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Rapid Multiplicity Rise as Function of Momentum
Transfer in Multicomponent Models.*

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It is shown that the average multiplicity exhibits a rapid rise as a function of t (the momentum transfer between the initial and leading particle) in multicomponent models in marked contrast to multi-peripheral models where a slow decrease is predicted.

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Recent data on multiplicity distributions have shown some interesting new features of multiparticle production processes¹, the most conspicuous of which is the energy dependence of the width of the multiplicity distribution. We can express this property most easily by means of the behavior of the variable D , given by

$$D = \langle n^2 \rangle / \langle n \rangle^2 - 1. \quad (1)$$

In multiperipheral type models (MPM) $D \rightarrow 0$ if the primary energy tends to infinity.² It was found that in the momentum range 50-300 GeV/c, D and similar combinations of the higher moments of the multiplicity distribution are constants within experimental errors.

Several models have been constructed to explain the data. Most of them have two or more physically different components, which are fairly energy-independent and give maxima at different multiplicities. The "interference" of these contributions gives rise to the effect described above. The two-component models³ (TCM) combine diffraction and multiperipheral type contributions to σ_n . Since pole-cut type models⁴ (PCM) combine the contributions of the Pomeron-pole exchange with multi-Pomeron exchange cuts in the elastic scattering amplitude, we refer to this description as a many component model. Here the multiplicity distributions are derived through elastic unitarity.

Since these models were invented to describe the energy dependence of the multiplicity distributions correctly, they can not be distinguished on the basis of this data alone. To attempt to differentiate among various models we study the dependence of the average multiplicity as a function of the momentum components of a leading particle, for example that of a proton

in proton-proton collisions. The average multiplicity can be defined through semi-inclusive cross sections⁵ as

$$N(s, t, M^2) = (\sum_n n d\sigma_n / dt dM^2) / (d\sigma / dt dM^2). \quad (2)$$

where $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$, $M^2 = (p_a + p_b - p_c)^2$; p_a and p_b are the momenta of the colliding particles, while p_c is the momentum of the leading particle in question. Alternatively, we can express $N(s, t, M^2)$ by inclusive cross sections as follows:

$$N(s, t, M^2) = \sum_d d^3 p_d (d\sigma / d^3 p_c d^3 p_d) / (d\sigma / d^3 p_c). \quad (3)$$

If correlations are neglected between the leading particle and other particles produced, then $N(s, t, M^2) = N(s)$, the integrated average multiplicity. Since the average number of the leading particles in one cone is 1, we have

$$\sigma^{-1} \int N(s, t, M^2) (d\sigma / dt dM^2) dt dM^2 = N(s). \quad (4)$$

For MPM the semi-inclusive cross section is given by

$$d\sigma_n / dt dM^2 = s^{-2} (s/M^2)^{2\alpha(t)} b(t) \sigma_{n-1}(M^2, t) \quad (5)$$

where $\sigma_{n-1}(M^2, t)$ is the $n-1$ particle production cross section for the collision of the exchanged system and particle b .

It is fairly easy to show that $\sigma_n(M^2, t)$ depends on t and M^2 only through the scaling variable $M^2/(m^2 + |t|)$ where m is some constant mass, and a possible overall t dependent factor is absorbed in $b(t)$. Since the average multiplicity depends logarithmically on M^2 , we end up with the following simple form of $N(s, t, M^2)$ in MPM:⁶

$$N(s, t, M^2) = a \ln(M^2/(m^2 + |t|)) + b. \quad (6)$$

We construct a TCM by making use of the inclusive Regge phenomenology⁷. We study $N(s, t, M^2)$ in the triple Regge limit. We add up to the contributions of PPf and ffP triple Regge couplings and neglect the contribution of PPP, where P and f stand for the Pomeron and secondary trajectories, respectively.⁸

The single particle cross section is given by

$$d\sigma/dtdM^2 = s^{-2} [a \exp \{bt + \alpha(0) \ln M^2 + 2\bar{\alpha}(t) \ln(s/M^2)\} + c \exp \{dt + \bar{\alpha}(0) \ln M^2 + 2\alpha(t) \ln(s/M^2)\}], \quad (7)$$

where $\alpha(t) = 1 + \rho t$ is the Pomeranchuk trajectory ($\rho \ll 1$), $\bar{\alpha}(t) = .5 + t$ is the secondary trajectory. The constant parameters appearing in (7) can be taken from a fit of experimental data by a similar formula.⁹ The average multiplicity corresponding to the first term of (7) (ffP) is approximately equal to the average multiplicity in proton-proton collisions at total cms. energy M . If the average multiplicity corresponding to the second term has a similar dependence on t and M^2 , then of course we obtain for $N(s, t, M^2)$ an expression similar to (6).

If, however, we assume with Jacob and Slansky¹⁰ that the average multiplicity corresponding to the diffraction term (PPf) is proportional to M , $N_{\text{diff.}} = 2 + \eta(M - m_0) \sim 1.7M$,¹¹ we arrive at a drastically different answer. Our results for $N(s, t, M^2)$ at 30 GeV/c for proton-proton interactions are plotted in Fig. 1. We made use of Eqs. (2) and (7) and the triple Regge expansion parameters of Ref. 6 to obtain the curves. At very large or very small values of M^2 , the diffraction or multiperipheral components disappear, respectively, and the t dependence vanishes. At intermediate values of M^2 , the t dependence of ffP is steeper than that of the diffractive component. Thus with increasing t , ffP will be suppressed compared to the PPf contribution. (Note that the average multiplicity associated with the diffraction component is larger than that of the multiperipheral component).

Finally we turn to the PCM. In this model the total cross section is given as the superposition of Pomeranchukon and multi-Pomeranchukon exchanges

$$\sigma \sim \sum_k c_k \exp \{[k\alpha(0) - k - 1] \ln s\}. \quad (8)$$

An additional logarithmic dependence of the terms on s is neglected, because the Pomeranchukon is assumed to have a very small slope.⁴ The coefficients c_k can be given in the physically reasonable form $c_k = a^{mk}/(mk)!.^2$ A fit to multiplicity distributions gives $a=4$, while m stays undetermined (a/m gives the average number of exchanged Pomeranchukons). The average multiplicity corresponding to the k Pomeranchukon exchange contribution is kn_0 , where n_0 is the average multiplicity corresponding to a single Pomeranchukon exchange. Next we shall find the inclusive cross section. Assuming that the multi-Pomeranchukon systems couple through a pair of f trajectories we obtain (using $\alpha(0)=1$)

$$d\sigma/dtdM^2 = s^{-2} \sum_k c_k (2 \cdot 2\bar{\alpha}(t)) M^2 \exp \{2\bar{\alpha}(t) \ln(s/M^2)\} f_k(t), \quad (9)$$

where the integral of $f_k(t)$ is normalized to 1. Our last chore is to estimate the t dependence of the coupling constants, $f_k(t)$. We shall show that it is reasonable to have $f_k(t) = (c/k) \exp\{ct/k\}$. Fig. 2 shows that through a duality type argument we can relate the exchange of the f trajectory between a proton and the systems which represents the "sides" of the ladders to the subsequent emission of these systems from an excited proton. Assuming that the proton momentum (p_i) distribution in the i -th excited system is fairly well described by a function $\exp\{ct_i\}$, where $t_i = (p_i - p_{i-1})^2$, we obtain the following t dependence for the vertices

$$[f_k(t)]^{1/2} = \int d^4p_1 d^4p_2 \dots d^4p_{n-1} \exp\{c(t_1 + t_2 + \dots + t_n)\} g(p_1, p_2, \dots, p_{n-1}), \quad (10)$$

where $p_0 = p_a$ and $p_n = p_c$ and g is assumed to be a slowly varying function (in comparison to the exponential functions) at negative momentum transfers. If we further assume that g allows a Wick rotation in the energy components of the four momenta then the integrations over p_i can be performed for a constant g and they give the above quoted form for $f_k(t)$. Physically it is clear that the impact of more particles on the excited proton system will result in a wider momentum transfer distribution of the final state proton.

It is possible to calculate now the average multiplicity. To get a rough idea about the t dependence of $N(s, t, M^2)$ we evaluate the sum over k in (9) by converting the sum into an integral and using the method of steepest descents. The M^2 and s dependences cancel out when we substitute (9) and a corresponding expression for $\sum_n n d\sigma_n/dtdM^2$ into (2), except for an overall multiplier depending on M^2 . The average multiplicity will have the form

$$N(s, t, M^2) = N(z)X(t)/4, \quad (11)$$

where $z = M^2 \exp \{1/(2-2\bar{\alpha}(t))\}$, a slowly varying function of t , and $X(t)$ is the solution of the implicit equation

$$|t| mc = X(t)^2 \ln(X(t)/4) + X(t). \quad (12)$$

The integrated average multiplicity at total c.m. energy $\sqrt{s}$ was introduced in Eq. 4 and denoted by $N(s)$, and m and c were defined earlier. We obtain the above form of z from the normalization condition (4), while (12) was obtained from the equation for the saddle point. The function $N(s, t, M^2)$ (using $\bar{\alpha}(t) = .65 + t$) is plotted on Fig. 3 at different values of the parameter mc . The M^2 and t dependences of $N(s, t, M^2)$ essentially factorize and the average multiplicity is a rising function of t . This behavior is the result of the enhancement of the multi-Pomeranchukon exchange contributions at higher values of t . Those contributions correspond to higher average multiplicities and we get a strong correlation between t and the average multiplicity.

Note that the inclusive cross section can be described within this framework as a linear combination of $f_k(t)$. At large values of t the terms with increasing k are responsible for the flattening of the t dependence

of the inclusive cross section. We would like to thank A. Kanofsky for a discussion on this point.

We did not take into account the diffraction dissociation contribution in the discussion of PCM. Since at fairly large M^2 and t the diffraction contribution can be neglected in comparison to Pomeron and multi-Pomeron contributions, the net effect of the inclusion of these contributions would only be a slightly wrong normalization of the curves of Fig. 3. When integrated they should give the average multiplicity for non-diffractive collisions, then the curves would run higher in Fig. 3.

In conclusion we want to emphasize that our result for rising multiplicity is more general than the specific models used to obtain our numerical answers. Rising average multiplicity is a general feature of any multi component model where the t dependence of the lower multiplicity components are steeper than those of the higher ones.

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Figure Captions

- Fig. 1. Charged particle multiplicity in 30 GeV/c p-p collisions in a two component model as a function of the missing mass and momentum transfer, the value of which is indicated on the curves in GeV^2 .
- Fig. 2. Duality inspired relation for the coupling of the proton to the multi-Pomeranchukon system through the exchange of f . Protons are denoted by straight lines, mesons by wiggly lines and Reggeons by double lines.
- Fig. 3. $N(s,t,M^2)$ for 30 GeV/c p-p collisions in PCM for $cm = 8$ (dashed curves) and $cm = 4$ (continuous curves). $M(\text{GeV})$ is indicated on the curves.

$N(s,t,M^2)$

$=$