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KNOLLS ATOMIC POWER LABORATORY

SCHENECTADY, NEW YORK

CONTROL AND ENDURANCE EVALUATION OF A POWER REACTOR
BY MEANS OF A CRITICAL EXPERIMENT

L. Tonks

September 18, 1953

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This document contains

Reactors - Research and Power

General Electric Company
KNOLLS ATOMIC POWER LABORATORY
Schenectady, New York

CONTROL AND ENDURANCE EVALUATION OF A POWER REACTOR
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CLASSIFICATION CANCELLED
DATE MAR 27 1957 *CE*
For The Atomic Energy Commission
H. R. Canale
Chief, Declassification Branch

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ABSTRACT

Critical experiments are useful for determining the safety margin and endurance and for calculating control characteristics of a power reactor, but it is not self-evident how experimental results should be interpreted to obtain these characteristics, and there has been no systematic exposition of this matter. The subject report establishes a rigorous theoretical basis for using experimental results to derive the desired characteristics.

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CONTROL AND ENDURANCE EVALUATION OF A POWER REACTOR
BY MEANS OF A CRITICAL EXPERIMENT

L. Tonks

INTRODUCTION

For some time we have been using critical experiments for determining the control characteristics, safety margin, and the endurance of a power reactor. The approach has been for the most part a common-sense one avoiding a critical analytical evaluation of the exact meaning of procedures used and results obtained. A more critical approach to the problem is now proposed to gain a better understanding and possibly correct errors and interpretations which may have crept in. The first step in this direction has already been taken by P. L. Hofmann and H. Hurwitz of this Laboratory.* Their approach was almost exclusively geometric, and it appears that a combination of geometry and analysis is more direct and clear-cut. A discussion with H. B. Stewart of this Laboratory inspired the following analysis.

For our purposes the fact must be recognized that positive reactivities of more than approximately 50 cents have no direct meaning, since such conditions imply a dangerously supercritical condition. Likewise, reactivities that are less than unity by more than 50 cents have meaning only in relation to the safety shutdown of a reactor. In our analysis for the most part we must confine ourselves to a narrow zone in the neighborhood of $k = 1$. Thus, when we say that a control drum has an excess reactivity of \$16 we cannot really mean this but may possibly mean something which can be interpreted, but not naively, in terms of fuel burnup and poison accumulation. It is proposed that for such nonrigorous use of the word "reactivity" as just cited we reserve the term "surreactivity" (the first syllable arising from the root of surrogate which refers to one who serves in the place of another). This is one of the matters to be examined in a rigorous way below.

THE DETERMINABLE POWER REACTOR CHARACTERISTICS

There are three operating characteristics of the power reactor upon which the critical experiment can give us information:

The first characteristic is the safety margin, which is the reactivity difference between unity and that of the reactor with its control set at minimum reactivity. This reactivity difference, which can be of the order of \$5, has basic engineering significance in that it determines the rate at which the power generation dies down.

The second is the endurance of the reactor, which correlates the total range of the control mechanism with the corresponding loss of fuel through fission, accumulation of fission-product poisons, and the maximum xenon transient that must still be overridden. In this normal working range of

*Unpublished note on the three-dimensional control space.

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the control k never deviates more than a few tenths of a dollar from unity.

The third is the control sensitivity in this very operating range, namely, $\partial k / \partial \theta$, where θ is the position variable of the control mechanism. This is important in the design of the control equipment, it being necessary that significant changes in k be obtainable with neither too small nor too great changes in θ .

THE CRITICAL EXPERIMENT VARIABLES

In a critical assembly we cannot follow exactly what happens in the reactor when it is generating power. There are certain limitations, but there are also certain new possibilities that constitute the advantages of the critical assembly.

1. There is a fixed gross fuel load denoted by M (in mols) in the critical experiment, whereas in the power reactor the fuel load is the variable quantity $M_0 - m$, the initial loading less the burnup. To change the fuel load of one of our critical experiments is a semimajor operation.

2. In the critical experiment it is possible to add nuclear poison (cadmium) in variable quantities and to a considerable amount denoted by Y (in mols). In the power reactor the poison accumulation is directly linked through fission products to the fuel burnup and through xenon to the power-level history.

3. The critical experiment has control rods used for experimental purposes independently of the mockup of the power-reactor control element. Such control rods are accurately and sensitively positionable. The control-rod position will be denoted by c .

4. In the critical experiments with which we have worked until now, only predetermined gross changes in control-mechanism positions have been possible. For example, "control drums" of the preliminary pile assemblies mocking up the SIR Mark A can be changed in steps of 20° in angle θ from 6° up. In the new Proof Test Reactor θ will be continuously variable. For analyzing experimental results it would be more desirable to use the variable cosine θ in place of θ because the dependence of various quantities on cosine θ is more nearly linear than the dependence on θ itself. This has not been done in the following theoretical treatment, but the procedure is obvious.

5. It is also possible in the critical experiment to make incremental changes in fuel and poison which can be compensated by changes in control rod position to maintain $k = 1$.

6. Another technique which has been found useful has been to change the escape probability of neutrons by introducing hydrogen locally, the amount being denoted by H (in mols).

Inevitably, there are over-all differences in reactivity between the critical experiment and the power reactor arising from differences in structure and in geometry. These differences are important in making an over-all comparative

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evaluation but are handled by very different techniques than those discussed here. This has been thoroughly discussed in KAPL-837.*

The reactivity of a critical assembly is dependent upon many variables from which we selected those already enumerated. Thus:

$$k = k(\theta, M, Y, H, c) \quad (1)$$

Then, for small variations in these quantities we have:

$$dk = \frac{\partial k}{\partial \theta} d\theta + \frac{\partial k}{\partial M} dM + \frac{\partial k}{\partial Y} dY + \frac{\partial k}{\partial H} dH + \frac{\partial k}{\partial c} dc \quad (2)$$

Actually, the differences in fuel and poison with which we shall be concerned are apt to be larger than "small" quantities, and we can think of extending the expansion to higher-order terms. However, to make use of such an expansion, it is necessary to evaluate the second-order partial differential coefficients. In the present state of the art it is not possible to do this directly with meaningful accuracy, but indirectly it may be possible to use a partial expansion to the quadratic terms, as will be discussed later.

A geometric representation of Equation (2) would involve more dimensions than we can readily portray, but it will appear that in most cases we can make useful representations without having to resort to more than three dimensions.

The quantities that are of basic importance in our analysis are the reactivity coefficients. (In the past we have called them "danger coefficients," this being the historic term based on the early fear that a reactor would not go critical, so that at that time the depressing effect of a particular reactor constituent upon pile reactivity constituted a hazard and was a focal point of attention. Today when we look at the problem in a more matter-of-fact fashion and consider reactivity as a positive quantity, it is fitting that we denote it in a more appropriate way.) The reactivity coefficients are just the differential coefficients in Equation (2). For fuel, poison, and hydrogen, the units are appropriately cents per mol; for the control mechanism, cents per radian or degree; and for a control rod, cents per inch. The key coefficient is that of the control rod which is determined directly in a rod-drop experiment with all other independent variables fixed. The change dk in k arising from a small abrupt change dc in c is found from the resulting fractional change in power level. Since in these experiments dk is of the order of \$0.5, the approximation of dk/dc to the true differential coefficient $\partial k/\partial c$ is good.

The other differential coefficients are then obtained from $\partial k/\partial c$ by making small experimental changes in each variable, one at a time, and maintaining $k = 1$ by compensating with c . Using the reactivity coefficient of fuel as an example, we add a quantity dM , compensating by a change dc , and since in Equation (2), $dk = d\theta = dY = dH = 0$ we have:

*KAPL-837, "Reactivity and Endurance Analysis of SIR Mark A," by H. B. Stewart, H. F. Schulz, and J. A. Bistline, November 14, 1952.

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$$0 = \frac{\partial k}{\partial M} dM + \frac{\partial k}{\partial c} dc$$

$$\frac{\partial k}{\partial M} = - \frac{\partial k}{\partial c} \frac{dc}{dM} \quad (3)$$

It follows similarly from experiments in which poison and hydrogen are added that,

$$\frac{\partial k}{\partial Y} = - \frac{\partial k}{\partial c} \frac{dc}{dY} \quad (4)$$

$$\frac{\partial k}{\partial H} = - \frac{\partial k}{\partial c} \frac{dc}{dH} \quad (5)$$

The same technique has not been applicable in the case of θ because the least step possible with it creates a reactivity change that exceeds the range of the control rods. In the new Proof Test Reactor, however, in which θ is continuously variable, this technique will be applicable to θ .

SAFETY MARGIN

The safety margin is the decrease of the reactivity below unity when the control drums are in their least reactive position θ_s . In the power reactor, the amount of poison is zero when we are interested in the safety margins, but in the critical experiment some poison may be present in order to have all drum positions the same at criticality.

Two experimental methods have been used. One depends on inverse multiplication measurements only; the other requires the addition of moderator to the core as the control drums are turned from θ_c (critical) to θ_s (shutdown), using the measured reactivity coefficient of the moderator to judge the reactivity contribution compensated by the drums. The reactivity coefficient is used as if it applied away from $k = 1$ and at a drum position different from that at which it was measured. The result of such a lack of rigor can only be "surreactivity." Any poison which had been added to balance the drum positions can, of course, be removed in the same process, and its reactivity coefficient can be figured in.

In the first method, by having the source in the reactor and measuring the counting rate n in a neutron detector, $k - k_c$ can be found at each θ from

$$k - k_c = -G/n \quad (6)$$

where G is a determinable numerical constant which depends on source strength, geometry, counter sensitivity, etc. G is determined at the subcritical condition by dropping a previously calibrated safety rod which lies in a region of the core that is relatively unaffected by the gross reactivity change. The resulting change in $1/n$ is thus associated with a known change in reactivity, and the proportionality constant is G . Thus, $k - k_c$ can be determined as a function of θ .

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This dependence is schematically shown as PQ in Figure 1. By reading this curve at the shutdown value of θ_s the safety margin is given directly. In the figure the safety margin $k_c - k_s$ is \$6. Theoretically, this method of obtaining PQ is exact. Experimentally, the conversion of counting rates to over-all multiplication is beset with uncertainties and difficulties arising from low counting rates.

In the second method hydrogen in the form of polyethylene has been added to the central portion of a reflector-controlled core, and θ has been varied in step with the increase in H to maintain criticality. Ideally, this process would follow a curve such as PS of Figure 1; actually, only the end points need to be determined. The reactivity coefficients $\partial k / \partial H$ can be measured both at θ_c and at θ_s (and at intermediate points), but these slopes of traces of the reactivity surface on θ -equal-constant planes bear no necessary direct relation to the evaluation of $k - k_c$ at Q. We might almost as well assume that the reactivity coefficient of hydrogen at S remains constant along the trace SQ so that at Q we should have

$$k_s - k_c = -H_s \left. \frac{\partial k}{\partial H} \right|_S \quad (7)$$

where H_s is the value of H at S.

A better procedure may be to use inverse multiplication to follow Curve SQ as hydrogen is withdrawn from the core, with the advantage here over the first method that rotation of control drums does not now introduce disturbing geometric changes.

This same reasoning would apply if θ_s were reached by a reduction in poison; it is seen that it also applies if θ_s is reached by a combined reduction in poison and addition of hydrogen, even though the graphical representation of the removal of both poison and moderator would require four dimensions instead of three.

It will be clearer if the designation θ_c is reserved for the value of θ at criticality when $Y = 0$, even though this angle may not actually be set on in the critical experiment. Thus we consider that in reaching θ_s from θ by the successive removal of poison and the addition of hydrogen, the value θ_c is traversed as the last poison is removed. It will be apparent that actually setting on θ_c is not essential to determining the safety margin.

A method of determining θ_c is illustrated in Figure 1 where Point 4 represents criticality with poison of an amount given by the ordinate scale to the right. In the course of determining endurance other $k = 1$ points, Point 5, etc., will be determined which will fix Curve PU. Supplementing this, the addition of polyethylene will fix Points 3, 2, and S, the H content of the core being given by the ordinate scale at the left. This establishes Curve PS. It follows that P and hence θ_c will be rather accurately established by the confluence of PS and PU on the θ -axis.

~~CONFIDENTIAL~~REACTOR ENDURANCE

The reactor endurance depends upon the control range. The one method that has been found usable in exploring endurance in the critical experiment is to balance rotation of the control drums toward increasing θ by the addition of distributed poison in the core, usually cadmium. The result of such an experiment can be embodied in a curve like PU in Figure 1 in which the mols of poison are plotted against control-rod position as criticality is maintained. This, then, describes the functional relation embodied in Equation (1) with $k = 1$, $H = 0$, and c and M constant.

This curve is to be used to derive a corresponding curve for the power reactor which will show burnup as a function of θ . Here M is to be replaced by $M_0 - m$ where m is the burnup so that $m/(1 + \alpha)$, α being the capture-to-fission ratio, measures the total energy generated. The quantity Y which refers to a specific poison, cadmium in our case, is to be replaced by $\rho_Z Z + \rho_X X$ where Z and X are mols of fission products and Xe, respectively, and the ρ 's are the effective cross sections in the reactor spectrum of these poisons relative to the cross section of cadmium. The quantity H is zero and c is small, being used for trimming only. If necessary, we can distinguish between steady state xenon and peak xenon concentrations by the subscripts s and p , respectively. Thus, we must write for the power reactor

$$k = k(\theta, M_0 - m, \rho_Z Z + \rho_X X, 0, 0) \quad (8)$$

where the functional relationship is the same as in Equation (1).

If now we consider that we move from a point characterized by the critical experiment variables (with no H and only small c for trimming) to a point characterized by the power reactor variables, we can do this by a Taylor Expansion, and if in addition we restrict ourselves to the same value of θ and to the critical condition in both reactors, we arrive at the relation:

$$(M_0 - m_1 - M) \partial k / \partial M + (\rho_Z Z + \rho_X X - Y) \partial k / \partial Y - c \partial k / \partial c = 0 \quad (9)$$

This has, for the present, been carried only to first-order terms in the expansion and the substitution of m_1 for m recognizes this in exactitude. Since the differential coefficients are reactivity coefficients and they apply to the point characterizing the critical experiments, their values can be determined experimentally.

Two fission-product nuclei replace each fuel nucleus in fissioning so that

$$Z = \frac{2 m_1}{1 + \alpha} \quad (10)$$

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and the substitution of this in Equation (9) gives us

$$(M_0 - M - m_1) \partial k / \partial M + [2\rho_Z m_1 / (1 + \alpha) + \rho_X X - Y] \partial k / \partial Y - c \partial k / \partial c = 0 \quad (11)$$

Solving for m_1 leads to

$$m_1 = \frac{(M_0 - M) \partial k / \partial M + (\rho_X X - Y) \partial k / \partial Y - c \partial k / \partial c}{\partial k / \partial M - 2\rho_Z (\partial k / \partial Y) / (1 + \alpha)} \quad (12)$$

which gives the burnup at a particular control drum position in terms of the known quantities Y , M_0 , m , c , and α ; the experimentally derivable reactivity coefficients $\partial k / \partial M$, $\partial k / \partial Y$, and $\partial k / \partial c$; derivable cross section ratios ρ ; and the derivable xenon concentration X . It thus becomes possible to derive the energy-vs- θ curve for the actual reactor with either steady state or peak xenon from experiments on the critical experiment.

This problem has been formulated heretofore in a somewhat inexact fashion. The right member of Equation (2) with $\partial H = \partial c = 0$ has been considered as being made up of individual reactivity contributions from each variable.

$$dk = dk_\theta + dk_M + dk_Y \quad (13)$$

$$dk_\theta = (\partial k / \partial \theta) d\theta, \text{ etc.}$$

with the possibility of integrating each term individually, using experimentally determined values of the reactivity coefficients. As a consequence, for a reactor which remains critical as θ , M , and Y are varied, we should have

$$0 = \int dk_\theta + \int dk_M + \int dk_Y \quad (14)$$

It would appear that the reactivity contribution from each variable is independent and additive.

This is fallacious because they are not independent but are linked by the requirement that the integration of each be carried out over the same path in the multidimensional space of θ , M , and Y . Actually, for the path traversed experimentally and for which the reactivity coefficients are known, M is constant, but for the path of interest in the case of the power reactor, all three variables undergo change. Calculation of "excess reactivity" ER , such as has been made in the past by:

$$ER = \int dk_Y = \int (\partial k / \partial Y) dY \quad (15)$$

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is without valid meaning, as can be readily seen if the attempt is made to represent this quantity geometrically in the three dimensions θ , Y , and k as shown in Figure 2. Curve PU is the experimentally determined trace of the k surface on the $Y\theta$ plane, and it is along this curve that the partial derivative $\partial k / \partial Y$ can be measured. The curve PT is the trace of the k surface on the $k\theta$ plane. It cannot be measured experimentally, but it represents true excess reactivity. It is apparent that mathematically there is no relationship between the curve PT, the curve PU, and the partial differential coefficient $\partial k / \partial Y$ at the curve PU. Nor will PT have any mathematical relationship with other reactivity coefficients for which other dimensions would have to be introduced in the representation. Actually, Equation (15) has no simple meaning in terms of anything shown in the figure.

When critical experiments are done with two fuel loadings instead of one, it is possible to obtain more accurate results than those given by the use of Equations (9) and (12), because second-order variations of k can be taken into account. It is justifiable then to include second-order terms in a Taylor Expansion bridging the difference in fuel and poison loading between the two experiments. Thus, in place of Equation (9) we can write in a "shorthand" style:

$$0 = \Delta M k'_M + \Delta Y k'_Y + \frac{1}{2} [(\Delta M)^2 k''_{MM} + 2(\Delta M)(\Delta Y) k''_{MY} + (\Delta Y)^2 k''_{YY}] \quad (16)$$

where

$$\Delta M = M_0 - m - M, \quad \Delta Y = \frac{2m\rho_Z}{1 + \alpha} + \rho_X X - Y \quad (17)$$

$$k'_M = \partial k / \partial M, \text{ etc.}$$

$$k''_{MM} = \partial^2 k / \partial M^2, \text{ etc.}$$

It will be seen that by separating out m we obtain a quadratic equation for it in the form

$$0 = (\alpha m + \beta) a_1 + (\gamma m + \delta) a_2 + \frac{1}{2} [(\alpha m + \beta)^2 a_{11} + 2(\alpha m + \beta)(\gamma m + \delta) a_{12} + (\gamma m + \delta)^2 a_{22}] \quad (18)$$

Since the validity of the Taylor Expansion presupposes that the quadratic term is small compared with the linear term, we shall make the assumption that the root of Equation (18) differs only slightly from the value of m given by Equation (12) (although the quadratic equation could be solved in a straightforward fashion) and shall, therefore, write

$$m = m_1 + m_2 \quad (19)$$

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Figure 2

Traces of k Surface, $M = \text{const}$

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where m_1 is given in Equation (12) and m_2 is the correction arising from the quadratic term. Under these assumptions it is justifiable to neglect m_2 in the quadratic term. Solving Equation (18) with the obvious substitutions for α , β , etc., gives:

$$m_2 = \frac{\left[Y - \rho_X X - \frac{2\rho_Z(M_0 - M)}{1 + \alpha} \right]^2}{2 \left(k'_M - \frac{2\rho_Z k'_Y}{1 + \alpha} \right)^3} \left[k_M'^2 k_{YY}' + 2k_M' k_{MY}' k_Y' + k_Y'^2 k_{MM}' \right] \quad (20)$$

Of course, this procedure requires that the values of the second-order differential coefficients be known, and although the experimental information will not completely determine them, it may be sufficient to obtain meaningful results.

We have to apply Equation (16) to the two loadings available. The differences in reactivity coefficients of poison and fuel, $\Delta k'_Y$ and $\Delta k'_M$, respectively, at a fixed θ are available. They are related through Taylor Expansions to the second-order coefficients by:

$$\Delta k'_M = \Delta M k_{MM}' + \Delta Y k_{MY}', \quad (21a)$$

$$\Delta k'_Y = \Delta M k_{MY}' + \Delta Y k_{YY}' \quad (21b)$$

We have, accordingly, three equations, namely Equation (16), (21a), and (21b), with three unknowns, but these equations are not independent. The second-order coefficients can be eliminated between them, yielding:

$$\Delta M k_M' + \Delta Y k_Y' + \frac{1}{2} (\Delta M \Delta k_M' + \Delta Y \Delta k_Y') = 0 \quad (22)$$

or in terms of the reactivity coefficients for the two loadings designated by the subscripts 1 and 2:

$$\Delta M (k_M' \big|_1 + k_M' \big|_2) + \Delta Y (k_Y' \big|_1 + k_Y' \big|_2) = 0 \quad (23)$$

This equation can serve as an experimental test on the accuracy of the extended Taylor Expansion.

If on theoretical grounds one of the second-order coefficients can be evaluated or related to another quantity, then all of them can be determined. For instance, we might assume that

$$k_M' = \text{const}/M, \quad k_{MM}' = -k_M'/M \quad (24)$$

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Then by obvious algebraic manipulation we shall have

$$k_{MY}^{\sim} = \frac{\Delta k_M' - \Delta M k_{MM}^{\sim}}{\Delta Y} \quad (25)$$

and

$$k_{YY}^{\sim} = \frac{\Delta k_Y' - \Delta M k_{MY}^{\sim}}{\Delta Y} = \frac{\Delta Y \Delta k_Y' - \Delta M (\Delta k_M' - \Delta M k_{MM}^{\sim})}{(\Delta Y)^2} \quad (26)$$

with $k_{MM}^{\sim}$ presumably known. These values can then be substituted into Equation (20) permitting the calculation of m_2 .

MAXIMUM REACTOR ENDURANCE BY CHANGING FUEL LOAD ONLY

The more fuel that is put into the clean fresh reactor the greater the endurance, but the less the initial safety margin. Maximum endurance is achieved by adding fuel until k_S is reduced to the maximum safe value.

Experimentally it can be determined in the critical experiment that the fuel mass M corresponds to a shutdown reactivity of k_{S1} . The loading that would have the desired maximum shutdown reactivity k_S would then be

$$M_0 = M_1 + \frac{k_S - k_{S1}}{\partial k / \partial M} \quad (27)$$

where the reactivity coefficient of the fuel is the value at θ_S in the shutdown condition. This may perhaps be obtainable with sufficient accuracy by extrapolation from values determined at other values of θ .

An alternative determination of M_0 depends on having had two different loads in the critical experiment, M_1 and M_2 , for which the shutdown reactivities are found to be k_{S1} and k_{S2} , respectively. Linear interpolation then leads to

$$M_0 = M_1 + \frac{(M_2 - M_1)(k_S - k_{S1})}{k_{S2} - k_{S1}} \quad (28)$$

Also, linear interpolation for the corresponding angle θ_S leads to

$$\theta_S = \theta_{S1} + \frac{(\theta_{S2} - \theta_{S1})(k_S - k_{S1})}{k_{S2} - k_{S1}} \quad (29)$$

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CONTROL SENSITIVITY

The control sensitivity $\partial k / \partial \theta$ of the control mechanism is the value of the partial differential coefficient at points on the actual operating curve of burnup (or total energy) vs θ . This differential coefficient, denoted for simplicity by k'_θ , is a function of the same variables as k itself:

$$k'_\theta = k'_\theta (\theta, M, Y) \text{ in the critical experiment} \quad (30)$$

$$k'_\theta = k'_\theta (\theta, M_0 - m, \frac{2m\rho_Z Z}{1 + \alpha} + \rho_X X) \text{ in the power reactor} \quad (31)$$

where H and c have been omitted.

It has already been pointed out that control sensitivity cannot be determined in the critical experiment in the same way as the other reactivity coefficients, but another method has been possible. The slope of Curve PU of Figure 2 is

$$\left. \frac{\partial Y}{\partial \theta} \right|_{M \text{ const}, k = 1}$$

whence, applying Equation (2) with

$$dk = dM = dH = dc = 0$$

we have

$$\partial k / \partial \theta = - (\partial k / \partial Y) (\partial Y / \partial \theta) \quad (32)$$

to give the control sensitivity.

For a particular load M_1 , we denote the control sensitivity by $k'_{\theta 1}$; for the power reactor at the same θ we denote it by $k'_{\theta P}$.

By using a Taylor Expansion we get:

$$k'_{\theta P} = k'_{\theta 1} + (M_0 - m - M_1) (\partial k'_{\theta 1} / \partial M) + \left(\frac{2m\rho_Z Z}{1 + \alpha} + \rho_X X - Y \right) (\partial k'_{\theta 1} / \partial Y) \quad (33)$$

Unfortunately, there is no practical way to determine the second-order differential coefficients which appear here.

Slight progress accrues from the use of two fuel loads in the critical experiment. Then we have

$$k'_{\theta 2} - k'_{\theta 1} = (M_2 - M_1) k'_{\theta M} + (Y_2 - Y_1) k'_{\theta Y} \quad (34)$$

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Insofar as M_1 and M_2 bracket M_0 and $M_0 - m$ at full burnup, the above difference establishes an outside limit to the range of k_0 .

It had been hoped to apply the approach outlined above to the actual measurements on PPA-18 and PPA-19, but it seems better to await the results on PPA-20 and let others treat them all together. In addition, new results on fission-product poisons will affect ρ_Z .

A matter not covered here and which must, of course, be taken into account is the proper averaging of individual local measurements of reactivity coefficients to obtain the distributed coefficient, which is the quantity pertinent to the analysis. This will be dealt with later in connection with the PPA results mentioned above.

Although the present report would be more complete and satisfactory if additional experimental material were included, it was felt that the theory alone is of sufficient interest to warrant its immediate presentation.

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