculations

Pressure Loss Calculations:

for T = 100°F

$$\mu = .72 \text{ cp} = .000484 \text{ #/ft. sec}$$

$$\rho = 62.0 \text{ #/ft}^2$$

V = velocity between plates (ft/sec)

Flow Areas:

	<u>M/W = .7</u>	<u>M/W = 1.3</u>
In fuel plate section (3"x3" = .0625 ft ²)	.0368 ft ²	.0272 ft ²
In 2.250" I.D. pipe	.0276 ft ²	.0276 ft ²
In 2.625" I.D. pipe	.0376 ft ²	.0376 ft ²
In 3.00" x 2.75" area	.0574 ft ²	.0574 ft ²

Velocity:

	<u>M/W = .7</u>	<u>M/W = 1.3</u>
In fuel plate section	1.0 V	1.0 V
In 2.250" I.D. pipe	1.402 V	.985 V
In 2.625" I.D. pipe	1.028 V	.723 V
In 3.00" x 2.75" area	.674 V	.474 V
Above and below assembly	.10 V	.10 V

Pressure Loss:

In upper end box

$$\text{Entrance loss } \Delta P = \frac{k_e u^2 \rho}{2(144) g_c}$$

$$\Delta P(0.7) = \frac{(.3)(1.028 \text{ V})^2(62.0)}{2(144) 32.2} = 0.00211 \text{ V}^2$$

$$\Delta P(1.3) = \frac{(.3)(.985 \text{ V})^2(62.0)}{2(144) 32.2} = 0.00195 \text{ V}^2$$

Friction in 2.250" I.D. pipe:

$$Re = \frac{D_e u \rho}{\mu}$$

$$Re(0.7) = \frac{(2.25/12)(1.402 V)(62.0)}{(.000484)} = 3.37 \times 10^4 V$$

$$Re(1.3) = \frac{(2.25/12)(.985 V)(62.0)}{(.000484)} = 2.363 \times 10^4 V$$

$$\Delta P = \frac{2f u^2 \rho L}{144 D_e g_c}$$

$$\Delta P(0.7) = \frac{2f(1.402 V)^2(62.0)(7/12)}{(2.250/12)(32.2)(144)} = .1640 V^2 f$$

$$\Delta P(1.3) = \frac{2f(.985 V)^2(62.0)(7/12)}{(2.250/12)(32.2)(144)} = .0808 V^2 f$$

V(ft/sec)	M/W = 0.7			M/W = 1.3		
	Re	f	ΔP	Re	f	ΔP
1	3.37×10^4	.0058	.000952	2.36×10^4	.0063	.000509
3	1.02×10^5	.0045	.00665	7.09×10^4	.0048	.00349
5	1.69×10^5	.0040	.0164	1.08×10^5	.0050	.0101
10	3.37×10^5	.0036	.0590	2.36×10^5	.0038	.0307
20	6.74×10^5	.0032	.210	4.72×10^5	.0033	.107
30	1.02×10^6	.0027	.399	7.09×10^5	.0031	.226

Plotting Δp against V on log-log paper establishes the relationship:

$$\Delta P(0.7) = 0.00095 V^{1.79}$$

$$\Delta P(1.3) = 0.00051 V^{1.79}$$

Friction if 5.50" long expander:

$$\Delta p = \frac{K_E(u_H - u_L)^2}{2 g_c(144)}$$

$$\Delta P(0.7) = \frac{(.15)(1.402 V - 1.028 V)^2 (62.0)}{2(32.2)(144)} = .000140 V^2$$

$$\Delta P(1.3) = \frac{(.15)(.985 V - .723 V)^2 (62.0)}{2(32.2)(144)} = .0000686 V^2$$

Total Head Loss, Upper Box:

$$\Delta P(0.7) = .00342 V^2 + .00095 V^{1.79}$$

$$\Delta P(1.3) = .00267 V^2 + .00051 V^{1.79}$$

Velocity Head Change:

$$\Delta P = \frac{(u_p^2 - u_1^2) \rho}{2 g_c (144)}$$

$$\Delta P(0.7) = \frac{[(.674 V)^2 - (.10 V)^2] (62.0)}{2(32.2)(144)} = .00297 V^2$$

$$\Delta P(1.3) = \frac{[(.474 V)^2 - (.10 V)^2] (62.0)}{2(32.2)(144)} = .00144 V^2$$

Net Pressure Loss, upper end box:

$$\Delta P(0.7) = .00639 V^2 + .00095 V^{1.79}$$

$$\Delta P(1.3) = .00411 V^2 + .00051 V^{1.79}$$

In Fuel Plate Section:

Entrance Loss

$$\Delta P = \frac{K U^2 \rho}{2 g_c (144)}$$

$$\Delta P(0.7) = \frac{(.25)(V^2)(62.0)}{2(32.2)(144)} = .00167 V^2$$

$$\Delta P(1.3) = \frac{(.30)(V^2)(62.0)}{2(32.2)(144)} = .00200 V^2$$

Friction in Plates

$$R_e = \frac{D_e u \rho}{\mu}$$

$$Re(.250) = \frac{(.500)(V)(62.0)}{12(.000484)} = 5.34 \times 10^3 V$$

$$Re(.075) = \frac{(.150)(V)(62.0)}{12(.000484)} = 1.61 \times 10^3 V$$

where $Re(.250)$ = Reynold's Number within .250" plate spacing

$Re(.075)$ = Reynold's Number within .075 plate spacing

$$\Delta p = \frac{2fu^2 \rho L}{g_c D_e (144)}$$

$$\Delta p(.250) = \frac{2f V^2 (62.0)(2.0)}{(32.2)(.15/12)(144)} = 1.285 f V^2$$

$$\Delta p(.075) = \frac{2f V^2 (62.0)(2.0)}{(32.2)(.15/12)(144)} = 4.28 f V^2$$

V(ft/sec)	0.250" Plate Spacing			0.015" Plate Spacing		
	Re(.250)	f	$\Delta p(.250)$	Re(.075)	f	$\Delta p(.075)$
1	5.34×10^3	.0093	.012	1.61×10^3	.013	.0056
3	1.6×10^4	.0068	.0785	4.83×10^3	.0096	.370
5	2.67×10^4	.0063	.203	8.05×10^3	.0081	.866
10	5.34×10^4	.0053	.680	1.61×10^4	.0071	3.04
20	1.07×10^5	.0046	2.360	3.22×10^4	.0060	10.25
30	1.60×10^5	.0044	5.10	4.83×10^4	.0057	21.95

Against the relationship between Δp and V is found to be

$$\Delta p(.250) = .012 V^{1.79}$$

$$\Delta p(.075) = .05 V^{1.79}$$

Exit Loss

$$\Delta p = \frac{(u_H - u_L)^2 \rho}{2g_c (144)}$$

$$\Delta p(0.7) = \frac{(V - .674 V)^2 (62.0)}{2(32.2)(144)} = .00071 V^2$$

$$\Delta p(1.3) = \frac{(V - .474 V)^2 (62.0)}{2(32.2)(144)} = .00185 V^2$$

Total Head Loss in Fuel Section (also net pressure loss):

For M/W = 0.7	.250" plate spacing	.015" plate spacing
Δp	$.00238 V^2 + .012 V^{1.79}$	$.00238 V^2 + .05 V^{1.79}$
For M/W = 1.3		
Δp	$.00385 V^2 + .012 V^{1.79}$	$.00385 V^2 + .05 V^{1.79}$

In Lower End Box:

Sudden Contraction Loss -

$$\Delta p = \frac{K u^2 \rho}{2g_c(144)}$$

$$\Delta p(0.7) = \frac{(.20)(.674 V)^2 (62.0)}{2(32.2)(144)} = .000607 V^2$$

$$\Delta p(1.3) = \frac{(.20)(.474 V)^2 (62.0)}{2(32.2)(144)} = .000310 V^2$$

Friction Loss (using some Re as in upper end box):

$$\Delta p = \frac{2 f u^2 \rho L}{g_c D_e(144)}$$

$$\Delta p(0.7) = \frac{2 f V^2 (1.402)^2 (62.0) (12.5/12)}{(2.25/12)(32.2)(144)} = .293 f V^2$$

$$\Delta p(1.3) = \frac{2 f V^2 (.985)^2 (62.0) (12.5/12)}{(2.25/12)(32.2)(144)} = .145 f V^2$$

V(ft/sec)	M/W = 0.7			M/W = 1.3		
	Re	f	Δp	Re	f	Δp
1	3.37×10^4	.0058	.00170	2.36×10^4	.0063	.00912
5	1.69×10^5	.0040	.0293	1.08×10^5	.0050	.0180
10	3.37×10^5	.0036	.1055	2.36×10^5	.0038	.0550
20	6.74×10^5	.0032	.376	4.72×10^5	.0033	.1910
30	1.02×10^6	.0027	.712	7.09×10^5	.0031	.404

The relationship between Δp and V is:

$$\Delta p(0.7) = .0017 V^{1.79}$$

$$\Delta p(1.3) = .0010 V^{1.79}$$

Exit Loss:

$$\Delta p = \frac{(u_H - u_L)^2 \rho}{2g_c(144)}$$

$$\Delta p(0.7) = \frac{(1.402 V - .10 V)^2 (62.0)}{2(32.2)(144)} = .0113 V^2$$

$$\Delta p(1.3) = \frac{(.985 V - .10 V)^2 (62.0)}{2(32.2)(144)} = .00523 V^2$$

Total Head Loss, Lower End Box:

$$\Delta p(0.7) = .0119 V^2 + .0017 V^{1.79}$$

$$\Delta p(1.3) = .00554 V^2 + .0017 V^{1.79}$$

Velocity Head Change:

This is the negative of the upper end box change -

$$\Delta p(0.7) = -.00297 V^2$$

$$\Delta p(1.3) = -.00144 V^2$$

Net Pressure Loss in Lower End Box:

$$\Delta p(0.7) = .00894 V^2 + .0017 V^{1.79}$$

$$\Delta p(1.3) = .00410 V^2 + .0010 V^{1.79}$$

Total Pressure Loss in Fuel Element:

$$M/W = 0.7$$

In upper end box
In lower end box
Total end box loss

$$\begin{array}{r} .00639 V^2 + .00095 V^{1.79} \\ .00894 V^2 + .0017 V^{1.79} \\ \hline .01533 V^2 + .00265 V^{1.79} \end{array}$$

Fuel plates (.250" plate opening)	$.00238 V^2 + .012 V^{1.79}$
Total with .250" plate spacing	$.01771 V^2 + .01465 V^{1.79}$
Fuel plates (.075" plate spacing)	$.00238 V^2 + .050 V^{1.79}$
Total with .075" plate spacing	$.01771 V^2 + .05265 V^{1.79}$

$$M/W = 1.3$$

In upper end box	$.00411 V^2 + .00051 V^{1.79}$
In lower end box	$.00410 V^2 + .0010 V^{1.79}$
Total end box loss	$.00821 V^2 + .00151 V^{1.79}$
Fuel plates (.250" plate spacing)	$.00385 V^2 + .012 V^{1.79}$
Total with .250" plate spacing	$.01206 V^2 + .0135 V^{1.79}$
Fuel plates (.075" plate spacing)	$.00385 V^2 + .050 V^{1.79}$
Total with .075" plate spacing	$.01206 V^2 + .0515 V^{1.79}$

Sample Calculation of Maximum Surface Temperature

For a maximum heat flux of 5×10^5 BTU/ft²hr the total heat transferred from a foot-wide strip of fuel is

$$\begin{aligned}
 H &= 3.241 \times 10^{-4} \\
 &= (3.241 \times 10^{-4})(5 \times 10^5)(2.781) \\
 &= 452 \text{ BTU/ft. sec.}
 \end{aligned}$$

For a heat capacity of 1.00 BTU/#°F the rise in the temperature of water flowing past the fuel plate is

$$\begin{aligned}
 T &= H/G \cdot C_p \\
 &= 452/G
 \end{aligned}$$

G is the mass flow rate of water past a foot-wide strip of plate. This value may be related to the mass flow G' , per square foot of flow area by:

$$G' = \frac{G (12)}{(D'/2)}$$

The average bulk water temperature is found by

$$t = \frac{T_{w1} + (T_{w1} + \Delta T)}{2}$$

and
$$h = \frac{5.6(1 + 0.011t)G^{0.8} 12^{0.8}}{(2)^{0.2}(0.25)}$$

for a 0.25" plate spacing and a water inlet temperature of 100°F and a G of 5 #/ft.sec.

$$h = \frac{5.6 (1 + 0.011)(145) 5^{0.8} 12^{0.8}}{(2)^{0.2} (0.25)} = 1,337 \frac{\text{BTU}}{\text{hr.ft}^2\text{°F}}$$

The point of maximum surface temperature is now determined by

$$\tan \frac{\pi x}{L} = \frac{2033 G}{h}$$

for the point chosen here this is

$$\tan \frac{\pi x}{L} = \frac{(10.165)}{1,337} = -7.60$$

for this tangent

$$\sin \frac{\pi x}{L} = .991$$

and
$$\cos \frac{\pi x}{L} = -.130$$

With this information the maximum surface temperature is found from

$$\begin{aligned} T_m &= T_{w1} + \frac{2Q_o L}{3600 G C_p} (0.9165 - \cos \frac{\pi x}{L}) + \frac{Q_o}{h} \sin \frac{\pi x}{L} \\ &= 100 + 5 \times 10^5 \left[\frac{(0.9165) + (.130)}{2033 G} + \frac{1}{n} \sin \frac{\pi x}{L} \right] \\ &= 100 + 5 \times 10^5 \left[\frac{(0.9165) + (.130)}{(2033)(5)} + \frac{1}{1337} (991) \right] = 357^\circ\text{F} \end{aligned}$$

The inner velocity is then found by the relation

$$V = \frac{G'}{\rho} = \frac{(5)(12)}{(0.25)(61.0)} = 3.93 \text{ ft/sec}$$

5.E. Appendix V. Pu Buildup

Since this reactor is designed to have a high (10^{14}) average thermal flux and since $4/5$ of the fuel atoms are U^{238} , it was thought that the buildup of Pu^{49} would be appreciable and help in cycling time. The following calculation shows this not to be the case; since the "p" is high enough that this is not important.

The equations describing the buildup are:

$$\frac{dN^{29}}{dt} = \frac{A_{1c}}{V} - (\lambda_{29} + \sigma_a^{29} \phi) N^{29}$$

$$\frac{dN^{39}}{dt} = \lambda_{29} N^{29} - (\sigma_a^{39} \phi + \lambda_{39}) N^{39}$$

$$\frac{dN^{49}}{dt} = \lambda_{39} N^{39} - \sigma_a^{49} \phi$$

where A_{1c}/V is the resonance absorption in the core per unit volume.

The first two equations can be easily solved assuming 29 and 39 equilibrium (an overestimate of buildup):

$$N^{29} = \frac{A_{1c}/V}{\lambda_{29} + \sigma_a^{29} \phi} = \frac{A_{1c}}{V \lambda}$$

$$N^{39} = \frac{\lambda_{29} N^{29}}{\sigma_a^{39} \phi + \lambda_{39}} = \frac{\lambda_{29} N^{29}}{\lambda_{39}}$$

$$N^{39} = A_{1c}/V \lambda_{39}$$

and solving the third equation:

$$N^{49} = \frac{\lambda_{39} N^{39}}{\sigma_a^{49} \phi} [1 - e^{-\sigma_a^{49} \phi t}]$$

$$N^{49} = \frac{A_{1c}/V}{\sigma_a^{49} \phi} [1 - e^{-\sigma_a^{49} \phi t}]$$

The 2G2R core used gives a normalized A_{1c} which is easily converted to the A_{1c} at designed flux levels. The core also gives a value for Σ_a^{25} .

The effect of buildup is best demonstrated by the ratio of fissions in 25 and 49.

Thus:

$$\frac{\sigma_f^{49N49}}{\sigma_f^{25N25}} = \frac{\frac{\sigma_f^{49}}{\sigma_a^{49}} \frac{A_{1c}}{V}}{\frac{\sigma_f^{25}}{\sigma_a^{25}} \Sigma_a^{25}}$$

Looking at a sample system:

Core: 50H₂O - 50D₂O; 30 cm radius

Reflectors: 50H₂O - 50D₂O; 15 cm thick

$$A_{1c} = 1.3$$

$$\Sigma_a^{25} = 0.0412$$

Using these figures in the above equation, the fission ratio is 0.03 at

$t = \infty$ and at $t = 18$ days, 0.005.

6. LIST OF REFERENCES

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