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OF TWO-COMPONENT MIRROR REACTORS

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ABSTRACT

An extensive study has been made of the energetics, equilibrium, and
stability of two-component mirror systems. Collisional target plasmas
are described by equilibrium fluid flow equations in the Marteni computer
code. The Marteni code is coupled to a particle simulation code for
application to an arc target plasma. The particle code determines the net
transfer of directed beam energy to the plasma (typically 10 to 15% maxi­
mum) and this produces input for the power balance of the Marteni code
that determines $T_e$.

For two-component systems supported by neutral injection, we couple
the Marten i code to Fokker-Planck (MFP) codes that describe the colli­
sional decay of the injected species. A radial transport code that gives
information on the trapping fraction of the neutral beam and on volume
erosion of the target plasma is included. In the simplest cases the codes
produce predictions of $T_e$ that are a factor of 2 to 3 lower than previous
estimates. Self-consistent magnetic field effects are calculated with a
particle simulation code that shows long-lived reversed field configura­
tions are possible. The MFP codes predict that velocity space instabili­
ties are definitely present during the build up phase, will occur even in the
steady state for low mirror ratios, $R \approx 2$, and can possibly be stabilized

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by high mirror ratios $R \geq 10$. Particle simulation codes predict a spreading of the hot ion energy spectrum.

INTRODUCTION

We have made an extensive study using a variety of theoretical and computational models of the energetics, equilibrium, and stability of two-component mirror reactor concepts, and of laboratory experiments to check the theoretical scaling laws. We used six basic codes on different aspects of the problems. The Marteni code describes collisional plasma flow along a variable magnetic flux tube; Fokker-Planck codes give the distribution function for hot injected ions; a radial transport code shows charge-exchange erosion of the plasma volume; the Superlayer code computes the self-consistent magnetic field of the hot injected ions; particle simulation codes examine velocity space instability of the distribution functions.

COLLISIONAL TARGET PLASMAS

We present a simple theory of cool plasma flow in a flux tube, including sheath formation at electrodes, cold gas input, and the effects of magnetic mirrors on the flow. Master curves are given for the simplest cases, relating magnetic field \( B(z) \), flow velocity \( U(z) \), and plasma flux \( P = nU/B \). The Marteni code elaborates on the model and includes surface physics calibrated by comparison with our experiments on a baseball field.

It is important for target experiments, magnetized arcs, and two-component mirror reactors to predict the axial variations of cold plasma flow, energy flow, and electrostatic plasma potentials. The simplest approach treats the cool ion component as a collisional fluid tied to the field lines with a large flow velocity as compared with the thermal velocity. Quasineutrality yields the electron flow and plasma potential. When the electron mean free path is large compared with the length, the electrons are confined electrostatically by wall or electrode sheaths. We find basic characteristics of cool plasma flow in a flux tube with cross section \( a \propto 1/B(z) \) from conservation of flux, \( nv/B \), ion energy, \( 1/2Mv^2 + e\phi \), and electron energy, \( e\phi - kT_e \log n \). Here \((n, v)\) are the plasma density and axial flow velocity, and \((\phi, T_e)\) the potential and electron temperature.

Combining these conservation laws in a region having no sources gives the master equation,

\[
\left( \frac{B}{B_0} \right)^2 = \left( \frac{v}{v_0} \right)^2 \exp \left[ \frac{(-v_0^2 + v^2)}{v_s^2} \right],
\]

relating \((v, B)\) to \((v_0, B_0)\) at some point in the plasma. The ion sound speed is \(c_s = (kT_e/M)\). Conservation of momentum, across a thin plasma source, \(G\), such as a beam or gas jet gives the jump condition,

\[
F_1 \left( v + \frac{c_s^2}{v} \right) = F_2 \left( v + \frac{c_s^2}{v} \right),
\]

where the fluxes \(F_1, F_2 = (nv/B)_{1,2} \) differ by \(G = F_2 - F_1\). Finally the Bohm boundary conditions at end walls are such that plasma cannot leave a surface faster, or arrive slower, than the sound speed \(c_s\). The above considerations give \((n, v, \phi)\) in an arbitrary thin flux tube with any distribution of narrow sources or end flows.
We note that the $B(V)$ has a maximum at $V = c_s$, and therefore mirror threats (field maxima) tend to act as nozzles where the flow becomes sonic. We first applied the Marten model to a voltage-driven arc plasma that is planned as a target plasma in a two-component experiment in 2XII. The electron temperature of an arc plasma is determined by the energy transfer from the directed energy in the electron current to the thermal energy in the dense plasma electrons. The mechanism of transfer is the weak beam-plasma instability. Since $\tau_B^2 \ll \tau_{e\varphi}$, we can simulate the instability with a collisionless model that shows that sometime after the instability saturates, a quasi-steady state occurs with the energy lost from the directed energy appearing in wave energy and in hot suprathermal plasma electrons. Since the arc time scale $\tau_0 = l/c_s \approx \tau_{e\varphi}$, we assume all of this transferred energy will eventually appear as thermal energy. Typical infinite model calculations show 10 to 15% energy transfer. Spatially bounded model calculations can give somewhat less.

The electron temperature is found from the total power balance in the arc. The power in is simply $J\Delta \varphi$ and the power out, in steady state, is

$$\dot{\Delta \varphi} = \left( \frac{M}{2} c_s^2 - \Delta \varphi_A + \epsilon_i \right) + \frac{B_A}{B_c} \left( \frac{M}{2} v_e^2 - \Delta \varphi_e + \epsilon_i + \frac{5}{2} T_e \right) + n_AV_cT_e + \xi J_e \Delta \varphi,$$

where $F_c = n_eV_e$ is the ion flux at the cathode, $|F_s|$ is the flux of ions falling back on the anode from the ionization sheath, $\Delta \varphi_A$, $\Delta \varphi_e$ are the anode and cathode sheath drops, $V_e = (T_e/n^2\pi)^{1/2}$ is mean velocity of the electron through the sheath and $\xi J_e \Delta \varphi$ is the amount of directed (beam) electron energy flux at the anode. This last term would be zero if all of the beam energy were delivered to the plasma. The equation can now be solved for $T_e$. In the case where there was no gas supply at the anode, we treated the anode terms in the same way as the cathode. Each ion lost had to be created at an ionization cost of $\epsilon_i \approx 33$ eV.

**TWO COMPONENT SYSTEMS**

For two-component systems, the Marten model is coupled to a numerical solution of the Fokker-Planck equation, which calculates the distribution function $f(\epsilon, \theta, t)$ of injected ions as a function of energy, $\epsilon$, pitch angle, $\theta$, and time. This allows predictions to be made of the performance of scaled two-component experiments in 2XII, as well as parameter searches in regimes of reactor interest. The calculations for 2XII included the 2XII target plasma, a suprathermal species formed from neutral injection, and a flowing Maxwellian target plasma formed from ionization of gas input outside the mirrors. We explored the entire gas range from the limit where the Maxwellian target density $n_M \ll n_{hot}$ to the other extreme $n_M \gg n_{hot}$. We also included a radial gas input from the neutral injectors. A radial transport code provided information on charge exchange, ionization, and classical diffusion at the surface of the plasma. We considered a general mirror system consisting of a hot neutral injected species, a collisionless initial target plasma, and a collisional flowing Maxwellian target plasma supported by cold gas feeds just outside the mirrors. The system parameters are injection current $I$ in amperes of its volume particle source equivalent $J_H$ (particles/cm$^3$ s) = $I/(e\lambda \Lambda)$, hot ion density $N_H$, cold ion density $N_C$, electron density $N_e$, injection energy $E_i$. 


hot ion energy \( W_{HH} \), cold ion temperature \( T_i \), electron temperature \( T_e \), and mirror ratio \( R \). With symmetric \( \alpha \)-drifts, the cold target plasma takes the form of a stagnant \((v = 0)\) constant density fluid between the mirrors and an outward flowing fluid with \( v \cdot c_s = \left( T_e + T_i \right) c_s M^{1/2} \) outside the mirrors.

In steady state the system must satisfy a power balance:

\[
\dot{E} \cdot v = 2 N C_s \frac{A}{R} \left( \frac{M}{2} \cdot \frac{v^2}{v_s} \cdot \frac{5}{2} T_e \right) + 2 F_H \frac{A}{R} \left( \phi + W_{\text{out}} \right) + 2 F_e \frac{A}{R} T_e + W_{\text{ex}}.
\]

The first three terms on the right-hand side represent for each species the flux out times the cross sectional area at the ends, times the energy carried out per particle. The last term represents the rate of energy lost caused by charge exchange. The \( 1/R \) factor occurs because of the construction of the cross sectional area from its midplane value \( A \) to its mirror value. The potential \( \phi \) is established at the end sheaths. In steady state we have \( 2 F_H R = J_H \cdot L \) and \( F_e = N_e c_s - F_H \). The factor \( s \) on the left-hand side represents the trapping efficiency of the neutral beam. To calculate an approximate value of \( T_0 \), we make the following substitutions:

\[
\phi = \frac{J_H}{T_e} \cdot W_{\text{out}} = T_e \cdot T_e = T_e, \quad \text{and find:}
\]

\[
T_e = \frac{J_H \cdot L (E - 5 T_e) - W_{\text{ex}} A}{18 N_e c_s R}.
\]

This approximate formula is to be compared with a similar expression given by Post et al.\cite{1} Aside from differences in small terms, their expression for \( T_e \) appears to be a factor of three too high. Even if we took \( T_e \ll T_i \), our equations still predict \( T_e \) to be a factor of two lower than theirs, the difference apparently caused by their accounting for the power put only at one end.

We now assume that \( 5 T_e \ll E, W_{\text{ex}} \) negligible, \( n_i = 1 \) and calculate typical numbers for planned experiments in 2XH: \( E = 20 \, \text{keV} \), \( L = 100 \, \text{cm} \), \( c_s = 5 \, \text{cm} \), \( J_H = 4 \times 10^{14} \, \text{A} \). If we assume \( N_e = 10^{14} \), we get \( T_e = 100 \, \text{eV} \) for \( R = 2 \), or \( 300 \, \text{eV} \) for \( R = 10 \). The hot ion density depends on the hot particle loss time \( \tau_H \): \( N_{HI} = J_H \tau_H \). If we take \( \tau_H \) equal to the hot ion slowing-down time, we get \( N_{HI} = 3 \times 10^{13} \) for \( R = 2 \) or \( 2.3 \times 10^{14} \) for \( R = 10 \). In our example, \( N_{HI} \ll N_e \) and \( W_{\text{ex}} \) is not negligible. Redoing the calculation with \( N_e = 4 \times 10^{14} \) we get \( T_e = 10 \, \text{eV}, N_{HI} = 10^{12} \) for \( R = 2 \) and \( T_e = 120 \, \text{eV}, N_{HI} = 5 \times 10^{13} \) for \( R = 10 \).

Similarly, for the reactor scale system considered in Ref. 1, \( E = 200 \, \text{keV} \), \( L = 10 \, \text{cm} \), \( c_s = 1 \, \text{cm} \), \( J_H = 2000 \, \text{A} \), \( N_e = 10^{12} \), we calculate \( T_e = 400 \, \text{eV} \) for \( R = 2 \), or \( 1000 \, \text{eV} \) for \( R = 10 \) (as contrasted with the 4 kV predicted in Ref. 1). The MFP code and the radial transport code solve the full-time dependent equation and give more accurate steady state results than the above estimates while also giving information on the transient start up phase. The radial transport code is first run to give an accurate value for the trapping efficiency \( n_i \) which is then used in the MFP.

In the 200-keV example, the code results are:
In both cases we note that $T_e$ is considerably lower than $T_\nu$ and $T_\nu$ is higher than the approximate estimate.

**SELF-MAGNETIC FIELD EFFECTS**

The self-consistent magnetic fields occurring in a high-beta two-component mirror reactor are determined by a particle simulation code. Of particular interest is the expected marked gains in the containment of the target plasma caused by the self-well improvement of the mirror ratio. We also examined regimes of field reversal that occur for certain methods of injection. In addition, there are possible problems of non-adiabaticity and loss of confinement of the hot particles. These various aspects are examined by taking initial $f(v)$ as calculated by the MFP codes, and calculating the self-consistent fields caused by a slowly increasing current density.

In cases where the energetic injected particles have gyro/radii on the order of the plasma radius, $\alpha_i/r_p \approx 1$, simple estimates indicate that there can be a current density high enough to reverse the field. A rough criterion is that the linear current density $j_H (A/cm) = B_0 (Gauss)$, or $1\pi/(\tau_c L) \approx B_0$, where $\tau$ is the slowing down time and $\tau_c$ is the cyclotron period. Within a factor of two this is the same criterion as $\beta \approx 1$. In 2X11 with $1 = 600 A$ and $\tau \approx 10^{-4} s$, we satisfy this criterion if $I \leq 50 cm$.

The initial set of particle simulation calculations, carried out in the range $\alpha_i/r_p \approx 1$, confirm that reversed field configurations are formed and persist. We used injection primarily perpendicular to the magnetic field over an axial length $L_{inj}$ on the order of 5 to 10 times the gyro radius. We attempted to meet the condition that the current density rate of increase is slower than a mean axial bounce frequency. We found that as the current density grows, a certain amount of axial focusing occurs so that the final state typically has a mean length somewhat less than $L_{inj}$. This axial focusing can be prevented with sufficient pitch angle dispersion on the injected beam, in which case the final mean length can be nearly equal to $L_{inj}$. For the present cases, we find less than 10% of the injected particles are lost. These lost particles are those that exit axially in a few bounce periods. The remaining particles appear to remain confined for long periods of time. The question of adiabaticity is not clear. The magnetic moment is not conserved since $\alpha_i \nabla B, R \approx 1$. Furthermore, adiabaticity is not obviously related to the confinement issue. The degree to which long-time confinement persists in reversed field configurations is being examined by following a representative set of particles in steady state reversed fields for the appropriate length of time.

These long time simple particle orbit calculations are similar to others previously carried out for Astron reversed field configurations. The results of the Astron calculations showed that long time confinement did exist for most of the orbits considered. We found that typical orbits would show differences from one bounce period to the next but that there was an overall long time restriction to a region of phase space. We expect...
similar results for the present saturation although the old Astron calculation considered only axis-circulating orbits while the present calculations must consider a wider class of orbits.

To roughly approximate the effects of the self-magnetic fields on the containment of the target plasma, we used the $B_{\max}/B_{\min}$ as calculated from the particle code as an enhanced mirror ratio for the MFP codes. At or beyond reversal, $B_{\min} = 0$ and this procedure produces $R \to \infty$ or perfect containment of the collisional target plasma, perhaps to be expected in the closed field lines of a reversed field configuration. Needless to say this completely changes the character of two-component mirror systems. The calculations to date have ignored completely the response of the collisional plasma to the time changing fields so that the question of containment of the collisional plasma needs further examination.

MICROINSTABILITIES

Effects of velocity space instabilities (primarily double hump modes) on the energy distribution of the injected hot species of a two-component reactor are determined with particle simulation computer codes. Previous analytic and numerical results lead us to suspect that the primary effect of such instabilities is to degrade the energy spectrum of the hot particles, a detrimental effect on the two-component concept. However, there is conjecture that a sizable fraction of the hot particles may actually increase in energy, leading to a possible net gain. Information on the double hump nature of the velocity distribution during the buildup phase is obtained from MFP codes, and is used as input for initializing the particle codes. Spatially uniform models are used to explore the relevant parameter range $n$, $n_{\gamma}$, $k_{ij}$, $k_{e}$. The results of the MFP code show that $f(v_{ij})$ is unquestionably unstable by infinite medium stability criteria during the buildup phase. We conclude that the transient buildup will take place via a sequence other than that predicted by the MFP code. If the steady state $f(v_{ij})$ from the MFP code is stable, we can conclude this instability activity is only a transient phenomenon. A more serious problem arises if the MFP steady state $f(v_{ij})$ is unstable. The instability activity then would be expected to be a continual problem during the entire lifetime of the experiment. We conclude that the energy of hot injected species will degrade with periodic bursts of instability activity so that $f(v_{ij})$ becomes stable at any given point in time during injection. A minimal requirement is that we calculate the effects on the energetics. The particle codes for $k_{ij} = 0$ resonant modes,

$$\left(\frac{w_{p}^{2} + w_{c}^{2}}{2}\right)^{1/2} = k_{c},$$

show a scattering both down and up in energy, with $f(v_{ij})$ evolving to a roughly stable distribution. To the extent that the instability does not dramatically increase the energy carried out of the system by the hot particles lost there is little direct effect on the power balance. The primary effect would be to create an enhanced energy transfer from the hot ions to the cold ions since the energy transferred from the hot particles via the instability initially goes into wave energy and then collisionally to the cold ions. There is a less easily calculable effect on the Q of the system since the $f(v)$ of the hot particles is spread both up and down in energy. The particle codes in some cases show a large spreading upwards in energy that may well enhance the Q, but these results are for a system that is strongly unstable whereas the instability activity actually envisaged is one
where a weak instability is excited whenever the \( f(v) \) evolves to a marginally unstable distribution.

The stability question depends on all the details of the system. The \( \text{MIFP} \) steady state \( f(v_f) \) for the previously quoted examples are unstable for \( R = 2 \), nearly stable for \( R = 10 \), and become stable at even higher mirror ratios such as might result from large self-magnetic fields. Also, still to be examined are finite geometry effects, expected to be large since \( a_i/r_p \) \( a_i(\nabla B/B) \approx 1 \).

**REFERENCE**


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**NOTICE**

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