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 CRITICAL EXPERIMENT AND THE APPR

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628-1

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-1-

TWO-GROUP CALCULATIONS FOR A STEEL-WATER CRITICAL EXPERIMENT AND THE APPR

ABSTRACT

Two-group constants have been prepared for a range of steel-water volume ratios. A calculation of critical mass and flux distribution has been made for an existing steel-water critical experiment. In order to check the experimental critical mass the age was increased. The effect of this adjustment on flux distribution was noted. The critical mass and flux distribution were calculated for the clean APPR critical experiment using the adjusted curve of age vs steel-water volume ratio. The calculated critical mass of the clean APPR critical experiment was 9.04 kg of U-235.

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Two-Group Calculations for Metal-Water Systems*

Two-group calculations have been applied to the aluminum-water MTR and the zirconium-water STR reactors. In the ANL-WAPD calculations the age was adjusted to fit one of the ZPR experiments and this curve used in all subsequent calculations. Excellent agreement with experiments has been obtained by WAPD for the STR Mark I and II.

The two-group method used in Ref. 1 for the APPR considered absorption and multiplication in the fast group (modified two-group method). It is of interest to check the validity of the straight two-group method when applied to a steel-water critical experiment.

Two-Group Constants for Steel-Water Mixtures

Age, τ . A number of different method of computing the age for metal-water mixtures are available. The Marshak formula⁽²⁾ was used⁽³⁾ to calculate the age of fission neutrons in a one-to-one mixture of steel and water at 460° F and 1200 psia. The result was $\tau = 67.5 \text{ cm}^2$. It can be shown⁽⁴⁾ that Marshak formula may be written in the form

$$\tau = \frac{f(\alpha)}{(1-\theta)^2 N_H^2}$$

*The many discussions of calculation techniques with A. M. Perry and R. R. Bate are gratefully acknowledged.

1. A Conceptual Design of a Package Power Reactor, A. L. Boch, et al, ORNL-1613.
2. Theory of the Slowing Down of Neutrons by Elastic Collision with Atomic Nuclei, Rev of Mod Phys, Vol. 19, No. 3, July 1947.
3. Reactor Design and Feasibility Problem - MTR Type Power Producers for a Remote Location, T. M. Hallman, et al, ORNL CF 52-8-22.
4. Intrinsic and Operational Characteristics of the Mark I STR, Part I, Inventory of Materials, Reactor Constants and Reflector Savings for the ZPR and Mark I STR, H. L. Garabedian, WAPD-RM-87.

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where

- N_H = number of hydrogen atoms per cm^3 of water
- θ = fraction of total volume occupied by metal
- $f(\alpha)$ = function of α involving integrations with respect to energy
- α = atomic density ratio of metal-to-hydrogen

At the point where the age was calculated, 460° F and 1200 psia ($\rho = 0.8221 \text{ gm/cc}$), steel-water volume ratio, $n = 1$ and $\alpha = 1.5544$. At 68° F , $n = 1.21639$ for the same α and $\tau = 56.024 \text{ cm}^2$. The calculations of τ for a range of zirconium-water⁽⁴⁾ and aluminum-water⁽⁵⁾ volume ratios with Marshak's formula indicate that τ vs metal-water volume ratio is very close to a straight line. The curves of τ vs steel-water volume ratio for 68° F and 14.7 psia and 450° F and 1200 psia are shown in Fig. 1.

The age used in Ref. 1 was calculated by Tittle's method⁽⁶⁾. It was reported in Ref. 1 that inclusion of inelastic scattering in iron lowered the age by 10%. The ages used in Ref. 1 are shown in Fig. 1.

The age for neutrons from a Ra- α -Be source to indium resonance in various steel-water mixtures is reported in Ref. 7. The experimental points are plotted in Fig. 1. The peak of the neutron spectrum for Ra- α -Be source is about 5.5 Mev compared to about 1 Mev for fission neutrons. The increased effect of inelastic scattering in iron for slowing down from the higher energy may

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5. Age to Thermal Energy of Fission Neutrons in $\text{H}_2\text{O-Al}$ Mixtures, Nancy D. Dismuke and M. Ruth Arnette, MonP-219.
 6. Nuclear Shielding Studies I, C. W. Tittle, NP-1418.
 7. Slowing Down Distribution to Indium Resonance of Neutrons from a Ra- α -Be Source in Water-Iron Mixtures, NACA-RM-#54HO4.

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-4-

account for the difference in slope of the experimental age curve from that based on Marshak's formula. There is a need for measurement of age of fission neutrons in steel-water mixtures.

Fast Diffusion Coefficient, D_f

The equation commonly used to evaluate D_f is:

$$D_f = \frac{\int_0^{u_{th}} D(u)\phi(u)du}{\int_0^{u_{th}} \phi(u)du}$$

where

$D(u)$ = diffusion coefficient at lethargy u

$\phi(u)$ = flux at lethargy u

The purely proton moderated flux spectrum taken from Ref. 8 (see Equation 7A, page 128 of Ref. 1) was used to compute D_f for the range of steel-water mixtures. The results are shown in Fig. 1, along with values from Refs. 1 and 4.

In Ref. 1 the above procedure was used to find D_f for the core. D_f for the reflector was found by averaging $D(u)$ for water over the flux for proton moderation of the core neutron leakage spectrum. This method yields $D_f(H_2O) = 1.54$ cm compared to $D_f(H_2O) = 1.36$ cm using the fission spectrum as source in the slowing down equation.

A value of $D_f(H_2O) = 1.143$ has been used for ANL-WAPD calculations⁽⁹⁾ because it gave good agreement with experiment⁽¹⁰⁾. This value was originally reported in Ref. 10 where the asymptotic expression for the age theory slowing down density from a fission source $f(u)$ given by:

8. The Fast Neutron Flux Spectrum of a Proton Moderated Thermal Reactor, T. C. Brooks and H. L. Glick, WAPD-45.
9. Quarterly Report, June 1 through August 31, 1949, Naval Reactor Division ANL-4337.
10. Critical Mass and Neutron Distribution Calculations for H_2O Moderated Reactors with D_2O , H_2O and Be Reflectors, C. Greuling and B. Spinrad, MonP-402.

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-5-

$$\phi(u) = \int_0^u \frac{f(u)du}{\int \Sigma_S(u)}$$

was used to average $D(u)$. This expression includes only the collided flux and not the uncollided or virgin flux.

Slow Diffusion Constant, D_S

The equation used to compute D_S is:

$$D_S = \frac{1}{3 \left[\sum_{tr}(H_2O) + \sum_{tr}(SS) \right]}$$

A method suggested by A. M. Weinberg⁽¹¹⁾ was used to compute $\sum_{tr}(H_2O)$ from the experimental L^2 for water (8.12 cm^2)⁽¹²⁾ and $\sum_a(H_2O)$ (0.01955 cm^{-1}) averaged over a Maxwellian flux. The result is $\sum_{tr}(H_2O) = 2.0997 \text{ cm}^{-1}$ and $D_S(H_2O) = 0.1588 \text{ cm}$. This is to be compared with $D_S(H_2O) = 0.1425 \text{ cm}^{(1)}$, $0.167 \text{ cm}^{(10)}$, $0.162 \text{ cm}^{(4)}$, and $0.160 \text{ cm}^{(11)}$. It appears that $D_S(H_2O)$ used in Ref. 1 is low. This is believed due to the value of $L^2 = 7.29 \text{ cm}^2$ ($L = 2.7 \text{ cm}$) used in its calculation. It is reported in Ref. 1 that this was taken from Radkowsky's curve⁽¹³⁾ of L vs temperature; however, this could not be verified.

A value for $\sum_{tr}(H_2O)$ at 450° F was found in the following manner: Radkowsky's calculated curve of L vs temperature was adjusted to $L = 2.85 \text{ cm}$ ($L^2 = 8.12 \text{ cm}^2$) at 300° K , at 450° F (505° K) the adjusted curve gave $L = 3.63 \text{ cm}$ for water of unit density. Correcting for density of water at 450° F and 1200 psia ($\rho = 0.8289 \text{ gm/cc}$) $L = 4.376 \text{ cm}$. Using $\sum_a(H_2O) = 0.01234 \text{ cm}^{-1}$ at 450° F , $D_S(H_2O) = 0.237 \text{ cm}$.

11. Calculations of Slow Constants for the MTR, A. M. Weinberg, ORNL CF 53-3-107.
12. Neutron Cross Sections, BNL-170.
13. Quarterly Report, April through June 1950, Physics Division, ANL-4476.

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-6-

Fig. 2 shows D_S for the steel-water mixtures together with the values from Refs. 1 and 4. It has been pointed out⁽¹⁴⁾ that the thermal diffusion length is quite sensitive to the shape of the neutron energy distribution and it should be calculated for the particular distribution rather than taken from experiments performed with different neutron energy distributions.

Cross Sections^(15,16)

The macroscopic cross sections of uranium (assumed to be 93.5% U-235 by weight), stainless steel (type 304) and water have been calculated. For use in the two-group equations the macroscopic cross sections are multiplied by the respective volume fractions.

Uranium (93.5% U-235)

$$\begin{aligned} 68^\circ \text{ F (0.025 ev)} \quad \rho &= 18.7 \text{ gm/cc} \\ \text{U-235} \quad \sigma_f &= 580\text{b} \quad 1 + \alpha = 1.184 \quad \sigma_a = 687\text{b} \quad \overline{\sigma_s} = 10\text{b} \\ &\quad \beta = 2.08 \text{ non } \frac{1}{\beta} = 0.981 \\ \text{U-238} \quad \sigma_a &= 2.75\text{b} \quad \overline{\sigma_s} = 10\text{b} \\ 450^\circ \text{ F (0.0435 ev)} \quad \rho &= 18.7 \text{ gm/cc} \\ \text{U-235} \quad \sigma_f &= 395\text{b} \quad 1 + \alpha = 1.184 \quad \sigma_a = 467.7\text{b} \quad \overline{\sigma_s} = 10\text{b} \\ &\quad \beta = 2.08 \text{ non } \frac{1}{\beta} = 0.993 \\ \text{U-238} \quad \sigma_a &= 2.09\text{b} \quad \overline{\sigma_s} = 10\text{b} \end{aligned}$$

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14. Diffusion Length of Thermal Neutrons in Water, M. Danzker, et al, TID-2008.
 15. Neutron Cross Sections, BNL-250.
 16. Neutron Cross Sections, AECU-2040.

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-7-

Steel (304 SS)

68° F (0.025 ev) $\rho = 7.93 \text{ gm/cc}$

<u>Composition</u>	<u>Wt %</u>	<u>σ_a^b</u>	<u>σ_{tr}^b</u>
C	0.08	0.0045	4.5
Mn	2.00	12.6	2.3
P	0.04	0.19	4
S	0.03	0.49	1
Si	1.00	0.13	2.2
Ni	8.00	4.5	17
Cr	18.00	2.9	4
Fe	70.85	2.43	11

450° F (0.0453 ev) $\rho = 7.93 \text{ gm/cc}$

$\frac{1}{v}$ correction factor = 0.7616 (applied to σ_a)

Water

68° F (0.025 ev) $\rho = 1 \text{ gm/cc}$

H $\sigma_a = 0.33$

O $\sigma_a = 0.002$

450° F (0.0435 ev) $\rho = 0.8289 \text{ gm/cc}$

$\frac{1}{v}$ correction factor = 0.7616 (applied to σ_a)

Macroscopic Cross Sections

<u>Material</u>	<u>68° F</u>			<u>450° F</u>		
	<u>Σ_a</u>	<u>Σ_{tr}</u>	<u>λ</u>	<u>Σ_a</u>	<u>Σ_{tr}</u>	<u>λ</u>
Uranium (93.5% U-235)	26.78	0.4775	2.08	18.44	0.4775	2.08
Stainless 304	0.2185	0.8524		0.16642	0.8524	
Water	0.01955	2.0997		0.01234	1.40885	

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-8-

Application of Two-Group Calculation to ANL-Critical Experiment

The only critical experiment available that is similar to the APPR is the ANL steel-water critical experiment⁽¹⁷⁾. Characteristics of the assembly of chief interest are:

Core composition

Water	48.13%	H ₂ density 0.1111 gm/cc
Polyethylene	21.84%	H ₂ density 0.1314 gm/cc
Steel	30.03%	

Effective H₂ density in core 0.1164 gm/cc

Fuel	93.15% enriched uranium; metal foils 0.003" x 0.75" x 20"
Critical mass	8.087 kg of U-235
Radius of Equivalent circle	6.99 in.
Active core height	20 in.
Radial reflector water	> 8 in. thick
Axial reflector	30% steel in water

The calculation assumed that the core was 30% steel and 70% water and neglected the effect of self-shielding, F , in the 0.003 in. thick foil ($F = 0.9345$ based on self-shielding equation in Ref. 1).

The two-group equations were solved⁽¹⁸⁾ for a finite cylinder with an infinite radial water reflector with an assumed axial reflector savings. Once the critical mass was determined, the radial reflector savings were calculated and the critical mass determined for an equivalent bare cylindrical reactor reflected on the ends with a 6 in. reflector (30% steel and 70% water). The axial reflector savings were computed and this procedure repeated until there was no change in reflector savings and critical mass.

17. Critical Studies with G.E. Type Fuel Elements by ZPR-1 Group, T.H. Martens and G. F. Helfrich, ANL-5108.

18. Handbook of Two-Group Formulas, H. L. Garabedian, WAPD-RM-167.

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-9-

The results of using the two-group constants given by Figs. 1 and 2 and the macroscopic cross sections previously listed are:

Critical mass	7.0 kg U-235
Radial reflector savings	$S_r(\infty) = 6.34$ cm
Axial reflector savings	$S_z(t) = 6.07$ cm

The age used in this calculation was $\uparrow = 40.5$ cm². The calculated radial thermal and epithermal flux are given in Figs. 3 and 4, respectively along with the measured fluxes. The experimental flux plots were made using iron manganese wires as thermal neutron detectors. Cadmium covered wires were used for the epithermal flux.

A comparison of the calculated and measured thermal flux indicates that the peak thermal flux in the reflector is overestimated by the two-group equations. Good agreement is obtained between calculated and measured epithermal flux. The calculated and measured axial thermal flux distributions are shown in Fig. 5. The 30% steel in the axial reflector can be seen to suppress the peak in the reflector, the axial leakage is also much less than the radial leakage because of the L/D of 1.43.

The approach used by ANL-WAPD to fit the ZPR experiments was to adjust the age until the correct critical mass is obtained. Adjusting the age in the ANL critical experiment calculation, it is found that:

$\uparrow$ (30% steel and 70% water)	$= 46$ cm ²
Critical mass	$= 8.09$ kg of U-235
Radial reflector savings	$= S_r(\infty) = 6.60$
Axial reflector savings	$= S_z(t) = 6.15$

The effect on the radial flux distribution of increasing the age from 40.5 cm² to 46 cm² to obtain the correct critical mass is seen in Fig. 3. The mass is increased by increasing the fast leakage and as a result the thermal peak in the reflector is increased.

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-10-

Observation of Fig. 3 indicates that to obtain the correct critical mass and flux distribution the non-fissioning absorptions in the core should be increased. Application of the modified two-group such as outlined in Ref. 1 should handle the epithermal absorptions and fissions correctly. However, using the method for calculating resonance escape and fast multiplication outlined in Ref. 1 in the modified two-groups equations lowers the critical mass (15% lower for the APPR⁽¹⁾) from that calculated using straight two-groups. Using the method of Ref. 1 for the ANL critical experiment⁽¹⁹⁾ gave a critical mass of 6 kg of U-235. It has been pointed out⁽²⁰⁾ that absorption in the steel should be included in computing resonance escape and fast multiplication. The application of the modified two-group to the ANL and the APPR critical experiment (including epithermal absorption in the steel as well as uranium) is presently being made by R. Johnson and H. Geisler at Alco Products, Incorporated.

Prediction of the APPR Clean Critical Mass

The approach used herein will be to adjust the curve of $\uparrow$ vs steel-water volume ratio to fit the ANL critical experiment and use a $\uparrow$ from this curve (at 20.32% steel by volume) for the APPR. The $\uparrow$ curve adjusted to fit the ANL critical experiment is shown in Fig. 1. The critical mass has been calculated for the clean APPR critical experiment (end boxes off). The characteristics of the calculated reactor are:

Core composition

Steel	20.43%
Water	79.57%

19. Personal Communication A. M. Perry.

20. A Conceptual Design of an Army Package Power Reactor, Supplement I, R. R. Bate and A. M. Perry, ORNL-1613, Suppl. I.

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-11-

Fuel (assumed to be 93.5% U-235)

Metal foils 0.002" x 2.5" x 22" incased in 0.011" stainless

Radius of equivalent circle	11.1 in.
Active core height	22 in.
Radial reflector	water, 8 in. thick
Axial reflector	5.9% steel in water, 6 in. thick

The results of the calculation using the adjusted age curve and neglecting self-shielding of the 2-mil uranium foils are:

Critical mass	9.04 kg U-235
Radial reflector savings	6.55 cm
Axial reflector savings	6.15 cm

The cold clean critical mass reported in Ref. 1 is 7.26 kg of U-235 based on a modified two-group calculation. The three-group Oracle code and the 30-group Univac Program gave 7.40 and 8.03 kg of U-235, respectively⁽²⁰⁾.

The calculated radial thermal and epithermal flux distribution are shown in Fig. 6. In comparing Figs. 3 and 6 it is of interest to note that the two-group fast flux in the ANL critical experiment (for $\bar{\Gamma} = 46 \text{ cm}^2$) is 13.0 times the thermal flux compared to 5.3 times in the APPR. This indicates that the neutron spectrum in the two reactors is much different.

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-12-

APPENDIX

Calculation of SIR Fuel Cage Critical Experiments

The criticality of several arrangements of the SIR fuel cages in water (SIR fuel elements without the Be center rod) has been checked⁽²¹⁾. One critical assembly consisted of a close packed hexagonal array of 19 fuel cages. Each cage consisted of 60 pins: 55 fuel pins and 5 structural pins. The fuel pins were 0.060 in. I.D. and 0.080 in. O.D. with 27.25 in. fuel length. The fuel pins were filled with 68.4% UO_2 and 31.6% MgO by weight⁽²²⁾. The characteristics of the critical assembly are:

Core composition

Steel	5.687%
UO_2	1.655%
MgO	2.271%
Void	2.114%
Water	88.273%

Critical mass	4.52 kg of U-235
Radius of equivalent circle	3.95 in.
Length of active core	27.25 in.
Radial reflector	Water
Axial reflector	5.68% steel and 94.313% water

The calculated reactor had the following core composition:

Steel	5.687%
Water	94.313%

21. SIR Program Progress Report, June and July 1954, KAPL-1171.

22. SIR Project Basic Design Data, August 1, 1953, KAPL-788.

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628 48

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13

The results of the calculation are:

Critical mass	2.62 kg of U-235
Radial reflector savings	$S_r(\infty) = 6.43$ cm
Axial reflector savings	$S_z(t) = 6.03$ cm

This is felt to be a very severe⁽¹⁹⁾ test of the theory due to the very high L/D of the core (3.45). A slight error in estimated the radius of the circle equivalent to the hexagonal array would cause a large error in critical mass. Also the calculated reactor contained too much water.

It was pointed out by R. A. Charpie⁽²³⁾ that two-group calculations are poor for small assemblies. For aqueous homogeneous cylindrical systems, about 10 in. diameter, the two-group calculated mass was 82% of the experimental mass. Much better agreement is obtained between two-group masses and experiments for MTR type reactors with both H₂O and D₂O reflectors.

The results of this calculation together with those reported in Ref. 23 for the aqueous homogeneous system tend to indicate that in order for the two-group calculation to give good agreement with experiment over the complete range of steel-water volume ratios the age curve should be raised over the entire range. This implies that the calculation for the APPR using the $\uparrow$ curve (at 20.43% steel) fitted to the ANL experiment at 30% steel by volume and the experimental age in pure water may result in a low critical mass. This neglects the effect of changes in neutron spectrum with changes in size.

23. Lecture, Criticality of Water Moderator Systems, R. A. Charpie, ORNL, March 28, 1955.

J. G. Gallagher:vj
September 28, 1955

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628

14

TWO GROUP CONSTANTS STEEL WATER MIXTURES

STEEL 8% Ni 18% Cr & 74% Fe

ORNL 1613 GROUP CONSTANTS

WARD-ANL

AGE TO IN RESONANCE
FOR RA-X-EE NEU.

Fig. 1

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SLOW DIFFUSION CONSTANT (D_s) STEEL WATER MIXTURES

STEEL 8% Ni 18% Cr & 74% Fe

● ORNL 1613

● WARD-ANL

Fig. 2

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MEASURED AND CALCULATED THERMAL FLUX DISTRIBUTION IN ANL STEEL-WATER CRITICAL EXPERIMENT

Calculated Reactor

Core

6.99" dia. x 20" high
30% steel (by volume)
70% water (by volume)

Reflector

Radial - infinite water
Axial - 6" thick, 30% steel
and 70% water

Critical Mass

Age τ

7.0 kg U-235

40.5 cm²

8.09 kg U-235

46 cm²

**MEASURED AND CALCULATED EPITHERMAL FLUX DISTRIBUTION
IN ANL STEEL-WATER CRITICAL EXPERIMENT**

Fig. 4

Radius r - Inches

MEASURED AND CALCULATED THERMAL FLUX DISTRIBUTION
IN ANL STEEL-WATER CRITICAL EXPERIMENT

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CALCULATED THERMAL AND EPITHERMAL FLUX DISTRIBUTIONS IN AFR. CRITICAL EXPERIMENT

Fig. 6