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THE PION-NUCLEUS INTERACTION* †

Carl B. Dover

Brookhaven National Laboratory, Upton, New York 11973

I. INTRODUCTION

I would like to start out by providing an outline of the topics I hope to cover. The first half of the talk will be rather general, with emphasis on selected phenomena which illustrate the potential richness of the field of pion-nucleus interactions. The second half will be devoted to a discussion of some recent theoretical ideas which address a central question: How does the presence of the nucleus modify the effective πN scattering amplitude in the medium?

The outline of the talk is as follows:

1. General comments on the interaction of external probes with nuclei. Here I would like to provide some quite general perspectives on the problem of extracting nuclear structure information from the interaction of various probe particles with nuclear targets. My aim

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will be to give a general overview in the light of which later more specific comments on pion-nucleus interactions may be better appreciated.

2. Classes of Probes and Their Uses. Here I compare and contrast electromagnetic and hadronic probes, with emphasis on some of the theoretical methods which recur in the analysis of different types of data.

3. Types of Nuclear Structure Information obtainable from an Analysis of Processes Involving Pions. I will try to answer two questions: a) What could we in principle learn about nuclear structure from pion reactions? b) What have we learned so far? (evaluation of the present status of our knowledge).

4. Theoretical Methods - the conventional Wisdom. Here we give a qualitative sketch of some of the main theoretical methods which have been brought to bear on the pion-nucleus problem.

5. Application of the Techniques of Many-Body Field Theory. We first exhibit the connection between the pion optical potential and the self energy $\pi(q)$ of a pion in a field theory. Various model calculations of $\pi(q)$ can be used to derive the pion-nucleus potential appropriate for pionic atoms, including terms arising from two nucleon correlations. A generalization of Chew-Low theory to pion-nucleus scattering will also be presented, in order to treat the problem of pion-nucleus scattering in the (3,3) resonance region.

6. Overview. We summarize some of the theoretical roadblocks which must be overcome if reliable structure information is to be extracted. We also indicate some possible directions for further theoretical work.

Now let us proceed to the task at hand.

1. General Comments on Probes

In discussing the interactions of external probes with nuclei, we are faced with two main questions:

- a) How well do we know the interaction between the probe and a single isolated nucleon?
- b) Do we have a proper many-body theory in which to embed this input information on the elementary probe-nucleon interaction?

As we shall see in the following discussion, it is not always possible to disentangle these two problems.

What sort of nuclear structure information do we hope to extract?

a) In elastic scattering studies involving strongly interacting probes, it is usual to replace the influence of the complicated many body system on the probe by a one body potential ("optical potential"). In the simplest sort of density expansion, the first order term in this potential is obtained by summing the elementary probe-nucleon interactions over the nucleus. Thus we would expect elastic scattering data to be sensitive to the target neutron and proton densities $\rho_n(r)$ and $\rho_p(r)$. If this first order term is anomalously small (as is the case for pion-nucleus interactions at very low energy, for instance), the probe-nucleus interaction will be unusually sensitive to correlations between target nucleons.

Inelastic processes will be more sensitive to detailed aspects of nuclear structure, such as wave functions of particular excited states. For example, for pion inelastic scattering, matrix elements of the type $\langle 1M_L | \Delta V | 0 \rangle$ enter, where $\langle 1M_L |$ and $\langle 0 |$ are wave functions for an excited state with spin L and the ground state, respectively, and ΔV is a perturbing potential (proportional to the deformation parameter β_L) which

induces the transition. Another example is provided by the reaction ${}^3\text{He}(\gamma, \pi^+) {}^3\text{H}$; here the nuclear structure enters via a reduced matrix element $\langle \psi_{{}^3\text{He}} || \theta || \psi_{{}^3\text{H}} \rangle$, where θ is related to the elementary $\gamma N \rightarrow \pi N$ amplitude.

2. Classes of Probes

Let us now consider various classes of probes, in order to exhibit their similarities and differences.

a) Electromagnetic Probes (γ , e^- , μ^-)

The main advantage here is that the elementary interaction is very well known, so it is easy to disentangle its effects from the nuclear structure. For calculating γ -ray absorption from nuclei, for instance, one may make use of the linear response theory. In this approach, the γ -nucleus interaction is represented graphically as in Fig. 1.

FIG. 1

In linear response theory, we treat the γ -nucleus interaction to order $\alpha = e^2/\hbar c$. Thus the γ only enters in the two external vertices; the γ serves to excite a particle-hole excitation in the nucleus, which then propagates (designated by F_{ph} in Fig. 1) in a manner determined by the nuclear dynamics in the absence of the γ -ray. Thus we get a clean separation between the probe-nucleon interaction and the nuclear structure. Such a neat separation is not the case for pions. Since part of the nuclear force is generated by pion exchange, the external probe is identical

to virtual pions inside the nucleus. Pions are thus intimately involved at both of the external vertices in Fig. 1, as well as in the nuclear dynamics which governs F_{ph} .

Elastic electron scattering and transition energies in μ^- -mesic atoms are mainly sensitive to the charge density $\rho_p(r)$, and have been used successfully to extract the charge radii and surface thicknesses of a variety of nuclei. These electromagnetic probes have not given us very much information concerning two nucleon correlations, however. In this respect, pionic probes may in principle be better.

b) Nucleons and nuclei as probes

Under this heading, we include the usual probes of nuclear physics, such as p, n, d, ^3He , t, α and heavy ions. For all such strongly interacting probes, one introduces an optical potential to represent the effect of the nucleus on the incident probe particle. This potential distorts the incident and outgoing plane waves. Cross sections for particular processes (one or two particle transfer, say) are usually calculated in the distorted wave Born approximation (DWBA), which involves an overlap integral of the distorted waves with a form factor which contains the nuclear structure information (for instance, the bound state wave functions of the transferred particles). DWBA methods are also applicable to processes involving pions, inelastic scattering, for instance.

c) More Exotic baryonic probes

Here we include probes such as $\bar{p}$, Σ , and Λ . In such cases, rather little was initially known about the elementary interaction. Thus instead of using the probe to deduce structure information, we make simple assumptions about the nucleus (for instance, the density) and turn the process

around to obtain information about the probe-nucleon interactions. Probes such as $\bar{p}$ and Σ^- are very strongly absorbed (for instance, the $\bar{p}p$ annihilation cross section into pions is large), so they only probe the tail of the nuclear density distribution. The probe-nucleus potential is thus essentially given by an effective scattering length times the nuclear density (i.e., the first term in a density expansion is probably adequate). An analysis of data on $\bar{p}$ and Σ^- Bohr atoms then yields values for the effective scattering length.

d) Mesonic probes

We consider in particular pions and kaons. Since these particles are pseudoscalar bosons, they can be absorbed (by processes such as one or two nucleon emission, etc.). This absorption is quite strong, so these probes do not generally penetrate into the interior of the nucleus. Typically kaons would be absorbed in regions of the nuclear surface where the density is of order 10-15% of central densities; for pions, 50% is a typical value. In principle, we could map out the behavior of the density $\rho(r)$ for large r by combining the information obtained from an assembly of probes such as π , $\bar{K}$, Σ^- , $\bar{p}$, etc. This program has only met with limited success in practice.

3. Nuclear Structure Information Obtainable from Processes Involving

Pions

Before considering more detailed nuclear structure aspects, we first point out several of the most striking features of pion-nucleus interactions, which illustrate some of the unique features of pions as probes of nuclei.

The first point is rather obvious. Since pions have isospin $T=1$, we can have $\Delta T_z = \pm 2$ charge exchange reactions. Thus one could produce exotic nuclei not readily accessible by other means, for example,

$^{12}\text{C}(\pi^+, \pi^-)^{12}\text{O}$. Analog states of this type have not yet been seen in pion double charge exchange [1]. One has also looked for bound states of light systems with only neutrons or only protons, for instance $^4\text{He}(\pi^-, \pi^+)4\text{n}$ [1].

The second point is a bit more subtle. This involves the observation that pions are the ideal probe of meson degrees of freedom in the nucleus, in particular of retardation effects involving the time delay between the emission and absorption of virtual pions. Let us consider an illustrative example. In calculating contribution to the binding energy of nuclear matter in perturbation theory, one considers graphs like the one shown in Fig. 2,

FIG. 2

where the wiggly lines indicate pion exchange and the solid lines are particle-hole excitations. The propagator for the pion labelled by $\vec{q}, \omega$ is $(\vec{q}^2 + \mu^2 - \omega^2)^{-1}$. The energy ω is of the order of a particle-hole excitation energy $\epsilon_{p+q} - \epsilon_p$, where $\epsilon_p = p^2/2m$. Since $\epsilon_{p+q} - \epsilon_p \sim \epsilon_F \sim 40$ MeV, we usually have $\omega \ll \mu$, where $\mu = 140$ MeV is the pion mass. Hence it is a good approximation to replace the full retarded propagator by its static limit $(\vec{q}^2 + \mu^2)^{-1}$, which is the Fourier transform of the usual Yukawa potential. For the usual nuclear physics applications, we can thus eliminate meson degrees of freedom in favor of static potentials, since virtual pions propagate far off the energy shell $\omega^2 = \mu^2 + \vec{q}^2$.

Now consider the case where we bring in an external pion to probe the nucleus. In this case, one type of graph which we must include is shown in Fig. 3:

FIG. 3

The propagator for the virtual pion in Fig. 3 is

$$1/(\mu^2 + (\vec{k} + \vec{q} - \vec{k}')^2 - (\epsilon_k + \omega_q - \epsilon_{k'})^2)$$

Now for $\vec{q} \rightarrow 0$ (the limit appropriate for pionic atoms), we have $\omega_q \rightarrow \mu$ and so we get $(|\epsilon_k - \epsilon_{k'}| \ll \mu)$ the result $1/|\vec{k} - \vec{k}'|^2$, i.e., a long range propagator. Equivalently, the virtual pion in Fig. 3, because of the presence of the on-shell external pion, propagates close to the energy shell (not localized in space). Thus it would be incorrect to replace the virtual pion in Fig. 3 by a static one pion exchange potential. Such retardation effects will only be important in processes involving external pions.

Now let us examine in more detail some selected processes involving pions. For each case, we will try to indicate which aspects of nuclear structure are being probed.

a) Pionic Atoms

Experimentally, one measures the energy ΔE and line-width Γ of Bohr transitions in π^- -atoms. The deviations of ΔE and Γ from electromagnetic values are due to the strong interactions of pions and nucleons. Representing these effects by an optical potential, Krell and Ericson [2] have determined empirically a potential which fits the pionic atom data. We discuss later the form of this potential, as well as attempts to calculate its parameters theoretically. At this point, we note only that the potential is mainly sensitive to the neutron and proton densities $\rho_n(r)$ and $\rho_p(r)$. Also, since the term linear in the total density is small (this is a consequence of the soft pion theorem) the pionic atom results are unusually sensitive to nucleon-nucleon correlations, represented phenomenologically by terms of second order in the density. An important point is that the pionic atom data determine rather well the empirical form of the pion-nucleus interaction at threshold. This knowledge serves as an important constraint on theoretical models.

b) Elastic Scattering and Total Cross Sections

We first consider pion-nucleus scattering in the region of the first $J = 3/2$ $T = 3/2$ pion-nucleon resonance at 180 MeV (the region from 100-300 MeV, say). The main feature of scattering in this region is the strong absorption in the pion-nucleus interaction, related to the resonance in the elementary interaction. In this energy region, the nucleus acts qualitatively like an opaque disc; there is thus little sensitivity to the details of nuclear structure. The elastic scattering is mainly sensitive to global properties of the nucleus such as the density $\rho(r)$. From one viewpoint, this feature is an advantage, since the (3,3) resonance in nuclei provides us with a means of studying the

structure of the optical potential itself in the presence of a strong resonance, with minimal sensitivity to nuclear structure aspects. An understanding of the potential for elastic scattering is of course crucial if we are to deduce nuclear structure information from inelastic and production processes such as (γ, π) for example, which are sensitive both to nuclear wave functions and to final state interactions of the outgoing pion.

In addition to elastic scattering, there are also some experimental data on reaction cross sections for pions on nuclei. At the (3,3) resonance, these reaction cross sections peak at values close to the modified geometrical limit $\pi(R+\lambda)^2$. Note that the wavelength $\lambda = 1/p \approx 0.7$ fm of a pion in the resonance region ($p \approx 300$ MeV/c) is not negligible compared to the radii R of light nuclei, so the observed cross sections considerably exceed the strict geometrical limit πR^2 . The most intriguing feature of reaction cross sections is perhaps the position of the resonance peak as a function of mass number A . For the cases which have been studied, the peak in pion-nucleus scattering is found to be shifted downward with respect to the resonance in pion-nucleon scattering; in ^{12}C this shift amounts to about 30 MeV (lab). Simple optical potentials linear in the nuclear density $\rho(r)$ generally predict a shift of the resonance which is too large. For instance, the results of Ref. [3], which represents one of the most complete calculations utilizing a first order potential, predicts a peak at about 115-120 MeV, compared to the experimental peak position of 145 MeV. As we shall see later, the peak position is rather sensitive to higher order terms in the potential. If one regards the (3,3) resonance as an elementary particle (a Δ), then these correction terms to the first

order potential can be viewed as corrections to the mass and width of the Δ when it propagates in the nuclear medium. The shift of the peak in the reaction cross section from its predicted position in the first order calculation may provide a rather clear signature of such density dependent modifications of the Δ (assuming that there are not large ambiguities in the results of first order calculations, due to different off-shell prescriptions, etc.).

The story is somewhat different for reaction cross sections at higher energy (500 MeV-1.5 GeV). In this case, the simple first order potential linear in the densities ρ_n and ρ_p should be a much better approximation than for the (3,3) resonance region (assuming that the potential concept retains its validity). The Rutherford collaboration [4] has measured pion-nucleus cross sections in the energy range 600 MeV-1200 MeV. For instance, they have looked in the energy region around the higher N^* resonances ($T = 1/2$). Since $T = 1/2$, only $\pi^- p$ and $\pi^+ n$ interactions will be resonant, and the ratio of π^- to π^+ cross sections is sensitive to the difference $\rho_n - \rho_p$ of neutron and proton densities. The analysis of pion reaction cross sections has thus been used to extract neutron radii [4]. Because of the remaining ambiguities due to the form of the potential and of the wave equation, the neutron radii which one extracts are probably not more reliable than those obtained by most other methods [Hartree-Fock calculations, alpha scattering, etc.] and certainly less reliable than neutron radii extracted from sub-Coulomb pickup reactions [5].

c) Charge Exchange Reactions

These have already been mentioned briefly in connection with $\Delta T_z = \pm 2$ reactions, leading to exotic nuclei. Here we would like to mention some other aspects of charge exchange reactions involving pions.

First let us consider single charge exchange reactions. In the simplest type of impulse approximation (with no rescattering), the (π^+, π^0) reaction would occur on neutrons and the (π^-, π^0) reaction on protons. The ratio $\sigma(\pi^+, \pi^0)/\sigma(\pi^-, \pi^0)$ would thus be quite sensitive to $\rho_n(r) - \rho_p(r)$. This means of getting information about neutron densities seems to be rather unexploited as yet, due to the paucity of data.

Instead of trying to detect the uncharged π^0 (by looking for e^+e^- pairs), one may also try to look for delayed proton emission from the analog state. This method has been successfully applied by Yavin et al. [6] in the case $^{91}\text{Zr}(\pi^+, \pi^0 p) ^{90}\text{Nb}$. In this case, the analog state lies below the neutron threshold, so it must decay by proton emission. More recently, other experiments of this type have been performed [7].

Double charge exchange was mentioned previously in connection with the production of exotic nuclei, for example $\pi^+ + ^{12}\text{C} \rightarrow \pi^- + ^{12}\text{O}$. However, in such a reaction only a very small (and in fact not yet observed) part of the cross section goes into the coherent formation of an analog state (the two body final state above). For 200 MeV incident pions, the average energy release $(E_{\pi^+} - E_{\pi^-})$ is about 60 MeV, so the residual nucleus tends to break up [1], resulting in three or more body final states. Particular final states would be of interest for nuclear structure studies. For example, a large cross section for $\pi^+ + ^{12}\text{C} \rightarrow \pi^- + 2\alpha + 4p$ could be interpreted as evidence in favor of an α -particle model for ^{12}C . Similarly, the reaction $\pi^+ + ^{12}\text{C} \rightarrow \pi^- + ^{10}\text{C} + 2p$ could be used to study two hole states in the ^{12}C core; the results could be compared to those from (p,t) reactions.

d) Inelastic Scattering

The (π, π') reaction provides another constraint on model wave functions for excited nuclear states in a microscopic theory, to supple-

ment information from the usual nuclear reactions such as (p,p') , (α,α') etc. In a macroscopic theory, on the other hand, in which the radius R is expanded in the form $R = R_0 (1 + \beta_2 Y_{20})$, an analysis of the (π,π') reaction to the first 2^+ excited state enables one to determine the deformation parameter β_2 [8]. The value of β_2 which one obtains is essentially independent of energy and consistent with values obtained by other means. For ^{12}C , the analysis has been done in both distorted wave Born approximation (DWBA) and the method of coupled channels [8]. The numerical differences are small in this case. In heavier deformed nuclei, however, there could be some interesting effects of strong channel coupling (second order processes).

e) Pion Absorption: (π,N) vs. (π,NN)

For low energies, pion absorption proceeds primarily through two nucleon emission (π,NN) . The one nucleon mechanism is suppressed for kinematical reasons. In the pion optical potential V_{opt} , this fact is expressed in terms of contributions linear and quadratic in the density; the coefficient of the linear term is essentially real near threshold, while the quadratic term is usually taken to be pure imaginary [2]. As we move towards the $(3,3)$ resonance, however, the term in V_{opt} linear in $\rho(r)$ becomes largely imaginary. This indicates that the (π,N) mechanism is relatively more important near the $(3,3)$ resonance than at threshold. An interesting question involves the competition between the (π,N) and (π,NN) absorption mechanisms as a function of incident pion energy. An experimental study of this question would shed light on the energy dependence of the corresponding terms of order ρ and ρ^2 in the optical potential.

In principle, the two nucleon absorption mechanism (π,NN) should provide us with information about two nucleon correlations in the

initial nuclear state. In practice, these correlations have proven very hard to unravel because of final state interactions (fsi). These fsi are due to scattering of the two continuum nucleons, as well as nucleon-nucleus interactions. The two nucleon fsi can be handled by using a two particle wave function which yields the correct NN phase shifts [9]; however, the nucleon-nucleus fsi involves one in a coupled channel 3-body problem [10].

f) Nucleon Knockout Reactions ($\pi, \pi N$)

If the simple impulse approximation were valid, one could use the nucleon knockout reaction to obtain information about the momentum distribution of a nucleon in the nucleus. Unfortunately, this process does not seem to be so simple. For instance the ratio $R = \sigma(\pi^-, \pi^- n) / (\sigma(\pi^+, \pi^+ n) + \sigma(\pi^+, \pi^0 p))$ should be 3 in the impulse approximation for the (3,3) resonance region. Instead, the experimental ratio is about 1 for light nuclei [11]. This suggests that final state interactions must be included, which of course greatly complicates the extraction of the momentum distribution from the data. Various forms of isospin dependent final state interactions for the nucleon have been discussed in the literature [12]. If one has a mechanism which preferentially excites states of a particular isospin [the T=1 giant dipole resonance in ref. [13]], the experimental ratio R can be reproduced.

In most of these treatments, one ignores the final state interaction of the pion and considers only the continuum nucleon-nucleus interaction. One could also hypothesize that the Δ resonance is formed by the first π -nucleon collision, and that the Δ then interacts with the nucleus as a whole before breaking up into π and nucleon. This sort of process is shown in Fig. 4

If the Δ -nucleus optical potential is assumed to be of the general form $V_{opt}^{\Delta} = V_0 + V_1 \vec{\tau}_{\Delta} \cdot \vec{T}$, then one can show that the ratio R remains 3 as in the impulse approximation. Thus it does not seem possible to interpret the data in terms of a simple optical potential for the Δ (treated as an elementary particle). Of course, the Δ might also excite a giant resonance, as for the nucleon in ref. [13]. In this case, excitation of a preferred isospin would be involved, and R would be different from 3.

g) Production of Pions

Here we have in mind pion production in reactions such as (γ, π) , $(e, e' \pi)$, $(N, N \pi)$ and perhaps even $(\pi, \pi \pi)$.

There is some recent experimental data on the reaction ${}^3\text{He}(\gamma, \pi^+) {}^3\text{H}$ in the (3,3) resonance region [14]. This reaction is of considerable theoretical interest, since the cross sections are very sensitive to the details of the three-body wave functions (S' , D admixtures into the symmetric S state), and an enormous effort has gone into the calculation of three body wave functions in variational or Faddeev frameworks. If the impulse approximation is valid, the (γ, π) reaction on nuclei is simply related to the single nucleon (γ, π) amplitude (which is very well known) and various reduced matrix elements

which contain the nuclear structure. In this ideal case, the (γ, π) cross sections could be viewed as providing a further constraint on wave functions for the three body system, to complement other information such as binding energy and charge form factor. Unfortunately, the simplest form of impulse approximation does not seem to be adequate [15]. The position of the (3,3) resonance is very sensitive to the choice of kinematics for the basic (γ, π) amplitude. In the usual sort of calculation, one works with three body wave functions in coordinate space, and evaluates the (γ, π) amplitude at some average nucleon momentum. This approximation is not necessary if one works directly with the three body wave function in momentum space. Such calculations have not yet been carried out [16]. Aside from questions of kinematics, there is the more thorny problem (as always) of final state interactions of the pion. Calculations indicate [15] that fsi are not negligible at low momentum transfer Q^2 and larger at high Q^2 . The reliable extraction of information about the three body wave function must be preceded by a careful estimate of these rescattering terms.

4. Theoretical Methods - The Conventional Wisdom

We have already mentioned the concept of an optical potential V_{opt} in the description of pion-nucleus interactions. To find the distorted wave function $\psi_{\pi}(r)$ of a pion in the potential V_{opt} generated by the nucleus, we solve the Klein-Gordon equation

$$[\nabla^2 - \mu^2 + (E_{\pi} - V_{\text{Coul}})^2] \psi_{\pi}(r) = 2E_{\pi} V_{\text{opt}}(r) \psi_{\pi}(r) \quad (1)$$

where E_{π} is the total pion energy and $V_{\text{Coul}}(\pi)$ is the Coulomb potential.

The potentials which have been used in applications to pionic atoms and (3,3) resonance scattering are of the form

$$-2E_{\pi} V_{\text{opt}} = f_s - \vec{\nabla} f_p \vec{\nabla} \quad (2)$$

where f_s and f_p are given by

$$\begin{aligned} f_s &= b_0 \rho(r) + b_1(\rho_n(r) - \rho_p(r)) + b_2 \rho^2(r) \\ f_p &= c_0 \rho(r) + c_1(\rho_n(r) - \rho_p(r)) + c_2 \rho^2(r) \end{aligned} \quad (3)$$

The $\vec{\nabla} f_p \vec{\nabla}$ form was first used by Kisslinger [17], and reflects the p-wave part of the pion-nucleon interaction. The coefficients b_0 and c_0 are closely related to appropriate spin and isospin averages of s- and p-wave scattering amplitudes, respectively, and b_1, c_1 are related to charge exchange πN amplitudes. For pionic atoms, the above form of potential was used to explain the strong interaction line shifts and widths [2], with b_0, c_0, b_2, c_2 treated as free parameters. In the (3,3) resonance region, the same form of potential has been used by retaining only the first order terms involving b_0 and c_0 , which are taken from s and p-wave πN phase shifts at each energy [18].

A number of refinements have been made in the basic first order calculation of Sternheim and Auerbach [18]. A careful treatment of the kinematics of transforming the pion-nucleon amplitude from the πN c.m. frame to the π -nucleus c.m. frame gives some additional terms in the optical potential [19]. In addition, corrections to the free π -nucleon amplitude due to the binding of the nucleons in the nucleus have been considered [20]. We return to this point later in conjunction with

our discussion of the many-body approach.

One can also try to calculate the π -nucleus T-matrix directly from the Watson multiple scattering expansion [21]. The Glauber approximation [22] has also been applied to π -nucleus scattering in the (3,3) resonance region, with moderate success. These methods have been recently reviewed in exhaustive detail [23], so we only mention them here.

5. The Many Body Field Theory Approach

Many of the techniques which I will develop here are familiar from other areas of physics, namely high energy and solid state physics [24,25]. For instance, the formal aspects of the meson-nucleon and phonon-electron problems are rather similar, although the physical approximations which one must make are quite different.

The main theoretical construct which is relevant for the present discussion is the pion self energy $\pi(\vec{q},\omega)$. The function $\pi(\vec{q},\omega)$ is defined by the equation (for an infinite system)

$$D(\vec{q},\omega) = D_0(\vec{q},\omega) + D_0(\vec{q},\omega)\pi(\vec{q},\omega)D(\vec{q},\omega) \quad (4)$$

where $D(\vec{q},\omega)$ is the full propagator for a pion in the nucleus and $D_0(\vec{q},\omega) = (\mu^2 + \vec{q}^2 - \omega^2 - i\epsilon)^{-1}$ is the free space propagator.

In terms of many body Feynman diagrams [25], $\pi(\vec{q},\omega)$ is given by a sum of irreducible diagrams, which cannot be separated into two disjoint parts by cutting one internal pion line. Some typical diagrams contributing to $\pi(\vec{q},\omega)$ are shown in Fig. 5:

FIG.5

The solid circles in Fig. 5 represent particle-hole excitations produced by the absorption of the incident pion or the exchange of a virtual pion between two nucleons. Note that heavy mesons (the short range part of the nuclear force) can also enter as internal lines. Physically, the function $\pi(\vec{q}, \omega)$ represents the sum of all polarization processes (nuclear excitations) induced by the external pion.

In the present discussion, we shall consider an infinite nucleus, so that the self energy is only a function of one momentum variable $\vec{q}$ and the energy ω . This follows from translational invariance. For a finite system, π is a function of two momenta and an energy, or in coordinate space, a non-local energy dependent function $\pi(\vec{r}, \vec{r}'; \omega)$. In the following discussion, we will use a plane wave basis in which to evaluate the self energy, which is appropriate for an infinite system.

The approximations which we shall introduce can also be formulated in a straightforward way in any other basis of states, for example those of the shell model or the Hartree-Fock approximation. Some beginnings in this direction have been made [26]. However, the consequences of the non-locality of $\pi(\underline{r}, \underline{r}'; \omega)$ for pion-nucleus scattering have not been explored in general. Phenomenological potentials are usually taken to be local, or with a very simple non-locality of the Kisslinger type (see Eq. (2)).

What is the connection of the self energy π to the optical potential V_{opt} which is to be inserted in the Klein-Gordon equation? From Eq. (4), we see that the energy-momentum relation (the condition for $D(\vec{q}, \omega)$ to have a pole) for a pion propagating in the medium is given by

$$\omega^2 = \mu^2 + \vec{q}^2 - \pi(\vec{q}, \omega) \quad (5)$$

To make contact with Eq. (1), we use the prescription $\vec{q} \rightarrow -i\vec{\nabla}$ to pass to coordinate space. Comparing Eq. (5) with Eq. (1), we then find

$$\pi = -2E_{\pi} V_{\text{opt}} \quad (6)$$

Thus except for a multiplicative factor, the self energy π is equivalent to the usual optical potential V_{opt} for elastic scattering.

Now let us consider several approximate calculations of $\pi(q)$. We will treat the problems of the potential for pionic atoms and for the (3,3) resonance region separately, since these problems invite somewhat different approaches. The pionic atom data are very well explained by the optical potential of Eqs. (2) and (3), which includes only terms linear and quadratic in the density $\rho(r)$. This form of the potential is reminiscent of a Taylor-like expansion in powers of $\rho(r)$. Such an

expansion has been constructed [27] in the form

$$\begin{aligned} \pi(q) &= \sum_k n(k) \left. \frac{\delta \pi(q)}{\delta n(k)} \right|_{n(k)'s \rightarrow 0} + \frac{1}{2} \sum_{kk'} n(k)n(k') \left. \frac{\delta^2 \pi(q)}{\delta n(k) \delta n(k')} \right|_{n's \rightarrow 0} + \dots \\ &= \pi^{(1)}(q) + \pi^{(2)}(q) + \dots \end{aligned} \quad (7)$$

The occupation numbers $n(k)$ are 1 or 0 depending on whether $k < k_F$ or $k > k_F$, where k_F is the Fermi momentum. The derivatives of the form $\delta \pi / \delta n$ and $\delta^2 \pi / \delta n \delta n$ can be computed explicitly for any self energy diagram written in terms of pion propagators D_0 and nucleon propagators

$$G(k, \omega) = \frac{n(k)}{\epsilon_k - \omega + i\eta} + \frac{1-n(k)}{\epsilon_k - \omega - i\eta} \quad (8)$$

where $\epsilon_k = k^2/2m$ (or, more generally, ϵ_k is a Hartree-Fock single particle energy). The use of the propagator (8) ensures that the requirements of the Pauli principle are satisfied. Graphically, a functional derivative $\delta / \delta n$ is equivalent to cutting a single nucleon line in a self energy graph, $\delta^2 / \delta n \delta n$ is the same as cutting two lines, etc. Let us see how this process works for the simplest self energy graph (Fig. 6):

FIG. 6

We note that the single derivative $\delta/\delta n$ of the lowest order self energy graph yields the lowest order Born graphs for πN scattering. If we further carry out the instruction of Eq. (7) to set all occupation numbers $n(k)$ equal to zero after taking derivatives, we recover the free space πN Born graphs. The simple relationship between $\delta\pi/\delta n \Big|_{n \rightarrow 0}$ and free space πN scattering graphs persists to all orders, so we find the result

$$\left. \frac{\delta\pi(q)}{\delta n(k)} \right|_{n's \rightarrow 0} = 4\pi f_{\pi N}^{\text{free}}(kq; kq)$$

$$\pi^{(1)}(q) = 4\pi\rho f_{\pi N}^{\text{free}} \quad (9)$$

In lowest order, the density expansion method thus reproduces the familiar result of multiple scattering theory.

The second order term in Eq. (7) has also been evaluated explicitly [27,28]. This term is especially interesting, since it involves the two nucleon correlations in the nucleus. It can be written in a compact form in the limit $q \rightarrow 0$, appropriate for pionic atoms

$$\pi^{(2)}(q) = 4\pi[b_s \rho^2 + b_p q^2 \rho^2] \quad (10)$$

where $\rho = 2/3\pi^2 p_F^3$ is the density of nuclear matter. The coefficients b_s and b_p (s and p-wave parts) are related to various combinations of on and off-shell nucleon-nucleon T-matrices, averaged over the distribution of two nucleon relative momenta in the Fermi sea. For instance, we have [28]

$$\mu b_p = -6(f^2/4\pi)(\mu/p_F)^2 \int_0^1 dx P(x) [\tilde{t}_{st}(x, x; x^2) + \tilde{t}_{st}(x, x; x^2 + \gamma) + \tilde{t}_{st}(x, x; x^2 - \gamma) + (s \leftrightarrow t)] \quad (11)$$

In Eq. (11), $f^2/4\pi = 0.08$ is the renormalized pseudoscalar coupling constant, $x = p/p_F$ and the T-matrix is normalized so that on-shell we have $\tilde{t}(x, x; x^2) = (-4\pi p_F/mx)e^{i\delta} \sin\delta$. In (11), the subscripts st or ts refer to 1S_0 and 3S_1 NN channels; only s-waves are retained. The quantity $\gamma = \mu m/p_F^2$ measures the degree to which some of the t-matrix elements are off the energy shell. The function $P(x) = x^2(1 - \frac{3x}{2} + \frac{x^3}{2})$ gives the distribution of relative momenta in the Fermi gas. Typical diagrams which give the three types of t-matrix elements in Eq. (11) are shown in Fig. 7:

FIG. 7

The on-shell contributions can be obtained directly from experimental NN phase shifts. However, the calculation of the off-shell parts (and these matrix elements are far from being on-shell) requires a model of the nucleon-nucleon potential. The contributions of processes such as those depicted in Fig. 7 to the pion optical potential have been calculated [28] using a variety of realistic NN potentials. Such a calculation yields a complex pion potential contribution of order ρ^2 . There is good agreement between the imaginary part of the theoretical potential and the empirical potential of Krell and Ericson [2]. In addition, the model predicts a dispersive (real) contribution of order ρ^2 . Such a term has been included in some of the phenomenological analyses of pionic atom data, but has not previously been estimated theoretically. In summary, there seem to be strong indications that a systematic density expansion of the pion-nucleus potential at low energy can provide a first principles approach which at the same time makes quantitative contact with experimental data. The quantities which enter the discussion are natural ones over which one possesses some theoretical control, i.e., free pion nucleon scattering amplitudes and nucleon-nucleon T-matrices. There is no compelling need at present for terms in the low energy optical potential higher than order ρ^2 . Higher order terms can be calculated in the framework of the density expansion, but they involve higher order correlations between nucleons (three-body forces). The whole question of the possible role of three body forces in pion-nucleus interactions deserves investigation, however.

There are other forms of expansion rather similar in structure to the density expansion; one of these is the hole line expansion. It can

also be written in the form of Eq. (7)

$$\pi(q) = \sum_k n_{<}(k) \left. \frac{\delta \pi(q)}{\delta n_{<}(k)} \right|_{n_{<} \rightarrow 0} + \dots \quad (12)$$

This differs from the straightforward density expansion in that one treats the Pauli factors $n_{<}(k) = n(k)$ and $n_{>}(k) = 1-n(k)$ associated with hole and particle lines as independent variables. Thus, while the density expansion of Eq. (7) is analogous to a Taylor expansion of a function of one variable, and hence counts powers of the density $\rho = \sum_k n(k)$, the hole line expansion may be thought of as a Taylor expansion in one variable of a function of two variables. The successive terms of Eq. (12) thus involve 1,2,... hole lines, while the Pauli restrictions are kept to all orders on particle lines. Thus in the hole line expansion, the derivatives $\delta \pi / \delta n$ and $\delta^2 \pi / \delta n \delta n$ involve an effective πN interaction $\tilde{f}$ and an effective NN interaction (the usual G -matrix), respectively, in the nuclear medium, rather than the free space interactions which enter in the pure density expansion.

If one keeps only the first order term in Eq. (12), one recovers essentially the results of Bethe [29] and Eisenberg-Weber [30]:

$$\pi(q) \cong 4\pi\rho\tilde{f} \quad (13)$$

The effective amplitude $\tilde{f}$ is obtained in ref. [30] from a Pauli-modified Chew-Low equation, following the methods of ref. [31].

Assuming that the hole line expansion is appropriate to the pion-nucleus problem, one may study its properties for pionic atoms and for the (3,3) resonance region. A recent examination has been made of the

difference between the hole line and density expansions in the threshold region [32]. It was found that the Pauli modification of the p-wave optical potential is rather small, of order 15% or less. The main effect of the Pauli principle is to suppress the Born terms, with rather small effects on higher order processes. However, the appropriate spin-isospin weighted sum of the Born terms, whether Pauli damped or not, does not contribute to the optical potential. Thus the density and hole line expansions give very similar predictions for the optical potential at low energies, both in essential agreement with experiment. At energies near the (3,3) resonance, however, the density expansion converges slowly at best. The rapid convergence of the hole line expansion is thus also in some doubt [33]. As we shall see in the next section, there are techniques for dealing with the (3,3) resonance problem without resorting to any sort of density or hole line expansion.

Now let us proceed to a discussion of pion-nucleus scattering in the (3,3) resonance region. Before proceeding with the details of a particular theoretical model, it is appropriate to list at the outset some of the physical features that we would like to incorporate in any realistic model:

- 1) an exact treatment of the Pauli principle
- 2) the effects of the nuclear medium on the propagation of pions and nucleons in intermediate states (other than the Pauli principle). We have in mind here binding energy or "collision damping" effects,
- 3) a prescription for treating the off-shell propagation of a pion in the nucleus ($\omega^2 \neq \mu^2 + q^2$),
- 4) a way of avoiding density or hole line expansions in the resonance region.

The need for this last point requires some motivation. By studying the structure of the density expansion, one can deduce that one parameter which characterizes the convergence of this expansion (or at least the sum of terms involving only virtual pions rather than heavy mesons) is $p_F |f_{\pi N}|$. For pionic atoms, we have $|f_{\pi N}| \approx 0.1/\mu$, so $p_F |f_{\pi N}| \approx 0.2$, using $p_F = 270 \text{ MeV}/c \approx 2\mu$. The convergence of the density expansion thus seems assured at low energies. At the (3,3) resonance, on the other hand, we have $p_F |f_{\pi N}| = p_F/p_{\text{res}} \approx 1$, so the convergence of the expansion is in some doubt. More detailed calculations support this qualitative estimate. Although the hole line expansion converges somewhat more rapidly than the density expansion at the (3,3) resonance, the convergence is only assured if a rather low surface value for p_F is used.

Some other general comments are in order before we proceed further. We take the viewpoint that one should seek guidance from field theory whenever possible in treating the pion-nucleon interaction, even though this guidance is very far from being complete. The alternative approach, which is also viable, is to parametrize the pion-nucleon interaction in terms of an energy independent potential, usually taken to be separable. Such potentials have been determined directly from the on-shell πN scattering data by solving the inverse scattering problem [34]. The only point we wish to make here is that the πN t-matrix given by a field theory (for example, the Chew-Low theory) cannot be reproduced off-shell by an energy independent potential. One can, however, write an energy dependent one term separable potential which reproduces the full Chew-Low t-matrix. Since π -nucleus scattering depends on off-shell scattering of pions and nucleons, this distinction between field theory

amplitudes and amplitudes generated by a static potential may have some significance for practical calculations [32].

Now suppose that we have a model for the self energy $\pi(q)$. The essential physics of the (3,3) resonance region can then be exhibited by using a semi-classical approach, which shows the analogy of the pion-nucleus interaction to the corresponding phenomena of classical optics. We start by defining an index of refraction n for the pion as

$$n = q/p \quad (14)$$

where q is the complex pion wave number in the medium and $p = (\omega^2 - \mu^2)^{\frac{1}{2}}$ is the external beam momentum. The energy momentum relation (5) then becomes

$$n^2 - 1 = \pi(\vec{q}, \omega)/p^2 \quad (15)$$

The mean free path $\tilde{\lambda}$ for a pion can now be related to n by noting that the intensity of a pion wave which penetrates an absorptive medium is proportional to $|e^{iqZ}|^2 = e^{-(2\text{Im}q)Z}$. Equating this to $e^{-Z/\tilde{\lambda}}$, which defines $\tilde{\lambda}$, we obtain

$$\tilde{\lambda} = 1/2\text{Im} q = 1/2p\text{Im} n \quad (16)$$

In the semi-classical model, the pion-nucleus reaction (total inelastic) cross section can be obtained from the modified geometrical formula

$$\sigma_R = \pi(R+\tilde{\lambda})^2 (1-T) \quad (17)$$

Here $R+\tilde{\lambda}$ is the (energy-dependent) effective radius seen by a pion; R is

taken to be the equivalent spherical radius for the nucleus in question.

The transmission coefficient T is given by ($\tilde{R} \equiv R + \lambda$)

$$T = \frac{\tilde{\lambda}}{2\tilde{R}^2} \left[1 - \left(1 + \frac{2\tilde{R}}{\tilde{\lambda}} \right) \exp \left(-\frac{2\tilde{R}}{\tilde{\lambda}} \right) \right] \quad (18)$$

Equations (17) and (18) have been used to make estimates of reaction cross sections for finite nuclei [31]. The bulk density of the nucleus is approximate by $\rho_{\text{bulk}} = A / \frac{4\pi R^3}{3}$. Since the pion-nucleus interaction is strongly absorptive, a typical surface density is more appropriate for use in the nuclear matter calculation; we have used $\rho = \alpha \rho_{\text{bulk}}$, with $\alpha \approx \frac{1}{2} - \frac{2}{3}$ as a nominal value for the surface density.

Now let us describe a particular model for the self energy $\pi(q)$. This model is based on an extension of the familiar Chew-Low theory of low energy pion-nucleon scattering [35]. This model remains of some interest, since it is one of the few soluble (or almost soluble) field theories which is at the same time able to make quantitative contact with the low energy ($\lesssim 300$ MeV) data. Briefly, the Chew-Low approach yields an integral equation for the pion-nucleon amplitude f of the following structure:

FIG. 8

The first two graphs represent the Born approximation. The Chew-Low equation may be understood as the static limit of pion-nucleon dispersion relations [36], where only the one pion-one nucleon intermediate state is kept in the dispersion term (the right-hand term in Fig. 8).

The amplitude f in free space can be expanded in the form

$$f(q, q'; \omega) = \sum_{\alpha} P_{\alpha}(q, q') h_{\alpha}(\omega) \quad (19)$$

where P_{α} is a projection operator onto channel $\alpha = [J, T]$ and the amplitude h_{α} satisfies the following set of coupled equations [35]:

$$h_{\alpha}(\omega) = \frac{\lambda_{\alpha}}{\omega} + \frac{1}{\pi} \int_{\mu}^{\infty} d\omega_p p^3 v^2(p) \left[\frac{|h_{\alpha}(\omega_p)|^2}{\omega_p - \omega - i\delta} + \sum_{\beta} A_{\alpha\beta} \frac{|h_{\beta}(\omega_p)|^2}{\omega_p + \omega - i\delta} \right] \quad (20)$$

In Eq. (20), $A_{\alpha\beta}$ is the crossing matrix [35], $v^2(p)$ is a cutoff function and $\lambda_{\alpha} = (f_{\pi}^2/4\pi\mu^2)(2/3)(-4, -1, -1, 2)$ for the $[J, T] = [11, 13, 31, 33]$ channels, respectively. If we neglect crossing, Eq. (20) has the solution

$$h_{\alpha}(\omega) = \frac{\lambda_{\alpha}/\omega}{1 - \frac{\lambda_{\alpha}\omega}{\pi} \int_{\mu}^{\infty} \frac{d\omega_p}{\omega_p} p^3 v^2(p) \frac{1}{\omega_p - \omega - i\delta}} \quad (21)$$

which displays a resonance form in the (3,3) channel ($\lambda_{33} > 0$) and is a smooth non-resonant function of energy in the other channels ($\lambda_{\alpha} < 0$).

The cutoff function $v^2(p)$ can be chosen so as to correctly position the (3,3) resonance in free space. In Fig. 9, we show the π^+p cross section obtained by using a Gaussian cutoff $v^2(p) = e^{-p^2/p_0^2}$ with $p_0 = 13.8\mu$ and a constant cutoff $v^2(p) = 1$ for $p < 11.75\mu$, together with Eq. (21). We note that the Chew-Low theory, admitted with a rather

FIG. 9 (FROM REF. 31)

ad hoc cutoff procedure, gives a quantitative fit to the free space π^+p cross section. This is surely an essential feature if we are to study the modifications of the (3,3) amplitude in the nucleus in a meaningful way.

We now make the assumption that the class of diagrams summed in Fig. 8 is also appropriate to the problem of the effective πN amplitude $\tilde{f}$ in nuclear matter which has the same form as in Fig. 8, but using propagators D and G in the medium to describe pion and nucleon propagation in intermediate states. Approximations of increasing sophistication can then be embedded in the theory by using more complete models for D and G. Physical effects which could be incorporated into the theory by appropriate modifications of D and G include

- a) the effect of the Pauli principle restrictions on G;
- b) binding energy corrections to G
- c) "collision damping" of D, via excitation of particle-hole states, for example.

In the simple calculations described here, we include only the Pauli effect a), by using the nucleon propagator G of Eq. (8). The modified Chew-Low equation for the amplitude $\tilde{f}$ in the medium can still be expanded in the form

$$\tilde{f}(q, \rho) = \sum_{\alpha} P_{\alpha}(q) \tilde{h}_{\alpha}(q, \rho) \quad (22)$$

where $q = (\vec{q}, \omega)$. If we neglect crossing, and consider only the (3,3) channel of interest here, we find the following equation for the density dependent amplitude $\tilde{h}(q, \rho) = \tilde{h}_{33}(q, \rho)$

$$\tilde{h}(q, \rho) = \frac{\tilde{\lambda}}{\omega} + \frac{1}{\pi} \int_{\mu}^{\omega} \frac{d\omega_p p^3 v^2(p) |\tilde{h}(\omega_p)|^2}{\omega - \omega_p - i\delta} - 4\pi \tilde{I}(q, \rho) |\tilde{h}(q, \rho)|^2 \quad (23)$$

In Eq. (23), $\tilde{I}(q, \rho)$ is a known function [31], approximately equal to $\rho/4$ for low densities and $\tilde{\lambda}$ is the coupling constant in the (3,3) channel (we use $\tilde{\lambda} = 4/3 (f_{\pi}^2/4\pi\mu^2) \approx 0.11/\mu^2$ as in free space, although there is in principle also a density-dependent coupling constant renormalization [37]).

For $\rho \rightarrow 0$, Eq. (23) reduces to the free space Chew-Low equation (20). Since $\text{Re } \tilde{I} > 0$, the effect of the density dependent modification $4\pi \tilde{I} |\tilde{h}|^2$ in Eq. (23) is to quench the pion-nucleon amplitude in the medium. The solution to Eq. (23) can be approximately represented in the form

$$\tilde{h}(q, \rho) = \frac{\tilde{\lambda} \tilde{\omega}_0 / \omega}{\tilde{\omega}_0 - \omega - i\tilde{\Gamma}/2} \quad (24)$$

where $\tilde{\omega}_0$ and $\tilde{\Gamma}$ are the modified (3,3) resonance energy and width. If we include only the Pauli effect in intermediate states, we have $\tilde{\omega}_0 > \omega_0$ for $\omega < \omega_0$ and $\tilde{\Gamma} < \Gamma$, where ω_0 and Γ are the free space resonance energy and width. The effect of the Pauli principle is thus to increase the (3,3) resonance energy and decrease the width.

It remains to exhibit the connection between the effective amplitude $\tilde{f}$ and the self-energy $\pi(q)$. In general, this relation assumes the form

$$\delta\pi(q)/\delta G(k) = 4\pi i \tilde{f}(kq; kq) \quad (25)$$

Thus the derivation of $\pi(q)$ from $\tilde{f}$ involves in general a complicated

functional integration. However, for the special case of the static Chew-Low model, it turns out that $\tilde{f}$ is only a function of the total density ρ , so Eq. (25) reduces to a simple quadrature of the form [31]

$$\pi(q, \rho) = 4\pi \int_0^{\rho} d\rho' \tilde{f}(q, \rho') \quad (26)$$

In Eq. (26), we sum the effects of the nucleus to all orders in ρ ; there is no need for any sort of density expansion. This feature is of considerable interest, since simple density and hole line expansions converge slowly at best in the (3,3) resonance region.

Although Eq. (26) sums terms to all orders in ρ , it is equivalent to the usual multiple scattering result at low density. To see this, consider $\tilde{f}$ near threshold. The solution of the density dependent Chew-Low equation (23) is approximately

$$f = q^2 \left[\sum_{\alpha} v_{\alpha} c_{\alpha} - 8\pi\rho \sum_{\alpha} v_{\alpha} c_{\alpha}^2 \right] \quad (27)$$

where $v_{\alpha} = \{1, 2, 2, 4\}$ and c_{α} is the p-wave scattering volume in channel α . Using Eq. (26), this yields a self energy

$$\pi(q, \rho) = 4\pi\rho q^2 \left\{ \sum_{\alpha} v_{\alpha} c_{\alpha} - \pi\rho \sum_{\alpha} v_{\alpha} c_{\alpha}^2 \right\} \quad (28)$$

Equation (28) is exactly the multiple scattering result [38] to order ρ^2 . Thus the Chew-Low model for the p-wave part of $\pi(q)$ can be used both at threshold and in the resonance region.

Now let us look at some results of the Chew-Low model, working within the semi-classical framework of Eqs. (14)-(18). The Eq. (23) for

the amplitude $\tilde{h}$ has been solved numerically for a range of energies and densities. Equations (22) and (26) are then used to construct the self energy $\pi(\vec{q}, \omega; \rho)$. The index of refraction n obtained from Eq. (15) is shown in Fig. 10. The curves in Fig. 10 are labelled by the pion lab kinetic energy. The dashed curve shows the index of refraction calculated using the free space πN amplitude, i.e., $\pi(q) \approx 4\pi\rho f$. The solid curve shows the complete calculation. Because of the Pauli quenching effect, the increase of the momentum (below resonance) as the pion enters the medium is less than if one uses the first order potential; the medium acts as to limit the growth of the momentum. The curves of Fig. 10 are reminiscent of anomalous dispersion in classical optics.

Typical mean free paths $\tilde{\lambda}$ for pions in nuclei are shown in Fig. 11 [31]. The curves are labelled by the value of the effective density ρ in units μ^3 . The mean free path exhibits a minimum of the order of 1fm in the (3,3) resonance region, since the nucleus is increasingly absorptive. The position of the minimum shifts downward with increasing effective density.

The reaction cross section for $\pi^- + {}^{12}\text{C}$ is shown in Fig. 12 for an effective density of $\rho = 0.2\mu^3$, corresponding to an appropriate density in the surface region. The experimental data are taken from ref. [39]. The observed and calculated maximum reaction cross sections are seen to be very close to the modified geometrical area $\pi(R+\tilde{\lambda})^2$.

Predictions for the mass dependence of the (3,3) resonance energy are shown in Fig. 13. The resonance energy is defined as that energy where the reaction cross section has its maximum. This definition is not exactly equivalent to other possible definitions of the resonance energy (zero of total π -nucleus amplitude, phase shift passing through

FIG. 10 (FROM REF. 31)

FIG. 11 (FROM REF. 31)

FIG. 12 (FROM REF. 31)

$\pi/2$, etc.). In each case, the effective surface density ρ to be used in the Chew-Low calculation was taken to be $2/3 \rho_{\text{bulk}}$, where $\rho_{\text{bulk}} = A/(4\pi R^3/3)$, R being the equivalent spherical radius of the nucleus in question. The general trend of the resonance shift is downward with increasing mass number A . For heavier nuclei, the resonance is predicted to be considerably shifted downward relative to its position in free space (174 MeV lab kinetic energy). The curve of Fig. 13 includes all Pauli corrections to the effective πN amplitude $\tilde{f}$, as per Eqs. (23) and (26). A first order calculation with $\pi = 4\pi\rho f$ would predict even larger shifts. It would be very interesting to try to determine the size of the shift experimentally for heavier nuclei [40]. Note that the modifications of the amplitude $\tilde{f}$ tend to cancel out part of the quite large shift which one predicts by using a first order potential. The first order theory would be essentially unable to explain a very small shift, if such were the case experimentally.

The factors which influence the resonance position include the following:

- a) the Pauli effect. As we have seen, this tends to shift the resonance upward.
- b) collision damping. Here we mean self energy effects due to the interaction of pion and nucleon (in intermediate states) with the rest of the nucleus. On general grounds, one would expect some downward shift from this effect. It has not been included in the numerical calculations, however.
- c) multiple scattering effect. By this we mean the downward shift produced by iterating the optical potential via the Klein-Gordon equation. This is the largest single effect (≈ 55 MeV in ^{12}C), and is

included in the first order calculation using $\pi(q) = 4\pi\rho f$.

d) energy dependent effective radius. In the semi-classical framework, the energy dependence of the effective geometrical area $\pi(R+\lambda)^2$ produces a downward shift of the resonance from the energy corresponding to the minimum transmission coefficient T.

The Chew-Low model only represents a qualitative evaluation of some of these interesting effects. It is clear, however, that higher order corrections to the first order potential are likely to be important in the (3,3) resonance region. The difficulty of calculating these corrections in detail for a finite nucleus may ultimately limit the usefulness of the pion as a probe of nuclear structure. At present, the problem of calculating the pion optical potential remains a very difficult and challenging one. Hopefully, the techniques of many body field theory which have been sketched here will provide some indication of how to proceed from first principles. Such an approach is complementary to a phenomenological approach using separable potentials to replace the field theory description of the pion-nucleon interaction [41].

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