On the Coupled Channel T Operator Formalism and Federbush Disease

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ABSTRACT

The coupled equations for n-body rearrangement collisions derived previously using the reduction method and the channel coupling array method are explicitly shown to be free of Federbush disease.

In the preceding paper¹ we have investigated the connection between the reduction method of Hahn^{2,3} and the new coupled channel T operator formalism of Kouri and Levin^{4,5} for treating general n-body, N-channel collisions. It was explicitly shown for 2 and 3 arrangement cases that the Kouri-Levin approach yielded exactly the reduction method equations when one introduced specific channel coupling arrays^{1,6}. Furthermore, it was shown that in general, additional equations could be obtained which also satisfied the ansatz

$$|\psi\rangle = \sum |\psi_i\rangle, \quad (1)$$

where $|\psi\rangle$ is the Schrodinger state vector and the $|\psi_i\rangle$ are channel components of the state, as in Hahn's work^{2,7}. Finally, we note that both the Hahn and Kouri-Levin methods can be used to avoid well known problems in many body collisions due to disconnected diagrams^{1,8-14} (in the integral form of both approaches).

In this note, we point out that the two methods also avoid another common difficulty in many body collision theory. This is the problem of spurious bound state solutions or Federbush disease¹²⁻¹⁴. In order to demonstrate this result, we recall that $|\psi\rangle$ obeys the n-body, N-channel Schrodinger equation

$$(E - H)|\psi\rangle = 0, \quad (2)$$

where H is the full Hamiltonian, E the total energy, and where

$$H = H_i + V_i, \quad i = 1, 2, \dots, N. \quad (3)$$

That is, H may be split into arrangement channel Hamiltonians H_i , and their concomitant channel perturbations V_i , in N ways. We also note that the adjoint of (2) leads to

$$\langle\psi|(E - H)^\dagger = 0. \quad (4)$$

As usual, the full Hamiltonian H is self-adjoint in the space of the $\{|\psi\rangle\}$, so that

$$\langle\psi|(E-H)=0. \quad (5)$$

Either Eq. (2) or (5) is a valid form of the Schrodinger equation.

We first proceed to show that the reduction equations^{2,3}, combined with the channel coupling array⁴⁻⁶, possess the same spectra as the Schrodinger equation. We note that the channel coupling array W satisfies the condition⁶

$$\sum_{i=1}^N W_{ij} = \sum_{j=1}^N W_{ij} = 1. \quad (6)$$

From this condition, it is easily verified that

$$V_j = \sum_{i=1}^N W_{ij} V_j = \sum_{i=1}^N V_j W_{ji}. \quad (7)$$

We now introduce arrangement channel state components $|\psi_j\rangle$ satisfying the equation¹

$$(E-H_i)|\psi_j\rangle - \sum_{j=1}^N W_{ij} V_j |\psi_j\rangle = 0, \quad (8)$$

where for different choices of the channel permuting array discussed by Kouri and Levin^{4,5,7}, Eq. (8) above leads to the same form as Hahn's reduction method equations^{1-3,7}. Furthermore, in general for N arrangements, if we sum (8) over i and use (7), we see that

$$\sum_{j=1}^N (E-H_j - V_j) |\psi_j\rangle = 0. \quad (9)$$

By (3), this obviously reduces to the Schrodinger equation, (2), where

$$|\psi\rangle = \sum_{j=1}^N |\psi_j\rangle;$$

this is the same result as (1). We therefore conclude that (8) leads to precisely the same eigenvalues as the Schrodinger equation.

It is convenient for our next consideration to note that (8) may be written in matrix form as

$$(E \underline{1} - \underline{H}_0 - \underline{WV}) |\hat{\psi}\rangle = 0, \quad (10)$$

where

$$(\underline{H}_0)_{ij} = \delta_{ij} H_j, \quad (11)$$

$$(\underline{W})_{ij} = W_{ij}, \quad (12)$$

$$(\underline{V})_{ij} = \delta_{ij} V_j, \quad (13)$$

and

$$(|\hat{\psi}\rangle)_i = |\hat{\psi}_i\rangle. \quad (14)$$

We now consider the arrangement channel T operator equations. It has been shown¹ that associated with (8) is the set of equations¹⁴

$$\hat{T} = \underline{WV} (\underline{1} + \underline{G}_0^+ \hat{T}), \quad (15)$$

where

$$(\underline{G}_0^+)_{ij} = \delta_{ij} (E + i\epsilon - H_j)^{-1} \quad (16)$$

$$= ([E + i\epsilon] \underline{1} - \underline{H}_0)^{-1}_{ij}, \quad (17)$$

and where

$$(\hat{T})_{ij} = \hat{T}_{ij} \quad (18)$$

is the channel T operator connecting initial arrangement j to final arrangement i.

We wish to show that the zeros of $(\underline{1} - \underline{W} \underline{V} \underline{G}_0^+)$ are the same as those of the operator $(E - H)$ since otherwise, the equations (15) suffer from Federbush disease.

Thus, we consider

$$\underline{Q} = (\underline{1} - \underline{W} \underline{V} \underline{G}_0^+) \quad (19)$$

$$= [(\underline{G}_0^+)^{-1} - \underline{W} \underline{V}] \underline{G}_0^+, \quad (20)$$

and we note first that by (16), $\underline{G}_0^+$ has no zeros apart from that occurring

at infinite energy, which we ignore. Therefore, by the factored form (20),

$\underline{Q}$ can vanish only due to the zero eigenvalues of $(\underline{G}_0^+)^{-1} - \underline{W} \underline{V}$. However, comparing (8), (10)-(13) and (17) with (20), we see that

$$(\underline{G}_0^+)^{-1} - \underline{W} \underline{V} = (E + i\epsilon) \underline{1} - \underline{H}_0 - \underline{W} \underline{V}, \quad (21)$$

and we have already proved that the only zeros of this operator are those of $E - H$. We therefore conclude that the coupled $\underline{T}$ operator equations (15) do not suffer from Federbush disease.

We next consider the alternative coupled $\underline{T}$ operator equations of Kouri and Levin^{4,5} which are

$$\underline{T} = \underline{V} \underline{W} (\underline{1} + \underline{G}_0^+ \underline{T}). \quad (22)$$

In this case, we must analyze the zeros of

$$\hat{\underline{Q}} = \underline{1} - \underline{V} \underline{W} \underline{G}_0^+ \quad (23)$$

$$= [(\underline{G}_0^+)^{-1} - \underline{V} \underline{W}] \underline{G}_0^+. \quad (24)$$

Again we note that $\underline{G}_0^+$ has no zeros. We need only study the zeros of

$(\underline{G}_0^+)^{-1} - \underline{V} \underline{W} \equiv E \underline{1} - \underline{H}_0 - \underline{V} \underline{W}$. We do this by considering solutions of the

equation

$$\langle \hat{\psi} | (E - H_0 - V W) = 0. \quad (25)$$

This reads, for the i -th component,

$$\langle \hat{\psi}_i | (E - H_i) - \sum_{j=1}^N \langle \hat{\psi}_j | V_j W_{ji} = 0, \quad (26)$$

and we sum this over i to obtain

$$\sum_{i=1}^N \langle \hat{\psi}_i | (E - H_i) - \sum_{j=1}^N \langle \hat{\psi}_j | V_j W_{ji} = 0. \quad (27)$$

Then using (6) and (7) we finally obtain (on interchanging summation indices in the second term)

$$\sum_{i=1}^N \langle \hat{\psi}_i | (E - H_i - V_i) = 0, \quad (28)$$

or

$$\left(\sum_{i=1}^N \langle \hat{\psi}_i | \right) (E - H) = 0. \quad (29)$$

The interchange of summation indices is permitted since both sums over i and j are finite. We recognize (29) as the Schrodinger equation (5) and therefore conclude that the only zeros of $\hat{Q}$ in (23) are those of $E - H$. Again, there is no Federbush disease.

In summary, we conclude that not only does the coupled channel T formalism provide a computationally viable method for avoiding problems of disconnected diagrams in the general n -body N -arrangement channel problem, it also is free of Federbush disease.

The proof of the absence of Federbush disease presented here is consistent with the interrelationship between the formalisms of Faddeev, Hahn, and Kouri-

Levin; the reduced matrix equations ^{2,3} are by construction closely related to the Faddeev-type coupled equations, while the channel coupling array method ^{4,6} has been shown ¹ to be equivalent to the reduction procedure. Since the Faddeev formalism ⁸ is known to be free of spurious solutions, we would therefore expect this to be also the case with the coupled equations considered here.

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initial states, for Eq. (10) are $|\underline{\psi}\rangle = |\underline{\phi}\rangle + G_0^+ \underline{W} \underline{V} |\underline{\psi}\rangle$ where $(|\underline{\phi}\rangle)_{ij} = \delta_{ij} |\phi_i\rangle$ is an initial state in arrangement channel i . Then as usual, $\underline{T}$ is defined by $\underline{T} |\underline{\phi}\rangle = \underline{W} \underline{V} |\underline{\psi}\rangle$, which leads directly to Eq. (15).