HIC-1: A FIRST APPROACH TO THE CALCULATION OF HEAVY-ION REACTIONS AT ENERGIES ≥ 50 MeV/NUCLEON

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENT	1
ABSTRACT	3
INTRODUCTION	5
THE CALCULATION	7
Nuclear Properties	7
General Dynamical Features	9
Computational Sequence	11
DISCUSSION	21
COMPARISONS WITH EXPERIMENTAL DATA	25
SUMMARY	37
APPENDIX A	39
APPENDIX B	47
APPENDIX C	49
REFERENCES	54

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Abstract

A preliminary version of a heavy-ion-interaction model is described in detail. The basic assumption of the model is that heavy-ion reactions can be described by the interactions of the individual nucleons of the projectile and target with one another. Transitions to the eigenstates of the target and projectile are ignored. Encouraging results are obtained when comparisons with experimental data are made on an absolute basis, even though there are differences of factors of three or more in these comparisons for some of the data. Several ways of improving the model and extending its applicability to lower energies are also presented.

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INTRODUCTION

Continuum-state transitions between projectile and target in heavy-ion reactions have been calculated under the assumption that the reaction can be represented as the interaction of two Fermi gasses that pass through each other. The reaction is envisioned to take place as follows: during the passage of the projectile through the target, those individual nucleons of the projectile that are in the region of overlap undergo quasi-free reactions with the nucleons of the target. A cascade, i.e., a progressively increasing number of collisions among the individual nucleons, is thereby generated in both the target and projectile. This cascade takes place simultaneously in the target and in the projectile during the time that the projectile is passing through the target. The nucleons that have been jarred loose from their bound states in the projectile and in the target and that have also managed to survive capture during the development of the cascade escape from the projectile and the target. They are emitted as free nucleons in various directions and with a variety of energies. When the projectile energy is sufficiently high, pions will be produced during the development of the cascade, and they will also be emitted in a variety of directions and with diverse energies if they escape. The remaining fragments of the projectile and of the target move off in highly excited states emitting evaporation particles, and ultimately gamma rays, until this excitation energy is lost, at which time the reaction is completed. The calculation is carried out up to the point of gamma-ray emission, but it does not include this process.

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THE CALCULATION

The present version of the program, HIC-1 (Heavy Ion Code, version 1), contains calculations that only approximate some of the interactions just described. These will be discussed in the course of the detailed description that follows. Monte Carlo techniques are used throughout.

Nuclear Properties

Before the reaction takes place, both the target and the projectile nuclei are given properties that are described in detail elsewhere.¹ A description of the bulk of these properties is given here for purposes of completion.

The nuclei are made to have density distributions that approximate the Fermi-type charge-distribution function given by Hofstadter.² Each nucleus has a high-density spherical central core surrounded by two annular regions of diminishing densities, which thereby accounts for the diffuse edge of the nucleus. Within each region, the density is a constant, but this constant is different for each region. The region boundaries apply to both the neutrons and the protons that make up the nucleus. The relative magnitudes of the density in these regions are made to conform to the relative magnitudes of the volume-averaged value of the Hofstadter function over these regions. The ratio of the neutron-to-proton density is a constant for all regions and is equal to the ratio of neutrons to protons in the nucleus. The absolute values of the neutron and proton densities are determined by normalizing to the total number of neutrons and protons in the nucleus.

The radius of the outermost region for nuclei whose mass numbers are > 27 is determined by the value of the radius at which the density is 1% of that of the Hofstadter function at zero radius. For mass numbers of 27 or

less, 4% instead of 1% is used. This difference is discussed in more detail in the section on comparisons with experimental data. The radius of the outermost region determines the geometric cross section of the nucleus which is the essential peg upon which all of the absolute values of the results from the calculation are based. Such a method gives varying values of r_0 when utilizing the usual method of calculating the nuclear radius r ; i.e., $r = r_0 A^{1/3}$. For ^{12}C our radii give an $r_0 = 1.8 \text{ F}$, while for ^{208}Pb it is 1.5 F . However, in conjunction with the intranuclear-cascade approach, this method has led to predicted reaction cross sections for nucleons on nuclei that represent the experimental data quite well when the 1% value is used for all nuclei.^{1,3,4} Reducing the radii of the lighter elements, i.e., using the 4% value, is consistent with these comparisons since the calculated values were generally greater than the experimental data by about 15% for the light elements.^{1,4}

In each nucleus the bound nucleons are assumed to have energies corresponding to a zero-temperature Fermi energy distribution. In each region the zero-temperature Fermi energies of the neutrons and protons are determined by their densities.

Attractive single-particle potentials are assumed to exist in each region, one for the protons and one for the neutrons. The strengths of these potentials, i.e., their well depths, are equal to the respective neutron and proton zero-temperature Fermi energies plus 7 MeV. The 7 MeV represents an estimate of the average energy required to remove a few of the least-bound nucleons, and it applies to all regions and to all nuclei. In the central regions of the nucleus, the well depths are in the vicinity of 45 MeV.

General Dynamical Features

The change in the trajectory of the projectile caused by the Coulomb interaction with the target is only crudely approximated. For each initial impact parameter that is selected for the projectile when the target and projectile are at infinity, a distance of closest approach is calculated under the assumption that the heavy ions are point charges acting under the influence of Coulomb forces only. The impact parameter applied in the calculation itself is then set equal to this distance. An approximate calculation, consisting of relativistic expressions for the kinematics and classical expressions for angular momentum, was used to alter the incident trajectory in order to account for Coulomb deflection. The cascade portion of the program was unaltered, but the cascade and evaporation results were rotated to correspond to the deflection. Details are given in Appendix C.

As each nucleon of the projectile, or any cascade particle, crosses a region boundary of the target, it gains or loses a kinetic energy equal to the potential energy change from the old to the new region. Refraction or reflection at the region boundaries is not included.

Depletion of the local density as cascades develop within the nuclei is not yet included. This is considered to be one of the significant deficiencies of the present model.

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Computational Sequence

1. The nuclear properties, i.e., the region boundaries, densities, and well depths, are calculated for the projectile and for the target.
2. A center-to-center impact parameter is selected; it is chosen from a uniform distribution over the area of a circle whose radius is the sum of the target and projectile radii. A calculation is carried out to determine the distance of closest approach, described previously, and this distance is then used as the impact parameter of the reaction.
3. Position coordinates with respect to the center of the projectile are chosen from spherically symmetric distributions in ϕ and $\cos \theta$ and are assigned to each nucleon in the projectile. The number of protons assigned to region i of the nucleus is selected from the distribution

$$N_i / \left(\sum_{i=1}^3 N_i \right),$$

where N_i is the number of protons in region i calculated from the product of the proton density and volume of the region. The number of neutrons assigned to the region is similarly selected. Given that a nucleon is assigned to region i , the radial coordinate r of that nucleon is selected from the distribution defined by the solution to the equation

$$R = \frac{\int_{r_{i-1}}^{r_i} 4\pi r^2 dr}{\int_{r_{i-1}}^{r_i} 4\pi r^2 dr}, \quad i = 1, 2, 3,$$

where R is a random number chosen uniformly over the interval of 0 to 1 and r_i is the outer boundary of the region. Note that $r_0 = 0$. This distribution assigns equal numbers of particles to equal volumes of the region.

4. An "internal" energy is given to each nucleon of the projectile. This energy represents that due to its motion inside the projectile only and is selected from the appropriate Fermi energy distribution. It is used in the calculation of the binding energy, but it does not alter the direction of the projectile or the incident kinetic energy per nucleon of the projectile.

5. For each nucleon of the projectile, a calculation is made to determine if it will impinge upon the target or miss completely. It is assumed here that the target is at rest with the coordinate axis of the reaction located at its center. Each nucleon of the projectile is assumed to be moving in the positive z direction, and the paths of these nucleons are parallel to each other. If the path passes through the target, a standard intranuclear-cascade calculation is performed⁴ (with energy modifications described in step 6) for that nucleon on the target only, and an independent intranuclear-cascade calculation is carried out for each such nucleon of the projectile, ignoring any influence that the first cascade might have on those that follow. Note that at this stage it is assumed that the cascade takes place entirely in the target. Further discussion of this point will be given subsequently.

A record is kept of all of the individual properties of the cascade particles that escape, such as their type, energy, direction cosines, and the point within the nucleus at which they made their last collision. Since the intranuclear-cascade calculation accounts for pion production,⁴ the emission of charged and neutral π -mesons is included in this heavy-ion calculation when the energy is sufficiently high.

Because nuclear depletion is not taken into account, the calculation occasionally yields more nucleons knocked out than are physically possible.

To avoid this, a continuous record is kept of the number of nucleons knocked out as each incident nucleon of the projectile interacts. If and when the sum of the nucleons knocked out exceeds or equals the number that were initially present, the last contribution to this sum is ignored, and the nucleon causing this situation, plus all of the nucleons of the projectile that have not yet been treated as incident nucleons, are made to pass through the target without collision. These nucleons are then considered to be part of the projectile fragment.

6. The kinetic energy given to each incident nucleon of the projectile is the kinetic energy per nucleon of the heavy-ion reaction. To account for the fact that the nucleons of the projectile are bound, and hence not all of their incident total energy is available to the reaction, a quantity $(V_i - \epsilon)$ is subtracted from the available reaction energy. V_i is the potential well depth within the projectile of the region in which the nucleon is located and ϵ is the internal kinetic energy assigned to the nucleon and described in step 4. A diagrammatic representation of this is given in the sketch below.

ϵ_i^f = Fermi energy of region i
for the type of nucleon
considered.

Note that, as defined by the diagram, $(V_i - \epsilon)$ is always a positive quantity. This introduces an overestimate of the binding energy of the projectile because we employ the same unchanged single-particle potential diagram even though many of the bound nucleons may interact. In other words, if one were to calculate the average binding energy per nucleon by this technique, the value would be larger than that measured for most nuclei (~ 8 MeV). This error is not very large in general and is a maximum only when all of the nucleons of the projectile interact. To yield an incident total energy consistent with the relativistic kinematic equations that are employed, this reduction of available energy for each interacting incident nucleon is introduced as a change in the mass of the incident nucleon; i.e.,

$$E = KE + [m - (V_i - \epsilon)] = KE + m'$$

$$p^2 = E^2 - (m')^2 ,$$

where E , KE , m , and p are the total energy, kinetic energy per nucleon, rest mass, and momentum of the incident nucleon, respectively.

7. A record is kept of all of the nucleons from the projectile that miss the target completely and also that pass through the target without colliding. These nucleons make up the fragment that remains of the projectile. By assuming conservation of baryon number and charge, the A and Z of the target fragment are also known.

8. The excitation energy remaining in the projectile fragment is calculated from the number of holes left in the fragment; i.e.,

$$E_{PF}^* = \sum_{j=1}^J (\epsilon_j^f - \epsilon_j) ,$$

where ϵ_j is the internal kinetic energy of the j th nucleon of the projectile

that interacts (and is hence removed from the projectile), ϵ_j^f is the Fermi energy of the region from which this nucleon was removed, and J is the total number of projectile nucleons that have interacted.

The excitation energy remaining in the target fragment is calculated from the conservation of energy; i.e.,

$$E^* = \sum_{j_{\text{inc}}}^{N_{\text{collide}}} \left\{ (KE)_j - (\epsilon_{f,j} - \epsilon_j + 7) - 7(N_{\text{esc}} - 1)_j - \left(\sum_i T_{\text{esc},i} \right)_j - \left(\sum_k m_{\pi,k}^{\text{esc}} \right)_j \right\},$$

where

N_{collide} = the number of incident nucleons for each projectile that collide,

$(KE)_j$ = the kinetic energy (MeV) of each incident nucleon in the lab system,

$\epsilon_{f,j}$ = the Fermi energy within the projectile of the j th incident nucleon that collides,

ϵ_j = the kinetic energy with respect to the projectile of the incident nucleon that collides,

N_{esc} = the number of escaping nucleons for the incident nucleon that collides,

$T_{\text{esc},i}$ = the kinetic energy of each escaping nucleon or pion for the incident nucleon that collides, and

$m_{\pi,k}^{\text{esc}}$ = the rest energy of each pion that escapes.

9. A kinematics calculation is carried out to determine the laboratory vector momenta of the projectile and target fragments. The general equations employed, in which all quantities refer to the laboratory frame, are as follows:

Before the reaction takes place -

$$E_b = (KE)_{P,inc} + A_{P,b}(m_n - 7) + A_{T,b}(m_n - 7)$$

$$\begin{aligned} \bar{p}_b &= \sqrt{[(KE)_{P,inc} + A_{P,b}(m_n - 7)]^2 - [A_{P,b}(m_n - 7)]^2} \\ &= \hat{k} p_{b,z} \end{aligned}$$

where

E_b = the total initial energy of the heavy-ion system,

$(KE)_{P,inc}$ = the total incident kinetic energy of the projectile,

$A_{P,b}$ = the mass number of the projectile before the interaction,

$A_{T,b}$ = the mass number of the target before the interaction,

m_n = the rest mass of a nucleon (taken to be the same for neutrons and protons),

7 = the average binding energy per nucleon (MeV),

$\bar{p}_b$ = the vector momentum of the incident projectile, and

$\hat{k}$ = the unit vector in the positive z direction.

After the cascade reaction has taken place and before evaporation -

$$E_r = E_b - \sum_{i=1}^I [m_n + (KE)_n]_i - \sum_{j=1}^J [m_\pi + (KE)_\pi]_j ,$$

$$\bar{p}_r = \bar{p}_b - \sum_{i=1}^I \bar{p}_{n,i} - \sum_{j=1}^J \bar{p}_{\pi,j} ,$$

where

E_r = the total energy remaining to the target and projectile fragments after the reaction,

I = the total number of escaping cascade nucleons,

$(KE)_n$ = the kinetic energy of these nucleons,

m_n = the nucleon rest mass energy,

J = the total number of escaping cascade pions,

$(KE)_\pi$ = the kinetic energy of these pions,

m_π = the rest mass energy of these pions,

$\bar{p}_r$ = the total momentum remaining to the target and projectile fragments,

$\bar{p}_{n,i}$ = the vector momentum of the i th escaping cascade nucleon, and

$\bar{p}_{\pi,j}$ = the vector momentum of the j th escaping cascade pion.

The equations defining the kinematics of the target and projectile fragments are:

$$E_{T,f} + E_{P,f} = E_r$$

$$\bar{p}_{T,f} + \bar{p}_{P,f} = \bar{p}_r ,$$

where

$$E_{T,f} = \sqrt{(\tilde{p}_{T,f})^2 + (m_{T,f})^2}$$

$$E_{P,f} = \sqrt{(\tilde{p}_{P,f})^2 + (m_{P,f})^2} \quad ,$$

and

$$\begin{aligned} m_{T,f} &= \text{the rest mass energy of the target fragment} \\ &= A_{T,f}(m_n - 7) + E_{T,f}^* \quad , \end{aligned}$$

$$\begin{aligned} m_{P,f} &= \text{the rest mass energy of the projectile fragment} \\ &= A_{P,f}(m_n - 7) + E_{P,f}^* \quad . \end{aligned}$$

These energy momentum equations comprise six equations in eight unknowns. Two of the unknowns are taken to be the angles θ and ϕ . They are defined to be the polar and azimuthal angles of the projectile fragment in a system moving with the vector velocity of the incident projectile. These angles are assumed to be isotropically distributed in this system. Indications from recent high-energy heavy-ion experiments point to this assumption.⁵ The details of the kinematics equations are given in Appendix A.

A calculational difficulty arises from the selection of θ and ϕ . Not all θ and ϕ are available to the system because of the nature of the reaction. This becomes manifest in the kinematics calculation by the appearance of imaginary or negative values for magnitudes of the momenta. New θ and ϕ are selected when this occurs.

10. A standard evaporation calculation⁶ is employed, in the rest system of the fragment, to calculate the vector momenta of the particles that are evaporated from the excited target and projectile fragments. It is assumed that the evaporated particles are emitted isotropically in these rest systems. As each particle is evaporated, the evaporating fragment is allowed

to recoil, which changes the fragments' laboratory momenta. All such changes are taken into account. The vector momenta of the evaporated particles are then transformed back to the laboratory system (see Appendix B for further details). This completes the calculation for one incident projectile.

11. The calculation is repeated starting at step 1 until a sufficient number of incident projectiles has been taken to provide satisfactory statistics for the quantities of interest.

12. An inverse reaction is carried out; i.e., the projectile is assumed to be stationary while it is bombarded by the target, moving in the negative z direction, with a velocity equal to that of the initial incident projectile. The cascades are now assumed to take place in the projectile only (i.e., the roles of target and projectile are reversed) while steps 1 through 11 are carried out.

13. All momenta and energies calculated for these inverse reactions are transformed back to the initial laboratory frame.

This completes the calculation of the heavy-ion reaction. There are two computer programs that carry out this calculation. The first is used to calculate the cascade phase of the reaction, i.e., steps 1 through 6, for both the forward and inverse reactions. All of the data pertinent to the cascade and pertinent to the subsequent calculations are written on magnetic tape for temporary storage. The second program uses this tape as input to calculate the final laboratory vector momenta of the fragments after the evaporation has taken place and to calculate the momenta of the evaporated particles in the laboratory system, i.e., steps 7 through 10. It is at this stage that the rotation to account for Coulomb deflection is

applied, and it is applied to the cascade as well as to the evaporation results before writing both on magnetic tape.

A third code, an analysis code, is used to bin the final data from this tape into the ranges of angle and energy that are specified by the input to the code. This code will therefore calculate the absolute cross sections for the myriad of results one has at his disposal. Examples of the available calculated results are the absolute cross sections for the total reaction, for the energy-angle-correlated spectra of the nucleons, π -mesons, and residual fragments, for the total production of fragments in different A and Z ranges, etc. Machine plots of the various spectra will also be available from this program.

DISCUSSION

As presently formulated, the calculation only approximates a cascade that takes place simultaneously in the target and projectile during the time interval in which the projectile is passing through the target. The approximation consists of allowing the cascades to take place only in the target in the "forward" reactions (moving projectile on stationary target), and then allowing them to take place only in the projectile during the "inverse" reaction (stationary projectile and moving target). The contributions to the total reaction from the forward and inverse reactions are then weighted by one-half each. It would be possible to calculate a simultaneous cascade with the present codes, but they would require extensive remodeling. Effects which could be major, such as nuclear depletion and angular momentum, are not presently included; therefore, it was not felt that the extensive re-programming could be justified at this time, particularly since the present method is not an unreasonable approximation to a simultaneous cascade.

A cascade code that will account for local nuclear depletion as a result of the cascades is being programmed, and a method of estimating the effects of angular momentum on the reaction products is under study. Angular-momentum considerations may lead to a criterion for switching to compound nuclear formation in these reactions, and then the energy range over which the calculation might apply could be extended to smaller values. Earlier, a somewhat similar approach to the present version was tried for alpha particles on complex nuclei.⁷ The backward reaction was not included, and the fragment of the alpha particle remaining after the reaction was unchanged in direction and velocity. Agreement of the calculated results with experimental data was fair.

A few examples of the calculation of absolute cross sections using the present version are given here. For purposes of definition, it is to be kept in mind that in order to provide statistics for the calculation, hundreds of projectiles are made to impinge on a target in the forward reaction, and the same number of targets are made to bombard a stationary projectile in the inverse reaction. The interaction from one incident projectile in the forward reaction is calculated by bombarding the target with the individual nucleons of the projectile (similarly for the backward reaction). The terminology employed is such that the initial projectile nucleus is always referred to as the projectile whether it is moving or stationary, and the same applies to the target.

The total nonelastic or reaction cross section σ_r is given by

$$\sigma_r = \pi(R_p + R_T)^2 \frac{(\# \text{ coll})}{(\# \text{ inc})} ,$$

where

R_p, R_T = the outer radii of the projectile and target, respectively,
 ($\# \text{ coll}$) = the number of projectiles in the forward reaction in which at least one of the projectile nucleons initiated a cascade in the target, plus the number of targets in the inverse reaction in which at least one of the target nucleons initiated a cascade in the projectile,
 ($\# \text{ inc}$) = the total number of incident projectiles in the forward reaction plus the total number of incident targets in the inverse reaction. These two numbers are set equal in the calculation.

The energy spectrum at a particular angle for any particle of mass A and charge Z (and, in addition, π -mesons and particles with a range of ΔA and ΔZ) is obtained in the form of a histogram. An angular interval, $\Delta\mu$ ($\mu = \cos \theta$), about the particular angle is selected, and the energy range is divided into bins of size ΔE . The cross section for the emission of the particles into $\Delta\mu$ and each ΔE is calculated, and the histogram is constructed from these cross sections for all of the ΔE spanning the energy range. The cross section for the emission of particles of mass A and charge Z into $\Delta\mu$ and ΔE is defined as

$$\frac{d^2\sigma}{d\Omega dE} = \frac{\text{number of particles of mass } A \text{ and charge } Z \text{ emitted into } \Delta\mu \text{ and } \Delta E \text{ in both forward and inverse reactions}}{(2\pi \Delta\mu \Delta E) \times (\# \text{ inc})} \times \pi(R_P + R_T)^2. \quad (1)$$

The cross sections for other distributions, such as angular distribution, have corresponding definitions.

The definition for $d^2\sigma/d\Omega dE$ (Eq. 1) can be cast into the standard one; i.e.,

$$\frac{d^2\sigma}{d\Omega dE} = \lim_{\Delta\mu, \Delta E \rightarrow 0} \left(\frac{\text{number of particles of mass } A \text{ and charge } Z \text{ emitted into } \Delta\mu \text{ and } \Delta E \text{ per second per target nucleus}}{(2\pi \Delta\mu \Delta E) \times (\text{incident beam intensity})} \right),$$

by dividing both numerator and denominator of Eq. 1 by an arbitrary length of time, and observing that

$$\frac{\# \text{ inc}}{\pi(R_P + R_T)^2 \text{ sec}}$$

corresponds to the incident beam intensity. The limit of $\Delta\mu$ and ΔE is not taken, and hence Eq. 1 represents the average value of the standard cross section over $\Delta\mu$ and ΔE . The calculation is such that every incident projectile would interact (except for target transparency) with the target, and hence the normalization, per target nucleus, is automatically taken into account.

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COMPARISONS WITH EXPERIMENTAL DATA

Comparisons of preliminary results from the heavy-ion model with the data of Heckman *et al.*⁵ indicated that improvements in the agreement could be obtained if the outer radius of the nuclei were to be decreased for the light elements. Three nuclear configurations, illustrated in Fig. 1, were examined: the standard configuration, one with a smaller radius, and one with the smallest radius. Comparisons of the calculated results from each of the three with the data of Heckman *et al.* are shown in Table 1. The "smaller" configuration was selected to describe the light nuclei ($A \leq 27$) because it represented a reasonable compromise between this particular comparison and the need of avoiding a radical change from the standard configuration, which has been used successfully in the past for the intranuclear-cascade calculation.^{1,3,4} The standard configuration was used for all mass numbers > 27 .

The comparison in Table 1 represents an extremely stringent test of the model because both the angular and energy intervals for which the data apply are very small. Furthermore, the sum of the cross sections represents a relatively small fraction of the total reaction cross section. Considering the crudeness of the model, this comparison is very encouraging. In the calculations, a value of $\pm 2\%$ of the incident velocity was used for fragments with $A = 12$ and 13 , and $\pm 4\%$ was used for the remaining mass values. These values approximately correspond to the limits for which the measured velocity distributions of the fragments reached 10% of their peak values (see Fig. 2 in Ref. 5).

Tables 2 and 3 contain spallation-type data for various reactions and the agreement with the experimental results varies from excellent to poor,

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Fig. 1. Relative nuclear density vs nuclear radius for three nuclear configurations representing the ^{14}N nucleus. Solid line: standard configuration; dashed line: "smaller" configuration; dot-dashed line: "smallest" configuration; dashed curve: Hofstadter's Fermi-type distribution (ref. 2).

Table 1

Cross Section (mb) for the Production of Various Fragments with Velocities Within $\pm 2\%$ (for A Values of 12 and 13) and $\pm 4\%$ (for $A \leq 11$) of the Incident ^{14}N Velocity and Emitted into the Angular Interval of 0-4 mrad from the Reactions of ^{14}N at 2.1 GeV/Nucleon on ^{12}C

Fragment	Theoretical Results for the Indicated Nuclear Configurations			Experimental Data ^a
	Standard	Smaller	Smallest	
^7_3Li	0	0.35 ± 0.18	0.30 ± 0.15	9.1 ± 3
^7_4Be	0	0.09 ± 0.09	0.07 ± 0.07	8.0 ± 1.0
^9_4Be	0	0.09 ± 0.09	0.30 ± 0.15	3.0 ± 0.3
^8_5B	0	0	0	5.1 ± 0.6
$^{10}_5\text{B}$	13 ± 3	12 ± 1	9.8 ± 0.9	14.4 ± 2.9
$^{11}_5\text{B}$	8.5 ± 2.1	6.5 ± 0.8	4.9 ± 0.6	10.7 ± 1.3
$^{12}_5\text{B}$	0.5 ± 0.5	2.2 ± 0.4	1.3 ± 0.3	1.9 ± 0.4
^9_6C	0	0.18 ± 0.12	0	0.14 ± 0.02
$^{10}_6\text{C}$	0	0.35 ± 0.18	0.37 ± 0.17	0.96 ± 0.12
$^{11}_6\text{C}$	7.4 ± 2.0	5.4 ± 0.7	4.0 ± 0.5	11.3 ± 1.1
$^{12}_6\text{C}$	68 ± 6	58 ± 2	54 ± 2	46 ± 9
$^{13}_6\text{C}$	36 ± 4	21 ± 1	17 ± 1	9.2 ± 1.8
$^{12}_7\text{N}$	0.5 ± 0.5	0.79 ± 0.26	0.22 ± 0.13	0.63 ± 0.13
$^{13}_7\text{N}$	17 ± 3	11 ± 1	10 ± 1	7.7 ± 1.5
Sum	151 ± 9	118 ± 3	102 ± 3	128.13 ± 10.4
Total Reaction Cross Section	1112	966	918	
% $\frac{\text{Sum}}{\text{Total}}$	14%	12%	11%	

a. H. H. Heckman *et al.*, "Fragmentation of ^{14}N Nuclei at 29 GeV: Inclusive Isotope Spectra at 0° ," Phys. Rev. Letters 28, 926 (1972).

Table 2

Theoretical and Experimental Cross Sections for Producing All
Isotopes of Carbon and Nitrogen Emitted into the Angular
Interval of $0-7^\circ$ and with Velocities Within a Few
Percent of the Incident Projectile Velocity
from the Reactions of Oxygen at 33.6 GeV
(2.1 GeV/nucleon) on a Few Elements

Isotopes Produced	Total Isotope Production Cross Sections (mb)					
	Target					
	C		Cu		Pb	
	Theor.	Exp. ^a	Theor.	Exp. ^a	Theor.	Exp. ^a
Carbon	100 ± 5	130	93 ± 9	190	111 ± 17	258
Nitrogen	141 ± 6	104	165 ± 12	164	199 ± 23	323

- a. H. H. Heckman, Lawrence Berkeley Laboratory, Berkeley, California,
private communication May 18, 1972. No error bars on the data were
established at the time the experimental results were quoted.

Table 3
Theoretical and Experimental Spallation Cross Sections
from 920-MeV ${}^4\text{He}$ on Carbon, Oxygen, and Aluminum

Spallation Products	Cross Sections (mb)					
	Target					
	C		O		Al	
	Theor.	Exp. ^a	Theor.	Exp. ^a	Theor.	Exp. ^a
${}^7_4\text{Be}$	0.92 ± 0.92	20.0 ± 1.1				
${}^{11}_6\text{C}$	100 ± 9	48.9 ± 1.8	4.9 ± 2.2	18.5 ± 0.9		
${}^{13}_7\text{N}$			3.0 ± 1.7	6.75 ± 0.5		
${}^{15}_8\text{O}$			87 ± 9	46.6 ± 2.3		
${}^{18}_9\text{F}$					5.6 ± 2.5	12.5 ± 0.5

- a. Jonathan Rudin, "α-Spallation Cross Sections at 920 MeV (230 MeV/n) in ${}^{27}\text{Al}$, ${}^{16}\text{O}$, ${}^{12}\text{C}$, and ${}^9\text{Be}$ and Application to Cosmic Ray Transport," Dissertation, Department of Physics, New York University (October 1971).

but most of the predicted results are within a factor of three of the experimental data.

Table 4 and Fig. 2 contain comparisons of the fragmentation parameters that are predicted and those that are deduced from experimental data.⁸⁻¹² In general, the calculation overestimates the production of heavy fragments and underestimates that for light fragments, while the overall agreement is fair to poor.

The experimental data given in Table 4, however, are from interactions that have taken place over a broad energy range, i.e., over the spectrum of incident cosmic rays of medium weight, whereas the theoretical predictions are from interactions at one energy for purposes of convenience. Furthermore, the fractions of the cosmic-ray interactions that take place with each element in the atmosphere are only approximately known, and, in addition, the angular intervals covered by the experimental results were either not specified exactly or not specified at all.¹³ Hence, considering the uncertainties in the designation of the interactions, the comparisons illustrated in Table 4 indicate agreements which are not unreasonable.

The theoretical results in Table 4 are shown for two angular intervals, 0 to 180° and 0 to 15°, which illustrates a forward peaking. The calculations were done for incident ions with energies corresponding to 3 GeV/nucleon, and they were repeated for energies of 2 GeV/nucleon, and the results were essentially the same. The calculated results for both forward and inverse reactions are those from the projectile only. This was done to eliminate the low-energy contributions which would come from the target fragments. Charge differences cannot be distinguished in such low-energy contributions.

Table 4
Fragmentation Parameters for Air from Medium-
Weight Cosmic-Ray Primaries

Secondaries Produced	Fraction of Interactions Producing Secondaries				
	Theoretical ^a		Experimental ^b		
	0 - 180°	0 - 15°	Friedlander ^c	Waddington ^d	Daniel ^e
$6 \leq Z \leq 9$	0.21	0.17	0.11	0.16	0.04
$3 \leq Z \leq 5$	0.16	0.13	0.24	0.21	0.26
α -particles	0.76	0.61	1.00	0.93	0.78

- a. It was assumed that air was composed of 80% N and 20% O by number and that the medium-weight incident cosmic rays were made up of C (50%), O (27.8%), and N (22.2%) and were incident at 3-GeV/nucleon energies [M. O. Burrell and J. J. Wright, The Estimation of Galactic Cosmic Ray Penetration and Dose Rates, NASA TN D-6600, George C. Marshall Space Flight Center (1972)].
- b. Data quoted by W. R. Webber, p. 209 in *Encyclopedia of Physics*, Vol. XLV1/2, Cosmic Rays II, Ed. K. Sitte, Springer Verlag, 1967.
- c. M. W. Friedlander *et al.*, *Phil. Mag.* 8, 1691 (1963).
- d. C. J. Waddington, *Prog. Nucl. Phys.* 8, 1 (1960).
- e. R. R. Daniel and N. Durgaprasad, *Nuovo Cimento* 23, Suppl. 82 (1962).

ORNL-DWG 73-10180

Fig. 2. Fragment emission frequency for boron, oxygen, and magnesium cosmic-ray primaries on tissue nuclei. The experimental data are from Curtis, Doherty, and Wilkinson (ref. 12). The theoretical results are for a ^{14}N target with incident energies corresponding to 1.5 GeV/nucleon. Other details are given in the text.

The calculations for Fig. 2 were done for energies corresponding to 1.5 GeV/nucleon on a ^{14}N target, and the cross section for the emission of the secondaries was for the angular interval of 0 to 2° and for energies extending from 10 MeV/nucleon to maximum energies. These energy and angular restrictions approximately correspond to the measured values.^{14,15}

One of the most interesting predictions from the model is that the proton spectrum in the forward direction from heavy-ion interactions contains a significant number of protons emitted with energies greater than the incident energy per nucleon. This is illustrated in Fig. 3 for 2.1 GeV/nucleon nitrogen incident on carbon.

There are two peaks predicted at high energies. The peak centered at 1.7 GeV is made up primarily of cascade protons from the forward reaction (84%), with the remainder consisting of cascade protons from the inverse reaction (10%) and evaporation protons from the projectile in the inverse reaction (6%). The peak centered at 2.3 GeV consists mainly of protons evaporated from the projectile in the forward reaction (56%), with cascade and projectile evaporation protons from the inverse reaction contributing the remainder (16 and 28%, respectively).

The peak at 1.7 GeV is the "quasi-free" peak from the direct interactions of the nucleons of the projectile with those of the target. The peak is located at a smaller energy than expected (expected at 2.1 GeV for a 0- to 10-mrad angular interval) because of exclusion effects.

The last comparison is made at energies lower than the expected energy limit of the validity of the model, but it is made for illustrative purposes. Figure 4 illustrates the effect of the Coulomb correction on the calculated results. The theoretical results, with and without the Coulomb correction,

bracket the experimental data, and they approach each other as the energy increases.

ORNL-DWG 73-7357

Fig. 3. Energy spectrum of protons emitted in the angular interval of 0 to 10 mrad from 2.1 GeV/nucleon ¹⁴N on carbon. The arrow indicates the position of the incident energy per nucleon.

Fig. 4. Nonelastic cross section vs incident energy for oxygen on silver.

SUMMARY

The predictions in absolute magnitude from the heavy-ion model as it is presently constituted (HIC-1) are generally within a few factors of the experimental data, even when those data represent small fractions of the total reaction cross section and correspond to angle-energy-correlated results. A trend toward predicting cross sections for the production of heavy fragments that are too large while underestimating the cross section for the production of light fragments is discernible. The inclusion of the effects of angular momentum in the calculation (which are ignored in HIC-1) might compensate for this trend because when the angular momentum transferred to the heavy fragments is sufficiently high, the fragments will be unstable, and they will therefore break up into smaller ones. Crude estimates of the angular momentum transferred indicate that the magnitude can indeed be sufficient for the breakup to occur.

The further inclusion in the calculation of simultaneous cascades and nuclear depletion will make the model more nearly internally consistent and realistic.

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APPENDIX A

KINEMATICS CALCULATION FOR TARGET AND PROJECTILE FRAGMENTS

Definition of Terms

E = Total energy (MeV)

KE = Kinetic energy (MeV)

A = Mass number

m = Rest mass (MeV)

$\vec{p}$ = Vector momentum (MeV/c)

$\hat{k}$ = Unit vector in z direction in laboratory system

E^* = Excitation energy (MeV)

$\vec{V}$ = Vector velocity of incident projectile in laboratory system (units of c)

θ, ϕ = Polar and azimuthal angles of the projectile fragment in a system moving with velocity $\vec{V}$ and whose axes are parallel to the laboratory system axes

Subscripts and/or Superscripts

b = Before the reaction

P = Projectile

T = Target

inc = Incident

n = Nucleon

r = Remaining or left to the target and projectile after the reaction

cc = Cascade particles that escape

π = Pion

a = After the interaction

' (primes) = Quantities defined in the system moving with velocity $\bar{V}$

Note: These definitions are slightly different from those given on previous pages; i.e., $E_{p,a}^*$ refers to the projectile after the reaction (i.e., the projectile fragment), whereas previously it was defined as $E_{p,f}^*$, etc.

Before the cascade reaction, in the laboratory system:

$$E_b = (KE)_{P,inc} + A_{P,b}(m_n - 7) + A_{T,b}(m_n - 7)$$

$$\begin{aligned} \bar{p}_b &= \left\{ [(KE)_{P,inc} + A_{P,b}(m_n - 7)]^2 - [A_{P,b}(m_n - 7)]^2 \right\}^{\frac{1}{2}} \\ &= \hat{k} p_{b,z} \end{aligned}$$

After the cascade reaction, in the laboratory system:

$$E_r = E_b - \sum_{i=1}^{\text{all esc casc nucl}} [m_n + (KE)_{cc,n}]_i - \sum_{j=1}^{\text{all esc casc pions}} [m_\pi + (KE)_{cc,\pi}]_j$$

$$\bar{p}_r = \bar{p}_b - \sum_i \bar{p}_{n,i}^{cc} - \sum_j \bar{p}_{\pi,j}^{cc}$$

Let

$$m_T = A_{T,a}(m_n - 7) + E_{T,a}^* \equiv \text{mass of target fragment}$$

$$m_P = A_{P,a}(m_n - 7) + E_{P,a}^* \equiv \text{mass of projectile fragment}$$

The energy and momentum equations for the target and projectile fragments which are to be solved in the laboratory system after the reaction are

$$E_{T,a} + E_{P,a} = E_r$$

and

$$\bar{p}_{T,a} + \bar{p}_{P,a} = \bar{p}_r$$

Since the angular distribution of the projectile fragment is to be specified in the system moving with the velocity of the incident projectile, all quantities must be transformed to that system, and later they are transformed back to the laboratory system. The velocity of the moving system is

given by

$$\bar{V} = \frac{\bar{p}_b}{E_{P,b}} ,$$

where

$$E_{P,b} = (KE)_{P,inc} + A_{P,b}(m_n - 7) .$$

Until otherwise specified, all symbols that follow will refer to those quantities after the reaction, and hence the subscript "a" will be dropped.

Let

$$\gamma = \frac{1}{\sqrt{1 - V^2}} .$$

In the system moving with velocity $\bar{V}$ (primed system) whose axes are parallel to those in the laboratory system,

$$\bar{p}'_r = \bar{p}_r + \frac{\bar{V}(\bar{p}_r \cdot \bar{V})}{V^2} (\gamma - 1) - \gamma \bar{V} E_r$$

$$E'_r = \gamma(E_r - \bar{p}_r \cdot \bar{V}) ,$$

and in this system, the conservation equations are

$$E'_T + E'_P = E'_R$$

and

$$\bar{p}'_T + \bar{p}'_P = \bar{p}'_R .$$

The primes will now be dropped since all quantities will refer to the " $\bar{V}$ " system until specified otherwise. The polar and azimuthal angles θ and ϕ for the direction of the projectile fragment are selected randomly from an isotropic distribution in the $\bar{V}$ system. Then

$$p_{P,x} = p_P \sin \theta \cos \phi$$

$$p_{P,y} = p_P \sin \theta \sin \phi$$

$$p_{P,z} = p_P \cos \theta .$$

The equation for the unknown p_p is obtained by substituting into the conservation equations and solving; i.e.,

$$\bar{p}_T = \bar{p}_R - \bar{p}_P$$

with

$$[(\bar{p}_R - \bar{p}_P)^2 + m_T^2]^{\frac{1}{2}} + [\bar{p}_P^2 + m_P^2]^{\frac{1}{2}} = E_R$$

or,

$$[p_R^2 + p_P^2 - 2p_P(p_{R,x} \sin \theta \cos \phi + p_{R,y} \sin \theta \sin \phi + p_{R,z} \cos \theta) + m_T^2]^{\frac{1}{2}} + [p_P^2 + m_P^2]^{\frac{1}{2}} = E_R$$

Let

$$\beta = p_{R,x} \sin \theta \cos \phi + p_{R,y} \sin \theta \sin \phi + p_{R,z} \cos \theta$$

Substituting into E_R and squaring,

$$E_R^2 = p_R^2 + p_P^2 - 2p_P \beta + m_T^2 + p_P^2 + m_P^2 + 2[(p_R^2 + p_P^2 - 2p_P \beta + m_T^2)(p_P^2 + m_P^2)]^{\frac{1}{2}}$$

To condense, let

$$\alpha = E_R^2 - p_R^2 - m_T^2 - m_P^2$$

Then transposing and squaring,

$$(\alpha - 2p_P^2 + 2p_P \beta)^2 = 4(p_R^2 + p_P^2 - 2p_P \beta + m_T^2)(p_P^2 + m_P^2)$$

or

$$\alpha^2 + 4p_P^4 + 4p_P^2 \beta^2 - 4\alpha p_P^2 + 4p_P \alpha \beta - 8p_P^3 \beta =$$

$$4(p_R^2 p_P^2 + p_P^4 - 2p_P^3 \beta + m_T^2 p_P^2 + p_R^2 m_P^2 + p_P^2 m_P^2 - 2p_P m_P^2 \beta + m_T^2 m_P^2),$$

which reduces to

$$(\beta^2 - \alpha - p_r^2 - m_T^2 - m_p^2) p_p^2 + (\alpha \beta + 2 m_p^2 \beta) p_p + (\alpha^2/4 - p_r^2 m_p^2 - m_T^2 m_p^2) = 0 .$$

Letting

$$a = (\beta^2 - \alpha - p_r^2 - m_T^2 - m_p^2)$$

$$b = (\alpha \beta + 2 m_p^2 \beta)$$

$$c = (\alpha^2/4 - p_r^2 m_p^2 - m_T^2 m_p^2)$$

and, replacing primes, the solution for p'_p reduces to solving

$$a(p'_p)^2 + b p'_p + c = 0 ,$$

which yields two solutions. It turns out that all directions of p'_p are frequently not available for the fragment; i.e., some directions lead to negative or imaginary values for both solutions of p'_p . When this occurs, the angles θ and ϕ are resampled from an isotropic distribution until a physical solution is obtained. In addition, some directions lead to two positive solutions, and when this occurs, the larger value is selected half the time while the smaller is selected the other half of the time. These peculiarities arise from the statistical nature of the cascade. For example, if five escaping cascade particles are emitted with the same energy ϵ , the sum of their vector momenta can range continuously from $-5 \sqrt{\epsilon^2 - m^2}$ to $+5 \sqrt{\epsilon^2 - m^2}$, depending upon the direction of each cascade particle. Each cascade particle can be completely uncorrelated from the others, because each incident nucleon of the projectile gives rise to an independent cascade when it collides.

Having obtained an acceptable p'_p , one proceeds to the vector momenta of the projectile and target fragments by calculating

$$p'_{p,x} = p'_p \sin \theta \cos \phi$$

$$p'_{p,y} = p'_p \sin \theta \sin \phi$$

$$p'_{p,z} = p'_p \cos \theta ;$$

then

$$E'_p = [(p'_p)^2 + m_p^2]^{\frac{1}{2}} .$$

Finally, the lab system momenta of the fragments are given by (replacing subscript a)

$$\bar{p}_{p,a} = \bar{p}'_p + \frac{\bar{V}(\bar{p}'_p \cdot \bar{V})}{V^2} (\gamma - 1) + \gamma \bar{V} E'_p$$

and

$$\bar{p}_{T,a} = \bar{p}_r - \bar{p}_{p,a} ,$$

and their energies are given by

$$E_{p,a} = \gamma(E'_p + \bar{p}'_p \cdot \bar{V})$$

with

$$E_{T,a} = E_r - E_{p,a} .$$

In the inverse reaction, the same calculation is applied to the cascade results after the transformation back to the laboratory system has been made.

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APPENDIX B

CALCULATION OF THE LABORATORY SYSTEM ENERGY AND MOMENTUM OF AN EVAPORATED
PARTICLE AND OF THE RESIDUAL NUCLEUS FOLLOWING AN EVAPORATION
FROM AN EXCITED PROJECTILE OR TARGET FRAGMENT

Definition of terms

E = the total energy,

T = the kinetic energy,

m = the mass energy of the fragment - includes excitation energy,

m_e = the mass energy of evaporated particle,

$\vec{p}$ = the vector momentum of nucleus,

$\vec{p}_e$ = the vector momentum of evaporated particle.

Primes refer to quantities in the rest frame of the excited fragment before the evaporation. Unprimed quantities refer to the laboratory system values.

Subscripts "b" and "a" refer to quantities before and after the evaporation has taken place, respectively.

Subscript "e" refers to the evaporated particle.

Note: The following calculation applies only when the particles are evaporated isotropically in the rest system of the evaporating fragment. The momentum and mass of the excited fragment before the evaporation ($\vec{p}_b$ and m_b) are given, as well as the kinetic energy in the prime system and the mass of the evaporated particle. The latter quantities are calculated by the evaporation program.⁶ The quantities to be calculated are the vector momenta of the evaporated particle and recoiling fragment in the laboratory system.

The calculation is carried out sequentially as follows:

$$E'_e = T'_e + m_e ,$$

$$p'_e = \sqrt{(E'_e)^2 - m_e^2} ,$$

$$E'_a = m_b - E'_e , \text{ i.e., this is the total energy of the fragment after evaporation,}$$

$$m_a = \sqrt{(E'_a)^2 - (p'_e)^2} .$$

θ and ϕ are sampled from an isotropic distribution:

$$p'_{e,x} = p'_e \sin \theta \cos \phi ,$$

$$p'_{e,y} = p'_e \sin \theta \sin \phi ,$$

$$p'_{e,z} = p'_e \cos \theta ,$$

$$E_b = \sqrt{(\bar{p}_b)^2 + m_b^2} ,$$

$$E_e = \frac{E'_e + \bar{p}'_e \cdot \bar{V}}{\sqrt{1 - V^2}} = \frac{E'_e + \bar{p}'_e \cdot \bar{p}_b / E_b}{\frac{m_b}{E_b}}$$

$$= \frac{1}{m_b} (E_b E'_e + \bar{p}'_e \cdot \bar{p}_b) ,$$

$$E_a = E_b - E_e ,$$

$$T_e = E_e - m_e ,$$

$$\bar{p}_e = \bar{p}'_e + \frac{\bar{p}_b (\bar{p}'_e \cdot \bar{p}_b)}{(\bar{p}_b)^2} \left(\frac{E_b}{m_b} - 1 \right) + \frac{\bar{p}_b E'_e}{m_b} ,$$

$$T_a = E_a - m_a ,$$

$$\bar{p}_a = \bar{p}_b - \bar{p}_e .$$

APPENDIX C

MODIFICATIONS OF IMPACT PARAMETER AND PROJECTILE
MOMENTUM DUE TO COULOMBIC FORCES

The effect of the Coulomb force between incident projectile and target is introduced into the calculation in an approximate manner. For a given history, the impact parameter b is chosen randomly according to an r^2 distribution. A relativistic transformation is made to the center of mass of the system (projectile plus target). In this frame, the nonrelativistic equations of motion for the interaction of two charged particles are used to determine an effective impact parameter d , defined as the distance of closest approach.

The equation of motion is

$$E = \frac{\mu}{2} \left| \frac{d\vec{r}}{dt} \right|^2 + \frac{K}{r}, \quad (1)$$

where E is the kinetic-plus-potential energy, μ is the reduced mass of target plus projectile, $\vec{r}$ is the relative position of the particles, and $K = Z_p Z_T e^2$ in an appropriate set of units.

The incident angular momentum of the system is

$$\vec{L} = b p_0 \hat{\xi}, \quad (2)$$

where $\hat{\xi}$ is a vector perpendicular to the z direction, which is the direction of the incident projectile. Conservation of angular momentum then tells us that

1. motion is always in a plane perpendicular to the $\hat{\xi}$ direction

$$\left(\frac{d\phi}{dt} = 0 \right), \text{ and}$$

2. $r^2 \frac{d\theta}{dt}$ is a constant:

$$\vec{L} = \vec{r} \times \mu \frac{d\vec{r}}{dt} = \mu r^2 \frac{d\theta}{dt} \hat{\xi}, \quad (3)$$

or

$$\frac{d\theta}{dt} = \frac{b p_0}{\mu r^2}. \quad (4)$$

For this discussion, ϕ is the polar angle measured with respect to $\hat{\xi}$ and θ is the corresponding azimuthal angle in the plane perpendicular to $\hat{\xi}$. Since $\frac{d\phi}{dt}$ is a constant, the velocity squared may be written as

$$\left| \frac{d\vec{r}}{dt} \right|^2 = \left| \frac{dr}{dt} \hat{r} + r \frac{d\theta}{dt} \hat{\theta} \right|^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2. \quad (5)$$

Invoking conservation of energy ($E = \frac{p_0^2}{2\mu}$), Eq. 1 may then be written as

$$\frac{p_0^2}{2\mu} = \frac{\mu}{2} \left[\left(\frac{dr}{dt} \right)^2 + \frac{b^2 p_0^2}{\mu^2 r^2} \right] + \frac{K}{r}. \quad (6)$$

The distance of closest approach (d) is that distance at which $\frac{dr}{dt} = 0$.

Solving Eq. 6 with $\frac{dr}{dt} = 0$,

$$d = b \left\{ \frac{\mu K}{b p_0^2} + \sqrt{1 + \left(\frac{\mu K}{b p_0^2} \right)^2} \right\}. \quad (7)$$

In the intranuclear-cascade calculation (as well as in the evaporation calculation), it is assumed that the projectile approaches the target in a straight-line path tangent to the "Coulomb path" at the position of closest approach. This geometry is illustrated (greatly exaggerated) in Fig. C.1. Thus the Coulomb force has the effect of (1) lengthening the impact parameter and (2) rotating the entire interaction by the angle β .

The calculation of the effective impact parameter d to find the Coulomb rotation angle β has already been discussed. To find the Coulomb rotation angle β , it is necessary to solve the equations of motion to find r as a function of θ . From Eq. 4 it is found that

$$\frac{d}{dt} = \frac{b p_0}{\mu r^2} \frac{d}{d\theta}. \quad (8)$$

Substituting into Eq. 6 and making the change of variable $u = \frac{1}{r}$ yields

ORNL-DWG 73-10610

Fig. C.1. Schematic diagram of the quantities involved in the calculation of Coulomb rotation.

$$\left(\frac{du}{d\theta}\right)^2 + u^2 + \frac{2\mu K}{p_0^2 b^2} u - \frac{1}{b^2} = 0 . \quad (9)$$

The solution of this equation with the boundary condition that $u = 0$ ($r = \infty$) at $\theta = \pi$ is

$$u = \frac{1}{r} = \frac{1}{b} \left\{ \sqrt{1 + \left(\frac{\mu K}{p_0^2 b} \right)^2} \cos(\theta + \delta) - \frac{\mu K}{b p_0^2} \right\} , \quad (10)$$

where

$$\cos \delta = - \frac{\mu K}{\sqrt{b^2 p_0^4 + \mu^2 K^2}} \quad (11)$$

with

$$\cot \delta = \pm \frac{\mu K}{b p_0^2} .$$

The positive value of $\cot \delta$ is selected arbitrarily.

To find Θ , the scattering angle in the center-of-mass system, u is set equal to zero ($r = \infty$) and Θ is determined; i.e.,

$$\Theta = \pi - 2\delta . \quad (12)$$

From plane geometry, β is obtained in terms of Θ . As seen in Fig. C.1, $\beta = \Theta/2$. Therefore, the Coulomb rotation angle β is given by

$$\cot \beta = \cot\left(\frac{\pi}{2} - \delta\right) = \frac{1}{\cot \delta} , \quad (13)$$

or

$$\tan \beta = \frac{\mu K}{b p_0^2} . \quad (14)$$

A relativistic transformation back to the laboratory frame gives the rotation angle β_L actually used in the computations.

The entire calculation (cascade plus evaporation) is carried out using the assumed forward direction. Immediately prior to writing the results from

the evaporation code onto tape, subroutine ROTATE is called. This subroutine rotates the reaction coordinate system inward by the angle β_L .

REFERENCES

1. Hugo W. Bertini, Phys. Rev. 131, 1801 (1963), with erratum Phys. Rev. 138, AB2 (1965).
2. R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).
3. Hugo W. Bertini, Phys. Rev. 188, 1711 (1969); Phys. Rev. C5, 2118 (1972).
4. Hugo W. Bertini, Phys. Rev. C6, 631 (1972).
5. H. H. Heckman *et al.*, Phys. Rev. Letters 28, 926 (1972).
6. Miriam P. Guthrie, "EVAP-4: Another Modification of a Code to Calculate Particle Evaporation from Excited Compound Nuclei," ORNL-TM-3119 (1970).
7. T. A. Gabriel, R. T. Santoro, and R. G. Alsmiller, Jr., "An Approximate High-Energy Alpha-Particle-Nucleus-Collision Model," ORNL-TM-3153 (1970).
8. The data in Table 4 were quoted by W. R. Webber in Encyclopedia of Physics, Vol. XLVI/2, Cosmic Rays II, Ed. K. Sitte, Springer Verlag, 1967, p. 209.
9. M. W. Friedlander *et al.*, Phil. Mag. 8, 1691 (1963).
10. C. J. Waddington, Prog. Nucl. Phys. 8, 1 (1960).
11. R. R. Daniel and N. Durgaprasad, Nuovo Cimento 23, Suppl. 82 (1962).
12. S. B. Curtis, W. R. Doherty, and M. C. Wilkinson, "Study of Radiation Hazards to Man on Extended Near Earth Missions," NASA CR-1469 (1969).
13. A statement by C. J. Waddington (ref. 10) indicates that in the experimental data the bulk of the fragments was in the angular interval of 0 to 10°.

14. P. H. Fowler, R. R. Hillier, and C. J. Waddington, Phil. Mag. 2, 293 (1957).
15. V. Y. Rajopadhye and C. J. Waddington, Phil. Mag. 3, 19 (1958).
16. M. O. Burrell and J. J. Wright, "The Estimation of Galactic Cosmic Ray Penetration and Dose Rates," George C. Marshall Space Flight Center Report NASA TN D-6600 (1972).

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Internal Distribution

1-3.	L. S. Abbott	70-94.	RSIC
4.	F. S. Alsmiller	95-99.	R. T. Santoro
5.	R. G. Alsmiller, Jr.	100.	S. Siegel
6.	T. W. Armstrong	101.	J. B. Storer
7.	J. A. Auxier	102.	D. Sundberg
8.	J. Barish	103.	J. R. Totter
9-38.	H. W. Bertini	104.	J. E. Turner
39.	A. A. Brooks	105.	G. E. Whitesides
40.	H. P. Carter	106.	H. A. Wright
41.	K. C. Chandler	107.	A. Zucker
42.	G. T. Chapman	108.	H. Feshbach (consultant)
43.	C. E. Clifford	109.	P. F. Fox (consultant)
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