27
- 7-75
exp 7/7/75

UCID - 16789

This is an informal report intended primarily for internal or limited external distribution. The opinions and conclusions stated are those of the author and may or may not be those of the laboratory.

LAWRENCE LIVERMORE LABORATORY

University of California/Livermore, California

MASTER

MEASUREMENTS OF ELECTRICALLY EXPLODED TUBES

J. W. Shearer, C. W. Hartman, R. H. Munger,
R. L. Gullickson, and D. O. Trimble

and

D. Y. Cheng

(University of Santa Clara)

May 13, 1975

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

Prepared for U. S. Atomic Energy Commission under contract no. W-7405-Eng-48

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

MEASUREMENTS OF ELECTRICALLY EXPLODED TUBES

ABSTRACT

The dynamics of electrically exploded tubes were investigated, principally by means of current measurements and flash x-ray pictures. The pinch effect was observed on the tube motion. Pileup of the imploding tube metal was seen on axis. An approximate analytical model can be roughly fitted to the data, but a more complete fit can be obtained with detailed numerical codes. Application of the results to the planning of future gas-embedded Z-pinch experiments is discussed.

INTRODUCTION

The recent work of D. Y. Cheng^{1,2} suggests that exploding tubes can be useful in gas-embedded Z-pinch work.³ They create a large voltage spike that can be used for submicrosecond switching of the current. The tube explosion can rapidly compress the gas, thus creating various radial density profiles.

This work was done in an effort to recreate some of Cheng's highest neutron outputs.⁴ Although no neutrons were detected, valuable information on tube dynamics was obtained and is reported here.

Cnare briefly reported similar exploding-tube work some years ago.⁵

APPARATUS

The experiments were done in a two-week period when we had access to the B-Division plasma-focus bank in Building 341. This bank consists of 32 capacitors rated at 14 μ F, 20 kV. For most of the shots we used 16 kV, corresponding to a stored energy of 57 KJ at 448 μ F. The inductance of 50 of the shots was approximately 35 nH, consisting of the low inductance of the bank and its flat plate line, plus the coaxial inductance of the exploding tube and the return circuit bars shown in Fig. 1. An additional long coaxial inductance of 275 nH was added to the circuit on 12 of the shots, in order to match the ringing time of Cheng's bank.^{2,3}

The tubes were usually two or three inches long; they were mounted by means of brass compression tube fittings inside the cage of return circuit bars seen in Fig. 1. Various metals, diameters, and thicknesses were used, as shown in Fig. 2 and tabulated in the caption to that figure. In addition a few simple wires were exploded for comparison purposes. The tubes were surrounded by Livermore air and filled with deuterium gas at an initial density that ranged from 0.125-5 atm. However, the exploding tube dynamics appear to be unaffected by these pressure variations.

Fig. 1. Arrangement of exploding tube, return circuit bars, and flash x-ray diagnostics. Film holder not shown in photographs. The second photograph (open shutter) indicates the violence of the explosion.

Fig. 2. Metal tubes used in exploding tube experiments:

- a. 0.25-in. diam, 0.006-in. wall, stainless steel (type 304).
- b. 0.375-in. diam, 0.010-in. wall, stainless steel (type 304).
- c. 0.31-in. diam, 0.004-in. wall, aluminum (machined).
- d. 0.31-in. diam, 0.0065-in. wall, copper (machined).
- e. 0.085-in. o.d., 0.055-in. i.d., copper rod (machined).
- f. Some post-shot debris from aluminum and copper shots.

Figure 3 shows some sample scope traces. The current was measured by integration of the signal from two pickup coils already installed in the flat plate section of the capacitor bank. The voltage was monitored with a resistance divider installed across the exploding tube.

The flash x-ray source consisted of a 100-180 kV Flexitron* triggered power supply and tube head. The film was type 3000 Polaroid medical x-ray film (9 × 10 in.), mounted between intensifying screens. Using an available medical x-ray development fixture enabled us to view the picture immediately after each shot. This was useful for the purpose of adjusting the delay time of the flash x-ray picture. The x-ray emission time was 50 nsec, and the reproducibility of the shots permitted us to take x-ray pictures with an accuracy in time of about 100 nsec. The timing of the x-ray picture was automatically obtained on each shot because the flash x-ray pulser generated noise that could be seen on all of the current and voltage scope sweeps (see Fig. 3). The dimensions of the exploding tube in each x-ray picture were easily determined from the magnification factor M of the shadowgraph technique. From Fig. 1, this is seen to be:

$$M = \frac{47.5 + 8.5}{47.5} = 1.18 . \quad (1)$$

The shock wave from the tube explosion was of some concern, since it caused additional images on the x-ray film. Fortunately, these artifacts usually occurred at the edges of the film, leaving a good clear image in the center. An additional convenience of this film is that it produces a positive image, as will be seen in Fig. 4 and later figures.

EXPERIMENTAL RESULTS

Figure 4 shows the time history of the stainless steel tube, which was the most extensively studied. About half of the total number of flash x-ray pictures are shown, correlated with the current-time curve. The outer surface of the tube is seen to move inward slowly until blowup time (5 μsec) when it begins to expand slightly. The inner surface is seen to move inward rapidly at blowup time. Our interpretation of this behavior is that the pinch force contains the outer surface, preventing it from moving outward as freely as the inner surface implodes inward. From Fig. 4 we find that the early inward velocity of the outer surface is approximately 0.03 cm/μsec, and the inner surface implosion velocity is of the order of 0.3 cm/μsec.

The transverse striations seen at blowup time are commonly seen in exploding wires and exploding tube experiments.⁶ Additional examples will be seen later in this report. They are believed to be caused by a resistive instability. Any part of the metal that

* Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Energy Research & Development Administration to the exclusion of others that may be suitable.

Fig. 3. Scope traces for type 304 stainless steel tube experiment: 0.25-in. diam with 0.006-in. wall. All traces 2 $\mu\text{sec}/\text{cm}$.
 a. This dI/dt trace is useful for timing the blowup.
 b. Current trace.
 c. and d. Voltage traces.
 The start of the noise signal at 1.2 μsec after blowup time is the time of the flash x-ray picture.

Fig. 4. Radius and current history of 0.25-in.-diam, 0.006-in.-wall-thickness, type 304 stainless steel tube, exploded in the low inductance circuit. The lightly shaded bars represent the flash x-ray measurements of the exploding tube; the heavily shaded bars represent the axial pileup of the tube. The last flash x-ray picture in the sequence was taken at 12 μsec.

gets hotter than its surroundings will have more resistance, which heats it still more, etc.

A small bump is seen on the current-vs-time curve at blowup time; there is no "dark pause" as is seen in some exploding wire experiments.⁷ It should be noted that restrike of the current along the outer surface is plausible, because the inward velocity of that surface is about equal to the speed of sound in air, and because the metal heats the adjacent gas (see Appendix A). Thus there will be a rarified atmosphere at the outer boundary of the tube at blowup time, which would lead to lower breakdown potential. However, we have no direct experimental evidence of such a restrike.

One can also draw inferences about the dynamics of the inner deuterium gas fill from the data. The final inner surface implosion velocity is of the order of three times the sound velocity in deuterium, which implies that a Mach 3-4 shock will converge on the axis. For this velocity range there will be some dissociation, but no ionization of the deuterium.⁸ We are unable to infer the earlier time history of the gas, because the experimental measurement of the motion of the inner surface of the tube was not possible when the tube was very thin. Measurements made on setup x-ray pictures yielded radius values that were systematically less than the true radius. However, we have much more confidence in the inner radius measurements near blowup time when the tube is thicker. The axial "pileup" at late times mentioned in the Fig. 4 caption will be considered later in the discussion section of this report.

Figure 5 shows data taken with a larger stainless steel tube in the high-inductance circuit. In this case the fourth x-ray picture exhibits a much greater pinch effect near blowup time (22 μ sec) than the Fig. 4 experiment.

The first two x-ray pictures in Fig. 5 were the result of an attempt to make the exploding tube more resistive by hand-filing one section. This was done in an unsuccessful attempt to duplicate some of Cheng's experiments.⁴ However, the current curve from this experiment showed only minor differences from the others in this series, despite the obvious large striations in the flash x-ray picture.

Figures 6 and 7 show some data taken with short tubes of copper and aluminum. In both cases these more-conductive materials pinched before they blew up, even though this was the fast low-inductance circuit. It was more difficult to obtain good flash x-ray pictures of the aluminum shots; the pictures shown in Fig. 6 were done by lowering the x-ray tube voltage from 180 kV to 120 kV, and by judicious photocopying to enhance the contrast. Striations are detectable at the ends of the pinched aluminum column and on the surface of the copper column.

If the tube pinches before it blows up (Figs. 5, 6, 7), the gas fill can be assumed to be adiabatically compressed, because the tube pinch velocity is slower than the sound speed in these cases. For a detectable radial compression ratio of 10:1 in a diatomic gas (such as D_2), this again leads to a temperature of less than 3000 K, which means that the compressed gas may be dissociated, but not ionized.

Fig. 5. Radius and current data for 0.375-in.-diam, 0.010-in.-wall-thickness, type 304 stainless steel tube, exploded in the high inductance circuit. The first two flash x-ray pictures are the setup picture and the shot for a tube whose wall was hand-filed to roughly half of its initial 0.010-in. thickness. The last picture is marred by the shock wave artifacts mentioned in the text.

Fig. 6. Radius and current data for 0.31-in.-diam, 0.004-in.-wall-thickness, aluminum tube.

Fig. 7. Radius and current data for 0.31-in.-diam, 0.0065-in.-wall-thickness, copper tube.

Figures 8 and 9 compare two small-radius copper samples. The thick-wall-tube scope traces are similar to all of the copper and aluminum tube shots; very little voltage spike was observed. The wire shot, however, is seen to have a pronounced voltage spike, and an obvious dark pause. This is believed to be due to one or more of the following reasons: the wire was longer; the bank voltage was lower; the small-radius wire could expand and cool more rapidly than tubes or rods; and most of the surrounding atmosphere was at a higher density than the room air. This demonstrates that a wire fuse is more easily designed than a tube fuse.

The 0.25-in. stainless steel tube that was filled to the lowest hydrogen density ($1/8$ atm), produced a slightly smaller blowup spike than the other shots in that series (compare Figs. 3 and 10). Two hypotheses were suggested: first, the nonuniform blowup revealed by the x-ray photograph (Fig. 10) may have spread out the voltage and current derivative spikes in time; second, gas breakdown in the 0.125-atm-density, 2-in.-long axial channel could have shunted the resistive tube. Insufficient data are available to decide between these two hypotheses; no neutrons were detected from the shot, nor from any other in this series.

No detailed analysis has yet been made of the voltage traces (see Figs. 3, 8, 9). The exploding-wire shots produced voltage peaks that were from 2-5 times greater than the initial resistive voltage signal. Most of the exploding tube shots produced barely discernible bumps in the voltage signal. The largest voltage peaks were seen in the 0.25-in. stainless steel tube series (Fig. 3); they were about 1.2 times higher voltage than the initial voltage signal. These results are further indirect experimental evidence of possible voltage breakdown in the rarified air on the outside of the pinched tube.

CALCULATIONS

The data on current and radius vs time can be compared to numerical magnetohydrodynamics calculations. It is intended to report most of this work elsewhere; however, one preliminary comparison with the data is shown in Fig. 11, where the radius-vs-time history of the 0.25-in. stainless steel experiment (Fig. 3, 4) is compared to a numerical one-dimensional Lagrangian magnetohydrodynamic computation. Fair agreement is found; the current-vs-time agreement is much better than this - so close that it is unnecessary to show the plot.

The computation includes a formulation for the strength of materials of the tube - a property that delays the onset of the pinch motion. Hand calculations of the motion of the tube have also been done (by the method of Appendix B) for this tube and for the others also. The hand calculations neglected the strength of materials, and they usually predicted pinching before it actually occurred. This effect of the strength of materials is in agreement with the previous exploding-tube work.⁵

Fig. 8. Scope traces (as in Fig. 3) and flash x-ray pictures of the thick-walled hollow copper rod (see Fig. 2), showing very little pinching and well-developed striations. Also, virtually no voltage spike is seen at blowup time.

Fig. 9. Scope traces (as in Fig. 3) and a flash x-ray picture of a 40-mil-diam, 4-in.-long copper wire that was fired at a lower voltage (12 kV). Note the striations, the sharp voltage peak, and the current pause after blowup.

Fig. 10. Scope traces and flash x-ray picture of the lowest-density deuterium-fill shot, using the same tube as in Fig. 3. Note the smaller current dip and the unsymmetrical tube blowup. Unfortunately, the voltage scope traces were lost, and the x-ray picture is marred by the shock wave artifacts mentioned in the text.

Fig. 11. Comparison of the radius-vs-time flash x-ray data (Fig. 3) with a one-dimensional MHD numerical calculation done on the "MAGPIE" code.

The major discrepancy between the code calculation and the data is seen to be in the motion of the outside surface. Further work is needed with equations of state and two-dimensional codes in order to resolve this discrepancy. However, it should be reemphasized that the code calculation shows excellent agreement with the current curve and with the blowup time.

The "action integral" technique is widely known as a simple and useful idea for analyzing exploding metal wires, tubes, etc.⁷ We start with the ohmic heating equation:

$$\dot{P} = \eta(I/S)^2 = \rho C_H (dT/dt) \quad (2)$$

where we have neglected heat losses (see Appendix A), and where $\dot{P}$ is the power input per unit volume, η is the resistivity, I is the current, S is the area through which the current flows, ρ is the mass density, C_H is the specific heat of the metal, and T is the temperature. The conductor is assumed to be thin compared to the skin depth, so that the current density $J = I/S$ is a constant.

We now define the "action integral" A_I :

$$A_I \equiv \int_0^t I^2 dt = \int_{T_0}^{T_1} \frac{C_H \rho S^2}{\eta} dT \quad (3)$$

where T_0 is the initial temperature, and T_1 is the "blowup temperature." Examination of Eq. (3) shows why the action integral is such a useful concept. The right-hand side is calculable from the known properties of the metals and measured dimensions of the individual fuses vs their temperature; thus, it can be evaluated theoretically. For example, the product ρS^2 can be found from:

$$\frac{d(\rho S^2)}{dT} \approx \rho_0 S_0^2 \left(\frac{1}{\rho} \frac{d\rho}{dT} + \frac{2}{S} \frac{dS}{dT} \right) = \rho_0 S_0^2 \ell_e \quad (4)$$

where ℓ_e is the linear expansion coefficient:

$$\ell_e \equiv \frac{1}{x} \frac{dx}{dT} \quad (5)$$

where x is any linear dimension.

The left-hand side of Eq. (3) is calculable entirely in terms of the measured circuit constants. Thus, in principle, one can directly compare theory with experiment by means of the action integral A_I .

In practice, for our tube experiments, this comparison is not easy. The blowup is so rapid that the density ρ and the area S cannot be determined from static measurements of the coefficient ℓ_e , because the internal density does not have time to relax to its equilibrium value. In addition, the flash x-ray pictures do not give us good values of tube thickness at early times when the tube is thin. Furthermore, the blowup temperature T_1 is not easily determined, and the skin effect may be important.

We have used a constant-density approximation to simplify our analysis. We define the modified action integral A_J :

$$A_J \equiv \frac{A_I}{S_0^2} \approx \int_0^t J^2 dt \approx C_H \rho_0 \int_{T_0}^{T_1} \frac{dT}{\eta} \quad (6)$$

where J is the current density. Note that, in this approximation, the modified action integral is a single number for each metal independent of fuse dimensions. For temperatures greater than room temperature T_0 , the specific heat C_H can take on its classical value for metals:

$$C_H \approx 3R_0/A = 25/A \text{ joules/g}\cdot\text{K} \quad (7)$$

where R_0 is the gas constant per mole. To evaluate the right-hand side of Eq. (6), it is convenient to fit the data for stainless steel, copper and aluminum with a function for the resistivity:

$$\eta = \eta_0 \left(\frac{T}{T_0} \right)^{1+m} \quad (8)$$

Substituting into Eq. (6), and integrating:

$$A_J = \frac{C_H \rho_0 T_0}{\eta_0} \frac{1}{m} \left[1 - \left(\frac{T_0}{T_1} \right)^m \right] \quad (9)$$

Equation (9) has been evaluated for the three materials used in this work: type 304 stainless steel, aluminum, and copper. The resistivity-vs-temperature data to which Eq. (8) was fitted are displayed in Figs. 12 and 13. The temperature T_1 was chosen to be the handbook value of vaporization temperature plus the adjustment for heat of fusion described in the caption to Fig. 12. When the heat of vaporization ΔH_V is also included, then the temperature is T'_1 :

$$T'_1 = T_1 + \Delta H_V / C_H \quad (10)$$

The modified action integral A_J is shown in Table 1, together with the input parameters used to calculate it.

The results of Table 1 show that the predicted value of A_J is not sensitive to the choice of T_1 for aluminum and copper, but it is important for stainless steel. Without further comment at this time, let us proceed to compare these calculations with experimental values of the modified action integral.

The calculation of A_J from the experiments is based on the left-hand integral in Eq. (6), where we assume that there is no expansion of the metal. Using a numerical technique similar to Appendix B, one calculates:

$$A_J = \sum_{n=1}^N \left(\frac{I_{n-\frac{1}{2}}}{S_0} \right)^2 \Delta t_n \quad (11)$$

where N corresponds to the value of time t at blowup as determined from the scope traces described earlier. The results of these calculations are presented in Table 2.

Comparison of the two tables for the modified action integral A_J shows that both the wires and tubes blow up at values of A_J that are larger than would have been predicted,

Table 1. Calculations of modified action integral A_J (Eqs. (7) - (11) of text); $T_0 = 293$ K in all cases.

Parameter	Aluminum	Copper	Type 304 stainless steel
A (g/mol)	27	63.54	56
ρ_0 (g/cm ³)	2.7	8.95	8
η_0 (m Ω ·cm)	0.00283	0.0017	0.072
m	+0.3	+0.3	-0.6
T_1 (K)	3153	3379	3773
T_1' (K)	15,000	15,500	18,000
$A_J(T_1)^a$	4.4	10.5	0.88
$A_J(T_1')^a$	6.0	14.1	8.6

^aUnits are in the modified emu system (g-cm- μ sec). For ordinary emu, multiply by 10^6 . For MKS, multiply by 10^{11} .

Table 2. Evaluation of modified action integral A_J based on experiments (Eq. 11) using the same units as Table 1.

Experiments	Aluminum		Copper		304 stainless steel	
	Low L	High L	Low L	High L	Low L	High L
<u>Wires</u>						
0.025 in. diam	24	—	—	—	—	—
0.040 in. diam	—	29	64	52	—	—
0.047 in. diam	—	—	—	—	24	19
<u>Tubes</u>						
0.083 in. diam 0.015 in. wall	—	—	53	—	—	—
0.25 in. diam 0.006 in. wall	—	—	—	—	8	—
0.33 in. diam. 0.004 in. wall	43	18	—	—	—	—
0.33 in. diam. 0.0065 in. wall	—	—	32	—	—	—
0.375 in. diam. 0.010 in. wall	—	—	—	—	9	5
0.40 in. diam. 0.005 in. wall	13	—	—	—	—	—
<u>Calc. (Table 1)</u>						
Low value	4.4		10.5		0.88	
High value	6.0		14.1		8.6

Fig. 12. Handbook values of electrical resistivity for aluminum and copper plotted vs temperature. The "effective temperature" for liquids is the actual temperature plus the heat of fusion divided by the specific heat C_H .

Fig. 13. Handbook values of electrical resistivity for iron and type 304 stainless steel plotted vs temperature as in Fig. 12. "Mattieson's rule" for alloys simply adds a constant to the resistivity of the bare metal.

which corresponds to later times than would have been expected. The principal reason for this is probably the assumption of no expansion. This assumption makes the value of A_J too large in both Eq. (6) and Eq. (9) because we have held constant the parameters S_0 and ρ_0 , respectively. But, as can be seen from Eqs. (4) - (5), the error in Eq. (6) is larger than the error in Eq. (9), so that we obtain the disagreement shown. We did not carefully examine the expansion of the wires and tubes at early times, but other exploding wire studies have observed expansion before blowup time,^{6,7} as would be expected from the known expansion coefficients.

In addition, there is some evidence elsewhere^{6,7} that the resistivity values for liquid metals such as iron do not rise as rapidly with temperature as we have assumed here. This effect would also tend to make the experimental value of A_J larger than the calculated value.

Despite these obvious deficiencies in our application of the action integral principle to the experimental data, we believe that the results are useful to point out certain features of the experimental results. For example, we can see from Table 2 that on the average the blowup values of A_J are lower for tubes than for wires. This effect is particularly obvious in the stainless steel experiments. Again, the high-inductance, slow-discharge experiments have smaller action integrals (in two-wire and two-tube examples) than the low inductances, higher current, fast discharge cores. We simply report these features; explanations would be tenuous and problematical.

In addition, we can state from experience that the parameter A_J is useful for planning experiments when one changes dimensions, currents, and materials on a rapid time scale. It was particularly valuable for forecasting the correct circuit delay time for the flash x-ray exposure.

DISCUSSION

As previously mentioned, comparisons of these results with magnetohydrodynamic code calculations are in progress. However, we already have some useful knowledge that can be applied to proposed future experiments on the gas-embedded Z-pinch. The principal applications are to rapid current switching and to the establishment of a favorable density profile (one that has minimum density on axis).

One way to momentarily create such a "hollow" gas-fill density profile is with a tube that explodes before it pinches. The explosion causes sufficient inward motion of the metal to drive a shock toward the axis. Just before the shock reaches the axis, the unperturbed gas constitutes a low density channel. As mentioned earlier, it is probable that such channels were formed in the shots using the 0.25-in.-diam 0.006-in.-wall stainless steel tube (Fig. 4).

The other tubes studied in this work behaved differently; they pinched before they blew up. This sequence would not be expected to produce an appreciable gas density minimum on axis because the pinch velocity is slower than the acoustic velocity for the parameter ranges explored in these experiments.

A simple approximate criterion for determining whether a given tube will explode before it pinches can be derived for the case of a monotonically increasing current:

$$I = I_t t^n . \quad (12)$$

The blowup time t_B can then be obtained by integration of Eq. (6), assuming the modified action integral A_J is already known for the tube under consideration:

$$t_B^{2n+1} = 3A_J(S_0/I_t)^2 . \quad (13)$$

Similarly, we calculate the pinching motion of the tube, neglecting the strength of materials. The acceleration, a , is:

$$a = P/\rho\delta_0 = B^2/8\pi\delta_0\rho = I^2/r_0 S_0\rho . \quad (14)$$

Then the velocity v is obtained by substitution from Eq. (12) and integration, neglecting the change in radius r :

$$v = \int_0^{t_B} a dt = (I_t^2/r_0 S_0\rho) (t_B^{2n+1}/2n+1) . \quad (15)$$

Finally, the inward displacement Δr is obtained, for the case when $\Delta r \ll r_0$:

$$r_0 \gg \Delta r = \int_0^{t_B} v dt' = (I_t^2/r_0 S_0\rho) \left[\frac{t_B^{2n+2}}{(2n+1)(2n+2)} \right] . \quad (16)$$

Combining Eq. (13) and (16):

$$r_0 \gg A_J S_0 t_B / (2n+2) r_0 \rho . \quad (17)$$

Rearrangement of Eq. (17) gives the desired form for our approximate criterion for blowup before pinching:

$$1 \gg \left(\frac{1}{2n+2} \right) \frac{A_J S_0}{r_0^2 \rho} t_B = \frac{\sqrt{2n+1} A_J^{\frac{2n+2}{2n+1}} S_0^{\frac{2n+3}{2n+1}}}{(2n+2) r_0^2 \rho I_t^{2/2n+1}} . \quad (18)$$

Thus, we find that for a given current profile and a given cross-sectional area S_0 , the tube will blow up before it pinches provided r_0 is large enough. The reason for this result is that the pressure at a given current varies as $(1/r_0^2)$, while the thickness δ varies only as $(1/r_0)$, so that the acceleration varies as $(1/r_0)$.

Equation (18) was calculated for a few of the cases studied, with the results shown in Table 3. Values of A_J and t_B were taken from the experiments. The parameter n was chosen to give a rough fit to the observed current curves. We see that the results of Table 3, when compared to Eq. (18), correctly predict that the tubes of Figs. 5, 6, 7 will not explode before they pinch. The experiment of Fig. 4 marginally satisfies the criterion of Eq. (18); 0.38 is less than unity, but not much less, and the tube of Fig. 4 only pinches a slight amount before it explodes. Thus, one concludes that Eq. (18) is a useful guide for designing future exploding-tube experiments.

Table 3. Explosion vs pinch criterion (Eq. 18) calculated for several cases.

Parameter	304 stainless steel		Aluminum	Copper	
	Fig. 4	Fig. 5	Fig. 6	Fig. 7	
n	1	0	1	1	
r ₀ (cm)	0.32	0.48	0.39	0.39	
S ₀ (cm ²)	0.030	0.076	0.025	0.040	
ρ (g/cm ²)	8	8	2.7	8.95	
A _J (Table 2)	8	5	43	32	
t _B (μsec)	5	22	6	8	
$\frac{1}{2n + 2}$	$\frac{A_J S_0 t_B}{2 r_0 \rho}$	0.38	2.3	3.9	1.9

Next, let us consider the condition of the gas fill at late times, after tube blowup. It has been suggested² that a low-density axial channel can be formed in the gas after the implosion when the whole experiment is coming apart. The experimental data obtained under our experimental conditions are not encouraging for this suggestion. None of the flash x-ray pictures taken at late times show any cavity inside the exploded metal. In particular, Fig. 4 shows an actual pileup of the metal on axis, such that the maximum metallic density is on axis. One is left with the question: where did the gas fill go?

We believe that the answer is that the gas fill has been turbulently mixed with the metal vapor. In particular, we suspect that the gas-metal interface would be Rayleigh-Taylor unstable if the gas were compressed enough to decelerate the metal vapor near the axis. In order to show this, let us estimate the density $\rho_{m.v.}$ of the metal vapor, assuming that it is uniformly distributed throughout the volume given by the initial radius:

$$\rho_{m.v.} \approx \rho_0 \frac{2\pi r_0 \delta_0}{\pi r_0^2} \approx 2 \frac{\delta_0}{r_0} \rho_0 \quad (19)$$

For the stainless steel shots of Fig. 4, we find:

$$\rho_{m.v.} \approx (2) \left(\frac{0.006}{0.125} \right) (8) \approx 0.77 \text{ g/cm}^3 \quad (20)$$

Now compare this to the compressed gas fill density ρ_g assuming a maximum initial fill density of 5 atm of deuterium gas, and assuming a maximum radial compression factor of 10:

$$\rho_g \leq (10)^2 (5) \frac{4}{2.24 \times 10^4} = 0.089 \text{ g/cm}^3 \quad (21)$$

Comparison of Eqs. (20) and (21) shows that $\rho_g < \rho_{m.v.}$, so that the gas-metal interface would be Rayleigh-Taylor unstable when the lighter gas tries to decelerate the heavier metal vapor.⁹ In addition, the observed striations suggest that the metal vapor density is not uniform throughout the tube. Altogether, the observed pileup of metal on axis and the disappearance of the gas fill cavity appear to be quite plausible.

It is still possible, however, that under other experimental conditions the late time gas density minimum on axis might be formed. The considerations of the previous paragraph suggest the use of a much thinner exploding tube (smaller ratio of tube mass to gas fill mass). In that case, the gas fill/metal vapor density ratio at turnaround could be stable to Rayleigh-Taylor perturbations. This parameter range was not explored in the experiments reported here.

In summary, there seem to be several reasons for using thinner exploding tubes at larger radii for future gas-embedded Z-pinch studies. First, such tubes explode before they pinch, so that there is less chance for a low-gas-density-breakdown channel to develop on the outside to short circuit the voltage spike. Second, a tube that explodes before it pinches will create an imploding shock wave in the gas, producing a favorable initial density profile for a gas-embedded Z-pinch. Third, a tube of low enough mass may lead to a stabilized metal-vapor/gas interface at late times.

Such thinner imploding tubes thus offer the possibility of rapid current switching of the gas-embedded Z-pinch.

ACKNOWLEDGMENTS

We wish to thank William C. Mumper and Melvin K. Bainter for their assistance with the flash x-ray diagnostics source and detector.

The code calculation (Fig. 11) was done by James L. Eddleman.

REFERENCES

1. D. Y. Cheng, W. J. Loubsky, and V. E. Fousekis, Phys. Fluids **14**, 2328 (1971).
2. D. Y. Cheng, Nucl. Fusion **13**, 129 (1973).
3. C. W. Hartman, D. Y. Cheng, G. E. Cooper, J. L. Eddleman, and R. H. Munger, High Density Fusion and the Z-Pinch, Lawrence Livermore Laboratory, Rept. UCRL-75543, prepared for the IAEA Tokyo Conference, Nov. 1974 (IAEA CN-33/H-5-2).
4. D. Y. Cheng, University of Santa Clara, private communication, February 1975.
5. E. C. Chare, J. Appl. Phys. **32**, 1275 (1961).
6. K. S. Fansler and D. D. Shear, "Correlated X-Ray and Optical Streak Photographs of Exploding Wires," in Exploding Wires, W. G. Chase and H. K. Moore, Eds. (Plenum Press, New York, 1968), Vol. 4, pp. 185-193.
7. Exploding Wires, W. G. Chase and H. K. Moore, Eds. (Plenum Press, New York, 1959), Vol. 1.
8. R. A. Gross, Rev. Mod. Phys. **37**, 724 (1965).
9. G. Taylor, Proc. Roy. Soc. (London) **A201**, 192 (1950).

APPENDIX A

HEATING OF GAS IN CONTACT WITH EXPLODING WIRES AND FOILS

At blowup time the hot metal wire or foil heats up the surrounding gases by thermal conduction and by radiation. The heat conduction equation in the gas in cylindrical geometry is:

$$\frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D \frac{\partial T}{\partial r} \right) = D \frac{\partial^2 T}{\partial r^2} \quad ((r - r_0) \ll r_0) \quad (\text{A-1})$$

where the diffusivity D is given by:

$$D = \frac{K}{C_p \rho} = \frac{2}{7} \frac{A}{\rho} \frac{K}{R} \quad (\text{A-2})$$

where K is the thermal conductivity of the gas, ρ is the density, C_p is the specific heat at constant pressure, A is the molecular weight, and R is the gas constant. We use C_p , rather than the constant-volume specific heat, because the heat flow is slower than the velocity of sound, as will be shown below. Thus, the density ρ will be reduced at the surface of the metal because the temperature there is higher, and the pressure is a constant.

For an order-of-magnitude estimate of the distance of penetration Δx of the heat conduction wave into the gas we approximate the heat conduction equation, (A-1):

$$D \approx (\Delta x)^2 / \Delta t \quad (\text{A-3})$$

$\Delta t \approx 1 \mu\text{sec}$ at blowup time, and the diffusivity D is about $4 \text{ cm}^2/\text{sec}$ for air at 1 atm and 3000 K ($\sim 9 \text{ cm}^2/\text{sec}$ for hydrogen at 1 atm). Thus, the value of Δx is about $10 \mu\text{m}$ for air ($\sim 30 \mu\text{m}$ for hydrogen). One concludes that the thickness Δx of the conduction-heated gas layer is much smaller than the wire radii and tube thicknesses that we used, which ranged from 4-40 mil (0.1-1 mm).

Because the gas density is much less than the metal density, it also follows that the energy loss from the exploding wire or tube is negligible compared to the ohmic heating, as was assumed in Eq. (2) of the main text. One also notes that the average velocity of the heat flow in $1 \mu\text{sec}$ is much less than the sound velocity in both gases, as was assumed in Eq. (A-2). Also $\Delta x \ll r_0$, as assumed in Eq. (A-1).

The formation of this thin layer of heated, lower-density gas in close proximity to the exploding metal may, however, be an important mechanism for establishing current breakdown paths at blowup time, as discussed elsewhere.¹ However, when the metal finally explodes this thin low-density layer may disappear due to recompression and turbulent mixing.

Finally, let us briefly consider the radiative heat loss by the metal:

$$\frac{dE}{dA} \approx (\sigma T^4) \Delta t \quad (\text{A-4})$$

which is the well-known Stefan-Boltzmann law. As an approximate example choose $T = 3000$ K, and ignore the emissivity coefficients. Then we find that the metal radiates 0.5 mJ/cm^2 in 1 μsec . For a 0.004-in.-thick aluminum sample, this corresponds to a temperature drop of less than 0.05°K . Thus, we can neglect the heat radiation loss in Eq. (2) of the main text. The heat radiation will have little effect on the surrounding gas, which is transparent in the optical spectrum where most of the heat radiation is emitted.

APPENDIX B
 NUMERICAL INTEGRATION OF TUBE ACCELERATION (PINCHING)
 BASED ON CURRENT MEASUREMENTS AND NEGLECTING STRENGTH
 OF MATERIALS

Starting from the scope traces, one first tabulates a series of average current measurements $I_{n-1/2}$ in each time interval t_{n-1} to t_n . All time intervals ($\Delta_t \equiv t_n - t_{n-1}$) are equal.

$$I_{1/2}, I_{3/2}, I_{5/2} \dots I_{n-1/2} \dots \quad (B-1)$$

At $t_0 = 0$, we begin with the initial radius r_0 , the initial inward velocity $u_0 = 0$, and the initial mass per unit area $\sigma_0 = \rho \delta_0$ where ρ is the density and δ_0 is the initial thickness. Then at each time interval we go through the following sequence (emu units):

1. Compute the magnetic field $B_{n-1/2}$:

$$B_{n-1/2} = 2I_{n-1/2}/r_{n-1} \quad (B-2)$$

2. Compute the magnetic pressure $P_{n-1/2}$:

$$P_{n-1/2} = (B_{n-1/2})^2/8\pi \quad (B-3)$$

3. Compute the acceleration $a_{n-1/2}$:

$$a_{n-1/2} = P_{n-1/2}/\sigma_{n-1} \quad (B-4)$$

4. Compute the inward velocity u_n :

$$u_n = u_{n-1} + a_{n-1/2} \Delta t \quad (B-5)$$

5. Compute the new radius r_n :

$$r_n = r_{n-1} - u_{n-1} \Delta t - \frac{1}{2} a_{n-1/2} (\Delta t)^2 \quad (B-6)$$

6. Compute the new thickness σ_n :

$$\sigma_n = \sigma_0 r_0 / r_n \quad (B-7)$$

This is an explicit elementary scheme that is valid only for $\delta \ll r$ and for constant density ρ . However, it is useful for rapid hand calculation of rough estimates, where only 5-10 time intervals are involved. Such a hand calculation, on a WANG or hand-held Hewlett-Packard computer, takes only a few minutes.

As mentioned in the text, the major error in this purely inertial method is the neglect of the strength of materials.

NOTICE

"This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research & Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately-owned rights."

Printed in the United States of America
Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$ ____*; Microfiche \$2.25

<u>* Pages</u>	<u>NTIS Selling Price</u>
1-50	\$4.00
51-150	\$5.45
151-325	\$7.60
326-500	\$10.60
501-1000	\$13.60