ASAE-S-8

Division of Reactor Development
U. S. Atomic Energy Commission

Contract No. AT(04-3)-109

Project SIZE-UP

1 January 1958

PLUTONIUM RECYCLE
IN THE CALDER HALL TYPE REACTOR

By: L. J. Barbieri
J. W. Webster
K. T. Chow

Approved:
C. M. Rice, Manager
Reactor Engineering Department

Atomic Energy Division
American Radiator & Standard Sanitary Corporation

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

TABLE OF CONTENTS

	Page
Acknowledgement	
Abstract	
I Introduction	1
II Results and Discussions	
A Possible Recycle Schemes	3
B Comparison Case of Selling Plutonium	4
C Results of Scheme A	6
D Results of Scheme B	12
E Results of Scheme C	14
F Results of Scheme D	14
III Effect of Non-Uniform Flux	17
IV Reactor Parameters and Cross-Section Data	20
A Calder Hall and Its Reactor Physics Parameters	20
B The Cross Sections	24
C Numerical Calculations	43
D Comparison of Analytic and Numeric Codes	45
V Solutions of Equations for Isotopic Concentrations	
A The Differential Equations and Solutions	46
B Reactivity Lifetime	55
VI Solution for Reactivity Life	58
Appendix	
A Fabrication Cost of Scheme B Type Fuel Element	103

ACKNOWLEDGEMENT

The programming and machine-computing help of Mr. James Burr and Mr. Roy Morrison, the computational help of Miss Edna Williams, the consulting help from Mr. David Herron on reactor-power economics and from Mr. Ed Leshan, and Dr. Gene Thompson on numerical techniques, and the cost estimating on fuel-element fabrication by Dr. Donald Mash, are all gratefully acknowledged.

ABSTRACT

The report AECU-3398, "Comparison of Calder Hall and PWR Reactor Types", compared the estimated cost of electric power (in mills-per-kilowatt-hour) from a "second generation" British gas-cooled Calder Hall design with that from a "second generation" U.S. pressurized-water (Shippingport) design. In the economic computations of AECU-3398 the plutonium produced in the reactor is assumed to be sold at \$12 per gm, the price which will be paid by the U.S. for plutonium produced outside this country, and a plutonium credit (in mills-per-kilowatt-hour) deducted from the power cost accordingly.

It is the purpose of the present report to consider the economics and physics of plutonium recycle in the Calder Hall type reactor. This report is thus in the nature of a supplement to AECU-3398. Four possible schemes of recycle are considered. Scheme A is to blend the plutonium produced in a run with fresh natural uranium for a subsequent run; Scheme B is to alloy the plutonium with some diluent metal and fabricate the alloy into high heat-transfer elements more like MTR or PWR-seed type elements and "spike" a subsequent load of fresh natural uranium elements with these plutonium elements; Scheme C is to recycle half the spent uranium as well as the plutonium; and Scheme D is to take advantage of the extra reactivity gained from the recycled plutonium to decrease the lattice spacing, thereby increasing the conversion ratio with the hope of increasing the attainable exposure.

The economic calculations indicate that Schemes C and D are not as attractive as A and B. In Scheme A the conclusion is that if plutonium can be blended into uranium at a weight percent up to about 0.7% and the blended fuel fabricated into the same type elements as presently used in Calder Hall at a cost of less than about \$33 per kg of blend, then recycling will compete with the mode of operation where the plutonium is sold at the end of each run for \$12 per gm. In Scheme B the results indicate that if the plutonium can be alloyed with diluent metal and fabricated into high-heat-transfer elements for about \$4.50 per gm of contained plutonium, then recycling will compete. The estimate for these fabrication costs is \$18 per kg of blend for Scheme A; and \$11 per gm of plutonium in Scheme B if the diluent metal were aluminum but considerably more than this if it were zirconium.

The conclusions, based on these results, are that Scheme A, i.e. blending the plutonium with fresh natural uranium, will be the most economic means of recycle; recycle by Scheme A will compete very favorably with the mode of operation where the plutonium is sold at the end of each run for \$12 per gm; or viewed in another way, with natural uranium having its current value and lease charge, the fuel value of plutonium for recycling, with all costs considered, will be greater than \$12 per gm. Schemes B, C, and D do not look as attractive for the Calder Hall type reactor as Scheme A.

The results are predicated on the assumption that the fuel elements will withstand exposure levels achievable from the reactivity standpoint, which are as high as 8800 MWD/ton. This is beyond present experience but it is believed that it is not unrealistic to assume that such exposures will be achieved in the future with improved fuel elements.

I. INTRODUCTION

The report AECU-3398, "Comparison of Calder Hall and PWR Reactor Types", compared the estimated cost of electric power in mills-per-kilowatt-hour from a "second generation" British gas-cooled Calder Hall design with that from a "second generation" U.S. pressurized-water (Shippingport) design. In the economic computations of AECU-3398 the plutonium produced in the reactor was assumed to be sold at \$12 per gm and a plutonium credit in mills-per-kilowatt-hour deducted from the power cost accordingly.

Some countries will attach no value to plutonium for military uses. It is of interest, therefore, to consider the value of plutonium, produced by (n, U-238) in a reactor, as additional reactor fuel. It is generally recognized that, eventually, the most economic utilization of reactor-produced plutonium will be achieved in fast reactors since the ratio of capture to fission cross sections is much lower at high energies, and thus in the future a power-reactor complex will exist in most countries consisting of both thermal and fast power reactors. Another way to utilize plutonium produced in a reactor, however, is to recycle it in the same reactor or in other reactors of the same type; and it is clear that many countries will wish to do this, especially in the early years before power reactors become numerous and of varied types.

It is the purpose of this report to consider the economics of plutonium recycle in the Calder Hall type reactor. The report is thus in the nature of a supplement to AECU-3398. It is planned to issue at a later date a second supplement which will discuss plutonium recycle in the PWR.

Although the purpose or objective of such a study as this is one of economics, the bulk of the work and difficulty lies with the physics. For this reason, the emphasis is on the physics methods - the cross-section data, the differential equations of isotope concentration as a function of flux x time (or exposure) and their solution and the use of these solutions together with the buildup of fission products to determine the reactivity-life. The term "reactivity-life" is used here to mean the exposure in MWD/ton which the fuel has sustained at the time when the reactor can no longer be kept critical and it shuts itself down. Radiation damage is not considered as a limiting factor to attainable exposure in this study. It is recognized that the exposures calculated here from the reactivity standpoint,

which are as high as 8800 MWD/ton, represent a considerable advance over radiation damage exposure-limits being experienced at the present time. However, it is believed that through experience and technological advances, exposures of this magnitude will be achieved.

II. RESULTS AND DISCUSSIONS

A. POSSIBLE RECYCLE SCHEMES

Four ways of operating the Calder Hall type reactor as a plutonium recycle reactor are considered. They are as follows:

1. Scheme A

Scheme A is perhaps the simplest scheme of recycle and, as will be shown later, seems to offer the greatest chance of economic success. It is assumed that natural uranium would be used in the first run, without any plutonium being present at the start. When reactivity considerations decree the end of Run 1, then the spent fuel is removed and processed. The uranium which is recovered is sold if it has any value. The plutonium which is recovered is blended with fresh natural uranium for a core loading for Run 2. This process is repeated at the end of Run 2 and all succeeding runs. (It is only for Run 1 that the spent uranium has any value and even there it is small.)

It has been customary in plutonium recycle calculations to consider only the equilibrium situation where the plutonium has been recycled back an infinite number of runs and each run has become an exact repetition of the preceding run. In the Calder Hall type reactor, however, the power density is so low that it would take over a century for this condition to be reached. It was, therefore, decided to study, individually, the first six runs, which would take about 70 years for Scheme A with the reactor power at 345 MW (heat). This power output is that of Stage 1a. of AECU-3398.

2. Scheme B

Scheme B is the same as Scheme A, except that instead of blending the plutonium in with the uranium for the new core loading, the plutonium is alloyed with a diluent metal and fabricated into high-heat-transfer elements of the MTR or PWR-seed type. The plutonium would thus be in separate elements from the uranium. By using high-heat-transfer type elements it would be possible to have most of the reactor volume still occupied by natural uranium. The plutonium elements could (1) be uniformly placed throughout the core, (2) be placed primarily in

the middle of the reactor to enhance reactivity, or (3) be placed near the top and bottom or outer parts of the core to improve the power distribution.

3. Scheme C

In Scheme C one-half of the spent uranium is recycled along with the plutonium. The other one-half is sold if it has any value.

4. Scheme D

When plutonium is recycled, the available reactivity of the reactor is increased considerably. In Schemes A and B, this reactivity is used to offset depletion and fission-product poisoning throughout the run. In Scheme C, on the other hand, part of this reactivity is used to permit the re-use of part of the spent uranium. In Scheme D, part of the reactivity is used in a different way. The lattice pitch is decreased requiring a different reactor or a modification of the original reactor, thereby increasing the conversion ratio. It was hoped that this might increase the attainable exposure, but such did not prove to be the case as will be seen later.

B. COMPARISON CASE OF SELLING PLUTONIUM

In order to have an economic comparison of plutonium recycle with the mode of operation where the plutonium is sold at the end of each run, the economics of the latter will be reviewed at this point.

In ASAE-S-3,¹ the ground rules for estimating reactor power costs are simplified somewhat from those that were used in AECU-3398,² and the ASAE-S-3 procedure is used here throughout. The simplification comes mainly in the computation of the non-nuclear carrying charge with no meaningful accuracy really being lost.

The cost computations are based on a kilogram of fuel, and the following items of cost arise for the advanced Calder Hall reactor:

1. Manufacturing of fuel elements = \$10/kg
2. Shipping of fuel elements to reactor site = \$00.05/kg
3. Shipping of irradiated elements to processing plant = \$5/kg
4. Chemical processing = \$20/kg (two assumptions were used for this in AECU-3398, one higher and one lower)

¹American-Standard, "An Evaluation of Heavy-Water Reactors for Power", October 1957.

²American-Standard, "Comparison of Calder Hall and PWR Reactor Types", March 1957.

5. Shipping of the uranyl nitrate = \$00.05/kg
6. Conversion of the uranyl nitrate to UF_6 = \$1.90/kg (the uranium must be in the form of UF_6 in order to be sold back to the AEC)
7. Shipping of the UF_6 = \$00.15/kg
8. Reduction of plutonium nitrate to metal = \$1.50/gm of Pu
9. Credit for sale of Pu = \$12/gm of Pu (although plutonium produced domestically can, for the next few years, be sold for more than this, it is presumed that the value is to drop to \$12/gm in 1963)

The uranium is rented to the user at 4% per year based on the initial value. The fuel is assumed to be in the possession of the user for six months prior to insertion in the reactor, plus the time it is in the reactor (an 0.85 innage factor is assumed), plus six months after discharge from the reactor. At the time the spent uranium is returned to the AEC the difference between the initial value of \$40/kg and the value of the spent fuel in the form of UF_6 is paid. The value of this spent fuel is calculated from the formula:

$$(\$/kg \text{ of contained U}) = 37.48 \left[(2x-1) \ln \frac{449x}{1-x} + 449 (x-.00222) \right]$$

where x is the weight fraction U-235. In addition to these costs is a non-nuclear carrying charge of 12% per year on the money invested in the fabrication and shipment of the fuel elements. This carrying charge is taken to be in existence for six months prior to insertion of the fuel into the reactor and for one-half of the residence time of the fuel in the reactor. This is a rough way of accounting for the fact that this investment will be retired by means of the revenue gained from the sale of power throughout the run. It is not an important item of cost for the Calder Hall type reactor, although it is very important for reactors with expensive fuel elements.

Using these data and assumptions together with a figure of 2749 MWD/ton for the calculated exposure, 185 tons for the fuel mass, and 345 MW for the thermal power, a result of 2.9 mills/kwh is obtained for fuel-cycle costs.

The 2749 MWD/ton figure is computed for batch discharge and a uniform burnup assumption. This is probably low because it is shown in Section III that the non-uniform vertical flux distribution shortens the average exposure by slightly more than 10%, while the charge-discharge by radial zones should

more than offset this. Since the plutonium recycle cases would be prohibitively complicated to calculate if the charge-discharge by radial zones and non-uniform flux were taken into account, it was decided to base the comparison on the assumption of batch discharge and uniform burnup throughout the study.

Although the interest here is in the fuel-cycle costs only, it is worth pointing out that the capital cost plus site operating costs will add about 15-1/2 mills/kwh in the U.S., and about 6 mills/kwh in Great Britain (see AECU-3398) to the above fuel-cycle cost, giving about 18-1/2 mills/kwh and 9 mills/kwh, respectively, for the total power costs. (AECU-3398 gave slightly lower estimates than this due to a lower assumed value for the processing costs and a slightly higher assumed fuel exposure.)

C. RESULTS OF SCHEME A

As was mentioned previously, the calculations for Scheme A were made for the first six runs. This would correspond to an elapsed time of over 70 years with the reactor at 345 MW (heat). The Pu-239, -240, and -241 have reached equilibrium at this time, but Pu-242 is still far from equilibrium.

In computing the fuel-cycle costs for the six individual runs, there is a choice to be made between assigning processing costs, and other costs and credits associated with discharged fuel to the preceding run or to the succeeding run. It is really a matter of bookkeeping, but it was decided to assign all such costs and credits to the succeeding run on the grounds that the spent fuel is being processed primarily to recover the Pu to use it in the succeeding run and, therefore, the cost of recovering it should be charged to the succeeding run.

Table I gives the results for power costs when Pu is recycled using the Scheme A mode. Since estimates of the fabrication cost for the Pu impregnated uranium elements are considered to be quite unreliable, the power costs are computed for three assumed fabrication costs. It is found that if the fabrication costs is less than about \$33/kg of the fuel blend, then Pu recycling by the Scheme A mode will be competitive with the mode of operation where the Pu is sold at the end of each run for \$12/gm. The fabrication cost for the Scheme A-type elements is estimated to be \$18/kg of the blend.

TABLE I

FUEL-CYCLE COSTS WITH PLUTONIUM RECYCLE
(Scheme A)

Run Number	Reactivity- Life (MWD/Ton)	Pu Concentration (gms Pu/kg U)		Fabrication Cost of the Pu-impreg- nated U Elements (\$/kg of fuel)	Fuel-Cycle Costs (mills/kwh)
		Initial	Final		
1	2749 (Residence time ^a 4.7 years)	0	1.93	10 (No Pu)	3.27 ^b
2	5428 (Residence time 9.4 years)	1.93	3.67	20	2.87
				35	3.53
				50	4.18
3	7517 (Residence time 13 years)	3.67	4.89	20	2.46
				35	2.64
				50	3.53
4	8391 (Residence time 14.5 years)	4.89	5.70	20	2.36
				35	2.86
				50	3.37
5	8713 (Residence time 15.1 years)	5.70	6.31	20	2.33
				35	2.82
				50	3.31
6	8763 (Residence time 15.2 years)	6.31	6.81	20	2.33
				35	2.83
				50	3.31
1 - 6	(Total residence time - 72 years)	--	--	20	2.60 (avg)
				35	2.99 (avg)
				50	3.49 (avg)

^a0.85 innage factor assumed.^bIf the Pu is sold at \$12/gm, the cost is reduced to 2.94 mills/kwh.

To obtain this estimate, consideration is given to the equipment and labor requirements over and above those currently employed to manufacture normal uranium on a routine basis. The low Pu-U alloy possesses an air tolerance which is 10^2 higher than Pu and 10^4 lower than U-238 (approximately 5×10^{-9} gm/m³ for the alloy). Thus, additional containment and tight control of all potential dust or fume producing operations such as casting and machining is indicated. Based upon existing uranium production technology, it is reasonable to assume that the cost of producing the alloy slugs would be no more than double the current uranium slug cost. On this basis the cost of fabricating the Pu-U alloy fuel elements of the Calder Hall type is estimated to be about \$18/kg of blend. The conclusion is that recycle by Scheme A will compete very favorably with the mode of operation where the plutonium is sold at the end of each run for \$12/gm; or viewed in another way, with natural uranium having its current value and lease charge, the fuel value of Pu for recycling, with all costs considered, will be greater than \$12/gm.

Table II gives the physics results for the exposure, and the isotopic content of the fuel at the beginning and the end of each of the six runs.

Figure 1 shows the kilograms of plutonium per ton of fuel throughout the six cycles. It is seen that Pu-239, -240, and -241 are at equilibrium, but Pu-242 is still far from equilibrium.

The decay of the Pu-241, which has a half-life of 13 years, is not taken into account in these calculations, but is treated as a special correction in order to allow ready application of the results to other power levels. It amounts to having about a 16% error in the σ_a for Pu-241 for a reactor having the flux level of Calder Hall. The effect was investigated for Run 2 and was found to shorten the exposure by 11%. Figures 7 and 8 show the effect of applying the correction for the 13-year half-life on the isotopic concentrations and on reactivity lifetime.

Figure 2 shows the variation of the reactivity lifetime MWD/ton for the six runs. The reactivity lifetime has reached a maximum for the fifth and sixth runs. If further runs are calculated, the reactivity-life will show a decrease as Pu-242 builds up to a higher concentration. The majority of the graphs, which pertain to the individual runs, have been placed at the end of the report. Figures 9 through 14 show the behavior of the reactivity

TABLE II
PHYSICS RESULTS FOR PLUTONIUM RECYCLE
(Scheme A or B)

Run Number	Exposure		Isotope	Initial Isotopic Concentrations			Final Isotopic Concentrations			Ratio of Final to Initial Concentrations
	ϕt ($\times 10^{21} \text{ cm}^{-2}$)	MWD/Ton		Nuclei/Ton	KG/Ton	% of Total	Nuclei/Ton	KG/Ton	% of Total	
1 (No initial concentration of plutonium)	0.720	2749	U-235	1.659 (10) ²⁵	6.486	--	1.035 (10) ²⁵	4.047	--	0.624
			U-238	2.278 (10) ²⁷	900.714	--	2.273 (10) ²⁷	898.91	--	0.998
			Pu-239	0	0	--	3.936 (10) ²⁴	1.563	81.3	--
			Pu-240	0	0	--	7.118 (10) ²³	0.284	14.6	--
			Pu-241	0	0	--	1.825 (10) ²³	0.073	3.6	--
			Pu-242	0	0	--	1.742 (10) ²²	0.007	0.5	--
			TOTAL Pu		1.927					
2 (1st plutonium recycle)	1.085	5428	U-235	1.656 (10) ²⁵	6.464	--	0.813 (10) ²⁵	--	--	0.491
			U-238	2.273 (10) ²⁷	898.6	--	2.266 (10) ²⁷	--	--	0.997
			Pu-239	3.936 (10) ²⁴	1.563	81.3	5.877 (10) ²⁴	2.333	63.6	1.493
			Pu-240	7.118 (10) ²³	0.284	14.6	2.032 (10) ²⁴	0.810	22.1	2.855
			Pu-241	1.825 (10) ²³	0.073	3.6	1.015 (10) ²⁴	0.406	11.1	5.562
			Pu-242	1.742 (10) ²²	0.007	0.5	2.933 (10) ²³	0.118	3.2	16.837
			TOTAL Pu		3.667					
3 (2nd plutonium recycle)	1.300	7517	U-235	1.652 (10) ²⁵	6.449	--	0.705 (10) ²⁵	--	--	0.427
			U-238	2.269 (10) ²⁷	897.0	--	2.261 (10) ²⁷	--	--	0.996
			Pu-239	5.877 (10) ²⁴	2.333	63.6	6.543 (10) ²⁴	2.598	53.3	1.113
			Pu-240	2.032 (10) ²⁴	0.810	22.1	2.815 (10) ²⁴	1.122	22.8	1.385
			Pu-241	1.015 (10) ²⁴	0.406	11.1	1.809 (10) ²⁴	0.724	14.8	1.782
			Pu-242	2.933 (10) ²³	0.118	3.2	1.120 (10) ²⁴	0.450	9.1	3.819
			TOTAL Pu		4.894					

TABLE II (contd)

Exposure									Ratio of Final	
Run Number	ϕt	MWD/Ton	Isotope	Initial Isotopic Concentrations			Final Isotopic Concentrations			to Initial Concentrations
	($\times 10^{21}\text{cm}^{-2}$)			Nuclei/Ton	KG/Ton	% of Total	Nuclei/Ton	KG/Ton	% of Total	
4 (3rd plutonium recycle)	1.353	8391	U-235	1.650 (10) ²⁵	6.441	--	0.680 (10) ²⁵	--	--	0.412
			U-238	2.265 (10) ²⁷	895.5	--	2.257 (10) ²⁷	--	--	0.996
			Pu-239	6.543 (10) ²⁴	2.598	53.3	6.785 (10) ²⁴	2.694	47.3	1.037
			Pu-240	2.815 (10) ²⁴	1.122	22.8	3.112 (10) ²⁴	1.241	21.8	1.106
			Pu-241	1.809 (10) ²⁴	0.724	14.8	2.174 (10) ²⁴	0.870	15.2	1.202
			Pu-242	1.120 (10) ²⁴	0.450	9.1	2.221 (10) ²⁴	0.893	15.7	1.983
TOTAL Pu							5.698			
5 (4th plutonium recycle)	1.367	8713	U-235	1.649 (10) ²⁵	6.437	--	0.673 (10) ²⁵	--	--	0.408
			U-238	2.264 (10) ²⁷	895.1	--	2.255 (10) ²⁷	--	--	0.996
			Pu-239	6.785 (10) ²⁴	2.694	47.3	6.879 (10) ²⁴	2.731	43.3	1.013
			Pu-240	3.112 (10) ²⁴	1.241	21.8	3.223 (10) ²⁴	1.285	20.4	1.036
			Pu-241	2.174 (10) ²⁴	0.870	15.2	2.311 (10) ²⁴	0.925	14.7	1.063
			Pu-242	2.221 (10) ²⁴	0.893	15.7	3.393 (10) ²⁴	1.364	21.6	1.528
TOTAL Pu							6.305			
6 (5th plutonium recycle)	1.360	8763	U-235	1.647 (10) ²⁵	6.429	--	0.676 (10) ²⁵	--	--	0.410
			U-238	2.262 (10) ²⁷	894.3	--	2.253 (10) ²⁷	--	--	0.996
			Pu-239	6.879 (10) ²⁴	2.731	43.3	6.92 (10) ²⁴	2.75	40.7	1.006
			Pu-240	3.223 (10) ²⁴	1.285	20.4	3.26 (10) ²⁴	1.30	18.2	1.011
			Pu-241	2.311 (10) ²⁴	0.925	14.7	2.36 (10) ²⁴	0.94	7.8	1.021
			Pu-242	3.393 (10) ²⁴	1.364	21.6	4.51 (10) ²⁴	1.82	33.2	1.329
TOTAL Pu							6.81			

Figure 1

KILOGRAMS OF PLUTONIUM PER TON OF FUEL vs NUMBER OF FUEL CYCLES

Figure 2

during the six runs. Figures 15 through 30 show the isotopic constituency of the plutonium during the six runs. Figures 21 through 26 show the mass of plutonium per ton of fuel. Figures 27 through 32 show the U-235 weight percentage of the uranium and Figures 33 through 38 show the isotopic concentrations during the six runs.

D. RESULTS OF SCHEME B

Scheme B differs from Scheme A in that the plutonium is alloyed with a diluent metal and fabricated into high-heat-transfer elements of the MTR or PWR-seed type. The results of the calculations for Scheme A can be used for Scheme B under certain conditions. Since high-heat-transfer capability is assumed, a high concentration of plutonium can be used, and, therefore, most of the volume of the reactor is occupied by uranium just as in Scheme A. If the plutonium elements are short in length and dispersed uniformly throughout the core, the resonance-escape probability and fast effect for Schemes A and B can be assumed to be equal with little sacrifice of accuracy.

Table III shows the results of the calculations of power cost for recycling by Scheme B. Again, since it was believed that estimates of fabrication costs of Scheme B-type elements were unreliable, the computations were made for three different assumptions. By averaging power costs over the six runs, it was found that if the Scheme B-type elements could be fabricated for less than \$4.50/gm of contained plutonium, then recycling by Scheme B would be competitive with the mode of operation where the plutonium is sold at the end of each run (or competitive with recycling by Scheme A if the blended elements of the Scheme A-type cost \$33/kg of fuel material).

If the diluent material is Zr, then the fabrication cost of \$4.50/gm of Pu is not achievable in the foreseeable future. It is estimated, for example, that the second generation PWR-seed elements will cost \$18/gm of contained U-235. Even if Al can be used, it is very unlikely that Scheme B can compete. Based on data furnished by Mr. E. C. Stewart of Albuquerque Operations Office, fabrication costs of Scheme B-type elements made of Al would be about \$11/gm of contained plutonium. The components of this cost estimate are included in a letter from Mr. Stewart which is attached as Appendix A.

TABLE III

FUEL-CYCLE COSTS WITH PLUTONIUM RECYCLE
(Scheme B)

Run Number	Reactivity Life (MWD/Ton)	Total Pu including that in Diluent Alloy Elements as well as that generated in U (gms Pu/kg U)		Fabrication Costs		Fuel-Cycle Costs (mills/kwh)
		Initial	Final	Uranium Elements (\$/kg)	Pu-Diluent Alloy Elements (\$/gm of Pu)	
1	2749	0	1.93	10	--	3.27 ^a
2	5428 ^b	1.93	3.67	10	4 ^c	2.81 ^d
					6	2.99
					8	3.18
3	7517 ^b	3.67	4.89	10	4	2.68
					6	2.97
					8	3.26
4	8391 ^b	4.89	5.70	10	4	2.75
					6	3.11
					8	3.47
5	8715 ^b	5.70	6.31	10	4	2.82
					6	3.23
					8	3.64
6	8763 ^b	6.31	6.81	10	4	2.91
					6	3.37
					8	3.82
1 - 6	--	--	--	10	4	2.87(avg)
					6	3.15(avg)
					8	3.44(avg)

^aIf the Pu is sold at \$12/gm, the cost is reduced to 2.94 mills/kwh.

^bPart of this energy will have come from the Pu-Metal Diluent elements.

^cBy way of comparison, it cost about \$2/gm of U-235 to fabricate MTR assemblies.

^dThese figures are less accurate than those for Scheme A because of:

1. Uncertainty as to where the Pu elements will be placed in the reactor;
2. Uncertainty as to how much space will be taken by the Pu elements; and
3. The fast effect will be decreased a little by having Pu separate from uranium.

E. RESULTS OF SCHEME C

Scheme C, it will be recalled, is to recycle one-half the spent uranium in addition to the plutonium. Originally it was intended to define Scheme C as a recycle of all the spent fuel, i.e., just a removal of fission products. This was tried at the end of Run 1 and only a very small addition to exposure resulted. The behavior of the plutonium buildup has a much greater effect on reactivity life than the accrual of fission-product poisons in Run 1, at least. The fact that the slope of the reactivity vs exposure curve changes from upward to downward is due to the tendency of the plutonium to saturate. When the fission products are removed and the reactor restarted, the reactivity just continues on its downward path as soon as Xe has built up, and the reactor soon stops with little extra exposure having been gained. Thus, it was decided to investigate a recycle of only one-half the spent uranium under the title of Scheme C.

Table IV gives the results for power cost for Scheme C. The results were so unfavorable compared to Scheme A for Run 2 that Runs 3 through 6 were not calculated.

Figure 39 shows the behavior of reactivity in Run 2 for Scheme C.

It should be mentioned that in computing the power costs for Scheme C, the plutonium and uranium were considered to be separated into plutonium nitrate and uranyl nitrate at a chemical processing cost of \$20/kg of uranium as used before (the cost of separating the plutonium from the uranium is a small part of this \$20); then for the half which is to be re-used, the uranyl nitrate is converted to UF_4 at a cost of \$1.20/kg; then UF_4 is reduced to metal at \$1.50/gm of plutonium as before. The assumed fabrication cost would cover the blending of the plutonium with the uranium and fabricating the fuel elements as in Scheme A.

An alternate procedure might be to leave the uranyl nitrate and plutonium nitrate as a mixture and reduce it to the metal mixture, but it is not believed that costs would be smaller with this procedure.

F. RESULTS OF SCHEME D

In Scheme D, the lattice pitch is assumed to be reduced to seven inches between Runs 1 and 2 in order to increase the conversion ratio (at the expense of some of the extra reactivity gained from the recycled plutonium). This

TABLE IV

FUEL-CYCLE COSTS WITH THE PLUTONIUM RECYCLED AND HALF THE URANIUM RECYCLED
(Scheme C)

<u>Run Number</u>	<u>Reactivity- Life (MWD/Ton)</u>	<u>Pu Concentration (gms Pu/kg U)</u>		<u>Fabrication Cost of the Pu-impreg- nated U Elements (\$/kg of fuel)</u>	<u>Fuel-Cycle Costs (mills/kwh)</u>
		<u>Initial</u>	<u>Final</u>		
1	2749	0	1.93	10 (No Pu)	3.27 ^a
2	3295	1.93		20	3.68
				35	4.60
				50	5.53

^aIf the Pu is sold at \$12/gm, the cost is reduced to 2.94 mills/kwh.

was an academic investigation to see if the exposure might be increased this way. It was found that the exposure is actually decreased so that power costs could clearly not be reduced by Scheme D as compared to Scheme A.

Figures 40 through 44 show the behavior of the reactivity and the plutonium and U-235 concentrations.

III. EFFECT OF NON-UNIFORM FLUX

As mentioned in Section II, since the plutonium recycle cases would be prohibitively complicated to calculate if the charge-discharge by radial zones (which results in having different grades of plutonium throughout the reactor) and the non-uniform flux were taken into account, it was decided to base the comparison of recycling plutonium versus selling plutonium on the assumption of batch discharge and uniform flux (and burnout) throughout the study.

All six fuel elements in a vertical channel are discharged and replaced at once in the Calder Hall type reactor, even though radially the discharge-charge is by zones. Thus, the discharge-charge operation is "batch", vertically, but approaches the continuous discharge-charge idealization, radially. This makes the radial non-uniformity of flux not particularly objectionable, but the vertical non-uniformity of flux will tend to shorten the average exposure as compared to the case of flat vertical distribution. It is of interest then to calculate how large this effect of non-uniform vertical flux is. If it is small it will certainly be more than offset by the greater exposure due to the radial continuous discharge-charge as opposed to the batch discharge mode of operation that was assumed (radially as well as vertically).

We will attempt to do this by observing the buckling in Run 1 as a function of time, taking into consideration a chopped-off cosine function as governing the vertical flux distribution.

The critical equation may be written as

$$\frac{k e^{-B^2 \tau}}{1 + L^2 B^2} = 1$$

For large reactors this reduces to

$$B^2 = \frac{k-1}{M^2}$$

where

$$M^2 = L^2 + \tau$$

and

$$k = \eta \epsilon p f$$

In the situation in which the flux is not uniform we must divide the reactor into vertical regions and determine the buckling, B_i^2 , for each region. If, for each region, R_i , we use the weighting function

$$W(R_i) = \frac{\int_{R_i} \phi_i^2 dV}{\int_V \phi^2 dV}$$

we may write the buckling of the entire reactor as

$$B^2 = \sum_i B_i^2 \frac{\int_{R_i} \phi_i^2 dV}{\int_V \phi^2 dV}$$

Assuming M^2 , ϵ , and p to remain constant, there results

$$B^2 = \frac{p\epsilon}{M^2} \sum_i \left\{ (\eta f)_i - \frac{1}{p\epsilon} \right\} \frac{\int_{R_i} \phi_i^2 dV}{\int_V \phi^2 dV}$$

Now ηf varies with isotopic concentrations in the fuel and the low cross-section fission-product poisons (LCSFPP). These in turn are functions of the flux time. Thus (ηf) will vary with time and/or flux.

Let us consider the variation of (ηf) with flux only keeping the time constant at some value, $t = t_1$. This variation of (ηf) with flux is just what we seek for our assumption of the vertical cosine flux distribution. We divide the reactor into nine regions and take into account a reflector savings of 54.9 cm at each end of the cylinder. Then, using the cosine flux distribution, we can obtain the value of (ηf) for each region and for each time interval we select. Substitution into the equation above and weighting each region by the square of the flux in that region yields the non-uniform buckling curve as shown in Figure 3 which compares it with the uniform flux buckling curve. From the graph it can be seen that the effect of the non-uniform vertical flux is to reduce the reactivity life by about 14%. The increase in exposure gained from the radial continuous discharge-charge feature should more than offset this.

Figure 3

BUCKLING vs TIME
(Effect of Non-Uniform Flux)

IV. REACTOR PARAMETERS AND CROSS-SECTION DATA

In this section a brief description of Calder Hall from the reactor physics viewpoint will be given first, together with a compilation of the numerical values of the parameters of the reactor; second, a discussion will be given of the method of arriving at the effective cross sections of the various isotopes, as well as fission products; and third, a numerical investigation is presented of the effect of arbitrary percentage errors in the various cross sections.

A. CALDER HALL AND ITS REACTOR PHYSICS PARAMETERS

The most complete data on the Calder Hall reactor appear in the "Journal of the British Nuclear Energy Conference", Vol. 2, No. 2, April, 1957. This is a collection of the papers presented at the Symposium on "Calder Works Nuclear Power Plant" which was held November 22 and 23, 1956. Abstracts of some of these papers appeared earlier than the above April, 1957, publication in the December, 1956, issue of Nuclear Engineering. A rather complete compilation of parameters and performance data appeared on a foldout sheet in this latter reference. That part of this compilation pertaining to the physics parameters is given in Table V.

A good set of average values for the components of k_{∞} from the above data would be $\eta = 1.266$, $f = .929$, $p = .881$, and $\epsilon = 1.030$, giving $k_{\infty} = 1.067$. It is interesting to note that the η used here is much lower than the current "best value", $\eta = 1.34$. These values are evidently based on a "recipe" which makes k_{∞} agree with critical experiments. In AECU-3398 the Metallurgical Laboratory "recipe" was used. The results, which should be about comparable with those above, were $\eta = 1.315$, $f = .905$, $p = .858$, $\epsilon = 1.031$, $k_{\infty} = 1.053$. It appears an η of 1.333 would give better agreement with the British, if used together with the result of the Metallurgical Laboratory recipe. This value would incidentally be in much better agreement with the "best value", $\eta = 1.34 \pm .02$. However, the above British values for the plutonium recycle work of this report are used on the grounds that they are in a better position to know the best parameters for Calder Hall due to Dr. Mummery's extensive set of exponential experiments.

Other data used are:

$$B^2 = 3.830 \times 10^{-5}$$

$$e^{-\tau B^2} = 0.9777$$

TABLE V

CALDER HALL PHYSICS DATA

Heat rating: 182 MW

Fuel

Material: Natural uranium

Description: Cast bars 1.15" diameter, 40" long

Density: 18.7 g/cm³

Elements in vertical channels: Six per channel

Total weight of fissile material: 120 tons per reactor

Number of fuel-element channels: 1696

Diameter of channels: 256 at 4.16"

576 at 3.95"

864 at 3.61"

Average temperature: 425°C

Canning

Material: Magnesium alloy - Magnox C, He filled

Wall thickness: 0.072"

Diameter: 1.30"

Extended surface: Single start helical fin 0.125" pitch

Over-all diameter: 2.125"

Design maximum can surface temperature: 408°C

Moderator

Material: Graphite

Core size: 21' high, 31' diameter

Over-all size including reflector: 24 sided regular prism, 27' high,
36' across corners

Total weight: 1,146 tons

Blocks: 8" square, 25" long

Average temperature: 250°C

TABLE V (contd)

Lattice

Description: Regular square, 8" pitch

	Core		
	Zone A	Zone B	Zone C
Inner radius, ft:	0	6.02	10.85
Outer radius, ft:	6.02	10.85	15.49
Channel diameter, in:	4.16	3.95	3.61
Number of channels:	256	576	864
Thermal utilization factor, f:	0.93186	0.93010	0.92672
Resonance escape factor, p:	0.87533	0.87861	0.88398
Fast fission factor, ϵ :	1.02972	1.02972	1.02972
Thermal fission factor, η :	1.26595	1.26595	1.26595
Multiplication constant, k_{∞} :	1.06330	1.06527	1.06789
Radial diffusion area, cm^2 :	406.6	391.9	371.1
Axial diffusion area, cm^2 :	464.6	438.9	403.6
Radial slowing down area, cm^2 :	628.9	588.6	528.1
Axial slowing down area, cm^2 :	709.5	652.7	570.6

Control

Number of channels: 112
 Diameter of channels: 3.25"
 Coarse: Up to 60 rods ganged together
 Fine: Up to 4, manually operated
 Normal operation: Total of 40 rods
 Length: 22'
 In speed: 5 in/min and 50 in/min
 Out speed: 0.5 in/min
 Shutdown speed: 4 ft/sec

Reactivity

Total excess: 4%
 Control channel increment: 0.015%
 Increment per control rod $\cong$ 0.2%

Some other points, taken from the references mentioned above, which are of interest for the present study are:

- (1) "In a reactor the neutron flux varies with radius, and the irradiation received by different rings of fuel elements varies accordingly. For this reason, it is desirable to charge and discharge fuel elements in batches rather than simultaneously."
- (2) "It might seem that the total heat output obtainable in MWD would be roughly equal to the initial excess of fissile material in grams. In actual fact, with natural uranium more than this amount of heat can be drawn because some of the U-238 present is converted by neutron capture, into fissile Pu-239, which partly offsets the depletion of U-235. If the swapping rate is such that three atoms of Pu-239 are created for every four atoms of U-235 consumed, the life of the uranium will be extended by the factor $1/(1 - .75) = 4$. Information, which became available about mid 1950, suggested that the total heat which could be drawn per metric ton of natural uranium before the reactor ran down might exceed 2,000 MWD."
- (3) "The multiplication factor, of the largest natural uranium graphite reactor, could not exceed about 1.07."
- (4) "Table VI gives the total excess k of PIPPA for various central channel ratings, with assumed average graphite and uranium temperatures of 300 and 450°C, respectively, over the core."

Table VI

BREAKDOWN OF REACTIVITY

Reactivity, with central channel rating (mean), MW/ton, of:

	2.50	2.75	3.0
Xe poison, steady state	1540×10^{-5}	1620×10^{-5}	1680×10^{-5}
Xe poison, buildup	160	200	240
Ur. temp. effect	600	600	600
Graphite temp. effect	1590	1590	1590
Total built in excess k	3890	4010	4110

- (5) "The two charge and two discharge machines are designed for continuous shift operation, and on this basis, the complete reactor core can be recharged in about ten days."

- (6) "Core graphite density: 1.73 gm/cm³
Core graphite absorption cross section: 4.0 millibarns
Reflector graphite density: 1.60 gm/cm³
Reflector graphite absorption cross section: 4.8 millibarns
Absorption cross section of cans and supports,
cm²/ft of uranium: 0.51"
- (7) "An uncertainty of 2% in the neutron yield per neutron absorbed in U-235 will lead to an uncertainty of a factor of two in the critical size of a reactor of the Calder Hall type."
- (8) "It is believed that an irradiation level corresponding to a heat extraction of about 3000 MWD/ton of natural uranium is possible with a Calder Hall type reactor."
- (9) "In making the lattice calculations, the uranium rod must be assumed to be continuous along the channel, though in practice the channel contains several short uranium rods with the spaces between them filled by supporting structure. To simplify this picture for calculations, the uranium is assumed to be continuous though of reduced density, and the neutron absorbing power of the intermediate supporting structure is added to that of the main body of the Mg sheath and distributed uniformly over the length of the uranium."
- (10) "When the possibility of the core developing more reactivity than required was realized, the effect of using the excess to flatten the radial neutron-flux distribution was investigated. Because of limitations on blower power, the net effect on reactor heat output was an increase that was small compared with the change in the ratio of the mean channel heat output to the maximum. It was therefore decided not to flatten the distribution."
- (11) "Only certain zones of the core are recharged at any time."

B. THE CROSS SECTIONS

1. Theory of "Effective" Cross Sections

The cross sections used in this report in the calculation of isotopic changes with exposure are specified according to the conventions set down by C. H. Westcott of Chalk River Laboratories.^{3,4}

³Westcott, C. H., "The Specification of Neutron Flux and Nuclear Cross Sections in Reactor Calculations", AECL-194, April 29, 1955.

⁴Westcott, C. H., "Effective Cross Section Values for Well Moderated Thermal Reactor Spectra", CRRP-680, January 25, 1957.

A cross section, $\hat{\sigma}$, according to these conventions, is equal to the 2200 m/sec cross section for an equivalent 1/v-law substance which would give the same reaction rate as does the actual substance. In order to get the reaction rate, per atom, the cross section defined in this way is multiplied by the "conventional" flux which is the total neutron density multiplied by 2.2×10^5 cm/sec.

As discussed in Section V, when the more approximate set of equations for isotopic changes is used, the set is solved by dividing through all equations by the equation for U-235. When this is done, the flux disappears and is no longer a parameter. Furthermore, it is clear in this method of approach that it is only the ratio of cross sections that matters. When the improved set of equations, including Pu-242, etc., is used, it is more convenient to solve for isotopic concentrations in terms of flux X time. The exposure (MWD/ton) is then determined in the same units, and the output of the code expresses isotopic concentrations in terms of exposure as well as flux X time.

Regardless of the method of solution, it is clear that the cross sections need be only in the proper ratio to each other in order to obtain the correct result for concentration vs exposure level. However, it seems advisable to express all cross sections in the conventional way in order to utilize the work of and make comparison with other authors.

It may be worth while to summarize Westcott's discussions at this point.

In a thermal reactor, the spectrum can be approximately represented by a Maxwell-Boltzman distribution of neutrons composed of neutrons which have slowed down and are in thermal equilibrium with moderator nuclei together with a 1/E tail extending up to energies at which neutrons are born. In finding the total effective cross section, it is somewhat simpler to consider these two parts of the spectrum as separate but overlapping. First a component, $g\sigma_{2200}$, is determined. This is equal to the 2200 m/sec cross section of an equivalent 1/v-law substance which would give the same reaction rate as does the actual substances in a pure Maxwellian spectrum. Then a component, $rs\sigma_{2200}$, is found which is, roughly speaking, the 2200 m/sec cross section of

an equivalent $1/v$ -law substance which would give the same reaction rate as does the actual substance in the $1/E$ spectrum. A more precise formulation of g , s , and r will be given later. The total is $\hat{\sigma} = g\sigma_{2200} + r\sigma_{2200}$ and when this is multiplied by the "conventional" flux, as defined above, the proper activation is obtained.

Consider first the pure Maxwellian spectrum. This is given by

$$n'(E) = \frac{2n}{E_0^{3/2} \sqrt{\pi}} E^{1/2} e^{-E/E_0} \quad (1)$$

where $E_0 = kT$.

The true flux F is given by

$$\begin{aligned} F &= \int_0^{\infty} n'(v) v dv = \int_0^{\infty} \frac{2n}{E_0^{3/2} \sqrt{\pi}} \sqrt{\frac{2}{m}} E e^{-E/E_0} dE \\ &= \int_0^{\infty} f(E) dE \end{aligned} \quad (2)$$

where this amounts to a definition of $f(E)$ the distribution function of flux over energy. The reaction rate is given by

$$R = \int_0^{\infty} \sigma(E) f(E) dE \quad (3)$$

and an effective cross section can be defined

$$\sigma_{\text{eff}} = \frac{R}{F} = \frac{\int_0^{\infty} \sigma(E) f(E) dE}{\int_0^{\infty} f(E) dE} \quad (4)$$

This definition of cross section could be used in theoretical calculations for obtaining the activation due to the Maxwell or thermal component of the flux. However, it has become conventional, because of experimental considerations, to define cross section and flux in a different way. It has been customary to measure the flux in a thermal reactor by determining the reaction rate of a $1/v$ -law detector such as boron and dividing this reaction rate by the 2200 m/sec cross section of the detector. Since*

$$R = \int_0^{\infty} n'(v) v \sigma_{2200} \frac{v_0}{v} dv = n v_0 \sigma_{2200} \quad (5)$$

where $v_o = 2.2 \times 10^5$ cm/sec, it follows that

$$\frac{R}{\sigma_{2200}} = nv_o = \phi \quad (6)$$

and this is what is referred to as the flux of the reactor. It is the total neutron density multiplied by the velocity of 2.2×10^5 cm/sec. Thus, this "conventional" definition of flux is independent of the moderator temperature or the spectrum of the neutrons, etc.

It is worth while to remark in passing that

$$\phi_M = F_M \frac{v_o}{\bar{v}} = F_M \sqrt{\frac{\pi T_o}{4T}} \quad (7)$$

where

ϕ_M = Conventional flux for pure Maxwellian distribution

F_M = True flux

$v_o = 2.2 \times 10^5$ cm/sec

$\bar{v}$ = Average velocity of the Maxwellian distribution

T_o = Temperature corresponding to a velocity of 2.2×10^5 cm/sec ($^{\circ}\text{K}$)

T = The moderator temperature ($^{\circ}\text{K}$)

The reaction rate for pure Maxwellian spectrum is given either by

$$R = F_M \sigma_{\text{eff}} \text{ or } R = \phi_M \sigma_{2200}$$

Now if the flux, ϕ_M , is known and it is desired to find a cross section of a non-1/v detector which when multiplied by ϕ_M will give the correct reaction rate, this is

$$\begin{aligned} \sigma_n &= \frac{1}{\phi_M} \int_0^{\infty} \sigma(E) f(E) dE \\ &= \frac{F_M}{\phi_M} \sigma_{\text{eff}} \\ &= \sigma_{\text{eff}} \sqrt{\frac{4T}{\pi T_o}} \text{ for a Maxwellian distribution.} \end{aligned} \quad (8)$$

Up to this point only the Maxwellian neutron distribution has been considered. As mentioned earlier, the complete spectrum in a thermal reactor can be closely specified as the sum of a Maxwellian together

with a $1/E$ tail. The complete spectrum can be represented as

$$f(E) dE = f_M(E) dE + f_e(E) dE \quad (9)$$

The Maxwellian part, $f_M(E)$, is

$$f_M(E) = F_M \frac{E}{E_0^2} e^{-E/E_0} \quad (10)$$

The epithermal part shall be specified as

$$f_e(E) = \beta F_M \frac{1}{E} \quad (11)$$

where β is thus the epithermal flux per unit interval of $\ln E$ per unit true flux. Thus

$$f(E) dE = F_M \left[\frac{E^2}{E_0^2} e^{-E/E_0} + \beta \right] \frac{dE}{E} \quad (12)$$

To solve for β , it will be recalled that

$$q = \sum_s \Sigma_s f_e(u) \quad (13)$$

where

q = The thermal slowing down density in the moderator; i.e., number of neutrons per cm^3 in the moderator becoming members of the Maxwellian per sec.

Σ_s = Macroscopic cross section in moderator

ξ = Loss in $\ln E$ per collision in moderator

$f_e(u)$ = Flux per unit $\ln E$ interval in moderator (and, approximately, in the fuel)

The fluxes in the cell are given by

$$-D_f \nabla^2 F_{M_f} + \Sigma_{a_f} F_{M_f} = 0 \quad (14)$$

and

$$-D_m \nabla^2 F_{M_m} + \Sigma_{a_m} F_{M_m} = q = \sum_s \Sigma_s \text{ (arbitrary)}$$

when $f_e(u)$ has been arbitrarily set equal to unity. The F_M can thus be determined as a function of position and then average values of F_{M_f} and F_{M_m} found. These are $\bar{F}_{M_f}$ and $\bar{F}_{M_m}$ per unit $f_e(u)$. The reciprocals are β for fuel and moderator.

One more remark needs to be made about the expression arrived at above for the complete spectrum, $f(E)$. It was mentioned that there is some overlapping of the Maxwellian and the $1/E$ part. The question is how much; i.e., how far should the $1/E$ part extend down into the Maxwellian? It has been shown experimentally that if the $1/E$ part is started at about 4.3 kT for a graphite reactor, a good fit to experimental data is obtained. Dr. Westcott uses 5 kT for D_2O reactors. We used 5 kT for Calder Hall because there should be little effect on the cross sections due to this slight difference, and Dr. Westcott's work could then be utilized directly. The spectrum can be represented, therefore, as

$$f(E) dE = F_M \left[\frac{E^2}{E_0^2} e^{-E/E_0} + \Delta\beta \right] \frac{dE}{E} \quad (15)$$

where Δ is a unit step function which is zero up to approximately 5 kT.

The concept of ϕ , the conventional flux, for the Maxwellian will now be extended to the complete spectrum. Write

$$\phi = \phi_M + \phi_e \quad (16)$$

where again ϕ is v_0 times the total neutron density. The ϕ_M is the activation divided by σ_{2200} of a $1/v$ -law detector due to Maxwellian neutrons and ϕ_e is the activation divided by σ_{2200} of a $1/v$ -law detector due to the $1/E$ part. It has already been noted that

$$\phi_M = F_M \sqrt{\frac{\pi T_0}{4T}} \quad (17)$$

then ϕ_e would be

$$\phi_e = \frac{\beta F_M}{\sigma_{2200}} \int_{5E_0}^{E_f} \sigma_{2200} \sqrt{\frac{E_0}{E}} \frac{dE}{E} = \beta F_M \sqrt{\frac{4T_0}{5T}} \quad (18)$$

where E_f can be set equal to infinity in the integration without much error. Thus,

$$\frac{\phi_e}{\phi_M} = \beta \sqrt{\frac{4}{5} \frac{4}{\pi}} \cong 1.01 \beta = \beta' \quad (19)$$

and

$$\phi = \phi_M (1 + \beta') = F_M (1 + \beta') \sqrt{\frac{\pi T_0}{4T}} \quad (20)$$

A quantity r is used as the spectral index, and this is defined as

$$r = \frac{\phi_e}{\phi} = \frac{\phi_e}{\phi_e + \phi_M} = \frac{\frac{\phi_e}{\phi_M}}{\frac{\phi_e}{\phi_M} + 1} = \frac{1.01\beta}{1 + 1.01\beta} \approx \frac{\beta}{1 + 1.01\beta} \quad (21)$$

Since ϕ_e/ϕ_M is really the ratio of the epithermal neutron density to the thermal (Maxwellian) neutron density, it would seem at first thought that ϕ_e/ϕ_M would be temperature independent. However, since β is clearly not temperature independent, it follows that ϕ_e/ϕ_M , and also r , is not either. This apparent paradox is resolved when it is recalled that ϕ_e is defined, with respect to its lower limit, by the kT cutoff. This makes ϕ_e temperature dependent. Since ϕ_M is not temperature dependent, this makes ϕ_e/ϕ_M (and also r) actually temperature dependent.

It was mentioned earlier that the total effective cross section is given by

$$\hat{\sigma} = g\sigma_{2200} + r s \sigma_{2200} \quad (22)$$

Let us see exactly what g and s are. The activation rate is given by:

$$\begin{aligned} \phi \hat{\sigma} &= \phi [g\sigma_{2200} + r s \sigma_{2200}] \\ &= \int_0^{\infty} f(E) \sigma(E) dE \\ &= F_M \int_0^{\infty} \left(\frac{E^2}{E_0^2} e^{-E/E_0 + \Delta\beta} \right) \sigma \frac{dE}{E} \end{aligned} \quad (23)$$

Thus

$$g\sigma_{2200} + r s \sigma_{2200} = \frac{F_M}{\phi} \int_0^{\infty} \left(\frac{E^2}{E_0^2} e^{-E/E_0 + \Delta\beta} \right) \sigma \frac{dE}{E} \quad (24)$$

Using (17) and (18) and the fact that $\phi = \phi_M + \phi_e$, this becomes

$$\begin{aligned} g\sigma_{2200} + r s \sigma_{2200} &= \sqrt{\frac{4T}{\pi T_0}} \left[\frac{1}{1 + 1.01\beta} \int_0^{\infty} \frac{E^2}{E_0^2} e^{-E/E_0} \sigma \frac{dE}{E} \right. \\ &\quad \left. + \int_{5kT}^{\infty} \frac{\beta}{1 + 1.01\beta} \sigma \frac{dE}{E} \right] \end{aligned} \quad (25)$$

Let

$$\frac{1}{1 + 1.01\beta} \cong 1 - 1.01\beta + \overline{1.01}^2 \beta^2 \quad (26)$$

Using (26) in (25), the latter becomes

$$\begin{aligned} g_{\sigma_{2200}} + rs_{\sigma_{2200}} &= \sqrt{\frac{4T}{\pi T_o}} \int_0^{\infty} \frac{E^2}{E_{\theta}^2} e^{-E/E_{\theta}} \sigma \frac{dE}{E} \\ &\quad - 1.01\beta \sqrt{\frac{4T}{\pi T_o}} \int_0^{\infty} \frac{E^2}{E_{\theta}^2} e^{-E/E_{\theta}} \sigma \frac{dE}{E} \\ &\quad + \overline{1.01}^2 \beta^2 \sqrt{\frac{4T}{\pi T_o}} \int_0^{\infty} \frac{E^2}{E_{\theta}^2} e^{-E/E_{\theta}} \sigma \frac{dE}{E} \\ &\quad + \sqrt{\frac{4T}{\pi T_o}} \int_{5kT}^{\infty} \frac{\beta}{1 + 1.01\beta} \sigma \frac{dE}{E} \end{aligned} \quad (27)$$

Using (21) and defining

$$g = \frac{1}{\sigma_{2200}} \sqrt{\frac{4T}{\pi T_o}} \int_0^{\infty} \frac{E^2}{E_{\theta}^2} e^{-E/E_{\theta}} \sigma \frac{dE}{E} \quad (28)$$

it follows that

$$\begin{aligned} rs_{\sigma_{2200}} &= - 1.01\beta g_{\sigma_{2200}} + \overline{1.01}^2 g_{\sigma_{2200}} \\ &\quad + \sqrt{\frac{4T}{\pi T_o}} \int_{5kT}^{\infty} \frac{\beta}{1 + 1.01\beta} \sigma \frac{dE}{E} \end{aligned} \quad (29)$$

But from (21)

$$\begin{aligned} - 1.01\beta g_{\sigma_{2200}} + \overline{1.01}^2 g_{\sigma_{2200}} &= g_{\sigma_{2200}} \left(- 1.01\beta + \overline{1.01}^2 \right) \\ &= g_{\sigma_{2200}} \left[- 1.01r (1 + 1.01\beta) \right. \\ &\quad \left. + \beta \overline{1.01}^2 r (1 + 1.01\beta) \right] \\ &\cong - g_{\sigma_{2200}} (1.01r) \end{aligned}$$

Using this and (21) in (29) it follows that

$$s = \sqrt{\frac{4T}{\pi T_0}} \frac{\Sigma}{\sigma_{2200}}$$

where

$$\Sigma = -1.01 g \sigma_{2200} \sqrt{\frac{\pi T_0}{4T}} + \int_{5kT}^{\infty} \sigma \frac{dE}{E}$$

In CRRP-680, Westcott presents values of g and s which he has evaluated at various temperatures for a large number of isotopes including those of uranium and plutonium.

A low lying resonance will effect both g and s . For an isotope like Pu-240 which has a high resonance lying above the principal range of the Maxwellian, the contribution of the $rs\sigma_{2200}$ term of $\hat{\sigma}$ can be a great deal more than that of the $g\sigma_{2200}$ term.

2. Evaluation of r

It was shown above that

$$\hat{\sigma} = g\sigma_{2200} + rs\sigma_{2200} \quad (30)$$

where g and s are obtained from CRRP-680 according to the moderator temperature. It is necessary to evaluate r for Calder Hall and this will be done in this section. It was defined above that

$$r = \frac{\beta}{1 + 1.01\beta} \quad (31)$$

where β is the epithermal flux per unit interval of $\ln E$ per unit true thermal flux. The β will be determined from the equations

$$q = \int \Sigma_s f_e(u) \quad (32)$$

$$-D_f \nabla^2 F_{M_f} + \Sigma_{a_f} F_{M_f} = 0 \quad (33)$$

$$-D_m \nabla^2 F_{M_m} + \Sigma_{a_m} F_{M_m} = q_m \quad (34)$$

where $f_e(u)$ is arbitrarily equal to unity. Then for Calder Hall graphite

$$q = \int \Sigma_s = 0.06468 \quad (35)$$

A Calder Hall cell is taken to be

where

r_o = radius of fuel rod and inside radius of hole (the clad
can be neglected here without much error)

r_g = outer radius of hole and inside radius of graphite

r_c = outer radius of cell

The boundary conditions are

$$F_{M_f}(r_o) = F_{M_m}(r_g) \quad (36)$$

$$D_f \left(\frac{dF_{M_f}}{dr} \right)_{r_o} = \frac{r_g}{r_o} D_m \left(\frac{dF_{M_m}}{dr} \right)_{r_g} \quad (37)$$

and

$$\left(\frac{dF_{M_m}}{dr} \right)_{r_c} = 0 \quad (38)$$

The solutions to (33) and (34) are

$$F_{M_f} = A I_o(\kappa_f r) \quad (39)$$

$$F_{M_m} = C \left[I_o(\kappa_m r) K_1(\kappa_m r_c) + K_o(\kappa_m r) I_1(\kappa_m r_c) \right] + \frac{q}{\Sigma_{a_m}} \quad (40)$$

where (38) has been used.

Application of (36) and (37) yields

$$\frac{1}{A} = \frac{\Sigma_a}{q} \left(I_o(\kappa_f r_o) - \frac{r_o}{r_g} \frac{D_f \kappa_f I_1(\kappa_f r_o)}{D_m \kappa_m} \right. \\ \left. \frac{\left[I_o(\kappa_m r_g) K_1(\kappa_m r_o) + K_o(\kappa_m r_g) I_1(\kappa_m r_c) \right]}{\left[I_1(\kappa_m r_g) K_1(\kappa_m r_c) - K_1(\kappa_m r_g) I_1(\kappa_m r_c) \right]} \right)$$

and

$$C = \frac{A D_f \kappa_f I_1(\kappa_f r_o)}{\frac{r_g}{r_o} D_m \kappa_m \left[I_1(\kappa_m r_g) K_1(\kappa_m r_c) - K_1(\kappa_m r_g) I_1(\kappa_m r_c) \right]}$$

The cross sections for use here should be averages over a Maxwellian rather than values at the most probable velocity, i.e., σ_{eff} rather than σ_o in Westcott's terminology. This leads to

$$\begin{aligned} D_f &= 0.806 \\ \Sigma_{af} &= 0.244 \\ \kappa_f^2 &= 0.302 \\ D_m &= 0.981 \\ \Sigma_{am} &= 0.000269 \\ \kappa_m^2 &= 0.000275 \\ r_o &= 1.4605 \text{ cm} \\ r_g &= 5.08 \\ r_c &= 11.46 \end{aligned}$$

This leads to

$$\begin{aligned} \overline{F_{M_f}} &= 12.26 \\ \overline{F_{M_m}} &= 15.24 \end{aligned}$$

where these are spatial averages of F_M over fuel and moderator under our arbitrary value of unity for $f_e(u)$. It follows that

$$\beta_f = \frac{f_e(u)}{\overline{F_{M_f}}} = 0.0816$$

$$\beta_m = 0.066$$

and

$$r_f = 0.075$$

$$r_m = 0.062$$

These results are in reasonable agreement with Westcott's statement that, "In the NRX, r varies from about 0.03 in the moderator to 0.07 in the fuel" 'and' "Hughes quotes $\beta = 0.0625$ for a graphite reactor."

It is the r in the fuel that should be used to evaluate the $\hat{\sigma}$ for essentially all isotopes required. This is again

$$r = 0.075$$

3. The Results for Heavy Isotopes and High Cross-Section Poisons

Using the tables of CRRP-680, $r = 0.075$, and $\theta = 250^\circ\text{C}$ and the formulas

$$\hat{\sigma} = (g + rs) \sigma_{2200}$$

$$\hat{\eta} = \eta_{2200} \left(\frac{g_f + rs_f}{g_a + rs_a} \right)$$

the following values in barns were obtained for cross sections:

U-235

$$g_a = 0.9312, s_a = 0.106$$

$$g_f = 0.9375, s_f = -0.001$$

Using $\sigma_{a_{2200}} = 697.8$, $\sigma_f = 581.6$, $\eta_{2200} = 2.070$ it follows that

$$\hat{\sigma}_a = 655, \hat{\sigma}_f = 545, \hat{\sigma}_c = 110, \text{ and } \hat{\eta} = 2.066$$

U-238

$$g_c = 1.008$$

Using $\sigma_{c_{2200}} = 2.75$ it follows that $\hat{\sigma}_c = 2.77$

The s is taken as zero for U-238 because the capture of neutrons slowing down is taken care of separately in the differential equations of isotopic concentration.

Pu-239

$$g_a = 1.459, s_a = 2.108$$

$$g_f = 1.342, s_f = 1.664$$

Using $\sigma_{a_{2200}} = 1027.2$, $\sigma_{f_{2200}} = 738.0$, $\eta_{2200} = 2.090$ it follows that

$$\hat{\sigma}_a = 1661, \hat{\sigma}_f = 1083, \hat{\sigma}_c = 578, \hat{\eta} = 1.896$$

Pu-240

$$g_c = 1.095, s_c = 49.3$$

Using $\sigma_{c_{2200}} = 260$, it follows that

$$\hat{\sigma}_c = 1246$$

It is interesting to note here that about three fourths of the Pu-240 capture is due to slowing down neutrons.

Pu-241

$$g_f = 1.250, s_f = 0.615$$

Using $\sigma_{f_{2200}} = 971.1$ it follows that

$$\hat{\sigma}_f = 1259$$

Westcott advises

$$\sigma_a / \sigma_f = 1.365$$

so

$$\hat{\sigma}_a = 1718$$

$$\hat{\sigma}_c = 459$$

and

$$\hat{\eta} = 2.23$$

In computing $\hat{\eta}$ here a value of $\nu = 3.05$ was used as given in the January 1957 supplement to BNL-325.

Pu-242

Very little data are available for Pu-242. We assumed

$$\hat{\sigma}_c = \sigma_{c_{2200}} = 64$$

according to the supplement to BNL-325.

Xe-135

Very little is known about the epithermal cross section of Xe-135 so that a value of s has not been determined.

Assuming a spin factor of $5/8$ and using E. C. Smith's data,

$$g_c = 1.24$$

Using $\sigma_{c2200} = 2.76 \times 10^6$, it follows that

$$\hat{\sigma} = 3.42 \times 10^6$$

Sm-149

$$g_c = 2.147, s_c = -2.002$$

Using $\sigma_{c2200} = 6.6 \times 10^4$ it follows that

$$\hat{\sigma}_c = 1.32 \times 10^5$$

The following table summarizes these results.

	Summary				Source
	$\hat{\sigma}_f$	$\hat{\sigma}_c$	$\hat{\sigma}_a$	$\hat{\eta}$	
U-235	545	110	655	2.066	Westcott
U-238	0	2.77	2.77	0	Westcott
Pu-239	1083	578	1661	1.896	Westcott
Pu-240	0	1246	1246	0	Westcott
Pu-241	1259	459	1718	2.23	Westcott and BNL-325 supplement 1
Pu-242	0	64	64	0	BNL-325 supplement 1
Xe-135	0	3.42×10^6	3.42×10^6	0	Westcott and BNL-325 supplement 1
Sm-149	0	1.32×10^5	1.32×10^5	0	Westcott and BNL-325 supplement 1

It is well to emphasize again that these values give the correct reaction rate per nucleus for Calder Hall when multiplied by the "conventional" flux as determined by measuring the reaction rate of a $1/v$ absorber in the reactor and dividing this reaction rate by the 2200 m/sec cross section of the $1/v$ absorber.

4. The Low Cross-Section Fission Product Poisons

In addition to the high cross-section fission product poisons, Xe-135 and Sm-149, there are a host of low cross-section fission-product poisons (LCSFPP) that accumulate with reactor operation and, therefore, limit the reactivity-life and attainable exposure. The results of IDO-16100⁵ were used in accounting for the LCSFPP. The customary way of calculating the macroscopic absorption cross section of these LCSFPP is to determine the total number of fissions ($\times 10^{-24}$) that have taken place in unit volume of fuel slug up to any time, t , and to multiply this by the "barns per fissioned nuclei", p . This quantity, p , is computed in the following way. Suppose all the fission products are stable isotopes and are formed directly at the instant of fission. Then each one builds up as

$$\Sigma_i(t) = \Sigma_f y_i (1 - e^{-\phi \sigma_i t}) \quad (41)$$

where

$\Sigma_i(t)$ = the macroscopic capture cross section of fission product i at time t in the fuel slug

Σ_f = macroscopic fission cross section of the slug

y_i = yield of fission product i

σ_i = microscopic capture cross section of fission product i

The total macroscopic capture cross section of fission products at time t is

$$\sum_i \Sigma_i(t) = \Sigma_f \sum_i y_i (1 - e^{-\phi \sigma_i t}) \quad (42)$$

The quantity p , the barns per fissioned nucleus, is

$$p = \frac{10^{24} \sum_i \Sigma_i(t)}{\phi \Sigma_f t} = \frac{10^{24}}{\phi t} \sum_i y_i (1 - e^{-\phi \sigma_i t}) \quad (43)$$

It will be noted that p is a function of ϕt .

⁵Webster, J. W., "The Low Cross-Section Fission-Product Poisons", IDO-16100.

Actually, nearly all the fission products are formed in unstable chains where the decay is by β -emission and the mass number stays the same. In most cases it is only the stable end product, and in a few cases the immediate parent, that remain in the reactor long enough to capture a significant number of neutrons. In these few cases where the parent is important, Equation (41) takes the more complicated form derived in IDO-16100,

$$\Sigma(t) = \frac{\phi \Sigma_f y}{\lambda_p + \phi \sigma_p} \left[\sigma_p - \sigma_p e^{-(\lambda_p + \phi \sigma_p)t} + \frac{\lambda_p}{\phi} \right. \\ \left. + \frac{\lambda_p \sigma_s e^{-(\lambda_p + \phi \sigma_p)t}}{\lambda_p + \phi \sigma_p - \phi \sigma_s} - \frac{\lambda_p (\lambda_p + \phi \sigma_p) e^{\lambda \phi \sigma_s t}}{\phi (\lambda_p + \phi \sigma_p - \phi \sigma_s)} \right] \quad (44)$$

where the subscript p refers to parent and s to daughter and where the subscript i has been omitted for brevity. Equation (43) will be accordingly a somewhat more complicated expression.

It is seen that for those fission products for which the more complicated expression (44) must be used, rather than (41), the concentration of that poison is dependent not only on the product ϕt but on ϕ by itself as well. Fortunately, however, these fission products do not contribute strongly to the total LCSFP poisoning and results of p quoted purely in terms of ϕt can be used with wide applicability.

Graphs of $p(\phi t)$, taken from IDO-16100, are shown on the next two pages. The contribution of U-236 is included in these results. It can be treated as a fission product, although it is actually not one, of course. In IDO-16100, double neutron captures were not considered; i.e., cases where a fission product captures a neutron and is converted into an isotope which again has significant cross section and half-life. Thus, the results shown are slightly optimistic.

A further assumption is made that the curve $p(\phi t)$ is the same for Pu-239, Pu-241, and fast-fission of U-238, as it is for U-235 fissions.

Figure 4
LOW CROSS-SECTION FISSION PRODUCT POISONING
(Low Exposures)

Figure 5

LOW CROSS-SECTION FISSION PRODUCT POISONING
(High Exposures)

5. Discussion

There are several points of uncertainty about the cross section values arrived at above:

- a. The data on the microscopic cross sections are incomplete, especially for the isotopes of plutonium. This is especially true of the above-thermal cross sections. To quote from Dr. Westcott, "For Pu^{239} the main term results from the first resonance; somewhat arbitrarily, the value for the resonance integral above 0.65 ev has been taken as 260 barns (fission) and 390 barns (absorption). For Pu^{241} the measurements above 1 ev are poor. . . .; we have therefore first calculated the result assuming the resonance integral above 0.4 ev is only 75 to 80 barns due to the tail of the 0.269 ev resonance. . . . For Pu^{240} the resonance integral has been measured by Cornish and Lounsbury but the thermal value is uncertain."⁶
- b. The representation of the energy distribution of the flux, as being Maxwellian plus a $1/E$ tail chopped off at $5kT$, is, of course, by no means perfect. It is better for the graphite and D_2O lattices, however, than for H_2O lattices. For H_2O lattices we feel it is better to abandon the general prescription used here and go to a multigroup plus Wigner-Wilkins procedure.
When a substantial concentration of an isotope having a sharp resonance peak like Pu-240 has built up, a depression of the flux, which has energy within the peak, will start to show up and reduce the "effective" cross section of this isotope.
- c. In addition to these uncertainties and changes of the energy distribution of the flux, with exposure, there are also errors introduced by the non-uniform spatial distribution of flux within the fuel rod. In the usual procedure of calculating isotopic concentration changes with exposure, the

⁶Westcott, C. H., "Effective Cross Section Values for Well Moderated Thermal Reactor Spectra", January 25, 1957.

assumption is made that the uranium and plutonium isotopes are all exposed to the same flux. Since the thermal flux dips in a fuel rod and also since resonance U-238 capture is partly a fuel-surface effect, the Pu-239 will tend to build up more rapidly near the rod surface than at the center. This means the Pu-239 is exposed to a greater average flux than is the U-235 for low exposure levels. Eventually, the Pu-239 tends to saturate at a roughly uniform concentration throughout the fuel rod, but by this time, the U-235 is non-uniform due to non-uniform burnout. The higher isotopes of plutonium will also build up non-uniformly, even more so than Pu-239, and, therefore, will be exposed to a different average flux than Pu-239, U-235, or each other. These spatial effects are probably within the limits of uncertainty of the fundamental data at present but will eventually need to be evaluated.

C. NUMERICAL CALCULATIONS

Both the differential equations (Equations (1) through (6), Chapter V) describing the isotopic concentration changes and their analytic solutions (Equations (32) through (37)) were applied specifically to the Calder Hall reactor. A code (IBM-650 Program Library File No. 4.0.005) for obtaining numerical solutions by the Milne method was used for the differential equations. The analytic solutions were coded by Mr. James Burr of the American-Standard, Atomic Energy Division. Both codes were run on the IBM-650.

A study was made using the numerical code to determine the effect of the Pu-241 feedback term. Figure 6 shows the effect of neglecting this term altogether.* The difference in the plutonium isotopic buildup even at such large values of flux-time as $3.5 \times 10^{21} \text{ cm}^{-2}$ is not very significant. For exposure times of the order of 5,000 MWD/ton the difference is on the order of one percent.

*The cross sections used in obtaining the results of Figure 6 were revised somewhat and are slightly different from those used in the remainder of this report. Hence no direct comparison should be made between it and Figure 33 in which the revised cross sections were used.

Figure 6

D. COMPARISON OF ANALYTIC AND NUMERIC CODES

The following table will serve to compare the analytic and numeric codes in obtaining the isotopic concentrations of reactor fuel as a function of flux time given the initial concentrations.

Flux Time $\times 10^{-21} \text{cm}^2$	U-235		U-238		Pu-239	
	$\times 10^{-25} (\text{nuclei/ton})^{-1}$		$\times 10^{-27} (\text{nuclei/ton})^{-1}$		$\times 10^{-24} (\text{nuclei/ton})^{-1}$	
	Analytic	Numeric	Analytic	Numeric	Analytic	Numeric
0	1.659	1.659	2.2783	2.2783	0	0
10	0.86184	0.86184	2.271997	2.271997	4.165254	4.165260
20	0.44772	0.44772	2.265713	2.265713	4.730811	4.730885
35	0.16764	0.16764	2.256319	2.256319	4.736828	4.736979

The two codes are in excellent agreement. The concentration of Pu-242 differs between the two codes by about one percent for exposures of the order of 10,000 MWD/ton. For other isotopes the discrepancy between the two is even less.

It was originally planned to develop the analytic code to a point where it would require merely the input of the initial isotopic concentrations for any given set of reactor parameters, but lack of sufficient funds forestalled this venture for the present. Aside from being faster than the numeric code, its output would also include, in addition to that of the numeric code, the k_{ij} coefficients, eigenvalues, and other constants of Equations (32) through (37) in Section V thus permitting the exact determination of isotopic concentrations for any value of flux-time by a simple hand calculation rather than being limited to the fixed points as given by the numeric code.

V. SOLUTIONS OF EQUATIONS FOR ISOTOPIC CONCENTRATIONS

A. THE DIFFERENTIAL EQUATIONS AND SOLUTIONS

At the start of the project with which this report is concerned, it was decided that an attempt be made to obtain general analytic expressions governing the changes in isotopic concentrations of uranium and plutonium in an operating reactor. This study was to include, as a variable in the expression for the buildup of Pu-239, that term representing the resonance absorption in U-238 of those neutrons which are produced by the fission of Pu-241. In the past, due to the complexity of the problem, this term was either neglected or considered constant. Interrelated with this term affecting the buildup of Pu-239 are the concentrations of the higher mass-number plutonium isotopes. Thus, the variations in concentration of Pu-240, Pu-241, and Pu-242 were taken into account in the system of equations used.

Certain simplifying assumptions were made such as, neglecting the buildup of U-237 resulting from neutron capture by U-236 and the subsequent decay of U-237 into Np-237. In addition, it is assumed that U-238 goes directly into Pu-239, the intervening steps involving the β -decay of U-239 and Np-239 being neglected. The buildup of Americium, and other less frequent reactions are also not considered. Only those isotopes explicitly involved in the differential equations are taken into account.

The differential equations governing changes in fuel concentrations of an operating reactor as a function of flux-time are:

$$\frac{dN^{25}}{d\theta} = - N^{25} \sigma_a^{25} \quad (1)$$

$$\frac{dN^{28}}{d\theta} = - N^{28} \sigma_a^{28} \quad (2)$$

$$\begin{aligned} \frac{dN^{49}}{d\theta} = & N^{28} \sigma_a^{28} + N^{25} \sigma_f^{25} \nu^{25} \epsilon e^{-B^2 \tau} (1 - p) + N^{49} \sigma_f^{49} \nu^{49} \epsilon e^{-B^2 \tau} (1 - p) \\ & + N^{41} \sigma_f^{41} \nu^{41} \epsilon e^{-B^2 \tau} (1 - p) - N^{49} \sigma_a^{49} \end{aligned} \quad (3)$$

$$\frac{dN^{40}}{d\theta} = N^{49}_{\sigma_c} - N^{40}_{\sigma_a} \quad (4)$$

$$\frac{dN^{41}}{d\theta} = N^{40}_{\sigma_a} - N^{41}_{\sigma_a} \quad (5)$$

$$\frac{dN^{42}}{d\theta} = N^{41}_{\sigma_c} - N^{42}_{\sigma_a} \quad (6)$$

where

$$\theta = \int_0^t \phi(T) dT \quad (7)$$

the flux time, θ , being the integral of the product of the flux, $\phi(T)$, and the time, t .

Let

$$\begin{aligned} C_{11} &= -\sigma_a^{25} \\ C_{22} &= -\sigma_a^{28} \\ C_{31} &= \sigma_f^{25} \nu^{25} e^{-B^2 \tau} (1 - p) \\ C_{32} &= \sigma_a^{28} \\ C_{33} &= \sigma_f^{49} \nu^{49} e^{-B^2 \tau} (1 - p) - \sigma_a^{49} \\ C_{35} &= \sigma_f^{41} \nu^{41} e^{-B^2 \tau} (1 - p) \\ C_{43} &= \sigma_c^{49} \\ C_{44} &= -\sigma_a^{40} \\ C_{54} &= \sigma_a^{40} \\ C_{55} &= -\sigma_a^{41} \\ C_{65} &= \sigma_c^{41} \\ C_{66} &= -\sigma_a^{42} \end{aligned} \quad (8)$$

All other $C_{ij} \equiv 0$

The system of differential Equations (1) through (6) can now be written in matrix form:

$$\dot{N}^i = \sum_j C_{ij} N^j \quad (9)$$

where $i, j = 1, \dots, 6$; it being hereafter understood that the following correspondence exists, namely,

$$N^1 \equiv N^{25}, N^2 \equiv N^{28}, N^3 \equiv N^{49}, N^4 \equiv N^{40}, N^5 \equiv N^{41}, \text{ and } N^6 \equiv N^{42}$$

The dot above the N^i indicates the total derivative with respect to the variable flux-time.

The eigenvalues λ_j of the matrix result from setting the determinant

$$\left| C_{ij} - \lambda_j \delta_{ij} \right| = 0 \quad (10)$$

This 6 x 6 determinant reduces to

$$(C_{11} - \lambda_1) (C_{22} - \lambda_2) (C_{66} - \lambda_6) \begin{vmatrix} (C_{33} - \lambda_3) & 0 & C_{35} \\ C_{43} & (C_{44} - \lambda_4) & 0 \\ 0 & C_{54} & (C_{55} - \lambda_5) \end{vmatrix} = 0 \quad (11)$$

Three of the eigenvalues are immediately obvious:

$$\lambda_1 = C_{11} \quad (12)$$

$$\lambda_2 = C_{22} \quad (13)$$

$$\lambda_6 = C_{66} \quad (14)$$

To evaluate the other three eigenvalues we must solve the 3 x 3 determinant for values of λ . From the determinant we obtain the cubic equation:

$$\lambda^3 + b\lambda^2 + c\lambda + d = 0 \quad (15)$$

where

$$b = - [C_{33} + C_{44} + C_{55}]$$

$$c = [C_{33}C_{44} + C_{33}C_{55} + C_{44}C_{55}]$$

$$d = - [C_{33}C_{44}C_{55} + C_{35}C_{43}C_{54}]$$

The roots of the general cubic (Equation (15)) are given by:

$$\lambda_3 = A + B - \frac{b}{3}$$

$$\lambda_4 = \omega A + \omega^2 B - \frac{b}{3}$$

$$\lambda_5 = \omega^2 A + \omega B - \frac{b}{3}$$

where:

$$\omega = -1/2 + (1/2) (\sqrt{3})(i)$$

$$A = \left(-\frac{q}{2} + R^{1/2} \right)^{1/3}$$

$$B = \left(-\frac{q}{2} - R^{1/2} \right)^{1/3}$$

$$R = \left(\frac{p}{3} \right) + \left(\frac{q}{2} \right)^2$$

$$p = c - \frac{b^2}{3}$$

$$q = d - \frac{bc}{3} + \frac{2b^3}{27}$$

Now the N's of Equation (9) may be written as linear combinations of functions involving the eigenvalues with appropriate coefficients, i.e.,

$$N^i = \sum_j k_{ij} e^{\lambda_j \theta} \quad (16)$$

where the λ 's are in terms of the C_{ij} 's. Differentiating Equation (16) with respect to θ ,

$$\dot{N}^i = \sum_j \lambda_j k_{ij} e^{\lambda_j \theta} \quad (17)$$

The substitution of Equation (16) into Equation (9) yields:

$$\dot{N}^i = \sum_m \left[\sum_j C_{ij} k_{jm} \right] e^{\lambda_m \theta} \quad (18)$$

Carrying out the matrix multiplication indicated in the brackets of Equation (18) and then equating the corresponding matrix elements of Equations (17) and (18) results in the determination of the k_{ij} coefficients of Equation (16), some of which are identically zero. All the non-zero coefficients can be obtained in terms of the six diagonal elements of the k_{ij} matrix of Equation (16) all of which are arbitrary and are determined by the boundary conditions. Hence, the coefficients may be written in the more convenient form:

$$k_{ij} = A_{ij} k_{jj} \quad (19)$$

where, obviously,

$$A_{jj} = 1$$

Written in this form the A_{ij} are independent of the boundary conditions and are functions only of the eigenvalues and the cross sections, etc., as given by Equation (8).

Equation (16) can now be written in matrix form as:

$$\begin{pmatrix} N^{25} \\ N^{28} \\ N^{49} \\ N^{40} \\ N^{41} \\ N^{42} \end{pmatrix} = \begin{pmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & 0 \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & 0 \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & 0 \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & 0 \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} \end{pmatrix} \begin{pmatrix} e^{\lambda_1 \theta} \\ e^{\lambda_2 \theta} \\ e^{\lambda_3 \theta} \\ e^{\lambda_4 \theta} \\ e^{\lambda_5 \theta} \\ e^{\lambda_6 \theta} \end{pmatrix} \quad (20)$$

Carrying out the indicated matrix multiplication and using Equation (19), there results:

$$N^{25} = k_{11} e^{\lambda_1 \theta} \quad (21)$$

$$N^{28} = k_{22} e^{\lambda_2 \theta} \quad (22)$$

$$N^{49} = A_{31} k_{11} e^{\lambda_1 \theta} + A_{32} k_{22} e^{\lambda_2 \theta} + k_{33} e^{\lambda_3 \theta} + A_{34} k_{44} e^{\lambda_4 \theta} + A_{35} k_{55} e^{\lambda_5 \theta} \quad (23)$$

$$N^{40} = A_{41}k_{11}e^{\lambda_1\theta} + A_{42}k_{22}e^{\lambda_2\theta} + A_{43}k_{33}e^{\lambda_3\theta} + k_{44}e^{\lambda_4\theta} + A_{45}k_{55}e^{\lambda_5\theta} \quad (24)$$

$$N^{41} = A_{51}k_{11}e^{\lambda_1\theta} + A_{52}k_{22}e^{\lambda_2\theta} + A_{53}k_{33}e^{\lambda_3\theta} + A_{54}k_{44}e^{\lambda_4\theta} + k_{55}e^{\lambda_5\theta} \quad (25)$$

$$N^{42} = A_{61}k_{11}e^{\lambda_1\theta} + A_{62}k_{22}e^{\lambda_2\theta} + A_{63}k_{33}e^{\lambda_3\theta} + A_{64}k_{44}e^{\lambda_4\theta} + A_{65}k_{55}e^{\lambda_5\theta} + k_{66}e^{\lambda_6\theta} \quad (26)$$

The determination of the k_{jj} by employing certain boundary conditions, follows. For example, at flux-time $\theta = 0$, let $N^{49} = N_o^{49}$. Then:

$$k_{33} = N_o^{49} - (A_{31}k_{11} + A_{32}k_{22} + A_{34}k_{44} + A_{35}k_{55}) \quad (27)$$

Similarly, one obtains k_{11} , k_{22} , k_{44} , k_{55} , and k_{66} . But it is to be noted that, with the exception of k_{11} and k_{22} which are identical with the initial concentrations of U-235 and U-238, respectively; that each k_{jj} is in terms of all the others. Thus, there needs to be solved a system of four linear, independent, simultaneous equations. These are:

$$k_{33} + A_{34}k_{44} + A_{35}k_{55} + 0 = N_o^{49} - A_{31}N_o^{25} - A_{32}N_o^{28} = M_1 \quad (28)$$

$$A_{43}k_{33} + k_{44} + A_{45}k_{55} + 0 = N_o^{40} - A_{41}N_o^{25} - A_{42}N_o^{28} = M_2 \quad (29)$$

$$A_{53}k_{33} + A_{54}k_{44} + k_{55} + 0 = N_o^{41} - A_{51}N_o^{25} - A_{52}N_o^{28} = M_3 \quad (30)$$

$$A_{63}k_{33} + A_{64}k_{44} + A_{65}k_{55} + k_{66} = N_o^{42} - A_{61}N_o^{25} - A_{62}N_o^{28} = M_4 \quad (31)$$

This system of equations can be solved by use of determinants and the resulting expressions for k_{33} , k_{44} , k_{55} , and k_{66} are functions only of the physical parameters of Equation (8) and the initial concentrations of the uranium and plutonium isotopes. With the values of k_{jj} thus obtained, Equations (21) through (26) give the uranium or plutonium isotopic concentrations for any flux-time, θ .

Return, now, to the cubic equation which yielded the eigenvalues λ_3 , λ_4 , and λ_5 . It is known that a general cubic equation will have either none or one pair of complex conjugate roots only. Assuming a pair of complex conjugate eigenvalues exist; if λ_3 is defined as the real eigenvalue,

and λ_4 and λ_5 the complex conjugate pair in the form $p + qi$ and $p - qi$, respectively, then the Equations (21) through (26) may be put into a more convenient form, viz:

$$N^{25} = k_{11} e^{\lambda_1 \theta} \quad (32)$$

$$N^{28} = k_{22} e^{\lambda_2 \theta} \quad (33)$$

$$N^{49} = k_{31} e^{\lambda_1 \theta} + k_{32} e^{\lambda_2 \theta} + k_{33} e^{\lambda_3 \theta} + 2e^{p\theta} \left(G_{34} \cos q\theta - H_{34} \sin q\theta \right) \quad (34)$$

$$N^{40} = k_{41} e^{\lambda_1 \theta} + k_{42} e^{\lambda_2 \theta} + k_{43} e^{\lambda_3 \theta} + 2e^{p\theta} \left(G_{44} \cos q\theta - H_{44} \sin q\theta \right) \quad (35)$$

$$N^{41} = k_{51} e^{\lambda_1 \theta} + k_{52} e^{\lambda_2 \theta} + k_{53} e^{\lambda_3 \theta} + 2e^{p\theta} \left(G_{55} \cos q\theta - H_{55} \sin q\theta \right) \quad (36)$$

$$N^{42} = k_{61} e^{\lambda_1 \theta} + k_{62} e^{\lambda_2 \theta} + k_{63} e^{\lambda_3 \theta} + 2e^{p\theta} \left(G_{64} \cos q\theta - H_{64} \sin q\theta \right) + k_{66} e^{\lambda_6 \theta} \quad (37)$$

The results, obviously, are real.

It is to be noted that when $\lambda_4 = \overline{\lambda_5}$, then

$$\begin{aligned} A_{34} k_{44} &= \overline{A_{35} k_{55}} \\ k_{44} &= \overline{A_{45} k_{55}} \\ A_{54} k_{44} &= \overline{k_{55}} \\ A_{64} k_{44} &= \overline{A_{65} k_{55}} \end{aligned} \quad (38)$$

in Equations (23) through (26). The aforementioned are the only complex quantities which may occur. All others are real. The remaining quantities for obtaining solution to Equations (32) through (37) follows

$$A_{31} = \frac{C_{31}}{(\lambda_1 - C_{33})} \left(1 + \frac{C_{35} C_{43} C_{54}}{(\lambda_1 - C_{33})(\lambda_1 - C_{44})(\lambda_1 - C_{55}) - C_{35} C_{43} C_{54}} \right)$$

$$A_{32} = \frac{c_{32}}{(\lambda_2 - c_{33})} \left(1 + \frac{c_{35}c_{43}c_{54}}{(\lambda_2 - c_{33})(\lambda_2 - c_{44})(\lambda_2 - c_{55}) - c_{35}c_{43}c_{54}} \right)$$

$$A_{34} = \frac{(\lambda_4 - c_{44})}{c_{43}}$$

$$A_{35} = \frac{c_{35}}{(\lambda_5 - c_{33})}$$

$$A_{41} = \frac{c_{31}c_{43}}{(\lambda_1 - c_{33})(\lambda_1 - c_{44})} \left(1 + \frac{c_{35}c_{43}c_{54}}{(\lambda_1 - c_{33})(\lambda_1 - c_{44})(\lambda_1 - c_{55}) - c_{35}c_{43}c_{54}} \right)$$

$$A_{42} = \frac{c_{32}c_{43}}{(\lambda_2 - c_{33})(\lambda_2 - c_{44})} \left(1 + \frac{c_{35}c_{43}c_{54}}{(\lambda_2 - c_{33})(\lambda_2 - c_{44})(\lambda_2 - c_{55}) - c_{35}c_{43}c_{54}} \right)$$

$$A_{43} = \frac{c_{43}}{(\lambda_3 - c_{44})}$$

$$A_{45} = \frac{c_{35}c_{43}}{(\lambda_5 - c_{33})(\lambda_5 - c_{44})}$$

$$A_{51} = \frac{c_{31}c_{43}c_{54}}{(\lambda_1 - c_{33})(\lambda_1 - c_{44})(\lambda_1 - c_{55}) - c_{35}c_{43}c_{54}}$$

$$A_{52} = \frac{c_{32}c_{43}c_{54}}{(\lambda_2 - c_{33})(\lambda_2 - c_{44})(\lambda_2 - c_{55}) - c_{35}c_{43}c_{54}}$$

$$A_{53} = \frac{(\lambda_3 - c_{33})}{c_{35}}$$

$$A_{54} = \frac{(\lambda_4 - c_{33})(\lambda_4 - c_{44})}{c_{35}c_{43}}$$

$$A_{61} = \left(\frac{c_{65}}{\lambda_1 - c_{66}} \right) \left(\frac{c_{31}c_{43}c_{54}}{(\lambda_1 - c_{33})(\lambda_1 - c_{44})(\lambda_1 - c_{55}) - c_{35}c_{43}c_{54}} \right)$$

$$= \frac{c_{65}}{(\lambda_1 - c_{66})} A_{51}$$

$$A_{62} = \left(\frac{C_{65}}{\lambda_2 - C_{66}} \right) \left(\frac{C_{32}C_{43}C_{54}}{(\lambda_2 - C_{33})(\lambda_2 - C_{44})(\lambda_2 - C_{55}) - C_{35}C_{43}C_{54}} \right)$$

$$= \frac{C_{65}}{(\lambda_2 - C_{66})} A_{52}$$

$$A_{63} = \frac{C_{65}}{C_{35}} \frac{(\lambda_3 - C_{33})}{(\lambda_3 - C_{66})} = \frac{C_{65}}{(\lambda_3 - C_{66})} A_{53}$$

$$A_{64} = \frac{C_{65}}{C_{35}C_{43}} \frac{(\lambda_4 - C_{33})(\lambda_4 - C_{44})}{(\lambda_4 - C_{66})} = \frac{C_{65}}{(\lambda_4 - C_{66})} A_{54}$$

$$A_{65} = \frac{C_{65}}{(\lambda_5 - C_{66})}$$

$$p = \operatorname{Re} \lambda_4 \equiv \operatorname{Re} \lambda_5$$

$$q = \operatorname{Coeff} \operatorname{Im} \lambda_4 = - \operatorname{Coeff} \operatorname{Im} \lambda_5$$

$$G_{34} = \operatorname{Re} k_{34}$$

$$G_{44} = \operatorname{Re} k_{44}$$

$$G_{55} = \operatorname{Re} k_{55}$$

$$G_{64} = \operatorname{Re} k_{64}$$

$$H_{34} = \operatorname{Coeff} \operatorname{Im} k_{34}$$

$$H_{44} = \operatorname{Coeff} \operatorname{Im} k_{44}$$

$$H_{55} = \operatorname{Coeff} \operatorname{Im} k_{55}$$

$$H_{64} = \operatorname{Coeff} \operatorname{Im} k_{64}$$

$$k_{11} = N_o^{25}$$

$$k_{22} = N_o^{28}$$

$$k_{33} = \frac{1}{\Delta} \left[M_1 (1 - A_{45}A_{54}) + M_2 (A_{35}A_{54} - A_{34}) + M_3 (A_{34}A_{45} - A_{35}) \right]$$

$$k_{44} = \frac{1}{\Delta} \left[(M_2 - M_3A_{45}) + M_1 (A_{45}A_{53} - A_{43}) + A_{35} (A_{43}M_3 - A_{53}M_2) \right]$$

$$k_{55} = \frac{1}{\Delta} \left[(M_3 - M_2A_{54}) + A_{34} (M_2A_{53} - M_3A_{43}) + M_1 (A_{43}A_{54} - A_{53}) \right]$$

$$k_{66} = \frac{1}{\Delta} \left\{ M_4 \Delta - M_3 \left[\left(A_{65} - A_{45} A_{64} \right) + A_{34} \left(A_{45} A_{63} - A_{43} A_{65} \right) + A_{35} \left(A_{43} A_{64} - A_{63} \right) \right] \right. \\ \left. + M_2 \left[\left(A_{54} A_{65} - A_{64} \right) + A_{34} \left(A_{63} - A_{53} A_{65} \right) + A_{35} \left(A_{53} A_{64} - A_{54} A_{63} \right) \right] \right. \\ \left. - M_1 \left[A_{43} \left(A_{54} A_{65} - A_{64} \right) + \left(A_{63} - A_{53} A_{65} \right) + A_{45} \left(A_{53} A_{64} - A_{54} A_{63} \right) \right] \right\}$$

$$\Delta = \left(1 - A_{45} A_{54} \right) + A_{34} \left(A_{45} A_{53} - A_{43} \right) + A_{35} \left(A_{43} A_{54} - A_{53} \right)$$

$$M_1 = N_o^{49} - N_o^{25} A_{31} - N_o^{28} A_{32}$$

$$M_2 = N_o^{40} - N_o^{25} A_{41} - N_o^{28} A_{42}$$

$$M_3 = N_o^{41} - N_o^{25} A_{51} - N_o^{28} A_{52}$$

$$M_4 = N_o^{42} - N_o^{25} A_{61} - N_o^{28} A_{62}$$

B. REACTIVITY LIFETIME

In determining the active life of the Calder Hall reactor, it was assumed that p , the resonance escape probability, and ϵ , the fast-fission factor, remains constant during the activity of the reactor. Thus the reactivity will then be a function of ηf . In calculating ηf , the reactor was assumed hot and poisoned with the equilibrium values of the Xenon and Samarium poisons included at $\theta = 0$. In the recycle case the initial isotopic concentrations of plutonium were taken into account. A value of $(\eta f)_0 = 1.13913$ was used as calculated from published British reports on Calder Hall.

1. Low Cross-Section Fission Product Poisons

In addition to the equilibrium Xenon and Samarium poisons the low cross-section fission product poisons as defined in IDO-16100, "Low Cross-Section Fission Product Poisons", by J. W. Webster, were considered as follows:

$$N_j^{F.P.} = \sum_{\alpha} N_j^{\alpha} (\sigma_f^{\alpha}) (\Delta\theta)_j + \sum_{k=1}^{j-1} N_k^{F.P.}$$

where

j = time interval in units of $0.1 \times 10^{21} \text{ cm}^{-2}$
 $\alpha \rightarrow \text{U}^{235}, \text{Pu}^{239}, \text{Pu}^{241}$

σ_f^α = cross sections for fission of each of the fuels

$\Delta\theta = 0.1 \times 10^{21} \text{ cm}^{-2}$

The cross sections for these low cross-section fission-product poisons vary with exposure and are taken from the aforementioned report as follows:

<u>jth time interval</u>	<u>$\sigma_a^{F.P.}$</u>
0 ($\times 10^{21} \text{ cm}^{-2}$)	83.5 ($\times 10^{-24} \text{ cm}^2$)
0.1 ($\times 10^{21} \text{ cm}^{-2}$)	83.5 ($\times 10^{-24} \text{ cm}^2$)
0.2 ($\times 10^{21} \text{ cm}^{-2}$)	72.5 ($\times 10^{-24} \text{ cm}^2$)
0.3 ($\times 10^{21} \text{ cm}^{-2}$)	65.3 ($\times 10^{-24} \text{ cm}^2$)
0.5 ($\times 10^{21} \text{ cm}^{-2}$)	62.6 ($\times 10^{-24} \text{ cm}^2$)
1.0 ($\times 10^{21} \text{ cm}^{-2}$)	56.3 ($\times 10^{-24} \text{ cm}^2$)
1.5 ($\times 10^{21} \text{ cm}^{-2}$)	51.0 ($\times 10^{-24} \text{ cm}^2$)
2.0 ($\times 10^{21} \text{ cm}^{-2}$)	47.9 ($\times 10^{-24} \text{ cm}^2$)
2.5 ($\times 10^{21} \text{ cm}^{-2}$)	45.4 ($\times 10^{-24} \text{ cm}^2$)
3.0 ($\times 10^{21} \text{ cm}^{-2}$)	41.4 ($\times 10^{-24} \text{ cm}^2$)

2. Energy Output

The energy generated by a reactor is proportional to the number of fissions. So,

$$\frac{dE}{d\theta} = \gamma \frac{F}{f} \left(N^{25}_{25} \sigma_f^{25} + N^{49}_{49} \sigma_f^{49} + N^{41}_{41} \sigma_f^{41} \right)$$

or, integrating between the flux-times, θ_1 and θ_2 ,

$$E = \gamma \frac{F}{f} \int_{\theta_1}^{\theta_2} \left(N^{25}_{25} \sigma_f^{25} + N^{49}_{49} \sigma_f^{49} + N^{41}_{41} \sigma_f^{41} \right) d\theta$$

where

E = energy in MWD/ton

$\frac{F}{f}$ = ratio of total fissions to thermal fissions

γ = a constant depending on the units

VI. SOLUTION FOR REACTIVITY LIFE

A large reactor like Calder Hall can be well represented by the one group model:

$$-D\nabla^2\phi + \phi\Sigma_a - \phi\Sigma_a k_{\infty} = 0 \quad (1)$$

where the first term is the leakage rate per unit volume, the second is the absorption rate, and the third is the production rate. The flux, ϕ , may be specified as the conventional flux as defined in Section IV. The quantity, D , is not the true diffusion coefficient, but is a quantity which when multiplied by $\nabla^2\phi$ gives the leakage rate. It is taken as

$$D \cong M^2 \Sigma_a \quad (2)$$

where

$M^2 \equiv$ the migration area of a neutron from birth to absorption

$\Sigma_a \equiv$ the macroscopic absorption cross section

Other definitions are:

$k_{\infty} \equiv \eta f \epsilon p$

$\eta \equiv$ the fertility

$f \equiv$ the utilization

$\epsilon \equiv$ the fast-fission factor

$p \equiv$ the resonance escape probability

In the evaluation of Σ_a , η , and f , the effective cross sections as defined in Section IV should be used.

Using Equation (2), Equation (1) can be written

$$\nabla^2\phi + \frac{k_{\infty} - 1}{M^2} \phi = 0 \quad (3)$$

Defining

$$B_m^2 = \frac{k_{\infty} - 1}{M^2} \quad (4)$$

Equation (3) is

$$\nabla^2\phi + B_m^2\phi = 0 \quad (5)$$

where B_m^2 is the material buckling.

If the reflected reactor is approximated by the equivalent-bare, cylindrical reactor, the solution to (5) is

$$\phi \propto J_0(\alpha r) \cos Bz \quad (6)$$

where

$$B_m^2 = \alpha^2 + \beta^2 \quad (7)$$

Since the flux must be zero at the boundaries of the equivalent-bare reactor,

$$\alpha = \frac{2.405}{R} \quad (8)$$

and

$$\beta = \frac{\pi}{H} \quad (9)$$

when R and H are the radius and height. Thus in the critical reactor

$$B_m^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 \quad (10)$$

Now suppose the size is fixed and the reactor is non-critical. Then (10) no longer is true. Define

$$B_g^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 \quad (11)$$

as the geometrical buckling. If the reactor is critical, then

$$B_m^2 = B_g^2 \quad (12)$$

but if the reactor is supercritical, then

$$B_m^2 > B_g^2 \quad (13)$$

where B_m^2 is given by (4) and B_g^2 by (11). It follows that

$$B_m^2 - B_g^2 \quad (14)$$

can be taken as a measure of the excess reactivity of the reactor. It is usually quoted in terms of "microbucks" of excess buckling.

To determine the reactivity life of a fuel charge the change in B_m^2 is calculated as a function of exposure as U-235 burns out and the plutonium isotopes and fission products build up. When B_m^2 becomes equal to B_g^2 , the core must be reloaded.

The approximation that leads to the simplest computations is to assume that B_m^2 changes uniformly over the core with time, i.e., that the flux is flat over the core or at least that the spatial average of the flux can be applied at points of the core.

Usually it can be assumed that M^2 is not affected much by the changes that take place with irradiation. This can also certainly be said for p and ϵ . It follows that changes in B_m^2 with exposure can be expressed approximately as

$$\Delta B_m^2 = \Delta \left(\frac{k_{\infty} - 1}{M^2} \right) \cong \frac{p\epsilon}{M^2} \Delta(\eta f) \quad (15)$$

Another way to measure reactivity in one-group theory is from the expression

$$k_{\text{eff}} = \frac{k_{\infty}}{1 + M_o^2 B_g^2} \quad (16)$$

Suppose the reactor is just barely critical when it is at its operating temperature and equilibrium Xe and Sm are present. This is very nearly the case for Calder Hall. Define $(k_{\infty})_o$ and M_o^2 as the k_{∞} and M^2 for this condition. Then

$$1 = \frac{(k_{\infty})_o}{1 + M_o^2 B_g^2} \quad (17)$$

Dividing (17) into (16) gives

$$k_{\text{eff}} = \frac{k_{\infty}}{(k_{\infty})_o} \frac{(1 + M_o^2 B_g^2)}{(1 + M^2 B_g^2)} \quad (18)$$

If $M_o^2 \cong M^2$, i.e., if M^2 does not change much with exposure; and if p and ϵ do not either, then (18) reduces to

$$k_{\text{eff}} = \frac{\eta f}{(\eta f)_o} \quad (19)$$

When this becomes unity, the core must be recharged. It may be noted that it is only necessary that $M_o^2 \cong M^2$ at the end of the irradiation for (19) to give as accurate a prediction of reactivity life as (18). It is (19) that we plot as a function of ϕt or exposure (MWD/ton).

Now let us define ηf more precisely. It is given by

$$\eta f = \frac{N^{25}_{\sigma_a} \eta^{25} + N^{49}_{\sigma_a} \eta^{49} + N^{41}_{\sigma_a} \eta^{41}}{\left(N^{25}_{\sigma_a} + N^{28}_{\sigma_a} + N^{49}_{\sigma_a} + N^{40}_{\sigma_a} + N^{41}_{\sigma_a} + N^{42}_{\sigma_a} \right.} \\ \left. + N^{Mg}_{\sigma_a} \frac{\bar{\phi}^{Mg}}{\bar{\phi}^f} + N^C_{\sigma_a} \frac{\bar{\phi}^C}{\bar{\phi}^f} + N^{Xe}_{\sigma_a} + N^{Sm}_{\sigma_a} + N^{FP}_{\sigma_a} \right) \quad (20)$$

where Mg refers to the clad and structural material, C to graphite, f to fuel, Xe to Xenon, Sm to samarium and FP to low cross-section fission-product pairs (see Section IV); and N^i refers to the number of nuclei of material i in or associated with a ton of fuel.

The N^i are all functions of ϕt with the exception of N^{28} (to a good approximation), N^{Mg} , N^C , N^{Xe} , and N^{Sm} (to a good approximation). The σ_a^{FP} is also a function of ϕt (see Section IV). The N^{41} is a function of t alone in addition to ϕt because of its relatively short half-life, 13 years. The solution of N^{25} , N^{49} , N^{40} , N^{41} , and N^{42} as a function of ϕt is discussed in Section V. Additional formulas are

$$N^{FP} = \int_0^t \phi \left(N^{25}_{\sigma_f} + N^{49}_{\sigma_f} + N^{41}_{\sigma_f} \right) dt \quad (21)$$

$$N^{Xe} = \frac{\phi \left(N^{25}_{\sigma_f} + N^{49}_{\sigma_f} + N^{41}_{\sigma_f} \right) Y^{Xe}}{\phi \sigma_a^{Xe} + \lambda^{Xe}} \quad (22)$$

$$N^{Sm} = \frac{Y^{Sm}}{\sigma_a^{Sm}} \left(N^{25}_{\sigma_f} + N^{49}_{\sigma_f} + N^{41}_{\sigma_f} \right) \quad (23)$$

when it is assumed, without any knowledge to the contrary, that the yields, Y^{Xe} and Y^{Sm} , are the same for plutonium fission as for uranium fission.

To determine the terms

$$N^{Mg}_{\sigma_a} \frac{\bar{\phi}^{Mg}}{\bar{\phi}^f}$$

and

$$N^C_{\sigma_a} \frac{\bar{\phi}^C}{\bar{\phi}^f}$$

we chose the following simple scheme. The British give the following average values for the hot, clean reactor:

$$\begin{aligned}\eta &= 1.266 \\ f &= .929\end{aligned}$$

or

$$(\eta f)_{hc} = 1.176$$

Equation (20) for the hot, clean reactor reduces to

$$(\eta f)_{hc} = \frac{N_o^{25} \sigma_a^{25} \eta^{25}}{N_o^{25} \sigma_a^{25} + N_o^{28} \sigma_a^{28} + \left(N_{c_a}^{Mg} \sigma_a^{Mg} \frac{\bar{\phi}^{Mg}}{\bar{\phi}^f} + N_{c_a}^C \sigma_a^C \frac{\bar{\phi}^C}{\bar{\phi}^f} \right)}$$

The quantity in parenthesis in (24) can then be determined. The σ_a^{25} , η^{25} , and σ_a^{28} should be evaluated here from Westcott's data for 250° (as was done in Section IV).

The $(\eta f)_o$ can then be evaluated from (20) as

$$(\eta f)_o = \frac{N_o^{25} \sigma_a^{25} \eta^{25}}{\left[N_o^{25} \sigma_a^{25} + N_o^{28} \sigma_a^{28} + \left(N_{c_a}^{Mg} \sigma_a^{Mg} \frac{\bar{\phi}^{Mg}}{\bar{\phi}^f} + N_{c_a}^C \sigma_a^C \frac{\bar{\phi}^C}{\bar{\phi}^f} \right) + N_{c_a}^{Xe} \sigma_a^{Xe} + N_{c_a}^{Sm} \sigma_a^{Sm} \right]} \quad (25)$$

where the cross sections are as given in Section IV for 250°. The ηf is determined as a function of ϕt and $\eta f / (\eta f)_o$ plotted vs ϕt . Since ϕt can be determined as a function of exposure (MWD/ton), it is also possible to plot $\eta f / (\eta f)_o$ vs exposure.

Now let us consider the effect of the non-uniform flux as well as the particular scheme of reloading that is used in Calder Hall. In Calder Hall the fuel channels are reloaded by zones, i.e., reloading is not so difficult that it is necessary to shut down completely and reload the entire core as a batch. However, it is necessary to reload all elements in a given channel - all six elements - at a time. In the vertical direction the elements near the midplane will have sustained a greater exposure, and greater change in material buckling, at the time of discharge than will those near the top and bottom. Since the average buckling is obtained by weighting the buckling as a function of position by the square of the flux, as will be derived below, the non-uniformity of the vertical distribution of the exposure results in less reactivity life than would

be the case for uniform exposure at the average flux of the vertical distribution. On the other hand, the possibility of reloading a radial zone at a time leads to higher exposure than would be the case for uniform radial exposure at the average flux of the radial flux distribution. This can be seen from the following reasoning. In the first run, the inner zone will have its $\eta f / (\eta f)_0$ return to unity first. At the time this happens, the $\eta f / (\eta f)_0$ in the other zones will still be greater than unity. This permits continuation of operation until the inner zone has its $\eta f / (\eta f)_0$ somewhat less than unity and, therefore, the fuel elements in the inner zone receive an exposure greater than they would for a flat flux, batch loading situation. The same argument follows for slugs in the zone adjacent to the inner one, etc.

The loading scheme for Calder Hall may be thought of as approaching continuous charge-discharge insofar as complete channels are concerned, but batch charge-discharge for any given channel.

To determine the exposure achieved by each fuel element at the time of its discharge under this loading scheme, and with the plutonium being recycled, is a tremendous problem of bookkeeping. This is so because the time at which the fuel elements from any given channel should be discharged is determined by the composition at that moment of all other fuel elements in the reactor, as well as the location and composition of the channel in question. In general, the plutonium will be of a different grade in different channels, especially with regard to Pu-242 content. (This situation would be simplified, of course, when the plutonium which is being recycled has reached an equilibrium ratio of isotopes. This will not occur, however, for a very long time in Calder Hall, perhaps a century.)

In principle, though, the condition of criticality that must exist at any time can be expressed for the non-uniform reactor rather easily. Glasstone and Edlund⁷ show that if there are n regions in the reactor, each with a material buckling $B_{M_n}^2$ characteristic of that region at any time the over-all average

⁷Glasstone and Edlund, "The Elements of Nuclear Reactor Theory", p. 378.

buckling is given by

$$\bar{B}_M^{-2} = \frac{B_{M_1}^2 \phi_1^2 V_1 + B_{M_2}^2 \phi_2^2 V_2 + \dots + B_{M_n}^2 \phi_n^2 V_n}{\phi_1^2 V_1 + \phi_2^2 V_2 + \dots + \phi_n^2 V_n}$$

where ϕ_n is the average flux in region n, and V_n is the volume of region n.

Rather than attempting to study the effect on exposure level of the radial and axial non-uniform flux and the zone discharge and charge, which would be a very complex undertaking, especially with plutonium being recycled, it was decided to be content with a study of the effect of the non-uniform axial flux. This is reported in Section III.

Figure 7

EFFECT OF Pu-241 HALF-LIFE ON ISOTOPIC CONCENTRATION
1st PLUTONIUM RECYCLE

Figure 8

EFFECT OF Pu-241 HALF-LIFE ON REACTIVITY LIFETIME
1st PLUTONIUM RECYCLE

- 99 -

Figure 9

REACTIVITY LIFETIME

Figure 10

REACTIVITY LIFETIME
1st PLUTONIUM RECYCLE

Figure 11

REACTIVITY LIFETIME
2nd PLUTONIUM RECYCLE

Figure 12

REACTIVITY LIFETIME
3rd PLUTONIUM RECYCLE

Figure 13

REACTIVITY LIFETIME
4th PLUTONIUM RECYCLE

Figure 14

REACTIVITY LIFETIME
5th PLUTONIUM RECYCLE

Figure 15

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM

Figure 16

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
1st PLUTONIUM RECYCLE

Figure 17

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
2nd PLUTONIUM RECYCLE

Figure 18

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
3rd PLUTONIUM RECYCLE

Figure 19

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
4th PLUTONIUM RECYCLE

Figure 20

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
5th PLUTONIUM RECYCLE

Figure 21

KILOGRAMS OF PLUTONIUM PER TON OF FUEL

Figure 22

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
1st PLUTONIUM RECYCLE

Figure 23

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
2nd PLUTONIUM RECYCLE

Figure 24

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
3rd PLUTONIUM RECYCLE

Figure 25

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
4th PLUTONIUM RECYCLE

Figure 26

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
5th PLUTONIUM RECYCLE

Figure 27

WEIGHT PERCENT OF U-235

Figure 28

WEIGHT PERCENT OF U-235
1st PLUTONIUM RECYCLE

Figure 29

WEIGHT PERCENT OF U-235
2nd PLUTONIUM RECYCLE

Figure 30

WEIGHT PERCENT OF U-235
3rd PLUTONIUM RECYCLE

Figure 31

WEIGHT PERCENT OF U-235
4th PLUTONIUM RECYCLE

Figure 32

WEIGHT PERCENT OF U-235
5th PLUTONIUM RECYCLE

Figure 33

ISOTOPIC CONCENTRATION vs FLUX-TIME

Figure 34
ISOTOPIC CONCENTRATION vs FLUX-TIME
1st PLUTONIUM RECYCLE

Figure 35

ISOTOPIC CONCENTRATION vs FLUX-TIME
2nd PLUTONIUM RECYCLE

Figure 36

ISOTOPIC CONCENTRATION vs FLUX-TIME
3rd PLUTONIUM RECYCLE

Figure 37

ISOTOPIC CONCENTRATION vs FLUX-TIME
4th PLUTONIUM RECYCLE

Figure 38

ISOTOPIC CONCENTRATION vs FLUX-TIME
5th PLUTONIUM RECYCLE

Figure 39

Figure 40

REACTIVITY LIFETIME

Figure 41

ISOTOPIC CONSTITUENCY OF THE PLUTONIUM
1st PLUTONIUM RECYCLE

Figure 42

KILOGRAMS OF PLUTONIUM PER TON OF FUEL
1st PLUTONIUM RECYCLE

Figure 43

WEIGHT PERCENT OF U-235
1st PLUTONIUM RECYCLE

Figure 44

ISOTOPIC CONCENTRATION vs FLUX-TIME
1st PLUTONIUM RECYCLE

APPENDIX A

FABRICATION COST OF SCHEME B TYPE FUEL ELEMENT

UNITED STATES
ATOMIC ENERGY COMMISSION
Albuquerque Operations Office
P.O. Box 5400
Albuquerque, New Mexico

In Reply Refer To:

ALP:ECS

November 6, 1957

Mr. J. W. Webster
Reactor Engineering Department
American-Standard, Atomic Energy Division
1682 Broadway
Redwood City, California

Dear Mr. Webster:

In reply to your letter of October 23, 1957, the following information is furnished with respect to our current estimate of costs of MTR-type plutonium containing fuel elements in order for you to complete your AEC Study Contract AT(04-3)-109. Your letter also expressed interest in one fuel form consisting of plutonium blended into uranium slugs. With respect to this latter form of fuel slug, Mr. Baker, LASL, replied to you as to the LASL capability. Our other facility at Rocky Flats Plant has no capability along this line, and works almost entirely for the weapons complex. Our capability in this area is probably less than that of Hanford.

Recently, we made a survey as to what it might cost to provide a capability for the fabrication of plutonium-bearing MTR-type fuel elements in one of our facilities. The survey revealed the data which I have presented below. If the figures are used, they should be qualified to reflect the added proper managerial and operational costs inherent with a commercial facility. Incidentally, we have no current thoughts on going beyond the "paper study" that was made.

Two assumptions were made on which to make the estimate. These were:

- I. Production of a single order of 50 complete fuel elements;
and
- II. Production of complete fuel elements at a rate of 100 per year
after the first 50.

November 6, 1957

The operational costs associated with each of the above assumptions are given in tabular form below. These tables are followed with tables covering equipment data and a summary table.

Table I

Costs of Plates for First 50 Elements

<u>Operation</u>	<u>Cost per Plate</u>
1. Casting	\$ 20.00
2. Rolling	20.00
3. Core Punching	5.00
4. Assembly and Fitting	20.00
5. Rolling Clad Plates	20.00
6. Inspection and Gaging	10.00
7. Shearing and Milling	10.00
8. Cambering	5.00
9. Chemical Recycle of Scrap	10.00
Total	\$ 120.00

Table II

Costs for Plates after First 50 Elements

<u>Operation</u>	<u>Cost per Plate</u>
1. Casting	\$ 11.00
2. Rolling	11.00
3. Core Punching	2.70
4. Assembly and Fitting	11.00
5. Rolling Clad Plates	11.00
6. Inspection and Gaging	5.50
7. Shearing and Milling	5.50
8. Cambering	2.80
9. Chemical Recycle of Scrap	5.50
Total	\$ 66.00

Table IIICosts for New Equipment for the Fabrication of Plates

1. Pot Furnace for Core Rolling	\$ 4,000
2. Furnace for Capsule Rolling	10,000
3. Flux Annealing Oven	10,000
4. Tools and Dies	20,000
5. Shear	1,000
6. X-Ray Fluoroscopic Unit	15,000
7. Cambering Press	15,000
Total	\$ 75,000

Table IVCosts for New Equipment for Fabrication of Other Parts

<u>Operation</u>	<u>Machine Required</u>	<u>Cost</u>
1. Fuel Plates	Gauges	
2. Side Plates	Milling Machine and Shear	1,515
3. Combs	Milling Machine and Saw	1,585
4. End Boxes	Die Mill and Vapor Degreaser	250
5. Fuel Assembly	Hayes and Brazing Furnaces	20,300
6. End Box	Tools and Gauges	520
7. Machine Final Assembly	3 Milling Machines	1,995
8. Final Inspection	Spacing Gauge	25,000
Total		\$ 51,405

Table VStart-up, Design, Tooling and Fabrication of Dies, Jigs,
and Fixtures Prior to First Assembly

<u>Operation</u>	<u>Cost</u>
1. Fuel Plates	\$ 3,264
2. Side Plates	8,466
3. Comb	2,091
4. End Box	3,774
5. Fuel Assembly	4,590
6. Assembly - End Box and Lattice	5,610
7. Machine Final Assembly	8,007
Total	\$ 35,802

November 6, 1957

Table VIOther Parts and Labor For Assembly of One Fuel Element

<u>Operation</u>	<u>Cost</u>
1. Fuel Plates - Inspection, Matching and Adjusting	\$ 306.00
2. Side Plates	130.05
3. Comb	28.05
4. End Box	48.45
5. Assemble Lattice to Braze	142.80
6. Assemble Lattice to End Box	33.15
7. Machine Final Assembly	175.95
8. Inspection of Final Assembly	<u>81.60</u>
Total	\$ 946.05

For the purposes of your report, the following summary tables may be used to provide a close approximation of the direct fabrication costs for the completed fuel elements:

Table VIISingle Order of 50 Elements - Assumption I

1. Plates - 50 Elements of 19 Plates each at \$120.00/plate	Table I	\$ 114,000
2. Capital Equipment for Plate Fabrication	Table III	75,000
3. Capital Equipment for other Parts	Table IV	51,405
4. Start-up, Tooling, Dies, Jigs, Fixtures	Table V	35,802
5. Other Parts and Labor for Assembly of Fuel Element		
50 Elements at \$946.05/Element		<u>47,302</u>
Total		\$ 323,509

Table VIIISummary of Costs for 100 Elements per Year After First 50
(Assumption II)

1. Plates - 100 Elements of 19 Plates Each at \$66.00/Plate, Table II	\$ 125,400
2. Other Parts and Labor for Assembly of Fuel Element, 100 at \$946.05	<u>94,605</u>
	\$ 220,005

Ltr to J. W. Webster

November 6, 1957

In addition to the above, we have estimated that the cost to industry to provide facilities similar to our own, and capable of performing the work, would be about \$500,000 for approximately 7000 square feet of floor space.

Since there would be no special contamination or material handling problem beyond the plate rolling stage, and recycle operations, the equipment cost to private industry might vary widely depending on the operating philosophy adopted.

I trust the above information will be of some assistance to you in preparing your report. It is not to be construed as a solicitation for work in any manner. Furthermore, this work was prepared as an account of government-sponsored work and we cannot make any warranty or representation, express or implied, with respect to the accuracy, completeness, or usefulness of the information presented. We cannot assume any liabilities with respect to the use of, or for damages resulting from the use of any of the information given.

If there is further information you desire, do not hesitate to write me.

Very truly yours,

(Signed) E. C. Stewart

E. C. Stewart
Office of Advance Planning