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RESONANCE ABSORPTION OF NEUTRONS

Chapter 3. Resonance Absorption in
Heterogeneous Media

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CHAPTER 3

RESONANCE ABSORPTION IN HETEROGENEOUS MEDIA

1. Introduction

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The previous chapters have dealt rather fully with the problem of resonance absorption in a homogeneous medium. The present chapter deals with resonance absorption in lumped absorbers, which is of considerable importance in reactor design.

2. Thin Resonances¹

Let us consider the case of absorption by a lumped absorber whose resonance lines fulfill the conditions of paragraph 4, Chapter 1, viz., a width in lethargy $\ll \frac{1}{\xi}$ and a separation $\gg \frac{1}{\xi}$. The lump is embedded in a moderating material which is assumed to have no absorption of its own. At energies between absorber resonances the flux per unit lethargy, ϕ , is continuous across the moderator-lump interface and does not vary in space. At energies near an absorber resonance the flux in the moderator is also very nearly equal to ϕ , since the neutrons which contribute to this flux originate from last collisions in a lethargy range very large compared to the resonance width. On the other hand, at energies near an absorber resonance the flux in the fuel lump is strongly depressed. We can see this by an argument similar to that used in paragraph 4, Chapter 1. If there were no resonance at all the number of neutrons leaving a small lethargy range Δu by collision is $N\sigma_p\phi\Delta u$, where N is the number of atoms per unit volume of the lump material, and σ_p is the potential scattering cross section per atom. These neutrons are supplied from last collisions far from the resonance in question; hence the rate of supply is independent of whether there is a resonance near Δu or not, and is equal to $N\sigma_p\phi\Delta u$. The rate at which neutrons leave Δu , however,

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is given by $N\phi'(\sigma_a + \sigma_s + \sigma_p) \Delta u$, where ϕ' is the local flux in Δu , σ_a and σ_s the resonance absorption and scattering cross sections, respectively. In the stationary state, then,

$$\phi' = \phi \frac{\sigma_p}{\sigma_a + \sigma_s + \sigma_p} \quad (2.1)$$

From the manner of derivation of Eq. (2.1) one sees that ϕ' is the local flux of neutrons which have made their last collision in the lump.

The rate of absorption of once-collided neutrons in the lump is then

$$A = NV\phi \int \frac{\sigma_p \sigma_a}{\sigma_a + \sigma_s + \sigma_p} du \quad (2.2)$$

where the integral is over the resonances of the absorbers and V is the lump volume. There is another contribution to the absorption in the lump, and that is due to neutrons which are absorbed on their first collision. These neutrons must enter the lump through its surface. The net current per unit area into the lump can be found by taking the difference between the inward current of moderator neutrons and outward current of lump neutrons, viz.,

$$\frac{\phi}{4} - \frac{\phi'}{4} = \frac{\phi}{4} \frac{\sigma_a + \sigma_s}{\sigma_a + \sigma_s + \sigma_p} \quad (2.3)$$

The total number of these neutrons which do not pass through the lump uncollided is

$$\frac{\phi}{4} \cdot 3 \int \frac{\sigma_a + \sigma_s}{\sigma_a + \sigma_s + \sigma_p} \left\{ 1 - \exp \left[-N(\sigma_a + \sigma_s + \sigma_p)l \right] \right\} r(l) dl \quad (2.4)$$

To obtain Eq. (2.4) we have integrated over the lump surface, S ; here ℓ is any chord of the lump and $f(\ell)$ is the normalized distribution of chord lengths in the lump, each chord length being weighted by the sum of the cosines of the angles it makes with the normals to the surface at its ends. Such a weighting is appropriate because the neutrons entering the lump have a cosine distribution.² Of these neutrons, a fraction $\sigma_a/\sigma_a + \sigma_s + \sigma_p$ are absorbed; the remainder are thrown out of the resonance range and then included among the once-collided neutrons. The absorption of hitherto uncollided neutrons is then

$$A' = \frac{\phi}{4} S \int du \int f(\ell) d\ell \frac{\sigma_a(\sigma_a + \sigma_s)}{(\sigma_a + \sigma_s + \sigma_p)^2} \left\{ 1 - \exp \left[-N(\sigma_a + \sigma_s + \sigma_p)\ell \right] \right\} \quad (2.5)$$

In the case of heterogeneous absorption the effective resonance integral, I , is defined as that cross section which would produce the same absorption if the flux per unit lethargy were everywhere the same as in the moderator, i.e., $NVI\phi = A + A'$. Combining this definition with Eq. (2.2) and (2.5) gives

$$I = \int \frac{\sigma_a \sigma_p}{\sigma_a + \sigma_p + \sigma_s} du + \frac{S}{4NV} \int du \int f(\ell) d\ell \frac{\sigma_a(\sigma_a + \sigma_s)}{(\sigma_a + \sigma_s + \sigma_p)^2} \times \left\{ 1 - \exp \left[-N(\sigma_a + \sigma_s + \sigma_p)\ell \right] \right\} \quad (2.6)$$

Equation (2.6) for I has some general properties worth explicitly writing down. If the lump volume increases the second integral approaches

zero as S/V does and the expression for I becomes the same as Eq. (3.3) of Chapter 2 which is applicable to an infinite homogeneous medium.

If, on the other hand, we allow V to approach zero, then so will the exponent in the second integral in Eq. (2.6). The bracket is then given by $N(\sigma_a + \sigma_s + \sigma_p)l$. The integral over l then replaces this last expression by $N(\sigma_a + \sigma_s + \sigma_p)\bar{l}$, where $\bar{l}$ is the average chord length in the lump. By a theorem of Gauss, $\bar{l} = 4V/S$. Substituting these expressions in Eq. (2.6) one obtains $I = \int \sigma_a du$, the value appropriate to infinitely diluted absorber.

3. Use of the Natural Line Shape

If σ_a and σ_s are described by the natural line shape the first integration indicated in Eq. (2.6) can be explicitly carried out; to carry out the second we set

$$1 - \exp \left[-N(\sigma_a + \sigma_s + \sigma_p)l \right] = \frac{N(\sigma_a + \sigma_s + \sigma_p)l}{1 + N(\sigma_a + \sigma_s + \sigma_p)l} \quad (3.1)$$

This replacement has the property that for $N(\sigma_a + \sigma_s + \sigma_p)l$ either $\ll 1$ or $\gg 1$ it is exact. For intermediate values it is, however, only approximate, the RHS being smaller than the LHS, sometimes by as much as 30%. After rearrangement the result of the integration is

$$I = \frac{\frac{\pi}{2} \frac{\sigma_0 \sqrt{\gamma}}{E_0}}{\left(1 + \frac{\sigma_0}{\sigma_p}\right)^{1/2}} \left[1 + (2N\bar{l}\sigma_p)^{-1} \frac{2}{1 + \left[1 + (N\bar{l}\sigma_p)^{-1}\right]^{1/2}} \right] \quad (3.2)$$

The expression (3.2) depends on the assumption that $N\sigma_0\bar{L} \gg 1$, which is nearly always true in cases of practical interest. The first term of Eq. (3.2), the limit in case $\bar{L} \rightarrow \infty$, follows also from Eq. (3.3) of Chapter 2 when the natural line shape is used.

When $N\bar{L}\sigma_p \gg 1$ both sides of Eq. (3.1) must always be near 1 no matter what the resonance cross sections are; hence, the error due to approximating the exponential will be small. If $N\bar{L}\sigma_p \ll 1$, on the other hand, both sides of Eq. (3.1) will take all values between 0 and 1 and the error due to approximating the exponential may be substantial. In this latter case it is possible to accurately estimate the error, for if we set $\sigma_p = 0$ we need not use Eq. (3.1) to perform the second integral in Eq. (2.6).³ When $\sigma_p = 0$, Eq. (2.6) becomes

$$I = \frac{1}{N\bar{L}} \frac{\Gamma_Y}{E_0} \int r(l) dl \int_0^\infty dx \left[1 - \exp\left(-\frac{N\sigma_0\bar{L}}{1+x^2}\right) \right] \quad (3.3)$$

The exponential in Eq. (3.3) is very small unless $x \gtrsim (N\sigma_0\bar{L})^{1/2}$, if $N\sigma_0\bar{L} \gg 1$, so that we can neglect 1 against x^2 in the exponential with small error. The integral can then be carried out easily and yields

$$I = \sqrt{\pi} \frac{(\sigma_0 \Gamma_Y^2)^{1/2}}{E_0} (N\bar{L})^{-1/2} \quad (3.4)$$

Equation (3.4) should really be multiplied by $\sqrt{\bar{L}}/\bar{L}$, but for shapes like spheres and long cylinders this fraction is .980 and .975, respectively. If in Eq. (3.2) we let $\sigma_p \rightarrow 0$ we obtain Eq. (3.4) exactly except the factor $\sqrt{\pi}$ is replaced by $\pi/2$. Hence, the expression for I based on Eq. (3.1) in the

limit $\sigma_p = 0$ is smaller than corresponding exact expression by a factor $\sqrt{\pi}/2$ or about by 10%. As $N\sigma_p \bar{V}$ increases from zero the error is expected to be less.

When the absorbing lump is large, or more correctly when its overall absorption is strong, it is no longer correct to treat the flux, ϕ , as being space independent. If there were no absorption of course, ϕ would be space independent. We argued before that even in the presence of absorption, since ϕ is produced by last collisions spread over an energy range in which the total absorption was small ($\bar{V} \ll \sum E$) it was space independent. To correct for the possibility of heavy absorption perhaps the simplest thing to do is to treat the resonance neutrons as a single monoenergetic group; and calculate its spatial variation with diffusion theory.⁴ In this event Eq. (3.2) should be written

$$I = \frac{\frac{\pi}{2} \frac{\sigma_0 \bar{V}}{E_0}}{\left(1 + \frac{\sigma_0}{\sigma_p}\right)^{1/2} d} \left[1 + \frac{d}{d_s} (2N\bar{V}\sigma_p)^{-1} \frac{2}{\left[1 + \left[H(N\bar{V}\sigma_p)^{-1}\right]^{1/2}\right]} \right] \quad (3.5)$$

where d , the volume disadvantage factor, is defined as the ratio of the average moderator flux to the average flux in the lump, and d_s , the surface disadvantage factor, is defined as the ratio of average moderator flux to the flux at the surface of the fuel lump. The ratio d/d_s is then the ratio of surface to average flux in the fuel lump.

Expressions for d and d_s based on diffusion theory have been given in the literature.⁵ Based on them and Eq. (3.5) the geometric dependence of the effective resonance integral of U^{238} has been analyzed in detail and good agreement with experiment has been obtained.⁶

4. The Doppler Effect in Heterogeneous Absorption

The Doppler effect has been treated sketchily for the case $N\sigma_p \bar{L} \gg 1$ and in considerable detail for the case $N\sigma_p \bar{L} \ll 1$. In the second case I is given by Eq. (3.3) with the Doppler broadened line shape, i.e.,

$$I = \frac{1}{N\bar{L}} \frac{\int \gamma}{E_0} \int_0^{\infty} r(b) dl \int_0^{\infty} dx \left\{ 1 - \exp \left[-N\sigma_0 l \psi(\theta, x) \right] \right\} \quad (4.1)$$

The inner integral (over x) has been studied numerically for the Doppler broadened line shape.⁷ As pointed out previously, it can be expressed in terms of Bessel functions of imaginary argument for the natural line shape. It has the property that for $N\sigma_0 \bar{L}$ either $\ll 1$ or $\gg 1$ it approaches the value determined by the natural line shape. In the first case, if the exponential is expanded the leading term is proportional to $\psi(\theta, x)$; the integral is then the same as for the natural line shape. The result for I is $\frac{\pi}{2} \frac{\sigma_0 \sqrt{\gamma}}{E_0}$ which is equal to $\int \sigma_a du$ over a Breit-Wigner resonance and is the $\bar{L} = 0$ limit of I . For $N\sigma_0 \bar{L} \gg 1$ the important part of $\psi(\theta, x)$ is in the wings and there it coincides with the natural line shape. Equation (3.4) is the result for I in this limit. The second integral has also been considered in detail for slabs, spheres, and cylinders.⁸

In the case $N\sigma_p \bar{L} \gg 1$ Eq. (3.5) can be written in a form similar to Eq. (3.3) of Chapter 2, viz.,

$$I = \frac{\sigma_p \int \gamma}{E_0} \frac{J(\theta, \rho)}{d} \left[1 + \frac{d}{d_s} (N\bar{L}\sigma_p)^{-1} \frac{K(\theta, \rho)}{J(\theta, \rho)} \right] \quad (4.2)$$

where

$$K(\theta, \rho) = \int_0^{\infty} \left(\frac{\gamma(\theta, x)}{\gamma(\theta, x) + \rho} \right)^2 dx \quad (4.3)$$

In obtaining Eq. (4.2) the exponential in Eq. (2.6) was equated to zero ($N\sigma_p \bar{l} \gg 1$). Again as $\rho \rightarrow 0$ or ∞ $K(\theta, \rho)$ approaches the value appropriate to the natural line shape, viz.,

$$K(\theta, \rho) = \frac{\pi/4}{(\rho + 1) \sqrt{\rho(\rho + 1)}} \quad (4.4)$$

For the natural line shape then

$$\frac{K(\theta, \rho)}{J(\theta, \rho)} = \frac{1}{2(1 + \rho)} \quad (4.5)$$

Also $K(\theta, \rho)/J(\theta, \rho) < 1$. Figure (3.1) is a plot of $K(\theta, \rho)/J(\theta, \rho)$ vs. ρ for several values of θ . The curve marked $\theta = \infty$ is Eq. (4.5). In many practical cases $\rho \ll 1$, and using the natural line shape is acceptable. Then, since $\rho \ll 1$, $K/J = 1/2$.

5. Thick Resonances

The foregoing theory has been based on the restrictive assumption that the absorption resonances are thin compared to the average lethargy loss in collision with absorber atoms. It is fortunate that in almost all practical situations this condition is well fulfilled. Less can be said when the resonances are thick, i.e., $\Gamma/E \gtrsim \xi$, but under certain conditions the problem can be treated as though the neutrons diffused in the lump monoenergetically.⁹

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Fig. 3.1

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E.g., if the resonance is so thick that even if the neutron makes several collisions its energy will not change enough to produce a significant variation in its cross sections. In such a case the absorption can be written

$$A = \frac{\phi}{4} S \int (1 - \beta) du \quad (5.1)$$

Here β is the albedo as a function of energy, calculated at each energy by monoenergetic diffusion (or transport) theory. All neutrons are presumed to remain at the energy they had when they entered the lump through the surface. A convenient expression for $I (= \frac{A}{NV\phi})$ can be obtained by noting that at each energy

$$\frac{\phi}{4} S(1 - \beta) = NV \sigma_a \bar{\phi} \quad (5.2)$$

where ϕ is the flux in the moderator, and $\bar{\phi}$ is the average flux in the lump.

I is then given by

$$I = \frac{1}{N\bar{L}} \int (1 - \beta) du = \frac{\int \sigma_a du}{(\phi/\bar{\phi})} \quad (5.3)$$

where $\phi/\bar{\phi}$ is the surface to average flux in the lump.

Let us consider now the case of a strongly absorbing resonance line ($\Gamma_r \gg \Gamma_n$) in a large lump ($N\sigma_p \bar{L} \gg 1$) with natural line shape. In the center of such a line, where $\sigma_a \gg \sigma_p$, $\beta \ll 1$. This last condition will prevail until $\sigma_a \sim \sigma_p$ which occurs at $x \sim (\sigma_0/\sigma_p)^{1/2}$. Hence, the contribution to I from the center of the line is

$$I = \frac{1}{N\sigma_p} \frac{\sigma_0 \Gamma_0 / E_0}{(\sigma_0 / \sigma_p)^{1/2}} \quad (5.4)$$

When $x > (\sigma_0 / \sigma_p)^{1/2}$, $\sigma_p \geq \sigma_a$. We can then calculate the contribution to I from the first equality in Eq. (5.3) choosing $\phi/\bar{\phi}$ from diffusion theory. For a slab geometry

$$\frac{\phi}{\bar{\phi}} = y \coth y \quad \text{where } y = \sqrt{3 \Sigma_a \Sigma_p} \frac{T}{2} \quad (5.5)$$

and T is the slab thickness and scattering has been assumed entirely due to Σ_p . Thus the contribution to I from the resonance wings, $x \geq (\sigma_0 / \sigma_p)^{1/2}$, can be written after rearrangement as

$$I = \frac{\sigma_0 \Gamma_0 / E_0}{(\sigma_0 / \sigma_p)^{1/2}} \frac{1}{\eta} \int_0^{\eta} \frac{dy}{y} \tanh y \quad (5.6)$$

where $\eta = 1/2 \sqrt{3} N\sigma_p T$. Since we have assumed $N\sigma_p T \gg 1$, Eq. (5.6) can be further simplified by noting that asymptotically the integral is given by $\ln \eta$. Similar results can be written for spherical and cylindrical geometries.

If $N\sigma_p T \ll 1$ and $\Gamma_0 \gg \Gamma_n$ then an expression for I can be obtained similarly by noticing that the condition $\sigma_a \sim \sigma_p$ should be replaced by the condition $N\sigma_a T \sim 1$. Then the contribution to I from the wings equals the contribution from the line center and the total I is given by

$$I = 2 \frac{(\sigma_0 \Gamma_0^2)^{1/2}}{E_0} (N\bar{L})^{-1/2} \quad (5.7)$$

Equations (5.4), (5.6), and (5.7) give the correct dependence of I on the resonance parameters, but do not have the right numerical factors, due to the arbitrary division of the line into center and wings. E.g., an exact solution to the second case we have considered, viz., $N\sigma_p L \ll 1$, $\Gamma_r > \Gamma_n$, is given by Eq. (2.6) with $\sigma_p' = \sigma_s = 0$. The result is Eq. (3.4) which is the same as Eq. (5.7) except for a factor of $\sqrt{\pi}$ instead of 2.

On the other hand consider a resonance which is not strongly absorbing, i.e., $\Gamma_r \lesssim \Gamma_n$ in a large lump ($N\sigma_p L \gg 1$) and with natural line shape. Again we split the line into a center and wings according as x is $<$ or $>$ $(\sigma_s/\sigma_p)^{1/2}$. In the center we use diffusion theory with $\phi/\bar{\phi}$ given by Eq. (5.5) (for slabs) with $y = \sqrt{3\Delta_a \Sigma_B} T/2$, while in the wings we use $y = \sqrt{3\Sigma_a \Sigma_p} T/2$. The center contribution is the same as Eq. (5.4) multiplied by $\left(\frac{\Gamma_r^2}{3\Gamma_r \Gamma_n}\right)^{1/2}$; the contribution from the wings is the same as Eq. (5.6).

The techniques employed in this section can be used to analyze resonance absorption in any situation and yield results which have the correct dependence on the resonance parameters and which are correct except for a numerical factor of the order of magnitude of unity. Indeed, these techniques applied to the thin resonance problem exactly produce Eq. (3.2) in the limit $N\sigma_p \gg 1$ with the exception that the second term is too large by a factor of $4/\pi$. Similar considerations apply to Eq. (5.7) as previously noted.

References

1. The theory of lumped resonance absorption for the case of thin resonances is due to E. P. Wigner, E. Creutz, H. Jupnik, and T. Snyder, J. Appl. Phys. 26 260 (1955).
2. Deviations from the cosine distribution due to mutual shielding of fuel lumps has been considered by S. M. Dancoff and M. Ginsburg, CP-2157, Oct. 21, 1944.
3. The x integral appearing in Eq. (3.3), which is the integral reference to, can be evaluated exactly in terms of Bessel functions of imaginary argument, R. Ladenburg and F. Reiche, Ann Physik 42 181 (1913). However, when $N\sigma_0 Z \gg 1$ we can use the approximate method subsequently given in the text, which is due to I. Gurevich and I. Pomeranchouk, Geneva Paper P/649 (1955). The result (3.4) is due to the latter authors.
4. R. F. Christy, A. M. Weinberg, and E. P. Wigner, CP-2062, 1944.
5. R. F. Christy, et al., loc. cit. Cf. also Reactor Handbook, Vol. I, AECD-3645, Ch. 1.5.
6. L. Dresner, Nuc. Sci. and Engr. 1 No. 4 (1956)(in press).
7. G. VonDardel and R. Persson, Nature 170 1117 (1952). E. Melkonian, W. W. Havens, Jr., and L. J. Rainwater, Phys. Rev. 92 702 (1953).
8. G. M. Roe, KAPL-1241, Oct. 15, 1954.
9. This problem has been treated by S. M. Dancoff and M. Ginsburg, CP-1589, April 17, 1944. The approach of this section largely parallels their own. Quite similar work by A. D. Galanin is reported by Gurevich and Pomeranchouk, loc. cit.

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