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**COMPUTER APPLICATIONS IN CONTROLLED
THERMONUCLEAR RESEARCH**

John Killeen

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Summary of a talk given at the
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COMPUTER APPLICATIONS IN CONTROLLED THERMONUCLEAR RESEARCH

Abstract

This report surveys those areas of plasma physics and controlled thermonuclear research in which computation has made significant contributions. Both

fluid and particle models are considered, and applications to specific confinement experiments and plasma theory are discussed.

Introduction

Numerical calculations have had an important role in controlled thermonuclear research since its beginning in the early 1950's, but the application of computers to plasma physics has advanced rapidly in the last few years. Since 1967 there have been six conferences devoted to the Numerical Simulation of Plasmas, and during that same period the CTR computer usage has increased by nearly an order of magnitude. One reason for this is the increasing sophistication of the mathematical models of plasma behavior, and another is the increased speed and memory of the computers which made it reasonable to consider numerical simulation of plasmas.

A general reference for this subject is my Scottish Universities Summer School lecture notes.¹ Although these lectures were given in 1968, the discussion of mathematical models of plasma confined by magnetic fields is applicable, and the numerical techniques, e.g., difference methods for solving partial differential equations, are now widely used. In 1970, Volume 9 of the series of books titled "Methods in Computational Physics" was devoted to plasma physics.² Most of the book is devoted to articles on the solution

of the Vlasov or collisionless Boltzmann equation, and these include the many-particle simulation techniques. There are two chapters on collisional plasma models -- one on the numerical solution of the Fokker-Planck equation for a plasma, and one on magnetohydrodynamic calculations. Another general reference is the "Proceedings of the Fourth Conference on Numerical Simulation of Plasmas"³ which appeared in 1971. (The proceedings of the other five Conferences contain only abstracts.)

In this survey we shall consider the following areas in which computation has made significant contributions:

1. Magnetohydrodynamics
2. Radial Transport in Tokamak Plasmas
3. Computational Work on Toroidal Pinches
4. Finite Beta Equilibria
5. Fokker-Planck Codes for the Determination of Endlosses in Mirror Machines
6. Collisionless Plasma Models to Study Mirror Instabilities
7. The Simulation of Parametric Instabilities
8. Numerical Simulation of Astron.

Magnetohydrodynamics

Many mathematical models are used to describe a plasma in a magnetic field, and these vary from the description of single-particle orbits to the kinetic theory of ionized gases. The fluid or magnetohydrodynamics (MHD) model is basic, and

any experiment is first analyzed to determine if MHD equilibria and stability exist. Within the fluid theory various degrees of complexity are considered. The so-called ideal MHD is an infinite conductivity approximation. In some

models the pressure is a scalar function, but in some problems it is necessary to consider it as a tensor with different values along and perpendicular to the magnetic field. The more realistic models include the transport coefficients, e. g., thermal conductivity and electrical resistivity, and these can also be scalars or tensors.

The article by Roberts and Potter⁴ gives a good review of the role of MHD computations and discusses methods for the solution of time-dependent problems. I have also written a review article⁵ for *Information Processing 71* which considers both time-dependent and time-independent problems.

The use of numerical calculations in the design and analysis of pinch experiments has been of central importance. In most of these computations the equations of magnetohydrodynamics are used. The earliest problems⁶ used the infinite conductivity theory in analyzing the linear pinch. At Livermore, to analyze some experiments we coupled the pinch dynamical equation with the external circuit equations.⁷ The resulting set of differential equations were solved numerically.

As the stability theory of the pinch advanced and more complex experiments evolved such as the diffuse "stabilized" pinch, hard core pinch and theta pinch, more elaborate numerical computations have been made. Fortunately, at this same time, the speed and capacity of computing machines have been increasing at a rapid rate.

Most of the above experiments have been analyzed with computations on a time-dependent, one-dimensional, fully ionized hydromagnetic model. Electrical resistivity and thermal conductivity of the plasma are included, and separate temperatures are assigned to the electrons and ions. The first programs with

this model were written by Hain and Roberts⁸ as a joint effort between AERE Harwell and the Max-Planck-Institut, Munich. In their work, the equations to be solved are given in Eulerian form. Other programs which have been written use the same model,⁹ but with equations in Lagrangian form.

The resulting partial differential equations which describe the initial-value problem are of both hyperbolic and parabolic type, and can be solved by difference methods which are described in Refs. 1, 4 and 5.

To give a better description of many of the experiments in CTR programs, a two-dimensional model is needed. We consider a cylindrical system with azimuthal symmetry. The variables are then functions of r , z and t . Because of the success of the earlier calculations in describing certain experiments, we shall again consider a two-fluid model (unequal electron and ion temperatures) with thermal conduction. The velocity has components v_r and v_z and the magnetic field has components B_r and B_z . This model is suitable for analyzing theta pinches, conical plasma guns, and the expansion of a laser-produced plasma in a magnetic field. A variety of difference schemes have been used and are described in Refs. 4 and 5. An alternating-direction implicit code¹⁰ (ADI) has been developed and used to study theta pinch implosion and the expansion and trapping of a laser-produced plasma by an initially uniform magnetic field.

Another version of the two dimensional (r, z) MHD equations use v_ϕ and B_ϕ instead of v_r , v_z , B_r , B_z for describing the plasma focus experiments. Considerable success has been achieved in simulating these experiments with both Eulerian⁴ and particle-in-cell (PIC)¹¹ schemes.

Radial Transport in Tokamak Plasmas

In the past few years, a considerable effort has been devoted to the numerical solution of radial transport equations for toroidal plasmas. This effort provides the only practical means of comparing theory with experiment, and, moreover, is invaluable in optimizing the parameters for the design of future devices. This effort was recently reviewed by Duchs¹² in a paper presented at the Garching Conference on Toroidal Confinement.

All of these computations are based on a description of the plasma by macroscopic variables such as particle density n , electron and ion temperatures T_e and T_i , and the poloidal and toroidal components of the magnetic field. The codes are one-dimensional and the cylindrical approximation is generally used, i. e., the magnetic surfaces are assumed to be concentric circles and the cylindrical coordinate r and the time t are the

independent variables. There is currently some effort going into generalizing these codes to noncircular cross section.

In 1967, we developed a code,¹³ to solve the transport equations for $n(r, t)$ and $T(r, t)$. The dependence of the transport coefficients on n , T and B was allowed to be quite arbitrary so that we could study the dependence of the plasma profile on the form of the coefficients, and also look for thermal instabilities.

In 1968 the neoclassical transport theory¹⁴ for Tokamak plasmas appeared which included the effects of trapped particles (banana orbits) on the transport coefficients. Since 1969 a large number of numerical calculations have been reported at meetings based on models of increasing complexity. These are listed in the Duchs paper.¹²

Computational Work on Toroidal Pinches

The objective of reversed field pinch experiments is to set up an MHD stable field configuration using field programming. Confinement is then limited primarily because field diffusion causes the configuration to decay until the stability properties are lost. In the analysis of both the setting up and decay phases, computational physics is playing an important role.

At the Garching meeting, Newton²¹ described calculations which simulated the field programming (compression) phase up to the time that the crowbar is applied. The code that is used for this purpose is a generalization²² of the original Hain-Roberts⁸ time-dependent, one-dimensional MHD code. The generalization is to a partially ionized gas, so it is a three-fluid model with classical transport coefficients except for anomalous resistivity. The code includes circuits for azimuthal and axial current with air core transformer coupling. Ionization and radiation are also included. Newton stated that the inclusion of the full circuit and neutral atoms is necessary for a self-consistent calculation. Good agreement was obtained between measured and computed field and density distributions.

The MHD stability of reversed field pinch configurations has been studied theoretically by Robinson²³ using the hydromagnetic energy principle. In a paper at the Garching meeting, Grossman²⁴ described numerical calculations of MHD instability growth rates for re-

In 1971-72 the most complete sets of neoclassical transport equations were derived^{15,16} for both the banana and plateau regimes of collision frequency. There are now five codes -- at Culham,¹⁷ Livermore,¹⁷ Princeton,¹⁸ Texas,¹⁸ and Oak Ridge¹⁹ -- which include this most recent theory. The calculations have been checked against each other for a standard choice of initial and boundary conditions, and give equivalent results.

Considerable effort is now under way to generalize these codes to include the effects of neutrals, impurities, and empirical models of plasma transport^{12,20} to give better agreement with experimental results from present day Tokamaks.

versed field profiles of the proposed Padua experiment which is under construction. The profiles considered correspond to various stages of the setting up (implosion) phase of the reversed field pinch. The profiles were obtained from a one-dimensional, time-dependent code of the type described in the previous paragraph. The instability growth rates were obtained by solving the linearized ideal MHD equations including compressibility. The effect of compressibility is to raise the growth rates of unstable modes.

In the analysis of the decay phase of reversed field pinches, several codes are in use at Culham. The results of these computations can be compared with the existing H β TX and can also aid in the design of future toroidal pinch experiments.²⁵ To study the effects of coupled field, particle and thermal diffusion, a new code originated by Killeen²⁶ has been developed at Culham and Livermore. It solves for $n(r, t)$, the number density of ions and electrons, $T_e(r, t)$ and $T_i(r, t)$, the electron and ion temperatures, and $B_\theta(r, t)$ and $B_z(r, t)$, the poloidal and toroidal components of the magnetic field. The classical transport coefficients have been used for the toroidal pinch calculations, but the code also includes the neoclassical (banana-plateau) coefficients. Results of this code, showing the evolution of an initially stable reversed field pinch, were presented by Robinson²⁷ at the Garching meeting. These results also

included a discussion of the thermal instability which shows up in some of the calculations.

Three codes are in use at Culham for studying the stability of the evolving field configurations of the previous paragraph. One code called NEWCOMB²⁸ uses the ideal MHD energy principle of the diffuse pinch²⁹ to determine marginal stability as a function of m , k and wall radius, R_w . Another code solves the linearized ideal MHD equations for the growth rates of unstable modes. The third code in use is RIPPLE IV³⁰ which solves for the resistive tearing mode in cylindrical geometry for an arbitrary $B_z(r)$ and $B_\theta(r)$. Results of these codes

for evolving reversed field profiles were presented by Robinson²⁷ at the Garching meeting, and also at the Innsbruck meeting.³¹

A new code³² which solves for the resistive tearing mode in cylindrical geometry, but also including compressibility, thermal conductivity, and viscosity, has been developed at Livermore. Preliminary results for reversed field configurations were presented at the Sherwood Theory Meeting in Austin, Texas, March, 1973.

All of the codes described above (both diffusion and stability) are applicable to Tokamaks³³ and other toroidal pinch plasmas.

Finite-Beta Equilibria

Experimental devices incorporating the idea of axial symmetry in a torus appear to be capable of plasma confinement for times which are of great interest. One reason for this is the assurance of equilibria in such devices, as predicted by MHD and guiding-center theories.³⁴ The choice of distribution function determines the type of equilibrium. With the guiding-center equations, one can choose the distribution function f arbitrarily as a function of flux function ψ , magnetic moment μ , and energy ϵ . The components of the pressure tensor $p_\perp$ and $p_\parallel$ are then determined as moments of f , and can be written as functions of ψ and magnitude of magnetic field B . The hydromagnetic equations then yield a nonlinear, second-order partial differential equation for $\psi(r, z)$. We developed³⁵ the technique for solving such free boundary equilibria problems. A detailed description of this finite-difference method for solving the ψ equation is given in Refs. 1 and 5.

Two-dimensional models for plasma equilibria have been used to compute the equilibrium fields for various containment schemes including Astron,³⁶ Levitron,³⁷ Tokamak, and the stuffed cusp.³⁵ Several containment schemes, including the stabilized mirror devices,

do not possess an ignorable coordinate, thus allowing a reduction of the equilibrium equations to two dimensions. A code³⁸ has been developed to handle the general case for open containment. This is a three-dimensional code that solves for plasma equilibria in open-field geometries which allows analysis of most minimum-B mirror systems. Open confinement, with $p = p(B)$ only, requires the hydromagnetic equilibrium equation to reduce to one elliptic, scalar partial differential equation involving a tensor pressure. The finite difference form of the equation is solved by an implicit iterative algorithm similar to the ADI (alternating direction implicit) method. Cylindrical grid coordinates are employed, and the mesh spacing is variable to allow representation of far boundary conditions. It is found that, when the threshold for hydromagnetic instability is passed, the hydromagnetic equilibrium equation becomes hyperbolic which leads to an ill-posed problem as predicted by Grad.³⁴ Equilibria are found for an increasing sequence of pressures until the instability threshold is reached. The present code utilized planes of inversion symmetry common to many minimum-B devices to allow a reduction in the domain to be studied; Life bar devices, Baseball II, 2X-II are representative.

Fokker-Planck Codes for Determining Endlosses and Distribution Functions in Mirror Machines

The proposition that a stable mirror plasma will yield net thermonuclear power hinges critically on the rate at which particles are lost out the ends of the device. In the absence of turbulence and at number densities and energies typical of mirror machines, the endlosses are due primarily to the scattering of charged particles into the loss cones in velocity space by weak, classical Coulomb collisions. The kinetic equation describing this process is the Boltzmann equation with the Fokker-Planck collision term.³⁹

We are confronted, in principle, with one such nonlinear, partial, integro-differential equation for each charged species in the plasma, and there are seven independent variables (three spatial coordinates, three velocities, and time). Such an equation, even for a single species, exceeds the capability of any extant computer. Several simplifying assumptions are therefore required to treat the problem at all. However, the parallel improvement of computers and Fokker-Planck codes over the past 15 years has led to the progressive elimination of the most questionable approximations, and has made it possible to specify error brackets for some of the others. At (LLL), in particular, the strong experimental mirror program and the unrivaled computing facilities have encouraged a major effort on the refinement of Fokker-Planck codes. The two complementary codes described in the following paragraphs are, to our knowledge, the most versatile and powerful Fokker-Planck programs in operation today.

MULTISPECIES CODE

This code⁴⁰ can handle an arbitrary number of species of charged particles, and electrons are treated on equal footing with ions. The code has seen extensive use in determining endlosses and distribution functions under a wide range of hypothetical reactor conditions for both D-T and D-³He systems, including reaction products.⁴⁰ (Runs have been carried out with as many as five species -- D, e, ³He, ⁴He, and H -- meaning the simultaneous solution of five coupled Fokker-Planck equations.)

The principal approximations in the multispecies code are as follows:

- The distribution functions are azimuthally and radially invariant in space and azimuthally invariant in velocity. This implies that the radius of the machine is much greater than a Larmor radius, and that the dominant loss process is along the field lines.
- The distribution functions are independent of the spatial coordinate along the field lines, as well. This follows if the mean free path of the particles greatly exceeds the length of the machine so that particles scattered into the loss cone in velocity space are ejected immediately, and if the spatial variation of the confining magnetic field and ambipolar potential occurs only near the ends of the device. In a calculation of this kind which represents conditions in the uniform central region of a long mirror machine, the existence of the magnetic mirrors and ambipolar potential enters only through the boundary conditions in velocity space. This is often called the "magnetic square well" approximation.

The numerical techniques by which the equations are solved have been described in detail elsewhere.⁴⁰ Briefly, the code employs an implicit finite-difference scheme, in which coefficients involving moments of the distribution functions are recomputed at each time step. The velocity mesh varies continuously to provide ample resolution over ion and electron distributions, and, in calculations seeking the equilibrium state corresponding to a given set of ion sources, the time step increases as the computation proceeds. An ambipolar potential is determined at each time step; it serves to equalize loss rates of ions and electrons and is computed by requiring charge neutrality (to within a specified tolerance) in the bulk of the plasma.

The LLL multispecies code incorporates, in addition to the capability of handling any number of species, several other improvements over previous, separated Fokker-Planck treatments. The difference form of the terms containing velocity derivatives is explicitly particle-conserving, and both energy and particle conservation for each species are

monitored in every run. The errors are usually less than 1% and always less than 5%. These results are useful not merely as diagnostics; they also provide insight into physical processes and may suggest routes to the optimization of reactor parameters. Specifically, matrices are printed out showing energy transfer rates among all species and the distribution of scattering losses of each species according to the species of the scatterers. Monitoring energy transfer in this way has already led to the correction of an error (concerning the treatment of the Coulomb logarithm) evidently made in all previous multispecies calculations.

Another improvement over previous multispecies codes is in the computation of the fusion reaction-rate parameter, σv . A rigorous computation would require integration over the full velocity space of both reacting ion species. Previous work with multispecies codes settled for approximating the first five integrations and performing only the last, over v for one distribution numerically. The LLL multispecies code employs a triple integration incorporating the distributions of both reactants (variables kept are the magnitudes of the two velocities and the magnitude of the relative velocity).

To date, the multispecies code has been used to determine equilibrium conditions, distribution functions and values of Q (= fusion power/injection power) for D-T systems with injection energies from 100 to 800 keV, and for D- ^3He systems with injection energies from 300 to 1500 keV and a variety of ^3He fractions. Sensitivity to mirror ratio, fusion cross-section data, and the inclusion of reaction products have been tested.⁴⁰ The code is also equipped to treat buildup and decay problems, charge exchange, and multiple ion beam injection at different energies. Output quantities displayed graphically include the evolution in time of densities, mean energies, and the distribution functions themselves, as well as the energy spectrum of the particles actually escaping through the mirrors. The latter information is essential to the detailed design of direct conversion schemes.

TWO-DIMENSIONAL CODE

A two-dimensional Fokker-Planck code⁴¹ is in routine operation at LLL. This code solves the unseparated Fokker-Planck equation in v and θ for a single ion species, under the assumption that the electrons can be represented by a Maxwellian distribution function with loss cone removed. The assumption of azimuthal and radial invariance in space and azimuthal invariance in velocity is retained.

The ion equation is solved using the alternating-direction implicit (ADI) finite difference scheme⁴¹ on a variable mesh in v and θ , and the coefficients containing moments over both ion and electron distribution functions are recomputed at each ion time step. The number density and mean energy characterizing the Maxwellian distribution of electrons are computed on a faster time scale by solving a pair of ordinary differential equations that include the effects of ion-electron interactions and electron endlosses and injection conditions. The ambipolar potential is determined by iteration on the electron equations in such a way as to equalize the positive and negative charge densities, as in the multispecies code.

The two-dimensional code can also treat realistic spatial dependence of the magnitude of the confining magnetic field (z -dependence), as well as spatially distributed sources. In the z -dependent version of the code, the spatial variation of the ambipolar potential is a computed quantity.⁴¹

To date, the two-dimensional code has been used extensively in buildup calculations corresponding to the Alice experiment, in decay calculations corresponding to the 2X experiment, and finally, in steady-state calculations under proposed reactor conditions comparable to those investigated with the multispecies code. Some of these results have been described elsewhere.⁴¹ Results as yet unpublished include the effects of charge exchange and of time-varying magnetic fields.

Collisionless Plasma Models to Study Mirror Instabilities

The Vlasov equation is one of the principal mathematical models used in plasma physics. Techniques for the

numerical solution of a Vlasov system in both one and two dimensions are described in Refs. 1 and 2. Most of the

codes use a Lagrangian scheme, i.e., the plasma is represented by a large number of "particles" which carry charge and mass, and the self-consistent electric and magnetic fields are computed from the calculated motion of the particles. Some applications of these codes to the study of plasma instabilities in mirror machines are discussed in the following.

NONLINEAR SATURATION AND MODIFICATION OF ION ORBITS PERPENDICULAR TO B

One group of codes primarily described the ion dynamics perpendicular to B for a variety of flute modes, all expected or actually observed in actual mirror devices: Dory-Guest-Harris loss cone, double hump (due to impurities or the presence of a cold target plasma), and drift cone. The flute modes are expected to persist in any mirror device, regardless of its length. The drift cone mode, in particular, remains as a major theoretical stumbling block to stable mirror confinement.

Most of the results obtained contributed to basic plasma physics, specifically the understanding of nonlinear saturation mechanisms. In particular, the codes confirmed in detail the theoretical calculations for the saturation of certain modes. In addition, the codes determined the saturation mechanisms for the other flute modes not amenable to analysis.^{42,43}

These codes also confirmed the presence of and described the saturation of 3-wave and 2-wave particle nonlinear explosive instabilities. Of significance here was the finding that these modes are extremely sensitive to resonance conditions, and hence are unlikely to be a threat to confinement.⁴⁴

One major code result aided in the interpretation of the rf spectra obtained from 2X, which were dominated by signals near the ion cyclotron harmonics. The code results showed that the nonlinear saturation always involved a definite change away from the linear regime real frequency, and that frequently there occurs a spectrum dominated by signals near the harmonics, despite the presence of modes whose linear real frequencies were nonharmonic. Thus, whatever the limitations of the model and whatever the precise saturation mechanism in the actual experiment, the code results show that the presence or absence of spectra specifically tied to linear theory is likely not to be a useful

diagnostic in proving the existence or absence of specific modes.⁴⁵

In sum, these codes provided the first detailed look at the microturbulence resulting from high-frequency mirror instabilities, and specifically:

1. Produced direct ties to analytical calculations.
2. Produced significant results unobtainable by other means on nonlinear saturations mechanisms, and predicted that the nonlinear saturation levels obtained by flute modes saturated by ion orbit changes would likely be disastrous for confinement.
3. Predicted the relative unimportance of the "explosive" nonlinear instabilities, despite their presence under highly idealized conditions.
4. Had direct meaning for interpretation of 2X rf spectra.

FINITE GEOMETRY EFFECTS

Another group of codes examined finite geometry effects on the flute modes. A 2-1/2 dimensional particle simulation code examined the nonlinear saturation of the modified negative mass in finite length models.⁴⁶ This mode requires rather singular velocity distributions, and can easily be cured in practice.

A linearized code examined finite geometry effects on flute modes in idealized slab geometry,⁴⁷ with the following results:

1. Confirmed the idea that short lengths do not seriously affect flute modes;
2. showed that flute mode potentials hold up to points where the density is a very small fraction of its mid-plane value; and
3. produced some information on the behavior of the drift cone mode in a system with a finite cross section.

A one-dimensional code examined the nonlinear behavior of electrons responding along the B lines to an unstable flute mode.⁴⁸ As expected, gross modification of the mode occurs when $e\phi_{\text{wave}} \approx T_e$; more work is required to determine whether this is a true saturation mechanism. The basic effect is expected to operate in any unstable flute mode, and will have to be accounted for in future models.

A single-particle code followed particle orbits in mirror magnetic fields with arbitrarily assumed fluctuating

electric fields. The main result of this code was that particle confinement can persist for long times (many bounce times), despite the presence of flute-like electric fields that produce changes in the particle's magnetic moment. These changes momentarily place the velocity vector in the loss cone. Also, strongly nonflute fields produce rapid loss of particles. This result provides one possible

explanation for the long life of 2X despite conditions that are strongly unstable to the drift cone mode; that is, if the mode is present at some undetermined nonlinear level but remains flute-like, the particles will still be contained for long times. The time scale for confinement depends in detail on the strength and frequency spectrum of the fluctuating electric fields.⁴⁹

Simulation of Parametric Instabilities

Parametric instabilities involve a periodic modulation of the plasma by an electromagnetic wave or modulated electron beam. They are of interest primarily as heating mechanisms. The external energy source ("pump") drives two waves in the plasma whose frequencies and wave vectors add up to those of the pump. In the pseudo-quantum mechanical description, a pump quantum decays into two lower-frequency wave quanta. The matching of wave vectors and frequencies corresponds to the conservation of the momentum and energy of the decaying pump quantum. There also are analogies to an important class of electronic amplifiers.

Early simulations were of the ion-acoustic decay and oscillating two-stream instabilities in which the pump is a (nearly) uniform oscillating external electric field.⁵⁰⁻⁵⁵ This represents the electromagnetic wave when it has traveled into the "critical density" surface where the laser frequency equals the local plasma frequency. Plasma oscillations grow and heat electrons by trapping or quasilinear diffusion, depending on the amplitudes and spectrum of the plasma waves. This was the first heating process, other than collisions (inverse bremsstrahlung), considered in laser-induced fusion, and is sometimes called "anomalous" heating. Some ion heating also can occur. Simulations are used to estimate heating rates and the resultant electron energy spectrum.

Of particular interest to CTR applications are parametric processes in which the pump frequency is much lower, around ion wave frequencies. Examples include driving a lower-hybrid oscillator,⁵⁶ and the modified two-stream.⁵⁷ Another example is the decay of a whistler.⁵⁸ Such processes offer the possibility of heating ions in magnetically confined plasmas.

Parametric instabilities also occur in underdense plasmas. One wave may be an electron oscillation or an ion-acoustic wave, and the other is an electromagnetic wave. These cases are commonly called stimulated Raman and Brillouin scattering, in analogy to processes in other media. They are of great interest to laser fusion because they may cause a large part of the laser energy to be lost by scattering, and because of the electron heating by Raman's electron decay wave.^{59,60} Decay into two electromagnetic waves is not possible in an unmagnetized plasma because frequencies and wave vectors cannot simultaneously be matched. Another process is the decay into two electron plasma oscillations, which provides an alternative heating mechanism occurring at one-quarter critical density.^{61,62} In the presence of self-focusing or filamentation of the laser light, additional decay possibilities arise.⁶³ Simulations are used to study rates and efficiencies of heating, effects of density gradients, laser frequency bandwidth, etc.

Numerical Simulation of Astron

A time-dependent numerical model of the Astron, with which injection and trapping can be studied in detail, has been developed. The effects due to the resistors and neutralization have been in-

cluded. The model is axially symmetric. The E-layer electrons are simulated by many thousands of finite-size superparticles, which move in the r - z domain and have velocity components v_r , v_θ , and v_z .

The model is relativistic and the electromagnetic fields are obtained by solving four wave equations -- three for the vector potential and one for the scalar potential. The E-layer current and the current induced in the resistor wires are included in the above field equations. The computed self-fields are added to the external field to give the field configuration as a function of time. Results of multiple pulse injection have been published.⁶⁴

The code results have been continually and intimately involved in interpretations of and decisions about the Astron experiments.⁶⁵

- In the very first series of runs on the Superlayer code, a phenomenon of saturation of the field reversal, ζ was discovered. This phenomenon was due to an axial lengthening during pulse stacking under charge neutralized conditions. This result directly stimulated other theoretical work, and the two together played a large part in the decision to turn the Astron experiment to stacking in vacuum, which was continued for more than a year. This lengthening phenomenon was discovered first by the code, and only later was observed in the experiment.

- The code reproduced several experimental results of trapping efficiency in vacuum; in particular, the trapping efficiency in a particular magnetic well configuration was sensitive to parameter changes (resistor location, injection current) in the same way as the experiment, thus providing the assurance of a simple explanation unrelated to difficulties peculiar to the experiment.

- The code results show that stacking in vacuum ultimately causes difficulty at sufficiently high ζ , in the range 0.5 to 1.0, due to increased radial expansion. This has yet to be confirmed

by the experiment since they are still at low ζ , but this result is one factor that must be considered in any plans to stack to reversal in vacuum. This result is modified under conditions of partial neutralization, such as exists experimentally even under the best vacuum.

- Reversal is obtained starting with vacuum stacking of 1000-A pulses up to the point of incipient radial loss followed by adiabatic charge neutralization (which compresses the layer and nearly doubles the field reversal), but this route may well not be successful when the idealized code conditions are relaxed.

- The code shows that sufficiently large current (only 4 times that previously available in the experiment and possibly only 2 times that available with the split pulse) can produce field reversal in one pulse (in charge neutralized environment and in a short system). This route does not appear sensitive to relaxation of idealized code conditions.

- The efficiency of single-pulse trapping in the code for a wide variety of conditions is nearly perfect. This fact explicitly tells us that the experimental poor trapping efficiency is due to nonideal conditions such as instabilities due to r, θ motion (see below) or to combined effects of poor beam quality and asymmetries in the magnetic fields. This latter effect has recently been confirmed in the experiment where the poor trapping was found in part to be caused by gross field asymmetries at the injector

The results obtained from an (r, θ) code show that for a moderate value of $\zeta \geq 0.1$, in an unneutralized layer, the negative mass instability will result in large radial loss, and thus prevent buildup in vacuum to reversal levels.⁶⁶ Other versions of this code are capable of examining nonideal effects such as z -variation and complex equilibrium orbits.⁶⁶

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