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UNIVERSAL CURVES FOR PREDICTING EQUILIBRIUM RADII OF RELATIVISTIC
ELECTRON BEAMS INCLUDING THE EFFECTS OF ATTRACTIVE SELF FORCE,
EXTERNAL B_z , INITIAL BEAM QUALITY AND SCATTERING IN THE GAS ENTRY FOIL

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March 1975

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OF RELATIVISTIC ELECTRON BEAMS INCLUDING THE EFFECTS OF
ATTRACTIVE SELF FORCE, EXTERNAL B_z , INITIAL BEAM
QUALITY AND SCATTERING IN THE GAS ENTRY FOIL*

Eugene J. Lauer

The beam passes from vacuum into gas through a thin foil as shown in Fig. 1. The beam radius is independent of z in region 2 where the beam is space charge neutralized and at the surface of the foil in region 1 (vacuum). We incorporate the effect of foil scattering into a previous analysis.⁽¹⁾ At the surface of the foil in region 1 the average value of $p_{\perp}^2/p^2$ for beam particles is

$$\langle \theta_{1,}^2 \rangle = \frac{1}{2} \frac{q_1^2}{a^2} \quad (1)$$

where q_1 is the beam quality before foil scattering. The beam energies from the foil with

$$\langle \theta_{2,}^2 \rangle = \frac{1}{2} \frac{q_2^2}{a^2} \quad (2)$$

The vector addition of many individual small changes of $\bar{p}_{\perp}$ in random directions has the net result of cumulating the squares of the magnitudes of the individual vectors

$$\langle \theta_{2,}^2 \rangle = \langle \theta_{1,}^2 \rangle + \langle \theta_s^2 \rangle \quad (3)$$

In region 2 the adiabatic invariant is

* Work performed jointly under the auspices of the U.S. Energy Research & Development Administration, and the Department of the Navy under the contract number NAonr 13-74.

¹E.P. Lee, Physics of Beam Compression Experiment, Memo of 26 Jan., 1973.

$$q_2 = \sqrt{\frac{k_c^2}{4} + k_\beta^2} a^2 \quad (4)$$

(k_β is understood to have the subscript 2, but it is left off for simplicity.)

In Appendix I the envelope equation, which gives Eq. 4 for the special case of a independent of z , is derived using an idealized model. k_c and $k_\beta^2 a^2$ are defined in Appendix I. Combining Eqs. 1, 2, 3 and 4 and defining $x = a^2/q_1^2$, results in the quadratic equation in x ,

$$\left(\frac{q_1^2 k_c^2}{4}\right) x^2 + (k_\beta^2 a^2 - 2\langle\theta_s^2\rangle)x - 1 = 0 \quad (5)$$

(This is Eq. 50 of ref. 2)

For $k_\beta^2 a^2 > 0$ and $q_1^2 k_c^2 = 0$, the solution is

$$x = \frac{1}{(k_\beta^2 a^2 - 2\langle\theta_s^2\rangle)}$$

For $k_\beta^2 a^2 > 0$ and $q_1^2 k_c^2 > 0$ the physically meaningful solution is

$$x = \frac{-(k_\beta^2 a^2 - 2\langle\theta_s^2\rangle) + \sqrt{(k_\beta^2 a^2 - 2\langle\theta_s^2\rangle)^2 + q_1^2 k_c^2}}{\left(\frac{q_1^2 k_c^2}{4}\right)} \quad (7)$$

B_z is the same in regions 1 and 2. In region 2 the self-force is directed radially inward; in region 1 it is directed radially outward and the magnitude is $(\beta\gamma)^{-2}$ times smaller. Therefore, in most cases of interest the beam is not in equilibrium in region 1, the net inward force being too weak if

$$2\langle\theta_s^2\rangle < k_\beta^2 a^2 (1 + 1/\beta^2 \gamma^2).$$

(2) E.P. Lee, "Envelope Equation of a Charged Particle Beam" UCID 16490, April 15, 1974.

On Fig. 2, $\sqrt{x}$ is plotted vs $\langle \theta_s^2 \rangle$ for parameters of interest in the Livermore beam experiments. Note that for $k_c = 0$, (a/q) diverges as $(k_\beta^2 a^2 - 2\langle \theta_s^2 \rangle) \rightarrow 0$. Finite k_c removes the divergence.

In Appendix II numerical values of $\langle \theta_s^2 \rangle$ are calculated for some common foils. In Appendix III numerical values of $\langle \theta_s^2 \rangle$ are given for some common gases and the most important formulas for increase of beam radius with gas scattering are stated for completeness.

FIG. 1

Beam Passing from Vacuum into Gas Through a Thin Foil. In Gas the Beam has the Equilibrium Radius a .

APPENDIX I

A.1 Derivation of the Equation of Motion for the Beam Radius

We use cylindrical coordinates (r, θ, z) . We use an idealized model in which the fields are independent of θ , $v_{\perp}^2 \equiv v_r^2 + v_{\theta}^2 \ll v^2$, where v is the particle total velocity, and there is no plasma current. The radial equation of motion of a beam electron is,

$$-e(E_r + \frac{v_{\theta}}{c} B_z - \frac{v_z}{c} B_{\theta}) = \gamma m_0 (\ddot{r} - \frac{v_{\theta}^2}{r}) \quad (8)$$

We use Gaussian units, e is the magnitude of electronic charge $v = \beta c$, $\gamma = (1 - \beta^2)^{-1/2}$ and dot means time derivative.

$$B_{\theta} = - \frac{2(I/c)r}{a^2} \quad (9)$$

where (I/c) is the magnitude of beam current in emu, and $v_z > 0$. The current density is uniform inside the beam radius a , and a varies slowly with z .

$$E_r = - \frac{2(I/c)r}{a^2 \beta} (1-f) \quad (10)$$

where f is the fraction of electrostatic field neutralized by plasma space charge. Substituting Eqs. 9 and 10, Eq. 8 can be written,

$$\frac{\ddot{r}}{\beta^2 c^2} = \left(\frac{v_{\theta}}{\beta c} \right)^2 \frac{1}{r} - k_c \left(\frac{v_{\theta}}{\beta c} \right) - k_{\beta}^2 a^2 \frac{r}{a^2} \quad (11)$$

where,

$$k_c = \frac{B_z}{\beta \gamma \left(\frac{m_0 c^2}{e} \right)} \quad (12)$$

(we neglect the contribution of the beam current to B_z) .

and,

$$k_{\beta a}^2 = \frac{2(I/c) (f-1/\gamma^2)}{\beta \gamma \left(\frac{m_0 c^2}{e} \right) \beta^2} \quad (13)$$

With axial symmetry, the canonical angular momentum P_θ is a constant. This gives a first integral of the azimuthal equation of motion.

$$P_\theta = \gamma m_0 r v_\theta - \frac{e}{c} r A_\theta - \gamma m_0 \beta c q \quad (14)$$

where $\beta c q$ is the value of $r v_\theta$ when $r A_\theta = 0$.

Since,

$$\int \vec{A} \cdot d\vec{\ell} = \int (\vec{\nabla} \times \vec{A}) \cdot d\vec{s}$$

$$2\pi r A_\theta \approx \pi r^2 B_z \quad (15)$$

Combining Eqs. 12, 14 and 15 gives,

$$\frac{v_{\theta}}{\beta c} = \frac{q}{r} + \frac{1}{2} k_c r \quad (16)$$

Substituting Eq. 16 into 11 gives,

$$\frac{\ddot{r}}{\beta^2 c^2} = \frac{q^2}{r^3} - \frac{k_c^2 r}{4} - (k_{\beta}^2 a^2) \frac{r}{a^2} \quad (17)$$

If the velocity distribution consists of particles with $\dot{r} < \dot{r}_0$, then the beam surface follows the motion of a particle with $r = a$ and,

$$\frac{\ddot{a}}{\beta^2 c^2} = \frac{q^2}{a^3} - \frac{k_c^2}{4} a - (k_{\beta}^2 a^2) \frac{1}{a} \quad (18)$$

APPENDIX II

A.1 Foil Scattering

For electrons passing through $n_Z L$ atoms/cm² of atomic number Z , the cumulated mean squared scattering angle is,

$$\langle \theta_s^2 \rangle = \frac{8\pi e^4 Z(Z+1)n_Z L}{m_0^2 \gamma^2 \beta^4 c^4} \ln \frac{\theta_{\max}}{\theta_{\min}} \quad (19)$$

For $\gamma = 11$,

$$\langle \theta_s^2 \rangle = 1.68 \times 10^{-26} Z(Z+1)n_Z L \ln \frac{\theta_{\max}}{\theta_{\min}}$$

For the "thin target" case (which is pertinent to the current beam experiment),

$$\frac{\theta_{\max}}{\theta_{\min}} = \frac{h}{\sqrt{\pi} m_0 c \beta} Z^{2/3} (n_Z L)^{1/2} \quad (20)$$

For $\gamma = 11$,

$$\frac{\theta_{\max}}{\theta_{\min}} = 1.38 \times 10^{-10} Z^{2/3} (n_Z L)^{1/2}$$

Table I gives numerical values for several common foils.

Table I Coulomb Scattering of Electrons in Thin Foils, $\gamma=11$

	L	ρ	W	$\frac{\rho A}{W}$	Z	Z(Z+1)	$n_Z L$	$Z^{2/3}$	$\ln \frac{\theta_{\max}}{\theta_{\min}}$	$\frac{\theta_{\max}}{n_Z L(Z+1) \theta_{\min}}$	$\langle \theta_s^2 \rangle$
	(.00254) cm	(gm cm ⁻³)	(gm)	(10 ²³ cm ⁻³)			(10 ²⁰ cm ⁻²)			(10 ²² cm ⁻²)	(10 ³)
Be	2	1.85	9.01	1.24	4	20	6.30	2.520	2.16	2.72	0.456
C	5	2.2	12.01	1.10	6	42	13.97	3.302	2.83	16.6	2.78
Al	2	2.702	26.98	0.6031	13	182	3.06	5.529	2.59	14.4	2.41
Ti	1	4.5	47.90	0.566	22	506	1.44	7.851	2.56	18.7	3.14
					6	42	1.045	3.302	1.535		
					1	2	0.238	1	—		
KAPTON 1	1	1.42	457.	.0187	8	72	0.475	4	1.333	1.15	0.193
(C ₂₂ H ₅ O ₁₀ N ₂)					7	56	0.095	3.3659	0.438		
					6		3.13		2.08		
					1		0.712		0.15		
KAPTON 3	3				8		1.425		1.88	4.82	0.808
					7		0.285		0.99		

APPENDIX III

A.1 Gas Scattering

In the case of slow gas scattering where the beam is always near equilibrium, the most important special cases are⁽²⁾:

Case I, $k_c = 0$, $k_{\beta}^2 a^2 = \text{constant}$

$$\frac{a}{a(0)} = \exp \frac{\langle \theta_s^2 \rangle}{k_{\beta}^2 a^2}$$

Case II, $k_{\beta}^2 a^2 = 0$, $k_c = \text{constant}$

$$a^2 - a^2(0) = 4 \frac{\langle \theta_s^2 \rangle}{k_c^2}$$

In table II, numerical values are given which are particularly pertinent to Case I for a beam of a few hundred amperes.

Table II Coulomb Scattering of Electrons in Gases, $\gamma=11$

	L (m)	P (torr)	n (10^{18} cm^{-3}) 20°C	nL (10^{20} cm^{-2})	Z	Z(Z+1)	$Z^{2/3}$	$\ln \frac{\theta_{\max}}{\theta_{\min}}$	$\langle \theta_s^2 \rangle$ (10^{-3})
H ₂	10	600	38.88	388.8	1	2	1	3.30	4.31
N ₂	1	200	12.96	12.96	7	56	3.659	2.90	3.53
He	10	400	12.96	129.6	2	6	1.587	3.22	4.18
Ne	1	200	6.48	6.48	10	110	4.642	2.79	3.33
Ar	1	60	1.944	1.944	18	342	6.868	2.58	2.87
Kr	1	15	0.486	0.486	36	1332	10.90	2.35	2.55
Xe	1	7	0.2268	0.2268	54	2970	14.29	2.24	2.53

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