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QUARK MODEL PREDICTIONS FOR HADRON DECAYS

II. ELECTROMAGNETIC DECAYS⁺

by

A. Dar^{*}

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QUARK MODEL PREDICTIONS FOR HADRON DECAYS

II. ELECTROMAGNETIC DECAYS

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A. Dar

Recently Johnson, Low and Suura¹⁾ and independently Cabibbo Maiani and Preparata²⁾ have shown that in order to build a theory of hadrons based on elementary constituents such that the radiative corrections to their β decays are finite, it is sufficient that the following conditions be satisfied:

- 1) All the elementary constituents (which are assumed to be spin $1/2$ particles) have charge 0 or ± 1 .
- 2) The transition current in β decay is of the form

$$\gamma_\mu (1 - \gamma_5) \text{ for } (Q=0) \rightarrow (Q=+1)$$

$$\gamma_\mu (1 + \gamma_5) \text{ for } (Q=1) \rightarrow (Q=0)$$

In view of the experimental situation such a model should also give rise to the well established hadron multiplets (the 56 Baryon multiplet and the 36 Meson multiplet) and to the well established part of the current algebra i.e. the local chiral $SU(3) \times SU(3)$.

Such a model was suggested²⁾ by Cabibbo et al.³⁾. The model is based on three fermion triplets which these authors called S, U and B; where S and U are Sakata triplets, while B is displaced downward by one unit of charge. Their quantum numbers are listed in Table I.

The Gell-Mann Zweig quark model⁴⁾ does not satisfy conditions 1) and 2). However, as pointed out¹⁾, conditions 1), and 2) are sufficient but not necessary and the radiative corrections to the β decays may be finite in the GZ quark model due, for example, to high energy modifications of conventional quantum electrodynamics and/or weak couplings.

The purpose of this note is to compare the predictions of the two models for the electromagnetic decay modes of the fundamental pseudoscalar (P) and vector meson (V) nonets, and of the fundamental baryon octet (B) and decuplet (D).

A non relativistic quark model⁵⁾ was used to calculate the relevant coupling constants entering the relativistic formulae for the partial decay widths. The electromagnetic vertices that were considered divide into two types: In the first type a quark reverses its spin direction and emits a photon. In the second type a quark-antiquark pair annihilates into a photon. In the nonrelativistic limit the quark spin flip vertex involves only the effective magnetic moment of the quark (see Tables I, II). The annihilation vertex involves the charges (see Tables I, II) and the relative wave function of the quark-antiquark pairs⁶⁾ (at the origin).

Radiative transitions

Five types of radiative transitions were considered: $V \rightarrow P + \gamma$, $P \rightarrow V + \gamma$, $D \rightarrow B + \gamma$, $B \rightarrow B' + \gamma$ and $P \rightarrow P' + 2\gamma$. The partial decay widths for the first four transitions are given by

$$\Gamma(V \rightarrow P + \gamma) = \frac{1}{3\pi} \mu_{VP}^2 k^3, \quad (1a)$$

$$\Gamma(P \rightarrow V + \gamma) = \frac{1}{\pi} \mu_{VP}^2 k^3, \quad (1b)$$

$$\Gamma(D \rightarrow B + \gamma) = \frac{1}{2\pi} \mu_{DB}^2 \frac{E_B}{M_D} k^3, \quad (1c)$$

$$\Gamma(B \rightarrow B' + \gamma) = \frac{1}{\pi} \mu_{BB'}^2 \frac{E_{B'}}{M_B} k^3. \quad (1d)$$

The quark model predictions for the transition moments are given by⁷⁾

$$\mu_{VP} = \left\langle P \left| \sum_q \mu_q \sigma_{3q} \right| V(m_J = 0) \right\rangle, \quad (2a)$$

$$\mu_{DB} = \left\langle B(m_J=1/2) \left| \sum_q \mu_q \sigma_{3q} \right| D(m_J=1/2) \right\rangle, \quad (2b)$$

$$\mu_{BB'} = \left\langle B'(m_J=1/2) \left| \sum_q \mu_q \sigma_{3q} \right| B(m_J=1/2) \right\rangle, \quad (2c)$$

and are summarized in Tables III, IV. These tables also include a comparison between the experimental results and the theoretical predictions of the GZ and CMP quark models.

A two stage mechanism was assumed⁸⁾ for the $P \rightarrow P' + 2\gamma$ decay mode:

$P \rightarrow \gamma + (V \rightarrow P' + \gamma)$. The associated diagram is described in Figure 1.

The corresponding matrix element is given by:

$$M = \frac{1}{\sqrt{2}} \left[\sum_V 2\mu_{VP} \epsilon^{\alpha\beta\sigma\mu} k_{1\alpha} \epsilon_{\beta}^{(\gamma_1)}(k_1) \left[\frac{g_{\mu\nu} + q_{\mu} q_{\nu} / m_V^2}{q^2 + m_V^2} \right] 2\mu_{VP'} \epsilon^{\alpha'\beta'\sigma'\nu} k_{2\alpha'} \epsilon_{\beta'}^{(\gamma_2)}(k_2) q_{\sigma'} \right. \\ \left. + \sum_V 2\mu_{VP} \epsilon^{\alpha\beta\sigma\mu} k_{2\alpha} \epsilon_{\beta}^{(\gamma_2)}(k_2) \left[\frac{g_{\mu\nu} + q_{\mu} q_{\nu} / m_V^2}{q^2 + m_V^2} \right] 2\mu_{VP'} \epsilon^{\alpha'\beta'\sigma'\nu} k_{2\alpha'} \epsilon_{\beta'}^{(\gamma_1)}(k_1) q_{\sigma'} \right]. \quad (3)$$

No analytic form has been found for the $P \rightarrow P' + 2\gamma$ partial decay width. For the $\eta \rightarrow \pi^0 + 2\gamma$ decay mode, a numerical calculation yields

$$\Gamma(\eta \rightarrow \pi^0 + 2\gamma) = \begin{cases} 0.5 \text{ eV} & \text{GZ quark model,} \\ 0.5 \text{ eV} & \text{CMP quark model,} \end{cases} \quad (4)$$

These numbers are far below the experimental estimate.

Annihilation transitions

$$\underline{V^0 \rightarrow l^+ + l^-}$$

In the Nambu-Sakurai model⁹⁾ the vector mesons are coupled directly to the photon via the effective coupling

$$e m_V^2 f_{V\gamma} \epsilon_{\mu}^{(V)} \epsilon_{\mu}^{(\gamma)}, \quad (5)$$

where $f_{V\gamma}$ is a dimensionless coupling constant. The decay $V^0 \rightarrow l^+ + l^-$

is considered then as a two stage process, $V^0 \rightarrow \gamma \rightarrow \ell^+ + \ell^-$, and the partial decay width is given by⁹⁾

$$\Gamma(V^0 \rightarrow \ell^+ + \ell^-) = \frac{4\pi}{3} \left(\frac{e^2}{4\pi} \right)^2 f_{V\gamma}^2 m_V \left[1 - \left(\frac{2m_\ell}{m_V} \right)^2 \right]^{1/2} \left[1 + 2 \left(\frac{m_\ell}{m_V} \right)^2 \right]. \quad (6)$$

To illustrate the quark model calculation of $f_{V\gamma}$ we specialize to the GZ quark model and the $\rho^0 \rightarrow \ell^+ + \ell^-$ decay. The diagram associated with the decay is described in Figure 2. The ρ^0 is considered as a bound quark-anti-quark pair. Its nonrelativistic relative motion is described by a properly normalized spatial wave function $\Psi_{\rho^0}(r)$. Using the ρ^0 wave function and noting that only its ± 1 polarization states contribute to the decay, the effective matrix element can be written as (assuming that only small relative momenta are important),

$$M(M_J = \pm 1) = e^2 A_{\rho^0} \bar{V}_q(0, \pm 1/2) \gamma_\mu U_q(0, \pm 1/2) \frac{1}{k} \bar{U}_\ell(\vec{p}_f, r) \gamma^\mu V_\ell(\vec{q}_f, r'), \quad (7)$$

where

$$A_{\rho^0} = \psi_{\rho^0} / \sqrt{2}. \quad (8)$$

The partial decay width is given then by Eq. (6) with

$$f_{\rho^0\gamma} = \sqrt{2} \psi_{\rho^0}(0) / m_\rho^{3/2}. \quad (9)$$

Using the $\psi_\rho(0)$ value⁶⁾ one obtains

$$f_{\rho^0\gamma} = \frac{m_\pi}{m_\rho} = 0.175 . \quad (10)$$

The ρ dominance model for the isovector form factor of the pion¹⁰⁾ yields

$$f_{\rho^0\gamma}^{-1} = f_{\rho^0\pi\pi}^{-1} , \quad (11)$$

where $f_{\rho\pi\pi}$ can be determined from the ρ^0 width

$$\Gamma(\rho^0 \rightarrow \pi^+ + \pi^-) = \frac{2}{3} \left(\frac{f_{\rho^0\pi\pi}^2}{4\pi} \right) \frac{k^3}{m_\rho^2} . \quad (12)$$

From the mass and width of the ρ

$$f_{\rho^0\pi\pi}^{-1} = 0.172 , \quad (13)$$

which is in excellent agreement with (10). The other vector meson-photon couplings can be derived in the same way and are listed in Table V. A comparison between the theory and the experimental results is also summarized in Table V.

$P \rightarrow 2\gamma$

Following Gell-Mann, Sharp and Wagner¹¹⁾ a two stage mechanism is associated with the $P \rightarrow 2\gamma$ decay; $P \rightarrow \gamma + (V \rightarrow \gamma)$.

The partial decay width is given by¹²⁾

$$\Gamma(P \rightarrow 2\gamma) = \frac{1}{4} \left(\frac{e^2}{4\pi} \right) m_P^3 \left[\sum_V \mu_{VP} f_{V\gamma} \right]^2, \quad (14)$$

with the values of μ_{VP} and $f_{V\gamma}$ listed in Tables III and V, respectively. A comparison with the experimental results is summarized in Table VI.

$$\underline{\omega \rightarrow \pi^+ + \pi^-, \quad \phi \rightarrow \pi^+ + \pi^-}$$

The ω and ϕ vector mesons cannot decay strongly into 2π because of G parity conservation in strong interactions. However, these decays may proceed, as pointed out by Glashow¹³⁾ by a small $\rho\omega$ and $\rho\phi$ electromagnetic mixing: $\omega, \phi \rightarrow \gamma \rightarrow \rho \rightarrow 2\pi$. The partial decay width is given then by⁹⁾

$$\Gamma(V_{e.m.}^0 \rightarrow \pi^+ + \pi^-) = c^2 \frac{\pi}{3} \left(\frac{e^2}{4\pi} \right) f_{V\gamma}^2 m_V \left[1 - \frac{4m_\pi^2}{m_V^2} \right]^{3/2} \left| \frac{m_\rho^2}{m_\rho^2 - m_V^2 - im_\rho \Gamma_\rho} \right|^2, \quad (15)$$

$$c = f_{\rho\gamma} f_{\rho\pi\pi}.$$

The theoretical predictions are summarized in Table VII.

"Mixed Transitions"

$$\underline{\eta \rightarrow 2\pi + \gamma}$$

Following Gell-Mann, Sharp and Wagner¹¹⁾ a two stage mechanism is associated with the $\eta \rightarrow 2\pi + \gamma$ decay; $\eta \rightarrow \gamma + (\rho \rightarrow 2\pi)$. The Feynman

diagram representing this mechanism is described in Figure 3. The corresponding matrix element is given by

$$M = 2\mu_{\rho\eta} \epsilon^{\alpha\beta\sigma\mu} k_{\alpha} \epsilon_{\beta}^{(\gamma)} q_{\sigma} \left[\frac{g_{\mu\nu} + q_{\mu} q_{\nu} / m_{\rho}^2}{q^2 + m_{\rho}^2} \right] (p^+ - p^-)^{\nu} f_{\rho\pi\pi} . \quad (16)$$

The partial decay width is given by⁹⁾

$$\Gamma(\eta^0 \rightarrow \pi^+ + \pi^- + \gamma) = 0.39 \mu_{\rho\eta}^2 \left(\frac{f_{\rho\pi\pi}^2}{4\pi} \right) \frac{m_{\pi}^5 m_{\eta}^2}{3\pi^2 m_{\rho}^4} = 96 \text{ eV}. \quad (17)$$

$$\underline{\eta \rightarrow 3\pi^0, \eta \rightarrow \pi^+ \pi^- \pi^0}$$

The simplest mechanism that can be associated with these decay modes is probably a mechanism where an electromagnetic mixing of the η^0 and π^0 particles precedes or follows a pole-type-diagram decay.¹⁴⁾ This mechanism is described in Figure 4. It introduces a new nonet of $\pi\pi$ S-wave resonance (σ) and thus new parameters. However, even with reasonable values for the mass and width of the σ particle the $\eta\pi$ mixing angle as calculated from a naive quark model¹⁵⁾ is too small to account for the magnitudes of the $\eta \rightarrow 3\pi$ partial decay widths.

As evident from Tables III-VII the radiative transition moments are correctly predicted by both models whereas the annihilation transitions are more in agreement with the predictions of the GZ quark model than with the predictions of the CMP quark model.

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TABLE I
CABIBBO-MAIANI-PREPARATA QUARK MODEL

	S_p	S_n	S_λ	U_p	U_n	U_λ	B_p	B_n	B_λ
Q	1	0	0	1	0	0	0	-1	-1
T	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
C^*	1	1	1	1	1	1	-2	-2	-2
μ	μ_P	0	0	μ_P	0	0	0	$-\mu_P$	$-\mu_P$

* The charge is related to T_3 , Y and the charm quantum number C
(see M. Y. Han and Y. Nambu, Phys. Rev. 139 (1965) B1006)
through $Q = T_3 + \frac{Y}{2} + \frac{C}{3}$.

TABLE II
GELL-MANN ZWEIG QUARK MODEL

	P	n	λ
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
T	$\frac{1}{2}$	$\frac{1}{2}$	0
T_3	$\frac{1}{2}$	$-\frac{1}{2}$	0
Y	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$
μ	$\frac{2}{3}\mu_P$	$-\frac{1}{3}\mu_P$	μ_Λ^*

* We introduce u spin breaking effects by taking

$\mu_\lambda = \mu_\Lambda$ rather than the symmetric value

$$\mu_\lambda = -\frac{1}{3}\mu_P$$

TABLE III

Transition moments and partial widths for the radiative transitions within the fundamental 36 meson multiplet of SU(6).

Decay Mode	μ_{VP} (Transition Moment)		Partial Width		
	GZ	CMP	GZ	CMP	Experiment
$\rho^+ \rightarrow \pi^+ + \gamma$	$-\frac{1}{3} \mu_P$	$-\frac{1}{3} \mu_P$	0.12 MeV	0.12 MeV	
$\rho^0 \rightarrow \pi^0 + \gamma$	$-\frac{1}{3} \mu_P$	$-\frac{1}{3} \mu_P$	0.12 MeV	0.12 MeV	< 0.6 MeV
$\rho^- \rightarrow \pi^- + \gamma$	$-\frac{1}{3} \mu_P$	$-\frac{1}{3} \mu_P$	0.12 MeV	0.12 MeV	
$\rho^0 \rightarrow \eta^0 + \gamma$	$-\frac{1}{\sqrt{3}} \mu_P$	$-\frac{1}{\sqrt{3}} \mu_P$	35 KeV	35 KeV	
$K^{*+} \rightarrow K^+ + \gamma$	$-\frac{2}{3} \mu_P - \mu_\Lambda$	$-\frac{1}{3} \mu_P$	88^{+32}_{-26} KeV	70 KeV	
$K^{*0} \rightarrow K^0 + \gamma$	$\frac{1}{3} \mu_P - \mu_\Lambda$	$\frac{2}{3} \mu_P$	$0.19^{+0.03}_{-0.05}$ MeV	21 MeV	
$\bar{K}^{*0} \rightarrow \bar{K}^0 + \gamma$	$\frac{1}{3} \mu_P - \mu_\Lambda$	$\frac{2}{3} \mu_P$	$0.19^{+0.03}_{-0.05}$ MeV	21 MeV	
$K^{*-} \rightarrow K^- + \gamma$	$-\frac{2}{3} \mu_P - \mu_\Lambda$	$-\frac{1}{3} \mu_P$	88^{+32}_{-26} KeV	70 KeV	
$\omega^0 \rightarrow \pi^0 + \gamma$	$-\mu_P$	$-\mu_P$	1.17 MeV	1.17 MeV	1.15 ± 0.25 MeV
$\omega^0 \rightarrow \eta^0 + \gamma$	$-\frac{1}{3\sqrt{3}} \mu_P$	$-\frac{1}{3\sqrt{3}} \mu_P$	6.2 KeV	6.2 KeV	< 0.2 MeV
$\varphi^0 \rightarrow \eta^0 + \gamma$	$2\sqrt{\frac{2}{3}} \mu_\Lambda$	$-\frac{2}{3}\sqrt{\frac{2}{3}} \mu_P$	$0.14^{+0.06}_{-0.07}$ MeV	21 MeV	< 0.4 MeV
$\varphi^0 \rightarrow x^0 + \gamma$	$\frac{2}{\sqrt{3}} \mu_\Lambda$	$-\frac{2}{3\sqrt{3}} \mu_P$	$0.29^{+0.18}_{-0.14}$ KeV	44 KeV	
$x^0 \rightarrow \rho^0 + \gamma$	$\sqrt{\frac{2}{3}} \mu_P$	$\sqrt{\frac{2}{3}} \mu_P$	0.26 MeV	0.26 MeV	< 1.0 MeV
$x^0 \rightarrow \omega^0 + \gamma$	$\frac{1}{3}\sqrt{\frac{2}{3}} \mu_P$	$\frac{1}{3}\sqrt{\frac{2}{3}} \mu_P$	18 KeV		

TABLE IV

Transition moments and partial widths for the radiative transitions within the fundamental 56 Baryon multiplet of SU(6).

Decay Mode	Transition Moments		Partial Width		
	GZ	CMP	GZ	CMP	Experiment
$N^{*+} \rightarrow P + \gamma$	$\frac{2\sqrt{2}}{3} \mu_P$	$\frac{2\sqrt{2}}{3} \mu_P$	0.40 MeV	0.40 MeV	0.55 ± 0.10 MeV
$N^{*0} \rightarrow N + \gamma$	$-\frac{2\sqrt{2}}{3} \mu_P$	$-\frac{2\sqrt{2}}{3} \mu_P$	0.40 MeV	0.40 MeV	
$Y^{*+} \rightarrow \Sigma^+ + \gamma$	$\frac{2\sqrt{2}}{3} \left(\frac{2}{3} \mu_P - \mu_\Lambda \right)$	$\frac{2\sqrt{2}}{3} \mu_P$	0.11 ± 0.02 MeV	0.13 MeV	
$Y^{*0} \rightarrow \Lambda^0 + \gamma$	$\sqrt{\frac{2}{3}} \mu_P$	$\sqrt{\frac{2}{3}} \mu_P$	0.26 MeV	0.26 MeV	
$Y^{*0} \rightarrow \Sigma^0 + \gamma$	$\frac{\sqrt{2}}{3} \left(\frac{1}{3} \mu_P - 2\mu_\Lambda \right)$	$\frac{\sqrt{2}}{3} \mu_P$	25^{+6}_{-4} KeV	32 KeV	
$Y^{*-} \rightarrow \Sigma^- + \gamma$	$-\frac{2\sqrt{2}}{3} \left(\frac{1}{3} \mu_P + \mu_\Lambda \right)$	0	$1.0^{+2.9}_{-1.0}$ KeV	0	
$\Xi^{*0} \rightarrow \Xi^0 + \gamma$	$-\frac{2\sqrt{2}}{3} \left(\frac{2}{3} \mu_P - \mu_\Lambda \right)$	$-\frac{2\sqrt{2}}{3} \mu_P$	0.19 ± 0.03 MeV	0.22 MeV	
$\Xi^{*-} \rightarrow \Xi^- + \gamma$	$-\frac{2\sqrt{2}}{3} \left(\frac{1}{3} \mu_P + \mu_\Lambda \right)$	0	$1.6^{+4.1}_{-1.6}$ KeV	0	
$\Sigma^0 \rightarrow \Lambda + \gamma$	$-\frac{1}{\sqrt{3}} \mu_P$	$-\frac{1}{\sqrt{3}} \mu_P$	8.9 KeV	8.9 KeV	> 0.06 eV

TABLE V

Coupling constants and partial decay widths for the Dalitz-pair decay mode of the vector mesons.

Decay Mode	$f_{V\gamma}$		Partial Width		
	GZ	CMP	GZ	CMP	Experiment
$\rho^0 \rightarrow \mu^+ + \mu^-$	$\frac{m_\pi}{m_\rho}$	$\frac{\sqrt{3} m_\pi}{m_\rho}$	5.8 KeV	17.4 KeV	$(5.5^{+2.6}_{-1.1})$ KeV
$\rho^0 \rightarrow e^+ + e^-$			5.8 KeV	17.4 KeV	$(5.5^{+2.3}_{-2.3})$ KeV
$\omega^0 \rightarrow \mu^+ + \mu^-$	$\frac{1}{3} \frac{m_\pi}{m_\omega}$	$\frac{1}{\sqrt{3}} \frac{m_\pi}{m_\rho}$	0.63 KeV	1.89 KeV	(57^{+27}_{-22}) KeV
$\omega^0 \rightarrow e^+ + e^-$			0.63 KeV	1.89 KeV	
$\phi^0 \rightarrow \mu^+ + \mu^-$	$\frac{-\sqrt{2}}{3} \frac{m_\pi}{m_\rho}$	$-\sqrt{\frac{2}{3}} \frac{m_\pi}{m_\phi}$	0.97 KeV	2.91 KeV	< 10 KeV
$\phi^0 \rightarrow e^+ + e^-$			0.97 KeV	2.91 KeV	

TABLE VI

Partial decay widths for the $P \rightarrow 2\gamma$ decay mode.

Decay Mode	Partial Width		
	GZ	CMP	Experiment
$\pi^0 \rightarrow 2\gamma$	13.5 eV	40.5 eV	$9. \pm 2.5$ eV
$\eta^0 \rightarrow 2\gamma$	$\begin{pmatrix} 0.48 & +0.08 \\ & -0.10 \end{pmatrix}$ KeV	0.73 KeV	1.0 ± 0.5 KeV
$\omega^0 \rightarrow 2\gamma$	$\begin{pmatrix} 11.0 & +0.5 \\ & -0.4 \end{pmatrix}$ KeV	53 KeV	

TABLE VII

Partial decay widths for the e.m. decay mode $V^0 \rightarrow \pi^+ + \pi^-$.

Decay Mode	Partial Width		
	GZ	CMP	Experiment
$\omega^0 \rightarrow \pi^+ + \pi^-$	4.5 KeV	40.5 KeV	
$\phi^0 \rightarrow \pi^+ + \pi^-$	0.31 KeV	2.79 KeV	

FOOTNOTES AND REFERENCES

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