2

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University of California/Livermore, California

MASTER

STABILIZATION OF LOW M-NUMBER INTERCHANGE MODES IN SIMPLE MIRRORS
BY AXIALLY INJECTED ELECTRONS

D. E. Baldwin and G. Rowlands

October 1974

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STABILIZATION OF LOW M-NUMBER INTERCHANGE MODES IN SIMPLE MIRRORS
BY AXIALLY INJECTED ELECTRONS *

D. E. Baldwin and G. Rowlands

Lawrence Livermore Laboratory, University of California

October 28, 1974

It is shown that electrons injected axially along a simple mirror machine will stabilize low m-value interchange modes. Currents required to stabilize such a machine under reactor conditions would be of order 100 amps/cm².

* Work performed under the auspices of the U.S. Atomic Energy Commission

There have been a number of investigations of line-tying as a means of stabilizing interchange modes in simple mirror machines. A general feature of this history is the lack of agreement between theory and experiment, sometimes differing by orders of magnitude in the plasma density external to the trap necessary for line-tying stabilization. A common result of a number of theories is that the greatest contribution to the impedance seen by axial electron flow occurs at the electron sheath which forms where plasma loaded magnetic lines enter a conductor. This sheath can be reduced by the injection of an electron current comparable to the natural escaping electron current; however, to do so also removes the thermal insulation provided by the sheath which would be vital to any energy-economic mirror reactor. We wish to point out that these previous investigators have considered only the effect of injected current on the d.c. sheath and the concomitant effect on the escape of otherwise trapped electrons. They have not considered the direct electrical effect of the injected electrons on the interchange mode itself. We find upon including this direct effect that a relatively weak injected current is sufficient to stabilize an $m = 1$ interchange mode in a simple mirror. Whether this effect could ultimately be considered as a means of stabilizing a simple mirror depends of course, on the results of further investigation. In this note we wish to demonstrate how this stabilization comes about so as to open discussion as to its possible utility.

Because of the thermal insulation of the sheath, we will take as a requirement that any injected current be sufficiently weak compared to $n_w v_{Te} |e|$ that the magnitude of the sheath be not significantly affected. (n_w is the plasma density near the wall just outside the sheath, and v_{Te} is the electron thermal velocity.) Under normal circumstances the ambipolar potential increases monotonically from the wall to the mid-plane. Thus, the injected electrons are accelerated to the mid-plane

where, for injection energy low compared to the ambipolar potential Φ , they have an axial velocity $v_{||} \propto \sqrt{\Phi}$ ($\Phi = 0$ at the wall). At the midplane the weak current requirement translates to the requirement that the density of injected electrons be small compared with that of the ions (or that of the trapped electrons). [Since the injected electrons will leave a symmetric plasma with their original velocity we will refer to them as "passing electrons".]

To investigate the effect of passing electrons, we consider an interchange instability in a three component plasma: ions, trapped electrons, and passing electrons. It is necessary to treat the passing electrons differently from the trapped particles. Because of the low frequencies involved, we can treat the trapped particles in the drift kinetic description, i.e., a particle of species σ has only motion parallel to $\underline{b} \equiv \underline{B}/B$ and a perpendicular drift

$$\underline{v}_D = \frac{1}{\Omega_\sigma} \underline{b} \times \left[\left(\frac{v_\perp^2 + 2v_{||}^2}{2B} \right) \nabla B + \frac{e_\sigma}{m_\sigma} \nabla \phi + \frac{e_\sigma}{m_c \Omega_\sigma} \underline{b} \times \underline{v} \frac{\partial \phi}{\partial t} \right] \quad (1)$$

where Ω_σ is the (signed) cyclotron frequency. We will neglect the velocity dependence of the ∇B drift terms by replacing $v_\perp^2$ and $v_{||}^2$ by the appropriate thermal velocities. This approximation is valid as long as there is not a resonance between a significant number of drifting particles and the phase velocity of the wave. As a model we treat an otherwise uniform plasma with density and B variation in x only. More properly, this model stems from a bounce average of the drift kinetic equation where the perturbed ϕ is taken as constant over the extent of the bounce of the trapped particles, and the density variable becomes the number of particles on a flux tube—thus the motion parallel to $\underline{b}$ drops out. Introducing a perturbation of the form $\exp i(k_x x + k_y y - \omega t)$ and neglecting the electron ∇B drift and dielectric contributions, the perturbed charge density for trapped electrons and ions become

$$4\pi\rho_t = -k^2 \frac{\omega_{*t}}{\omega} \phi \quad (2)$$

$$4\pi\rho_i = -\frac{\omega_{pi}^2}{\Omega_i^2} k^2 \phi - k_{Di}^2 \frac{\omega_{*i}}{(\omega - \omega_{Di})} \phi \quad (3)$$

where

$$\omega_{Di} \equiv \frac{k}{\Omega_i} \cdot \underline{b} \times \frac{(v_\perp^2 + 2v_{Ti}^2)}{2B} \nabla B \quad (4)$$

$$\rightarrow \frac{3}{2} k_y \frac{v_{Ti}^2}{\Omega_i} e_{\underline{x}} \cdot \nabla \ln B ,$$

$$\omega_{*o} \equiv v_{T\sigma}^2 \frac{k}{\Omega_\sigma} \cdot \underline{b} \times \nabla \ln \omega_{p\sigma}^2 \rightarrow k_y \frac{v_{T\sigma}^2}{\Omega_\sigma} e_{\underline{x}} \cdot \nabla \ln \omega_{p\sigma}^2 , \quad (5)$$

and

$$k_{D\sigma}^2 \equiv \omega_{p\sigma}^2 / v_{T\sigma}^2 .$$

The subscript t refers to trapped electrons; and, below, the subscript p to passing electrons.

The response of passing electrons to a perturbing field is quite different. Trapped electrons and ions are moving in regions of no axial dependence of ϕ , and a change in magnitude of ϕ does not give rise to a change in particle energy due to parallel motion. However, the mode cannot be flute all the way to the wall; all of the axial variation required to satisfy the $\phi = 0$ boundary condition will take place in the sheath near the wall. Passing electrons which are drawn into the positive ambipolar potential will thus be accelerated by the instantaneous value of the perturbed potential. In fact, because of the short transit time of the passing electrons compared to the MHD growth time they see an effectively static perturbed potential. Thus, the number of such passing electrons on a tube of flux varies as the time of transit. The linearized charge density resulting from a perturbed ϕ is thus

$$4\pi\rho_p = \frac{m_e \omega_{pp}^2}{|e| \Phi} \phi \quad (6)$$

where ω_{pp} is the plasma frequency of passing electrons.

Notice that the sign of the perturbed passing electron charge density is opposite to the familiar expression for the change in charge density by Boltzman distributed electrons in a positive perturbed potential ϕ . Physically, because the assumptions leading to Eq. (6) are equivalent to a constant current source, an increase in $\phi \rightarrow \phi + \phi$ results in higher parallel velocity and thus lower passing electrons density, or a positive change in charge density. It is this behavior of passing electrons which leads to the new physical effects we describe:

Introducing (1)-(3) into Poisson's equation gives the resulting dispersion relation

$$k^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2}\right) = -k_{Di}^2 \frac{\omega_{*i}}{\omega - \omega_{Di}} - k_{Dt}^2 \frac{\omega_{*t}}{\omega} + \frac{m_e \omega_{pp}^2}{|e| \Phi} \quad (7)$$

Introducing $\epsilon = \omega_{pi}^2 / \Omega_i^2$ and $\alpha = n_p / n_i$, and using $\omega_{*}^t = -\omega_{*}^i T_e / T_i$ and neutrality condition $n_i = n_t + n_p$, (7) becomes

$$k^2 a_i^2 \left(1 + \frac{1}{\epsilon_i}\right) = -\frac{\omega_{*t}}{\omega - \omega_{Di}} + (1 - \alpha) \frac{\omega_{*i}}{\omega} + \alpha \frac{T_i}{|e| \Phi} \quad (8)$$

with the solution

$$\omega = \frac{1}{2 \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right)} \left\{ \omega_{Di} \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right) - \alpha \omega_{*i} \right. \\ \left. \pm \left[\left(\omega_{Di} \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right) - \alpha \omega_{*i} \right)^2 - 4 \omega_{Di} \omega_{*i} (1 - \alpha) \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right) \right]^{1/2} \right\} \quad (9)$$

The effects of passing electrons enter in two distinct ways: (i) The final term of Eq. (8) occurs due to the direct contribution of passing electron to Poisson's equation, and (ii) the α in the second term of Eq. (8) arises due to the reduction of trapped electrons to maintain equilibrium charge neutrality. These may be distinguished in Eq. (9) by whether or not α is multiplied by T_i / Φ . From Eq.

(9) the first effect gives a sufficient condition for stability in a simple mirror, where $\omega_{Di} \omega_{*i} > 0$, namely

$$\alpha > \frac{|e|}{T_i} \frac{\phi}{k^2 a_i^2} (1 + \epsilon_i^{-1}) . \quad (10)$$

To apply this result to cylindrical geometry we take $k \rightarrow m/R_p$ where m is the azimuthal mode number and R_p the plasma radius. Then Eq. (9) gives the possibility of stabilizing low m values. For high m values finite Larmor radius (FLR) effects will have to be invoked. Under reactor condition, the right side of Eq. (10) might be 10^{-3} .

We may understand the mechanism by which the passing electrons bring about this dramatic effect by rewriting Poisson's equation as

$$\nabla_{\perp}^2 \left(1 + \frac{\omega_{pi}^2}{\Omega_i^2} \right) \nabla \phi + \frac{m}{|e|} \frac{\omega_{pi}^2}{\Omega_i^2} \frac{\phi}{\phi} = -4\pi(\rho_t + \rho_i) + \nabla \cdot \frac{\omega_{pi}^2}{\Omega_i^2} \nabla \phi .$$

In this form we have placed on the right hand side those contributions to be perturbed charge density which result from the density and magnetic gradient drifts. On the left side we have put the trapped ion dielectric and the passing electron contributions. If we consider a sinusoidal perturbed charge density, the resulting perturbed potential will be reversed in sign if the second term on the left is greater than the first. In fact, this simple condition leads to Eq. (10). Similarly, electron fields resulting from a given charge separation of trapped particles will be reversed. For example, in the Rosenbluth - Longmire picture of the simple interchange, the electric field resulting from the ∇B drift of particles in a perturbation tends to increase the perturbation. However, passing electrons will reverse the sign of the electric field and the associated drift, provided the wavelengths of the perturbation is long enough, and hence they can lead to stability. This effect of passing electrons changes the Poisson equation from elliptic to hyperbolic.

The second effect of passing electrons, due to the reduction of the number of trapped electrons, is related to the effects of FLR in conventional theory.

In fact, were we to include FLR terms, Eq. (9) would become

$$\omega = \frac{1}{2 \left[k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right]} \left\{ \omega_{Di} \left[k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right] - (\alpha - \underline{k^2 a_i^2}) \omega_{*i} \right. \\ \left. \pm \left[\left(\omega_{Di} \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right) - (\alpha - \underline{k^2 a_i^2}) \omega_{*i} \right)^2 - 4 \omega_{*i} \omega_{Di} (1 - \alpha) \left(k^2 a_i^2 (1 + \epsilon_i^{-1}) - \alpha \frac{T_i}{|e| \Phi} \right) \right]^{1/2} \right\} \quad (10)$$

It should be noted that the FLR terms, which are double underlined in Eq. (10), enter only in the combination $\alpha - \underline{k^2 a_i^2}$. This has a simple physical explanation. FLR stabilization occurs because the ion diamagnetic drift current, which is reduced due to FLR by a factor $(1 - k^2 a_i^2)$, no longer quite cancels the electron diamagnetic current. However, in our model the trapped electron current is reduced by a factor $(1 - \alpha)$, so that these two reduced currents may again be equal if $\alpha = \underline{k^2 a_i^2}$; and the FLR effect would then disappear. However, examination of Eq. (10) shows that there is a value of α such that stability results for all k . (In this demonstration, k in the FLR terms should properly be replaced by $(m - 1)/R_p$, so that they do not effect the $m = 1$ mode.)

To achieve a steady state density of passing electrons of even .1% under reactor conditions requires what would seem to be rather large currents. For example, along a tube of constant field strength a density of 10^{11} cm^{-3} of electron accelerated through a potential of 10^4 keV , requires a current of about 10^4 amps/cm^2 . Assuming this current would be injected where the magnetic field was reduced by 100, the current is still 10^2 amps/cm^2 . (However, the power required could be fairly low since they can be injected at low energy and recovered

after passing through the machine.) To reduce the total current required, it might be sufficient to pass current only along those flux tubes on the surface of the plasma.

Finally, we have not considered other effects of these passing electrons, e.g., two stream instabilities. Presumably growth rates for these could be minimize by spreading the distribution of injected electrons. We also have not considered finite β effects. The simplest, that of the direct magnetic field due to the current of the passing electrons, could be removed by counter-streaming.

Perhaps even more startling than the above result is the effect of passing electrons in a good well where $\omega_{Dj}\omega_{*j} < 0$. In this case the good well is destabilized when Eq. (10) is satisfied. One good well which is known to be unstable with drift-type frequencies is BBII with the neutral beam on and ϵ_j above a certain threshold. Although no passing electrons are injected into this device it is interesting to speculate whether there may be some mechanism which provides an effective source of passing electrons. Such a source with an α of a few percent would by Eq. (10) give rise to instability in the observed ϵ_j range which is independent of ion energy. In addition, by the argument given above showing electric field reversal, the plasma would drift upstream in the neutral beam rather than be pushed downstream as in the usual picture. This would be consistent with Porter's measurements.

The source of electrons in BBII which could give rise to an effective α of a few percent is not at all clear. Using reasonable secondary emission coefficients, it appears that both the hot ion axial loss by scattering and the direct flux of hot ions ionized with large $v_{||}$ are insufficient in number. Also, it is hard to imagine even pointed structures in the fans creating the electric field strengths of order 10^5 volts/cm. necessary for cold field emission. It is possible

that a copious source of electrons generated initially by ionization of a cold background gas might have an electrical effect similar to that of passing electrons, but a detailed mechanism is not apparant at this time.

In summary, the physics of passing electron merit study as a possible means of stabilizing simple mirrors and as a possible cause of anomolous behavior in minimum-B wells.

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