UNCLASSIFIED

EXTERNAL TRANSMITTAL AUTHORIZED

OAK RIDGE NATIONAL LABORATORY
Operated By
UNION CARBIDE NUCLEAR COMPANY

1

UCC

POST OFFICE BOX P
OAK RIDGE, TENNESSEE

ORNL
CENTRAL FILES NUMBER

CF-56-11-82

DATE: November 21, 1956

SUBJECT: THE INTEGRAL SPECTRUM METHOD FOR GAMMA HEATING
CALCULATIONS IN NUCLEAR REACTORS

FROM: L. G. Alexander

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission to the extent that such employee or contractor prepares, handles or distributes, or provides access to, any information pursuant to his employment or contract with the Commission.

Photostat Price \$ 6.30

Microfilm Price \$ 3.00

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

UNCLASSIFIED

417-001

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

THE INTEGRAL SPECTRUM METHOD FOR GAMMA HEATING
CALCULATIONS IN NUCLEAR REACTORS

INTRODUCTION

Apparently, Hurwitz et al. (9) were the first to investigate the applicability of the integral spectrum method of estimating gamma heating rates in nuclear reactors. The method is based upon two principal assumptions: (a) photons are degraded in energy but not deflected by scattering collisions, and (b) the spectrum of the photon flux does not vary appreciably with distance from the source of photons. As shown in the next section, these assumptions lead directly to the formulation of a simple exponential attenuation law having an attenuation coefficient appropriately defined in terms of the properties of the medium and the spectrum of the source.

The reliability of the method needs thorough investigation in order to delimit its area of applicability. No such general study has as yet been made. Hurwitz (9), however, has compared the integral-spectrum method with the results of a multi-group, straight-ahead-scattering calculation in which the spectrum was allowed to vary and found good agreement between the two methods and agreement with the results of experimental measurement for a typical case.

ATTENUATION LAW FOR THE INTEGRAL BEAM

In the integral spectrum method, a current of photons is treated as a mono-energetic beam of photons of energy $\bar{E}_\gamma$ having an effective energy absorption coefficient $\bar{\mu}_e$, both means being defined by appropriate integrations over the spectrum of the source (1).

Consider a gamma source of strength $s_\gamma(\vec{\rho}_s)$ photons per second located at a point in space having vector coordinates denoted by $\vec{\rho}_s$. Let the spectral distribution be denoted by $P_E(E_\gamma)$, and the angular distribution by $P_\Omega(\alpha, \beta, \gamma)$

3.

where $P_{\Omega}(\alpha, \beta, \gamma) d\Omega$ is the probability that a photon will be emitted into differential solid angle $d\Omega$ oriented along a line whose direction cosines are (α, β, γ) . Let the number of those photons emitted by the source per second into a particular elementary solid angle $\delta\Omega$ be designated by $\delta n_{\gamma}(\vec{p}_s)$. It follows that

$$\delta n_{\gamma}(\vec{p}_s) = s_{\gamma}(\vec{p}_s) P_{\Omega}(\alpha, \beta, \gamma) \delta\Omega$$

1

Figure 1

In the straight-ahead-scattering model, photons are permitted to lose energy during Compton scattering collisions, but are not permitted to deviate from their original flight directions. Thus, once they are emitted in the direction α, β, γ , the photons must continue in that direction until they are absorbed. The number flux is attenuated only by absorptions, but the energy flux is attenuated by both absorptions and Compton collisions. The energy carried by the beam at any point along the beam is found by multiplying the number current $\delta n_{\gamma}(\vec{p})$ at that point by the mean energy of the photons in the beam.

$$\text{Energy flow, mev/sec} = \delta n_{\gamma}(\vec{p}) \bar{E}_{\gamma}(\vec{p})$$

2

where $\bar{E}_{\gamma}(\vec{p})$ is found by integrating the photon energy E_{γ} over the spectrum of the beam at $\vec{p}$.

447-003

$$\bar{E}_\gamma(\vec{r}) = \int_0^\infty P_E(E_\gamma, \vec{r}) E_\gamma dE_\gamma$$

The spectral distribution $P_E(E_\gamma, \vec{r})$ is a function of position because the spectrum is progressively altered away from the source by absorptions and scatterings.

The energy deposited in the medium by the beam is readily formulated by taking an energy balance in the beam entering and leaving an elementary volume

$$\rho^2 \delta\Omega \delta\rho.$$

$$\text{Energy deposition rate, mev/sec} = -\frac{d}{d\rho} [\delta n_\gamma(\vec{r}) \bar{E}_\gamma(\vec{r})] \delta\rho. \quad 4$$

This is equated to the energy deposition rate calculated by consideration of the physics of the collision processes. This rate is found by multiplying the number $\delta n_\gamma(\vec{r}) P_E(E_\gamma, \vec{r})$ of photons of a given energy E_γ in the beam by the total collision probability $\mu_t(E_\gamma, \vec{r})$, the energy E_γ of the photons, the average fraction $f_t(E_\gamma)$ of their energy E_γ deposited locally by such photons in collision of all types, followed by integration of the product over all energies.

$$\frac{d}{d\rho} [\delta n_\gamma(\vec{r}) \bar{E}_\gamma(\vec{r})] = \int \delta n_\gamma(\vec{r}) P_E(E_\gamma, \vec{r}) dE_\gamma \mu_t(E_\gamma, \vec{r}) E_\gamma f_t(E_\gamma) dE_\gamma \quad [5]$$

where $\vec{r}$ denotes the medium surrounding the point designated by the coordinates $\vec{r}(\rho, \alpha, \beta, \gamma)$.

Now let an "energy absorption" coefficient be defined by

$$\mu_e(E_\gamma, \vec{r}) = \mu_t(E_\gamma, \vec{r}) f_t(E_\gamma, \vec{r}) \quad 6$$

and let a mean value be defined by

$$\bar{\mu}_e(\vec{r}) = \frac{1}{\bar{E}_\gamma(\vec{r})} \int_0^\infty \mu_e(E_\gamma, \vec{r}) P_E(E_\gamma, \vec{r}) E_\gamma dE_\gamma \quad 7$$

Applying these definitions to Equation 5, we have

$$\frac{d}{d\rho} [\delta n_\gamma(\vec{r}) \bar{E}_\gamma(\vec{r})] = -\bar{\mu}_e(\vec{r}) \delta n_\gamma(\vec{r}) \bar{E}_\gamma(\vec{r}) \quad 8$$

Integrating, one has

$$\delta n_\gamma(\vec{r}) \bar{E}_\gamma(\vec{r}) = \delta n_\gamma(\vec{r}_0) \bar{E}_\gamma(\vec{r}_0) \exp \left[-\int_{\rho_0}^{\rho} \bar{\mu}_e(\vec{r}') d\rho' \right] \quad 9$$

which is the attenuation law for the "integral" beam. If now the spectral distribution $P_E(E_r, \vec{\rho})$ varies widely along the beam, $\bar{\mu}_e(\vec{\rho})$ will be a strong but unknown function of position. We will thus not be able to evaluate the integral in the exponent of Equation 9, and the integral beam method fails for this case. If, however, the spectral distribution of the photons varies little along the beam, the spatial dependence of $\bar{\mu}_e$ arises mainly from variations in the nature of the medium from point to point. The integration is readily performed for this case, yielding,

$$S_{n_r}(\vec{\rho}) \bar{E}_r(\vec{\rho}) = S_{n_r}(\vec{\rho}_s) \bar{E}_r(\vec{\rho}_s) \exp \left[- \sum_{n=1}^{n=m} \bar{\mu}_e(\vec{\rho}) \Delta \rho_n \right] \quad 10$$

where the mean energy absorption coefficients $\bar{\mu}_e$ are evaluated separately for each medium by integration of the μ_e characteristic of that medium over the spectrum of the source, i.e.,

$$\bar{\mu}_e(\vec{\rho}_n) = \frac{1}{\bar{E}_r(\vec{\rho}_s)} \int_0^{\infty} \mu_e(E_r, n) P_E(E_r, \vec{\rho}_s) E_r dE_r \quad 11$$

Equation 10 is the attenuation law to be used in the formulation of the gamma heating rate using the straight-ahead-scattering, integral spectrum model.

THE GAMMA HEATING EQUATION

Equation 4 gives the heat released in mev/sec in the volume $\rho^2 \delta \Omega \delta \rho$ about $\vec{\rho}$ by photons originating in the source $S_r(\vec{\rho}_s)$ at $\vec{\rho}_s$. The corresponding contribution δG to the specific heating rate is obtained by dividing Equation 4 by the volume of the element. Substituting from

Equations 10 and 1, and performing the indicated operations, one has

$$\delta G = s_\gamma(\vec{\rho}_s) P_\Omega(\alpha, \beta, \gamma) d\Omega \bar{E}_\gamma(\vec{\rho}_s) \bar{\mu}_e(\vec{\rho}) \exp \left[- \sum_{n=1}^{n=m} \bar{\mu}_e(M_n) \Delta \rho_n \right] \frac{1}{\rho^2 \delta \Omega \delta \rho} \quad 12$$

The source $s_\gamma(\vec{\rho}_s)$ may be in general represented by the product of a volume distributed source of strength $S_\gamma(\vec{\rho}_s)$ photons/cc. sec, and an element of volume $\delta V(\vec{\rho}_s)$ about $\vec{\rho}_s$.

$$s_\gamma(\vec{\rho}_s) = S_\gamma(\vec{\rho}_s) \delta V(\vec{\rho}_s) \quad 13$$

Making this substitution, and going to the limit as $\delta V \rightarrow 0$, one obtains the gamma heating equation,

$$\frac{dG(\vec{\rho})}{dV(\vec{\rho}_s)} = \frac{S_\gamma(\vec{\rho}_s) \bar{E}_\gamma(\vec{\rho}_s) P_\Omega(\alpha, \beta, \gamma) \bar{\mu}_e(M_m)}{\rho^2} \exp \left[- \sum_{n=1}^{n=m} \bar{\mu}_e(M_n) \Delta \rho_n \right] \quad 14$$

where dV is located at $\vec{\rho}_s$ in medium M_1 , the heating, dG , is at $\vec{\rho}$ in medium M_m , and media M_n , interposing path lengths $\Delta \rho_n$, lie between. The total heating rate, G , is obtained by integrating Equation 14 over all gamma sources present.

INTEGRATION OF THE GAMMA-HEATING EQUATION

Cylindrical Sources

The estimation of the gamma-heating rate at various points in the moderator of a heterogeneous reactor is a typical problem. Consider a cubical reactor of the X-10 Graphite Reactor type having fuel slugs 1-inch O.D. in fuel-coolant channels 1.2 inches I.D., 1.3 inches O.D., by 30 feet long. Let there be 900 channels arranged on a 12-inch square lattice and extending in a direction parallel to the Z axis. Suppose the neutron flux distribution $\phi_n(x, y, z)$ is known. Let it be required to compute the heating rate due to prompt fission

7.
 gammas in mev./cc. sec. at some point in the reactor. Now, the total heating rate can be found by summing the contributions from each of the individual fuel channels. Thus, the problem is solved if we can extract from Equation 14, an expression for a single channel that can be handled without excessive labor.

Isotropic Line Source

Let the origin of coordinates be taken on the axis of a fuel channel in a plane, perpendicular to the z axis, that contains ^{the point} P at which it is desired to evaluate the heating. In the first approximation, we might be willing to assume that the fuel slugs themselves absorb a negligible fraction of the prompt gamma radiation. If we neglect the end caps between slugs and treat the fuel as a continuous rod, dV in Equation 14 may be replaced by $\pi R^2 dz$, where R is the radius of the slugs (exclusive of clad), and dz is an elemental length of rod.

Fig. 2

In the absence of significant self-absorption, the radiation from any elemental length of rod dz will be isotropically distributed and $P_{\Omega}(\theta) = 1/4\pi$. The contribution to the heating at P due to that part of the fuel lying along the positive z-axis may be formulated:

$$\delta G = \int_0^{fL} \frac{\phi_n(z) \Sigma_f N_f \pi R^2 dz \bar{E}_r \bar{\mu}_{e,m}}{4\pi \rho_m^2} \exp[-\bar{\mu}_{e,m} \rho_m] \quad 15$$

where L is the length of the channel, and f is that fraction lying above the x-y plane.

Several cases are now readily distinguished. Suppose first that x is small compared to fL. The radiation path ρ_m in the exponential term will vary from a minimum value equal to x to a maximum value greater than fL; hence $\rho_m(\max) \gg \rho_m(\min)$. It follows that the gamma sources near the end of the rod make much smaller contributions to the heating at P than do those in the vicinity of the origin. In fact, those sources lying at distances along the z axis greater than 2a often make only negligible contributions. If now, the variation of the flux $\phi_n(z)$ in the vicinity of the origin is not great, Equation 15 will be well approximated by taking $\phi_n(z)$ equal to $\phi_n(0)$.

$$\delta G = \frac{\phi_n(0) \Sigma_f N_f \pi R^2 \bar{E}_r \bar{\mu}_{e,m}}{4\pi \rho_m^2} \int_0^{fL} \exp[-\bar{\mu}_{e,m} \rho_m] dz \quad 16$$

Making use of the relations

$$\rho_m = a \sec \theta$$

and

$$dz = a \sec^2 \theta$$

Equation 16 takes the form

$$\delta G = \frac{\phi_n(0) \sum f R^2 N_r \bar{E}_r \bar{\mu}_{e,m}}{4a} F[\theta(fL), b] \quad 17$$

where

$$b = \bar{\mu}_{e,ma}$$

$$\theta(fL) = \arctan(fL/a)$$

and

$$F(\theta, b) = \int_0^\theta \exp[-b \sec \theta'] d\theta' \quad 18$$

The function $F(\theta, b)$ has been graphed for a wide range of variables by Foderaro and Obenshain (2).

If the point P happens to be in a plane lying below the end of the fuel channel, as shown in Fig. 2, the solution takes the form

$$\delta G = \frac{\phi_n(z_1) \sum f R^2 N_r \bar{E}_r \bar{\mu}_{e,m}}{4a} [F(\theta_2, b) - F(\theta_1, b)] \quad 19$$

Because the flux frequently varies rapidly with z near the ends of fuel channels, the postulate of uniform flux distribution will give much less reliable results for this case.

By way of example, we will compute the heating at the midpoint of the reactor described above by prompt gammas coming from the 36 nearest fuel channels for the case where the total power of the reactor is 100 mw. and the ratio of maximum to average flux (ϕ_m/ϕ_{av}) is 4.0. The power output is related to the average flux by the following equation.

$$Q = \phi_{av} \sum_f N_f \pi R^2 L E_f \quad (\text{mev./sec.}) \quad (\text{C.F.}) \quad 20$$

where

Q = total reactor heat, megawatts.

ϕ_{av} = average thermal neutron flux in the reactor, neutron-ft./ft.³sec

f = fission cross section for thermal neutron in fuel, $\frac{\text{fissions}}{\text{neutron-ft.}}$

N_f = total number of fuel-coolant channels

C.F. = 1.60×10^{-19} mw. sec./mev.

E_f = energy release per fission, 200 mev.

Setting $\phi_{av} = \phi_m/4$ and solving for $\phi_m \sum_f \pi R^2$, one has

$$\begin{aligned} \phi_m \sum_f \pi R^2 &= \frac{4 \times 100 \text{ mw.}}{900 \times 30 \text{ ft.} \times 200 \frac{\text{mev.}}{\text{fission}} \times 1.60 \times 10^{-19} \text{ mw. sec./mev.}} \\ &= 4.63 \times 10^{14} \text{ fissions/sec.ft. of fuel} \end{aligned}$$

According to Gamble(9),

$$N_f \bar{E}_\gamma = 7.5 \text{ mev/fission} = 11.4 \times 10^{-16} \text{ BTU/fission}$$

The mean absorption coefficient $\bar{\mu}_e$ for graphite is about 1.4 ft.^{-1}

Hence

$$\begin{aligned} \frac{\phi_m \sum_f \pi R^2 N_f \bar{E}_\gamma \bar{\mu}_e (\text{mod})}{4\pi} &= 4.63 \times 10^{14} (\text{fissions/ft. sec}) \times 11.38 \times 10^{-16} \\ &\times \frac{\text{BTU/fission} \times 1.4 \text{ ft.}^{-1} \times 3600 \text{ sec/hr}}{4\pi} \\ &= 212 \text{ BTU/ft}^2 \text{ hr.} \end{aligned}$$

Table I

Number of channels, N_n	a_n ft.	$\tan \theta_n(fL)$	$\theta_n(fL)$	b_n	$F(\theta_n, b_n)$	$N_n F(\theta_n, b_n) / a_n$
4	$\sqrt{2} / 2$	$30 / \sqrt{2}$	87.3	0.99	0.330	1.87
8	$\sqrt{10} / 2$	$30 / \sqrt{10}$	84.0	2.22	0.082	0.42
4	$3\sqrt{2} / 2$	$30 / 3\sqrt{2}$	82.0	2.97	0.032	0.06
8	$\sqrt{26} / 2$	$30 / \sqrt{26}$	80.2	3.57	0.0165	0.05
8	$\sqrt{34} / 2$	$70 / \sqrt{34}$	79.0	4.08	0.0094	0.03
4	$5\sqrt{2} / 2$	$30 / 5\sqrt{2}$	76.7	4.95	0.0036	0.00

Total 2.43

Hence,

$$G = 212 \times 2 \times 2.43 = 976 \text{ BTU/hr/ft}^3$$

22

We suppose that the flux is reasonably uniform in the central region of the reactor so that this value of the coefficient of the F's in Equation 19 may be used for all 36 of the adjacent fuel channels. A cross section of the central region is shown in Fig. 4.

Fig. 4

It is seen that there are six sets of channels, containing 4, 8, 4, 8, 8, and 4 channels respectively, and that the members of a set are all equidistant from the point P. The common distances, a , in feet, are given in Table I. Since the channels extend 15 feet in both directions from the midplane, each channel contributes two integrals to Equation ²⁰ with limits of 15 feet each.

$$\frac{G_{BTU}}{hr. ft.^3} = 212 \times 2 \sum_{n=1}^{n=6} \frac{N_n}{a_n} F(\theta_n, b_n) \dots 21$$

Where N_n is the number of fuel channels in the n th group, a_n the corresponding distance.

A second case of the application of Equation 15 is where "a" is comparable to "fL" in magnitude, such as would be the case if the point P at which it is desired to estimate the heating rate were located near the outer boundary of the reflector. The variation in the gamma source strength along the fuel channels will now be taken into consideration. By way of illustration of the method, the case treated above will be treated again with $\phi(z)$ retained as a variable; the results of this slightly more rigorous procedure will be shown to justify the approximation of using a uniform source strength in that example.

Suppose the neutron flux to be distributed in the axial direction like the central two-thirds of a cosine function. Then

$$\phi_n(z) = \phi_n(0) \cos \left[\frac{z/2}{3L/2} (\pi/2) \right] \quad 23$$

On evaluating the ratio of maximum to average flux by integrating this equation over the length of the core, it is found that $\phi_{av}/\phi_n(0)$ is 2.41. The ratio of maximum to average flux in the transverse direction must therefore be $4/2.41$.

The maximum flux $\phi_n(0)$ is the same as before. Substituting Equation 23 into Equation 15, summing with respect to the six groups of fuel channels, and putting in the numbers, one has,

$$G = 414 \sum_{n=1}^{n=6} \frac{N_n}{a_n} \int_0^{\text{Arctan}(L/2x)} \cos(2\pi z/3L) \exp \left[-\bar{\mu}_{e,m} a \sec \theta \right] d\theta \quad 24$$

But, from Fig. 1, we see that $z = a \tan \theta$. For the first set of channels, $N_n = 4$, $a_n = \sqrt{2}/2$, $L/2a = 21.2$, $\arctan L/2a = 87.3^\circ$, $2\pi a/3L = 0.0494$, $\bar{\mu}_{e,m} a = 0.990$. Using these numbers, Table II was computed.

TABLE II

(1) θ°	(2) 0.0494 Tan θ	(3) 0.990 Sec θ	(4) $\exp-(3)$	(5) Cos (2)	(6) (4) X (5)
0	0	0.990	0.371	1.000	0.371
10	0.00869	1.005	0.365	1.000	0.365
20	0.0180	1.052	0.348	1.000	0.348
30	0.0285	1.14	0.319	1.000	0.319
40	0.0415	1.29	0.275	0.999	0.272
50	0.0586	1.54	0.215	0.998	0.214
60	0.0853	1.98	0.137	0.996	0.136
70	0.135	2.98	0.056	0.991	0.055
80	0.278	5.70	0.003	0.961	0.003
85	0.565	11.32	0.00002	0.844	0.000

Graphical integration of the function tabulated in column six versus the angle θ in radians yields the value 2.72. Accordingly, the first term of the sum in Equation 24 is $4 \times 2.72 / (\sqrt{2}/2) = 1.87$ which agrees with the first value tabulated in the last column of Table I. Thus no significant improvement was obtained in this case by taking into account the axial variation of the flux. An examination of the fifth column of Table II shows that diminution in the source strength becomes appreciable at angles below 60 degrees. But the corresponding multiplier ($\exp[-b \text{Sec } \theta]$) in the fourth column is so small that the significant figures in the product (column 6) are not affected by the diminution. A similar calculation for the second set of channels at $x = \sqrt{10}/2$ again shows no effect on the significant figures of the numbers in Table II.

where $\rho = \rho_f + \rho_m$

26

By trigonometry

$$\rho_m = \frac{a \cos \xi - \sqrt{a^2 \cos^2 \xi - (a^2 - R^2)}}{\cos \theta} \quad 27$$

and $dV = \rho \cos \xi d\xi d\theta d\rho$ 28

Using these relations, Taylor and Obenshain were able to reduce the triple integral to a single integral whose integrand is readily evaluated numerically. Their results are here presented in modified form.

$$S_G = \frac{\phi_n \sum_f N_f \bar{\mu}_{e,m} \bar{E}_f}{\pi \bar{\mu}_{e,f}} F \left[\bar{\mu}_{e,f} a ; \bar{\mu}_{e,m} a ; R/a \right] \quad 29$$

Values of $F \left[\bar{\mu}_{e,f} a ; \bar{\mu}_{e,m} a ; R/a \right]$ are tabulated and graphed in the cited reference.

The method of Taylor and Obenshain will be applied to the case studied in the previous example. Taking the fuel rod radius R as $1/4$ ft., $\bar{\mu}_{e,f}$ for uranium to be 31.3 ft^{-1} , and the distance "a" to be $\sqrt{2}/2$ ft., we find, using previous calculations, that $\phi_n \sum_f = 3.06 \times 10^{20}$ fissions/hr.ft³; $N_f \bar{E}_f = 11.4 \times 10^{-16}$ BTU/fission, $\bar{\mu}_{e,f} a = 22.1$, $\bar{\mu}_{e,m} a = 0.99$, and $R/a = 0.06$. Since F is not tabulated for $\bar{\mu}_{e,f} a$ greater than five we can conclude only that $F > 8 \times 10^{-3}$. Substituting these numbers into Equation 29 gives $G \geq 40$ BTU/hr.ft³ for the contribution to the heating at P from one fuel channel. For comparison, we take the first number from the last column of Table I, divide by 4 and multiply by 2×424 (the fuel extends in both directions from the midplane) and obtain

198 BTU/hr.ft³ from the heating when self-absorption is neglected. The difference is of the order of a factor of five, and we conclude that self shielding in the present system may be important.

We can get a better estimate of the self shielding effect by using some other calculations of Taylor and Obenshain for the case of a cylindrical source behind a slab shield, as shown below in Fig. 6-a.

Fig. 6-a

Fig. 6-b

Fig. 6-b is for the case in hand; it is seen that these two cases do not differ appreciably, especially since the absorption coefficient of the fuel is so much larger than that of the moderator. While the curves presented by Taylor and Obenshain may be used, it is more convenient to use their results in the form presented by Foderaro and Obenshain (2). The latter authors correlate the calculated results in terms of an equivalent line source located within the fuel, as shown in Fig. 7.

Fig. 7

The heating at P is given by the following relation, modified from Foderaro and Obenshain:

$$\delta G = \frac{\phi_n \sum_f N_f \bar{E}_\gamma \bar{\mu}_{e,m} R^2}{4 (a - R + Z)} \left[F(\theta_1, b) + F(\theta_2, b) \right] \quad 30$$

when F is the function graphed by Foderaro and Obenshain (2). 31

$$b = \bar{\mu}_{e,f} R + \bar{\mu}_{e,m} (a - R)$$

When $a/R \geq 10$, $\bar{\mu}_{e,f} Z$ is a function of $\bar{\mu}_{e,m} R$ and when $a/R < 10$, $\bar{\mu}_{e,f} Z/m = f(a/R, \bar{\mu}_{e,f} R)$ and $m = f(\bar{\mu}_{e,f} a, a/R)$. All functions are graphed in the cited reference. For the case in hand $R = 1/24$ ft., $a = 0.707$ ft., $a/R = 17$,

$\bar{\mu}_{e,f} R = 1.30$. Hence, $\bar{\mu}_{e,f} Z = 0.63$, from which $Z = 0.023$ ft., or about one half of the radius. It follows that $(a - R + Z) = 0.688$ ft., $b = 1.56$, $\theta_1 = \theta_2 = 90^\circ$. From (2), $F(90^\circ, 1.56) = 0.162$. Substituting into Equation 30, δG is found to be about 100 BTU/hr.ft.^3 , which is to be compared to the value of 198 BTU/hr.ft.^3 obtained by treating the fuel rod as a line source and neglecting self absorption. Thus about half of the energy of the fission gammas is absorbed within the fuel.

Taylor and Obenshain, by comparing their results to those of an earlier study (3), conclude that the case of finite channel length may be treated satisfactorily, provided $\bar{\mu}_{e,m}(a-R) \ll 1$, and $R/a \leq 1/2$, by multiplying the right side of Equations 29 and 30 by the ratio $F(\theta, b)/F(\frac{\pi}{2}, b)$, where $\theta = \arctan(L/2R)$. In the present instance, the correction for the finiteness of length of the fuel channel is negligible.

So much for the case of uniform distribution of gamma sources. Actually, the neutron flux and the prompt gamma source vary both radially and axially in fuel slugs. Suppose the flux distribution to be given by the product of functions of r and z , viz.:

$$\phi_n(r, z) = \phi_0 f(r) f(z) \quad 32$$

By definition, the average value of the flux over any cross section is represented by

$$\bar{\phi}_r(z) = \phi_0 \overline{f(r)} f(z) \quad 33$$

where the superbar indicates the averaging operation and the subscript indicates the variable with respect to which the averaging was performed. Application of diffusion (4) theory yields the result that

$$f(r) = f(0) I_0(\chi r) \quad 34$$

where I_0 is the modified Bessel function of the first kind of order zero, and $\chi = \sqrt{3\Sigma_a\Sigma_{tr}}$, where Σ_a is the thermal neutron absorption cross section and Σ_{tr} is the transport cross section. χ has a value of about 0.7 cm. in natural uranium; however, a better fit of the experimental data is obtained if a value of about 1 cm. is used. It follows from Equation 34 that

20.
35

$$\phi(r, z) = \frac{\bar{\phi}_r(z) (\mathcal{L} R) I_0 (\mathcal{L} r)}{2 I_1 (\mathcal{L} R)} \quad 36$$

Fig. 8

The ray connecting the elementary volume $r d\theta r dz$ to the point P lies partly within the fuel, partly within the moderator. Denoting the projections on the $r-\theta$ plane of these sub-rays by ρ_f and ρ_m respectively, Equation 14 takes, for the case in hand, the form

$$\delta G = 2 \int_0^{\frac{L}{2}} 2 \int_0^{\pi} \int_0^R \frac{\phi_r(z) (\mathcal{L}D/2) I_0(\mathcal{L}r) \Sigma_f N_s \bar{E}_s \bar{\mu}_{e,m}}{2I_1(\mathcal{L}D/2) 4 (\cos\theta)^2} \exp \left[\frac{-\rho_f \bar{\mu}_{e,f} - \rho_m \bar{\mu}_{e,m}}{\cos\theta} \right] r d\theta r dz \quad 38$$

where R is the radius of the fuel slugs, L the length of the channel and $\rho = \rho_f + \rho_m$. It is assumed that the axial distribution of the flux $\phi_r(z)$ is symmetrical about the midplane. The influence of the radial variation of ϕ on the gamma heating at P will be investigated by setting $\theta = 0$ and evaluating the double integral with respect to r and ψ for two values of $\mathcal{L}$, namely 1.00 cm and 0.00 cm. The latter value of $\mathcal{L}$ corresponds to the case of uniform radial distribution of the flux.

$$\text{Int. (1)} = 2 \int_0^{\pi} \int_0^R \frac{(\mathcal{L}D/2) I_0(\mathcal{L}r)}{2I_1(\mathcal{L}D/2)^2} \exp \left[\frac{-\rho_f \bar{\mu}_{e,f} - \rho_m \bar{\mu}_{e,m}}{1} \right] r d\psi r \quad 39$$

Now $\frac{(\mathcal{L}D/2) I_0(\mathcal{L}r)}{2I_1(\mathcal{L}D/2)}$ approaches 1.00 as $\mathcal{L}$ approaches zero, hence

$$\text{Int. (2)} = 2 \int_0^{\pi} \int_0^R \frac{\exp \left[\frac{-\rho_f \bar{\mu}_{e,f} - \rho_m \bar{\mu}_{e,m}}{\rho^2} \right]}{\rho^2} r d\psi r \quad 40$$

Let the fuel rod diameter be one inch as before, and take the distance "a" as 1.0 inch. By numerical integration, $\text{Int. (1)} = 0.422$ and $\text{Int. (2)} = 0.406$.

Although the ratio of surface to average flux level over the section was 1.20, the gamma heating at P, due to gammas originating in the x-y plane, is increased merely by a factor of $0.422/0.406 = 1.04$. This difference is probably smaller than the errors introduced by use of the integral-spectrum, straight-ahead-scattering approximation itself, and it appears that it is probably not worth while to take the non-uniformity of the radial flux distribution in fuel rods into account in computing gamma heating in the moderator.

On the other hand, the self-shielding in the rod is quite pronounced. Treating the fuel rod as a line source by setting $dV = \pi R^2 dz$ in Equation 14, and comparing with Equations 38 and 39, it follows that,

$$\frac{\pi R^2 P_{\Omega}(0) \exp[-a \bar{\mu}_{e,m}]}{a^2} = \frac{\text{Int. (1)}}{4}$$

Introducing the value of Int. (1) computed above, it is found that $P_{\Omega}(0) = 0.0461$, whereas for an isotropic line source (as used in Eq. 15 et seq.), $P_{\Omega}(0) = 1/4\pi = 0.0795$. Thus the heating at P, due to gammas originating in the x-y plane, is reduced about one-half by absorptions of gammas in the fuel itself, in agreement with the results obtained in the previous example.

We now consider a method of treating the case where the gamma source strength varies in the axial direction. The effect of the variation will not be important at low values of the ratio a/R , except near the ends of fuel rods. For that case, numerical integration of Equation 38 with kappa taken as zero is recommended. For the case where a/R is large (greater than 10), the following procedure has been developed. The heating at P (Fig. 9) is given by the integral,

$$\delta G \approx \int_{z_1}^{z_2} \int_{y=-R}^{y=+R} \int_{x=-\sqrt{R^2-y^2}}^{x=+\sqrt{R^2-y^2}} \frac{\bar{\phi}_r(z) \sum_f \bar{N}_f \bar{E}_f \bar{\mu}_{e,m}}{4 \rho^2 \pi} dx dy dz \exp[-\bar{\mu}_{e,f} \rho_f - \bar{\mu}_{e,m} \rho_m] \quad 41$$

Since a/R is large, the projection onto the x-y plane of the ray from any point within the fuel rod will be nearly parallel to the x-axis.

From the figure we see that

$$R_f = (x_R - x) / \cos \theta$$

$$x_R = \sqrt{R^2 - y^2} = \sqrt{R^2 - y^2}$$

$$\rho \cong \frac{a}{\cos \theta} \cong \rho_m$$

Using these relations, and rearranging Eq. 41 gives

$$\delta G = \int_{z_1}^{z_2} \frac{\phi_r(z) \sum_f N_f \bar{E}_f \bar{\mu}_{e,m} \exp[-a/\cos \theta] dz}{4\pi (a/\cos \theta)^2} \times$$

$$\int_{y=-R}^{y=+R} \int_{x=-\sqrt{R^2-y^2}}^{x=+\sqrt{R^2-y^2}} \exp\left[-\frac{\bar{\mu}_{e,f} \sqrt{R^2-y^2}}{\cos \theta}\right] dy \exp\left[\frac{\bar{\mu}_{e,f} x}{\cos \theta}\right] dx \quad 42$$

The double integral in x and y may be expressed in terms of known functions.

$$\iint_{x,y} = \frac{\pi R}{\bar{\mu}_{e,f}} \left[I_1\left(\frac{(2\bar{\mu}_{e,f} R)}{(\cos \theta)}\right) + H_1\left(\frac{(i2\bar{\mu}_{e,f} R)}{(\cos \theta)}\right) \right]$$

where I_1 is the modified Bessel function of the first kind or order one, and H_1 is Struve's function (5, p. 167, Eq. 146 and p. 68, Eq. 41) of order one with imaginary argument, which does not seem to have been tabulated anywhere.

Substituting into Equation 42

$$\delta G \cong \int_{z_1}^{z_2} \frac{\phi_r(z) \sum_f N_f \bar{E}_f \bar{\mu}_{e,m} R}{4\bar{\mu}_{e,f}} \left[I_1\left(\frac{(2\bar{\mu}_{e,f} R)}{(\cos \theta)}\right) + H_1\left(\frac{(i2\bar{\mu}_{e,f} R)}{(\cos \theta)}\right) \right] \exp[-a/\cos \theta] dz \quad 43$$

where

$$\theta = \text{Arctan}(|z|/a)$$

Struve's function of order one with imaginary argument may be expressed in series form (5).

$$-H_1(ix) = \frac{2}{\pi} \left[\frac{x^2}{1^2 \cdot 3} + \frac{x^4}{1^2 \cdot 3^2 \cdot 5} + \frac{x^6}{1^2 \cdot 3^2 \cdot 5^2 \cdot 7} + \dots \right] \quad 44$$

It will be instructive to derive from Equation 45 a formulation of $P_{\Omega}(\alpha, \beta, \gamma)$ appearing in Equation 14. If we set $dV = \pi R^2 dz$ for an equivalent line source, and then compare Equations 14 and 43, it follows that

$$P_{\Omega}(\theta) = \frac{I_1\left(\frac{2\bar{\mu}_{e,f}R}{(\cos \theta)}\right) + H_1\left(\frac{i2\bar{\mu}_{e,f}R}{(\cos \theta)}\right)}{4\pi R\bar{\mu}_{e,f}} \quad 45$$

For the case we have been studying, $R = 1/2$ inch, $\bar{\mu}_{e,f} = 2.61 \text{ in.}^{-1}$ and $2\bar{\mu}_{e,f}R = 2.61$. By taking the first five terms of Equation 47, $H_1(2.61 i) = -2.245$. Now $I_1(2.61) = 2.780$. On substituting into Equation 48, it is found that $P_{\Omega}(0) = 0.0326$, which is to be compared with $1/4 = 0.0795$ for an unshielded, isotropic line source.

Note that the value 0.0326 obtained by this method is less than the value of 0.0461 obtained from Equation 41 for the case where $a/R = 0.5$. In that case the path lengths inside the fuel are all longer than those in the present case, and the shielding would be expected to be correspondingly greater; however, this effect is more than offset by the fact that, when $a/R = 0.5$, a considerable fraction of the gamma sources are appreciably closer to P than implied by the use of an equivalent line source located at the center of the fuel and thus the "geometric" attenuation ($1/a^2$) is overestimated and more than compensates for the increased path lengths.

Slab Sources

Gamma Heating At External Points

While Equation 14 may be applied to cylindrical sources, it is more convenient to treat slabs in terms of a current of photons with appropriate angular distribution at the interface between the source and the medium, as shown in Fig. 10.

Fig. 10

Consider the photons in a current passing through an element of surface area $dy dz$ of a slab source at the point $(0, y, z)$ whose flight paths lie in the element of solid angle $d\Omega$ inclined at an angle θ to the normal to the surface of the slab. Under the straight-ahead scattering approximation, only those photons originating in the projection of $d\Omega$ into the source volume can have flight paths in $d\Omega$ outside the slab. Now referring to Fig. 10

$$dV = \rho_f^2 d\Omega dy dz$$

Since we may make $d\Omega$ as small as desired, we may treat dV as a point source with respect to the element of surface $dy dz$. This element of surface subtends a solid angle of magnitude $\cos \theta dy dz / \rho_f^2$ with respect to dV . The energy current in appropriate units (e.g., mev. per cm.² per sec. per steradian) is

now readily formulated:

$$I(\theta) = \frac{\int_{\rho_f=0}^{\rho_f=\rho_f^{(max)}} \frac{\phi_n(x) \sum_f N_f \bar{E}_f \rho_f^2 d\rho_f}{4\pi} \exp\left[-\bar{\mu}_{e,f} \rho_f\right] \frac{\cos \theta dy dz}{\rho_f^2}}{d\Omega dy dz}$$

46

where the neutron flux level ϕ_n is presumed to vary only with x .

For the case where ϕ_n is also uniform in x , Equation 46 is readily integrated,

$$I(\theta) = \frac{\phi_n \sum_f N_f \bar{E}_f \cos \theta}{4\pi \bar{\mu}_{e,f}} \left[1 - \exp\left[-\bar{\mu}_{e,f} \rho_f^{(max)}\right] \right] \quad 47$$

For thick sources, the exponential term becomes negligible and

$$I(\theta) = \frac{\phi_n \sum_f N_f \bar{E}_f \cos \theta}{4\pi \bar{\mu}_{e,f}} \quad 48$$

For thin slabs of thickness b , $\exp\left[-\bar{\mu}_{e,f} \rho_f^{(max)}\right] \cong 1 - \left[\frac{\bar{\mu}_{e,f} b}{\cos \theta}\right]$, and

$$I(\theta) \cong \frac{\phi_n \sum_f N_f \bar{E}_f b}{4\pi} \quad 49$$

as expected. For slabs of intermediate thickness, the next higher term in the expansion of the exponential may be retained, giving

$$I(\theta) = \frac{\phi_n \sum_f N_f \bar{E}_f b}{4\pi} \left[1 - \frac{1}{2} \left(\bar{\mu}_{e,f} b / \cos \theta \right)^2 \right] \quad 50$$

Another case of interest is that of an infinitely thick source with the neutron flux increasing linearly away from the surface.

Figure 11

$$\phi_n(x) = \phi_o(1 - cx)$$

Putting this into Equation 45, setting $x = \rho_f \cos \theta$, and taking the upper limit of integration at infinity, one obtains

$$I(\theta) = \frac{\phi_o \sum_f N_f \bar{E}_f \cos \theta}{4\pi \bar{\mu}_{e,f}} \left[1 - \frac{c \cos \theta}{\bar{\mu}_{e,f}} \right] \quad 51$$

The above cases will be sufficient for the treatment of most problems of interest.

The gamma heating at a point P located in the moderator at a distance "a" from the interface of a slab of thickness "b" containing gamma sources uniformly distributed can now be formulated by reference to Fig. 12 and Equation 47.

Fig.12

$$\delta G = \int_{\psi=0}^{\psi=2\pi} \int_{r=0}^{r=\infty} \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m} \cos \theta}{4\pi \bar{\mu}_{e,f}} \left[1 - \exp(-\bar{\mu}_{e,f} \rho) \exp(-\bar{\mu}_{e,m} \rho_m) \right] \frac{r dr d\psi}{\rho_m^2} \quad 52$$

But

$$\begin{aligned} \cos \theta &= a/\rho_m \\ \rho_f(\max) &= b/\cos \theta = b \rho_m/a \\ r dr &= \rho_m d\rho_m \end{aligned}$$

Making these substitutions, changing the limits of integration to correspond to the new variable of integration and integrating with respect to ψ gives

$$\delta G = \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,f}} \int_{\rho_m=a}^{\rho_m=\infty} \left\{ 1 - \exp\left[-b \rho_m \bar{\mu}_{e,f}/a\right] \right\} \exp\left[-\bar{\mu}_{e,m} \rho_m\right] \frac{d\rho_m}{\rho_m^2}$$

Integration with respect to ρ_m gives

$$\delta G = \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,f}} \left[E_2(\bar{\mu}_{e,m} a) - E_2(\bar{\mu}_{e,m} a + \bar{\mu}_{e,f} b) \right] \quad 53$$

where E_2 denotes the "exponential integral" of order two tabulated by Placzek (6)

by and Foderaro and Obenshain (2). The functions of order two may be reduced by means of the recurrence formula

$$E_n(x) = \frac{1}{n-1} \left[e^{-x} - x E_{n-1}(x) \right] \quad ; \quad n > 1 \quad 54$$

Applying this to Equation 53 gives

$$\begin{aligned} \delta G = \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,f}} & \left[(\bar{\mu}_{e,m}^a + \bar{\mu}_{e,f}^b) E_1(\bar{\mu}_{e,m}^a + \bar{\mu}_{e,f}^b) - (\bar{\mu}_{e,m}^a) E_1(\bar{\mu}_{e,m}^a) \right. \\ & \left. + \exp[-\bar{\mu}_{e,m}^a] - \exp[-(\bar{\mu}_{e,m}^a + \bar{\mu}_{e,f}^b)] \right] \quad 55 \end{aligned}$$

When $\bar{\mu}_{e,f}^b$ is small compared to $\bar{\mu}_{e,m}^a$, Equation 52 is conveniently evaluated by expanding the exponential in $\bar{\mu}_{e,f}^b \rho_m^b / a$, dropping terms of order higher than three, and integrating the result term by term.

$$\delta G \approx \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}^b}{2} \left[E_1(\bar{\mu}_{e,m}^a) - (\bar{\mu}_{e,f}^b / 2) E_0(\bar{\mu}_{e,m}^a) \right] \quad 56$$

E_1 and E_0 are also tabulated by Placzek. However, $E_0(x) = e^{-x}/x$.

The extension of the above equations to the case where the fuel is covered by a non-radiating clad and several fuel plates lie between the point P and the plate whose contribution to the heating at P it is desired to compute is readily treated by modifying the second exponential term in Equation 54.

Fig. 13

The total attenuation path length outside the fuel zone is made up of segments the lengths of which are $a_n / \cos \psi = a_n \rho / a$. Accordingly, the product $\bar{\mu}_{e,m} \rho_m$ in the second exponential term in Equation 52 is to be replaced by the sum

$\sum_n \bar{\mu}_{e,n} a_n \rho / a$. The result is

$$\delta G = \frac{\phi_n \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,f}} \left[E_2(\bar{\mu}_{e,n} a_n) - E_2\left(\sum_n \bar{\mu}_{e,n} a_n + \bar{\mu}_{e,f} b\right) \right] \quad 57$$

where $\bar{\mu}_{e,n}$ represents the mean energy absorption coefficient for the nth region.

^{Other}
An important case is that of estimating the "core gamma" heating in a pressure shell or thermal shield. It is supposed that the system is so large that the curvature of the interface between shield and fuel zone may be neglected, and that the finiteness of the system may also be neglected. The neutron flux is assumed to increase linearly away from the interface, so that Equation 50 applies. Using Equation 51, Equation 52 becomes,

$$\delta G = \frac{\phi_o \sum_f N_f \bar{E}_f \bar{\mu}_{e,m}}{2 \bar{\mu}_{e,f}} \left[E_2(\bar{\mu}_{e,m} a) - \frac{c}{\bar{\mu}_{e,f}} E_3(\bar{\mu}_{e,m} a) \right] \quad 58$$

Gamma Heating at Internal Points

While the "core" gamma heating computed in the previous section may be significant, Chapman (7) has shown that the major contribution to the heating in pressure shells and thermal shields is made by "capture" gammas resulting from the absorption of thermal neutrons in the shell or shield.

The integral in ρ may be expressed in terms of the exponential integral E_1 .

$$\delta G = \frac{\mathcal{H} J N_r \bar{E}_r \bar{\mu}_e}{2} \int_{x=0}^{x=b} \exp(-\mathcal{H} x) E_1 \left[\frac{(a-x) \bar{\mu}_e}{\bar{\mu}_e} \right] dx \quad 63$$

The integral in x may be transformed into double exponential integrals discussed by Foderaro and Obenshain (2) as follows. In the interval where $x < a$, let $(a-x) \bar{\mu}_e = Z_1$ so that $x = a - Z_1 / \bar{\mu}_e$, $dx = -dZ_1 / \bar{\mu}_e$, $Z_1 = a \bar{\mu}_e$ when $x = 0$, $Z_1 = 0$ when $x = a$. In the interval where $x > a$, let $(x-a) \bar{\mu}_e = Z_2$, etc. Introducing the new variables, Equation 63 becomes

$$\begin{aligned} \delta G = & \frac{\mathcal{H} J N_r \bar{E}_r \bar{\mu}_e}{2} \int_{Z_1 = \bar{\mu}_e a}^{Z_1 = 0} \exp \left[-(a - Z_1 / \bar{\mu}_e) \right] E_1(Z_1) (-dZ_1 / \bar{\mu}_e) \\ & + \frac{J N_r \bar{E}_r \bar{\mu}_e}{2} \int_{Z_2 = 0}^{Z_2 = (b-a) \bar{\mu}_e} \exp \left[-(a + Z_2 / \bar{\mu}_e) \right] E_1(Z_2) (dZ_2 / \bar{\mu}_e) \quad 64 \end{aligned}$$

which, in terms of F-integrals, becomes

$$G = \frac{\mathcal{H} J N_r \bar{E}_r}{2} \exp(-\mathcal{H} a) \left\{ F_1 \left[\bar{\mu}_e a; \mathcal{H} / \bar{\mu}_e \right] + F_1 \left[\bar{\mu}_e (b-a); -\mathcal{H} / \bar{\mu}_e \right] \right\} \quad 65$$

The F-integrals have been graphed by Foderaro and Obenshain⁽²⁾ for values of the first argument ranging from zero to 14 and the second from -4 to 4.

Concluding Remarks

The above examples illustrate the application of the straight-ahead-scattering, integral-spectrum method to the estimation of gamma heating in nuclear reactors for cases of relatively simple geometry and source distribution. In principle, any case howsoever complicated may be treated by numerical evaluation of Equation 14. However, the computational labor may become excessive. Several cases of somewhat greater complexity than those considered here have been treated by Foderaro and Obenshain (2) from the standpoint of shielding calculations; their results may be readily modified for heating calculations.

REFERENCES

1. Alexander, L. G., "The Gamma Energy-Absorption Coefficient", ORNL-CF-56-8-219
2. Foderaro, A., and Obenshain, F., "Fluxes from Regular Geometric Sources", WAPD-TN-508 (55)
3. Project Handbook, Vol. III, CL-697 (45)
4. Alexander, L. G., "Effect of Non-Uniform Neutron Flux Distribution on the Temperature Distribution in Thick Fuel Elements", ORNL-CF-55-10-118 (1955)
5. MacLachlan, N. W., "Bessel Functions for Engineers", Oxford University Press, London (1934)
6. Placzek, G., "The functions $E_n(x)$ ", MT-1 (1946)
7. Chapman, R. H., "Analysis of a Spherical Pressure Vessel Having an Energy Source within the Wall", ORNL-CF-54-10-76 (1954)
8. Hurwitz, H., Jr. et al, KAPL-753 (52) Classified
9. Gamble, R., and Francis, J. E., "Prompt Fission Gamma Rays", ORNL Physics Division Semiannual Progress Report, ORNL-1879, p. 21 (1955)

Symbols

- a Distance from origin of coordinates and point P where it is desired to compute the gamma heating rate.
- b Thickness of slab source of gamma radiation, also parameter in Eq. 17.
- c Slope of neutron flux distribution in slab source of gamma radiation, Figure 9.
- (C.F.) Conversion Factor: 1.60×10^{-19} Mw. sec/mev
- E_0, E_1, E_2, E_n Exponential integrals, Equation 54.
- E_f Energy of fission, 200 mev./fission
- E_γ Energy of gamma photons, mev/photon
- $\bar{E}_\gamma$ Energy of gamma photons averaged over the spectrum, mev/photon, Equation 3.
- $F(\theta, b)$ Incomplete exponential integrals, Equation 18.
- F_1 Double exponential integral of order one, Equation 65.
- f Fraction of length L of fuel channel lying above origin of coordinates, Equation 15.
- $f(r)$ Function describing radial distribution of neutron flux distribution in fuel slug.
- $f(z)$ Function describing axial distribution of neutron flux distribution in fuel slug.
- $f_t(E_\gamma, n)$ Average fraction of their energy, E_γ , deposited locally as heat in collision of all kinds by gamma photons per collision.
- G Local heat release rate in nuclear reactors, mev/cc.sec. or BTU/hr.ft.³
- δG Contribution to local heat release rate made by one fuel channel or element.
- H_1 Struve's function of order 1, Equation 44

$I(\theta)$	Distributed gamma current at surface of slab source, photons/ft. ² sec. steradian
I_0, I_1	Modified Bessel functions of the first kind.
J	Net current of neutrons at surface of a slab, neutron/cm. ² sec or neutron/ft. ² sec.
L	Length of fuel channel, ft.
M	Medium
N_f	Total number of fuel channels
N_n	Number of fuel channels of nth type.
N_γ	Number of gamma photons emitted per fission or per neutron capture.
n	Number of gamma photons in a beam passing a given point, photons/sec.
P	Point at which it is desired to estimate the gamma heating rate.
$P_E(E_\gamma, \vec{\rho})$	Spectral distribution of beam of gamma photons at point $\vec{\rho}$, fraction of photons at $\vec{\rho}$ having energy E_γ per mev about E_γ .
$P_\Omega(\alpha, \beta, \gamma)$	Angular distribution of photon source, probability that photon will be emitted along line having direction cosines α, β, γ per unit solid angle about that line.
Q	Total reactor heat output, mw.
R	Radius of fuel rods or slugs.
r	Radial coordinate in polar system.
S_γ	Strength of volume gamma source, photons/sec.cc.
V	Volume of source, cm ³
x	Cartesian coordinate, normal to slab or rod axis
y	Cartesian coordinate, parallel to slab surface.
z	Cartesian coordinate parallel to slab surface, parallel to rod axis.

Z Distance from surface of rod gamma source at which line source will be equivalent to the rod source, also variable of integration in Equation 64.

Greek

α, β, γ Direction cosines of line in space

κ Kappa, $\kappa = \sqrt{3 \Sigma_a \Sigma_{tr}}$

$\mu_t(E_\gamma, n)$ Gamma-photon absorption coefficient characteristic of medium n and photon energy E_γ , probability of collision of all kinds per cm. of travel.

$\mu_e(E_\gamma, n)$ Gamma-energy absorption coefficient characteristic of medium n and photon energy E_γ , Equation 6.

$\bar{\mu}_e(\vec{p})$ Energy absorption coefficient averaged over gamma spectrum at point $\vec{p}$, Equation 11.

$\bar{\mu}_{e,f}, \bar{\mu}_{e,m}, \bar{\mu}_{e,n}$ Gamma energy absorption coefficient characteristic of fuel, moderator, and medium n respectively, all averaged with respect to the spectrum of the gamma source, Equation 11.

ρ Radial coordinate in spherical system

ρ_f Radiation path length in fuel zone

ρ_m Radiation path length in moderator zone.

$\vec{p}$ Vector coordinate, $\vec{p} = \vec{p}(\rho, \alpha, \beta, \gamma)$

$\vec{p}_s$ Vector coordinates of element of gamma source volume.

$\Sigma_f, \Sigma_a, \Sigma_{tr}$ Macroscopic neutron cross sections for fission, absorption, and transport.

θ Angle between radiation path and the x-axis, or to normal to surface of rod.

ϕ_n Neutron flux, neutron-cm./cm³.sec.

ϕ_{av} Neutron flux level averaged over whole of reactor.

ϕ_m	Maximum neutron flux in reactor
ϕ_0	Neutron flux level at origin of coordinates.
$\bar{\phi}_r(z)$	Neutron flux level at coordinate z in fuel rod averaged over the cross section.
ϕ	Azimuth angle in spherical coordinate system, Fig. 6
ψ	Angular coordinate in polar system, radians
Ω	Solid angle, steradians

1-39

Handwritten notes in the right margin, including the word "SOME" and other illegible characters.

1/42