DETAILED PREDICTIONS FOR THE P-N STRUCTURE FUNCTIONS IN
THEORIES WITH COMPUTABLE LARGE MOMENTA BEHAVIOUR[†]

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ABSTRACT

If strong interactions are described by non-abelian gauge theories, detailed predictions are obtained for the q^2 dependence at fixed ω of the difference of the structure functions of the proton and neutron.

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It has been recently recognized that in a particular class of renormalizable theories the large momenta behaviour is computable from perturbation theory¹, at least for sufficiently small coupling constant. Some of the non-abelian gauge theories belong to this class^{2,3,4}. In these theories the Bjorken scaling law for deep inelastic scattering is satisfied⁵ apart from logarithmic corrections which can be computed.

We consider a model with 3×3 colored quarks: the interaction is carried by an octet of colored gluons and it is symmetric under $SU(3) \times SU(3)$. In this model the following relation holds:⁶

$$\int_1^{\infty} F_T^2(\omega, q^2) \omega^{-N} d\omega \xrightarrow[q^2 \rightarrow \infty]{} \sum_{i=1}^4 C_T^{i,N} (\lg q^2)^{-A^{i,N}} \quad (1)$$

where T stands for the target name and $A^{i,N}$ are computed numbers which are independent of the target.

Different power behaviour comes from the contributions of different operators. However, in the $I = 1$ t -channel, only one operator is present; a simple relation is satisfied by the difference between the proton and the neutron structure functions:⁶

$$\int_1^{\infty} F_{P-N}^2(\omega, q^2) \omega^{-N} d\omega \xrightarrow[q^2 \rightarrow \infty]{} C_{P-N}^N (\lg q^2)^{-A^N} \quad (2)$$

$$A^N = \frac{2}{27} \left\{ 1 - \frac{2}{N(N+1)} + 4 \left[\psi(N+1) + \gamma - 1 \right] \right\}$$

where ψ is the logarithmic derivative of the Euler Γ function and γ is the Euler-Mascheroni constant.

A very nice test of the validity of the theoretical scheme would be provided by a comparison of Eq. (2) with the experimental data.

The aim of this note is to point out that it is possible to derive from Eq. (2) simple consequences on the q^2 behaviour of the structure functions at fixed ω . The final result is:

$$\begin{aligned}
 D(\omega, q^2) &\equiv q^2 \lg q^2 \frac{\partial}{\partial q^2} F_{P-N}^2(\omega, q^2) = \\
 &= \frac{2}{27} \left\{ \left[3 + 4 \lg(1 - \frac{1}{\omega}) \right] F_{P-N}^2(\omega, q^2) + \right. \\
 &\quad \left. + \int_1^{\omega} \frac{d\mu}{\mu^2} \left[\left(2 - \frac{2}{\mu} + \frac{4}{\mu-1} \right) F_{P-N}^2\left(\frac{\omega}{\mu}, q^2\right) - \frac{4\mu}{\mu-1} F_{P-N}^2(\omega, q^2) \right] \right\} \quad (3)
 \end{aligned}$$

Eq. (2) and Eq. (3) are mathematically equivalent, however, we believe that the second equation is easier to test than the first one. A salient feature of Eq. (3) is that the integral in its r.h.s. is done from 1 up to ω : information on the high ω behaviour of the structure functions is not needed.

Simpler relations are valid in the Regge region. The coupling constant of a Regge pole of spin j is simply proportional to $(\lg q^2)^{-A^j}$. If $0 < j < 1$, A^j is negative; e.g., $A^{1/2} = -.24$, $A^{1/3} = -.42$.

As a simple example we have computed the function $D(\omega, q^2)$ assuming as input in Eq. (3): $F_{P-N}^2(\omega, q^2) = \omega^{-4}(\omega-1)^3$ which is a good fit of the experimental data in the present q^2 region. In Fig. 1 we have plotted the function $D(\omega, q^2)/F_{P-N}^2(\omega, q^2)$. Using

the same technique, the ratio $R_{P-N}(\omega, q^2) = \sigma_{P-N}^L(\omega, q^2) / \sigma_{P-N}^T(\omega, q^2)$ can also be computed.

The proof of Eq. (3) is rather simple; we only sketch its derivation.⁷ The analytic continuation of the l.h.s. of Eq. (2) up to non-integer N is the Mellin transform $(\tilde{F}(\sigma, q^2))$ of the structure function. It satisfies the following differential equation:

$$q^2 \lg(q^2) \frac{\partial}{\partial q^2} \tilde{F}(\sigma, q^2) = -A(\sigma) \tilde{F}(\sigma, q^2) \quad (4)$$

where $A(\sigma)$ is the analytic continuation in N of A^N . The convolution theorem for the Mellin transform implies Eq. (3).

We stress that these considerations can be applied only in the region where $\log q^2$ is a large number, corrections of order $(\log q^2)^{-1}$ being neglected. Nevertheless, it may be interesting to extend this kind of prediction to the structure functions of the proton itself. Phenomenological models of the type described in Ref. 8 can be used to fix the relative weight of different operators in Eq. (1).

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FIGURE CAPTIONS

Figure 1: Plot of the function $D(\omega, q^2)/F_{P-N}^2(\omega, q^2)$, computed under the assumption $F_{P-N}^2(\omega, q^2) = \omega^{-4}(\omega-1)^3$.

Figure 1