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TSAAS: Finite Element Thermal and Stress Analysis of Axisymmetric Solids with Orthotropic Temperature-Dependent Material Properties

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TSAAS
FINITE ELEMENT THERMAL AND STRESS ANALYSIS
OF AXISYMMETRIC SOLIDS WITH ORTHOTROPIC
TEMPERATURE-DEPENDENT MATERIAL PROPERTIES

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ABSTRACT

The finite element method is used to determine the temperatures, displacements, stresses, and strains in axisymmetric solids with orthotropic, temperature-dependent material properties under axisymmetric thermal and mechanical loads. The mechanical loads can be surface pressures, surface shears, and nodal point forces as well as an axial or centripetal acceleration. The continuous solid is replaced by a system of ring elements with triangular or quadrilateral cross sections. Accordingly, the method is valid for solids that are composed of many different materials and that have complex geometry. Nonlinear mechanical behavior as typified by plastic, locking, "no tension," or creeping materials can be approximated. Two-dimensional mesh generation, plotting, and editing features allow the computer program to be readily used.

In addition to a stress analysis program that is based on a modified version of the SAAS code, TSAAS can carry out a transient thermal analysis with the finite element mesh used in stress analysis. An implicit time differencing scheme allows the use of arbitrary time steps with consequent fast running times. At specified times, the program will return to SAAS for thermal stress analysis. Nonlinear thermal properties and Arrhenius reaction kinetics are also incorporated into TSAAS.

I. INTRODUCTION*

The finite element terminology as applied to the analysis of continua was first used in 1960.¹ Before 1960, two-dimensional elements were used in conventional methods of structural analysis as a means of improving the stiffness idealization of complex aircraft structures.² Since the introduction of the finite element method for the stress analysis of plane stress bodies, the technique has been successfully applied to plates,³ axisymmetric solids,^{4,5,6} axisymmetric shells,^{7,8} three-dimensional solids,^{9,10} and other boundary value problems (e. g., Refs. 11 and 12). Reference 13 is an up-to-date text on the application of the finite element method to many linear and nonlinear field problems in engineering science.

In the finite element approximation of solids, the continuum is replaced by a system of elements that are interconnected at their corners (nodes). In the case of an axisymmetric solid, the nodes are actually circles and are called nodal circles. A finite element idealization of a simple axisymmetric solid using triangular elements is shown in Fig. 1. For axisymmetric loading, two equilibrium equations, which are expressed in terms of unknown nodal circle displacements, are derived for each node of the finite element system. A solution of the resulting set of linear algebraic equations constitutes an equilibrium solution to the finite element approximation of the axisymmetric solid.

The advantages of the finite element method over other methods of analysis are numerous. The finite element method affords nearly complete generality in the specification of geometrical and material properties, i. e., geometrically complex bodies of many different materials are easily represented. Displacement or stress boundary

Fig. 1. The finite element idealization.

conditions can be specified at any node of the finite element system. Thermal and mechanical loads can be specified at nodes and, if the number of nodes is sufficient, nearly arbitrary distributions of the loads can be represented. In addition, the body as a whole can be subjected to an axial acceleration and/or an angular velocity.

The present report is concerned with transient thermal stress analysis using the finite element method. In Section II, the finite element method is applied to the determination of stresses, strains, and displacements in arbitrary axisymmetric bodies with orthotropic, temperature-dependent material properties subjected to arbitrary axisymmetric mechanical and thermal loading. The mechanical loads can be surface pressures, surface shears, and nodal point forces as well as an axial acceleration or angular velocity. Using triangular and quadrilateral finite elements, the equilibrium equations for a

*Portions of Sections I and II and Appendix A have been lifted from the interim report, SAAS IIa, Finite Element Stress Analysis of Axisymmetric Solids with Orthotropic, Temperature-Dependent Material Properties, Sandia Corporation.

finite element system are derived from the minimum potential energy theorem for elastic solids.

In Section III, the finite element method is applied to the determination of transient temperature fields in axisymmetric solids with time-dependent boundary conditions starting from specified initial conditions. Again temperature-dependent thermal properties as well as nonlinear temperature-dependent heat generation (such as provided by reaction kinetics) are incorporated into the analysis.^{14,15} The finite element discretized equations are again based on triangles and quadrilaterals allowing the use of a single finite element mesh to be used in both problems. Finally, an implicit time differencing scheme allows the use of an arbitrary time increment without loss of numerical stability in the discretized equations; the net result is a great saving in computer time compared with finite element methods that are restricted by a stability requirement.

TSAAS, Thermal Stress Analysis of Axisymmetric Solids, incorporates the theory of Sections II and III into a practical code. The stress analysis portion of TSAAS is based on the SAAS code¹⁶ together with two-dimensional automatic mesh generation. This is an important aid to the user since it drastically reduces the number of input data cards. The thermal analysis portion of TSAAS has been written by the authors and uses several initial condition and boundary condition options that will be an important aid to the user. The overlay program SAASPL provides a plotting and editing capability for the enormous amount of data (displacements, temperatures, stresses, and strains) generated in TSAAS. In addition, TSAAS has a multiple case capability whereby a problem can be run using a previously generated mesh.

Section IV discusses various special features of TSAAS such as mesh generation, skew boundaries, and orthotropic elastic properties. In Section V are discussed the various material

nonlinearities that have been adapted to TSAAS. Basically, TSAAS carries out a linear elastic analysis, but by changing elastic material properties for each element, forming a new stiffness matrix, and then re-solving, nonlinear material behavior can be approximated. In this way plastic, locking, and "no tension"¹⁷ material behavior have been approximated in TSAAS. In addition, the initial strain method has been incorporated into a version of TSAAS which allows for the calculation of both thermal and creep behavior of axisymmetric solids.

Input instructions for TSAAS are described in Appendix A while Appendix B describes the TSAAS computer output. Finally, Appendix C illustrates several examples of stress and thermal problems solved by using TSAAS.

II. METHOD OF STRESS ANALYSIS

The finite element method is described in Ref. 13 and the general equations that govern the equilibrium of an elastic system are given there. However, for completeness and in order to define the various terms involved, the equations are briefly rederived here. A standard matrix notation will be used with $\{v\}$ representing a vector, $[A]$ a matrix, etc.

A. Equilibrium Equations

The potential energy for an elastic solid is given by

$$V = U - \int_{\text{Vol}} \{v\}^T \{F\} dV - \int_{\text{Area}} \{v\}^T \{P\} ds, \quad (1)$$

where U is the total strain energy for the solid, and is given by the integral

$$U = \int_{\text{Vol}} \left[\int_0^{\epsilon} \{\sigma\}^T \{d\epsilon\} \right] dV. \quad (2)$$

In Eqs. (1) and (2), $\{v\}$ is the displacement vector, $\{F\}$ is the body force vector, and $\{P\}$ is the surface traction. For a finite element system that is composed of M elements, the potential energy can be written as a summation over all the elements

$$V = \sum_{m=1}^M \left[U_m^m - \int_{Vol} \{F\}_m^T \{v\}_m dV - \int_{Area} \{P\}_m^T \{v\}_m ds \right]. \quad (3)$$

The surface integral exists only if the m th element is on the boundary of the body and subjected to surface tractions $\{P\}$.

In the finite element displacement approach, it is necessary to assume a solution for the displacement field in each element, that is, in general, taken so as to satisfy compatibility between elements of the system. For an axially symmetric system, for instance, the radial displacement in the m th element $u_m(r, z)$ can be written down

$$u_m(r, z) = [A(r, z)]_m \{u\}_m, \quad (4)$$

where $[A]_m$ is the so-called shape function matrix and $\{u\}_m$ is the vector of the radial nodal displacements. In the displacement approach, an expression for the element strains $\{\epsilon\}_m$ is obtained from the assumed displacement field

$$\{\epsilon\}_m = [B]_m \{\delta\}_m, \quad (5)$$

where $[B]_m$ is the strain-displacement matrix and $\{\delta\}_m$ is the vector of all of the element nodal point displacements

$$\{\delta\}_m = \begin{Bmatrix} u_i \\ w_i \\ u_j \\ w_j \\ \vdots \\ \vdots \end{Bmatrix}. \quad (6)$$

In Eq. (6), u_i, w_i represent the radial and axial components of displacement of node i of element m , etc. In transposed form, Eq. (5) becomes

$$\{\epsilon\}_m^T = \{\delta\}_m^T [B]_m^T. \quad (7)$$

For an elastic material, the element stresses are expressed in terms of the corresponding strains and thermal effects by the linear thermo-elastic stress-strain relationship

$$\{\sigma\}_m = [C]_m \{\epsilon\}_m - \{\tau\}_m, \quad (8)$$

where $\{\tau\}$ are thermal stresses.

Using Eq. (8) and some manipulation with transposes, the strain energy, Eq. (2), can be written in matrix form as

$$U^m = \frac{1}{2} \int_{Vol} \{\epsilon\}_m^T [C]_m \{\epsilon\}_m dV - \int_{Vol} \{\epsilon\}_m^T \{\tau\}_m dV. \quad (9)$$

The total potential energy, Eq. (3), can then be written in matrix form as

$$V = \frac{1}{2} \{\delta\}^T [K] - \{\delta\}^T \{Q\}, \quad (10)$$

$[K]$ is the stiffness matrix for the assembled finite element system:

$$[K] = \sum_{m=1}^M [k]_m. \quad (11)$$

The individual element stiffness matrix is

$$[k]_m = \int_{Vol} [B]_m^T [C]_m [B]_m dV, \quad (12)$$

and the load vector $\{Q\}$ is defined as

$$\{Q\} = \sum_{m=1}^M \{L\}_m + \sum_{m=1}^M \{R\}_m. \quad (13)$$

The element body force is

$$\{L\}_m = \int_{Vol} \left[[A]_m^T \{F\}_m + [B]_m^T \{\tau\}_m \right] dV, \quad (14)$$

and the element load vector due to surface forces is

$$\{R\}_m = \int_{Area} [A]_m^T \{P\}_m ds. \quad (15)$$

The potential energy is made stationary by requiring that

$$\frac{\partial V}{\partial \delta_i} = 0 \quad i = 1 \dots N \quad (16)$$

where N is the total number of unknown nodal point displacements. After consideration of the rules for matrix differentiation with respect to a vector, the application of Eq. (10) yields the following linear system equations for the equilibrium of a finite element system:

$$[K]\{\delta\} = \{Q\} \quad (17)$$

B. Boundary Conditions

Before the displacement boundary conditions are imposed, the stiffness matrix K is singular and, thus, a unique solution does not exist for the displacements. The minimum number of boundary conditions that must be used are those sufficient to restrain the rigid body motion of the structure.

The method used herein is that which has been used by Wilson, and that is the diagonal term of the stiffness matrix of the corresponding restraint is replaced by unity, while the remaining off-diagonal elements in that row and column are set to zero. The corresponding force term is the prescribed displacement while the other force terms are modified by subtracting from them the product of the corresponding off-diagonal term of the stiffness matrix and the prescribed displacement. This is obviously done before the off-diagonal terms are set to zero. The subroutine MODIFY, which is used to perform these steps, consists of only 17 FORTRAN statements.

Incidentally, this technique does not destroy the band width and, in fact, the boundary conditions are imposed on the matrix that is stored in a rectangular array. Using this method, the number of equations that must be "solved" is equal to the total number of degrees of freedom; however, the number of restraints is usually relatively small and the band solution proceeds at a fast rate.

C. Triangular Element

The cross section of a typical triangular ring element is shown in Fig. 2. The displacements (u, w) in the r-z plane in the element are assumed to be of the following form:

$$u(r, z) = b_1 + b_2 r + b_3 z \quad (17a)$$

$$w(r, z) = b_4 + b_5 r + b_6 z \quad (17b)$$

This linear displacement field assures continuity between elements since lines which are initially straight remain straight in their displaced position.

If Eqs. (17a) and (17b) are evaluated at the three vertices (r_i, z_i) , (r_j, z_j) , (r_k, z_k) of the triangular ring element for the nodal point displacements (u^i, w^i) etc., then, the following matrix equation is obtained

$$\begin{bmatrix} u^i & w^i \\ u^j & w^j \\ u^k & w^k \end{bmatrix} = \begin{bmatrix} 1 & r_i & z_i \\ 1 & r_j & z_j \\ 1 & r_k & z_k \end{bmatrix} \begin{bmatrix} b_1 & b_4 \\ b_2 & b_5 \\ b_3 & b_6 \end{bmatrix} \quad (18)$$

A solution of Eq. (18) yields the following expression for the constants b_i ($i = 1, 2, \dots, 6$) in terms of the displacements at the vertices:

$$\begin{bmatrix} b_1 & b_4 \\ b_2 & b_5 \\ b_3 & b_6 \end{bmatrix} = [D] \begin{bmatrix} u^i & w^i \\ u^j & w^j \\ u^k & w^k \end{bmatrix} \quad (19a)$$

where

$$[D] = \frac{1}{\lambda} \begin{bmatrix} r_j z_k - r_k z_j & r_k z_i - r_i z_k & r_i z_j - r_j z_i \\ z_j - z_k & z_k - z_i & z_i - z_j \\ r_k - r_j & r_i - r_k & r_j - r_i \end{bmatrix} \quad (19b)$$

in which $\lambda = r_j(z_k - z_i) + r_i(z_j - z_k) + r_k(z_i - z_j)$. Equation (19a) can be written in an expanded form where the constants and displacements are column vectors,

$$\{b\} = [h]\{\delta\} \quad (20a)$$

Fig. 2. Triangular Element

and the shape function matrix $[A]$ given in Eq. (4) is seen to be

$$[A(r, z)]^T = [D]\{e\} \quad (20b)$$

where

$$\{e\}^T = \{1, r, z\}$$

The element strains are obtained from Eqs. (17a) and (17b). Letting θ be the hoop direction,

$$\epsilon_{rr} = \frac{\partial u}{\partial r} = b_2$$

$$\epsilon_{zz} = \frac{\partial w}{\partial z} = b_6$$

$$\epsilon_{\theta\theta} = \frac{u}{r} = \frac{1}{r} b_1 + b_2 + \frac{z}{r} b_3$$

$$\epsilon_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} = b_3 + b_5$$

which can be written in matrix form as

$$\begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{\theta\theta} \\ \epsilon_{rz} \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{1}{r} & 1 & \frac{z}{r} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix} \quad (21)$$

or, symbolically,

$$\{\epsilon\} = [g]\{b\} \quad (22)$$

The substitution of Eq. (20a) into Eq. (22) yields

$$\{\epsilon\} = [g][h]\{\delta\} \quad (23)$$

Thus, the strain-displacement transformation matrix $[B]$, as defined in Eq. (5), is

$$[B] = [g][h] \quad (24)$$

With this definition of $[B]$, the element stiffness matrix, Eq. (12), is rewritten as

$$[k] = \int_{Vol} [h]^T [g]^T [C] [g] [h] dV \quad (25)$$

Since $[h]$ is not a function of r and z , Eq. (25) for the element stiffness matrix becomes

$$[k] = [h]^T \left[\int_{Vol} [g]^T [C] [g] dV \right] [h] \quad (26)$$

and the body force vector, Eq. (14), reduces to

$$\{L\} = [h]^T \int_{Vol} [g]^T \{\tau\} + \{e\}^T \{F\} dV \quad (27)$$

where again $\{e\}^T = \{1, r, z\}$.

For axisymmetric bodies with orthotropic material properties whose principal axes are not aligned with the r - z coordinates, the stress-strain relations in the principal N-S coordinates are

$$\begin{Bmatrix} \sigma_{NN} \\ \sigma_{SS} \\ \sigma_{\theta\theta} \\ \sigma_{NS} \end{Bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} & 0 \\ C'_{12} & C'_{22} & C'_{23} & 0 \\ C'_{13} & C'_{23} & C'_{33} & 0 \\ 0 & 0 & 0 & C'_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_{NN} \\ \epsilon_{SS} \\ \epsilon_{\theta\theta} \\ \epsilon_{NS} \end{Bmatrix} + \begin{Bmatrix} \tau_{NN} \\ \tau_{SS} \\ \tau_{\theta\theta} \\ 0 \end{Bmatrix} \quad (28)$$

where

$$\begin{aligned} \tau_{NN} &= T (C'_{11} \alpha_N + C'_{12} \alpha_S + C'_{13} \alpha_\theta) \\ \tau_{SS} &= T (C'_{12} \alpha_N + C'_{22} \alpha_S + C'_{23} \alpha_\theta) \\ \tau_{\theta\theta} &= T (C'_{13} \alpha_N + C'_{23} \alpha_S + C'_{33} \alpha_\theta) \end{aligned} \quad (29)$$

and T is the element temperature. The N-S coordinates are defined in terms of the r - z coordinates in Fig. 3.

Fig. 3. N-S and r-z coordinates.

Equation (28) can be abbreviated as

$$\{\sigma\}_{NS} = [C]_{NS} \{\epsilon\}_{NS} - \{\tau\}_{NS} \quad (30)$$

where the subscripts refer to the coordinate system in which the quantities are expressed. The stresses in the local N-S coordinates are transformed into the body r-z coordinates by use of the transformation

$$\{\sigma\}_{rz} = [t]^T \{\sigma\}_{NS} \quad (31)$$

where $[t]^T$ is the transpose of

$$[t] = \begin{bmatrix} \cos^2 \beta & \sin^2 \beta & 0 & \sin \beta \cos \beta \\ \sin^2 \beta & \cos^2 \beta & 0 & -\sin \beta \cos \beta \\ 0 & 0 & 1 & 0 \\ -2 \sin \beta \cos \beta & 2 \sin \beta \cos \beta & 0 & \cos^2 \beta - \sin^2 \beta \end{bmatrix} \quad (32)$$

The strains in the local N-S coordinates are transformed into the body r-z coordinates by use of the transformation

$$\{\epsilon\}_{rz} = [t]^{-1} \{\epsilon\}_{NS} \quad (33)$$

The inverse of Eq. (38) is

$$\{\epsilon\}_{NS} = [t] \{\epsilon\}_{rz} \quad (34)$$

Upon substitution of Eqs. (31) and (34) in Eq. (30), it is seen that

$$\{\sigma\}_{rz} = [C]_{rz} \{\epsilon\}_{rz} - \{\tau\}_{rz} \quad (35)$$

where

$$[C]_{rz} = [t]^T [C]_{NS} [t] \quad (36)$$

$$\{\tau\}_{rz} = [t]^T \{\tau\}_{NS} \quad (37)$$

Equation (35) can be expanded to read

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{zz} \\ \sigma_{\theta\theta} \\ \sigma_{rz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{12} & C_{22} & C_{23} & C_{24} \\ C_{13} & C_{23} & C_{33} & C_{34} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} \begin{Bmatrix} \epsilon_{rr} \\ \epsilon_{zz} \\ \epsilon_{\theta\theta} \\ \epsilon_{rz} \end{Bmatrix} - \begin{Bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \end{Bmatrix} \quad (38)$$

where

$$\begin{aligned} \tau_1 &= T(C_{11} \alpha_r + C_{12} \alpha_z + C_{13} \alpha_\theta) \\ \tau_2 &= T(C_{12} \alpha_r + C_{22} \alpha_z + C_{23} \alpha_\theta) \\ \tau_3 &= T(C_{13} \alpha_r + C_{23} \alpha_z + C_{33} \alpha_\theta) \\ \tau_4 &= T(C_{14} \alpha_r + C_{24} \alpha_z + C_{34} \alpha_\theta) \end{aligned} \quad (39)$$

The coefficients of thermal expansions, α_r , α_z , and α_θ , are in the r, z, and θ directions, respectively, and T is the temperature change within the element. For this type of material, the terms under the integral in Eq. (26) are

$$[g]^T [C] [g] = \begin{bmatrix} \frac{1}{r^2} C_{33} & \frac{1}{r} (C_{13} + C_{33}) & \frac{z}{r^2} C_{33} + \frac{1}{r} C_{34} & 0 & \frac{1}{r} C_{34} & \frac{1}{r} C_{23} \\ & C_{11} + 2C_{13} + C_{33} & \frac{z}{r} (C_{13} + C_{33}) + C_{14} + C_{34} & 0 & C_{14} + C_{34} & C_{12} + C_{23} \\ & & \frac{z^2}{r^2} C_{33} + \frac{2z}{r} C_{34} + C_{44} & 0 & \frac{z}{r} C_{34} + C_{44} & \frac{z}{r} C_{23} + C_{24} \\ & \text{Symmetrical} & & 0 & 0 & 0 \\ & & & & C_{44} & C_{24} \\ & & & & & C_{22} \end{bmatrix} \quad (40)$$

and

$$[g]^T \{\tau\} + [e]^T \{F\} = \begin{bmatrix} F_r + \frac{1}{r} \tau_3 \\ rF_r + \tau_1 + \tau_3 \\ zF_r + \frac{z}{r} \tau_3 + \tau_4 \\ F_z \\ rF_z + \tau_4 \\ zF_z + \tau_2 \end{bmatrix} \quad (41)$$

In the case of an angular frequency, ω , the body force in the r-direction is $F_r = m\omega^2 r$ and, for an axial acceleration, a_z , the body force in the z-direction is given by $F_z = -ma_z$ where m is the mass density of the material.

Equations (40) and (41) can be programmed directly, since the integrals over the volume of the element for the various functions are most conveniently evaluated numerically in the computer program. The element stiffness matrix, Eq. (26), and the body force vector, Eq. (27), are then formed by standard matrix operations.

D. Quadrilateral Element

A typical quadrilateral element is composed of four triangular elements as illustrated in Fig. 4. The ten equilibrium equations for the quadrilateral are developed by the application of Eq. (10) and can be written in the following matrix form

$$\begin{bmatrix} k_{aa} & k_{ab} \\ k_{ba} & k_{bb} \end{bmatrix} \begin{Bmatrix} u_a \\ u_b \end{Bmatrix} = \begin{Bmatrix} q_a \\ q_b \end{Bmatrix} \quad (42)$$

where u_a and q_a are associated with points 1 to 4 and u_b and q_b are associated with point 5.

Equation (42) can be written as two matrix equations,

$$[k_{aa}] \{u_a\} + [k_{ab}] \{u_b\} = \{q_a\} \quad (43)$$

$$[k_{ba}] \{u_a\} + [k_{bb}] \{u_b\} = \{q_b\} \quad (44)$$

Equation (44) can be solved for the displacements u_b :

$$\{u_b\} = -[k_{bb}]^{-1} [k_{ba}] \{u_a\} + [k_{bb}]^{-1} \{q_b\} \quad (45)$$

If Eq. (45) is substituted in Eq. (43), an expression is found which relates the forces at points 1 to 4 to the unknown displacements at points 1 to 4 and the known thermal loads.

Fig. 4. Quadrilateral Element

$$\{\bar{q}_a\} = [\bar{k}_{aa}] \{u_a\} \quad (46)$$

where the quadrilateral stiffness matrix is

$$[\bar{k}_{aa}] = [k_{aa}] - [k_{ab}] [k_{bb}]^{-1} [k_{ba}] \quad (47)$$

and the modified load matrix is

$$\{\bar{q}_a\} = \{q_a\} - [k_{ab}] [k_{bb}]^{-1} \{q_b\} \quad (48)$$

The use of the quadrilateral as a separate element is desirable since the resulting set of equilibrium equations has fewer unknowns for a given number of triangular elements.

III. METHOD OF THERMAL ANALYSIS

A. Basic Equations

Development of the matrix equations governing finite element transient heat conduction may proceed in several ways as typified in Refs. 12, 14, or 18. Since the potential energy concept was employed for elastic stress analysis in Section II, our approach here will use "temperature potentials." Adopting this approach we begin, as before, by approximating the temperature field $\phi(r, z, t)$ over the finite element shown in Fig. 2 with functions of the form

$$\phi_m(r, z, t) = [A(r, z)]_m \{\phi(t)\}_m, \quad (49)$$

where $[A]_m$ is the shape function matrix (for the triangle, $[A]$ is given by 19b and 20b) and $\{\phi(t)\}_m$ are the nodal point temperature values that are to be determined. The conductivity potential V_k and heat capacity potential V_c can be written¹⁸ in matrix notation as

$$V_k = \frac{1}{2} \{\phi\}^T [K] \{\phi\} - \{\phi\}^T \{Q\} \quad (50)$$

$$V_c = \frac{1}{2} \{\dot{\phi}\}^T [C] \{\dot{\phi}\}, \quad (51)$$

where a $\{\dot{\phi}\}$ represents the time derivative of $\{\phi\}$. In Eqs. (50) and (51), $[K]$ and $[C]$ are the conductivity and heat capacity matrices for the assembled finite element system and $\{Q\}$ is the thermal flux (load) vector for the assembled system:

$$[K] = \sum_{m=1}^M [k]_m, [C] = \sum_{m=1}^M [c]_m, \{Q\} = \sum_{m=1}^M \{q\}_m. \quad (52)$$

The individual element conductivity and heat capacity matrices are

$$[k]_m = \frac{1}{2} \int_{VOL} [a]_m^T [\eta]_m [a]_m dV, \quad (53)$$

$$[c]_m = \frac{1}{2} \int_{VOL} \rho_m c_m [A]_m^T [A]_m dV, \quad (54)$$

where $[\eta]_m$ is the element thermal conductivity matrix

$$[\eta]_m = \begin{bmatrix} \eta_{rr} & \eta_{rz} \\ \eta_{rz} & \eta_{zz} \end{bmatrix}, \quad (55)$$

$\rho_m c_m$ is the product of the element density and the specific heat capacity, and $[a]_m$ is the matrix of gradients of the shape functions,

$$[a]_m = \begin{bmatrix} \frac{\partial [A]_m}{\partial r} \\ \frac{\partial [A]_m}{\partial z} \end{bmatrix}. \quad (56)$$

Finally, the element thermal flux vector $\{q\}_m$ is given by

$$\{q\}_m = - \int_{VOL} \rho_m [A]_m^T Q^* dV - \int_{AREA} [A]_m^T q^* ds, \quad (57)$$

where Q^* is the internal heat generation per unit of mass and q^* is the surface flux intensity (note that the second integral occurs only when the element possesses a side on the boundary).

The stationary conditions of the variational approach to finite element transient heat conduction then require that

$$\frac{\partial V_k}{\partial \phi_i} + \frac{\partial V_c}{\partial \dot{\phi}_i} = 0, \quad i = 1, 2, \dots, N, \quad (58)$$

for the N nodal point temperatures. Invoking these conditions on the potentials defined in Eqs. (50) and (51) then leads to the matrix differential equation for transient heat condition

$$[C] \rightarrow \{\dot{\phi}\} + [K]\{\phi\} = \{Q\} \quad (59)$$

B. Triangular Element and Quadrilateral Element

The temperature distribution within the element is given, in terms of the nodal point values, by Eq. (49) where the shape function matrix $[A(r, z)]$ for triangular finite elements has been defined previously by Eqs. (19b) and (20b). From Eq. (19b) it is easily seen that the gradient matrix $[a]_m$ is constant in each element and is of the form

$$[a]_m = \frac{1}{\lambda} \begin{bmatrix} (z_j - z_k) & (z_k - z_i) & (z_i - z_j) \\ (r_k - r_j) & (r_i - r_k) & (r_j - r_i) \end{bmatrix} \quad (60)$$

where (r_i, z_i) , (r_j, z_j) , (r_k, z_k) are the nodal coordinates of the triangle and $\lambda = r_j(z_k - z_i) + r_i(z_j - z_k) + r_k(z_i - z_j)$ as before.

Observing that the gradient matrix $[a]_m$ does not depend on r or z and using the representation (20b) for the shape function matrix, we can write down expressions for the element conductivity and heat capacity matrices as well as the thermal flux vector:

$$[k]_m = \frac{1}{2} [a]_m^T [\eta]_m [a]_m \cdot \text{Vol} \quad (61)$$

$$[c]_m = \frac{1}{2} \rho_m c_m [D] \left[\int_{\text{VOL}} \{e\} \{e\}^T dV \right] [D]^T \quad (62)$$

$$\begin{aligned} \{q\}_m &= -\rho_m [D] \int_{\text{VOL}} \{e\} Q^* dV \\ &\quad - [D] \int_{\text{AREA}} \{e\} q^* dS \quad (63) \end{aligned}$$

For the quadrilateral element composed of four triangular elements shown in Fig. 4, nodal point reduction is carried out as discussed in Section II. D and an equivalent 4 by 4 element conductivity matrix $[k]_m$ and thermal load vector $\{q\}_m$ are derived. For the heat capacity matrix, $[c]_m$ the capacitance of the fifth interior node is equally distributed among the quadrilateral corner nodes.

The element thermal conductivity matrix $[\eta]$ for orthotropic materials is expressed along principal N-S coordinates. In terms of r, z coordinates, the element conductivity matrix is

$$[\eta]_{rz} = [\hat{t}]^T [\eta]_{NS} [\hat{t}] \quad (64)$$

where the transformation matrix $[\hat{t}]$ is

$$[\hat{t}] = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix} \quad (65)$$

where β is again the inclination of the N-S coordinate system to the $r-z$ coordinate system.

C. Boundary and Initial Conditions

A well-posed problem of transient heat conduction requires the specification of initial values of the temperature field ϕ_i , $i = 1, 2, \dots, N$. This is done by setting the values at $t = 0$.

In addition to initial conditions, boundary conditions must be specified. For temperature boundary conditions subroutine MODIFY is used as discussed in Section II. B. For flux boundary conditions, the appropriate thermal "load" vector is given by second integral in Eq. (63). Finally, for many practical situations the boundary condition

$$\eta_r \frac{\partial \phi}{\partial r} + \eta_z \frac{\partial \phi}{\partial z} + \gamma(\phi - \phi_a) = 0, \quad (66)$$

typifying Newton cooling into ambient air at temperature ϕ_a with surface heat transfer coefficient γ , is useful. For this case to the element conductivity matrix $[k]_m$ and the flux vector $\{q\}_m$ must be added the terms

$$\gamma \int_{\text{AREA}} [A]_m^T [A]_m dS$$

and

$$-\gamma \phi_a \int_{\text{AREA}} [A]_m^T dS, \quad (67)$$

respectively. Of course, these contributions are taken only if the element possesses a side along the boundary where Eq. (66) applies.

D. Transient Solution by the Crank-Nicolson Method

In order to solve the transient equation

$$[K]\{\phi\} + [C]\{\dot{\phi}\} - \{Q\} = \{0\}, \quad (68)$$

with specified initial condition $\{\phi\} = \{\phi\}_0$ at $t = 0$, we proceed by discretizing time pointwise, i. e., we consider a sequence of points in time $t_0 = 0, t_1, t_2, \dots, t_n, \dots$. At a time t_n , say, we can write a midinterval approximation for the vector $\{\dot{\phi}\}$,

$$\{\dot{\phi}\}_{n+\frac{1}{2}} = \frac{\{\phi\}_{n+1} - \{\phi\}_n}{\Delta t}$$

where $\Delta t = t_{n+1} - t_n$. If we evaluate the remaining terms in Eq. (68) at the midinterval (for instance, $\{\phi\}_{n+\frac{1}{2}} = \frac{1}{2}(\{\phi\}_n + \{\phi\}_{n+1})$, etc.) we obtain the equation

$$\begin{aligned} \frac{1}{2}[K](\{\phi\}_{n+1} + \{\phi\}_n) + \frac{[C]}{\Delta t}(\{\phi\}_{n+1} - \{\phi\}_n) \\ - \frac{1}{2}(\{Q\}_n + \{Q\}_{n+1}) = \{0\}. \end{aligned} \quad (69)$$

We now make the further assumption that the conductivity and heat capacity matrices $[K]$ and $[C]$ and the flux vector $\{Q\}$ do not vary greatly with ϕ so that they can be regarded as constant with respect to the change in ϕ occurring in the time interval Δt . Equation (69) can then be rearranged, putting the components of $\{\phi\}_{n+1}$ on one side, to give

$$\begin{aligned} ([K] + \frac{2}{\Delta t}[C])\{\phi\}_{n+1} = -([K] - \frac{2}{\Delta t}[C])\{\phi\}_n \\ + 2\{Q\}_n \end{aligned} \quad (70)$$

where the right-hand side is known since $\{\phi\}_n$ is known at t_n .

Equation (70) provides the algorithm for advancing in time from the initial $\{\phi\}_0$ in solving the transient Eq. (68). In contrast to explicit methods of solution, this algorithm is unconditionally stable for linear problems and corresponds to the Crank-Nicolson method of Ref. 19 for finite difference equations. This stability feature is very important, since it allows the use of arbitrary size time steps Δt .

In addition, the truncation error associated with the Crank-Nicolson procedure is of order $(\Delta t)^2$ contrasted with (Δt) for the usual explicit time differencing method.

IV. SPECIAL COMPUTER PROGRAM FEATURES

Certain special computer program features which are not implied by the report title are discussed in this section. The special features include finite element mesh generation, skew boundaries, material properties, and a transient thermal loading capability. In addition, it can be noted from Appendix A that a constant temperature can be specified for the body (a feature which can be utilized in cool-down problems), or a constant material temperature can be specified and free thermal strains can be input as an alternative to coefficients of thermal expansion.

A. Finite Element Mesh Generation

The mesh generator incorporated in TSAAS has been designed to simplify the meshing process for fairly complex bodies and to allow for quickly modifying an existing mesh. The boundaries of the region may be described in terms of straight or circular line segments, as well as general spline curves. All internal nodal point locations can be determined automatically by the mesher, and nodal points can be allowed to slide along the boundary curves, providing a more uniform mesh than obtainable with uniform nodal point spacing. All boundary conditions may be specified over rows of

nodal points. The mesh generation instructions and most boundary conditions are saved, and on subsequent cases, only changes to the previous information is input to define a different mesh.

This mesher is an elaboration of one written by Perry Stern of Lockheed Palo Alto Research Laboratory and originally adapted to SAAS at Sandia Laboratories.

The first step in the mesh generation process is to define and number all of the external and internal material boundaries of the two-dimensional region. These line segments may be defined as straight lines, circular, circi-linear curves, or spline curves. Additional spline curves may be defined as being a constant distance from a previously defined reference spline. (Offsets from other curve types are handled in the nodal point definitions, see below.)

The second step in the generation process is to establish a mapping from a logical or I-J index space to the real r-z space. In this mapping, straight lines between I-J nodes are associated with the previously defined line segments. The nodes can be uniformly spaced along the curve, or they can be constrained to lie on the curve with their actual location selected by the mesher. In addition, a row of nodes may be offset by a constant distance from a given nonspline type reference curve, greatly simplifying the meshing of shell geometries.

The material type for each element is initialized to 1. Changes to other material numbers can be made for rectangular blocks of elements (in the I-J space) by giving the indices of diagonally opposed corner nodes on a material block card.

Triangular elements may be specified either in the interior or on the boundary. In the I-J space, a square or diagonal row of squares is split into one or two triangular elements, depending on whether it is an internal or a boundary element. With the currently used nodal point

numbering scheme, there is little advantage to triangular elements and their use is not recommended.

Given the above information, the mesher can generate the geometrical information needed by TSAAS. The final location of all unspecified interior nodes and constrained nodes are determined by an iterative process which approximates the construction of a conformal mapping from the I-J space to the r-z space, subject to the given constraints.

Figures 5-8 show an example mesh. Normally a drawing or sketch of the material boundaries of the region to be meshed, as shown in Fig. 6, is the starting point. An estimate of the required number of elements in the mesh is made, and a sketch of the I-J logical mesh is made as shown in Fig. 5. All the lines should be numbered except lines offset from linear, circular or circi-linear curves. Thus, the external curve is to be a spline curve and a reference for spline curve 5; however, only one of the inner circular curves is numbered and the other is specified as offset from it on the nodal point definition card. Figures 7 and 8 show the card deck and resulting mesh. Other examples are given in the test problems in Appendix C.

To generate one dimensional nodal point and element numbers in accordance with the requirements of TSAAS, nodal points are assigned sequential numbers starting with J = 1 and going in the increasing I-direction, then numbering the J = 2 row, and so forth. This means that the bandwidth of the equations in TSAAS is determined by the width of the mesh in the I-direction. The range of J values is limited to 200 by the mesher, however, this can be readily increased by changing some array dimensions in the program. The more realistic limitation is 1600 total nodal points, the limit for TSAAS.

Because the mesher automatically fills in the interior of any specified boundary, it is not possible to create a multiply connected mesh. Interior voids must be included in the mesh and then assigned a

Fig. 5. I-J representation of the example, a thick-walled composite vessel.

Fig. 6. Material boundaries in r - z coordinates. Circles on outer boundary indicate spline knots.

material with properties approximating those of a void. Also, the filling-in is done by assuming the existence of nodes from the smallest value of I to the largest I, for each J that appears on a nodal point definition card (but not material block or other cards). This implies that indentations in the logical mesh in the J-direction must be handled like interior voids as mentioned above.

Mechanical and thermal boundary conditions can be assigned over rows or blocks of nodal points in the I-J index space. For proper operation, pressure shear, flux and convective conditions should be applied to rows of nodal points. Nodal point constraints and temperature conditions can be applied to blocks of nodal points as required. The mechanical conditions and initial thermal conditions are saved and can be reused in multiple case problems. The conditions following the TSAAS control card, that give time varying boundary conditions, are not saved and must be input for each case.

Concave indentations in a region can result in meshes with distinct nodal points occupying the same location or even negative area elements. Although this problem is aggravated by the use of the sliding boundary node option, it is basically caused by the iterative procedure used to locate unconstrained interior points. A nodal point is located at the average location of its four neighbors in the I and J-direction. In a concave region, if the node spacing is too great, this average location may fall outside the desired boundary. To fix this problem, either more nodes must be used in the region or else additional constraints must be applied to the interior nodes.

B. Skew Boundaries

Mixed constraints can be specified in a sloping direction, for example, as required by a sliding sloping boundary. To use this option the value of CODE, usually set to 0, 1, 2, or 3, should be set to a negative value, giving the slope in degrees. When this is done the constraint values are

interpreted as follows:

XR is the specified load in the 1-direction.

XZ is the specified displacement in the 2-direction.

Furthermore, the displacements printed by the program are given with respect to the rotated frame, as follows:

U_r is the displacement in the 1-direction.

U_z is the displacement in the 2-direction

The stresses and strains are calculated in the r and z-directions, as usual. (See Example 1.)

C. Orthotropic Material Properties

Material properties versus temperature are input in tabular form for each material. The properties for each element in the system are then evaluated by interpolation according to the temperature of the element. The mass density of a material is required. The coefficients of thermal expansion are required only for thermal stress analysis. Finally, the yield stresses are required only if nonlinear material properties are specified.

The orthotropic mechanical properties are specified in the principal N-S directions of the material in terms of the definitions displayed in the following stress-strain relation:

$$\begin{bmatrix} \epsilon_{NN} \\ \epsilon_{SS} \\ \epsilon_{\theta\theta} \\ \epsilon_{NS} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_N} & -\frac{\nu_{SN}}{E_S} & -\frac{\nu_{\theta N}}{E_\theta} & 0 \\ -\frac{\nu_{NS}}{E_N} & \frac{1}{E_S} & -\frac{\nu_{\theta S}}{E_\theta} & 0 \\ -\frac{\nu_{N\theta}}{E_N} & -\frac{\nu_{S\theta}}{E_S} & \frac{1}{E_\theta} & 0 \\ 0 & 0 & 0 & \frac{1}{G_{NS}} \end{bmatrix} \begin{bmatrix} \sigma_{NN} \\ \sigma_{SS} \\ \sigma_{\theta\theta} \\ \sigma_{NS} \end{bmatrix} + T \begin{bmatrix} \alpha_N \\ \alpha_S \\ \alpha_\theta \\ 0 \end{bmatrix} \quad (71)$$

The above matrix is symmetrical; therefore,

$$E_i \nu_{ji} = E_j \nu_{ji}, \text{ see Ref. 20.}$$

The orthotropic thermal conductivity matrix is specified in the principal N-S directions of the material as

$$\begin{bmatrix} k_N & 0 \\ 0 & k_S \end{bmatrix} \quad (72)$$

The thermal conductivity matrix and the material heat capacity are also input in tabular form as a function of temperature.

D. Transient Thermal Loading Capability

A practical problem of thermal stress analysis is the situation where a body is subjected to a time-dependent temperature or Newton cooling (see III. C) boundary condition. Practically, this situation is simulated in so-called environmental tests. One can model this situation using TSAAS by two different methods. In the first method, new boundary condition cards (flux, temperature, or Newton cooling) can be read in before each entry into the transient thermal calculation subroutine; these boundary conditions are then taken to be constant over the time interval until exit from the subroutine. In the second method, a set of boundary condition cards is read in, and N (≤ 100) sets of multiplicative thermal factors and times are read in. During any time interval linear interpolation is used to determine the ambient air temperature (Newton cooling) or the surface temperature. This method provides for a continuous variation in temperature boundary conditions. The method is shown schematically in Fig. 9.

E. Plane Strain and Plane Stress Options

TSAAS has been modified to carry out the thermal stress analysis of plane two dimensional solids. The changes to TSAAS include modification of the incremental volume dV and incremental area dS occurring in such equations as (1)-(3), (9), (12) etc., in Section II as well as (53), (54), and (57) of Section III and changes in the elastic constitutive matrix Eq. (71) discussed in Section IV. C. For the plane stress case the matrix corresponding to Eq. (71) is

$$\begin{bmatrix} \epsilon_{NN} \\ \epsilon_{SS} \\ \epsilon_{NS} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_N} & -\frac{\nu_{SN}}{E_S} & 0 \\ -\frac{\nu_{NS}}{E_N} & \frac{1}{E_S} & 0 \\ 0 & 0 & \frac{1}{\sigma_{NS}} \end{bmatrix} \begin{bmatrix} \sigma_{NN} \\ \sigma_{SS} \\ \sigma_{NS} \end{bmatrix} \quad (73)$$

while for the plane strain case TSAAS uses the constitutive matrix described by Eq. (71) and sets the hoop strain to zero. In both cases the conductivity matrix Eq. (72) is unchanged. The remaining modifications to TSAAS consist of changes to heading formats. All of these changes are accomplished by an update deck.

Fig. 9. Specification of continuously varying ambient air temperature time - thermal load factors (t_i, f_i) , $i = 1, 2, \dots, N$ and initial temperature T_0 .

V. NONLINEAR MATERIAL PROPERTIES

In this section, an attempt is made to define the behavior of orthotropic and isotropic materials which have a bilinear effective stress-effective strain curve. This work should not be construed as being anything more than a rough approximation since practical orthotropic materials may or may not be representable by the effective stress function used here. Accordingly, this theory should be applied with considerable reservation when orthotropic materials are treated. However, for isotropic materials, the effective stress function is well founded (it is related to the von Mises yield criterion), and can be used with confidence. This section concludes with a discussion of creep analysis using the initial strain method.

A method of successive approximation is used to solve for displacements, stresses, and strains in bodies with nonlinear material properties. Basically, this involves the repeated solution of the following equation for the displacements of the system:

$$\{Q\}_{i-1} = [K]_{i-1} \{u\}_i$$

where

$[K]_{i-1}$ = an estimate of the effective stiffness of the system based on the previous solution ($i - 1$)

$\{Q\}_{i-1}$ = an estimate of the loads acting on the system (since the thermal loads are a function of the stiffness)

$\{u\}_i$ = the displacements of the system for the i th approximation.

The load and stiffness used in the first approximation ($i = 1$) are based on the initial linear material properties. Since deformations are assumed to be small, the development of the effective stiffness depends only on the estimation of an effective stress-strain relationship for each element in the system.

It is apparent that this approach has certain disadvantages. First, the procedure is not guaranteed to converge. However, experience has indicated that, for systems where the nonlinear effects

are small compared to the initial linear analysis, the procedure does converge. Second, to obtain a unique solution, the method is restricted to elastic materials, in other words, materials with single-valued stress-strain relationships. Incremental plasticity is not approximated by this method and the plasticity capability of TSAAS corresponds to the so-called deformation theory of plasticity. Thus, proportional loading problems are done correctly, however, unloading is not handled at all; see Ref. 21.

A. Plastic Materials

The von Mises yield condition for isotropic material is given by

$$\sigma_y = \frac{1}{\sqrt{2}} \sqrt{(\bar{\sigma}_1 - \bar{\sigma}_2)^2 + (\bar{\sigma}_1 - \bar{\sigma}_3)^2 + (\bar{\sigma}_2 - \bar{\sigma}_3)^2} \quad (74)$$

where $\bar{\sigma}_1$, $\bar{\sigma}_2$, and $\bar{\sigma}_3$ are the principal stresses, and σ_y is the uniaxial yield stress. Equation (74) can be rewritten in normalized form:

$$1 = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{\bar{\sigma}_1}{\sigma_y} - \frac{\bar{\sigma}_2}{\sigma_y}\right)^2 + \left(\frac{\bar{\sigma}_1}{\sigma_y} - \frac{\bar{\sigma}_3}{\sigma_y}\right)^2 + \left(\frac{\bar{\sigma}_2}{\sigma_y} - \frac{\bar{\sigma}_3}{\sigma_y}\right)^2} \quad (75)$$

For orthotropic materials, Eq. (75) is modified without justification to

$$1 = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{\bar{\sigma}_1}{\sigma_{1y}} - \frac{\bar{\sigma}_2}{\sigma_{2y}}\right)^2 + \left(\frac{\bar{\sigma}_1}{\sigma_{1y}} - \frac{\bar{\sigma}_3}{\sigma_{3y}}\right)^2 + \left(\frac{\bar{\sigma}_2}{\sigma_{2y}} - \frac{\bar{\sigma}_3}{\sigma_{3y}}\right)^2} \quad (76)$$

where σ_{1y} , σ_{2y} , and σ_{3y} are the yield stresses in the principal directions.

Now, for all values of stress, the "normalized effective stress" for orthotropic materials is defined as

$$\sigma = \frac{1}{\sqrt{2}} \sqrt{\left(\frac{\sigma_1}{\sigma_{1y}} - \frac{\sigma_2}{\sigma_{2y}}\right)^2 + \left(\frac{\sigma_1}{\sigma_{1y}} - \frac{\sigma_3}{\sigma_{3y}}\right)^2 + \left(\frac{\sigma_2}{\sigma_{2y}} - \frac{\sigma_3}{\sigma_{3y}}\right)^2} \quad (77)$$

The normalized effective stress-strain relationship is shown in Fig. 10. An approximate consideration of strain hardening can be included in

the formulation as shown in the figure. The solution procedure for the nonlinear plasticity is as follows:

1. After each approximate solution, the normalized effective stress is calculated for each element (linear properties are used in the first approximation).
2. The ratio of nonlinear properties to linear properties for each element for a given approximation i is defined by

$$R_i = \frac{\sigma_i}{e_i} \quad (78)$$

Therefore, the corresponding normalized effective strain is

$$e_i = \frac{\sigma_i}{R_i} \quad (79)$$

(R_i is set equal to 1 for the first approximation.)

3. If the strains are assumed not to change, an estimation of the plasticity ratio is

$$R_{i+1} = \frac{1 + n(e_i - 1)}{e_i} \quad (80)$$

4. In the next approximation, the linear properties for E_N , E_θ , E_S , and G_{NS} are multiplied by the ratio in Eq. (80) before they are used. The Poisson's ratios are changed according to the following relation:

$$\nu_{\alpha\beta_{i+1}} = 1/2 - (1/2 - \nu_{\alpha\beta_i}) * R_{i+1} \quad (81)$$

Fig. 10. Effective stress-strain relationship.

The procedure must be repeated until convergence is obtained. The displacements, stresses, and strains are not printed by the program after each approximation. The number of approximations required will depend on the specific problem and must be specified as computer input. Hence, a certain amount of experience is required in using the nonlinear option. An arbitrary number of approximations can be requested, but the program will stop when R_{i+1} for all elements changes by 0.5 percent or less.

B. Hardening or Locking Materials

In TSAAS, nonlinear hardening or locking material behavior can be simulated by using a plasticity ratio, greater than one. The material properties E_N , E_S , E_θ , and G_{NS} are multiplied at each iteration by the new plasticity ratios whereas the Poisson's ratios are unchanged [i.e., Eq. (81) is suppressed].

C. Limited Tension Materials

Limited tension type material behavior may be simulated in TSAAS by using a negative plasticity ratio. If the stress in the β direction, σ_N , exceeds the specified yield stress in that direction, σ_{ly} , then the modulus E_N is reduced by dividing by the absolute value of the elastic plastic ratio. For example, an elastic plastic ratio of -1000., results in a modulus of $E_N = E_{N0}/1000.$ after σ_N exceeds the yield stress. Note that the mesh generator option to automatically calculate β angles during element generation simplifies the use of this option.

D. Creep Behavior

TSAAS provides for an initial strain distribution, which can be caused, for example, by differential thermal expansion. Thus, TSAAS is easily adapted to the determination of stresses and deformation in materials exhibiting creep behavior where the creep strain-rates are determined from the stress, temperature, and time. In particular, TSAAS uses a time-hardening creep law of the form

$$\dot{\epsilon}_e = \sum_{i=1}^M A_i \sigma_e^{m_i} \varphi^{n_i} t^{r_i} \quad (82)$$

where σ_e is the effective stress [given by Eq. (74), for example], φ is the temperature, and t is time. The components of the element creep strain-rates are then calculated by means of

$$\begin{aligned} \dot{\epsilon}_r^c &= \frac{\dot{\epsilon}_e}{\sigma_e} [\sigma_r - \nu_c (\sigma_\theta + \sigma_z)] \\ \dot{\epsilon}_\theta^c &= \frac{\dot{\epsilon}_e}{\sigma_e} [\sigma_\theta - \nu_c (\sigma_r + \sigma_z)] \\ \dot{\epsilon}_z^c &= \frac{\dot{\epsilon}_e}{\sigma_e} [\sigma_z - \nu_c (\sigma_r + \sigma_\theta)] \\ \dot{\epsilon}_{rz}^c &= \frac{2\dot{\epsilon}_e}{\sigma_e} (1 + \nu_c) \sigma_{rz} \end{aligned} \quad (83)$$

where ν_c is the effective Poisson's ratio in creep.

The creep version of TSAAS (called CSAAS) combines the stress and thermal analysis programs of TSAAS with the element creep Eqs. (82) and (83) into one program. The element creep strains that occur during any time interval are added as initial strains to the load vector (see Eqs. (14) and (27) and elastic re-solution is performed. CSAAS adjusts the time intervals so that both temperatures and stresses vary only within prescribed limits over the given time interval, otherwise a halving or doubling of the time interval takes place. Long increments in time indicate that steady creep is occurring.

These modifications to TSAAS are carried out by use of an update deck.

APPENDIX A COMPUTER PROGRAM INPUT INSTRUCTIONS

The first step in the analysis of an axisymmetric solid is to select a finite element representation of the two-dimensional cross section of the solid. The definition of positive directions of the r-z coordinate system is shown in Fig. 1 in the main text. The I-J coordinate system used by the mesher must have the same orientation as the r-z system, in order to generate elements with positive areas. The cards described below define the problem.

If stress calculation alone is desired (i.e., a 1 in Column 3 of the problem control card) then the setup deck is described in Appendix A I. For transient thermal analysis (with or without accompanying stress analysis) the problem setup deck must also include the cards described in Appendix A II. After each data case, a card with END OF CASE beginning in Column 1 must appear. After the last data case, a card with END OF DATA must appear (after END OF CASE).

Plotting control cards are described in Appendix A III.

I. PROBLEM DECK - SAAS

NUMBER OF CASES CARD

Format (I5)

Cols. 1-5	The number of cases to be run.
-----------	--------------------------------

TITLE CARD

Format (8A10)

Col.	Case Title
------	------------

CONTROL CARD

Format (20I1, 6I5, 3F10.0)

Cols. 1-20	Output option switches. A 1 in the corresponding column enables the option, a 0 suppresses the option.
------------	--

Card Column	Parameter	
1	ISTART	Start after mesh description using previous mesh.
2	IMESH	Generate mesh.
3	ISAAS	Stress calculation only.
4	NUMTC	Material temperature initialization. Set to 0 to set all temperatures to TREF, set to 1 to initialize by material, see p. 26.
5-6	5-6	Not used.
7	IPT2	Write calculation results on Tape 11 — for material boundary plots, contour plots, or edited stress and strain listings produced by SAASPL.
8	IPT3	Suppress printing of mesh generation information.
9	IPT4	Suppress printing of nodal point listing.
10	IPT5	Suppress printing of element listing.
11	IPT6	Suppress printing of displacement.
12	IPT7	Suppress printing of stresses.
13	IPT8	Suppress printing of strains.
14	IPT9	Print (nonsuppressed items) after every iteration, and dump results on Tape 11 every iteration if enabled.

41-45	NSB	Number of shear boundary conditions.
46-50	NQB	Number of flux condition card (initial values).
51-55	NCB	Number of convective condition cards (initial values).
56-60	NTB	Number of temperature boundary condition cards (initial values).

*Note: On multiple case problems when MESHOP = 1, the numbers in Cols. 11-60 refer to the number of additional line segments or description cards to be added to the previous mesh definition. In all cases, contradictory definitions result in the last definition being used. The main limitation is that the logical I-J mesh size cannot be reduced and triangles can not be deleted from a mesh.

LINE SEGMENT CARDS

These cards define the various basic curves required to define the boundaries and fixed nodal point locations required in a mesh. If an object has constant thickness shells in it, then only one shell surface need be defined in detail; the other may be specified as a constant thickness away from the basic surface.

Format (2I5, 6E10.0)

Cols. 1-5	IN	Curve index number.
6-10	IT	Type of space curve (see definition below). Make negative to indicate values are R- θ (in degrees) polar coordinates, but do not use with special curves.
11-20	R1	R-coordinate of first point (if both R1 and Z1 are zero, then end point of previous line segment is used).

21-30	Z1	Z- or θ -coordinate (If both R1 and Z1 are zero, then end point of previous line segment is used.)
31-40	R2	R-coordinate of second point.
41-50	Z2	Z- or θ -coordinate.
51-60	R3	R-coordinate of third point.
61-70	Z3	Z- or θ -coordinate.

The curve type number in Cols. 6-10 can take the following values:

- IT = 1 straight line, input first and second points only.
- IT = 2 circle specified by first and second points at the ends of the arc and the third point at some intermediate point (e.g., the mid-point) of the arc.
- IT = 3 circle specified by first and second points at the ends of the arc, and the third point at the center of the circle. The arc runs counter clockwise from 1 to 2.
- IT = 4 as in 3, except arc runs clockwise from 1 to 2.
- IT = 5 circle specified by radius (R1), angle in degrees from R-axis about center to first end (Z1), angle to second end (R2), and center coordinate Z2 and R3. Arc is counter clockwise from 1 to 2. Do not use negative type number.
- IT = 6 as in 5, except that arc runs clockwise from 1 to 2.
- IT = 7 Not used.
- IT = 8 Not used.
- IT = 9 a spline curve parallel to a previously defined spline curve. For this particular case the line segment card format is:

Cols. 1-5	IN	Curve index number.
6-10	IT	9
11-20	R1	Index of reference spline curve.

21-30 Z1 Normal offset from reference spline curve. Positive if to right of reference, negative if to left, as seen looking from first point to last point.

IT = 10 piecewise circilinear curve. That is, a curve composed of a sequence of straight lines and circular arcs. On this card R1 (Cols. 11-20) designates the number of lines and arcs in the curve. The lines and arcs are defined on R1 number of following continuation cards. The continuation cards follow the same format as used for the line segment types 1-6 with two exceptions. First, the curve index number (IN) is ignored, and secondly the coordinates of the first point (R1 and Z1) are not used except on the first continuation card. Thus, the first point of every straight line or circular arc (except for the first one) is automatically defined as the second point of the previous line or arc.

IT = 11 spline curve — in this case R1 contains the number of points (NUMSG) defining the curve. The points are given on succeeding continuation cards. A continuation card for spline definition contains either one or two points as follows:

Format (I5, 5X, 6E10.0)

Cols. 1-5		Blank or zero.
11-20	R1	R-coordinate of first point.
21-30	Z1	Z- or θ -coordinate.
31-40	R2	Slope of curve (in degrees) at this point. A value is required at the first and last point. For intermediate points, a zero value (or blank) can be given and a cubic spline curve will be generated.

41-50	Z2	Same as column 11-40
51-60	R3	except define next point.
61-70	Z3	If <u>all</u> values are zero, then these fields are ignored.

Note that NUMSG specifies the number of points to be read not the number of cards on which points are defined.

NODAL POINT DEFINITION CARDS

These associate lines of nodal points, in the logical I-J space, with actual R-Z locations as specified by the preceding space curve cards. The line of nodal points may be specified as being uniformly spaced along the curve (this is only approximate for the spline curves) or can simply be constrained to have all nodal points lie along the curve with the first and last nodal points located at the ends of the curve. At least enough cards must be used to define the outer boundary of the region. Interior nodal points are normally constrained only along material boundaries.

Format (4E5.0, 2I5, 2E10.0)

Cols. 1-5	I1	I-coordinate of first nodal point.	} If both are zero, then I, J, coordinate of last point (Col. 11-20) from previous card are used. Initialized to (1, 1).
6-10	J1	J-coordinate.	

11-15	I2	I-coordinate of last nodal point.
16-20	J2	J-coordinate.
21-25	IM	Space curve index number. If not positive, then fixes one node (I1, J1).
26-30	IC	Constraint type number (see below).

- 31-40 T Normal offset from reference space curve. Positive if to left as seen looking from first point to last point. If IM = 0 use as R-coordinate of I1, J1.
- 41-50 If IM = 0 use as Z-coordinate of I1, J1.

Constraint type number—the number in Cols. 26-30 can be:

- IC = 1 then the nodal points are uniformly spaced along the curve.
- IC = 2 the nodal points are constrained to lie along the curve.

MATERIAL BLOCK ASSIGNMENT CARDS

Each card assigns a material definition number to a block of elements defined by the I-J coordinates. The default assignment is to material 1 with $\beta = 0$. The angle β , defined by Fig. 3, if specified as -1, is set to the average I-direction through the element. (See Fig. A-1.)

Cols. 1-5	Material definition number.
6-10	Minimum I.
11-15	Maximum I.
16-20	Minimum J.
21-25	Maximum J.
26-30	Reserved.
31-40	Material property angle β , in degrees $0 \leq \beta \leq 360$. (If $\beta = -1$ then mesher sets β to the average I-direction in element.)

Fig. A-1. Automatic definition of β .

TRIANGLE DEFINITION CARDS

A diagonal line in the I-J space is specified by giving a nodal point and a count of elements to be split. This creates one triangular element for each partly defined boundary quadrilateral or two triangular elements for a fully defined quadrilateral. (See examples.)

Format (3I5)

Cols. 1-5	I-coordinate of nodal point.
6-10	J-coordinate of nodal point.
11-15	Count of number of logical elements to be split — <u>positive</u> if line goes upward and rightward, <u>negative</u> if upward and leftward.
- Example A: Circles show previously defined nodal points.

Card image: _ _ _ _ 1 _ _ _ 1 _ _ _ 1

Example B:

Card image: _ _ _ _ 4 _ _ _ 2 _ _ _ -3

NODAL POINT CONSTRAINT CARDS

Format (4E5.0, 3F10.0)

Cols.	1-5	I1	I-coordinate of first point.
	6-10	I2	I-coordinate of last point (if zero set equal to I1).
	11-15	J1	J-coordinate of first point.
	16-20	J2	J-coordinate of last point (if zero set equal to I2).
	21-30	CODE	Boundary condition code (see below).
	31-40	XR	Radial boundary condition.
	41-50	XZ	Axial boundary condition.

The boundary condition codes are

CODE = 0	{	XR is the specified R-load and XZ is the specified Z-load.
CODE = 1	{	XR is the specified R-displacement and XZ is the specified Z-load.
CODE = 2	{	XR is the specified R-load and XZ is the specified Z-displacement.
CODE = 3	{	XR is the specified R displacement and XZ is the specified Z displacement.

All loads are total forces acting on a one radian segment. If CODE is other than 0, 1, 2, or 3, it is taken as the angle of a sloping boundary restraint, as discussed in Section IV. B.

PRESSURE SHEAR, FLUX, CONVECTIVE, AND TEMPERATURE BOUNDARY CONDITIONS

Format (4E5.0, E10.0)

Cols.	1-5	I1	First I.
	6-10	I2	Last I (if zero, assumed equal to I1).
	11-15	J1	First J.
	16-20	J2	Last J (if zero, assumed equal to J1).
	21-30		Pressure, shear, flux or temperature value. Not used for convective condition.

Note: Pressure, shear, and flux conditions are assumed to apply to the element(s) to the

left of the nodal points, when going from the first nodal point to the last nodal point.

ELEMENT CARDS

These cards are not included if a mesh is generated.

One card is required for each element in numerical sequence.

Format (6I5, F10.0)

Cols.	1-5	Element number.	1. Order nodal points counter clock-wise (see Fig. 1 of main text) around element.
	6-10	Nodal point I.	}
	11-15	Nodal point J.	
	16-20	Nodal point K.	
	21-25	Nodal point L.	
	26-30	Material identification.	2. Maximum difference between nodal point numbers must be less than 40.
	31-40	Material angle β - in degrees.	

If element cards are omitted, the program generates the omitted elements by incrementing by 1 the preceding I, J, K, and L. The material identification code for the generated elements is the value specified on the first nongenerated card. The last element card must be supplied.

Triangular elements are specified by repeating the last nodal point number, i.e., I, J, K, K.

NODAL POINT CARDS

One card is required for each nodal point in numerical sequence. These cards are not included if a mesh is generated.

Format (I5, F5.0, 5F10.0)

Cols.	1-5	Nodal point number.
	6-10	Boundary condition code.
	11-20	R-coordinate.
	21-30	Z-coordinate.
	31-40	XR.
	41-50	XZ.
	51-60	Temperature (if not given on temperature cards or on tape).

If the number in Cols. 6-10 is

0. $\left\{ \begin{array}{l} \text{XR is the specified R-load and} \\ \text{XZ is the specified Z-load.} \end{array} \right.$
1. $\left\{ \begin{array}{l} \text{XR is the specified R-displacement and} \\ \text{XZ is the specified Z-load.} \end{array} \right.$
2. $\left\{ \begin{array}{l} \text{XR is the specified R-load and} \\ \text{XZ is the specified Z-displacement.} \end{array} \right.$
3. $\left\{ \begin{array}{l} \text{XR is the specified R-displacement and} \\ \text{XZ is the specified Z-displacement.} \end{array} \right.$

All loads are total forces acting on a one-radian segment.

If nodal point cards are omitted, nodal points will be generated at equal intervals along a straight line between the defined nodal points. The boundary condition code is then 0, along with XR and XZ. The temperatures are linearly interpolated from the defined temperatures.

TEMPERATURE INITIALIZATION BY MATERIAL

If material temperature initialization is specified (i. e., a 1 in Col. 4 of the problem control card) then a constant value of temperature is given on a single card and all nodes are initialized to this temperature. In addition, NMAT material blocks can be initialized to different temperatures.

Format (F10.0, I10)

First card

- Cols. 1-10 Initial temperature T_0
- 11-20 Number of material blocks, NMAT, initialized differently from T_0 .

Next NMAT cards

- Cols. 1-10 Temperature.
- 11-20 Material number.

Material Property Information

The following group of cards must be specified for each of a maximum of twenty materials.

MATERIAL IDENTIFICATION CARD

Format (2I5, 2F10.0)

- Cols. 1-5 Material identification number.
- 6-10 Number of temperatures for which properties are given (six maximum).
- 11-20 Mass density of material (if required).
- 21-30 Thermal expansion parameter (if 1, free thermal expansions are on the material property cards. Otherwise, coefficients of thermal expansion are on the material property cards.)

MECHANICAL PROPERTY CARDS*

Two cards are required for each temperature. Cards are to be in order of ascending temperature.

First card

Format (8F10.0)

- Cols. 1-10 Temperature
- 11-20 Modulus of elasticity, E_N .
- 21-30 Modulus of elasticity, E_S^{**} .
- 31-40 Modulus of elasticity, E_θ^{**} .
- 41-50 Poisson's ratio ν_{SN} .
- 51-60 Poisson's ratio, $\nu_{\theta N}^{**}$.
- 61-70 Poisson's ratio, $\nu_{\theta S}^{**}$.
- 71-80 Shear modulus, G_{NS}^{**} .
- (For an isotropic elastic material $E_N = E_S = E_\theta = E$,

$$\nu_{SN} = \nu_{\theta N} = \nu_{\theta S} = \nu \text{ and } G = \frac{E}{2(1 + \nu)}$$

Second card

Format (7F10.0)

- Cols. 1-10 Free thermal strain, $\alpha_N T$, or Coefficient of thermal expansion, α_N
- 11-20 Free thermal strain, $\alpha_S T$, or Coefficient of thermal expansion, α_S^{**} .

- 21-30 Free thermal strain, $\alpha_T T$, or coefficient of thermal expansion, α_T^{**} .
- 31-40 Yield stress, σ_N , or yield strain, ϵ_N , is greater than one.
- 41-50 Yield stress, σ_S , or yield strain, ϵ_S , is greater than one**.
- 51-60 Yield stress, σ_T , or yield strain, ϵ_T , is greater than one.
- 61-70 Ratio of effective plastic to elastic modulus, PEMR.
- This number is less than one for plastic materials and greater than one for locking materials. Negative for no-tension materials.

*In general, for an orthotropic material there are nine independent material constants, but due to symmetry conditions (axisymmetric body and loading) there are only seven independent constants — the input for first card described above. Note that the Poisson's ratios are defined by the following symmetry expression,

$$E_i \nu_{ji} = E_j \nu_{ij}$$

If the material is transversely isotropic with the N- θ plane being the isotropic plane then

$$E_N = \nu_{N\theta} E_\theta / \nu_{\theta N}, \quad \nu_{S\theta} = \nu_{NS} \frac{E_S}{E_\theta}$$

i.e., there are only five independent constants. Furthermore, [20],

$$|\nu_{SN}| < \left(\frac{E_S}{E_N} \right)$$

$$-1 < \nu_{N\theta} < 1 - 2 \left(\frac{E_N}{E_S} \right) \nu_{SN}^2$$

**If left blank then the value is automatically set to the isotropic value.

THERMAL PROPERTY CARDS

One card required for each temperature. Each card follows the two mechanical property cards. If a one appears in Col. 3 of the problem control card (i.e., stress calculation only) then the thermal property cards are omitted.

Format (7F10.0)

Cols. 1-10 Thermal conductivity in N-direction.

11-20 Thermal conductivity in S-direction**.

21-30 Heat capacity.

31-40 Internal heat generation rate per unit mass. If a chemically reactive material, this is QZ.

41-50 For chemically reactive materials, the activation energy divided by gas constant $\frac{E}{R}$. Use zero for nonreacting materials.

51-60 Maximum temperature (TMAX) change before recomputing conductivity and heat capacity matrices.

BOUNDARY PRESSURE CARDS

Used only if conditions are not input via automatic mesher cards. One card is required for each boundary element which is subjected to a normal pressure.

Format (2I5, F10.0)

Cols. 1-5 Nodal point I.

6-10 Nodal point J.

11-20 Normal pressure.

As shown in the sketch below, the boundary element must be on the left when progressing from I to J to obtain the correct sense of the pressure. Surface normal tensile force is input as a negative pressure.

BOUNDARY SHEAR CARDS

Used only if conditions are not input via automatic mesher cards. One card is required for each boundary element which is subjected to surface shear.

Format (2I5, F10.0)

Cols. 1-5 Nodal point I.
6-10 Nodal point J.
11-20 Surface shear.

As shown in the sketch below, the boundary element must be on the left when progressing from I to J. The positive sense of the shear is from I to J.

II. PROBLEM DECK - TSAAS

TSAAS requires the control cards, mesh description cards, and material property cards described above, together with the following additional cards:

TSAAS CONTROL CARD

Format (5I1, 5X, 6F10.0)

Cols. 1	IPT11	If 1 uses variable time step.
2	IPT12	If 1 uses thermal properties varying with temperature.
3	IPT13	If 1 all thermal loads are read from cards; otherwise uses load capability described in Section IV.D.
4	IPT14	If 1 determines ambient air temperature by subroutine.
5	IPT15	If 1 Flux cards have I-J indices.
6	IPT16	If 1 Convective cards have I-J indices.
7	IPT17	If 1 Temperature cards have I-J indices.
8-10		Not used.
11-20	DT	Starting value of time increment.
21-30	DTMIN	Minimum value of time increment.
31-40	TDMIN	Minimum nodal point temperature change per calculational cycle.
41-50	TDMAX	Maximum nodal point temperature change per calculational cycle.
51-60	TDEXP	Ignition temperature change per calculational cycle.

61-70 TZERO Zero offset for absolute temperature for reactive kinetics.

INCREMENT CONTROL

Format (15) NINC This card specifies the number of times the transient thermal subroutine TEM1 is entered. NINC can be at most 100 and NINC-1 entries into TEM1 take place.

OUTPUT CONTROL

Format (16I5) NOUT(I) This card (or cards) sets up the NOUT(I), $I = 1$, NINC array. Upon exit from TEM1 at increment I, a stress analysis is carried out using the temperature field calculated in TEM1 if NOUT (I + 1) = 1. Otherwise re-entry into TEM1 takes place. NOUT(1) controls whether a stress analysis is carried out for the initial temperature field.

THERMAL LOAD CONTROL

Format (I10, 2F10.0) NINC cards occur here only if a 1 does not occur in Col. 3 of the TSAAS control card.

Cols. 1-5 NUMQC Number of flux boundary cards, negative if old flux boundary conditions are used.

6-10 NUMSCC Number of convective boundary cards, negative if old convective boundary conditions are used.

11-15 NUMTB Number of temperature boundary cards, negative if old temperature boundary conditions are used.

16-20 NCYCLE Number of calculation cycles per print cycle.

20-30 FC Coefficient of surface heat transfer.

31-40 ATEM Ambient air temperature.

41-50 TIMMAX Time at which exit from TEM1 takes place. If equal to 0.0 only the equilibrium state is calculated.

Time-dependent boundary conditions can be given to TSAAS using either the TSAAS nodal point numbering scheme or the logical I-J numbering method used by the automatic mesher. If the TSAAS numbering is used, then one card is required for every element or nodal point subjected to the given constraint. Using the I-J logical numbering constraints can be placed on rows of nodal points with one card. The first series of card definitions below use the TSAAS numbering while the second series assume logical numbering. The selection of card format is controlled by the setting of Cols. 5-7 on the TSAAS control card, set to 0 for TSAAS numbering or 1 for logical numbering.

The following three formats are the boundary condition card formats for TSAAS nodal point numbering reference:

FLUX BOUNDARY CARDS*

NUMQC in number, use if Col. 5 on TSAAS control card is 0.

Format (2I5, F10.1)

Cols. 1-5 Nodal point I.
 6-10 Nodal point J.
 11-20 Flux.

CONVECTIVE BOUNDARY CARDS*

NUMSCC in number, use if Col. 6 on TSAAS
 control card is 0.

Format (2I5)

Cols. 1-5 Nodal point I.
 6-10 Nodal point J.

TEMPERATURE BOUNDARY CARDS*

NUMTB in number, use if Col. 7 on TSAAS
 control card is 0.

Format (I5, F10.0)

Cols. 1-5 Nodal point.
 6-15 Nodal point temperature (°C).

*Flux and convection are prescribed across the surface of an element, temperature is fixed on a nodal point.

The following three formats are for the boundary condition and card formats for logical I-J nodal point numbering reference:

FLUX BOUNDARY CARDS

NUMQC in number, use if Col. 5 on TSAAS
 control card is 1.

Format (4E5.0, E10.0)

Cols. 1-5 First I.
 6-10 Last I (if zero, assumed equal to min I).
 11-15 First J.
 16-20 Last J (if zero, assumed equal to min J).
 21-30 Flux value.

CONVECTIVE BOUNDARY CARDS

NUMSCC in number, use if Col. 6 on TSAAS
 control card is 1.

Format (4E5.0)

Cols. 1-20 As above.

TEMPERATURE BOUNDARY CARDS

NUMTB in number, use if Col. 7 on TSAAS
 control card is 1.

Format (4E5.0, E10.0)

Cols. 1-20 As above.

21-30 Temperature.

After the data cards for each case, an END OF CASE card must appear and after the last case, an END OF DATA card should appear.

The control cards for SAASPL, if any, immediately follow the END OF DATA card.

III. SAASPL CONTROL CARDS (optional, see Appendix B)

Format (3X, I2, 2E10.0, I5)

Cols. 1-5 Index number (1-19) of plot frame to be controlled. The frames are indexed in the plotting order, that is, temperature, r-displacement, z-displacement, stresses (σ_r , σ_z , σ_θ , σ_{rz} , $\sigma_{\max}$, $\sigma_{\min}$, σ_{avg} , $\sigma_{\text{max shear}}$) and the strain (same order as the stresses).

6-15 Minimum limit for contouring (used only if KON = 0).

16-25 Maximum limit for contouring (used only if KON = 0).

26-30 KON - control number. If negative, suppress plot. If 0, make 20 equally spaced contours between the specified minimum and maximum limits. If positive, generate contours only in the corresponding material type.

APPENDIX B

COMPUTER PROGRAM OUTPUT

In a computer program as comprehensive as the present program, it is easy to generate enough information to overwhelm the interpretive capabilities of the user. Consequently, much thought was given to the problem of making the assimilation task as easy as possible. There are two types of output: printed and plotted. The plotted output serves as a quick guide to the more extensive information which is printed by the program.

I. PRINTED OUTPUT

The following information can be printed by the program:

- a. Input data. (An image of each data card is printed prior to execution of any data case. Most input data is also printed out and described for each data case as it is executed.)
- b. Mesh generation information.
- c. Nodal point locations and interpolated temperatures.
- d. Element makeup (with respect to nodal points) and interpolated temperatures.
- e. Nodal point displacements.
- f. Stresses and mechanical strains (total strain minus free thermal strain) at the center of each element.
- g. The approximate value of the weight or mass and strain energy in each material as well as the total values.

II. PLOTTED OUTPUT: SAASPL

An associated plotting and editing program called SAASPL has been prepared. This program can produce a plot of the mesh, a plot of the material boundaries, and contour plots of the temperature field, displacements, stresses (σ_r , σ_z , σ_{θ} , σ_{rz} , $\sigma_{\max}$, $\sigma_{\min}$, σ_{avg} , $\sigma_{\text{max shear}}$), and strains as well as edited listings of the stresses and strains.

Two listings are prepared for each material type showing stress and strain components for each selected element. The elements selected

are those which have the largest positive value, the largest negative value, and the five largest absolute values of a stress component. A similar criteria is used to choose elements for the strain listing. On each of these listings there is a column marked SEL IND. There are eight octal numbers corresponding to the eight listed stresses or strains. If the number is in the range of 7 to 3, it indicates that the corresponding value is the largest down to the fifth largest in absolute value. A 2 or 1 indicates the largest positive and largest negative value, respectively, while a zero indicates that this value played no role in the selection of this element.

The primary inputs to SAASPL are the files TAPE 10 and TAPE 11 generated by SAAS (assuming the second option switch is set to 1). An optional card deck can be used to control the range values of the contour plots. This is done by making a card with the desired ranges for each plot that is to be controlled. The plots are considered to be indexed 1, 2, 3, ..., 19 in the order plotted, that is temperature, r-displacement, z-displacement, the stresses (σ_r , σ_z , σ_{θ} , σ_{rz} , $\sigma_{\max}$, $\sigma_{\min}$, σ_{avg} , $\sigma_{\text{max shear}}$), and the strains (same order as stresses). The control cards are read with a (3X, I2, 2E10.0, I5) format to obtain the index number, the minimum limit, and the maximum limit, and a control number. If the control number is 0, then the contours will be done at 20 equally spaced values between the specified limits. If the control number is negative, the frame is skipped. If the control number is positive, then the contour plotting is done only in the material with that number, with maximum and minimum limits automatically generated for that material. Plots for which there are no control cards will be automatically scaled as before.

APPENDIX C

EXAMPLE PROBLEMS

Two numerical examples are presented to illustrate the use of the various capabilities of the TSAAS program.

I. THICK SPHERICAL SHELL WITH STEADY HEAT FLOW

TSAAS results are compared with an exact solution for a thick spherical shell under a condition of steady heat flow. The shell has an internal radius of 10 and an external radius of 20; the material of the shell is elastic with an elastic modulus of 70 000, a Poisson's ratio of 0.33, and a coefficient of linear expansion of 0.0001. The thermal conductivity of the material is 0.00092. A constant flux of 0.02 is applied to the inner surface and the temperature of the external surface is fixed at 0.

The finite element solution is obtained for a wedge-shaped ring of the spherical shell so that full advantage is taken of the spherical symmetry

of the problem. A schematic diagram of the wedge-shaped ring is shown in Fig. C-1. The boundary conditions on the sloping lines of Fig. C-1 are zero shear stress and normal displacement.

The input cards for a 50-element wedge-shaped ring are shown in Fig. C-2. Note that the wedge-shaped ring must be treated by use of the skew boundaries capability described in Section IV. B. The computer output is given in Fig. C-3. Figure C-4 illustrates the results of SAASPL editing program described in Appendix B2.

Table C-1 shows the close agreement between the finite element and analytical results for the radial stress σ_R , the hoop stress σ_θ , and temperature T.

TABLE C-1

COMPARISON OF TSAAS AND EXACT VALUES

Radius	σ_R (Exact)	σ_R (TSAAS)	σ_θ (Exact)	σ_θ (TSAAS)	T (Exact)	T (TSAAS)
12.193	-173	-173	-316	-315	69.54	69.47
14.192	-173	-172	- 54	- 53	44.45	44.38
16.191	-124	-123	119	120	25.55	25.49
18.190	- 60	- 59	241	242	10.81	10.75

II. INTERNAL HEAT GENERATION BY A SPHERE IMBEDDED IN A CYLINDER WITH TEMPERATURE AND CONVECTIVE BOUNDARY CONDITIONS

This problem illustrates a time-dependent thermal stress calculation for a radioactive sphere of radius 8 embedded in an inert cylinder of radius 12 and height 48. The material (material 2) in the sphere has a heat generation rate of 0.005. The ends of the cylinder are fixed

at 0° and the side of the cylinder convects to air at 0° with a surface coefficient of heat transfer of 0.0001.

The input cards for this problem are shown in Fig. C-5. Table C-2 shows the elastic and thermal properties that were used in the analysis. Figure C-6 illustrates the finite element mesh and the thermal boundary conditions. Figure C-7 is a plot of isotherms at 5000 seconds, obtained from SAASPL, while Fig. C-8 illustrates the edited stresses at this time.

TABLE C-2

MATERIAL PROPERTY DATA FOR COMPOSITE PROBLEM

Material	Elastic Modulus	Poisson's Ratio	Coefficient of Thermal Expansion	Heat Capacity	Thermal Conductivity
1	13 000	0.3	50×10^{-6}	0.3	0.01
2	6 000 000	0.3	8×10^{-6}	0.03	0.1

Fig. C-1. Finite element mesh.

```

***** TSAAS-II DEC72 LASL-7600 VER 3 *****
      10      20      30      40      50      60      70      80
123456789012345678901234567890123456789012345678901234567890
THERMAL STRESS ANALYSIS OF A SPHERE
0101 110000000      1      1      0.
      1      0      3      0      0      2      0      0      1
      1      1      9.9778      0.6667      19.9555      1.3333      0
      2      1      10.0000      0.0000      20.0000      0.0000
      3      1      9.9778      -0.6667      19.9555      -1.3333
      1      1      1      26      1      1
      2      1      2      26      2      1
      3      1      3      26      3      1
      1      1      1      26-176.17727
      3      3      1      26      -3.82273
      1      3      1      1      .02
      1      3      26      26      0.
      1      1      2.
      1      0.      70.E3      .33
      .0001      700.
      .00092      .248      5.      1.
001
      1
      1
      2
END OF CASE

```

Fig. C-2. Data input to TSAAS.

CASE NO 1 EXECUTION STARTED AT 15.571

THERMAL STRESS ANALYSIS OF A SPHERE
OPTIONS 1=TRUE

START PARAMETER-----	2	NUMBER OF NODAL POINTS-----	-0
GENERATE MESH-----	1	NUMBER OF ELEMENTS-----	-0
STRESS CALCULATION ONLY----	2	NUMBER OF MATERIALS-----	1
BLOCK TEMP INITIALIZATION--	1	NUMBER OF PRESSURE CARDS-----	-0
MESH PLOT--(SUPPRESSED)----	-2	NUMBER OF SHEAR CARDS-----	-0
GEN. TAPE 11 FOR SAASPL----	1	NUMBER OF APPROXIMATIONS-----	1
NOPRINT MESH GEN. INFO----	1	REFERENCE TEMPERATURE-----	0.
NO PRINT NODAL PNT LISTING-	2	AXIAL ACCELERATION-----	-0.
NO PRINT ELEMENT LISTING---	2	ANGULAR VELOCITY-----	-0.
NO PRINT DISPLACEMENTS----	2		
NO PRINT STRESS LISTING----	2		
NO PRINT STRAIN LISTING----	2		
OUTPUT AFTER EVERY ITERATE-	2		
NO PRINT TEMPERATURES-----	2		

***** AUTOMATIC MESH GENERATOR -- TSAAS II VERSION, 28 JUN 73 *****

THERMAL STRESS ANALYSIS OF A SPHERE

SELECTED OPTIONS

TYPE OF ELEMENT (NSFR)	1
RESTART FROM LAST MESH (MESHOP)	0
NO. OF LINES OR CURVES (NLS)	3
NO. OF NODAL POINT DEF. CARDS (NNPD) ...	3
NO. OF MATERIAL BLOCK CARDS (NMB)	0
NO. OF TRIANGLE DEF. CARDS (NTD)	0
NO. OF NODAL PNT. CONSTRAINTS (NNCP) ...	2
NO. OF PRESSURE BOUNDARY COND. (NPB) ...	0
NO. OF SHEAR BOUNDARY COND. (NSB)	0
NO. OF FLUX CONDITION CARDS (NQB)	1
NO. OF CONVECTIVE COND. CARDS (NCB)	0
NO. OF TEMPERATURE CARDS (NTB) : : :	1

MESH DEFINITION STORAGE (IS) USED FOR THIS RUN IS 63 WORDS.

NODAL POINT COORDINATES CALCULATED AFTER 1 ITERATIONS.

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

I	J	NP	TYPE	R-ORDINATE	Z-ORDINATE	R LOAD OR DISPLACEMENT	Z LOAD OR
1	1	1	-176.2	9.977800	.666700	-0.	-0.
2	1	2	0.0	10.000000	0.000000	0.	0.
3	1	3	-3.8	9.977800	-.666700	-0.	-0.
1	2	4	-176.2	10.376908	.693364	-0.	-0.
2	2	5	0.0	10.400000	0.000000	0.	0.
3	2	6	-3.8	10.376908	-.693364	-0.	-0.
1	3	7	-176.2	10.776016	.720028	-0.	-0.
2	3	8	0.0	10.800000	0.000000	0.	0.
3	3	9	-3.8	10.776016	-.720028	-0.	-0.
1	4	10	-176.2	11.175124	.746692	-0.	-0.
2	4	11	0.0	11.200000	0.000000	0.	0.
3	4	12	-3.8	11.175124	-.746692	-0.	-0.
1	5	13	-176.2	11.574232	.773356	-0.	-0.
2	5	14	0.0	11.600000	0.000000	0.	0.
3	5	15	-3.8	11.574232	-.773356	-0.	-0.
1	6	16	-176.2	11.973340	.800020	-0.	-0.
2	6	17	0.0	12.000000	0.000000	0.	0.
3	6	18	-3.8	11.973340	-.800020	-0.	-0.
1	7	19	-176.2	12.372448	.826684	-0.	-0.
2	7	20	0.0	12.400000	0.000000	0.	0.
3	7	21	-3.8	12.372448	-.826684	-0.	-0.
1	8	22	-176.2	12.771556	.853348	-0.	-0.
2	8	23	0.0	12.800000	0.000000	0.	0.
3	8	24	-3.8	12.771556	-.853348	-0.	-0.
1	9	25	-176.2	13.170664	.880012	-0.	-0.
2	9	26	0.0	13.200000	0.000000	0.	0.
3	9	27	-3.8	13.170664	-.880012	-0.	-0.
1	10	28	-176.2	13.569772	.906676	-0.	-0.
2	10	29	0.0	13.600000	0.000000	0.	0.
3	10	30	-3.8	13.569772	-.906676	-0.	-0.
1	11	31	-176.2	13.968880	.933340	-0.	-0.
2	11	32	0.0	14.000000	0.000000	0.	0.
3	11	33	-3.8	13.968880	-.933340	-0.	-0.
1	12	34	-176.2	14.367988	.960004	-0.	-0.
2	12	35	0.0	14.400000	0.000000	0.	0.
3	12	36	-3.8	14.367988	-.960004	-0.	-0.
1	13	37	-176.2	14.767096	.986668	-0.	-0.
2	13	38	0.0	14.800000	0.000000	0.	0.
3	13	39	-3.8	14.767096	-.986668	-0.	-0.
1	14	40	-176.2	15.166204	1.013332	-0.	-0.
2	14	41	0.0	15.200000	0.000000	0.	0.
3	14	42	-3.8	15.166204	-1.013332	-0.	-0.
1	15	43	-176.2	15.565312	1.039996	-0.	-0.
2	15	44	0.0	15.600000	0.000000	0.	0.
3	15	45	-3.8	15.565312	-1.039996	-0.	-0.
1	16	46	-176.2	15.964420	1.066660	-0.	-0.
2	16	47	0.0	16.000000	0.000000	0.	0.
3	16	48	-3.8	15.964420	-1.066660	-0.	-0.
1	17	49	-176.2	16.363528	1.093324	-0.	-0.
2	17	50	0.0	16.400000	0.000000	0.	0.
3	17	51	-3.8	16.363528	-1.093324	-0.	-0.
1	18	52	-176.2	16.762636	1.119988	-0.	-0.
2	18	53	0.0	16.800000	0.000000	0.	0.
3	18	54	-3.8	16.762636	-1.119988	-0.	-0.
1	19	55	-176.2	17.161744	1.146652	-0.	-0.

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

I	J	NP	TYPE	R-ORDINATE	Z-ORDINATE	R LOAD OR DISPLACEMENT	Z LOAD OR
2	19	56	0.0	17.200000	0.000000	0.	0.
3	19	57	-3.8	17.161744	-1.146652	-0.	-0.
1	20	58	-176.2	17.560852	1.173316	-0.	-0.
2	20	59	0.0	17.600000	0.000000	0.	0.
3	20	60	-3.8	17.560852	-1.173316	-0.	-0.
1	21	61	-176.2	17.959960	1.199980	-0.	-0.
2	21	62	0.0	18.000000	0.000000	0.	0.
3	21	63	-3.8	17.959960	-1.199980	-0.	-0.
1	22	64	-176.2	18.359068	1.226644	-0.	-0.
2	22	65	0.0	18.400000	0.000000	0.	0.
3	22	66	-3.8	18.359068	-1.226644	-0.	-0.
1	23	67	-176.2	18.758176	1.253308	-0.	-0.
2	23	68	0.0	18.800000	0.000000	0.	0.
3	23	69	-3.8	18.758176	-1.253308	-0.	-0.
1	24	70	-176.2	19.157284	1.279972	-0.	-0.
2	24	71	0.0	19.200000	0.000000	0.	0.
3	24	72	-3.8	19.157284	-1.279972	-0.	-0.
1	25	73	-176.2	19.556392	1.306636	-0.	-0.
2	25	74	0.0	19.600000	0.000000	0.	0.
3	25	75	-3.8	19.556392	-1.306636	-0.	-0.
1	26	76	-176.2	19.955500	1.333300	-0.	-0.
2	26	77	0.0	20.000000	0.000000	0.	0.
3	26	78	-3.8	19.955500	-1.333300	-0.	-0.

THERMAL STRESS ANALYSIS OF A SPHERE

FLUX BOUNDARY CONDITIONS (INITIAL VALUES)

N	I	J	FLUX
1	1	2	.020
2	2	3	.020

THERMAL STRESS ANALYSIS OF A SPHERE

TEMPERATURE BOUNDARY CONDITIONS (INITIAL VALUES)

N	I	TEMP
1	76	0.0
2	77	0.0
3	78	0.0

EQUILIBRIUM TEMPERATURE----- 0.

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

EL	I	J	K	L	MATERIAL	ANGLE	TEMPERATURE
1	1	2	5	4	1	0.0	0.000
2	2	3	6	5	1	0.0	0.000
3	4	5	8	7	1	0.0	0.000
4	5	6	9	8	1	0.0	0.000
5	7	8	11	10	1	0.0	0.000
6	8	9	12	11	1	0.0	0.000
7	10	11	14	13	1	0.0	0.000
8	11	12	15	14	1	0.0	0.000
9	13	14	17	16	1	0.0	0.000
10	14	15	18	17	1	0.0	0.000
11	16	17	20	19	1	0.0	0.000
12	17	18	21	20	1	0.0	0.000
13	19	20	23	22	1	0.0	0.000
14	20	21	24	23	1	0.0	0.000
15	22	23	26	25	1	0.0	0.000
16	23	24	27	26	1	0.0	0.000
17	25	26	29	28	1	0.0	0.000
18	26	27	30	29	1	0.0	0.000
19	28	29	32	31	1	0.0	0.000
20	29	30	33	32	1	0.0	0.000
21	31	32	35	34	1	0.0	0.000
22	32	33	36	35	1	0.0	0.000
23	34	35	38	37	1	0.0	0.000
24	35	36	39	38	1	0.0	0.000
25	37	38	41	40	1	0.0	0.000
26	38	39	42	41	1	0.0	0.000
27	40	41	44	43	1	0.0	0.000
28	41	42	45	44	1	0.0	0.000
29	43	44	47	46	1	0.0	0.000
30	44	45	48	47	1	0.0	0.000
31	46	47	50	49	1	0.0	0.000
32	47	48	51	50	1	0.0	0.000
33	49	50	53	52	1	0.0	0.000
34	50	51	54	53	1	0.0	0.000
35	52	53	56	55	1	0.0	0.000
36	53	54	57	56	1	0.0	0.000
37	55	56	59	58	1	0.0	0.000
38	56	57	60	59	1	0.0	0.000
39	58	59	62	61	1	0.0	0.000
40	59	60	63	62	1	0.0	0.000
41	61	62	65	64	1	0.0	0.000
42	62	63	66	65	1	0.0	0.000
43	64	65	68	67	1	0.0	0.000
44	65	66	69	68	1	0.0	0.000
45	67	68	71	70	1	0.0	0.000
46	68	69	72	71	1	0.0	0.000
47	70	71	74	73	1	0.0	0.000
48	71	72	75	74	1	0.0	0.000
49	73	74	77	76	1	0.0	0.000
50	74	75	78	77	1	0.0	0.000

Fig. C-3. Output from TSAAS. (continued)

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

MATERIAL= 1 ,NO. OF MATERIAL TEMPERATURE CARDS= 1 ,MASS DENSITY= .2000E+01
 TEMP= 0 EN= 70000 FS= 70000 ET= 70000 NUSN= .330 NUTN= .330 NUTS= .330 GNS= 26316
 AN= .100E-03 AS= .100E-03 AT= .100E-03 YN= 700 YS= 700 YT= 700 PEMR= 1.000
 TCN= .0009 TCS= .0009 CP= .2480 HOD=-0. A.HT.FAC= -0. TM= .5E+01

BAND WIDTH = 10

THERMAL STRESS ANALYSIS OF A SPHERE
 OPTIONS 1=TRUE

VARIABLE TIME STEP-----	0	INITIAL TIME STEP-----	-0.
VARIABLE PROPERTY OPTION---	0	MINIMUM TIME STEP-----	-0.
THERMAL LOADS ON CARDS-----	1	MINIMUM TEMP STEP/CYCLE-----	.10
AIR TEMP FROM SUBROUTINE---	-0	MAXIMUM TEMP STEP/CYCLE-----	1.00
ARRHENIUS KINETICS-----	-0	IGNITION TEMP STEP/CYCLE-----	500.00
		ZERO OFFSET FOR ARS TEMP-----	0

NO OF THERMAL LOAD INCREMENTS= 0

OUTPUT OPTIONS

1

THERMAL STRESS ANALYSIS OF A SPHERE

NO. OF FLUX BOUNDARY CARDS ---- 0
 NO. OF CONV BOUNDARY CARDS ---- 0
 NO. OF TEMP BOUNDARY CARDS ---- 0
 TEMP CALCS PER PRINT CYCLE --- 2
 FILM COEFFICIENT ----- -0.0000
 AMBIENT AIR TEMPERATURE ----- -0.0
 TOTAL CALCULATIONAL TIME -----0.

Fig. C-3. Output from TSAAS. (continued)

THERMAL LOAD INC NO -1 ENTRY INTO TEM1 AT 15.767
THERMAL STRESS ANALYSIS OF A SPHERE

	0	1	2	3	4	5	6	7	8	9
0		108.6	108.6	108.6	100.3	100.3	100.3	92.5	92.5	92.5
10	85.3	85.3	85.3	78.7	78.7	78.7	72.4	72.4	72.4	66.6
20	66.6	66.6	61.1	61.1	61.1	56.0	56.0	56.0	51.1	51.1
30	51.1	46.6	46.6	46.6	42.2	42.2	42.2	38.2	38.2	38.2
40	34.3	34.3	34.3	30.6	30.6	30.6	27.2	27.2	27.2	23.8
50	23.8	23.8	20.7	20.7	20.7	17.7	17.7	17.7	14.8	14.8
60	14.8	12.1	12.1	12.1	9.4	9.4	9.4	6.9	6.9	6.9
70	4.5	4.5	4.5	2.2	2.2	2.2	0.0	0.0	0.0	
MATERIAL	EL.NO.	MAXIMUM TEMPERATURE			EL.NO.	MINIMUM TEMPERATURE			ENERGY	
1	2	154.39			50	1.10			0.	

EXIT FROM TEM1 AT 16.361

THERMAL STRESS ANALYSIS OF A SPHERE
STRESS ANALYSIS AT TIME = 0.000

MAT NO WEIGHT
1 3905.16
SYSTEM WEIGHT = 3905.16

INCREMENT NUMBER 1
STIFF TIME= .260 CP= 16.623
SOLV TIME= .016 CP= 16.641
APPROX. STRAIN ENERGY = .220E+05 CONVER. PARAMETER AT EL NO 0 = 0.0000
MATERIAL STRAIN ENERGIES
1 22019.08
THE SYSTEM CONVERGED IN 1 ITERATIONS

THEMAL STRESS ANALYSIS OF A SPHERE

NODAL POINT	R	Z	UR	UZ
1	9.978	.667	-.031062	0.000000
2	10.000	0.000	.031061	.000000
3	9.978	-.667	.031062	0.000000
4	10.377	.693	-.037904	0.000000
5	10.400	0.000	.037903	-.000000
6	10.377	-.693	.037904	0.000000
7	10.776	.720	-.043732	0.000000
8	10.800	0.000	.043731	-.000000
9	10.776	-.720	.043732	0.000000
10	11.175	.747	-.048689	0.000000
11	11.200	0.000	.048688	-.000000
12	11.175	-.747	.048689	0.000000
13	11.574	.773	-.052892	0.000000
14	11.600	0.000	.052891	.000000
15	11.574	-.773	.052892	0.000000
16	11.973	.800	-.056440	0.000000
17	12.000	0.000	.056440	.000000
18	11.973	-.800	.056440	0.000000
19	12.372	.827	-.059417	0.000000
20	12.400	0.000	.059417	.000000
21	12.372	-.827	.059417	0.000000
22	12.772	.853	-.061893	0.000000
23	12.800	0.000	.061893	.000000
24	12.772	-.853	.061893	0.000000
25	13.171	.880	-.063927	0.000000
26	13.200	0.000	.063927	-.000000
27	13.171	-.880	.063927	0.000000
28	13.570	.907	-.065571	0.000000
29	13.600	0.000	.065571	-.000000
30	13.570	-.907	.065571	0.000000
31	13.969	.933	-.066869	0.000000
32	14.000	0.000	.066869	-.000000
33	13.969	-.933	.066869	0.000000
34	14.368	.960	-.067859	0.000000
35	14.400	0.000	.067859	.000000
36	14.368	-.960	.067859	0.000000
37	14.767	.987	-.068573	0.000000
38	14.800	0.000	.068572	.000000
39	14.767	-.987	.068573	0.000000
40	15.166	1.013	-.069040	0.000000
41	15.200	0.000	.069039	.000000
42	15.166	-1.013	.069040	0.000000
43	15.565	1.040	-.069285	0.000000
44	15.600	0.000	.069285	.000000
45	15.565	-1.040	.069285	0.000000
46	15.964	1.067	-.069330	0.000000
47	16.000	0.000	.069330	.000000
48	15.964	-1.067	.069330	0.000000
49	16.364	1.093	-.069194	0.000000
50	16.400	0.000	.069194	.000000
51	16.364	-1.093	.069194	0.000000
52	16.763	1.120	-.068895	0.000000
53	16.800	0.000	.068895	.000000
54	16.763	-1.120	.068895	0.000000
55	17.162	1.147	-.068447	0.000000

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE
 NODAL

POINT	R	Z	UR	UZ
56	17.296	0.000	.068447	-0.000000
57	17.162	-1.147	.068447	0.000000
58	17.561	1.173	-.067864	0.000000
59	17.606	0.000	.067864	-0.000000
60	17.561	-1.173	.067864	0.000000
61	17.965	1.200	-.067158	0.000000
62	18.000	0.000	.067158	-0.000000
63	17.965	-1.200	.067158	0.000000
64	18.359	1.227	-.066339	0.000000
65	18.405	0.000	.066339	-0.000000
66	18.359	-1.227	.066339	0.000000
67	18.758	1.253	-.065416	0.000000
68	18.805	0.000	.065416	-0.000000
69	18.758	-1.253	.065416	0.000000
70	19.157	1.280	-.064399	0.000000
71	19.205	0.000	.064399	-0.000000
72	19.157	-1.280	.064399	0.000000
73	19.556	1.307	-.063294	0.000000
74	19.605	0.000	.063294	-0.000000
75	19.556	-1.307	.063294	0.000000
76	19.955	1.333	-.062110	0.000000
77	20.000	0.000	.062110	-0.000000
78	19.955	-1.333	.062110	0.000000

Fig. C-3. Output from TSAAS. (continued)

NODAL POINT	REACTIONS AT RESTRAINED NODES		REACTION R	REACTION Z
	UR	UZ		
1	-.03106	0.00000	-.0	-9753.3
3	-.03106	0.00000	.0	-9753.3
4	-.03790	0.00000	.0	-18144.0
6	.03790	0.00000	.0	-18144.0
7	-.04373	0.00000	.0	-16070.5
9	.04373	0.00000	-.0	-16070.5
10	-.04869	0.00000	0.0	-14112.7
12	.04869	0.00000	-.0	-14112.7
13	-.05289	0.00000	-.0	-12254.8
15	.05289	0.00000	.0	-12254.8
16	-.05644	0.00000	-.0	-10483.8
18	.05644	0.00000	.0	-10483.8
19	-.05942	0.00000	.0	-8788.5
21	.05942	0.00000	-.0	-8788.5
22	-.06189	0.00000	-.0	-7159.6
24	.06189	0.00000	.0	-7159.6
25	-.06393	0.00000	.0	-5589.2
27	.06393	0.00000	-.0	-5589.2
28	-.06557	0.00000	.0	-4070.6
30	.06557	0.00000	-.0	-4070.6
31	-.06687	0.00000	-.0	-2597.8
33	.06687	0.00000	-.0	-2597.8
34	-.06786	0.00000	.0	-1165.8
36	.06786	0.00000	.0	-1165.8
37	-.06857	0.00000	0.0	229.6
39	.06857	0.00000	.0	229.6
40	-.06904	0.00000	.0	1592.4
42	.06904	0.00000	-.0	1592.4
43	-.06928	0.00000	-.0	2925.7
45	.06928	0.00000	.0	2925.7
46	-.06933	0.00000	.0	4232.6
48	.06933	0.00000	.0	4232.6
49	-.06919	0.00000	.0	5515.6
51	.06919	0.00000	-.0	5515.6
52	-.06890	0.00000	.0	6777.0
54	.06890	0.00000	.0	6777.0
55	-.06845	0.00000	0.0	8018.6
57	.06845	0.00000	.0	8018.6
58	-.06786	0.00000	.0	9242.3
60	.06786	0.00000	.0	9242.3
61	-.06716	0.00000	0.0	10449.8
63	.06716	0.00000	-.0	10449.8
64	-.06634	0.00000	.0	11642.3
66	.06634	0.00000	-.0	11642.3
67	-.06542	0.00000	.0	12821.1
69	.06542	0.00000	-.0	12821.1
70	-.06440	0.00000	.0	13987.4
72	.06440	0.00000	-.0	13987.4
73	-.06329	0.00000	-.0	15142.2
75	.06329	0.00000	.0	15142.2
76	-.06211	0.00000	.0	7635.5
78	.06211	-0.00000	.0	7635.5

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

Fig. C-3. Output from TSAAS. (continued)

EL	MTL	R	Z	EPR	SIGMAR	SIGMAZ	SIGMAI	SIGMRZ	SIGMMX	SIGMMN	ANGLE	SIGMAN	SIGMAS	SIGMNS	SIGAVG	SIGMXS
1	1	10.189	.340	1.00	-30	-751	-751	24	-29	-752	1.91	-30	-751	24	-511	361
2	1	10.189	-.340	1.00	-30	-751	-751	-24	-29	-752	-1.91	-30	-751	-24	-511	361
3	1	10.588	.353	1.00	-79	-643	-643	19	-79	-643	1.91	-79	-643	19	-455	282
4	1	10.588	-.353	1.00	-79	-643	-643	-19	-79	-643	-1.91	-79	-643	-19	-455	282
5	1	10.988	.367	1.00	-116	-546	-546	14	-116	-547	1.91	-116	-546	14	-403	216
6	1	10.988	-.367	1.00	-116	-546	-546	-14	-116	-547	-1.91	-116	-546	-14	-403	216
7	1	11.387	.380	1.00	-143	-461	-461	11	-142	-461	1.91	-143	-461	11	-355	159
8	1	11.387	-.380	1.00	-143	-461	-461	-11	-142	-461	-1.91	-143	-461	-11	-355	159
9	1	11.787	.393	1.00	-161	-384	-384	7	-161	-384	1.91	-161	-384	7	-310	112
10	1	11.787	-.393	1.00	-161	-384	-384	-7	-161	-384	-1.91	-161	-384	-7	-310	112
11	1	12.186	.407	1.00	-173	-315	-315	5	-173	-315	1.91	-173	-315	5	-268	71
12	1	12.186	-.407	1.00	-173	-315	-315	-5	-173	-315	-1.91	-173	-315	-5	-268	71
13	1	12.586	.420	1.00	-180	-252	-252	2	-180	-252	1.91	-180	-252	2	-228	36
14	1	12.586	-.420	1.00	-180	-252	-252	-2	-180	-252	-1.91	-180	-252	-2	-228	36
15	1	12.986	.433	1.00	-183	-196	-196	0	-183	-196	1.92	-183	-196	0	-191	6
16	1	12.986	-.433	1.00	-183	-196	-196	-0	-183	-196	-1.92	-183	-196	-0	-191	6
17	1	13.385	.447	1.00	-182	-144	-144	-1	-144	-182	88.09	-182	-144	-1	-157	19
18	1	13.385	-.447	1.00	-182	-144	-144	1	-144	-182	88.09	-182	-144	1	-157	19
19	1	13.785	.460	1.00	-178	-97	-97	-3	-96	-178	88.09	-178	-97	-3	-124	41
20	1	13.785	-.460	1.00	-178	-97	-97	3	-96	-178	88.09	-178	-97	3	-124	41
21	1	14.184	.473	1.00	-172	-53	-53	-4	-53	-173	88.09	-172	-53	-4	-93	60
22	1	14.184	-.473	1.00	-172	-53	-53	4	-53	-173	88.09	-172	-53	4	-93	60
23	1	14.584	.487	1.00	-165	-13	-13	-5	-13	-165	88.09	-165	-13	-5	-64	76
24	1	14.584	-.487	1.00	-165	-13	-13	5	-13	-165	88.09	-165	-13	5	-64	76
25	1	14.983	.500	1.00	-156	24	24	-6	24	-156	88.09	-156	24	-6	-36	90
26	1	14.983	-.500	1.00	-156	24	24	6	24	-156	88.09	-156	24	6	-36	90
27	1	15.383	.513	1.00	-146	58	58	-7	58	-146	88.09	-146	58	-7	-10	102
28	1	15.383	-.513	1.00	-146	58	58	7	58	-146	88.09	-146	58	7	-10	102
29	1	15.782	.527	1.00	-135	90	90	-7	90	-135	88.09	-135	90	-7	15	112
30	1	15.782	-.527	1.00	-135	90	90	7	90	-135	88.09	-135	90	7	15	112
31	1	16.182	.540	1.00	-123	119	120	-8	120	-123	88.09	-123	119	-8	39	121
32	1	16.182	-.540	1.00	-123	119	120	8	120	-123	88.09	-123	119	8	39	121
33	1	16.582	.553	1.00	-111	147	147	-9	147	-111	88.09	-111	147	-9	61	129
34	1	16.582	-.553	1.00	-111	147	147	9	147	-111	88.09	-111	147	9	61	129
35	1	16.981	.567	1.00	-98	173	173	-9	173	-98	88.09	-98	173	-9	83	136
36	1	16.981	-.567	1.00	-98	173	173	9	173	-98	88.09	-98	173	9	83	136
37	1	17.381	.580	1.00	-85	197	197	-9	198	-85	88.09	-85	197	-9	103	141
38	1	17.381	-.580	1.00	-85	197	197	9	198	-85	88.09	-85	197	9	103	141
39	1	17.780	.593	1.00	-72	220	220	-10	220	-72	88.09	-72	220	-10	123	146
40	1	17.780	-.593	1.00	-72	220	220	10	220	-72	88.09	-72	220	10	123	146
41	1	18.180	.607	1.00	-59	241	242	-10	242	-59	88.09	-59	241	-10	141	150
42	1	18.180	-.607	1.00	-59	241	242	10	242	-59	88.09	-59	241	10	141	150
43	1	18.579	.620	1.00	-46	262	262	-10	262	-46	88.09	-46	262	-10	159	154
44	1	18.579	-.620	1.00	-46	262	262	10	262	-46	88.09	-46	262	10	159	154
45	1	18.979	.633	1.00	-32	281	281	-10	281	-33	88.09	-32	281	-10	176	157
46	1	18.979	-.633	1.00	-32	281	281	10	281	-33	88.09	-32	281	10	176	157
47	1	19.378	.647	1.00	-19	299	299	-11	299	-19	88.09	-19	299	-11	193	159
48	1	19.378	-.647	1.00	-19	299	299	11	299	-19	88.09	-19	299	11	193	159
49	1	19.778	.660	1.00	-6	316	316	-11	316	-6	88.09	-6	316	-11	209	161
50	1	19.778	-.660	1.00	-6	316	316	11	316	-6	88.09	-6	316	11	209	161

Fig. C-3. Output from TSAAS. (continued)

THERMAL STRESS ANALYSIS OF A SPHERE

EL	MTL	EPSR	EPSZ	EPST	EPSRZ	EPSMAX	EPSMIN	ANGLE	EPSN	EPSS	EPSNS	TEMPERATURE
1	1	.6654	-.7046	-.7052	.0915	.6669	-.7061	7.61	.6654	-.7046	.0915	104.39
2	1	.6654	-.7046	-.7052	-.0915	.6669	-.7061	-7.61	.6654	-.7046	-.0915	104.39
3	1	.4926	-.5775	-.5780	.0715	.4938	-.5786	7.61	.4926	-.5775	.0715	96.35
4	1	.4926	-.5775	-.5780	-.0715	.4938	-.5786	-7.61	.4926	-.5775	-.0715	96.35
5	1	.3495	-.4681	-.4685	.0546	.3504	-.4690	7.61	.3495	-.4681	.0546	88.90
6	1	.3495	-.4681	-.4685	-.0546	.3504	-.4690	-7.61	.3495	-.4681	-.0546	88.90
7	1	.2306	-.3735	-.3739	.0404	.2313	-.3742	7.61	.2306	-.3735	.0404	81.97
8	1	.2306	-.3735	-.3739	-.0404	.2313	-.3742	-7.61	.2306	-.3735	-.0404	81.97
9	1	.1317	-.2914	-.2916	.0283	.1322	-.2918	7.61	.1317	-.2914	.0283	75.50
10	1	.1317	-.2914	-.2916	-.0283	.1322	-.2918	-7.61	.1317	-.2914	-.0283	75.50
11	1	.0493	-.2196	-.2197	.0180	.0496	-.2199	7.61	.0493	-.2196	.0180	69.47
12	1	.0493	-.2196	-.2197	-.0180	.0496	-.2199	-7.61	.0493	-.2196	-.0180	69.47
13	1	-.0193	-.1566	-.1567	.0092	-.0192	-.1567	7.61	-.0193	-.1566	.0092	63.81
14	1	-.0193	-.1566	-.1567	-.0092	-.0192	-.1567	-7.61	-.0193	-.1566	-.0092	63.81
15	1	-.0765	-.1011	-.1011	.0016	-.0765	-.1011	7.63	-.0765	-.1011	.0016	58.51
16	1	-.0765	-.1011	-.1011	-.0016	-.0765	-.1011	-7.63	-.0765	-.1011	-.0016	58.51
17	1	-.1241	-.0520	-.0520	-.0048	-.0519	-.1242	-172.40	-.1241	-.0520	-.0048	53.52
18	1	-.1241	-.0520	-.0520	.0048	-.0519	-.1242	172.40	-.1241	-.0520	.0048	53.52
19	1	-.1636	-.0084	-.0083	-.0104	-.0083	-.1637	-172.39	-.1636	-.0084	-.0104	48.82
20	1	-.1636	-.0084	-.0083	.0104	-.0083	-.1637	172.39	-.1636	-.0084	.0104	48.82
21	1	-.1962	.0304	.0305	-.0151	.0306	-.1965	-172.39	-.1962	.0304	-.0151	44.38
22	1	-.1962	.0304	.0305	.0151	.0306	-.1965	172.39	-.1962	.0304	.0151	44.38
23	1	-.2231	.0651	.0653	-.0192	.0654	-.2234	-172.39	-.2231	.0651	-.0192	40.19
24	1	-.2231	.0651	.0653	.0192	.0654	-.2234	172.39	-.2231	.0651	.0192	40.19
25	1	-.2451	.0962	.0964	-.0228	.0966	-.2455	-172.39	-.2451	.0962	-.0228	36.22
26	1	-.2451	.0962	.0964	.0228	.0966	-.2455	172.39	-.2451	.0962	.0228	36.22
27	1	-.2629	.1242	.1244	-.0259	.1246	-.2633	-172.39	-.2629	.1242	-.0259	32.46
28	1	-.2629	.1242	.1244	.0259	.1246	-.2633	172.39	-.2629	.1242	.0259	32.46
29	1	-.2771	.1494	.1497	-.0285	.1499	-.2776	-172.39	-.2771	.1494	-.0285	28.88
30	1	-.2771	.1494	.1497	.0285	.1499	-.2776	172.39	-.2771	.1494	.0285	28.88
31	1	-.2883	.1722	.1726	-.0308	.1727	-.2888	-172.39	-.2883	.1722	-.0308	25.49
32	1	-.2883	.1722	.1726	.0308	.1727	-.2888	172.39	-.2883	.1722	.0308	25.49
33	1	-.2969	.1929	.1933	-.0327	.1934	-.2974	-172.39	-.2969	.1929	-.0327	22.26
34	1	-.2969	.1929	.1933	.0327	.1934	-.2974	172.39	-.2969	.1929	.0327	22.26
35	1	-.3032	.2117	.2121	-.0344	.2122	-.3038	-172.39	-.3032	.2117	-.0344	19.18
36	1	-.3032	.2117	.2121	.0344	.2122	-.3038	172.39	-.3032	.2117	.0344	19.18
37	1	-.3076	.2288	.2292	-.0358	.2294	-.3082	-172.39	-.3076	.2288	-.0358	16.24
38	1	-.3076	.2288	.2292	.0358	.2294	-.3082	172.39	-.3076	.2288	.0358	16.24
39	1	-.3104	.2444	.2449	-.0371	.2450	-.3110	-172.39	-.3104	.2444	-.0371	13.43
40	1	-.3104	.2444	.2449	.0371	.2450	-.3110	172.39	-.3104	.2444	.0371	13.43
41	1	-.3117	.2587	.2592	-.0381	.2593	-.3124	-172.39	-.3117	.2587	-.0381	10.75
42	1	-.3117	.2587	.2592	.0381	.2593	-.3124	172.39	-.3117	.2587	.0381	10.75
43	1	-.3119	.2718	.2723	-.0390	.2724	-.3125	-172.39	-.3119	.2718	-.0390	8.18
44	1	-.3119	.2718	.2723	.0390	.2724	-.3125	172.39	-.3119	.2718	.0390	8.18
45	1	-.3109	.2838	.2843	-.0397	.2845	-.3116	-172.39	-.3109	.2838	-.0397	5.72
46	1	-.3109	.2838	.2843	.0397	.2845	-.3116	172.39	-.3109	.2838	.0397	5.72
47	1	-.3091	.2948	.2954	-.0403	.2955	-.3098	-172.39	-.3091	.2948	-.0403	3.36
48	1	-.3091	.2948	.2954	.0403	.2955	-.3098	172.39	-.3091	.2948	.0403	3.36
49	1	-.3065	.3050	.3056	-.0409	.3057	-.3072	-172.39	-.3065	.3050	-.0409	1.10
50	1	-.3065	.3050	.3056	.0409	.3057	-.3072	172.39	-.3065	.3050	.0409	1.10

STRESS TIME= .395

CP= 17.038

Fig. C-4. SAASPL edit.

THERMAL STRESS ANALYSIS OF A SPHERE

MATERIAL 1			PRINT CYCLE 1				TIME		0.000000			
EL	R LOC	Z LOC	SIG R	SIG Z	SIG TH	SIG RZ	SIG MX	SIG MN	SIG AV	SIG MS	SEL IND	
1	10.188677	.340016	-30	-751	-751	24	-29	-752	-511	361	07670777	
2	10.188677	-.340016	-30	-751	-751	-24	-29	-752	-511	361	06760666	
3	10.588231	.353348	-79	-643	-643	19	-79	-643	-455	282	05550554	
4	10.588231	-.353348	-79	-643	-643	-19	-79	-643	-455	282	04440445	
5	10.987785	.366680	-116	-546	-547	14	-116	-547	-403	216	03330330	
6	10.987785	-.366680	-116	-546	-547	-14	-116	-547	-403	216	00000003	
14	12.586001	-.420008	-180	-252	-252	-2	-180	-252	-228	36	30000000	
15	12.985555	.433340	-183	-196	-196	0	-183	-196	-191	6	70001000	
16	12.985555	-.433340	-183	-196	-196	-0	-183	-196	-191	6	60000000	
17	13.385109	.446672	-182	-144	-144	-1	-144	-182	-157	19	50000000	
18	13.385109	-.446672	-182	-144	-144	1	-144	-182	-157	19	40000000	
45	18.978865	.633320	-32	281	281	-10	281	-33	176	157	00003000	
47	19.378419	.646652	-19	299	299	-11	299	-19	193	159	00005000	
48	19.378419	-.646652	-19	299	299	11	299	-19	193	159	00004000	
49	19.777473	.659984	-6	316	316	-11	316	-6	209	161	02207020	
50	19.777473	-.659984	-6	316	316	11	316	-6	209	161	00006000	

THERMAL STRESS ANALYSIS OF A SPHERE

MATERIAL 1			PRINT CYCLE 1				TIME		0.000000			
EL	R LOC	Z LOC	EPS R	EPS Z	EPS TH	EPS RZ	EPS MX	EPS MN	EPS AV	EPS MS	SEL IND	
1	10.188677	.340016	.6654	-.7046	-.7052	.0915	.6669	-.7061	-.2481	.6865	67676777	
2	10.188677	-.340016	.6654	-.7046	-.7052	-.0915	.6669	-.7061	-.2481	.6865	76767666	
3	10.588231	.353348	.4926	-.5775	-.5780	.0715	.4938	-.5786	-.2209	.5362	44454454	
4	10.588231	-.353348	.4926	-.5775	-.5780	-.0715	.4938	-.5786	-.2209	.5362	55545545	
5	10.987785	.366680	.3495	-.4681	-.4685	.0546	.3504	-.4690	-.1957	.4097	00330030	
6	10.987785	-.366680	.3495	-.4681	-.4685	-.0546	.3504	-.4690	-.1957	.4097	33003303	
15	12.985555	.433340	-.765	-.1011	-.1011	.0016	-.0765	-.1011	-.0929	.0123	00001000	
44	18.579311	-.619988	-.3119	.2718	.2723	.0390	.2724	-.3125	.0774	.2925	10000000	
49	19.777473	.659984	-.3065	.3050	.3056	-.0409	.3057	-.3072	.1014	.3065	02200020	

EXECUTION OF SAASPL ENDS AT 21.128

Fig. C-6. Finite element mesh.

Fig. C-7. Temperature isotherms.

INTERNAL HEAT GENERATION WITH TEMP. AND CONV. BOUNDARY CONDITIONS

MATERIAL 1			PRINT CYCLE 1				TIME		5000.000349			
EL	R LOC	Z LOC	SIG R	SIG Z	SIG TH	SIG RZ	SIG MX	SIG MN	SIG AV	SIG MS	SEL IND	
9	8.432104	.401206	96	-487	-652	-289	215	-607	-348	434	04310004	
21	8.372572	1.196990	274	-543	-601	-114	290	-559	-290	445	06007007	
33	8.240516	1.974658	281	-556	-594	46	284	-558	-290	439	27005006	
45	8.045621	2.726241	252	-526	-585	165	286	-559	-286	435	05006005	
57	7.796846	3.448070	196	-476	-576	252	280	-560	-285	428	03004003	
69	7.503847	4.143193	121	-420	-567	309	261	-560	-289	414	00003000	
81	7.177761	4.827304	30	-364	-557	335	222	-555	-297	389	00050000	
93	6.832034	5.546862	-78	-321	-544	319	142	-540	-314	343	00030000	
97	.450689	8.335646	-697	52	-698	45	55	-700	-448	377	70700770	
98	1.341767	8.260222	-666	48	-687	127	69	-688	-435	379	60600660	
99	2.204295	8.093947	-619	22	-676	203	80	-678	-425	379	50500550	
100	3.024365	7.852897	-557	-15	-664	267	95	-666	-412	380	40400440	
101	3.792660	7.554405	-483	-60	-649	317	109	-652	-397	381	30000330	
102	4.505995	7.219555	-405	-111	-631	348	120	-636	-382	378	00060000	
103	5.170362	6.877370	-334	-165	-609	356	116	-615	-370	365	00070000	
104	5.809560	6.579637	-290	-212	-580	328	79	-581	-361	330	00040000	
108	11.327914	7.028688	-4	135	2	-1	135	-4	45	69	02000000	
132	11.383459	9.195735	-3	122	41	-12	123	-4	53	64	00000020	
133	.453784	10.938010	-194	-124	-196	3	-123	-194	-171	36	00001000	
156	11.432712	11.439257	-1	81	50	-14	84	-4	43	44	00200000	
249	8.448484	20.228531	5	7	17	-2	8	3	10	7	00000200	

INTERNAL HEAT GENERATION WITH TEMP. AND CONV. BOUNDARY CONDITIONS

MATERIAL 2			PRINT CYCLE 1			TIME			5000.000349			
EL	R LOC	Z LOC	SIG R	SIG Z	SIG TH	SIG RZ	SIG MX	SIG MN	SIG AV	SIG MS	SEL IND	
8	7.391933	.406469	-12	-61	148	115	81	-155	25	152	11020100	
44	6.993374	2.761650	494	430	592	-131	597	327	505	135	04000400	
56	6.735016	3.490011	513	445	621	-144	627	331	526	148	06354540	
68	6.427771	4.184605	521	436	633	-156	640	317	530	162	45466350	
73	.443278	6.259771	401	45	401	-15	401	44	282	179	00000007	
74	1.323469	6.167676	405	58	410	-32	408	55	291	177	00000006	
75	2.185702	6.049378	416	85	429	-53	425	77	310	176	00000004	
76	3.021078	5.869926	435	122	458	-74	452	106	338	176	00000003	
77	3.824279	5.643935	465	169	497	-92	491	143	377	177	00000005	
78	4.594724	5.387281	515	237	554	-94	543	208	435	173	30000000	
79	5.338817	5.117356	579	426	655	-84	617	389	554	133	63600660	
80	6.076543	4.851203	512	423	637	-167	640	294	524	173	00575030	
90	4.473294	6.230050	487	268	553	-136	553	203	436	175	00030000	
91	5.107302	5.864797	548	336	616	-140	618	267	500	175	50043000	
92	5.674401	5.510455	729	614	783	-120	804	539	708	133	77707770	

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