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THE ROLE OF CREEP IN THE GAS-PRESSURE-BONDING PROCESS

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Stephen D. Beck and Michael A. Gedwill, Jr.

A theoretical method of determining the time, temperature, and pressure required for pressure bonding was proposed and then evaluated experimentally. The analysis considered only the mechanical deformation of the bonding surfaces induced by the external pressure. It was postulated that such deformations obey the power law for steady creep.

The application of the law to the closure of thick-walled aluminum cylinders under gas pressure was investigated. The behavior was compared with that predicted from creep data obtained from short-time torsion creep tests. Only fair agreement was established.

The creep law was then applied to the calculation of the closure of voids at the interface between bonding surfaces. It was assumed that the voids exist as isolated spherical or cylindrical holes. The law failed to predict the total closure which occurred.

Apparently, the creep law does not hold in the almost microscopic domain of the interfacial voids, although the disagreement may have been due to the oversimplification of the geometric representation. Resolution of the ambiguities probably will be attained only when sufficient experimental data to establish an empirical relationship of closure with bonding conditions are available.

INTRODUCTION

The purpose of this investigation has been to develop a simple procedure for estimating the conditions of time, temperature, and pressure which are required for pressure bonding. The term pressure bonding is used in this report to refer to a fabrication process in which gas pressure is applied to effect the joining of the component parts of an assembly. After preliminary surface treatment, the components are assembled and placed in a container which is then evacuated and sealed. The container is placed in an autoclave and subjected to a high gas pressure at an elevated temperature for a period of several hours. Under the action of the external hydrostatic pressure the mating surfaces deform until there is intimate contact between them. The surface may then form a bond, if other conditions are satisfactory for bond information.

The pressure-bonding process has been successfully used for the cladding of fuel elements for nuclear reactors.⁽¹⁻³⁾ The process is useful for the fabrication of assemblies of complicated shape or those in which brittle materials are involved.

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(1) References at end of text.

An examination of the pressure-bonding question may be roughly divided into two parts: that of the mechanical process by which the mating surfaces deform sufficiently to effect intimate contact, and that of the metallurgical processes by which the contacting surfaces form a bond. This investigation has been concerned solely with the process of mechanical deformation.

There is a wealth of literature on conventional pressure-bonding processes. (4-11) These are essentially different from the hydrostatic process in that the pressure is applied in only one direction, normal to the mating surfaces, with the welded material free to deform. The change in dimensions of an assembly being fabricated by the conventional process is substantial, ranging anywhere from 5 to 95 per cent. With these large deformations there is substantial extrusion and the mating surfaces are radically distorted, and as a consequence are readily forced into contact.

The gas-pressure-bonding process, on the other hand, is characterized by pressure being applied hydrostatically. The over-all deformations in this case are negligible. Therefore the interface is not subjected to the high degree of upset prevalent in the conventional processes and the problem of obtaining complete compliance of the mating surfaces becomes significant.

The mating surfaces are initially rough. They may display large numbers of asperities which are distributed in a random fashion, or they may be composed of more or less regular patterns of ridges and grooves caused by machining operations. When the surfaces are placed in contact the interface will contain points or areas of contact and void spaces. During the pressure-bonding operation, the voids must close and the area of contact become complete.

The problem seems to be resolved into two parts. The first question is to find a general law which describes how a material deforms under the action of hydrostatic pressure. Given such a law, the next concern is to apply it to the problem of surfaces attaining the necessary intimate contact. Two models, therefore, are required for the theoretical analysis. A rheological model is needed to describe permanent deformations under the influence of the bonding conditions. A geometrical model is needed to describe the physical configuration of the interfacial region.

The combined model must be physically reasonable as well as mathematically tractable. Many combinations of physical and rheological models have been tried during the course of this investigation. Most of these were discarded because either they were unrealistic or they led to insurmountable mathematical difficulties. The combined model which was eventually used still does not give complete success. There is at least a little evidence that the rheological part of the model applies reasonably well to data on closure of cylindrical holes, but the application of the complete model to the study of mating of surfaces has not proved successful. It is not clear whether this failure arises from deficiencies of the geometrical model or from using the rheological model for conditions to which it does not apply.

THEORETICAL BACKGROUND

Creep Law

The rheological model which offers the most promise for analyzing the deformations of pressure bonding is a simple power law for creep. The creep law used relates the creep rate to the stress by means of parameters which are constant for a given material at a given temperature. The use of the power law implies that the material has no memory of a previous state. In other words, the behavior of the body depends only on its instantaneous conditions of stress, rather than on any previous condition. Such a law takes into account the time dependence of the deformation and the variation of stresses throughout the body. On account of its generality, it is applicable to a variety of physical problems.

The creep law which is used has certain recognized shortcomings. It is, strictly speaking, a representation of second-stage or constant-rate creep. That is, it does not account for the dependence of creep rate on total creep or time. Furthermore, it does not account for the lack of isotropy or homogeneity of the material, nor does it account for changes in the rheological behavior of the material during deformation. As with all creep laws it fails to represent exactly the physical behavior of the body, even in the second stage of creep, and the parameters obtained from one type of test may not give very accurate results when analyzing a different type of problem. Finally, the practical difficulties of the analysis limit its application to relatively simple problems.

As shown in Appendix A, the creep law is conveniently expressed

$$\dot{\epsilon} = A\sigma^n, \quad (1)$$

where $\dot{\epsilon}$ and σ are the second invariants of the strain rate and stress deviation tensors. The parameters A and n are constants which characterize the creep behavior of the material at a certain temperature.

Equation (1) follows from a generalization of the familiar power law for tensile creep, which is similar in appearance. In the case of tensile creep, the law reduces to

$$\dot{\epsilon}_x = \left(2/3^{(n+1)/2} \right) A \sigma_x^n, \quad (2)$$

where $\dot{\epsilon}_x$ and σ_x are the axial strain rate and stress.

Physical Representation of the Mating Surfaces

A physical model is needed to represent the interface between the two elements to be bonded. One reasonable model may be obtained by thinking of the void spaces at the interface as a collection of hollow spheres in a medium with an infinitely thick wall. It is assumed that there is no interaction between the voids. Each void space is treated as a hollow sphere with hydrostatic pressure p at infinity. The creep law, Equation (1),

may then be used to express the ratio of the instantaneous sphere radius a to the original radius a_0 as a function of pressure and time. It can be shown then, that

$$\frac{a}{a_0} = \exp \left[- \frac{A 3^{(n-1)/2} p^{nt}}{(2n)^n} \right]. \quad (3)$$

Since creep deformations are assumed permanent, Equation (3) may be used to prescribe the factor p^{nt} needed for a desired final void size. It should be noted that, according to the creep description, the void diameter may be made arbitrarily small, but never reduced to zero.

If the voids are assumed to be long circular cylinders instead of spheres, a formula analogous to Equation (3) is found. In this case

$$\frac{a}{a_0} = \exp \left[- \frac{A p^{nt}}{n^n} \right]. \quad (4)$$

Since n is a positive constant, the cylinder deforms more readily than the sphere. Equation (4) is less conservative than Equation (3). It may be used for an alternative prediction of the required factor p^{nt} , if the deformation is more rapid than the spherical model implies.

The maximum shearing stress at the surface of the void is of interest in the case of either model. For the sphere and cylinder, respectively,

$$\tau_{\max} = \begin{cases} 3p/4n & (\text{sphere}) \\ p/n & (\text{cylinder}). \end{cases} \quad (5)$$

It is recognized that neither model is completely realistic. A void may certainly be expected to be influenced to some extent by its neighbors. Furthermore, in the early stages, the areas of contact may be only a small part of the total area of the interface. In such a case, the squashing or extrusion of the contact sites may proceed much more rapidly than spherical holes would close.

The spherical model is weak also because this shape is one of the most difficult to deform. Variations from the spherical shape for the void will induce stress-concentration effects which will stimulate creep. Therefore that model is quite conservative. The cylindrical hole is somewhat more easily deformed. However, it too leads to predictions which are much too conservative. It is felt that the ellipsoidal hole would have been a more appropriate physical model. Unfortunately, however, no success was achieved in the attempts to solve the mathematical equations for creep of the ellipsoid.

Since the actual voids between surfaces have shapes different from spheres or cylinders, the only shapes for which closure theories are available, these theories cannot be expected to be strictly applicable. However, it should be noted that both of those theories lead to a relation of the form

$$\text{Void space} = \text{Initial void space} \times \exp(-A' p^{nt}),$$

the only variation being in the constant A' . One may hope that this kind of relation applies broadly to closure of typical voids by making appropriate adjustments in A' . Unfortunately, the bonding data of the present tests are insufficient to test this hypothesis.

Other complications may arise as the hole size becomes small, so that the creep mechanism would then fail to provide a realistic representation of the deformation. Surface tension and various metallurgical phenomena may supersede or supplement the creep mechanism, effecting complete closure of the hole.

METHODS OF CREEP ANALYSIS

Two kinds of experiments were performed in an effort to determine the creep parameters and to examine the validity of the creep law which is used. The collapse of thick-walled hollow circular cylinders under hydrostatic pressure was studied because it bears a resemblance to the problem of the closure of the voids found in the interface between two surfaces to be pressure bonded. As an alternative method of determining creep parameters a study was made of the torsion of solid shafts. The latter method was considered because of its relative simplicity compared to the first. Both methods are characterized by variable stress distribution.

Collapse of Cylinders

An analysis for thick-walled circular cylinders was made for the case of zero internal pressure and constant radial pressure on the surface. Plane strain was assumed. This assumption is slightly in error since the normal stress in the axial direction must, at the ends of the cylinder, be equal to the pressure. However, as is pointed out in Appendix A, the resultant axial force is in equilibrium with the pressure. Furthermore, plane strain implies that there is no deformation in the axial direction. This is in agreement with the experimental evidence.

Let a thick-walled circular cylinder have a concentric axial hole. If radial pressure is applied to the external surface, the internal radius must satisfy the equation

$$I(x) \equiv \int_x^1 \frac{1 - (kx)^{2/n}}{x} dx = \frac{Ap^n t}{n^n} \quad (6)$$

where

$$x = a/a_0$$

$$k = a_0/b.$$

The function I may be evaluated for integral values of n , using Equation (A-27), (A-32), or (A-35) in Appendix A, as appropriate. Table 1 shows the values of I for $k = 1/3$ and integral n from 1 to 10 at selected values of a/a_0 . These points are sufficient to draw a family of curves showing the variation of a/a_0 with I . This family is shown in Figure 1 with the values of I plotted logarithmically as abscissas.

TABLE 1. THE CLOSURE FUNCTION, $I(x)$

$$I(x) = \int_x^1 \frac{[1 - (kx)^{2/n}]^n}{x} dx$$

$$k = 1/3$$

Value of x	Theoretical Closure Function for Indicated Value of $n(a)$									
	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$	$n = 10$
0.9	9.480 (-2)	4.925 (-2)	1.622 (-2)	3.872 (-3)	7.229 (-4)	1.111 (-4)	1.447 (-5)	1.633 (-6)	1.138 (-7)	1.461 (-8)
0.8	2.031 (-1)	1.098 (-1)	3.793 (-2)	9.540 (-3)	1.883 (-3)	3.065 (-4)	4.229 (-5)	5.082 (-6)	3.774 (-7)	5.164 (-8)
0.6	4.753 (-1)	2.797 (-1)	1.077 (-1)	3.060 (-2)	6.893 (-3)	1.289 (-3)	2.066 (-4)	2.896 (-5)	3.455 (-6)	4.074 (-7)
0.4	8.696 (-1)	5.630 (-1)	2.475 (-1)	8.197 (-2)	2.188 (-2)	4.909 (-3)	9.530 (-4)	1.632 (-4)	2.489 (-5)	3.485 (-6)
0.2	1.556	1.129	5.918 (-1)	2.423 (-1)	8.191 (-2)	2.369 (-2)	5.995 (-3)	1.350 (-3)	2.737 (-4)	5.054 (-5)
0.1	2.248	1.758	1.046	5.024 (-1)	2.034 (-1)	7.141 (-2)	2.214 (-2)	6.143 (-3)	1.540 (-3)	3.723 (-4)

(a) Numbers in parentheses indicate the power of ten by which the entry should be multiplied.

Because of the equality of the first and third members of Equation (6), I serves as a dimensionless time variable. Thus a/a_0 is an implicit function of t which may be evaluated directly from the appropriate curve of Figure 1.

The relationship between a/a_0 and t does not lend itself to a precise analytic procedure for determining the creep parameters from experimental data. However, the parameters may be determined by a method which combines inspection with a quasi-graphical procedure.

Notice that a/a_0 varies with $p^n t$. Data for a given temperature are obtained for the values of a/a_0 for more than one value of pressure and for various times. For each of several trial values of n a plot is made with $(p/p_0)^n t$ as the abscissa on a logarithmic scale and a/a_0 as the ordinate. One value of n is chosen by inspection as giving a plot most suitable for fitting to the corresponding curve of Figure 1. The set of points is then compared to the curve to find the best fit, again by inspection. There will then be a one-to-one correspondence between points on the abscissa of the $(p/p_0)^n t$ scale and the $A(p/n)^n t$ scale. Say that the points

$$(p/p_0)^n t = d_1$$

* p_0 is any convenient number, chosen to simplify the calculations. Ordinarily it will be the lowest pressure used in the experiment. The results will be independent of the choice of p_0 .

FIGURE 1. THEORETICAL CLOSURE OF TUBES

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and

$$(Ap/n)^{n_t} = d_2$$

correspond. Then, p^{n_t} may be eliminated and A is found.

$$A = (n^n d_2) / (p_0^n d_1). \quad (7)$$

It can be seen that the procedure for finding the creep parameters from cylinder collapse data is somewhat tedious, and since it involves a rather undependable inspection procedure, it may be inaccurate. In addition, a large number of data may be needed for even an approximate analysis. A much simpler and more straightforward method is discussed next.

Torsion of Shafts

A method is needed to determine the creep parameters A and n in a manner which is both simple and general. The stresses associated with the closure of voids vary throughout the body. Therefore, it is appropriate that the procedure for determining the creep parameters should be one in which there is a variable stress distribution. It was decided that the determination of the creep parameters from torsion tests of solid shafts was a method worth investigating. It is recognized that the variation of stress is not similar to that of the cylinder under hydrostatic pressure. An advantage of this is that data from torsion tests, together with data from closure tests, would simultaneously provide an indication of the generality of the creep laws.

The torsion tests have the additional advantage of providing creep data in a short time. The complete data for a creep torsion test may be obtained in a few hours or less.

For a solid shaft subjected to pure torsion the creep law, Equation (1), may be used to relate the angular velocity and the torque, giving the result of Equation (A-46) in Appendix A.

$$\dot{\alpha} = \left(\frac{3n+1}{2\pi n} \right)^n \frac{2\ell A}{b^{3n+1}} T^n. \quad (8)$$

Equation (8) may be expressed linearly by taking logarithms of both sides. That is, the equation

$$\log \dot{\alpha} = \log \left[\left(\frac{3n+1}{2\pi n} \right)^n \frac{2\ell A}{b^{3n+1}} \right] + n \log T \quad (9)$$

is a straight-line relationship between $\log \dot{\alpha}$ and $\log T$. Since data of $\dot{\alpha}$ versus T are obtained, a least-squares fit⁽¹²⁾ may be used. Thus A and n are found from

$$n = \frac{N \sum (\log \dot{\alpha})(\log T) - (\sum \log \dot{\alpha})(\sum \log T)}{N \sum (\log T)^2 - (\sum \log T)^2}, \quad (10)$$

and

$$\log \left[\left(\frac{3n+1}{2\pi m} \right)^n \frac{2\ell A}{b^{3n+1}} \right] = \frac{\sum \log \dot{\alpha} - n \sum \log T}{N}, \quad (11)$$

where N is the number of data.

If the behavior of the shaft is governed by the power law for creep as assumed, then the creep constants may be determined without ambiguity from torsion data. Furthermore, the number of data points required in the torsion method is much smaller than that required for the same amount of information from the cylinder collapse method.

The maximum shearing stress in the shaft occurs at the outer surface. It has the value

$$\tau_{\max} = \frac{(3n+1) T}{2\pi m b^3}. \quad (12)$$

Comparison of the maximum shearing stress in the shaft with that in the hollow sphere or cylinder, Equation (5), indicates that in the case of the shaft there is only a slight dependence of the maximum stress on the value of n .

ANALYSIS OF CREEP BEHAVIOR

An experimental program (discussed more fully in Appendix B) was performed on 2S aluminum in conjunction with the analytic portion of the investigation. The purpose of the tests was to examine the validity of the postulated rheological model and to explore the possibilities of predicting the behavior of materials under pressure-bonding conditions.

The choice of 2S aluminum was made not particularly to study this specific material, but rather to provide a material which could be used at a relatively low temperature to examine the premises of the investigation. If the results were conclusive and favorable, it would then be relatively simple to extend the investigation along the same lines to materials actually proposed for pressure-bonding applications. As will be pointed out, the results did not warrant this direct extension.

Closure of Hollow Cylinders

Hollow circular cylinders of 2S aluminum were subjected to gas pressures up to 6000 psi while their temperatures were at 500 and 700 F. The cylinders, nominally identical, were characterized by the ratio of inner radius to the outer radius of one-third. They were prepared for pressurizing by evacuating the interior and sealing the ends to establish the condition of zero pressure at the inner surface. Two specimens at a time were placed in the autoclave. The temperature was elevated to the prescribed value and the pressure applied. The temperature and pressure were maintained at a nominally constant level for the prescribed time interval. At the end of the cycle each specimen was tested for leaks. Those which were discovered to have leaked were discarded. The remaining specimens were sectioned, and their diameters were measured.

The ratios of the final to the original radii of the holes are listed in Table 2. The data from the tests were analyzed by the method described earlier. The following tabulation shows the values determined for the creep parameters:

Test Temperature, F	Value of Creep Parameter	
	n	$A, (\text{in.}^2/\text{lb})^n \text{ hr}^{-1}$
500	2	$2.8 (10)^{-8}$
700	3	$7.0 (10)^{-11}$

Figures 2 and 3 show as solid curves the calculated curves for closure using the creep parameters obtained here. It is clear that the fit of the data is at best only approximate.

TABLE 2. DATA FROM CYLINDER-CLOSURE EXPERIMENTS

Test Time, hr	a/a_0 Obtained at Temperature and Pressure Shown				
	500 F		700 F		
	4000 Psi	6000 Psi	3000 Psi	4000 Psi	6000 Psi
1/2	--	0.66	--	--	0.00
	--	0.69	--	--	0.00
1	0.84	0.71	--	--	0.00
	--	--	--	--	0.00
2	0.67	0.52	0.35	0.50	0.00
	--	0.50	0.35	0.50	0.00
3	0.80	--	--	0.32	--
	0.60	--	--	0.29	--
	--	--	--	0.34	--
4	0.66	0.16	0.38	0.25	0.00
	0.26	0.15	0.38	0.24	0.00
	0.26	--	0.31	--	--
	--	--	0.34	--	--
6	0.34	0.10	0.27	0.22	0.00
	0.36	--	0.27	0.22	0.00

FIGURE 2. CLOSURE OF CYLINDRICAL HOLES IN 2S ALUMINUM AT 500 F

$$a_0/b = 1/3.$$

FIGURE 3. CLOSURE OF CYLINDRICAL HOLES IN 2S ALUMINUM AT 700 F

$$a_0/b = 1/3.$$

FIGURE 4. SECTIONED CYLINDERS AFTER CLOSURE EXPERIMENTS

It may be noticed that there is substantial scatter in the data. In several cases the final hole size was not monotonically dependent on the time or the pressure. In the several cases where both specimens from a cycle were retained, the difference between their measured deformations was slight. This indicates that the scatter is more likely due to unavoidable fluctuations in the pressurizing conditions than to differences among the specimens.

The complete closure which occurred at 700 F at 6000 psi is inconsistent, in terms of the creep law, with what would have been expected from a comparison of the results at 3000 and 4000 psi. This inconsistency introduces substantial doubt into the accuracy of the values of the creep parameters.

In Figure 4, the cross sections of the specimens after pressurizing are shown. The circular cross section of the hole, in many cases, became distorted during the deformation process. Figure 5 shows a photomicrograph of a completely closed hole. The network of black lines is the oxide which coated the original circular hole. There evidently was extreme distortion in the final stages of closing. The departure from the assumed geometry must contribute to the invalidity of the creep model as the hole becomes small, and to the uncertainties of the creep parameters determined from the data.

FIGURE 5. EXAMPLE OF A COMPLETELY CLOSED HOLE

The network of black lines is the oxide which was originally the inner surface. Note the substantial distortion which occurred as the hole closed. The experiment was run for 1/2 hr at 700 F and 6000 psi.

Torsion of Shafts

An independent determination of the creep parameters A and n was made for 2S aluminum at 500 and 700 F from torsion tests of shafts. The torsion was performed with the specimen inside a furnace to maintain the prescribed temperature. The shafts were 3 in. long and 3/4 in. in diameter. Each specimen was twisted at a constant angular velocity, and the torque was measured periodically during the cycle.

It was observed that the torque increased with angle of twist for the early part of the cycle, and then attained a more or less level value which was maintained through several revolutions. The curve of torque versus angle of twist for each constant angular velocity was inspected to choose the portion of essentially constant torque. The arithmetic mean of the measured torque was then used for the tabulated value. Table 3 shows the values of $\dot{\alpha}$ and T which were used in the calculations.

TABLE 3. TORSION OF SHAFTS MEASURED IN EXPERIMENTAL DETERMINATION OF CREEP PARAMETERS

Test Temperature, F	Angular Velocity, radians per hr	Torque, in-lb
500	3.67	258
	17.58	280
	37.47	304
	86.35	380
700	2.08	117
	10.29	129
	29.05	139
	96.78	172

Equation (9) was fitted to the data by the least-squares technique as described in the previous section. Figures 6 and 7 show the linear fit with the measured points. The creep parameters determined from the torsion tests were as follows:

Test Temperature, F	Value of Creep Parameter	
	n	$A, (\text{in.}^2/\text{lb})^n \text{ hr}^{-1}$
500	7.7	$3.1 (10)^{-27}$
700	9.6	$1.4 (10)^{-30}$

It can be noticed that the relationship between $\log \dot{\alpha}$ and $\log T$ is not precisely linear as assumed. In fact, there is a very systematic curvature associated with the points. Although an empirical relationship could very easily be fitted to the data, it would be inaccurate to express such a relation in terms of the fundamental assumed creep law, since the assumption that the creep parameters are constant is explicit in the derivations. Furthermore, many more data would be needed to justify the conclusion that the curvature is due to factors other than experimental error.

FIGURE 6. TORSION OF 2S ALUMINUM SHAFTS AT 500 F

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FIGURE 7. TORSION OF 2S ALUMINUM SHAFTS AT 700 F

Comparison of Results

The numerical values of A and n obtained from the two methods of testing are substantially different. The accuracy of such values, therefore, cannot be assumed. However, a comparison of the results may be made by using the creep parameters determined from the torsion tests to predict the closure of cylinders.

Figure 1 is used to calculate the amount of closure as a function of pressure and time from the results of the torsion tests. For the given value of n , not an integer, a curve may be drawn to interpolate between the two adjacent integral values. The value of $A(p/n)^n$ is computed and the value of $I = A(p/n)^n t$ for each value of time is determined. The value of a/a_0 versus t may be drawn. The curves for 500 and 700 F are shown as dashed lines in Figures 2 and 3, respectively.

The agreement between the calculated curves and the experimental data is, at best, only approximate. However, even this approximate agreement is significant, considering that the methods of testing are of substantially different character.

Analysis of Stress

The intensity of the stresses varies with positions in both the hollow cylinder and the shaft. In the case of the cylinder, the stresses vary with the amount of deformation as well. The maximum shearing stresses may be calculated from Equation (12) or (A-17) for the shaft or the hollow cylinder. In either case, the stress depends on the value of n . Since there are two experimentally determined values for n at each temperature, the maximum stress is calculated assuming each of the values. For the hollow cylinder, the calculation is made both for the original hole size ($a/b = 1/3$), and for the infinitesimally small hole ($a/b = 0$). The calculated values of the stresses are shown in Table 4 for each intensity of loading which occurred in the test program. It will be noticed that the maximum shearing stresses in both the tube and the shaft are of the same order of magnitude.

Experimental Applications to Pressure Bonding

The ultimate test of the applicability of the theory rests with its generalization to the case of actual surfaces prepared for pressure bonding. 2S aluminum plates were prepared for pressurizing by machining the mating surfaces to prescribed roughness of 250 or 750 rms. The surfaces were oriented with their directions of machining normal to one another to give a pattern of areas of contact and void spaces.

If the voids are assumed to be isolated cylindrical holes, Equation (4) may be used to calculate the reduced hole size after pressurizing. Specimens with surfaces of each roughness were pressurized at 700 F and 4000 psi for 2, 4, and 6 hr to examine the time dependence of closure. In addition, a specimen of each roughness was pressurized at 700 F for the few minutes needed to bring the pressure up to 4000 psi, at which time the pressure was released. The object of the latter procedure was to remove the extreme irregularities of the surfaces and to eliminate any separation of the

TABLE 4. CALCULATED MAXIMUM SHEARING STRESSES

Temperature, F	Assumed Value of n	Shaft		Pressure, psi	Cylinder	
		Torque, in-lb	$\tau_{\max}$, psi		$\tau_{\max}$, psi	
					a/b = 1/3(a)	a/b = 0(b)
500	7.7	258	2400	4000	2100	500
		280	2600	6000	3100	800
		304	2900	--	--	--
		380	3600	--	--	--
	2	258	2700	4000	3000	2000
		280	3000	6000	4500	3000
		304	3200	--	--	--
		380	4000	--	--	--
700	9.6	117	1100	3000	1500	300
		129	1200	4000	2000	400
		139	1300	6000	3100	600
		172	1600	--	--	--
	3	117	1100	3000	1900	1000
		129	1300	4000	2600	1300
		139	1400	6000	3900	2000
		172	1700	--	--	--

(a) Hole size before deformation.

(b) Infinitesimally small hole.

plates. It was assumed that measurements of the void size could be made after this zero-time pressurizing in order to have an initial hole size a_0 to use in the calculations for longer times.

It turned out, however, that as far as could be determined all the voids closed, even at zero time. This was the case even though there were many points, prior to pressurizing, where the maximum distance between the surfaces was about 1 or 3 mils, for the 250- or 750-rms surfaces, respectively. If the cylindrical-hole model were valid, the reduction in hole size should have been measurable. Although uncertainties of the creep parameters make a prediction of the reduced hole size unreliable, use may be made of the parameters determined from either the closure of cylinders or the torsion tests to give an indication of the reduced hole size ratios expected. For example, the results for cylinder closure and torsion predict that a/a_0 after 5 min should be 0.986 and 0.999998 and after 6 hr 0.37 and 0.99988, respectively. If the spherical hole were used to describe the void, even less deformation would be predicted.

DISCUSSION OF RESULTS

The experimental results indicated that the theoretical method proposed is inadequate to describe the deformations of pressure bonding.

Creep Law

The deformation analysis in this investigation was based on the assumption that the power law for creep governs the behavior of a solid subjected to load. The experimental behavior of 2S aluminum at 500 and 700 F was approximately as described by the creep law, but did not support the model with great precision.

The geometrical configurations which were examined provide a severe test of the generality of the creep law. Most creep investigations are concerned with conditions in which the stress for a given test is constant. Usually the stress is tensile, but even in cases of combined stress the conditions are ordinarily chosen such that the stress does not vary with position or time. In this investigation, the stress distributions are more complicated. For both methods the stress varies with position. For the closure of cylindrical holes, there is variation with time also, due to the changing geometry of the specimens. The cylinder closure introduces an additional measure of uncertainty into the analysis, since only the final deformation is observed. In other words, no information about creep rates may be observed directly. Therefore, if first-stage (transient) creep contributes significantly to the deformation, the neglect of its effect may be a source of error.

The analytic description of the cylinder-closing process requires the assumption that the circular hole does not change its shape. The experimental evidence shows that this assumption is incorrect for the later stages of closing.

The stress distributions in the shaft and in the tube are dissimilar. In the shaft, the largest and smallest and principal stresses are equal in magnitude and opposite in sign. In the cylinder, on the other hand, all the principal stresses are compressive.

Each torsion test was performed at a nominally constant angular velocity. It was observed that the torque did not remain constant. In the earliest stages of the cycle there was a brief period of increasing torque corresponding to transient creep. Afterward, for the remainder of the test the torque fluctuated in an apparently random manner. The fluctuations of torque were ignored, and an arithmetic average torque was calculated. An element of uncertainty is associated with the averaging since the second-stage region is not clearly defined.

An examination of the deformation curves of Figures 2 and 3, which were made using the creep parameters determined from the torsion tests, indicates that there is a resemblance between the calculated curves and the measured deformations. In light of the complexities of the stress distributions and considering the experimental uncertainties, it is felt that the fit is sufficiently accurate to support the power law. It is not felt that the accuracy of the creep parameters has been assured. However, the torsion test appears to be more accurate than the closure test as an indication of the values of the creep parameters.

There are clearly limitations on the model which invalidate it for very small holes. It has not been determined what this limit might be. When the hole is small, grain-boundary slip, recrystallization, diffusion, surface tension, and other mechanisms may contribute to the deformation process in ways beyond their contribution to second-stage creep.

In brief, then, it is concluded that the power law for creep describes in a fairly adequate manner the deformation process for 2S aluminum at 500 and 700 F. The creep parameters have not been determined with a high degree of accuracy.

Application to Pressure Bonding

The major shortcoming of the investigation is felt to lie in the description of the behavior of the mating surfaces. The pressure-bonding conditions of 4000 psi at 700 F used for the plate specimens were chosen such that the predicted deformations would have been moderate. Included was a loading which lasted only a minute or two, just long enough to raise the pressure to the prescribed amount and release it. Complete contact was apparently attained instantaneously.

There may be two reasons for the gross error in the prediction. The first is the assumption that creep is the controlling mechanism. It is reasonable to expect that the description of macroscopic creep behavior fails to apply to deformations in the almost microscopic regions of the interfacial voids. The other source of error is the representation of the geometric configuration of the interface. The hollow sphere and cylinder are among the more difficult shapes to compress. The interface is actually composed of peaks and valleys with the contacting points relatively free to deform. If a creep criterion is to be used, it may need to account more accurately for the nature of the interfacial region.

Under the conditions used here, the deformation was plastic flow, occurring instantaneously. It seems to be indicated that a desirable criterion for choosing the pressure-bonding conditions may be a static condition. The pressure and temperature conditions would be chosen to produce effectively instantaneous contact. Time would be considered only to the extent that the process of forming a bond requires time, and that residual elastic strains may need time to be removed by annealing.

FUTURE WORK

In the course of this investigation, two areas have been indicated as being in need of further study.

Deformations of Pressure Bonding

The deformations associated with the pressure-bonding process should be examined from a different point of view than that used in this investigation. Hindsight indicates that it was perhaps unwise to try to develop a general deformation model which would then be applicable to the pressure-bonding process.

In an alternative approach, the emphasis should be placed on examining actual pressure-bonding situations. The first step should be to find experimentally a range of pressure-bonding conditions for which the attained contact varies from partial to complete. The conditions can then be varied to determine how they affect the extent of deformation.

When the deformation behavior of the test material is understood, it may be possible then to formulate empirical rules to describe its closure process. It will be possible, for example, to specify the significance of the time variable, hence to decide whether to establish a static or dynamic criterion for deformation.

Deformation might be characterized in one or both of two ways. In the first, the approach of the mean surfaces of the components would be considered. That is, under bonding conditions, surfaces which were initially separated by some average distance approach one another as the separation becomes zero. The alternative is to consider the fraction of the surface area which is in contact, and to examine the conditions by which the fractional contact becomes unity.

When such a deformation relationship has been determined the next phase would be an attempt to correlate the behavior with the appropriate mechanical properties of the material and characteristics of the surface. Both experimental and theoretical correlation would be desirable. If this latter phase were successful it could provide the fabricator with the needed information for choosing pressure-bonding conditions to assure complete contact.

Creep Law

Disregarding its role in the pressure-bonding process, there is clearly a need for further analytic study of creep theory. There are several representations of the stress-strain rate law for second-stage creep. The power law was used in this investigation. The exponential and hyperbolic sine laws are others. The application of these laws to various situations involving variable stress distributions has not been widespread. To a great extent, the nonlinearity of the associated differential equations complicates the problems. The path to solutions of many of these problems may possibly be found using variational techniques, in a manner similar to that of Patel, et al.,⁽¹³⁾ for analyzing the problem of torsion of prismatic bars.

A potentially more difficult problem is to include primary creep in the analysis. However, if total creep, instead of creep rate, is to be considered, the effect of primary creep may be significant.

NOMENCLATURE

A, n = creep parameters

$\dot{\epsilon}$ = second invariant of the strain rate tensor

σ = second invariant of the stress deviation tensor

$\dot{\epsilon}_{ij}$ = the ij component of the strain rate

σ_{ij} = the ij component of the stress

$\tilde{\sigma}_{ij}$ = the ij component of the stress deviator

σ_i = the i component of normal stress

a = inner radius of hollow sphere or cylinder

a_0 = initial inner radius of hollow sphere or cylinder

b = outer radius of hollow finite cylinder; radius of solid shaft

p = pressure

p_0 = arbitrary pressure

t = time

τ = shearing stress

$I(x)$ = closure function

$x = a/a_0$

$k = a_0/b$

$\dot{\alpha} = \text{angular velocity of shaft}$

$T = \text{torque}$

$\ell = \text{length of shaft}$

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APPENDIX A

MATHEMATICAL ANALYSIS

APPENDIX A

MATHEMATICAL ANALYSISFormulation of the Creep Law

The creep law used in this investigation is a generalization of the power law for steady tensile creep. The law was formulated by Marin⁽¹⁾ in terms of the principal components of stresses and strain rates. Nye⁽²⁾ expresses the same law in a somewhat more general form in terms of the general components. The formulation used here is precisely that of Nye with some minor notational changes.

The creep law is obtained from the postulate that the ij components of the creep rate are given by

$$\dot{\epsilon}_{ij} = A \sigma^{n-1} \tilde{\sigma}_{ij}, \quad (\text{A-1})$$

where the subscripts i and j may be independently chosen as 1, 2, or 3. Here $\tilde{\sigma}_{ij}$ is the general component of the stress deviator tensor defined by

$$\tilde{\sigma}_{ij} = \begin{cases} \sigma_{ij}, & \text{if } i \neq j, \\ \sigma_{ij} - (1/3)(\sigma_{11} + \sigma_{22} + \sigma_{33}), & \text{if } i = j, \end{cases} \quad (\text{A-2})$$

and σ , the second invariant of the stress tensor is defined to be

$$\sigma = \frac{1}{\sqrt{2}} \left(\sum_{i=1}^3 \sum_{j=1}^3 \tilde{\sigma}_{ij}^2 \right)^{1/2} \quad (\text{A-3})$$

A and n are the creep parameters, and are constant for a given material at a given temperature. The incompressibility condition,

$$\dot{\epsilon}_{11} + \dot{\epsilon}_{22} + \dot{\epsilon}_{33} = 0, \quad (\text{A-4})$$

follows directly from Equation (A-1). The second invariant of the strain-rate tensor, the so-called effective strain rate, corresponds to Equation (A-3), and is defined as

$$\dot{\epsilon} = \frac{1}{\sqrt{2}} \left(\sum_{i=1}^3 \sum_{j=1}^3 \dot{\epsilon}_{ij}^2 \right)^{1/2} \quad (\text{A-5})$$

Both invariants are always taken as positive. It is seen that squaring both sides of Equation (A-1) and summing over i and j gives the formula

$$\dot{\epsilon} = A \sigma^n. \quad (\text{A-6})$$

---

(1) References at end.

Although this equation is not as broad as Equation (A-1), it is nevertheless sufficient, when used with Equation (A-4), for all the applications of the present study. Equation (A-6) is therefore a concise three-dimensional expression of the power law for second-stage creep.

Closure of Holes

Cylindrical Holes

The power law for creep has been applied by Coffin⁽³⁾ as well as by Nye to the behavior of long hollow cylinders under uniform radial stress. Plane strain is assumed. In other words, it is assumed that there are no strains in the longitudinal direction.

For the case of a long hollow cylinder subjected to uniform radial tension q on the inner surface $r = a$, and stress free at $r = \infty$, Nye obtains the radial velocity and the stresses. The radial velocity u is given by

$$u = - \frac{a^2 A q^n}{r n^n}, \quad (A-7)$$

and the stresses are

$$\sigma_r = q (a/r)^{2/n}, \quad (A-8)$$

$$\sigma_\theta = q (a/r)^{2/n} (1 - 2/n), \quad (A-9)$$

and

$$\sigma_z = q (a/r)^{2/n} (1 - 1/n). \quad (A-10)$$

The shearing stresses vanish on account of radial symmetry. If a uniform hydrostatic pressure q is superposed on all the normal stresses the velocity will not be affected, since the strain rates depend on the stress invariant and the stress deviators, which are unchanged. Therefore Equation (A-7) also gives the radial velocity due to a uniform pressure of q at $r = \infty$. The normal stresses may be obtained from Equations (A-8) - (A-10) by diminishing each value by q .

The case of the thick-walled cylinder subjected to uniform pressure at $r = b$ may also be obtained from Equation (A-7). For this case, however, q will not be constant. From Equation (A-8), the radial stress due to pressure q at $r = \infty$ is

$$\sigma_r = q \left[(a/r)^{2/n} - 1 \right]. \quad (A-11)$$

But at $r = b$, $\sigma_r = -p$. Therefore

$$q = \frac{p}{1 - (a/b)^{2/n}}. \quad (A-12)$$

Thus, for cylinder of inner radius a and outer radius b , subjected to uniform radial pressure p on the outer surface and stress free on the inner surface, the radial velocity and stresses are

$$u = -\frac{a^2 A}{r n^n} \left[\frac{p}{1 - (a/b)^{2/n}} \right]^n, \quad (\text{A-13})$$

$$\sigma_r = \frac{p}{1 - (a/b)^{2/n}} \left[(a/r)^{2/n} - 1 \right], \quad (\text{A-14})$$

$$\sigma_\theta = \frac{p}{1 - (a/b)^{2/n}} \left[(a/r)^{2/n} (1 - 2/n) - 1 \right], \quad (\text{A-15})$$

and

$$\sigma_z = \frac{p}{1 - (a/b)^{2/n}} \left[(a/n)^{2/n} (1 - 1/n) - 1 \right]. \quad (\text{A-16})$$

The maximum shearing stress occurs at the surface of the hole. The maximum and minimum principal stresses are σ_r and σ_θ . The maximum shearing stress is one-half their difference:

$$\tau_{\max} = \frac{p}{n [1 - (a/b)^{2/n}]}. \quad (\text{A-17})$$

It is noticed that the value of σ_z given by Equation (A-16) is not equal to $-p$ as would be expected for true hydrostatic pressure. The value is a variable as a consequence of the plane-strain assumption. It is of interest to calculate the resultant force acting on a section normal to the longitudinal axis. The total force P is given by

$$P = \frac{2\pi p}{1 - (a/b)^{2/n}} \int_a^b \left[(a/r)^{2/n} (1 - 1/n) - 1 \right] r dr. \quad (\text{A-18})$$

Integration gives the resultant force,

$$P = \pi b^2 p. \quad (\text{A-19})$$

P is therefore precisely the total force acting on the area within $r = b$. Since the ends of the cylinder are sealed to effect zero pressure on the inner surface, the effective area over which the pressure acts is πb^2 . Thus the plane-strain approximation leads to calculated longitudinal stresses which are in static equilibrium with the applied pressure.

The rate at which the hole closes is obtained from Equation (A-13) by evaluating at $r = a$. Since a is the instantaneous radius of the hole, the velocity at $r = a$ is

$$u = \frac{da}{dt} = - \frac{aA}{n^n} \left[\frac{p}{1 - (a/b)^{2/n}} \right]^n. \quad (A-20)$$

In other words,

$$\left[\frac{1 - (a/b)^{2/n}}{a} \right]^n \frac{da}{dt} = - A (p/n)^n. \quad (A-21)$$

It is convenient to express Equation (A-15) in terms of a dimensionless hole size x , and a dimensionless ratio k , where a_0 is the original hole radius and where

$$x = a/a_0,$$

and

$$k = a_0/b.$$

Equation (A-21) may now be rewritten

$$I(x) \equiv \int_x^1 \frac{[1 - (kx)^{2/n}]^n}{x} dx = A (p/n)^n t. \quad (A-22)$$

The first equality is a definition of the function $I(x)$. This function may not, in general, be evaluated in terms of elementary functions in closed form.

For the special case of the infinite medium, $k = 0$. In this case, the integration may be performed. Thus, the reduced radius of the long cylinder in an infinite medium subjected to uniform pressure p at $r = \infty$ (or uniform tension p at $r = 0$) is

$$a/a_0 = \exp [- A (p/n)^n t]. \quad (A-23)$$

For the finite cylinder, the assumption is made that the outer radius is constant.* A formal evaluation of $I(x)$ may be made in terms of the incomplete beta function.⁽⁴⁾ It may be shown that

$$I(x) = \frac{n}{2} \left[B_{k^{2/n}}^{2/n} (0, n+1) - B_{(kx)^{2/n}}^{2/n} (0, n+1) \right], \quad (A-24)$$

where

$$B_{\xi}^{\mu} (\mu, \nu) = \int_0^{\xi} \xi^{\mu-1} (1 - \xi)^{\nu-1} d\xi. \quad (A-25)$$

*The assumption of a constant value for b is reasonable since the total change in b is slight. For example, if the original radius of the hole is one-third the outer radius, complete closure of the hole will reduce b to $\sqrt{8/3}$ times its original size.

Equation (A-24) may be used to the extent that tabulated values of the incomplete beta function are available. It was not used in this investigation however. Instead, the function $I(x)$ is evaluated directly for the special case of n being an integer. For nonintegral values of n , interpolation may be used to evaluate the function. The integration of $I(x)$ may be done directly by writing the expansion of the binomial factor on the left side. In integral form then, write,

$$I(x) = \int_x^1 \frac{1}{x} \sum_{i=0}^n (-1)^i \frac{n!}{(n-i)! i!} (kx)^{2i/n} dx. \quad (A-26)$$

Integration gives

$$I(x) = -\ln x + \sum_{i=1}^n (-1)^i \frac{n \cdot n!}{2i (n-i)! i!} k^{2i/n} (1-x)^{2i/n}. \quad (A-27)$$

Equation (A-27) is an exact formula for $I(x)$. However in the form shown it is difficult to evaluate. An equivalent integration may be performed by an alternative method. Make the substitution

$$y = 1 - (kx)^{2/n}$$

in Equation (A-22). Then

$$I(x) = -\frac{n}{2} \int_{1-(kx)^{2/n}}^{1-k^{2/n}} \frac{y^n dy}{1-y}. \quad (A-28)$$

Separate into two terms as follows:

$$I(x) = \frac{n}{2} \int_{1-(kx)^{2/n}}^{1-k^{2/n}} \left[\frac{1-y^n}{1-y} - \frac{1}{1-y} \right] dy. \quad (A-29)$$

Note that

$$\frac{1-y^n}{1-y} = \sum_{i=0}^{n-1} y^i. \quad (A-30)$$

The integral may be now evaluated termwise giving

$$I(x) = \frac{n}{2} \left[\sum_{i=1}^n \frac{1}{i} y^i + \ln(1-y) \right] \bigg|_{1-(kx)^{2/n}}^{1-k^{2/n}}. \quad (A-31)$$

Evaluating at the limits gives

$$I(x) = -\ln x + \frac{n}{2} \sum_{i=1}^n \frac{1}{i} \left\{ (1-k^{2/n})^i - [1-(kx)^{2/n}]^i \right\}. \quad (A-32)$$

Equation (A-32) may readily be shown to be identical to Equation (A-21). However, for evaluating $I(x)$ for given values of x , (A-32) is more convenient, since Equation (A-27) contains large binomial coefficients.

It turns out that for the larger values of n , however, Equation (A-32) also becomes unsuitable for evaluation, since the terms in the brackets approximate $\ln x$. This difficulty may be overcome by noting the series formula,⁽⁵⁾

$$\ln(1-y) = - \sum_{i=1}^{\infty} \frac{1}{i} y^i. \quad (\text{A-33})$$

Within the brackets of Equation (A-31) are the first n terms of this series. Therefore, the substitution of

$$- \sum_{i=n+1}^{\infty} \frac{1}{i} y^i = \ln(1-y) + \sum_{i=1}^n \frac{1}{i} y^i \quad (\text{A-34})$$

at either or both of the limits may be made. The infinite series formula for $I(x)$ is

$$I(x) = \frac{n}{2} \left\{ \sum_{i=n+1}^{\infty} \frac{1}{i} [1 - (kx)^{2/n}]^i + \sum_{i=1}^n \frac{1}{i} (1 - k^{2/n})^i \right\}. \quad (\text{A-35})$$

The series converges most rapidly for the values at which the closed form of Equation (A-32) is most difficult to evaluate.

Spherical Holes

The analysis for the spherical cavity is similar to that for the cylinder. Nye⁽²⁾ shows that the velocity of closure for the spherical hole in an infinite medium with uniform pressure at infinity is

$$u = - \frac{a^3 3^{(n-1)/2} A p^n}{r^3 (2n)^n}. \quad (\text{A-36})$$

The rate at which the hole closes is found by evaluating at $r = a$.

$$\frac{1}{a} \frac{da}{dt} = - 3^{(n-1)/2} A (p/2n)^n. \quad (\text{A-37})$$

The diminished hole size may be found by integration.

$$a/a_0 = \exp [- 3^{(n-1)/2} A (p/2n)^n t]. \quad (\text{A-38})$$

Nye also shows that the stresses are

$$\sigma_r = p [(a/r)^{3/n-1}], \quad (\text{A-39})$$

and

$$\sigma_{\theta} = \sigma_{\phi} = p \left[(a/r)^{3/n} (1 - 3/2n) - 1 \right], \quad (\text{A-40})$$

where r , θ , and ϕ are the spherical coordinates.

Torsion of Shafts

The creep law may be used to relate the angular velocity to the torque in the case of solid shafts.

Relationship Between Torque and Angular Velocity

The usual assumption is made for inelastic torsion of a circular shaft. That is, the only nonvanishing strain rates are $\dot{\epsilon}_{z\theta}$ and $\dot{\epsilon}_{\theta z}$ which are equal, and given by

$$\dot{\epsilon}_{z\theta} = \dot{\epsilon}_{\theta z} = \frac{r\dot{\alpha}}{2\ell} \quad (\text{A-41})$$

The effective strain rate, from Equation (A-5), also is

$$\dot{\epsilon} = \frac{r\dot{\alpha}}{2\ell} \quad (\text{A-42})$$

As a consequence of the vanishing of most of the strain rates, the only nonzero stresses are $\sigma_{\theta z}$ and $\sigma_{z\theta}$. These are equal, and it follows from Equation (A-3) that

$$\sigma = \sigma_{\theta z} \quad (\text{A-43})$$

Substitution of Equations (A-41) and (A-42) into Equation (A-6) gives

$$\frac{r\dot{\alpha}}{2\ell} = A\sigma \quad (\text{A-44})$$

The torque T acting on the shaft is obtained by integrating the shearing stresses. Thus

$$T = \int_0^{2\pi} \int_0^b r\sigma_{\theta z} r dr d\theta = 2\pi \int_0^b r^2 \left(\frac{r\dot{\alpha}}{2\ell A} \right)^{1/n} dr. \quad (\text{A-45})$$

Since $\dot{\alpha}$ and the dimensions are constants they may be taken outside of the integral. Integration then gives the result

$$\dot{\alpha} = \left(\frac{3n+1}{2\pi n} \right)^n \frac{2\ell A}{b^{3n+1}} T^n. \quad (\text{A-46})$$

This result has also been obtained by Patel, Venkatraman and Hodge⁽⁶⁾ using variational techniques. The above method is more direct, although their method may also be applied to noncircular prismatic bars.

Substitution into Equation (A-44) gives the shearing stress

$$\begin{aligned}\sigma_{\theta z} &= \left(\frac{r\dot{\alpha}}{2\ell A} \right)^{1/n} \\ &= \left(\frac{3n+1}{2\pi n b^3} \right) \left(\frac{r}{b} \right)^{1/n} T.\end{aligned}\tag{A-47}$$

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APPENDIX B

EXPERIMENTAL PROGRAM

APPENDIX B

EXPERIMENTAL PROGRAMIntroduction

The experimental portion of the investigation was divided into three phases. The first two of these, the cylinder-closure tests and the torsion tests provided data which were used for the evaluation of the creep law. The third was a set of pressure-bonding tests designed to provide an evaluation of the validity of applying the creep law to pressure-bonding specimens.

Closure of Thick-Walled CylindersPreparation of Specimens

Specimens for the cylinder-closure tests were prepared from commercial 2S aluminum extruded bar stock (39 DPH, 1-kg load). Tubular 5-in. -long specimens with a 0.750-in. outer diameter and a 0.250-in. inner diameter were machined from 1.25-in. -diameter bar stock. The holes were made by center boring and reaming to a depth of 4.75 in. , leaving one end solid.

Evacuation tubes were prepared from 3S aluminum tubing (1/4 in. in outer diameter with a wall thickness of 1/32 in.). An evacuation tube was welded to each cylinder. The weld was tested by forcing helium gas at a pressure of 100 psi into the cylinder with the specimen under water. If bubbles were observed the specimen was discarded.

A stainless steel insert 1.5 in. long by 1/8 in. in diameter was placed into each evacuation tube. The purpose of the insert was to prevent the relatively weak tube from rupturing during subsequent bonding operations. The specimen was then evacuated for at least 16 hr to a pressure of not more than 5×10^{-3} mm of mercury. The evacuation tube was then sealed.

The completed specimen was again tested for leaks. This time the assembly was subjected to 1000 psi for 15 min. The pressure was released quickly and the assembly was placed in an alcohol bath immediately. If no bubbles were observed the completed specimen was retained. A sectional view of the prepared specimen is shown in Figure B-1.

A-31771

FIGURE B-1. SECTIONAL VIEW OF A 2S ALUMINUM THICK-WALLED-CYLINDER SPECIMEN

Pressurizing Procedure

The pressurizing operations were performed in a high-pressure hot-walled autoclave. The 6-in. -OD by 2-in. -ID by 32 5/8-in. -long autoclave was machined from an Inconel X forging. The unit is designed to operate at working pressures of 5000 psi at 1500 F. The autoclave was heated in a Marshall furnace rated at 220 v at 30 amp. The temperature of the heater and specimens was regulated by six thermocouples to within 20 F of the nominal temperature.

Two specimens at a time were placed side by side within the autoclave. The specimens were packed with copper shot to increase the mass, and thereby reduce the temperature fluctuations during the operation. The specimens were heated to the desired temperature. Then helium gas, which was stored in the storage vessel of the autoclave pump at about 2000 psi above the desired pressure was introduced into the autoclave. The temperature and pressure conditions were maintained for the prescribed time. At the end of the operation the pressure was released and the specimen was allowed to cool. The tested specimens were again checked for leaks. Those for which no leaks could be detected were retained for data.

Experimental Results

Measurements of final hole size were made on a transverse slice (about 3/8 in. in length) removed from the center of the specimen. The faces of the specimen were polished and the holes very carefully burred. The hole, in most cases, was distorted from the originally circular shape. To overcome this two diameters were measured, and an average used. The results did not significantly depend on the orientation of the measured diameter.

Table B-1 shows the measured final diameter of the pressurized specimens for which data were obtained. The information is arranged so that two specimens pressurized in a single operation appear together.

There was no measurable change of length for any of the specimens.

Torsion of Shafts

Preparation of Specimens

Specimens for the torsion tests were prepared from 2S aluminum obtained in the form of 1-1/2-in. -diameter bar stock. The over-all length of the specimen was 7-1/4 in. The center 3-in. section was machined to 0.750 ± 0.002 in. in diameter. A length of 1-1/2 in. at each end was machined square for clamping. Large-radius fillets provide the transition from the square ends to the circular center section. The details of the specimen are shown in Figure B-2.

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FIGURE B-2. SPECIMEN EMPLOYED IN TORSION EXPERIMENTS

The angle of twist was measured at the center 3-in. section.

Testing Procedure

The torsion specimen was heated within an electric furnace. A Foxboro controller, connected to a Chromel-Alumel thermocouple on the specimen, was used to regulate the heat input. The temperature along the reduced section was maintained to within 5 F of the nominal temperature.

A torsional load was applied to one end of the specimen by twisting the end at a constant angular velocity. The torque was measured by four electric-resistance strain gages mounted on the surface of a cylindrical bar in series with the specimen. The gages were connected in the form of a four-arm bridge and the signal was fed into a strain indicator.

TABLE B-1. RESULTS OF CLOSURE EXPERIMENTS ON 2S ALUMINUM HOLLOW CYLINDERS

Test Conditions			Initial Outer Diameter, in.	Inner Diameter, in.		Test Conditions			Initial Outer Diameter, in.	Inner Diameter, in.	
Temperature, F	Pressure, psi	Time, hr		Initial	Final	Temperature, F	Pressure, psi	Time, hr		Initial	Final
500	4000	1	.744	.252	(a)	700	3000	2	.748	.250	.087
			.745	.253	.210				.751	.250	.088
		2	.750	.250	.167			4	.749	.250	.094
			.750	.250	(a)				.749	.250	.096
		3(b)	.749	.253	.201			4	.749	.250	.078
		3(b)	.749	.253	.150				.749	.250	.086
		4	.750	.251	.164			6	.750	.250	.068
			.750	.252	(a)				.749	.250	.068
		4	.752	.251	.064		4000	2	.750	.250	.125
			.751	.251	.064				.750	.250	.124
		6	.751	.250	.085			3	.750	.251	.081
			.751	.250	.091				.750	.250	.072
	6000	1/2	.751	.251	.166			3(b)	.750	.250	.086
			.751	.250	.173			4	.750	.251	.063
		1	.750	.250	.178			6	.751	.253	.054
			.749	.250	(a)				.751	.252	.054
		2	.750	.250	.131		6000	1/2	.750	.250	.000
			.750	.249	.126				.750	.250	.000
		4	.750	.251	.040			1	.749	.250	.000
			.750	.251	.038				.749	.250	.000
		6	.748	.250	.026			2	.750	.250	.000
			.750	.250	(a)				.751	.250	.000
		4	.749	.250	.000			4	.749	.250	.000
			.749	.250	.000				.749	.250	.000
		6	.749	.250	.000			6	.749	.250	.000
			.749	.250	.000				.749	.250	.000

(a) Specimen failed by leaking.

(b) Single specimen pressurized in operation.

The angle of twist was measured for the 3-in. gage length only. Specially designed extensometers were mounted at the ends of the 3-in. center section. A disk and pointer were used to indicate the angle of twist.

Experimental Results

Data were recorded showing time, angle of twist, and torque. The torque-twist data are shown in Figures B-3 and B-4 for 2S aluminum at 500 and 700 F, respectively. It may be seen that the torque did not attain a constant value. Instead, it oscillated significantly. However, the portion of the curve for which the torque seemed most nearly to oscillate about a constant value was chosen visually. An arithmetic mean of the recorded data was taken. The angular velocity was calculated by dividing the total change in angle of twist during the chosen interval by the time of the interval. The resultant data relating angular velocity and torque were used in the further calculations.

It should be noted that the creep data for one test are obtained in a very short time, from less than an hour to a few hours at most.

Pressure-Bonding Tests

Preparation of Specimens

Specimens were prepared for the pressure-bonding tests from 5/16-in. 2S aluminum flat-plate stock (44 DPH). The mating surfaces of the flat-plate stock were machined to the prescribed surface roughness of 250 or 750 rms*. The directions of the machining were oriented normal to each other to guarantee that the maximum separation between the plates would be about 1 and 3 mils for the 250- and the 750-rms surfaces, respectively, when the ridges of the plates were in contact. A photograph of the machined surfaces is shown in Figure B-5.

The plates were assembled and welded together at a flange which surrounded the specimen. A shallow hole at the interface provided the site for mounting the evacuation tube. Figure B-6 is a sectional view of the assembled specimen. The specimens were inspected for leaks before and after evacuation.

Examination of Specimens

In order to examine the specimens, a cut was made in a plane normal to the plane of bonding and at about 30 deg from the long axis of the plate. The resulting cross-sectional view then avoided the possibility of coincidence with a ridge line or a depression, and permitted a comparison to be made among the various specimens examined.

A section was made of a specimen of each roughness before any pressure bonding was performed. Figure B-7 shows typical interfaces for the 250- and 750-rms specimens.

*As a measure of surface roughness, rms refers to the square root of the mean square distance in microinches from the median surface to the peaks and valleys of the surface. Thus a surface of 250 rms has an average peak-to-valley distance of approximately 0.5 mil.

FIGURE B-3. TORQUE-TWIST CURVES FOR 2S ALUMINUM AT 500 F

FIGURE B-4. TORQUE-TWIST CURVES FOR 2S ALUMINUM AT 700 F

FIGURE B-5. TWO MACHINE-ROUGHENED 2S ALUMINUM PLATES PRIOR TO ASSEMBLY AS A PRESSURE-BONDING SPECIMEN

Note that the directions of machining are normal to one another.

Approximate Size

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FIGURE B-6. SECTIONAL VIEW OF A 2S ALUMINUM FLAT-PLATE SPECIMEN

100X

a. 250-RMS Specimen

N53518

100X

b. 750-RMS Specimen

N53517

FIGURE B-7. TYPICAL INTERFACES OF FLAT-PLATE SPECIMENS
BEFORE PRESSURE BONDING

a. Region of Complete Contact in 250-RMS Specimen

b. Region of Separation in 250-RMS Specimen

c. Region of Complete Contact in 750-RMS Specimen

d. Region of Separation in 750-RMS Specimen

FIGURE B-8. TYPICAL INTERFACE AREAS IN FLAT-PLATE SPECIMENS BONDED FOR ZERO TIME AT 700 F AND 4000 PSI

a. 750-RMS Specimen Bonded for 2 Hr

b. 250-RMS Specimen Bonded for 6 Hr

c. 750-RMS Specimen Bonded for 6 Hr

FIGURE B-9. BONDS OBTAINED WITH FLAT-PLATE SPECIMENS AFTER 2 AND 6 HR AT 700 F AND 4000 PSI

Pressure-Bonding Operation

The pressurizing experiments for the flat-plate specimens were performed in the same apparatus used for the cylinder-closure tests. Two specimens, one of each surface roughness, were placed in the autoclave for each operation. Tests were run at 700 F and 4000 psi for various time periods.

In order to determine the original separation, a zero-time operation was performed. In this operation, the pressure was elevated to the prescribed value and immediately released. It had been assumed that some closure would occur due to instantaneous plastic flow. The resulting hole size could then provide a basis for analyzing the reduced hole size after an extended time interval.

Results of Pressure Bonding

Examination of the specimens after the pressure-bonding operations indicated that complete contact was substantially obtained.

The zero-time specimen did exhibit an occasional void which did not completely close. In addition, there was a tendency for the plates to separate on sectioning, since only a negligible amount of bonding occurred. However, as shown in Figure B-8, the surfaces had clearly deformed to a position of contact over most of their area. The tendency to separate was more pronounced in the 250-rms specimen, and the contact was less complete, indicating that the rougher surface had deformed more readily.

Although the contact was not absolutely complete at zero time, the amount of closure was so nearly complete that a quantitative expression of initial void size is unrealistic.

The specimens bonded for longer times apparently did attain complete contact. Figure B-9 shows typical sectional views of the interfaces.