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PLASTIC COLLAPSE LOADS FOR
PIPE ELBOWS USING INELASTIC ANALYSIS*

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ABSTRACT

The results of an inelastic analysis are used to predict plastic collapse loads for in-plane bending of several pipe elbows. The analysis is made using a nonlinear finite element computer program which takes into account the effects of material strain hardening, stress redistribution, and ovalization of the elbow cross section. Various methods are used to determine plastic collapse loads based on the calculated load-deformation curves. The predicted collapse loads are compared to collapse loads determined by limit analysis and the method described for plastic analysis in Section III, Appendix F, of the ASME Boiler and Pressure Vessel Code. It is concluded that the current Code definition of collapse load is overly conservative for structural materials typical of high temperature application, and that the .2% offset method or the 2 times the angle of linear response method produce more realistic results.

PLASTIC COLLAPSE LOADS FOR PIPE

ELBOWS USING INELASTIC ANALYSIS

INTRODUCTION

The design of pipe elbows for high temperature application is of current interest for systems, such as the Liquid Metal Fast Breeder Reactor coolant pipe loops. Elbows are usually the most flexible members in piping systems, and severe strain concentrations may result from thermal expansion, dead-weight, and seismic loads. Care must be taken to avoid plastic and geometrical instability which might lead to strains or deformations in excess of the design limits. To assure experimental verification, component-pipe structures testing at elevated temperatures are being sponsored by the USAEC to develop structural design methods for fast reactor components. The work reported herein is in support of the design of a sodium test facility, MLTF (multi-loading test facility), being fabricated at Westinghouse for elbow and pipe testing.

Elbow collapse loads have been determined experimentally by Gross [1] and Bolt and Greenstreet [2], or by simplified analytical techniques such as Marcal and Turner's [3] approach using a linear elastic analysis and the Nuclear Piping Code [4] method using elbow stress intensity factors. The simplified analytical methods have generally neglected the effects of material strain hardening, stress redistribution and inelastic ovalization of the elbow cross section. An inelastic analysis, however, by Marcal [5] presented load-strain curves and estimated theoretical limit loads for comparison with Gross' experiments. With the recent development of nonlinear finite element structural analysis computer programs such as MARC [6], EPACA [7], and ANSYS [8], it is possible to calculate the detailed inelastic response of elbows including nonlinear effects previously neglected or treated very approximately. Such calculations can be used to predict more realistic plastic collapse loads for use in Code design analysis.

Care should be taken to define terms in current usage. The terminology used in Section III of the ASME Boiler and Pressure Vessel Code is based primarily on a limit analysis approach where the material behavior is taken

to be elastic-perfectly plastic. When the actual stress-strain relation is used in analysis, it is not clear that the Code definitions still apply in a practical sense. For a plastic analysis, the "collapse load" for a structure is defined as the load at which the distortion is 2 times the value at the calculated initial departure from linearity. For a material with a small range of linear behavior such as Type 304 stainless steel at high temperature, or for a structure where the change in geometry is significant, such as ovalization of the elbow cross section, this definition would lead to extremely low collapse loads. It seems appropriate, therefore, to consider alternate more realistic methods for determining the collapse load. The "plastic instability load" is defined in the Code as the load at which deformation increases without bound or the relation of force and deformation has a horizontal tangent. Neither "collapse load" nor "plastic instability load" as defined by the Code refer to geometrical instability or buckling. Although geometrical instability may occur at a load below the "collapse load" for some geometries, that mode of failure is not considered here. Collapse load as used here refers only to deformation due to plastic instability.

An inelastic analysis of large diameter (4.5, 6, 16, and 28-inch) long radius (1.5 diameter) elbows is presented here using the MARC* computer program [6]. The elbows are loaded by an in-plane moment acting so as to reduce the bend radius. The deflections and strains are calculated including the effects of material strain hardening, stress redistribution, and ovalization of the elbow cross section; and the resulting load-deflection and load-strain curves presented. The collapse load is calculated for each elbow using several different methods that have been proposed, and the results compared to collapse loads calculated by the ASME Code method. A comparison is also made between predicted and measured load-deflection relations for the 6-inch diameter elbow to demonstrate applicability of the analytical technique.

*A readily available Westinghouse version of the MARC program was used for this analysis.

INELASTIC ANALYSIS

The constant bending, three-node, pipe-bend element developed specifically for the MARC computer program to provide nonlinear analysis of complete piping loops at realistic cost, was used for the elastic-plastic analysis of the large diameter elbows. A complete description of the theoretical formulation of the element can be found in references [6] and [9]. Each pipe-bend element or section is built up using several sections of modified axisymmetric, thin-shell elements as shown in Figure 1. Each such section has a beam mode, with constant stretch and curvatures, superimposed on the axisymmetric shell modes so that ovalization of the cross-section is admitted.

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of approximately 4.5 pipe diameters from the nearer end of the elbow. The loading of interest is the in-plane bending moment at that end of the elbow. By selecting a relatively long moment arm, shear forces were small in comparison with the moments. The parameters for this model were chosen to duplicate the test parameters [2].

These analyses were performed using the properties for Type 304 stainless steel. For the 28 and 16-inch elbows, average value stress-strain curves were used at room temperature and 1100°F. For the 4.5-inch elbow, properties for the ORNL standard heat of 304SS (9T2796) at 1100°F were used to represent elbows to be tested by Battelle-Columbus [10]. For the 6-inch elbow, the actual stress-strain curve was obtained from specimens cut from the elbow after it had been tested. In MARC, the stress-strain curves are represented in multi-linear form. The representations used in the analyses presented here are shown in Figure 5.

ANALYSIS RESULTS

Load-displacement and elbow strain and stress distributions were obtained for each elbow analyzed. The 28-inch elbow at 1100°F was analyzed using both models no. 1 and no.2 (see Figures 2 and 3) and the results showing end deflection as a function of bending moment are shown in Figure 6. The model with 18 elements around the circumference of the elbow (model 2) gives better results than the model with 10 shell element around the circumference (model 1). It was noted that for the ten element model, gross discontinuities in stresses occurred around the circumference of the elbow, indicating the need for more elements. Figures 7 and 8 show the stress distributions for model no. 2 around the outside circumference of the 28-inch elbow at first departure from linearity ($M = 0.305 \times 10^6$ in-lbs, $S_y = 12,850$ psi). Smoothing of the analytical data is represented by the curves, showing fairly good continuity of stress around the circumference for the 18 shell element model. Also from Figure 6, it can be seen that the proportional increment of loading used in the stepwise inelastic analysis was varied from .05 to .10, but the larger increment seems sufficient and was used in succeeding analyses. Note from Figure 6, the highly nonlinear behavior of the elbow deformation which shows a relatively rapid transition from elastic to predominantly plastic behavior.

Load-deflection curves showing plots of load versus elbow deflection and rotation were obtained for all elbows, and are shown in Figures 9 through 14. For the first five analyses, in-plane bending moments are plotted versus elbow end rotations and vertical deflections. For the 6-inch elbow, a plot of vertically applied force versus elbow deformation is shown. In Figure 14, the experimentally obtained load-deflection curves [2] at points A and B are shown, and it can be seen that the analytical results compare favorably. As would be expected, deflections obtained analytically are greater than the measured deflections because the restraint of the straight pipe section at the elbow intersections is neglected in the MARC three-node, pipe-bend section (i.e., ovalization of the elbow cross-section is constant along the arc length of the pipe-bend section. This assumption is equivalent to the ASME Code where elbow flexibility factors assume no restraints on the curved pipe). It is realized that detailed finite element analysis of elbows using shell elements can be currently carried out with MARC and other programs where elbow boundary restraint affects can be accounted for. However, for the purpose of this analysis, the more economical MARC pipe-bend element gives satisfactory results with a modest expenditure of effort.

The load-deflection plots (Figures 9 thru 14) show that the MARC analyses have been carried out well into the plastic region. Stress versus strain data obtained in the analyses have followed the inputted stress-strain curves very closely.

For illustrative purposes, a representative plot of elbow load versus strain for in-plane bending obtained in the inelastic analysis of the elbows discussed in this paper is shown in Figure 15. This load-strain curve is plotted from the finite element in the elbow with the maximum stress. In all cases, as seen from Figures 7 and 8, the maximum stress occurred as a circumferential hoop stress at the neutral axis of bending for the cross section. The load-strain curves obtained in the analysis of the other elbows are very similar to the one shown here.

COLLAPSE LOAD DETERMINATION

The determination of the plastic collapse load for a given combination of loads on a given structure using load-deformation curves obtained by plastic

or tests for comparison with limit analysis calculations has been the subject of many discussions ([11], [12], [13] and [14]). According to the Code, a plastic analysis is that method which computes the structural behavior under given loads considering the strain-hardening characteristics of the materials, strain-rate effects, permanent deformations, and stress redistributions. To distinguish from plastic analysis, the method of limit analysis assumes an ideally plastic (nonstrain-hardening) material, and the maximum load is computed as the load at which deformations of the structure increase without bound. Relative to a limit analysis, this load is termed the collapse load [15]. In reality, the force-deformation curve for a given structure displays a region of linear response, a transition region where the behavior changes from mainly elastic to mainly plastic, and a region in which a small increase in load produces a large change in deformation. Hence when the actual stress-strain relation is used, and a complete plastic analysis is carried out, the selection of the collapse load for a real structure requires careful interpretation of the load-deflection relation.

The ASME Boiler and Pressure Vessel Code allows for determination of the collapse load for a given combination of loads on a given structure from a plastic analysis. Appendix F of the Code states that the collapse load shall be taken as the one at which the distortion is 2 times the value at the calculated initial departure from linearity (see Figure 16). For the elbows analyzed here, this definition leads to extremely low predicted collapse loads since the elbow material (Type 304 stainless steel) has a small range of linear behavior and the elbows ovalize causing a significant change in structural geometry.

It seems appropriate, therefore, to consider alternate methods for determining the collapse load from plastic analysis. Figure 16 shows two other suggested methods for determining collapse loads from plastic analysis: the tangent method and the 2 times the angle of linear response method. The tangent method, which makes use of the structure load-deflection curve, establishes the collapse load by using the point of intersection of a line drawn tangent to the initial portion of the load-deflection curve (the elastic response region) and a line drawn tangent to the straight-line portion of the curve in the plastic region. This method depends upon the

existence of a region in the predominantly plastic response range for which the deflection is directly proportional to the load. Hence, the MARC analyses for the elbows were carried out well beyond the transition region, assuming of course that geometrical instability effects, except for cross section ovalization are absent. For the elbows analyzed, the tangent method was applied to both the load-deflection and load-rotation curves. Collapse loads predicted from the latter were the lowest and are reported here.

The third technique shown on Figure 16, and the one currently proposed for inclusion in the ASME Boiler and Pressure Vessel Code, is the 2 times the angle of linear response method. For this method, the collapse load is taken at the intersection of the load-rotation curve and the line which forms an angle equal to two times the angle obtained from the constant slope of the initial linear portion of the load-deflection curve. The fourth method considered is the 0.2 percent offset method as shown in Figure 15. The collapse load is determined at 0.2 percent offset strain from a load-strain diagram, where the strains are the calculated strains from the plastic analysis in the high stress, or strain, regions of the elbow or structure. From Figure 8, it can be seen that the maximum stresses occur at the elbow neutral axis, and an average value of the peak stress or strain was used in the moment-strain plots as shown in Figure 15.

Using the ASME recommended procedure and the other three alternate methods described above, the collapse loads using the plastic analysis results were obtained for each elbow and are shown in Table 2. As a matter of interest, the load at which the load-deformation curve departed from linear response is given for each elbow. However, this load is highly dependent upon the multi-linear idealization of the actual stress-strain curve as shown in Figure 5. The calculated results in Table 2 are arranged in decreasing order of magnitude showing that the tangent method gives the highest collapse loads while the ASME, Section III method gives the lowest. The two times the angle of linear response method and the 0.2 percent offset method give results that are very close in magnitude.

The results obtained from the plastic analysis using the above mentioned methods were compared to the limit analysis collapse loads determined by the

method described in USAS B31.7 [4] and shown in the last column of Table 2. In that reference, the limit moment is defined as 4/3 of the moment to produce a local bending moment equal to the yield strength of the elbow material, or

$$M_{LL} = \frac{\pi t r^2 S_o}{B_2}$$

where M_{LL} is the collapse moment determined by limit analysis, t is the elbow thickness, r is the elbow cross-section mean radius, B_2 is the elbow stress intensity factor, and S_o is taken as 1.2 times the 0.2 percent offset yield strength of the elbow material. The limit-type collapse loads predicted using this technique are quite conservative for these elbows constructed from a very pronounced strain-hardening material. The results listed in Table 2, show that the limit-type loads are approximately equal to the collapse loads predicted by the ASME Code method as applied to plastic analysis, but these are considerably lower than those predicted using the three alternative techniques discussed in this paper for application to plastic analysis.

CONCLUSIONS AND FUTURE WORK

Inelastic analyses have been completed for a series of stainless steel elbows loaded by in-plane bending with the MARC nonlinear finite element program which takes into account the effects of material strain-hardening, stress redistribution, and ovalization of the elbow cross-section. For one 6-inch elbow, a comparison is made between predicted and experimentally measured load-deflection relations to demonstrate applicability of the analytical technique. Close agreement between analysis and test is shown, except that the MARC model is slightly more flexible since elbow end constraints are neglected.

Using the load-deformation behavior of the elbows calculated from plastic analysis, various methods were used to determine the plastic collapse load for each elbow. These collapse loads were compared to collapse loads determined from limit analysis techniques. The ASME Code recommended plastic analysis technique and the limit analysis method predict consistently low,

conservative collapse loads for the elbows with a limited range of linear response. It is concluded that the current Code method and the limit analysis technique of determining collapse load is unduly conservative for the strain-hardening material considered here, and that the 0.2% offset method or the two times the angle of linear response method should be used to determine collapse loads based on a plastic analysis of the structural behavior.

Finally, it may be added that the work completed and reported here, demonstrates the applicability of the MARC simplified, elbow finite element for plastic analysis using simple loadings. Further investigation of the applicability of this element to elbow creep analysis, to combined loading plastic instability analysis, to elbow geometrical buckling analysis, and to elbow creep buckling analysis should to be carried out.

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Specimen	Description	Nominal Dimensions				Material	Modulus of Elasticity (psi) (10^{-6})	.2% Offset Yield Stress (psi)
		D _o (in.)	D _i (in.)	t (in.)	D _i /t			
1	28" Schedule 20 LR	28	27.25	.375	72.67	304SS	28.3 (@ R.T.)	37,700
2	28" Schedule 20 LR	28	27.25	.375	72.67	304SS	21.7 (@ 1100°F)	18,300
3	16" Schedule 40 LR	16	15.25	.375	43.33	304SS	28.3 (@ R.T.)	37,700
4	16" Schedule 40 LR	16	15.25	.375	43.33	304SS	21.7 (@ 1100°F)	18,300
5	4.5" Schedule 10 LR	4.5	4.26	.120	35.50	304SS (9T2796)	21.7 (@ 1100°F)	10,600
6	6" Schedule 40 LR	6.625	6.065	.280	21.66	304SS	28.8 (@ R.T.)	37,700

TABLE 1
NOMINAL DIMENSIONS AND MATERIAL PROPERTIES
OF ANALYZED ELBOWS

Comp.	MARC Inelastic Analysis Results (Moments given in 10 ⁶ in-lbs.)					Collapse Load Using Limit Analysis Method (B 31.7)
	Mom. at Departure from Linear Response	Plastic Collapse Mom. Prediction			ASME, Section III Method	
		Tangent Method	2X Angle of Linear Response	0.2 Percent Offset		
28" Elbow at 1100°F	0.305	1.04	0.83	0.815	0.565	0.64
28" Elbow at R.T.	0.604	2.07	1.86	1.59	1.090	1.318
16" Elbow at 1100°F	0.152	0.476	0.42	0.378	0.280	0.301
16" Elbow at R.T.	0.301	0.98	0.89	0.742	0.560	0.620
4.5" Elbow at 1100°F	0.0024	0.00698	0.00575	0.00572	0.00412	0.00444
6" Elbow at R.T.	973 lbs. (Force)	4900 lbs.	4360 lbs.	3160 lbs.	1900 lbs.	2490 lbs.

TABLE 2
PLASTIC COLLAPSE LOADS USING SELECTED CRITERIA

Figure 1

Typical Pipe Bend Section

Figure 2

MARC Model Number 1: 28-Ton Elbow

Figure 3

Model Number 2; 16, 16 and 4.5-Inch Elbows

Figure 4
MARC Model Number 3; 6-Inch Elbow

Figure 5

Type 304 Stainless Steel Stress-Strain Curves

Figure 6

28-Inch Elbow at 1100°F, Load-Deflection Curve for Specimen No. 1

Figure 7
28-Inch Elbow at 1100°F, Outside Circumferential Axial Stress Distribution

Figure 8

28-Inch Elbow at 1100°F, Outside Circumferential Hoop Stress Distribution

6-1619

Figure 9

28-Inch Elbow at 1100°F, Load-Deflection/Rotation Curves for Specimen No. 1

Figure 10
28-Inch Elbow at R.T., Load-Deflection/Rotation Curves for Specimen No. 2

Figure 11

16-Inch Elbow at 1100°F, Load-Deflection/Rotation Curves for Specimen No. 3

Figure 12

16-Inch Elbow at R.T., Load-Deflection/Rotation Curves for Specimen No. 4

Figure 13

4.5-Inch Elbow at 1100°F, Load-Deflection/Rotation Curves for Specimen No. 5

Figure 14
6-Inch Elbow, Experimental vs. Analytical Load-Deflection Curves for Specimen No. 6

Figure 15

28-Inch Elbow at 1100°F, Load-Strain Plot

Figure 16

Collapse Load Determination Using Load-Rotation Curves