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SNAP-23A PROGRAM PHASE I

ADVANCED TERRESTRIAL RADIOISOTOPE POWER SYSTEMS CONVERTER DEVELOPMENT PROGRAM

QUARTERLY REPORT NO. 1

3M Isotope Power Products
MINNESOTA MINING & MANUFACTURING CO.
2501 HUDSON ROAD, ST. PAUL, MINN. 55119 PH. 631-2600

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SNAP-23A PROGRAM PHASE I

ADVANCED TERRESTRIAL RADIOISOTOPE POWER SYSTEMS CONVERTER DEVELOPMENT PROGRAM

QUARTERLY REPORT NO. 1

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MAY 22, 1967

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FOREWORD

The Minnesota Mining and Manufacturing Company (3M Company), as prime contractor to the United States Atomic Energy Commission under contract AT(30-1)-3418, designed, developed, fabricated and tested the SNAP-23 60-watt mockup radioisotope power supply system. The basic SNAP-23 Program was directed toward the development of high performance, reliable thermoelectric converter modules, using segmented thermoelectric legs, having a five-year design life.

As a result of the advancements made in the application of segmented thermoelectric legs during the preliminary SNAP-23 effort, the AEC initiated a follow-on development program designed to advance the SNAP-23 technology into a series of technically improved and economically competitive radioisotope power systems in the 25- and 100-watt power ranges with a ten-year design life objective.

The SNAP-23A development effort is directed toward the design, fabrication, and test of a thermoelectric converter design to be used as a component in the 60-watt SNAP-23A Terrestrial Radioisotope Power Supply System.

The 3M Company Converter Development Program will be performed under AEC contract AT(30-1)-3824, in conjunction with the SNAP-23A system contractor, Westinghouse Astronuclear Laboratory (WANL). While the converter development program is directed toward the development of a 60-watt converter and conceptual designs for 25- and 100-watt converters major emphasis will be concentrated on development of a design for thermoelectric couples which will perform reliably over a ten-year design life.

The specific requirements of the Phase I, SNAP-23A development effort are:

- Design, develop, fabricate and test 60-watt(e) converters
- Prepare conceptual designs for 25- and 100-watt(e) converters

- Develop and test thermoelectric couples to substantiate extrapolations of a ten-year life feasibility
- Conduct post-test analysis on SNAP-23 residual devices
- Integration of the SNAP-23A converter into SNAP-23A system

Documents governing the design, development, fabrication and testing of the SNAP-23A 60-watt(e) thermoelectric converter and conceptual design of the 25- and 100-watt converters include:

- AEC Specification for SNAP-23A Thermoelectric Converter, dated 30 November 1966.
- Design baseline criteria developed by 3M Company during execution of AEC Contract AT(30-1)-3418, SNAP-23.
- WANL/3M Interface Document, SNAP-23A.

The converter hardware development effort will be performed concurrently with the technical development program.

The basis of the technical development program will be:

1. Bell jar tests, to verify the design of the thermoelectric couple on an initial performance basis.
2. Ten-couple module tests, to verify couple performance characteristics under a controlled variable testing matrix.
3. Post-test analysis of the SNAP-23, to establish the characteristics and magnitude of existing and potential failure or degradation mechanisms.

The 3M Converter Development Program is arranged in three primary tasks – 60-watt converter development, based on the SNAP-23 Program baseline technology; thermoelectric couple improvement program, designed to incorporate material and fabrication technology improvements into thermoelectric units capable of achieving the desired 10-year life objective of the SNAP-23A Program;

and the 25- and 100-watt conceptual design effort, leading to the design of technically-improved and commercially-competitive, economical converters, in a wide variety of power ranges, which could be produced by any competent converter manufacturer.

Monthly and quarterly reporting by 3M Company under the SNAP-23A Program will discuss the technical progress of the Converter Development Program. The reader is referred to progress reports issued by WANL for a discussion of the SNAP-23A System Program.

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1.0 SUMMARY

The effort conducted during the initial two month reporting period consisted mainly of program plan preparation, establishment of firm liaison with the SNAP-23A system contractor (Westinghouse Astronuclear Laboratory), and initiation of technical efforts for design, fabrication, test of the Experimental Mockup Converter.

Results of program progress are outlined in Sections 2, 3, and 4 of this report. Detailed analyses are included in the Appendices.

Significant highlights occurring during this reporting period are summarized below:

- The 3M-Westinghouse Interface Document was completed during February.
- Design was initiated on the Experimental Mockup Converter.
- A Parametric Performance study program was initiated on the converter to establish the optimum system operating point.
- Design was initiated on the ten-couple modules and the test program was planned in detail.
- Computer data was prepared and submitted to Westinghouse for use in preparation of a total system computer analytical program.
- A stress analysis was conducted to determine the dynamic capability of the converter structure under increased system imposed shock and vibration conditions.

2.0 PROGRAM MANAGEMENT

2.1 LIAISON

2.1.1 INTERFACE DOCUMENT

Coordination and liaison activities were performed with Westinghouse to define the interface between the 3M-furnished thermoelectric converter hardware and services and the SNAP-23A hardware and services which are provided by Westinghouse.

The Interface Document, which defines and governs the 3M-Westinghouse interface, was developed during a series of meetings between 3M, Westinghouse, and the Commission. The final document was completed and signed by both contractors on 28 February 1967, and approved by the Commission on 8 March 1967.

2.1.2 INTERFACE ACTIVITIES

A meeting was held at 3M on 16 and 17 March 1967 for the purpose of discussing the design analyses and integration of the SNAP-23A converter and system. Highlights of this first system-converter interface workshop are as follows:

1. The experimental mockup converter would be designed for end-of-life temperature conditions of 180°F cold junction and 880°F hot junction temperature. However, a parametric study would be performed on the converter for various outer case temperatures to provide tradeoff data for system optimization.

2. The dynamic integrity of the (Conax) gland electrical feedthrough under shock and vibration exposure would be investigated. An investigation would be made to incorporate hermetic terminals of glass- (or ceramic) to-metal construction for greater device reliability.
3. No allowance for degradation of the converter or system would be included in the design of the first converter. It is an objective of the Phase I Program to conduct tests to determine the exact degradation rate.
4. Portions of the 3M converter computer program were provided to Westinghouse for use in preparation of a complete SNAP-23A system-converter computer program. (This data is discussed in Appendix F.)
5. The radiator configuration, interface and methods of attachment to the converter were discussed. The two approaches considered for the experimental mockup converter are a right cylindrical and a tapered interface. Design analyses of the tapered concept are further discussed in Section 3.2.
6. The converter will be exposed to dynamic conditions from the system, as defined by Westinghouse:
 - a. Amplitude for vibration specified in MIL-STD-810A will be double in value.
 - b. The converter may be exposed to 10 g's during shock and vibration.
 - c. A radiator weight of 65 pounds is predicted. This weight will be entirely supported by the converter.

Other topics covered at that meeting included material considerations, establishment of preliminary thermal and electrical interface parameters, performance characteristics of SNAP-23 devices, and preliminary instrumentation requirements. This initial interface meeting established preliminary definition of many design areas necessary to initiate parametric studies and detailed converter design effort.

2.2 DOCUMENTATION AND REPORTS

Program planning documents which were under preparation during the first quarter include:

- SNAP-23A Program Plan
- Fabrication Plan
- Quality Control Plan
- Reliability Plan
- Post-test Analysis Plan
- Converter Specification

2.3 EFFORT PLANNED FOR NEXT QUARTER

During the forthcoming quarter, 3M and WANL will update the interface document to incorporate further SNAP-23A Program planning efforts or to reflect any apparent revisions to previously established interface requirements. Scheduled efforts anticipated for the second quarter include:

- Release of 60-watt converter design to meet program specifications.
- Completion of mechanical interface definition for the Experimental Mockup Converter.
- Release of drawings for items requiring long fabrication lead-time.
- Completion of post-test analysis for SNAP-23 prototype converter and 6-couple module.
- Perform liaison for integration of test activities with Westinghouse. Where possible, instrumentation and test readout will be standardized.
- Undertake preliminary efforts to establish acceptance test specifications for the Experimental Mockup Converter.

- Initiate efforts to define the Prototype Mockup Converter design criteria base.
- Determine design requirements for shipping container and accomplish preliminary design.

3.0 TASK II-60-WATT CONVERTER DEVELOPMENT

3.1 INTRODUCTION

The objective of this task is to design, develop, fabricate and test a thermoelectric converter that will interface with the SNAP-23 power supply system. As a component of the system, the converter is required to produce 60-watts under ten year end-of-life temperature conditions.

Two distinct converter designs will be prepared during the Phase I program; the first design, identified as the Experimental Mockup Converter, will generally use SNAP-23 technology as the design base; the second converter, identified as the Prototype Mockup Converter, will evolve from the first design and will incorporate data from thermoelectric module tests, converter performance and dynamic testing, and integrated system-converter testing.

All effort covered in this report is concerned with the Experimental Mockup Converter design.

3.2 DESIGN AND ANALYSIS

During the initial reporting period, design efforts were concentrated on parametric analyses and design analyses specifically required to establish and define the converter-system interface. Since the interface definition was not fully established during the period, the analyses presented in this report are investigative in nature and should be considered as preliminary.

3.2.1 CONVERTER STRUCTURAL ANALYSIS

The converter, when integrated as a component into the SNAP-23A system, must be capable of supporting the full load of the radiator and associated power conditioning apparatus.

An analysis was conducted to determine the capability of the converter structure under conditions of Dynamic Loading.

An assembly of the fins and fuel capsule spud mounted to the converter weldment is shown in Figure 3-1. The converter weldment consists of an outer case, support cones, lower cover, and hot frame. The support cone is a critical part of the weldment from both heat loss and strength standpoints. From a heat loss standpoint, it is desirable to have the support cone thickness, t , very thin. Heat escapes along a path formed by the upper cylindrical portion of the hot frame and the support cone. Since the support cone is in series with the upper cylindrical portion of the hot frame, the heat loss does not increase in proportion to increase in cone thickness. The only mechanical tie between the converter and the system is via the fuel capsule spud. When the system is vibrated per Mil-Std-810A, the converter is subjected to increased vibration amplitudes as a result of being attached to a truss rod supported fuel capsule which vibrates at approximately twice the specification amplitudes. Truss rod stiffness is related to fuel capsule heat loss in that an increase in stiffness produces a subsequent increase in heat loss. Therefore, the double amplitude (this also creates twice the acceleration forces as specified in Mil-Std-810A) represents a compromise in the Westinghouse-designed fuel capsule support system.

It is necessary to analyze the converter weldment stresses in conjunction with added deflections induced by vibration. Deflections can be magnified appreciably depending on how closely a resonant condition is approached. Therefore, the natural frequency of the system should be well below the excitation frequency which varies from 5 to 52 cycles per second. At an excitation frequency of 52 cps, the spud amplitude is 0.036 inch which corresponds to a 10 g acceleration. The support cone is the only structurally stiff member in the vertical (axial) direction. This permits thermal expansion of the hot frame with respect to the outer case, the lower cover being flexible in the vertical direction. However, in the horizontal

Figure 3-1. Converter-Fins-Fuel Capsule Spud Assembly

(transverse) direction, the lower cover adds considerable stiffness to the converter weldment. Consequently, the support cone stresses are much higher under vertical excitation than transverse, and therefore, the stress analysis is confined to the vertical direction after noting the high degree of stiffness in the transverse direction as shown by the spring rate analysis.

3.2.1.1 Converter Weldment Stress Analysis

Figure 3-2 shows the direction of the hoop (circumferential) stresses and the meridional stress in the support cone. The results of portions of the converter weldment analyzed from a stress standpoint are summarized in Table 3-1. (Supporting calculations for those items listed in this table are included in Appendix A.) As a result of the stress analysis, the support cone thickness was increased from 0.016 to 0.025 inch. Heat loss increased only 1.34 watts for EOL temperature conditions, the net effect being a decrease in overall converter efficiency from 7.70 to 7.68 percent. In the tabulation, stresses induced by the internal gas pressure were included if the stresses added to the dynamically induced stresses and neglected if the stresses subtracted.

Figure 3-2. Support Cone Stress Definition

Table 3-1. Converter Weldment Stresses

Item	Hoop Stress*			Meridional Stress*			Combined Stress *	Failure Mode	M. S. **
	Down Load	Up Load	Pressure Load	Down Load	Up Load	Pressure Load			
Support cone adjacent to its outer thickened ring	—	—	+11,220	+ 6,140	—	+5,610	+11,500	Tens. Yield	1.19
	—	—	+11,220	—	- 6,140	+5,610	—	Tens. Yield	1.20
Support cone adjacent to its inner thickened ring	—	—	+ 4,520	+15,280	—	+2,260	+15,750	Tens. Yield	0.16
	—	—	+ 4,520	—	-15,280	+2,260	—	Comp. Yield***	0.42
Outer thickened ring of support cone	Up Load Not as Critical as Down Load								
	-12,800	—	—	—	—	—	-12,800	Buckling	1.09
Inner thickened ring of support cone	—	-14,900	—	—	—	—	-14,900	Comp. Yield***	0.60
	+17,050	—	—	—	—	—	+17,050	Tens. Yield	0.07
Upper cylindrical portion of hot frame	—	—	+ 2,820	- 9,550	—	—	- 9,550	Comp. Yield***	0.83

* (+) Indicates tension, (-) indicates compression, all stresses in psi.

** M. S. = (allowable stress/design stress)⁻¹.

*** Not critical in buckling.

3. 2. 1. 2 Converter Support – Spring Rate Analysis

The natural vibration frequency and spring rate were analyzed both in the axial and transverse direction. The load, W , shown in Figure 3-2, causes a stretching in the meridional direction of the support cone which produces a downward deflection component. This same tension force causes the thickened outer ring to contract which creates a bending deflection in the support cone thereby adding to the downward deflection. In addition, the load W causes the upper cylindrical portion of the hot frame (shown in Figure 3-1) to shorten in the axial direction. It is thus seen that several individual deflections must be calculated before arriving at a total deflection.

The following items were evaluated in the determination of the spring rate and natural frequency of vibration in the transverse direction. The system was considered fixed at the lower end of the upper cylindrical portion of the hot frame (2.8 inches down from top of support cone) and supported by the lower cover. Expressions were written for deflections specified below.

1. Bending deflection of upper cylindrical portion of hot frame as a cantilever beam.
2. Shear deflection of upper cylindrical portion of hot frame as a cantilever beam.
3. Rotation of free end of upper cylindrical portion of hot frame center line (due to bending).
4. Bending deflection of support cone.
5. Shear deflection of support cone.
6. Change in slope of support cone center line due to bending.

Each of the above items was evaluated separately for a load of 105 pounds applied at the center of gravity of the system but with no support at the bottom, and an unknown reaction Q , acting at the bottom but no load at the center of gravity. The effects of items 1 through 6 for the two load conditions were evaluated at location of reaction Q . Deflection of the lower cover system was also evaluated in terms

of Q. From the fact that the relative deflection between the lower cover and the rest of the system must be equal to zero, a solution for Q was then obtained.

Spring rates, magnification factors, and natural frequencies calculated for both the axial and transverse directions are listed in Table 3-2. (Supporting analysis is presented in Appendix B.)

Spud acceleration forces, when multiplied by the magnification factors, yield the acceleration forces transmitted to the converter weldment. A magnification of 1.00 means that the weldment is infinitely stiff. The magnification factors are for a forcing frequency of 52 cps.

Table 3-2. Spring Rate Analysis Results

Direction	Spring Rate, lbs. per inch	Natural Fre- quency, Cycles per second	Magnification Factor
Axial	4.97×10^5	215	1.062
Transverse	2.69×10^6	500	1.01

3.2.2 OUTER CASE AND COLD FRAME ANALYSIS

The outer case, which is a thin walled stainless steel cylinder, is assembled to the cold frame by means of a shrink fit, the cold frame being a thick sectioned aluminum cylinder which is inside the outer case. For the Experimental Mock-up Converter the cold frame has 560 radially drilled holes which are subject to distortion when the radial band pressure, created by the stretching of the outer case, becomes excessive. Each radial hole contains a spring loaded piston (this part is referred to as a follower) which bears against a couple leg and establishes an electrical contact pressure at the hot and cold junction. Therefore, cold frame distortion is limited to approximately 0.0001 inch to avoid pinching the followers. The radial pressure exerted on the cold frame is limited by the thickness of the outer case wall and the yield point of the stainless steel as a result of the outer case being stretched beyond the yield point in the initial shrink fit assembly. Further stretching takes place in the heated condition when the aluminum cold frame expands more than the outer case due to the difference in the thermal coefficient of expansion between the two metals.

The outer case analysis involves a tapered design requested by Westinghouse, the design permitting a one piece fin assembly. This design was later discarded for a straight cylindrical shape to which a two piece fin assembly is clamped.

Results of this Design Analysis Task are summarized here; detailed calculations are presented in Appendix C.

3.2.2.1 Outer Case Design

An assembly of the fins mounted to the outer case is shown schematically in Figure 3-3. Because the outer case is tapered the fins must be installed from the bottom. The relation of the cold frame to the outer case and fins is also shown.

Figure 3-3. Outer Case-Fin Mounting

A top view of the outer case is shown in Figure 3-4 where it is noted that the wall thickness between the thickened taper portion is 0.60 inch. It is this thickness that limits the radial pressure on the cold frame.

Figure 3-4. Outer Case - Top View

The radial pressure exerted on the cold frame is analyzed by equating the hoop force in the thin section to the total force created by the radial pressure acting on the projected area of one half the outer case. The loading is shown in Figure 3-5.

Figure 3-5. Cold Frame - Outer Case Loading

By equating the hoop force (F) to the total pressure force (P), it is found that, for the hoop stress at the yield point, 389 psi is the maximum radial pressure that the outer case can exert on the cold frame.

3.2.2.2 Cold Frame Stress Analysis

The holes in the cold frame have very little metal between them. There are fourteen 0.219 inch diameter holes in a span of 3.75 inches (cold frame height) which leaves a web thickness between the holes of 0.0488 inch. Hole distortion is caused by the high stress in the thin webs. The stress in the webs is analyzed by equating the outer case hoop force to resisting web force. This loading is shown schematically in Figure 3-6. For simplification, only a diametrically opposite set of holes is shown in the cold frame sketch.

Figure 3-6. Cold Frame Force-Loading

By equating the outer case hoop force (F) to the total web forces (S_w), allowing for variation in web stress for changing web thickness, calculation of hole distortion indicates that the hole diametral shrinkage is approximately 0.00007 inch. This value is within the desired limit of 0.0001 inch.

3.2.3 HOT FRAME INTERFACE ANALYSIS

The hot frame interface has been analyzed from the standpoint of stress loads imposed during assembly and disassembly for 500 psi contact pressure, and for two separate cases of taper on the fuel container heat spud – the one case of a 2 degree taper; the other case, a 3 degree taper per side.

Results of this analysis are summarized below; detailed calculations and assumptions applied are presented in Appendix D. These results are presented in three sections. The first section considers the required assembly and disassembly forces. A second section involves a stress analysis of the jacking base threads which, by means of a special disassembly tool, are subjected to a static disassembly force. Bending stresses in the jacking base are analyzed in a third section.

3.2.3.1 Assembly and Disassembly Force Analysis

Figure 3-7 illustrates the engagement of the fuel capsule spud with the converter hot frame. The fuel capsule spud is pulled into the converter hot frame by means of the assembly bolt. Taper dimensions are defined by the top and bottom engagement radii, r_1 and r_2 ; and the taper height, h . The maximum allowable contact pressure of 500 psi will be used in the calculations. Forces acting on the fuel capsule spud during engagement are shown in Figure 3-8.

For assembly, the coefficient of friction is assumed to be 0.10. For disassembly, the coefficient of friction is assumed to be 1.0. This higher value for disassembly, though somewhat indeterminate, is based upon the probability of sticking after a long operating period at temperatures up to 1150°F – the use of heat transfer grease would probably have degraded lubricity; possible oxidation of the mating surfaces could tend to promote seizure.

Results of this analysis are listed in Table 3-3.

Figure 3-7. Converter Hot Frame - Fuel Capsule Spud Interface

Figure 3-8. Fuel Capsule Force Diagrams

Table 3-3. Assembly/Disassembly Forces for Two Taper Conditions

Taper, Degrees	Coefficient of Friction	Assembly Force, lbs.	Disassembly Force, lbs.
2	0.10	2840	-
3	0.10	3150	-
2	1.0	-	20200
3	1.0	-	19460

3.2.3.2 Jacking Base Threads Stress Analysis

The jacking base threads shown in Figure 3-7, have a pitch diameter of 1.75 inches and an engagement length of 0.500 inch which is the thickness of the jacking base. The number of threads per inch is 12. A possible disassembly technique involves screwing an internally threaded adapter into the jacking base. A long rod or bolt would then be screwed into the adapter until the bolt bears against the top of the fuel capsule spud. Spud disengagement is effected by turning the bolt and holding the adapter with a separate wrench so that the jacking base threads are not exposed to dynamic loading.

The shear strength of 347 stainless steel at room temperature is approximately 20,000 psi. At 1150°F, which is the approximate operating temperature, the shear strength is 47 percent of the room temperature value or 9,400 psi.

For a calculated shear area of 2 square inches (see Appendix D), the threads will theoretically shear when the disassembly force is 18,800 lbs.

3.2.3.3 Jacking Base Bending Stresses

The jacking base can be analyzed as a disc with a fixed periphery and a uniform load around the edge of hole as shown in Figure 3-9.

By solution for the maximum bending stress which occurs at the outer edge of the disc, and relating this stress to the total edge load around the hole, it is found that, for the condition of the jacking base at a temperature of 1150°F, the jacking base will yield when the disassembly force is approximately 7730 lbs. Since the theoretical disassembly force (19,460 lbs.) is considerably greater, the jacking base must either be thickened or else another technique must be considered for separating the fuel capsule spud from the converter.

Figure 3-9. Jacking Base Loading Diagram

3.2.4 THERMOELECTRIC COUPLE DESIGN

The design of the thermoelectric couples is based, to a large extent, on the geometry and technology used in the SNAP-23 converters. Many of these N- and P-leg parameters will be applied to the first converters built in the SNAP-23A Program. This includes leg diameters, materials of both the hot and cold segments, and the ratios of the cold segment length to total leg length. The SNAP-23 method of joining the cold segments to the hot segments will also be used in the manufacture of the legs.

Parameters which influence change in SNAP-23A leg geometry and number of couples include 60-watt power output requirement at end of 10-year design life, couple cold junction temperatures at beginning-of-life (BOL) and end-of-life (EOL), as established by the Westinghouse system heat sink design – couple hot junction temperatures, as established by the decay rate of the isotopic fuel; maximum allowable operating temperature for leg materials; and anticipated extraneous electrical resistance equal to 6.33 percent of total couple resistance.

A detailed design procedure for determining couple geometry and number of couples for a particular set of specifications is presented in Appendix E. The couple design procedure is performed with the application of two digital computer programs – a primary program which establishes couple geometry, and a secondary program which evaluates converter performance (of a given design) for conformance to the

given specifications. This latter computer program is similar to the analytical model for the computer program for converter performance – system integration studies which is described in Appendix F.

Table 3-4 shows the preliminary converter couple geometry established by this design procedure. Table 3-5 lists the performance characteristics calculated for the given converter design at both BOL and EOL conditions.

Table 3-4. Preliminary Converter Couple Geometry

Number of couples (N)	280
Length of P leg (L_p)	0.327 inch
Length of P leg segment (L_{cp})	0.114 inch
Diameter of P leg (D_p)	0.187 inch
Length of N leg (L_n)	0.479 inch
Length of N leg segment (L_{cn})	0.120 inch
Diameter of N leg (D_n)	0.187 inch

Table 3-5. Converter Performance Characteristics
Based on Preliminary Design Calculations

	Cold Junction Temper- ature °F	Hot Junction Temper- ature °F	Shunt Heat Loss Watts	Total Heat Input Watts	Con- verter Effi- ciency Per- cent	Load Voltage Volts	Open Circuit Voltage Volts	Power Output Watts
EOL	180	890	58.2	796	7.7	24.72	42.78	61.4
BOL	210	1075	75.2	1030	7.7	24	55.18	79.27

It should be noted that although the preliminary power output value at the end of 10 years is 61.4 watts, the exact design will be determined on the basis of further system integration. Information gathered on power degradation during the testing of this converter may affect the couple design for the Prototype Mockup Converter, depending upon the degradation allowance factored into that design.

The total heat input of 1030 watts represents the required heat input at BOL to achieve 79.3 watts. It is important that this figure be equal to or greater than the available heat from the isotopic fuel. In this case the capacity of the converter to absorb heat is greater than the available heat. Dividing the heat input at EOL of 796 watts by the 10-year decay factor for isotopic fuel of 0.782, results in 1018 watts available heat at BOL. Consequently, there is no danger of exceeding the maximum allowable operating temperature for the leg materials.

3.3 FABRICATION AND ASSEMBLY

The principal fabrication and assembly efforts were directed at long-term scheduling and planning. Toward this end, preliminary Manufacturing Planning Schedules were developed by the following sequence of work, as summarized in Figure 3-10.

A Master Fabrication Flow Plan for the converter was developed. This plan is a block-type fabrication chart which identifies all component parts by name and identifies the assembly and processing steps which will be required. This plan is used for overall comprehension of the subtask and for organization of fabrication details.

A preliminary Parts Release Requirements Listing was developed based upon the expected assembly and processing time and program schedule requirements. The preliminary Manufacturing Planning Schedule was developed which establishes the preliminary time base for each major function to be accomplished during the fabrication and assembly effort. These are:

1. Development of detailed fabrication flow plans
2. Material, parts orders
3. Material, parts received

"OLD" SNAP-23 CONVERTER DESIGN

Figure 3-10. Logical Development Sequence

4. Development and writing of Manufacturing Standard Instructions, Process Routings, and TPO's (Manufacturing Specifications)
5. Tooling Design
6. Tooling procurement, checkout
7. Parts manufacture, internal
8. Sub-assembly of converters
9. Final assembly of converters
10. Processing of converters (outgassing and backfilling)
11. Procurement of process and test stations
12. Procurement of vibration fixture
13. Design and procurement of shipping container

Based on the SNAP-23 development and known changes thereto, a preliminary list of the tooling which will be required was made. Early conceptual ideas for an improved assembly fixture were developed. The anticipated tooling list is shown in Table 3-6.

The potential problem areas were identified and action planned as follows:

Insulation Form and Processing Methods: If microquartz insulation is considered for use in the second generation converter design, a development study must be undertaken to determine the best processed form and processing methods for efficient packing. Min-K 1301 insulation will be used in the first converter design; no packing problems are anticipated with this insulation.

Process for Module Pinch-Off Tube: The method of pinching off the module tube must always provide an adequate seal. Design consideration has been requested on an alternate seal design.

Table 3-6. Anticipated Converter Tooling Requirements

Item	Tool Name	Quantity
1	Leg positioning jig, straight legs	1
2	Leg positioning job, angled legs	1
3	Assembly fixture, converter	1
4	Cold frame positioner	3
5	Couple rack	1
6	Mounting flange weld fixture	1
7	Pinch-off tool	1
8	Shrink fixture, outer case	1
9	Weld fixture or clamp, support cone to outer case	1
10	Weld fixture or clamp, support cone to outer case	1
11	Weld chill rings	1
12	Min-K packing tools	8
13	Process station manifolds	1
14	Feedthrough header leak test fixture	1
15	Outer case leak test fixture	1
16	Hot frame leak test fixture	1
17	Leak test fixture for checking one welded end of final assembly	1
18	Mold for Min-K or form cutting tool, depending on design selection	1
19	Min-K container manifold	1
20	Min-K processing container	3

Converter Cold Frame Machining Source: The Purchasing Department investigated sources for machining the converter cold frame.

Converter Cold Frame Tolerances: If the completed converter cold frame falls outside drawing tolerances, fabrication schedules could be jeopardized. Careful machine shop selection and specific instructions on tolerance levels will provide preventive action.

3.4 TEST AND ANALYSIS

During the first quarter, major test and analysis efforts centered around planning the test program for the SNAP-23A Program. The integrated test program planning was completed and incorporated into the SNAP-23A Program Plan. Briefly, the integrated test plan is divided into four areas. The first area, converter tests, will be discussed here; the remaining three areas are detailed in Task III – Thermoelectric Couple Improvement.

- Converter tests
- Post-test analysis plan
- Bell jar tests
- 10-couple module tests

The converter test program is intended to provide a comprehensive evaluation of the 60-watt SNAP-23A converter designs. Acceptance tests will be performed on the units destined for system integration by Westinghouse to validate the design of the converter and assure attainment of technical performance requirements. The converters which are retained by 3M will undergo various tests to determine their overall operational characteristics (including BOL, during and after exposure to design environmental conditions, and EOL), and to determine their long-term performance traits.

The test will be conducted by designing suitable bench test fixtures to support, thermally match, and electrically match the needs of the converter unit. Using a water jacket or other suitable cooling system and an electrical heat source, the hot and cold electrode temperatures will be adjusted to the appropriate values. Sufficient data will be recorded to yield electrical characteristics as a function of:

- Load resistance
- Heat input
- Temperature distribution throughout the converter

A detailed test matrix showing units to be tested and tests to be performed is presented in Table 3-7.

Table 3-7. SNAP-23A Converter Test Matrix (Preliminary)

Test Type	Experimental Mockup			Prototype Mockup		Remarks
	1 (3M)	2 (WANL)	3(3M)	1(WANL)	2(3M)	
Acceptance	C		C		C	
Vibration	C					MIL-STD-810A 514.1 Class 6.
Shock						MIL-STD-810A 516
Basic Design	C	System tests to be performed by Westinghouse		System tests to be performed by Westinghouse		Procedure I (5g's)
Transit Bench Handling						Procedure III
Environmental Humidity						Procedure VI
Pressure						
Salt Water						
Short Term Temperature						100%
Altitude						1 Atmosphere
Temperature Excursion			C			MIL-STD-810A (509)
Extended Long-Term Power Degradation	C C(P)		C C(P)		C C(F)	-80°F to +160° Air Transport (normal) One 100 hour at 1150°F

Legend: C = Converter
(P) = Preliminary
(F) = Final
(S) = System

3.5 RELIABILITY

During the first quarter primary reliability efforts consisted of planning and analyses for the following items:

- Reliability Program Plan
- Design Reviews, converter drawings
- Design Reviews, converter-system interface

A preliminary reliability plan was prepared. The "SNAP-23A Reliability Program Plan," which is in conformance with MIL-STD-785, "Requirements for Reliability Program," will be the governing 3M reliability document. The Reliability effort will provide participation in the SNAP-23 Converter Program:

- Early identification of potential problem areas
- A systematic procedure for selection of solutions to design problems which may occur
- Methods of monitoring and evaluating a design as it progresses from its initial concept
- A means for evaluation and selection of the best possible design trade-offs.

The converter-system interface drawing was reviewed as part of the design review activity. At the initial interface meeting, Westinghouse-proposed dimensions which did not agree with concurrent 3M design were determined. Possible changes to the converter design were then considered to comply with the preliminary system interface, and the differences which could not be resolved by 3M, due to design constraints, were defined and resolved at subsequent meetings.

Three drawing review meetings were held during the report period to review drawings prepared for the 10-couple modules and test fixtures. The first of these meetings defined the drawing review procedures. Reviews of twenty-five module drawings, all fixture drawings, were completed.

3.6 QUALITY CONTROL

During the first quarter, the Quality Program Plan was prepared and submitted to Westinghouse for concurrence. The Quality Program Plan outlines the procedures and documents to be used in assuring end-item acceptability. Quality inputs to engineering documents are made during the design cycle, prior to release, and into manufacturing control programs. Quality inputs to all other disciplines will also be made at times when they will be most effective. The quality program assures that all work affecting quality is prescribed in clear and complete documented instructions which are appropriate to a development effort.

Quality control surveys of prospective vendors was initiated. These surveys will be a continuing activity while establishing new vendors. Procedures for surveying, rating and establishing a list of qualified vendors are defined in the Quality Program Plan.

Preliminary effort was begun to establish Quality Assurance Standard Instructions for the SNAP-23A Program. These Standard Instructions will be used to supplement the Product Quality section procedures during their quality control functions.

4.0 TASK III-THERMOELECTRIC COUPLE IMPROVEMENT

4.1 INTRODUCTION

The thermoelectric couple improvement program will be conducted concurrently with the Baseline converter development program. This effort is intended to meet the following goals:

- Identify problem areas related to ten-year life thermoelectric leg stability.
- Develop alternate constructional features to eliminate problem areas.
- Test proposed alternates as candidate design inputs to the converter design effort.
- Document and record test data, methods, and test detail so that valid test analyses and conclusions may be generated for the converter design.

In order to meet these objectives, the thermoelectric couple improvement program will be conducted through three distinct approaches:

- Post-test analysis of SNAP-23 converter and module
- Bell jar testing to verify ingradient thermoelectric performance with respect to design objectives and specifications and to qualify new concepts for long-term performance evaluation

- 10-couple module testing for long-term performance evaluation of promising variables investigated in the bell jar tests.

The specific areas of investigation under this task are described in the following sections.

4.2 POST-TEST ANALYSIS PLAN

As a portion of the thermoelectric couple improvement effort, a post-test analysis will be conducted on SNAP-23 6-couple module A-2 and 252-couple prototype converter P-1. This examination will attempt to establish potential failure or degrading mechanisms within the converter and module. Pre-test, test and post-test data will be compiled and correlated for thermoelectric couples and adjacent hardware; conclusions and supporting data will be included in a final test report.

Principal areas of investigation include:

- Thermoelectric N- and P-leg performance changes
- P-leg hot junction electrical contact resistance
- Metallurgical changes within thermoelectric couple hardware
- Cold end thermal impedance
- Changes in gas environment
- Chemical and physical changes in the internal thermal insulation

Several secondary problem areas will be investigated, including long-term hot junction temperature measurement, electrical feedthroughs, and possible cold frame follower seizing.

The post-test analysis plan will be submitted as a separate document under the SNAP-23A Program.

4.3 BELL JAR TEST EFFORTS

The SNAP-23A bell jar tests are short-term tests designed to verify ingradient thermoelectric performance with respect to design objectives and specifications and to qualify new concepts for long-term performance evaluation in the 10-couple module portion of the development program.

Initial testing will be conducted on legs and couples of the baseline design, with subsequent testing conducted on new legs and/or couples evolving from development efforts. This effort will also include recharacterization of the thermoelectric properties of baseline materials (S , ρ , k), necessitated by improvements made in the manufacture of the materials.

Specific variables to be investigated on a short-term basis are:

- Leg manufacturing technique
- Soft-nose P-legs
- Hot junction contact pressure

As the development effort progresses, other variables may be investigated to determine thermoelectric reproducibility, stability and efficiency. These new ideas will be screened in the short-term test equipment.

Residual bell jar test fixtures were procured during the quarter. The bell jar test plan will be prepared in April and testing is scheduled to begin during the next reporting period.

4.4 10-COUPLE MODULES

This phase of the thermoelectric couple improvement program will consist of long-term performance evaluation tests conducted in 10-couple module test vehicles. The primary purpose of the long-term tests will be to determine the stability characteristics of both the Baseline design and new concepts.

A secondary objective of these tests will be to demonstrate reproducibility in a thermopile configuration. As discussed previously, short-term bell jar test work will be used to qualify new concepts for inclusion in this long-term stability test program.

4.4.1 TECHNICAL APPROACH – SUMMARY

Major effort during the first quarter was concentrated on test planning, design and analysis of the SNAP-23A 10-couple modules. The fundamental objectives of this effort have been:

1. To establish specific variables to be investigated and evaluated in the thermoelectric couple improvement program.
2. To design 10-couple modules which will enable determination of effects of variations in thermoelectric materials, leg fabrication techniques, related couple hardware and thermal insulation on overall thermoelectric performance.
3. To design test panels and related equipment which will provide meaningful data for the evaluation of the 10-couple module test parameters.

Twelve 10-couple modules will be fabricated and tested; specific variables to be investigated on a long-term basis include:

- Soft nose P-legs
- Thermal insulation
- Hot electrode temperature
- Hot junction contact pressure
- Hot electrode cap material – alternate

The SNAP-23A design will be based upon the 10-couple module designed for the SNAP-27 Program. That design will be modified to accept SNAP-23A legs, couples and related hardware and to incorporate any alternate leg, couple or hardware design changes suggested in the development program. The SNAP-27 10-couple

module heat sink will be altered to accommodate SNAP-23A hardware and to provide the proper cold junction temperature. The required instrumentation and test panel will be determined.

4.4.2 TEST PLAN DEVELOPMENT

In the initial test plan six major items (variables) were proposed for investigation and evaluation; a matrix of the combinations of these variables was proposed as they would be evaluated in ten 10-couple modules (Appendix G). Two additional modules were proposed – one module to be used for thermal analysis, and another module to incorporate the best features of modules 1 through 10.

Of the total of 12 modules proposed, four definitive combinations (modules) were established to represent both the baseline and the back-up designs and as controls or standards against which the remaining eight modules would be compared.

Further review of that test plan provided a well-defined, experimentally-sound analytical approach to the ten-couple module test plan and its definition of and combination of those variables under consideration in the thermoelectric couple improvement program. This plan is shown in Table 4-1, and lists the complete 10-couple module variables developed through full factorial test design. Through analytical treatment of test results, this plan should eliminate unintentional biasing of results and aid in better, more complete evaluation of the variables under consideration.

Table 4-1. 10-Couple Module Variable Matrix

Module No.	Couple No.	P-Leg	N-Leg	Thermal Insulation	Hot Contact Material	Contact Pressure	Hot Junction Temperature	Comments
1 & 2	1	SP, N	SP, N	MK	Fe	250	1100	Baseline Design Long-term design evaluation tool
	2	B, N	B, N					
	3	SP, N	SP, N					
	4	B, N	B, N					
	5	SP, N	SP, N					
	6	B, N	B, N					
	7	SP, N	SP, N					
	8	B, N	B, N					
	9	SP, N	SP, N					
	10	B, N	B, N					
3 & 4	1	SP, H	SP, N	MQ	Fe	150	1100	Backup Design Long-term backup design evaluation
	2	B, H	B, N					
	3	SP, H	SP, N					
	4	B, H	B, N					
	5	SP, H	SP, N					
	6	B, H	B, N					
	7	SP, H	SP, N					
	8	B, H	B, N					
	9	SP, H	SP, N					
	10	B, H	B, N					
5	1	SP, H	SP, N	MQ	XXX	XX	X	Factorial test design
	2	B, N	B, N		Fe			
	3	B, H	B, N		XXX			
	4	B, N	B, N		XXX			
	5	SP, N	SP, N		XXX			
	6	B, H	B, N		Fe			
	7	SP, N	SP, N		Fe			
	8	B, H	SP, N		XXX			
	9	B, N	SP, N		Fe			
	10	SP, H			Fe			

Table 4-1. 10-Couple Module Variable Matrix (Continued)

Module No.	Couple No.	P-Leg	N-Leg	Thermal Insulation	Hot Contact Material	Contact Pressure	Hot Junction Temperature	Comments
6	1	SP, N	SP, N	MQ	XXX	XX	1100°F	Factorial test design
	2	B, H	B, N		XXX			
	3	SP, N	SP, N		Fe			
	4	B, N	B, N		XXX			
	5	B, H	B, N		XXX			
	6	B, H	B, N		Fe			
	7	B, H	B, N		Fe			
	8	SP, N	SP, N		Fe			
	9	SP, H	SP, N		Fe			
	10	SP, H	SP, N		XXX			
7	1	B, H	B, N	MQ	Fe	250	X	Factorial test design
	2	SP, H	SP, N		XXX			
	3	SP, N	SP, N		Fe			
	4	SP, H	SP, N		Fe			
	5	B, N	B, N		Fe			
	6	B, H	B, N		XXX			
	7	B, H	B, N		XXX			
	8	SP, N	SP, N		Fe			
	9	SP, N	SP, N		XXX			
	10	B, N	B, N		XXX			
8	1	SP, N	SP, N	MQ	Fe	250	1100°F	Factorial test design
	2	B, H	B, N		Fe			
	3	SP, N	SP, N		XXX			
	4	B, N	B, N		Fe			
	5	SP, N	SP, N		Fe			
	6	B, H	B, N		XXX			
	7	B, H	B, N		XXX			
	8	B, N	B, N		XXX			
	9	SP, H	SP, N		Fe			
	10	SP, H	SP, N		XXX			

Table 4-1. 10-Couple Module Variable Matrix (Continued)

Module No.	Contact No.	P-Leg	N-Leg	Thermal Insulation	Hot Contact Material	Contact Pressure	Hot Junction Temperature	Comments
9	1	B, N	B, N	MK	Fe	XX	X	Factorial test design
	2	SP, H	SP, N		Fe			
	3	B, H	B, N		Fe			
	4	SP, N	SP, N		XXX			
	5	B, H	B, N		XXX			
	6	SP, N	SP, N		Fe			
	7	SP, H	SP, N		XXX			
	8	B, N	B, N		XXX			
	9	SP, N	SP, N		Fe			
	10	B, H	B, N		XXX			
10	1	SP, N	SP, N	MK	Fe	XX	1100°F	Factorial test design
	2	SP, H	SP, N		XXX			
	3	B, H	B, N		XXX			
	4	B, H	B, N		Fe			
	5	B, H	B, N		XXX			
	6	B, N	B, N		XXX			
	7	SP, N	SP, N		Fe			
	8	SP, H	SP, N		Fe			
	9	B, N	B, N		Fe			
	10	SP, N	SP, N		XXX			
11	1	SP, N	SP, N	MK	XXX	250	X	Factorial test design
	2	B, H	B, N		XXX			
	3	B, N	B, N		Fe			
	4	SP, H	SP, N		XXX			
	5	SP, N	SP, N		Fe			
	6	SP, H	SP, N		Fe			
	7	B, N	B, N		XXX			
	8	B, H	B, N		Fe			
	9	SP, N	SP, N		Fe			
	10	B, H	B, N		XXX			

Table 4-1. 10-Couple Module Variable Matrix (Continued)

Module No.	Contact No.	P-Leg	N-Leg	Thermal Insulation	Hot Contact Material	Contact Pressure	Hot Junction Temperature	Comments
12	1	SP, N	SP, N	MK	XXX	250	1100°F	Factorial test design
	2	B, H	B, N		Fe			
	3	B, N	B, N		XXX			
	4	B, H	B, N		XXX			
	5	B, N	B, N		Fe			
	6	SP, N	SP, N		Fe			
	7	SP, N	SP, N		Fe			
	8	SP, H	SP, N		Fe			
	9	B, H	B, N		XXX			
	10	SP, H	SP, N		XXX			

KEY:

SP – Baseline Leg Manufacturing technique

B – Alternate Leg Manufacturing Technique

N – Normal

H – Not Segmented

MK – MIN-K

MQ – Microquartz

Fe – Baseline (iron) hot contact caps

XXX – Alternate hot contact material (to be selected on basis of short-term test work)

XX – Alternate Hot contact pressure (to be selected on basis of short-term test work)

X – Alternate hot junction temperature (to be selected on basis of short-term test work)

4.4.3 10-COUPLE MODULE DESIGN

While a majority of the SNAP-23A 10-couple module hardware has been designed and is similar to that of the SNAP-27 10-couple module, several areas required evaluation and design effort. The following analyses pertinent to the design of the 10-couple module have been completed:

- Thermoelectric couples
- Follower springs
- Center post-structural support
- Heater block and electrical heaters
- Electrical feedthroughs
- Insulation materials

The results of these analyses are summarized in this section. Complete discussions of all of the above items are contained in Appendix H.

4.4.3.1 Thermoelectric Couples

The geometry of the SNAP-23A thermoelectric couples is nearly identical to the geometry of the SNAP-23 thermoelectric couples. Sixteen couple types were designed to determine the optimum performance possible from the combinations of two types of N-legs, four types of P-legs, and instrumentation variations. The combinations of these elements considered both baseline and alternate fabrication techniques.

4.4.3.2 Follower Springs

The objective of this design was to provide follower springs with nominal electrode contact pressures of 150 psi or 250 psi on each thermoelectric leg when assembled in the 10-couple module.

Follower spring design specifications were determined as shown in Table 4-2.

Table 4-2. Follower Spring Design Specifications

Contact Pressure	150 psi	250 psi
Material	Stainless steel wire, AISI – Type 302 spring temper.	
Working Conditions:		
To work in:	0.218 inch diameter hole (max.)	
To work over:	0.08 inch diameter shaft (min.)	
Load	4.12 lbs $\pm$ 0.40 lb. between length of 0.927 and 0.830 inch	6.87 lbs $\pm$ 0.55 lbs. between length of 0.927 and 0.830 inch
Maximum Solid Height	0.800 inch	
Maximum Free Length	1.500 inch	
Operating Temperature	250°F	
Type of Ends	Ground and Squared	
Reference Design Data:		
Wire Diameter	0.32 inch	
Coil Outside Diameter	0.203 inch	
Rate	8.0 lb/inch	11.3 lb/inch
Free Length	1.394 inch	1.487 inch
No. of Coils	24 (32 active)	18 (16 active)

4.4.3.3 Center Post

The center post configuration was evaluated for use with 250 psi follower springs pressure on the thermoelectric legs, with particular emphasis on strength in the critical region near the hot end, where post area is minimum (0.0154 in.²).

The structural strength of the SNAP-27 center post was increased through the use of 1/4 - 28 - UNF - 2A thread on each end, thereby increasing the nominal minimum diameter to 0.190 inch. Heat conductance through the post from hot to cold side is not increased because the diameter of the spacing portion of the center post remained the same.

The ultimate design factor of 1.80, based on a creep strength of 10,000 psi for 304 stainless steel and a calculated stress factor of 5,550 psi, assures that adequate strength is designed into the center post in the critical stress region near the hot end.

4.4.3.4 Heater Block and Electrical Heaters

In the SNAP-23A 10-couple module, the couples are arranged in four rows. To provide uniform heating to all couples, two heaters (3/8 inch diameter x 2.00 inch long) were provided, one for each two rows.

At the 1150°F module operating temperature, maximum heater watt density varies according to the "fit" or clearance between the hole and the heater. Allowable watt densities for the selected heaters are:

- 165 watts/in² at 0.002 inch fit, 1150°F
- 60 watts/in² at 0.009 inch fit, 1150°F

Thermal analyses indicated that a minimum of 50-60 watts heater block power was required. Assuming that each heater provides 30-watts, watt density is calculated to be approximately 24-watts/in². This is well below the allowable maximum fit (0.002 inch), indicating that under normal operating conditions, long heater life can be expected.

4.4.3.5 Electrical Feedthrough

Electrical feedthroughs were selected to provide leak-proof seals and electrical insulation for instrumentation and power leads. The design selected for the power seals uses an oversize transducer gland with bare wire, covered by a flexible teflon sleeve fed through the sealing gland. While voltage breakdown should be eliminated through the use of sleeving, proper sealing may still be a problem. This method, and alternate solutions, will be tested before actual use in the 10-couple modules.

4.4.3.6 Insulation Materials

Microquartz and Min-K insulation will be evaluated to determine their comparative effects on thermoelectric performance. To maintain specific thermal conductivity values, these materials must be packed to a specified density, and thus calculations were made to determine these values.

A total packing volume of 70.9 in³ was calculated for the 10-couple modules. Modules using Min-K insulation (which also use shredded microquartz near the top and bottom plates) will require 177 grams of insulation. Microquartz units will require 55 grams for insulation purposes.

4.4.4 10-COUPLE MODULE THERMAL ANALYSIS

The objective of this thermal analysis was to provide design criteria for the 10-couple module to assure that proper hot and cold junction temperatures will be maintained.

The basic SNAP-27 10-couple module proposed for use in this program, the SNAP-23A module design #1, and the SNAP-23A module design #2 were each analyzed in four steps, as follows:

- a. The operating temperatures and heat flow were determined for the total thermopile.
- b. Heating requirements and heat losses of the hot side were calculated.
- c. Heating requirements and heat losses of the cold side were determined.
- d. The heat sink was analyzed; heat balance among the hot end, thermopile, cold end and heat sink was determined.

A brief summary of the results of these analyses is presented in Table 4-3. Detailed calculations for each design are presented in Appendix I.

The additional heat sink capacity required for SNAP-23A designs #1 and #2 will be provided by the addition of 12 spines to the module base, extending them into pedestal cooling water. These spines will provide an additional 14.4 watts of sink capacity.

Table 4-3. Results of 10-Couple Module Thermal Analysis

	Q'_{in} (watts per couple)	Q'_{out} (watts per couple)	Q_{in} (total watts)	Q_{out} (total watts)	Additional Heat Sink Capacity Required (watts)
SNAP-27	2.71	2.86			--
Thermopile			38.2	34.8	
Upper Heater Block			51.1	38.2	
Lower Heater Block			35.8	30.8	
SNAP-23A DESIGN #1	3.00	2.75			1.4
Thermopile			42.6	37.6	
Upper Heater Block			51.5	42.6	
Lower Heater Block			37.6	37.4	
SNAP-23A DESIGN #2	3.00	2.75			15.2
Thermopile			49.2	42.2	
Upper Heater Block			72.2	49.2	
Lower Heater Block			42.2	41.6	

4.4.5 INSTRUMENTATION AND TEST PANEL DESIGN

4.4.5.1 Couple Instrumentation

To provide a maximum amount of meaningful data, the couples are instrumented for temperature on the hot and cold ends and each leg is instrumented for voltage. Cold end instrumentation is directly attached to the thermoelectric couple and hot end instrumentation is attached to the hot buttons which are in contact with the thermoelectric elements.

Thermocouples are placed in the upper heater block and cold frame for added reference points. Voltage taps, in addition to output leads, are attached to the terminal legs of the circuit. Figure 4-1 illustrates the instrumentation layout for the two independent odd and even circuits.

4.4.5.2 Test Panel

The test panel design is similar to the SNAP-27 10-couple panel with modifications in electrical hardware to account for differences in instrumentation and load circuit.

The cabinet hardware consists of a base plate, two side panels, a chassis, face plate, jack bracket, edging strips, heat shield and screws, as illustrated in Figure 4-2.

Electrical hardware consists of the following:

- 4 - 12 position - 2 pole switches
- 2 - variable transformers for hot and cold end heaters
- 2 - wattmeters for indicating hot and cold end power inputs
- 2 - shunts for measuring current in the odd and even circuits
- 1 - elapsed time meter
- 2 - potentiometers for even and odd circuit loads

Various accessories such as switches, terminals, knobs, jacks, screws, etc.

4 VOLTAGE TAPS 2P, 1P, 9N, 10N (4 LEADS)
 3 HOT FRAME T/C RELATIVE POSITION 5P, 4N, 2P (6 LEADS)
 1 COLD FRAME T/C
 10 HOT SHOE (20 LEADS)
 8 COLD CAP (16 LEADS)

Figure 4-1. Instrumentation Array 10-Couple Module SNAP-23A Phase I

Figure 4-2. 10-Couple Module Test Cabinet

One primary design problem was encountered in selection of the load rheostat. The desired item was a stable, high resolution, low resistance rheostat with a fairly high current capacity, to be used in determining performance of the 5-couple circuits.

A variable resistance R_2 is desired to allow plotting of performance (similar to the P-curve in Figure 4-3 and as listed in Figure 4-4). Current capacity and watt-dissipation of the rheostat are critical items.

The rheostat selected is a ten-turn potentiometer for high resolution. The maximum current rating for a single pot is 2.25 amps and power rating is 5 watts at 40°C (104°F). To obtain current values greater than 2.25 amps requires 2 ganged units connected in parallel, thereby reducing the current through a single unit to tolerable values.

Figure 4-3. 5-Couple Circuit Parameters versus Current

5-COUPLE CIRCUIT

R_2	$R_T = R_1 + R_2$	$E/R_T = I$	$V = IR_2$	$P = IV$
0.1	0.3	3.33A	0.333V	1.0 WATT
0.2	0.4	2.50A	0.500V	1.25 WATT
0.3	0.5	2.00A	0.600V	1.20 WATT
0.4	0.6	1.67A	0.666V	1.11 WATT
0.5	0.7	1.43A	0.715V	1.02 WATT
0.6	0.8	1.25A	0.750V	0.93 WATT
0.7	0.9	1.11A	0.775V	0.86 WATT
0.8	1.0	1.00A	0.800V	0.80 WATT
0.9	1.1	0.91A	0.825V	0.75 WATT
1.0	1.2	0.83A	0.832V	0.69 WATT

Figure 4-4. 5-Couple Circuit Performance Parameters versus Variable Resistance

4. 4. 6 FABRICATION AND ASSEMBLY

Primary fabrication and assembly emphasis during this quarter for the SNAP-23A 10-couple module was placed on planning and scheduling efforts.

A Master Fabrication Flow Plan was established as a block-type sequence chart showing all of the parts and the principal assembly and process operations which are needed for the fabrication of the SNAP-23A 10-couple module. At this stage parts are identified by name only and operations by one word description, i. e. , weld, assemble, etc. This chart is based on a previous design of the SNAP-27 10-couple module which was revised as the SNAP-23A design was finalized.

From the Master Fabrication Flow Plan, a preliminary list of 10-couple module parts and the time required for production or procurement of these parts was tabulated.

Based on program requirements, assembly and processing time estimates, and the parts fabrication time requirement, a preliminary Manufacturing Planning Schedule was detailed. This schedule time-framed the following principal activities of the IPP Manufacturing Section:

1. Establishing detailed Fabrication Flow Plans
2. Ordering Materials, parts
3. Receiving Materials, parts
4. Writing Manufacturing specifications (Manufacturing Standard Instructions, Process Routings, and TPO's).
5. Tooling Design
6. Tooling procurement and check out
7. Parts manufacture, internal
8. Subassembly of each module
9. Final assembly of each module

10. Processing of each module
11. Procurement of test cabinets

The parts production or procurement time requirement listing identifies the long-lead items. Orders for these parts were placed consistent with the Manufacturing Planning Schedule.

A list of the probable tooling requirements was established. Tooling used on the SNAP-17 10-couple module program will be used where possible and action has been taken to obtain the use of this tooling.

Preliminary detailed Fabrication Flow Plans were made for the assembly of the modules. These detailed plans will be the principal assembly instructions and fabrication document for 10-couple modules.

4.4.7 QUALITY CONTROL

Primary efforts during the first quarter were concerned with the planning stages of this program. A list of quality class and types of inspections was developed for use during the various phases of the thermoelectric couple improvement effort.

- Experimental Development
- Engineering Development and Test
- Manufacturing Development
- Production

Experimental Development

During this phase, raw material receiving inspection (Surveillance of Certification) and in-process inspection (limited to surveillance of operating parameters) will be performed. Materials evaluation testing will include EMF and resistivity tests to evaluate samples for continued processing, and physical, dimensional and electrical inspection of samples.

Data from this testing and surveillance will be used to formulate tentative in-process and acceptance specifications.

Engineering Development and Test

The engineering development and test phase is that part of the overall program whereby the tentative specifications are applied to the product and process for evaluation, leading to preliminary manufacturing specifications.

During this time, receiving inspection will include acceptance, certification, and re-certification as required. In-process inspection will include 100 percent weight verification and surveillance for material reaction and pertinent operating processes. Material acceptance tests for material batches incorporate surveillance on pressing and sintering material samples, documented with inspection buy-offs at various steps in the process, and performance of EMF and resistivity tests as required by design criteria. The inspection tests for the remaining batch material include surveillance of pressing and sintering on thermoelectric legs and surveillance of couple fabrication. All samples used for acceptance testing will be 100 percent inspected for physical and dimensional requirements, and will be electrically inspected to appropriate AQL's.

Data from testing, verification and surveillance will provide inputs for formulation of revised quality control in-process and acceptance specifications.

Manufacturing Development

The quality control aspect of manufacturing development will consist of that part of the overall program whereby the preliminary manufacturing specifications are administered and updated to obtain desired objectives and to derive the type of inspection necessary for the fabrication of item(s) in a production program.

Receiving inspection will include acceptance, certification and recertification as required from the sample held out. In-process inspection will include 100 percent weight verification, surveillance of material reaction and pertinent operating processes, and surveillance of couple fabrication, documented with inspection buy-off at various steps in the process. Material acceptance tests for material batches will include surveillance of pressing and sintering of material samples, and through appropriate sampling techniques, EMF and resistivity tests as defined by design requirements. The remaining batch material will be surveyed for pressing and

sintering of thermoelectric legs and couple fabrication. Electrical testing to appropriate AQL's and 100 percent physical and dimensional inspection will be performed.

From these three preceding phases, final classes and types of inspections will be prepared. As sufficient data is obtained to substantiate their approval and use, sampling plans will be prepared.

Production

The procedures and criteria used in the production phase have been developed from the preceding phases. They consist of that part of the overall program whereby the procedures for quality assurance have been adequately defined and implemented to assure the required level of quality is maintained.

Receiving and in-process inspection, material acceptance tests for material batches and processing procedures for remaining batch material, will follow the above descriptions.

5.0 TASK IV-25- AND 100-WATT CONCEPTUAL DESIGNS

No effort was expended on this task during this quarter.

APPENDIX A

CONVERTER WELDMENT STRESS ANALYSIS

An analysis of the effects of vibration at the maximum test frequency of 52 cps (MIL-STD-810) has been completed. It was found that for vibration in the vertical direction, a magnification factor of 1.062 could be experienced. For the transverse direction the factor was found to be approximately 1.01, and for the sake of uniformity in design, the 1.062 figure will be brought into the new design safety factor. In addition, a support cone thickness of 0.025 inch will be used for compatibility with the design safety factor considered in this analysis.

In accordance with MIL-STD-810, total test cycling time will be 2 hours. For a maximum frequency of 52 cps, assuming a constant frequency for the entire test duration (conservative since actual vibration is sinusoidal) gives total number of cycles = $52 \times 2 \times 60 \times 60 = 3.74 \times 10^5$.

From the International Nickel Company data book on Cr-Ni Stainless Steel, Table VII of Section 1 Bulletin A, lists the following values for tensile yield strength and "endurance limit" in bending.

<u>ALLOY</u>	<u>F_{TY}</u> <u>(psi)</u>	<u>END. LIMIT</u> <u>(psi)</u>	<u>END. LIMIT</u> <u>F_{TY}</u>
301	40,000	35,000	0.875
302	40,000	34,000	0.850
303	35,000	35,000	1.000
321	35,000	38,000	--
347	40,000	39,000	0.975
304	35,000	35,000	1.000

From the above, the minimum ratio of endurance limit to tension yield stress is 0.850. We will use this ratio for our design purposes even though our two materials are 304 and 347. These data are room temperature properties.

The number of cycles calculated for vibration tests is actually quite limited. If we use the material endurance limit as the basis for design, the number of cycles of stress reversal has no effect (by definition of endurance limit).

A basic safety factor of 1.5 will be used with the 0.85 factor reduction for fatigue, and the 1.062 magnification factor for vibration.

$$\frac{1.5 \times 1.062}{0.85} = 1.875$$

For convenience we arbitrarily increase this to 2.0 for our "Design" factor. This, together with employment of minimum material properties will give us reasonable confidence in the structural integrity of the generator-fin support system.

With a 10-g load factor, the design yield load becomes (for the 112-lb. weight of the supported system): $2 \times 10 \times 112 = 2240$ lbs.

For vertical acceleration this load is applied uniformly (W_o lb/in.) around the circumference of the outer cylinder as shown below, and resisted by a uniform reaction in the inner cylinder (W_i lb/in.).

$$W_o = \frac{2240}{2 \times 3.1416 \times 4.65} = 76.7 \text{ lb/in.}$$

$$W_i = \frac{2240}{2 \times 3.1416 \times 1.87} = 191 \text{ lb/in.}$$

Direction of forces shown are for downward acting acceleration and are reversed if acceleration acts in the opposite direction.

For conditions shown in the sketch, there will be tension loading in the direction of a cone element (meridional direction)

$$1) \text{ at radius } r_o, q_o = \frac{W_o}{\sin 30^\circ} = \frac{76.7}{0.50} = 153.4 \text{ lb/in.}$$

$$\text{giving a unit stress } \frac{q_o}{t} = \frac{153.4}{0.025} = 6,140 \text{ psi (T)}$$

$$2) \text{ at radius } r_i, q_i = \frac{W_i}{\sin 30^\circ} = \frac{191}{0.50} = 382 \text{ lb/in.}$$

$$\text{giving a unit stress } \frac{q_i}{t} = \frac{382}{0.025} = 15,280 \text{ psi (T)}$$

The next step is to determine the effect of internal pressurization in the annular space between the inner and outer cylinders. It is assumed that the converter could be subjected to less than sea level ambient pressure, and could also be subjected to g-loadings from vertical accelerations and/or vibration, therefore, the pressure effect will be combined with g-loading to arrive at maximum design stress levels. Assume an altitude of 40,000 ft on a "standard cold day" with ambient pressure of 2.29 psi. Under these conditions with an initial internal pressurization of 25 psia, the pressure differential will be $(25 - 2.29) = 22.71$. Both the initial pressure and its effects on the structure are more accurately defined than other loads, and the safety factor utilized need not be as high. Arbitrarily adopting 1.33 as the safety factor gives $1.33 \times 22.71 = 30.2$ psi as the design pressure differential.

Hoop (circumferential) stresses in the cone will be

$$f_H = \frac{pr}{t \sin 30^\circ} :$$

at radius r_o ;

$$f_{Ho} = \frac{30.2 \times 4.65}{0.025 \times 0.50} = 11,220 \text{ psi (T)}$$

at radius r_i ,

$$f_{Hi} = \frac{30.2 \times 1.87}{0.025 \times 0.50} = 4,520 \text{ psi (T)}.$$

Meridional stresses for the cone are given by

$$f_M = \frac{pr}{2 t \sin 30^\circ} :$$

at radius r_o ;

$$f_{Mo} = \frac{30.2 \times 4.65}{2 \times 0.025 \times 0.5} = 5,610 \text{ psi (T)}$$

at radius r_i ;

$$f_{Mi} = \frac{30.2 \times 1.87}{2 \times 0.025 \times 0.5} = 2,260 \text{ psi (T)}.$$

Total Meridional Stress = $f_M + \frac{q}{t}$: at r_o , $f_{xo} = 5,610 + 6,140 = 11,750 \text{ psi (T)}$

Hoop stress = $f_{Ho} = f_\phi = 11,220 \text{ psi (T)}$. f_x and f_ϕ are combined using the

Hencky-VonMises criterion (Maximum Distortion-Energy Theory):

$$f = (f_\phi^2 + f_x^2 - f_\phi f_x)^{1/2}$$

$$f_o = [(11,220)^2 + (11,750)^2 - (11,220 \times 11,750)]^{1/2} = 11,500 \text{ psi (T)}$$

at r_i ,

$$f_{xi} = 2260 + 15,280 = 17,540 \text{ psi (T)}$$

$$f_{\phi i} = 4,520 \text{ psi (T)}$$

$$f_i = [(4,520)^2 + (17,540)^2 - (4,520 \times 17,540)]^{1/2} = 15,770 \text{ psi (T)}.$$

Margins of safety for the condition shown in the sketch on page A-2 combined with internal pressure are calculated below. Material of the cone is annealed, type 304 CRES. The minimum guaranteed room temperature properties for this material are taken from MIL-HDBK-5 and are as follows:

$$\text{tension yield: } F_{TY} = 30,000 \text{ psi}$$

$$\text{compression yield: } F_{CY} = 35,000 \text{ psi}$$

Effects of temperature on the yield strengths of the material are not too well documented. From several available sources, information for several of the similar type 300 Series materials was examined and a lower envelope curve was established for tension and compression yield allowable as a percentage of room temperature values (Figure A-1). This design curve is conservative, and is presented on page A-6.

Allowable tension yield stress at 180° F:

$$F_{TY 180} = 0.84 \times 30,000 = 25,200 \text{ psi.}$$

Allowable tension yield stress at 600° F:

$$F_{TY 600} = 0.612 \times 30,000 = 18,360 \text{ psi.}$$

At r_o ,

$$M.S. = \frac{25,200}{11,500} - 1 = +1.19$$

Figure A-1. Effect of Temperature on Yield Strength of Annealed Series 300 Corrosion Res. Steel

At r_i ,

$$M.S. = \frac{18,360}{15,750} - 1 = +0.16$$

For the acceleration acting upward, internal pressure induced tension stresses, in both the hoop and meridional directions, will be the same as found on page A-4:

$$\text{at } r_o: f_{HO} = 11,220 \text{ psi (T)}; f_{MO} = 5,610 \text{ psi (T)}$$

$$\text{at } r_i: f_{HI} = 4,520 \text{ psi (T)}; f_{MI} = 2,260 \text{ psi (T)}.$$

Acceleration forces produce compression stresses in the meridional direction in the cone. Two cases must be examined for this condition.

1. The combination of reduced compression stress in the meridional direction plus hoop tension. (Note: In a bi-directional stress field, like stresses give a combined stress equal to or less than either one. Unlike stresses give a combined stress greater than either one.)
2. With internal pressure absent, maximum compression-stresses exist in the cone.

Case 1

At r_o , net meridional stress $f_{MO} = -6140 + 5160 = -530 \text{ psi (C)}$ and $f_{HO} = 11,220 \text{ psi (T)}$ as before.

Since consideration is given to both tension and compression and the allowable yield values are different, it is necessary to use the more fundamental form of the Hencky-Von Mises criterion:

$$M.S. = \frac{1}{(R_x^2 - R_x R_y + R_y^2)^{1/2}} - 1 \quad (\geq 0)$$

where

$$R_x = \frac{f_x}{F'} \quad \text{and} \quad R_y = \frac{f_y}{F''}.$$

For this case, F'_x = meridional stress = f_{MO} = 530 psi (C)

$$f' = \text{allowable compression yield stress at } 180^\circ \text{F} = 0.918 \times 35,000 = 32,100 \text{ psi.}$$

$$f_y = \text{hoop stress} = f_{HO} = 11,220 \text{ psi (T)}$$

$$F'' = \text{allowable tension yield stress at } 180^\circ \text{F} = 0.84 \times 30,000 = 25,200 \text{ psi.}$$

Thus,

$$R_x = \frac{-530}{32,100} = -1.65 \times 10^{-2};$$

$$R_y = \frac{11,220}{25,200} = 0.446;$$

$$M.S. = \frac{1}{[(-1.65 \times 10^{-2})^2 - (-1.65 \times 10^{-2} \times 0.446) + (0.446)^2]^{1/2}} - 1$$

$$M.S. = +1.20.$$

At r_i , net meridional stress $f_{Mi} = -15,280 + 2,260 = -13,020$ psi (C), and $f_{Hi} = 4,520$ psi (T);

$$\text{Temp.} = 600^\circ \text{F and } F' = 0.682 \times 35,000 = 23,900 \text{ psi,}$$

$$F'' = 0.612 \times 30,000 = 18,360 \text{ psi:}$$

$$R_x = \frac{-13,020}{23,900} = -0.545;$$

$$R_y = \frac{4,520}{18,360} = 0.246;$$

$$M. S. = \frac{1}{[(-0.545)^2 - (-0.545 \times 0.246) + (0.246)^2]^{1/2}} - 1 = +0.42.$$

Case 2 (no pressurization)

At r_o ,

$$f_{MO} = \frac{q_o}{t} = -6,140 \text{ psi (C)},$$

$$M. S. = \frac{32,100}{6,140} - 1 = +4.23.$$

At r_i ,

$$f_{MI} = \frac{q_i}{t} = -15,280 \text{ psi (C)},$$

$$M. S. = \frac{23,900}{15,280} - 1 = +0.56.$$

Buckling Strength of Cone

Two cases will be assessed:

Case 1.

Compression stress in the cone with no internal pressure. The method of evaluating the allowable axial compression load is taken from STL/TR-60-0000-19425, which gives $P_{cr} = 2\pi CE't^2 \cos^2 \alpha$, where C is determined from the curve in Figure A-2, and $E' = \sqrt{E_T E_S}$; both the tangent modulus E_T , and secant modulus E_S are dependent on stress level and temperature. We will conservatively adopt temperature and stress level for conditions at the inner cylinder at radius r_i for determining E_T and E_S . The graphs presented in Figures A-3, A-4, and A-5 give material properties applicable to types 304 and 347 CRES.

Figure A-2. Critical Buckling Loads for Cones in Compression

Figure A-3. Stress vs Strain Typical for Types 304 and 347 Cres. Room Temperature, Compression, Plate or Bar Stock, Transverse Direction

Figure A-4. Stress vs Tangent and Secant Moduli Typical for Types 304 and 347 Cres. at Room Temperature for Plate or Bar Stock in Compression in Transverse Direction

Figure A-5. Modulus of Elasticity vs Temperature Types 304 and 347
Corrosion Res. Steel

The applicable compression stress level at r_i is 15,280 psi (see page A-4, with opposite loading). From page A-12, $E_T = 2.52 \times 10^7$ psi and $E_S = 2.75 \times 10^7$ psi; thus,

$$\sqrt{2.52 \times 10^7 \times 2.75 \times 10^7} = 2.63 \times 10^7 \text{ psi at room temperature.}$$

From the curve on page A-13, at 600° F, $E = 2.5 \times 10^7$ psi. We approximate the effect of temperature on $\sqrt{E_S E_T}$ by the ratio

$$\frac{2.5 \times 10^7}{2.8 \times 10^7} \sqrt{E_S E_T} = E'$$

such that

$$E' = 2.45 \times 10^7 \text{ psi.}$$

The curve in Figure A-2 gives values of

$$C \text{ vs } \frac{r}{t \cos \alpha};$$

Therefore,

$$\frac{r_i}{t \cos \alpha} = \frac{1.87}{0.025 \times 0.50} \approx 150$$

such that

$$C = 0.315;$$

then

$$p_{cr} = 7,580 \text{ lb.}$$

The design yield load given previously is 2,240 lb; therefore, the M. S. against cone buckling $= \frac{7580}{2240} - 1 = +2.38$.

Case 2 (Compression stress in cone with internal pressure)

Since the M. S. for tension and compression stresses for elastic conditions as found on page A-8 is lower than the buckling M. S., no further check need be made, since this case is identical with the one on page A-8.

The next step is to check strength of rings, at both inner and outer perimeters of cone, in resisting radial components of cone loading.

1. Inertia load downward

(a) At r_o , meridional loading $= t f_{xo} = 0.025 \times 11,750 = 294 \text{ lb/in. (tension)}$

NOTE: Hoop stresses due to internal pressure do not require an external reaction.

The above meridional tension loading produces an inward acting radial loading on the ring, placing the ring in compression, thus the stress in the ring

$$f = \frac{P_{\text{rad}} r}{A} = \frac{294 \cos 30^\circ r}{A}$$

The "r" used is to the ring cross-section centroid (4.51 in.), and $A = 0.09 \text{ in.}^2$ approximately (ring cross-section area), such that

$$f = \frac{294 \times 0.866 \times 4.51}{0.09} = 12,800 \text{ psi compression.}$$

Then, the M. S. against elastic yield of material will be for an allowable compression yield at 180° F of 32,100 psi (page A-8); therefore,

$$\text{M. S.} = \frac{32,100}{12,800} - 1 = +1.51$$

Now, the stability of the ring against compression buckling is given by

$$P_{cr} = \frac{3E'I}{r^3}$$

(Ref: Roark, Table XV);

$$E' = \sqrt{E_S E_T} \quad (\text{at } 180^\circ \text{ F and } 12,800 \text{ psi stress level}) = 2.67 \times 10^7 \text{ psi};$$

I = moment of inertia of ring cross-section, as shown:

Thus,

$$I_{yy} = 1/12 (0.316) (0.285)^3 = 6.1 \times 10^{-4} \text{ in}^4;$$

Therefore, P_{cr} is calculated by

$$P_{cr} = \frac{3 \times 2.67 \times 10^7 \times 6.1 \times 10^{-4}}{(4.51)^3} = 532 \text{ lb/in.}$$

We have the actual loading = $294 \times 0.866 = 254 \text{ lb/in.}$, which then establishes the M.S. against ring buckling = $\frac{532}{254} - 1 = +1.09$.

(b) at ri,

$$\text{meridional loading} = 0.025 \times 17,540 = 438 \text{ lb/in.}$$

Then,

$$P_{\text{radial}} = 438 \times 0.866 = 379 \text{ lb/in.}$$

producing tension in the inner ring,

$$\text{ring stress} = f = \frac{P_{\text{rad}} r}{A},$$

where

$$r = \text{radius to inner ring centroid} = 1.94", \text{ and}$$

$$A = \text{cross-section area of inner ring} = 0.025 \text{ in}^2; \text{ then,}$$

$$f = \frac{379 \times 1.94}{0.025} = 29,400 \text{ psi.}$$

At 600° F the allowable tension yield value is 18,300 psi. This combination indicates that the inner ring cross-section area should be increased to

$$A = \frac{379 \times 1.94}{18,300} = 0.0402 \text{ in}^2 \text{ minimum.}$$

An increase of ring dimensions to 0.21 gives $A = 0.044$ and also increases r to 1.98 in., this results in a new

$$f = \frac{379 \times 1.98}{0.044} = 17,050 \text{ psi,}$$

such that

$$\text{M.S.} = \frac{18,300}{17,050} - 1 = +0.07$$

(Use of increased ring size for design is suggested).

2. Inertia load upward

(a) Internal pressure effect on meridional load is neglected for this condition. At r_o the loading is somewhat less than for condition 1 on page A-15 — the inner ring will be in compression and must be checked. For this case,

$$\text{meridional loading} = 0.025 \times 1528 = 382 \text{ lb/in}$$

and

$$P_{\text{rad}} = 382 \times 0.866 = 331 \text{ lb/in.}$$

Now, the

$$\text{compression stress in ring} = \frac{P_{\text{rad}} r'}{A} ; f = \frac{331 \times 1.98}{0.044} = 14,900 \text{ psi (C);}$$

and the allowable elastic yield stress at $600^\circ \text{F} = 23,900 \text{ psi}$; therefore, the

$$\text{M.S. against elastic yield} = \frac{23,900}{14,900} - 1 = +0.60.$$

For ring buckling,

$$E' = 2.36 \times 10^7 \text{ psi;}$$

$$I = 1.62 \times 10^{-4} \text{ in}^4;$$

Thus,

$$P_{\text{cr}} = \frac{3 \times 2.36 \times 10^7 \times 1.62 \times 10^{-4}}{(1.98)^3} = 1480 \text{ lb/in.}$$

Then, for a design radial loading = 331 lb/in., the

$$\text{M.S. against inner ring buckling} = \frac{1480}{331} - 1 = +3.47.$$

The top of inner cylinder ($r_i = 1.87''$) is subjected to a combination of compression buckling (downward acceleration) plus external pressure.

The compression hoop stress due to a pressure of 30.2 psi will be

$$\frac{pr}{t} = \frac{30.2 \times 1.87}{0.020} = 2820 \text{ psi.}$$

The compression stress in the axial direction due to an inertial load of 2240 lbs will be

$$\frac{P}{2\pi rt} = \frac{2240}{2\pi \times 1.87 \times 0.020} = 9550 \text{ psi.}$$

Tension stress in the axial direction due to pressure will be acting to oppose compression and will be conservatively neglected. Critical stress at which buckling will occur on cylinder with external pressure is given by

$$\sigma_{cr} = \frac{k_y \pi^2 E'}{12 (1-\mu^2)} \cdot \left(\frac{t}{L} \right)^2$$

(Ref: NACA TN 3783).

ky is found in Figure 35(a) in the referenced TN, where it is given as a function of

$$Z_L = \frac{L^2 (1 - \mu^2)^{1/2}}{rt} ;$$

and for

$$L = \text{cyl. length} = 2.5 \text{ in.};$$

$$\mu = \text{Poisson's ratio} = 0.3;$$

$$r = \text{cyl. radius} = 1.87 \text{ in.};$$

$$t = \text{cyl. wall thickness} = 0.020 \text{ in.};$$

$$Z_L = \frac{(2.5)^2 (1 - 0.30^2)^{1/2}}{1.87 \times 0.02} \approx 160, \text{ such that } ky = 28.$$

Now, $E' = \sqrt{E_S E_T}$ is corrected for temperature at the stress level of 2820 psi, (from curves on page A-12); $E_S = E_T = E' = 2.8 \times 10^7$ psi; therefore, the only correction will be for a temperature of 600° F at the top and 1020° F at the bottom. (The 1020° value will be used for computations.) At 600° F, $E = 2.5 \times 10^7$ psi, and at 1020° F, $E' = 2.26 \times 10^7$ psi; therefore,

$$\sigma_{cr} = \frac{28 \times 9.87 \times 2.26 \times 10^7}{12 (1 - 0.30^2)} \frac{(0.020)^2}{(2.5)^2} = 3.65 \times 10^4 \text{ psi.}$$

This is far in excess of the allowable compression yield of the material at the operating temperatures, therefore, elastic compression stress is the governing factor. Allowable compression yield for the cylinder will be based on the 1020° F temperature. From the curve on page A-6, $F'_\alpha = 0.50 \times 35,000 = 17,500$ psi. Then the

$$\text{M.S. for elastic yield for hoop stress} = \frac{17,500}{2,280} - 1 = +5.20.$$

Critical buckling stress for a thin cylinder subjected to axial loading is given by different expressions depending on the $\frac{L^2}{rt}$ ratio (Ref: NACA TN 3783); this ratio term is calculated to be

$$\frac{(2.5)^2}{1.87 \times 0.02} = 167$$

Since this value is greater than 100, we are in the so-called "long cylinder" range and $\sigma_{cr} = \eta C E_r^t$ where C is obtained from Figure 7 in the above referenced TN, and " η " is a plasticity reduction factor. Calculation of η is somewhat complex and uncertain due to a lack of material properties at higher temperature. Poisson's ratio for the elastic range and for the fully plastic range must be known at the design temperatures, in addition to modulus of elasticity, tangent modulus, and secant modulus at the given temperature. The actual stress level of 9550 psi is low enough so that this will not be in the plastic zone and hence, η will be assumed to be 1.0, but the reduction in E due to temperature will be included. As for the case on page A-20, $E' = 2.26 \times 10^7$ psi (at 1020°F).

From Figure 7 in NACA TN 3783, with

$$\frac{r}{t} = \frac{1.87}{0.02} = 93.5, C = 0.39,$$

giving

$$\sigma_{cr} = 0.39 \times 2.26 \times 10^7 \times \frac{0.02}{1.87} = 94,300 \text{ psi.}$$

This is also not at all critical because the elastic compression yield at operating temperature is much lower. Then the

$$\text{M. S. for axial loading} = \frac{17,500}{9,550} - 1 = +0.83$$

The result of combining both compression stresses would be to show a higher M. S. than for either one alone.

APPENDIX B

CONVERTER SUPPORT SYSTEM SPRING RATE ANALYSIS

Spring Rate of Support System in Vertical Direction

Assuming that the weight of supported mass = 105 lb = W (at the outer cylinder radius $r_o = 4.65$ in.), load/inch circumference $w_o = \frac{W}{2\pi r_o} = 8.93$ lb/in.

Vertical deflection of the inner cylinder for the total load of 105 lb, compressing 2.5 in. length of cylinder, is expressed by:

$$\Delta = \frac{WL}{AE},$$

where

$A = 2\pi r_i t = 0.235 \text{ in.}^2$; "E" will be approximated as the average value for a temperature at the top of 600°F and 1023°F at 2.5 in. down, such that

T (°F)	E(psi)	} E avg. = 23.76×10^6 psi
600	25×10^6	
1023	22.52×10^6	

Since the stress level is only $\frac{105}{0.235} = 447$ psi, no reduction in "E" due to plasticity is needed (only above the proportional limit does this become a factor). Thus,

$$\Delta = \frac{105 \times 2.5}{0.235 \times 23.76 \times 10^6} = 4.7 \times 10^{-5} \text{ in.}$$

Vertical deflection of outer (lower) edge of cone, relative to inner (upper) edge will be established by first considering a sector of the cone surface with unit length at the outer circumference, as illustrated by the following diagram:

$$t = 0.016 \text{ in.}$$

$$c = \frac{1.87}{4.65} \times 4.65 = 1.87 \text{ in.}$$

$$\text{Load} = P = 3.59 \text{ lb}$$

$$L = dx$$

$$A = \frac{0.402x}{1.87} t = 3.44 \times 10^{-3} (x) \text{ in.}^2$$

"E" will be approximated as the average value for temperatures of 600°F and 180°F; thus $E = 2.665 \times 10^7$ psi. Then, for $\Delta = PL/AE$, we have

$$\Delta = \int_{1.87}^{4.65} \frac{P dx}{AE} = \int_{1.87}^{4.65} \frac{3.59 dx}{3.44 \times 10^{-3} (x) (2.665 \times 10^7)}, \text{ resulting in}$$

$$\Delta = 3.57 \times 10^{-5} \text{ in. in the direction of a cone element.}$$

$$\text{Thus the vertical deflection component} = \frac{3.57 \times 10^{-5}}{2} = 1.79 \times 10^{-5} \text{ in.}$$

The ring tends to contract an amount Δr radially, due to the inward pull from the cone. Lengthening of the cone and contraction of the ring are treated separately and the total effect combined for total vertical deflection (this is conservative) in the following.

We have that $\Delta r = \frac{r}{E} f$, and with $f = \frac{pr}{A}$. This becomes $\Delta r = \frac{pr^2}{AE}$, where r is the mean radius of ring = 4.51 in.

Also, $E = 27.5 \times 10^6$ at 180°F for compression and $p = \frac{w_o}{\tan 30^\circ} = 6.22 \text{ lb/in.}$

$$\text{Finally, } \Delta r = \frac{6.22 (4.51)^2}{0.09 \times 2.75 \times 10^7} = 5.12 \times 10^{-5} \text{ in.}$$

Then for

$$h = \frac{(5.12 + 3.1) 10^{-5}}{0.577} = 14.25 \times 10^{-5} \text{ in. and}$$

$$h_2 = \frac{3.57 \times 10^{-5}}{2} = 1.79 \times 10^{-5} \text{ in. ,}$$

$$\text{total vertical deflection} = (4.70 + 14.25 + 1.79) 10^{-5} \text{ in.} = 2.074 \times 10^{-4} \text{ in.}$$

Now,

$$k = \frac{105}{2.074 \times 10^{-4}} = 5.07 \times 10^5 \text{ lb/in.}$$

$$\text{M. F.} = \frac{1}{1 - \frac{4 \times 9.87}{0.386} (52)^2 (2.074 \times 10^{-4})} = 1.062$$

$$f_n = 3.13 \sqrt{\frac{4.97 \times 10^5}{1.05 \times 10^2}} = 215 \text{ cps}$$

Spring Rate of Support System in Transverse Direction

Case I: Assume no support at lower end of outer cylinder.

The inner cylinder is assumed fixed as shown at 2.8 in. down from top. This is conservative since at approximately 2.5 in. down, the thickness increases very rapidly.

For the geometry as shown on the previous page, the total deflection at point "a" will be made up of the following:

1. Bending deflection of inner cylinder 2.8 in. long fixed as a cantilever beam. Loading will be as follows:

2. Shear deflection due to same set of loads.
3. Rotation of free end of cylinder (due to bending) which will affect point "a".
4. Bending deflection of cone loaded as shown.

5. Shear deflection of cone loaded as in 4.
6. Change in slope at end of cone due to bending.

For Item 1):

$$F \delta M_{cyl.} = \int_0^L \frac{mMdx}{EI}$$

where F is an auxiliary load applied at the free end of the cantilever beam and "m" is the resulting moment on beam. For F = 1 lb applied to left:

$$\delta M_{cyl.} = \int_0^{2.8} \frac{mMdx}{EI}$$

Where:

$$m = -x$$

$$M = M_{cyl.} - 105x = (453 - 105x)$$

$$E = \text{average of values at } x = 0 \text{ (600°F) and } x = 2.8 \text{ (1090°F) such that}$$

$$E = 2.36 \times 10^7 \text{ psi (values for compression were used since they are lower and hence more conservative for our purpose)}$$

$$I = \pi r^3 t = 3.1416 (1.87)^3 (0.020) = 0.410 \text{ in}^4. \text{ By substitution,}$$

$$\delta M_{cyl.} = \int_0^{2.8} \frac{(-x) (453 - 105x) dx}{2.36 \times 10^7 \times 0.410}$$

$$\delta M_{cyl.} = -1.045 \times 10^{-4}$$

The negative sign indicates deflection to right instead of as assumed.

For Item 2):

$$\delta S_{cyl.} = \frac{NPL}{AG},$$

where

N is a constant depending on cross-section geometry and is equal to 2.0 for a thin tube;

$$L = 2.8 \text{ in.};$$

$$P = 105 \text{ lb};$$

$$A = 2\pi r t = 2 \times 3.1416 \times 1.87 \times 0.020 = 0.235 \text{ in.}^2;$$

G = Shear modulus, which will be taken as the average at the two temperatures and assuming Poisson's ratio $\mu = 0.30$, such that

$$G = \frac{E}{2(1 + 0.3)} = \frac{2.36 \times 10^7}{2.6} = 9.06 \times 10^6 \text{ psi. Then,}$$

$$\delta S_{\text{cyl.}} = \frac{2 \times 105 \times 2.8}{0.235 \times 9.06 \times 10^6} = 2.76 \times 10^{-4} \text{ in.}$$

For Item 3):

Due to 105 lb load,

$$\theta_1 = \frac{PL^2}{2EI} = \frac{105 (2.8)^2}{2 \times 2.36 \times 10^7 \times 0.41} = 4.26 \times 10^{-5} \text{ radians}$$

Due to 453 in. /lb moment,

$$\theta_2 = \frac{ML}{EI} = \frac{453 (2.8)}{2.36 \times 10^7 \times 0.41} = 1.31 \times 10^{-4} \text{ radians}$$

Net angular deflection at free end of beam is $1.31 \times 10^{-4} - 0.426 \times 10^{-4} = 8.84 \times 10^{-5}$ radians.

For Item 4):

The cone is considered as a cantilever beam with variable moment of inertia.

Usual thin shell considerations will be neglected because stresses are very low and the cone will not be subject to buckling instability. Apply 1 lb load at "a" and use

$$\delta M_{\text{cone}} = \int_0^{1.6} \frac{mMdx}{EI}$$

For this case,

$$m = x;$$

$$M = 284.3 + 105x;$$

$$E = \text{Average for } 600^{\circ}\text{F and } 180^{\circ}\text{F} = (2.5 \times 10^7 + 2.75 \times 10^7)^{1/2} \\ = 2.625 \times 10^7 \text{ psi (used compression values);}$$

$$I = \pi r^3 t.$$

Now,

$$r = 4.65 - \frac{x}{\tan 30^{\circ}} = 4.65 - \frac{x}{0.577}, \text{ and}$$

$$t = \frac{0.016}{\sin 30} = \frac{0.016}{0.5} = 0.032, \text{ such that}$$

$$I = \frac{3.1416 \times 0.032}{0.577} (2.673 - x)^3 = 0.174 (2.673 - x)^3 \text{ in}^4.$$

Using these values gives

$$\delta M_{\text{cone}} = \int_0^{1.6} \frac{x(284.3 + 105x) dx}{2.625 \times 10^7 [0.174 (2.673 - x)^3]}, \text{ such that}$$

$$\delta M_{\text{cone}} = 2.3 \times 10^{-5} + 1.67 = 3.84 \times 10^{-5} \text{ in.}$$

For Item 5):

Since cross-section area is varying, it is necessary to integrate the expression $\frac{NPL}{AG}$. Then,

$$\delta S_{\text{cone}} = \frac{NP}{G} \int_0^{1.6} \frac{dx}{A},$$

where

$$N = 2;$$

$$P = 105 \text{ lb};$$

$$G = \frac{2.625 \times 10^7}{2(1 + 0.3)} = 1.01 \times 10^7 \text{ psi};$$

$$A = 2\pi rt;$$

$$r = 4.65 - \frac{x}{0.577};$$

$$t = \frac{0.016}{0.5} \quad (\text{Ref P7}).$$

Then,

$$\delta S_{\text{cone}} = \frac{2 \times 105}{1.01 \times 10^7} \int_0^{1.6} \frac{dx}{2 \times 3.1416 \left(4.65 - \frac{x}{0.577} \right) (0.032)},$$

which is evaluated to give

$$\delta S_{\text{cone}} = 5.45 \times 10^{-5} \text{ in.}$$

For Item 6:

Since the moment of inertia is varying we will use the "energy" method and integrate over the length. Then,

$$M^1 \theta_{\text{cone}} = \int_0^{1.6} \frac{mMdx}{EI}$$

where M^1 is unit moment applied at the location whose rotation is being sought.

In our case, we apply 1 in. /lb moment at the base of cone ($x = 0$). "m" is moment on beam due to application of M^1 , then

$$\theta_{\text{cone}} = \int_0^{1.6} \frac{Mdx}{EI}.$$

From page A-8,

$$M = 284.3 + 105x;$$

$$E = 2.625 \times 10^7 \text{ (psi);}$$

$$I = 0.174 (2.673 - x)^3 \text{ (in.}^4\text{)}.$$

Thus,

$$\theta_{\text{cone}} = \int_0^{1.6} \frac{(284.3 + 105x) dx}{2.625 \times 10^7 [0.174 (2.673 - x)^3]}$$

which is evaluated to give

$$\theta_{\text{cone}} = 2.30 \times 10^{-5} \times 1.404 = 3.23 \times 10^{-5} \text{ radians.}$$

The following steps sum up the effects of items 1) through 6).

For the cylinder the idealized beam is represented by its centerline as shown:

The upper end of the cylinder is deflected $(2.76 - 1.045) 10^{-4}$ or 1.715×10^{-4} in. to left of the centerline and is oriented at an angle of 8.84×10^{-5} radians as shown. Since the top end of the cylinder (free end of cantilever beam) is the base (fixed end) for the cone, we have the following situation:

At 1.6 in. down from top, the total horizontal deflection will be

$$\Delta = \delta_{cone} + \delta_{cyl.} + L_{cone} \theta_{cyl.} = (3.84 + 5.45) 10^{-5} + 1.715 \times 10^{-4} + 1.6 \times 8.84 \times 10^{-5} = 3.858 \times 10^{-4} \text{ in.}$$

Deflection at the center of gravity (4.313 in. down from top) is illustrated in the following diagram:

Total deflection in horizontal plane of center of gravity

$$\begin{aligned}
 &= (4.313 - 1.6) (3.23 \times 10^{-5}) + 3.858 \times 10^{-4} \\
 &= 4.734 \times 10^{-4} \text{ in.}
 \end{aligned}$$

$$k = \frac{105}{4.734 \times 10^{-4}} = 2.22 \times 10^5 \text{ lb/in.}$$

Thus, $f_n = 145$ cps, and the magnification factor = 1.17 at a forcing frequency of 52 cps.

The preceding analysis was based on the assumption of no support from the lower end cap or from the springs used in the generator. It is reasonable to expect that with the addition of these resisting forces that the deflection will be considerably reduced, the natural frequency will become considerably higher and the magnification factor will be much reduced.

Case II: Assume support at the top as before plus a reacting force due to generator springs plus a reaction at the base due to lower cover plate.

We assume that the flat portion of the bottom cover has "infinite" stiffness for loads in its own plane. We assume that this flat circular plate applies a uniformly distributed shear loading to the edge of a ring of 2.37 in. radius and 0.35 in. wide. The other edge of this ring is connected to a flat disc with 2.37 in. outer radius and 1.8 in. inner radius. The inner edge of the disk is connected to a ring of

1.87 in. radius and 0.35 in. wide. The equivalent beam configuration assumed is as shown below.

(Not to Scale)

All thickness = 0.030 in. to top of inner cylinder.

$$I_1 = 3.1416 (2.37)^3 (0.030) = 1.25 \text{ in.}^4$$

$$I_2 = 0.7854 \left(\frac{4}{2.37} - \frac{4}{1.87} \right) = 15.18 \text{ in.}^4$$

$$I_3 = 3.1416 (1.87)^3 (0.030) = 0.617 \text{ in.}^4$$

We will add up deflections due to beam bending for the two ring sections plus shear deflections of the rings and the disk.

For unit load at the bottom:

$$2.22 \times 10^7 \delta M = \int_0^{0.35} \frac{(x)(Qx) dx}{1.25} + \int_{0.35}^{0.70} \frac{(x)(Qx) dx}{0.617} = 0.173Q$$

Such that

$$\delta M = 7.78 \times 10^{-9} Q.$$

Now,

$$\delta S = \frac{N_1 L_1 Q}{A_1 G_1} + \frac{N_2 L_2 Q}{A_2 G_2} + \frac{N_3 L_3 Q}{A_3 G_3},$$

where:

$$N_1 = N_2 = N_3 = 2.0;$$

$$L_1 = 0.35 \text{ in.}; L_2 = 0.030 \text{ in.}; L_3 = 0.35 \text{ in.};$$

$$G_1 = G_2 = G_3 = \frac{2.22 \times 10^7}{2(1 + 0.30)} = 0.855 \times 10^7 \text{ psi};$$

$$A_1 = 2 \times 3.1416 \times 2.37 \times 0.03 = 0.447 \text{ in.}^2;$$

$$A_2 = 3.1416 (2.37^2 - 1.87^2) = 6.66 \text{ in.}^2;$$

$$A_3 = 2 \times 3.1416 \times 1.87 \times 0.03 = 0.352 \text{ in.}^2.$$

By substitution of these values,

$$\delta S = \frac{2 Q}{8.55 \times 10^6} \left[\frac{(0.35)}{0.447} + \frac{(0.03)}{6.66} + \frac{(0.35)}{0.352} \right] = 4.18 \times 10^{-7} Q$$

$$\text{Thus the total deflection} = (0.0778 + 4.18) 10^{-7} Q = 4.26 \times 10^{-7} Q.$$

The centerline of the spring force is very near load application, and with the extreme case of no support at the lower cover, the deflection (for 1G) was found previously to be 4.734×10^{-4} in. For a total spring constant (generator springs) of 5236 lb/in. this would be a force of $5236 \times 4.734 \times 10^{-4} \approx 2.5$ lb. Since the actual deflection will be much less we may neglect the spring forces and thus simplify the problem.

Making use of the previous analysis resulting in a deflection at the center of gravity, we next apply the reacting force "Q" at the lower end of the cantilever system and find the net deflection at the lower end of the beam (including the new deflection in terms of "Q").

The outer cylinder has both the cold frame and the fins attached and as a result may be assumed rigid between the bottom cover plate and the outer perimeter of the cone.

With only reaction "Q" applied: (Refer to pages A-4 through A-9 for analysis method)

Item 1):

$$\begin{aligned} \delta' M_{cyl.} &= \frac{QL^3}{3EI} - \frac{MoL}{EI} = \frac{Q(2.8)^3}{3(2.36 \times 10^7)(0.410)} - \frac{6.63 Q(2.8)}{(2.35 \times 10^7)(0.410)} \\ &= -1.163 \times 10^{-6} Q \quad (\text{direction is opposite of } \delta M_{cyl.} \text{ on page A-6}) \end{aligned}$$

Item 2):

$$\begin{aligned} \delta' S_{cyl.} &= \frac{NQ1}{AG} = \frac{2 Q(2.8)}{0.235(9.06 \times 10^6)} = 2.63 \times 10^{-6} Q \\ &(\text{direction is opposite of } \delta S_{cyl.} \text{ on page A-7}) \end{aligned}$$

Item 3:

Due to load Q,

$$\theta'_1 = \frac{QL^2}{2EI} = \frac{(2.8)^2 Q}{2 \times 2.36 \times 10^7 \times 0.41}.$$

Due to moment 6.63 Q,

$$\theta_2 = \frac{MoL}{EI} = \frac{6.63 \times 2.8 Q}{2.36 \times 10^7 \times 0.41}.$$

Then, the net

$$\theta' = \frac{2.8 Q}{2.36 \times 10^7 \times 0.41} \left(\frac{2.8}{2} - 6.63 \right) = -1.51 \times 10^{-6} Q$$

Item 4:

$m = x$; $M = (6.63 - 1.6) Q + QX = (5.03 + X) Q$; such that

$$\delta' M_{\text{cone}} = \int_0^{1.6} \frac{X (5.03 + X) Q dx}{(2.625 \times 10^7) [0.174 (2.673 - X)^3]},$$

which is evaluated to give

$$\delta' M_{\text{cone}} = 2.19 \times 10^{-7} \times 1.004 Q = 2.20 \times 10^{-7} Q$$

(opposite direction from δM_{cone} on page A-9).

Item 5:

$$\delta' S_{\text{cone}} = \frac{NQ}{G} \int_0^{1.6} \frac{dx}{A} = \frac{2Q}{1.01 \times 10^7} \int_0^{1.6} \frac{dx}{2 \times 3.1416 \times 0.032}$$

which is evaluated to give

$$\delta' S_{\text{cone}} = 0.907 \times 5.68 \times 10^{-7} Q = 5.15 \times 10^{-7} Q$$

(direction is opposite of δS_{cone} on page A-9).

Item 6:

$$\theta'_{\text{cone}} = \int_0^{1.6} \frac{M dx}{EI} = \int_0^{1.6} \frac{(5.03 + X) Q dx}{(2.625 \times 10^7) [0.174 (2.673 - X)^3]}$$

which is evaluated to give

$$\theta'_{\text{cone}} = 2.258 \times 2.19 \times 10^{-7} Q = 4.94 \times 10^{-7} Q$$

(direction is opposite of θ_{cone} on page A-10)

The following steps sum up the effects of items 1) through 6).

The upper end of the cylinder is deflected $(2.63 - 1.163) 10^{-6} Q = 1.467 \times 10^{-6} Q$ to right of the centerline and at a slope of $1.51 \times 10^{-6} Q$ radians as shown on page A-15.

Now, at 1.6 in. down from the top, the total horizontal deflection will be

$$\begin{aligned} \Delta' &= \delta'_{\text{cone}} + \delta'_{\text{cyl.}} + L_{\text{cone}} \theta'_{\text{cyl.}} = (2.20 + 5.15) 10^{-7} Q + \\ &1.467 \times 10^{-6} Q + 1.6 \times 1.51 \times 10^{-6} Q = 4.618 \times 10^{-6} Q \end{aligned}$$

The deflection at the center of gravity will be $2.716 \times 4.94 \times 10^{-7} Q + 4.618 \times 10^{-6} Q = 5.958 \times 10^{-6} Q$ to the right of the centerline.

We now transfer both deflections (for 105 lb acting alone, and for Q acting alone) to the location of reaction " Q ", as follows:

1) For 105 lb loading:

$$(6.63 - 4.313) 3.23 \times 10^{-5} + 4.734 \times 10^{-4} = 5.481 \times 10^{-4} \text{ to left of the centerline.}$$

2) For "Q" loading:

$$(6.63 - 4.313) 4.94 \times 10^{-7} Q + 5.958 \times 10^{-6} Q = 7.100 \times 10^{-6} Q \text{ to the right of the centerline.}$$

The sum of these deflections and the deflection of the bottom cover plate must be equal to zero; therefore, $-5.481 \times 10^{-4} + 7.100 \times 10^{-6} Q + 4.26 \times 10^{-7} Q = 0$, such that

$$Q = \frac{5.481}{7.526} \times 10^2 \approx 73 \text{ lb.}$$

This could be expected since the bottom cover plate is relatively stiff.

The next step is to determine the horizontal deflection in the plane of the center of gravity, given by

$$\delta = 5.958 \times 10^{-6} \times 73 - 4.734 \times 10^{-4} \approx 3.9 \times 10^{-5} \text{ in.}$$

Then, the natural frequency f_n is given by

$$f_n = 3.13 \sqrt{\frac{1}{8}} \approx 500 \text{ cps.}$$

Finally, the magnification factor is approximately 1.01 at a forcing frequency of 52 cps.

APPENDIX C

OUTER CASE AND COLD FRAME ANALYSIS

Outer Case Design Analysis

For the cold frame - outer case loading depicted in Section 3.2.2.1, Figure 3-5, the solution for the radial pressure (p) is determined from the following calculations:

Equating the hoop force to the total pressure force:

$$pA_p = 2F \quad (1)$$

where

F = hoop force in pounds

p = radial pressure in psi

A_p = projected area in square inches

The hoop force is found from the relation:

$$F = S_h t l \quad (2)$$

where

S_h = hoop stress in thin section in psi

t = wall thickness in inches

l = length of outer case in inches

Since the projected area, A_p , equals diameter, d , times the outer case length, l , equation (1) becomes

$$p d l = 2(S_h t l)$$

Solving for p:

$$p = \frac{2S_h t}{d} \quad (3)$$

substituting

$$S_h = \text{yield stress} = 30,000$$

$$t = 0.060$$

$$d = 9.25$$

$$p = \frac{2(30,000)(0.060)}{9.25}$$

$$p = 389 \text{ psi}$$

Since the hoop stress is at the yield point, 389 psi is the maximum radial pressure the outer case can exert on the cold frame.

Cold Frame Hole Distortion Analysis

For the cold frame force loading discussed in Section 3.2.2.2, Figure 3-6, the solution for hole diametral shrinkage is established by the following calculations:

Equating outer case hoop force to total web force:

$$2F = A_w S_w \quad (4)$$

where

$$A_w = \text{total web area in square inches}$$

$$S_w = \text{web stress in psi}$$

Since there are a total of 30 webs (15 on each side) having a length of 1.875 inches and a width of 0.0488 inch, the total web area is

$$A_w = 30 (0.0488) (1.875)$$

$$A_w = 2.745 \text{ square inches}$$

The total force acting on the thin section (shown as width a) of a single web is equal to the web stress, S_w , times the single web area, $A = a l$, where l is the length of the web in the direction perpendicular to the paper. Expressed mathematically,

$$F_w = S_w a l \quad (5)$$

This same force also acts on the element of area located at a distance X from the thinnest portion of the web. Since stress is equal to force divided by area, the stress on this element of area, noticing the width of the area in Figure C-1, is

$$S_x = \frac{S_w a l}{A_x}$$

or

$$S_x = \frac{S_w a l}{(a + 2r - 2\sqrt{r^2 - x^2})l} \quad (6)$$

The shortening of the element thickness, dx , caused by the compressive stress, S_w , is

$$d\delta = E dx$$

Since the unit strain, E , is equal to the stress, S_x , divided by the modulus of elasticity, E ,

$$d\delta = \frac{S_x dx}{E} \quad (7)$$

Substituting equation (6) in equation (7)

$$d\delta = \frac{S_w a}{(a + 2r - 2\sqrt{r^2 - x^2})E} dx \quad (8)$$

Integrating,

$$\delta_n = \frac{Sa}{E} \int_r^0 \frac{S_w a}{(a + 2r - 2\sqrt{r^2 - x^2})} dx$$

The expression is integrated by making the substitutions

$$M = \frac{a + 2r}{2r}$$

$$X = r \sin \theta$$

$$dx = r \cos \theta d\theta$$

Subsequently,

$$\delta_r = \frac{S_w a}{E} \left[\frac{\pi}{4} - \frac{M}{\sqrt{M^2 - 1}} \tan^{-1} \sqrt{\frac{M^2 - 1}{M - 1}} \right]$$

where

δ_r represents the radial deformation of the hole.

$$\text{With } M = \frac{a + 2r}{2r} = \frac{0.0488 + 2 \left(\frac{0.219}{2} \right)}{2 \left(\frac{0.219}{2} \right)} = 1.2228$$

$$\delta_r = 0.06885 \frac{S_w}{E}$$

Substituting $S_w = 4918$ psi and $E = 10,000,000$ for aluminum,

$$\delta_r = 0.06885 \frac{(4918)}{10,000,000}$$

$$\delta_r = 0.000034 \text{ inch}$$

Hole diametral shrinkage, δ_d , is twice this value.

$$\delta_d = 2(0.000034)$$

$$\delta_d = 0.000068 \text{ inch}$$

Therefore, hole shrinkage is below the established limit of 0.0001 inch.

APPENDIX D

HOT FRAME INTERFACE ANALYSIS

Assembly Force Analysis

A vector diagram of the spud forces with construction lines, corresponding to the force diagram (Figure 3-8) given in Section 3.2.3.1, is shown by the following:

Inspection of this vector diagram shows that the assembly force F_a is the vector sum of the two resultant forces, R . Therefore,

$$F_a = 2 R \sin (\alpha + \vartheta) \quad (1)$$

It is also noticed that R is the vector sum of N and Nf_a and since the friction force, Nf_a is at a right angle to the normal force, N , the right triangle gives the relation

$$R^2 = N^2 + (Nf_a)^2$$

or

$$R = N \sqrt{1 + f_a^2} \quad (2)$$

Substituting equation (2) in equation (1)

$$F_a = 2N \sqrt{1 + f_a^2} \sin (\alpha + \vartheta) \quad (3)$$

The normal force N is equal to one half the area of the engaging portion of the cone times the contact pressure, p. Tapered contact area is given by the formula

$$A = \pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2}$$

where r_1 , r_2 , and h are in inches and area A is in square inches.

Therefore,

$$N = \frac{A p}{2} = \frac{p \left(\pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2} \right)}{2} \quad (4)$$

where contact pressure, p is in lbs per square inch.

Substituting equation (4) in equation (3),

$$F_a = p \left[\pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2} \right] \sqrt{1 + f_a^2} \sin (\alpha + \phi) \quad (5)$$

As noted in the vector diagram,

$$\tan \alpha = \frac{N f_a}{N} = f_a$$

Therefore,

$$\alpha = \tan^{-1} f_a$$

For a coefficient of friction equal to 0.10,

$$\alpha = 5^\circ 43'.$$

For a 3 degree taper per side,

$$\phi = 3^\circ$$

$$r_1 = 1.842$$

$$h = 3.75$$

$$r_2 = 1.645$$

$$p = 500$$

Substituting these values in equation (5),

$$F_a = 500 \left[\pi (1.842 + 1.645) \sqrt{(3.75)^2 + (1.842 - 1.645)^2} \right] \sqrt{1 + 0.1^2} \sin(5^\circ 43' + 3^\circ)$$

$$F_a = 3,149 \text{ lbs.}$$

For a 2 degree taper per side,

$$\phi = 2^\circ$$

$$r_1 = 1.842$$

$$h = 3.75$$

$$r_2 = 1.711$$

$$p = 500$$

$$\alpha = 5^\circ 43'$$

Substituting these values into equation (5), the assembly force, F_a , is 2,840 lbs.

Disassembly Force Analysis

For this case, the vector diagram of the spud forces (as before) is the following:

Inspection of this vector diagram shows that the disassembly force F_d is the vector sum of the two resultant forces, R . Therefore,

$$F_d = 2 R \sin (\alpha - \theta) \quad (6)$$

Therefore, it is seen that equation (6) differs from equation (1) in the sign of the angle θ only. For disassembly, equation (5) becomes

$$F_d = p \left[\pi (r_1 + r_2) \sqrt{h^2 + (r_1 - r_2)^2} \right] \sqrt{1 + f_d^2} \sin (\alpha - \theta) \quad (7)$$

Since $\alpha = \tan^{-1} f_d$, a 1.00 coefficient of friction gives

$$\alpha = 45^\circ$$

For a 3 degree taper per side:

$$F_d = 500 \left[- (1.842 - 1.645) \sqrt{(3.75)^2 + (1.842 - 1.645)^2} \right] \sqrt{1 + 1^2} \sin (45^\circ - 3^\circ)$$

$$F_d = 19,462 \text{ lbs.}$$

For a 2 degree taper per side, the disassembly force, F_d , is 20,193 lbs.

Jacking Base Threads Stress Analysis

The shear area of the threads (represented in Figure 3-7, Section 3.2.3.1) is given by the relation (S.A.E. Handbook):

$$A_s = \pi n L_c D_{s_{\min.}} \left[\frac{1}{2n} + 0.57735 (D_{s_{\min.}} - E_{n_{\max.}}) \right]$$

where

A_s = shear area in square inches

n = number of threads per inch

L_c = length of engagement in inches

$D_{s_{\min.}}$ = minimum major diameter of external thread in inches

$E_{n_{\max.}}$ = maximum pitch diameter of internal thread in inches.

By substitution of values from a table of thread-data,

$$A_s = \pi 12 (0.5) (1.7368) \left[\frac{1}{2(12)} + 0.57735 (1.7368 - 1.7037) \right]$$

$$A_s = 2 \text{ square inches}$$

With a shear area of 2 square inches, the threads will theoretically shear when the disassembly force equals 18,800 lbs. Expressed mathematically,

$$\begin{aligned} F_{d_{\max}} &= A_s S_{\max} \\ &= 2 (9,400) \end{aligned}$$

$$F_{d_{\max}} = 18,800 \text{ lbs.}$$

Jacking Base Bending Stresses

Corresponding to the jacking base loading diagram given in Figure 3-9, Section 3.2.3.3, the equation for this case, as given in "Formulas for Stress and Strain" by R.J. Roark, is

$$s = \frac{3W}{2\pi t^2} \left[1 - \frac{2 m b^2 - 2 b^2 (m + 1) \ln \frac{a}{b}}{a^2 (m - 1) + b^2 (m + 1)} \right]$$

where

- s = maximum bending stress in psi which occurs at outer edge of disc
- W = total edge load around hole in lbs
- t = disc thickness in inches
- m = reciprocal of Piosson's ratio = $\frac{1}{0.3}$ for steel
- a = radius of outer edge of disc in inches
- b = radius of hole in inches

Substituting the values

$$a = 1.71$$

$$b = 0.85$$

$$t = 0.500$$

$$m = \frac{1}{0.3} = 3.333$$

$$s = \frac{3W}{2\pi (0.5)^2} \left[1 - \frac{6.666 (0.85)^2 - 2 (0.85)^2 (4.333) \ln \frac{1.71}{0.85}}{(1.71)^2 (2.333) + (0.85)^2 (4.333)} \right]$$

$$s = 1.824 W$$

When the jacking base is at 1150° F, the yield strength is 14,100 psi.

Substituting,

$$14,100 = 1.824 W$$

or

$$W = 7,730 \text{ lbs.}$$

APPENDIX E

THERMOELECTRIC COUPLE DESIGN PROCEDURES

The following series of calculations represents a typical design calculation procedure for the determination of couple geometry and number of couples for a particular set of specifications (thermopile design data base). Calculations were performed with two digital computer programs, as discussed in Section 3.2.4.

Before proceeding with the calculation steps, the assumptions and specifications followed in this analysis are listed.

Thermopile Design Data Base

1. Simultaneously pressed P- and N-leg materials are:

N-leg hot segment	M-223
N-leg cold segment	M-13
P-leg hot segment	M-76
P-leg cold segment	M-29

2. Maximum allowable P-leg cold segment temperature — 600° F.

3. Leg geometries are

P- and N-leg diameter	0.187 inch
P-leg L_{cp}/L_p ratio	To be determined
N-leg L_{cn}/L_n ratio	To be determined

where

L_{cp} = length of P-leg cold segment in inches

L_p = over-all length of P-leg in inches

L_{cn} = length of N-leg cold segment in inches

L_n = over-all length of N-leg in inches

4. Extraneous electrical resistance is assumed to be 6.33 percent of total resistance.

5. The BOL and EOL couple cold junction temperatures given as interface conditions will apply.

BOL cold junction temperature, T_c 210° F

EOL cold junction temperature, T_c 180° F

6. The BOL couple hot junction temperature given as an interface condition will apply.

Average hot junction temperature, T_h 1075° F

7. Heat input at EOL is 78.2 percent of heat input at BOL (because of isotopic fuel decay).

8. Calculations at EOL are for maximum efficiency condition.

9. Power output at EOL 60 watts

10. Load EMF at EOL 24 ± 1 volts

11. Load current at EOL 2.5 amp nominal

12. Load EMF at BOL 24 volts minimum

13. Load current at BOL 2.5 amp minimum

14. No heat is lost in axial (vertical) direction from upper and lower covers.

15. Heat losses at BOL and EOL are determined from Figure 4-1 of the SNAP-23 Program parametric study report No. MMM 3418-33; with anticipated insulation improvement and actual temperature measurements taken on the SNAP-23 converters, it is felt that Figure 4-1 is applicable for estimating converter heat losses. A mathematical equation fit to the Figure 4-1 curve is

$$Q_1 = 0.1097 [T_h - T_c - 600] + 46.11$$

where

- Q_1 = converter heat loss in watts
 T_h = couple hot junction temp in °F
 T_c = couple cold junction temp in °F

Regions of heat losses included in this equation are:

- a. Lower cover
- b. Support cone
- c. Around thermoelectric legs
- d. Lower microquartz insulation
- e. Lower Min-K and Xenon insulation
- f. Upper Min-K and Xenon insulation
- g. Upper microquartz insulation

Calculation Procedures

Steps used in calculating the number of couples and couple geometry are demonstrated as follows.

1. P-leg L_p/A_p ratio

With the cold junction temperature fixed at 180° F, it is necessary to assume a hot junction temperature before this ratio can be calculated. Trial and error showed the optimum hot junction temperature to be 890° F. From the

computer-calculated P-leg data for this temperature range, the current term, $I L/A$, is given as

$$I \frac{L_p}{A_p} = 29.4 \text{ amp/in.}$$

where

I = P-leg current in amperes

L_p = over-all P-leg length in inches

A_p = cross-sectional area of 0.187 diameter P-leg which equals 0.02761 square inch

Substituting the 2.5 ampere design specification current for EOL,

$$2.5 \frac{L_p}{A_p} = 29.4$$

$$\frac{L_p}{A_p} = 11.76$$

Over-all P-leg length is

$$L_p = 11.76 \times 0.02761 = 0.325 \text{ in.}$$

Selecting an L_{cp}/L_p ratio of 0.350 (this ratio satisfies the maximum allowable cold segment temperature of 600° F), the P-leg cold segment length is

$$\begin{aligned} L_{cp} &= 0.350 L_p \\ &= 0.350 \times 0.325 \\ &= 0.114 \text{ in.} \end{aligned}$$

2. P-leg power

From the P-leg data for the same temperature range, the power term, $P L/A$, is given as

$$P_p \frac{L_p}{A_p} = 1.231$$

where

P_p is P-leg power for one P-leg in watts

Substituting

$$\frac{L_p}{A_p} = 11.76 \text{ from step 1,}$$

$$P \times 11.76 = 1.231$$

$$P = 0.1047 \text{ watts}$$

3. P-leg heat input

The heat input term, $Q_s L/A$, is given as

$$Q_{sp} \frac{L_p}{A_p} = 15.26$$

where

Q_{sp} is heat input to one P-leg in watts

Substituting

$$\frac{L_p}{A_p} = 11.76 \text{ from step 1,}$$

$$Q_{s_p} \times 11.76 = 15.26$$

$$Q_{s_p} = 1.2976 \text{ watts}$$

$$4. \text{ N-leg } \frac{L_n}{A_n} \text{ ratio}$$

From the N-leg data, the current term, $I L/A$, is given as

$$I \frac{L_n}{A_n} = 41.35$$

where

$$L_n = \text{over-all N-leg length in inches}$$

$$A_n = \text{cross-sectional area of 0.187 diameter N-leg, which equals } 0.02761 \text{ square inch}$$

Substituting the 2.5 ampere design specification current for EOL

$$2.5 \frac{L_n}{A_n} = 41.35$$

$$\frac{L_n}{A_n} = 16.54$$

Over-all N-length is

$$L_n = 16.54 \times 0.02761 = 0.457 \text{ inch}$$

Computed data showed that the N-leg efficiency was highest at an $\frac{L_{cn}}{L_n}$ ratio of 0.300. Therefore, N-leg cold segment length is

$$\begin{aligned} L_{cn} &= 0.300 L_n \\ &= 0.300 \times 0.457 \\ &= 0.137 \text{ inch} \end{aligned}$$

5. N-leg Power

The N-leg power term, $P L/A$, is given as

$$P_n \frac{L_n}{A_n} = 1.9635$$

where

P_n = N-leg power for one N-leg in watts.

Substituting

$$\frac{L_n}{A_n} = 16.54 \text{ from step 4,}$$

$$P_n \times 16.54 = 1.9635$$

$$P_n = 0.1187 \text{ watts}$$

6. N-leg heat input

The heat input term, $Q_s L/A$, is given as

$$Q_{s_n} \frac{L_n}{A_n} = 22.925$$

Substituting

$$\frac{L_n}{A_n} = 16.54 \text{ from step 4,}$$

$$Q_{S_n} \times 16.54 = 22.925$$

$$Q_{S_n} = 1.3860 \text{ watts}$$

7. Total number of couples

Since 24 volts are specified for EOL operating conditions, the number of couples times the voltage per couple must equal 24 for couples connected in series. The number of couples is, therefore, found from the relation

$$N_c = \frac{24}{V_p + V_n}$$

where

$$N_c = \text{total number of couples}$$

$$V_p = \text{load voltage for one P-leg}$$

$$V_n = \text{load voltage for one N-leg}$$

As given by the calculated data, V_p equals 0.04190 volts and V_n equals 0.04745 volts.

Substituting

$$N_c = \frac{24}{0.04190 + 0.04745} = 269 \text{ couples}$$

8. Total power output

The power developed by one couple is equal to the power developed by one P-leg plus the power developed by one N-leg. Total power, then, is equal to the number of couples times the combined-couple power and is given by the expression

$$P_t = N_c (P_p + P_n)$$

where

P_t equals total power output in watts.

Substituting the values for P_p , P_n , and N_c from steps 2, 5, and 7, respectively,

$$P_t = 269 (0.1047 + 0.1187) = 60.09 \text{ watts.}$$

This checks quite closely with the required EOL power of 60.00 watts.

9. Total heat input to couples at EOL

Heat input is calculated in a manner similar to that for power and is given by the equation

$$Q_{s_t} = N_c (Q_{s_p} + Q_{s_n})$$

where

Q_{s_t} equals total heat input to couples.

Substituting the values for Q_{s_p} , Q_{s_n} , and N_c from steps 3, 6, and 7 respectively,

$$Q_{s_t} = 269 (1.2976 + 1.3860) = 721.9 \text{ watts}$$

10. Total converter heat input at EOL

Total heat input is equal to the total heat to couples, Q_{s_t} , plus the converter heat loss Q_1 .

From item 15, "Thermopile Design Data Base" (above),

$$Q_1 = 0.1097 (T_h - T_c - 600) + 46.11.$$

Substituting 890° F for T_h and 180° F for T_c ,

$$Q_1 = 0.1097 (890 - 180 - 600) + 46.11 = 58.2 \text{ watts}$$

Letting $Q_{t_{EOL}}$ equal total converter heat input at EOL,

$$Q_{t_{EOL}} = Q_1 \Big|_{\text{at EOL}} + Q_{s_t} \Big|_{\text{at EOL}}$$

Substituting the values for Q_1 and Q_{s_t} from steps 9 and 10,

$$Q_{t_{EOL}} = 58.2 + 721.9 = 780.1 \text{ watts}$$

11. Total converter heat input at BOL

The total converter heat input at BOL is given by the expression

$$Q_{t_{BOL}} = \frac{Q_{t_{EOL}}}{0.782}$$

where

0.782 is the decay factor for the fuel over a ten-year period.

Substituting the value for $Q_{t_{EOL}}$ from step 10,

$$Q_{t_{BOL}} + \frac{780.1}{0.782} = 997.5 \text{ watts}$$

12. Converter heat loss at BOL

The heat loss at BOL is greater than at EOL because of the higher operating temperatures. Hot and cold junction temperatures are 1075° F and 210° F, as stated in the design data base.

Substituting in the heat loss equation,

$$Q_{1_{\text{BOL}}} = 0.1097 (1075 - 210 - 600) + 46.11 = 75.2 \text{ watts}$$

13. Total heat input to couples at BOL

Using the expression given in step 10,

$$Q_{t_{\text{BOL}}} = Q_1 \Big|_{\text{at BOL}} + Q_{s_t} \Big|_{\text{at BOL}}$$

Substituting the values for $Q_{t_{\text{BOL}}}$ and Q_1 at BOL from steps 11 and 12,

$$997.5 = 75.2 + Q_{s_t} \Big|_{\text{at BOL}}$$

$$Q_{s_t} \Big|_{\text{at BOL}} = 922.3 \text{ watts}$$

14. P-leg segment temperature and hot junction temperature checks at BOL

In order to determine whether the above thermoelectric leg design will satisfy the maximum allowable segment temperature of 600° F and the maximum allowable average hot junction temperature of 1075° F, the design values calculated are applied as input data for the converter-performance digital computer program mentioned previously; results solved for by this program are compared to the given specifications for conformance.

Since the ability of the couples to absorb heat is at a maximum under a load voltage of 24 volts, this value of load voltage is applied as a constraint in the computer solution.

For the above design, the P-leg segment temperature was 575° F, which satisfies the segment temperature specification. Couple heat input was 926.7 watts and since this value is slightly greater than the available BOL heat to couples calculated in step 13, there is no danger in exceeding the maximum allowable average hot junction temperature of 1075° F. Expressing this mathematically,

$$\begin{array}{rcl} \text{Ability to absorb heat at BOL} & \geq & \text{Available heat at BOL} \\ 926.7 & > & 922.3 \text{ (watts)} \end{array}$$

15. Converter efficiency at EOL

Converter efficiency calculated for EOL conditions involves the electrical power output at EOL and total heat input to converter at EOL. Letting η designate converter efficiency in percent,

$$\eta = \frac{P_{tEOL}}{Q_{tEOL}} \times 100$$

Substituting the values for P_{tEOL} and Q_{tEOL} from steps 8 and 10,

$$\eta = \frac{60.09}{780.1} = 7.7 \text{ percent.}$$

APPENDIX F

COMPUTER PROGRAM FOR CONVERTER PERFORMANCE — SYSTEM INTEGRATION STUDIES

Objectives

The converter will be designed to be integrated with the overall system design effort being conducted by the SNAP-23A System contractor (WANL). Design objectives will not be limited to only the physical, thermal, and electrical interface requirements, but will also include studies to determine effects of varying parameters on the thermoelectric converter performance and system performance. Parametric studies will be conducted to provide data which will influence the physical, thermal, and electrical interfaces of the converter and the system.

Technical Approach

In order to accomplish the program technical objectives through parametric studies-system integration, a computer program is required for system performance analysis. A desirable computer program will be established which would contain three major elements:

1. A thermal subroutine to establish system temperature distribution, heat losses, and heat input to the converter.
2. A thermoelectric converter subroutine to establish converter performance as a result of the varying of either hot or cold interface temperatures.
3. A radiator subroutine to predict radiator performance under varying conditions of thermoelectric converter-system interface cold temperatures or ambient air (or coolant).

An analytical model of the overall system will be developed by the system contractor (WANL), 3M Company will provide data for the establishment of the converter performance code. System parametric analyses will be performed then by use of this system analytical model.

The data required for the converter portion of this analytical model includes thermoelectric property data (Seebeck, resistivity, and thermal conductivity coefficients) and equations for computer programming. 3M Company has developed and used computer programs of this nature (and has verified results experimentally) with other TE devices.

Effort during this first quarter was directed toward the preparation of a general-purpose computer program to meet the program technical objectives, using previously-developed computer programs as a base. As a result, a versatile master or general purpose program has been written (in FORTRAN compiler language) which allows flexibility, through slight modifications to the source program, to treat specific applications for parametric studies quite readily. In addition, this analytical model and the thermoelectric property data have been provided to the SNAP-23A system contractor for incorporation into the overall system analytical model.

The following material represents a general description of the analytical model, with a listing of the necessary input data, equations and calculations, and a logic flow-diagram. The degree of detail of necessary equations and calculations presented herein, though sufficient to describe the program, is not oriented toward any particular computer hardware and/or software. A further distinction of this flow diagram from the program as it exists is that no provision has been shown for the treatment of input/output data, nor for the handling of such items as temperature increments to repeat the cycle of calculations for parametric analyses. However, these matters are not considered necessary for the description of the type of calculation employed, and may be treated as specific items related to the particular computer software/hardware chosen.

The equations and/or calculations necessary for each block shown on the flow diagram are given by reference to a corresponding item-number designation.

ITEM NO.	REFERENCED PARAMETER	INPUT DATA - DESCRIPTION
1	No. Cpl	Number of couples
2	A_N	Area N-leg
3	A_P	Area P-leg
4	L_N	Total length N-leg
5	L_{CN}	Cold end length N-leg
6	L_P	Total length P-leg
7	L_{CP}	Cold end length P-leg
8	R_{Hshoe}	Hot shoe resistance
9	R_{Cshoe}	Cold shoe resistance
10	R_{misc}	Miscellaneous resistance (including contact)
11	T_C	Temperature cold junction
12	T_H	Temperature hot junction
13	$N^{K_C}(T)$	N-type cold-end thermal conductivity, given by the coefficients K_J , $J=1, \dots, 6$. $N^{K_C}(T) = K_1 + K_2T + K_3T^2 + K_4T^3 + K_5T^4 + K_6T^5$
14	$N^{K_H}(T)$	N-type hot-end thermal conductivity given by the coefficients K_J , $J=7, \dots, 12$. $N^{K_H}(T) = K_7 + K_8T + K_9T^2 + K_{10}T^3 + K_{11}T^4 + K_{12}T^5$
15	$P^{K_C}(T)$	P-type cold-end thermal conductivity given by the coefficients K_J , $J=13, \dots, 18$. $P^{K_C}(T) = K_{13} + K_{14}T + K_{15}T^2 + K_{16}T^3 + K_{17}T^4 + K_{18}T^5$
16	$P^{K_H}(T)$	P-type hot-end thermal conductivity given by the coefficients K_J , $J=19, \dots, 24$. $P^{K_H}(T) = K_{19} + K_{20}T + K_{21}T^2 + K_{22}T^3 + K_{23}T^4 + K_{24}T^5$

ITEM NO.	REFERENCED PARAMATER	INPUT DATA - DESCRIPTION
17	$N^{S_C}(T)$	N-type cold-end Seebeck coefficient given by the coefficients S_J , $J=1, \dots, 6$. $N^{S_C}(T) = S_1 + S_2T + S_3T^2 + S_4T^3 + S_5T^4 + S_6T^5$
18	$N^{S_H}(T)$	N-type hot-end Seebeck coefficient given by the coefficients S_J , $J=7, \dots, 12$. $N^{S_H}(T) = S_7 + S_8T + S_9T^2 + S_{10}T^3 + S_{11}T^4 + S_{12}T^5$
19	$P^{S_H}(T)$	P-type hot-end Seebeck coefficient given by the coefficients S_J , $J=19, \dots, 24$. $P^{S_H}(T) = S_{19} + S_{20}T + S_{21}T^2 + S_{22}T^3 + S_{23}T^4 + S_{24}T^5$
20	$N^{[\rho K(T)]_C}$	N-type cold-end resistivity-conductivity coefficient given by the coefficients $(\rho K)_J$, $J=1, \dots, 6$. $N^{[\rho K(T)]_L} = (\rho K)_1 + (\rho K)_2T + (\rho K)_3T^2 + (\rho K)_4T^3 + (\rho K)_5T^4 + (\rho K)_6T^5$
21	$N^{[\rho K(T)]_H}$	N-type hot-end resistivity-conductivity coefficient given by the coefficients $(\rho K)_J$, $J=7, \dots, 12$. $N^{[\rho K(T)]_H} = (\rho K)_7 + (\rho K)_8T + (\rho K)_9T^2 + (\rho K)_{10}T^3 + (\rho K)_{11}T^4 + (\rho K)_{12}T^5$
22	$P^{[\rho K(T)]_C}$	P-type cold-end resistivity-conductivity coefficient given by the coefficients $(\rho K)_J$, $J=13, \dots, 18$. $P^{[\rho K(T)]_C} = (\rho K)_{13} + (\rho K)_{14}T + (\rho K)_{15}T^2 + (\rho K)_{16}T^3 + (\rho K)_{17}T^4 + (\rho K)_{18}T^5$
23	$P^{[\rho K(T)]_H}$	P-type hot-end resistivity-conductivity coefficient given by the coefficients $(\rho K)_J$, $J=19, \dots, 24$. $P^{[\rho K(T)]_H} = (\rho K)_{19} + (\rho K)_{20}T + (\rho K)_{21}T^2 + (\rho K)_{22}T^3 + (\rho K)_{23}T^4 + (\rho K)_{24}T^5$
24	$P^{S_C}(T)$	P-type cold-end Seebeck coefficient given by the coefficients S_J , $J=13, \dots, 18$. $P^{S_C}(T) = S_{13} + S_{14}T + S_{15}T^2 + S_{16}T^3 + S_{17}T^4 + S_{18}T^5$
25	I_{fraction}	Fraction of matched-load current, a multiplicative factor chosen arbitrarily as an input parameter to analyze generator performance at conditions other than matched load ($I_{\text{fraction}} = 1$).

ITEM NO.

EQUATIONS AND/OR CALCULATIONS

- 1 Calculate ratio of N-leg cold-end length to total length.
Calculate ratio of P-leg cold-end length to total length.
- 2 $T_H = 5/9 (T_H + 459.7)$
 $T_C = 5/9 (T_C + 459.7)$
- 3 Do both $T_H(J) = T_H J$ and $T_C(J) = T_C J$ for $J = 1, 2, \dots, 6$.
- 4 Iterative loop for calculation of N segment temp.:
$$T_S \equiv T(I+1) = T(I) + [T(I-1) - T(I)] * [L_C/L \text{ N-leg}] - L(I) / [L(I-1) - L(I)]$$

for $I \equiv 1, 2, \dots$

with initial values for $I = 1$ defined to be

$$T(1) \equiv T_C$$

$$T(0) \equiv T_H$$

$$L(1) \equiv 0$$

$$L(0) \equiv 1$$

and $L(I)$ subsequently determined as trial L_C/L calculated at Item No. 10

- 5 Do $T_S(J) = T_S J$ for $J = 1, 2, \dots, 6$.
- 6 Do $T_S(J) - T_C(J)$ for $J = 1, 2, \dots, 6$.
- 7 Do $T_H(J) - T_S(J)$ for $J = 1, 2, \dots, 6$.
- 8
$$Q_A^L \Big|_{N\text{-cold}} = \int_{T_C}^{T_S} T_{N^K C}(T) dT$$
- 9
$$Q_A^L \Big|_{N\text{-hot}} = \int_{T_S}^{T_H} T_{N^K H}(T) dT$$

ITEM NO.

EQUATIONS AND/OR CALCULATIONS

10 L(I) trial $L_{C/L}$ for N-leg

$$L(I) = \left[Q \frac{L}{A} \right]_{N-cold} \bigg/ \left(\left[Q \frac{L}{A} \right]_{N-cold} + \left[Q \frac{L}{A} \right]_{N-hot} \right)$$

and calculate DIFFERENCE = $ABS \left(L(I) - L_{C/L} \right) \bigg|_{N-leg}$

11 Iterative loop for calculation of P segment temp.:

$$T_S \equiv T(J+1) = T(J) + [T(J-1) - T(J)] * \left[L_{C/L} \right]_{P-leg - L(J)} \bigg/ [L(J-1) - L(J)]$$

for J = 1, 2, ...

with initial values for J = 1 defined to be:

$$T(1) \equiv T_C$$

$$T(0) \equiv T_H$$

$$L(1) \equiv 0$$

$$L(0) \equiv 1$$

and L(J) subsequently determined as trial $L_{C/L}$ calculated at Item No. 17.

12 Do $T_S(J) = T_S^J$ for J = 1, 2, ..., 6.

13 Do $T_S(J) - T_C(J)$ for J = 1, 2, ..., 6.

14 Do $T_H(J) - T_S(J)$ for J = 1, 2, ..., 6.

$$15 \quad Q \frac{L}{A} \text{ P-cold} = \int_{T_C}^{T_S} P K_C(T) dT$$

$$16 \quad Q \frac{L}{A} \text{ P-hot} = \int_{T_S}^{T_H} P K_H(T) dT$$

17 L(J) $\equiv$ Trial $L_{C/L}$ for P-leg

$$L(J) = Q \frac{L}{A} \text{ P-cold} \bigg/ \left(Q \frac{L}{A} \text{ P-cold} + Q \frac{L}{A} \text{ P-hot} \right)$$

and calculate: Difference = $ABS \left(L(J) - L_{C/L} \right) \bigg|_{P-leg}$

ITEM NO.

EQUATIONS AND/OR CALCULATIONS

$$18 \quad Q_C \Big|_{N\text{-leg}} = \frac{A}{L} \Big|_{N\text{-leg}} * \left(Q_C \frac{L}{A} \Big|_{N\text{-cold}} + Q_C \frac{L}{A} \Big|_{N\text{-hot}} \right)$$

$$19 \quad Q_C \Big|_{P\text{-leg}} = \frac{A}{L} \Big|_{P\text{-leg}} * \left(Q_C \frac{L}{A} \Big|_{P\text{-cold}} + Q_C \frac{L}{A} \Big|_{P\text{-hot}} \right)$$

$$20 \quad Q_C = (\text{No. Cpl}) * \left(Q_C \Big|_{N\text{-leg}} + Q_C \Big|_{P\text{-leg}} \right)$$

$$21 \quad E \Big|_{N\text{-cold}} = \int_{T_C}^{T_S} T_N S_C (T) dT$$

$$22 \quad E \Big|_{N\text{-hot}} = \int_{T_S}^{T_H} T_N S_H (T) dT$$

$$23 \quad E \Big|_{P\text{-cold}} = \int_{T_C}^{T_S} T_P S_C (T) dT$$

$$24 \quad E \Big|_{P\text{-hot}} = \int_{T_S}^{T_H} T_P S_H (T) dT$$

$$25 \quad E = (\text{No. Cpl}) * \left(E \Big|_{N\text{-cold}} + E \Big|_{N\text{-hot}} + E \Big|_{P\text{-cold}} + E \Big|_{P\text{-hot}} \right)$$

$$26 \quad R Q_C \Big|_{N\text{-cold}} = \int_{T_C}^{T_S} T_N [\rho K(T)]_C dT$$

$$27 \quad R Q_C \Big|_{N\text{-hot}} = \int_{T_S}^{T_H} T_N [\rho K(T)]_H dT$$

$$28 \quad R_{N\text{-leg}} = \left(R Q_C \Big|_{N\text{-cold}} + R Q_C \Big|_{N\text{-hot}} \right) / Q_C \Big|_{N\text{-leg}}$$

$$29 \quad R Q_C \Big|_{P\text{-cold}} = \int_{T_C}^{T_S} T_P [\rho K(T)]_C dT$$

ITEM NO.

EQUATIONS AND/OR CALCULATIONS

$$30 \quad R_{Q_C} \Big|_{P\text{-hot}} = \int_{T_S}^{T_H} P[\rho K(T)]_H dT$$

$$31 \quad R \Big|_{P\text{-leg}} = \left(R_{Q_C} \Big|_{P\text{-cold}} + R_{Q_C} \Big|_{P\text{-hot}} \right) / Q_C \Big|_{P\text{-leg}}$$

$$32 \quad R_{mat} = (\text{No. Cpl}) * \left(R \Big|_{N\text{-leg}} + R \Big|_{P\text{-leg}} \right)$$

$$33 \quad R_{H, shoe} = (\text{No. Cpl}) * R_{H, shoe}$$

$$R_{C, shoe} = (\text{No. Cpl}) * R_{C, shoe}$$

$$34 \quad R_{total} = R_{mat} + R_{H, shoe} + R_{C, shoe} + R_{misc.}$$

$$35 \quad I_{M.L.} = E / \left(2 * R_{total} \right)$$

$$36 \quad I_{load} = I_{M.L.} * I_{fraction}$$

$$37 \quad Q_P \Big|_{N\text{-hot}} = N^{S_H}(T_H)$$

$$38 \quad Q_P \Big|_{P\text{-hot}} = P^{S_H}(T_H)$$

$$39 \quad Q_P \Big|_{Hot} = \left(Q_P \Big|_{N\text{-hot}} + Q_P \Big|_{P\text{-hot}} \right) * T_H * I_{load} * (\text{No. Cpl})$$

$$40 \quad Q_P \Big|_{N\text{-cold}} = N^{S_C}(T_C)$$

$$41 \quad Q_P \Big|_{P\text{-cold}} = P^{S_C}(T_C)$$

$$42 \quad Q_P \Big|_{cold} = \left(Q_P \Big|_{N\text{-cold}} + Q_P \Big|_{P\text{-cold}} \right) * T_C * I_{load} * (\text{No Cpl})$$

$$43 \quad Q_T \Big|_{N\text{-cold}} = \int_{T_C}^{T_S} T \frac{d}{dT} \left[N^{S_C}(T) \right] dT$$

$$44 \quad Q_T \Big|_{N\text{-hot}} = \int_{T_S}^{T_H} T \frac{d}{dT} \left[N^{S_H}(T) \right] dT$$

$$45 \quad Q_T \Big|_{P\text{-cold}} = \int_{T_C}^{T_S} T \frac{d}{dT} \left[P^{S_C}(T) \right] dT$$

ITEM NO.

EQUATIONS AND/OR CALCULATIONS

- 46 $Q_T \Big|_{P\text{-hot}} = \int_{T_S}^{T_H} T \frac{d}{dT} \left(P^{S_H}(T) \right) dT$
- 47 $Q_T = (\text{No. Cpl}) * I_{\text{load}} * I \left(Q_T \Big|_{N\text{-cold}} + Q_T \Big|_{N\text{-hot}} + \right. \\ \left. Q_T \Big|_{P\text{-cold}} + Q_T \Big|_{P\text{-hot}} \right)$
- 48 $Q_J = I_{\text{load}}^2 * R_{\text{total}}$
- 49 $Q_{\text{input}} = Q_C + Q_P \Big|_{\text{hot}} - 1/2 Q_J - 1/2 Q_T$
- 50 $Q_{\text{rejected}} = Q_C + Q_P \Big|_{\text{cold}} + 1/2 Q_J + 1/2 Q_T$
- 51 $V_{\text{load}} = E - I_{\text{load}} * R_{\text{total}}$
- 52 $P = V_{\text{load}} * I_{\text{load}}$
- 53 $\eta = P/Q_{\text{input}}$

APPENDIX G

DEVELOPMENT OF 10-COUPLE MODULE VARIABLES MATRIX

The following information describes the development of the matrix of variables for eight modules; the complete matrix for the 12 ten-couple modules is given in Section 4.4.2. (Table 4-1)

Table G-1 represents the initial 10-couple module variables matrix. Table G-2 represents a full factorial test design and is the result of arranging all possible combinations of two levels of N controlled variables yielding 64 test combinations or sets. The diagram is easily established by first coding the six variables to be considered as A through F, respectively, and then assigning + to represent one alternative and - the other alternative. Then the rows are arranged as follows:

1st column = alternate -'s and +'s

2nd column = alternate pairs of -'s and +'s

3rd column = alternate groups of four -'s and +'s

etc.,

An arrangement of this type is referred to as "standard order". All possible combinations of variables are contained in this diagram.

Although there are ten couples per module, the test lends itself best to variations of only eight couples per module. The remaining two couples will be discussed later. Notice that the diagram is divided into eight sections representing eight modules, each containing eight combinations of variables. Any one set of combinations would be applied to one couple. For example, for the given definition of variables with module #1, set 7, the affected couple would be a) pressure bonded, b) hard nose, c) iron hot electrode, d) less than 1100° T_h, e) less than 250 psi spring pressure, and f) microquartz insulation.

Because it is not feasible to vary the insulation within any one module, the F column represents the insulation variable. Note that no variation occurs in any one module for columns D, E, or F. The least flexible variables are assigned to

Table G-1. Initial 10-Couple Module Variables Matrix (Proposal)

Module	Circuit	Thermoelectric Couple	Thermal Insulation	Hot Electrode	Pressure	Temperature	Comments
1	A	N	MK	Fe	250	1100	Baseline
	B	N					
2	A	N	MK	Fe	250	1100	Baseline
	B	N					
3	A	H	MQ	Fe	250	1100	Backup
	B	H					
4	A	H	MQ	Fe	250	1100	Backup
	B	H					
5	A	N	MK	X	250	1100	Hot electrode variable
	B	H					
6	A	N	MQ	X	250	1100	Hot electrode variable
	B	H					
7	A	N	MK	Fe	K	1100	Pressure variable
	B	H					
8	A	N	MQ	Fe	X	1100	Pressure variable
	B	H					
9	A	N	MK	Fe	250	X	Temperature variable
	B	H					
10	A	N	MQ	Fe	250	X	Temperature variable
	B	H					

Thm Insl - Thermal Insulation

Mk - Min-K 1301

MQ - Microquartz

Ht Elec - Hot Electrode Button Material

Fe - Iron

X - To be selected on basis of short-term test work

Press - Electrode Contact Pressure
250 (psi)

X - To be selected on basis of short-term test work

Temp - Hot Electrode Temperature

1100° F Maximum

X - To be selected on basis of short-term test work

Circuit - Each module has two independent circuits
A or B

TE Couple - Thermoelectric Couple

H - Hot Segmented

N - Normal

Table G-2. Full Factorial Test Combinations

	A	B	C	D	E	F
1	-	-	-	-	-	-
2	+	-	-	-	-	-
3	-	+	-	-	-	-
4	+	+	-	-	-	-
5	-	-	+	-	-	-
6	+	-	+	-	-	-
7	-	+	+	-	-	-
8	+	+	+	-	-	-
9	-	-	-	+	-	-
10	+	-	-	+	-	-
11	-	+	-	+	-	-
12	+	+	-	+	-	-
13	-	-	+	+	-	-
14	+	-	+	+	-	-
15	-	+	+	+	-	-
16	+	+	+	+	-	-
17	-	-	-	-	+	-
18	+	-	-	-	+	-
19	-	+	-	-	+	-
20	+	+	-	-	+	-
21	-	-	+	-	+	-
22	+	-	+	-	+	-
23	-	+	+	-	+	-
24	+	+	+	-	+	-
25	-	-	-	+	+	-

Table G-2. Full Factorial Test Combinations (Continued)

	A	B	C	D	E	F
26	+	-	-	+	+	-
27	-	+	-	+	+	-
28	+	+	-	+	+	-
29	-	-	+	+	+	-
30	+	-	+	+	+	-
31	-	+	+	+	+	-
32	+	+	+	+	+	-
33	-	-	-	-	-	+
34	+	-	-	-	-	+
35	-	+	-	-	-	+
36	+	+	-	-	-	+
37	-	-	+	-	-	+
38	+	-	+	-	-	+
39	-	+	+	-	-	+
40	+	+	+	-	-	+
41	-	-	-	+	-	+
42	+	-	-	+	-	+
43	-	+	-	+	-	+
44	+	+	-	+	-	+
45	-	-	+	+	-	+
46	+	-	+	+	-	+
47	-	+	+	+	-	+
48	+	+	+	+	-	+
49	-	-	-	-	+	+
50	+	-	-	-	+	+

Table G-2. Full Factorial Test Combinations (Continued)

	A	B	C	D	E	F
51	-	+	-	-	+	+
52	+	+	-	-	+	+
53	-	-	+	-	+	+
54	+	-	+	-	+	+
55	-	+	+	-	+	+
56	+	+	+	-	+	+
57	-	-	-	+	+	+
58	+	-	-	+	+	+
59	-	+	-	+	+	+
60	+	+	-	+	+	+
61	-	-	+	+	+	+
62	+	-	+	+	+	+
63	-	+	+	+	+	+
64	+	+	+	+	+	+
+	= high		D+	= 1100°		
-	= low		D-	= <1100°		
A+	= SPP		E+	= 250		
A-	= PB		E-	= <250		
B+	= CONVENTIONAL		F+	= MIN-K		
B-	= SOFT NOSE		F-	= MICRO-Q		
C+	= Fe					
C-	= Mo					

these columns. In this test such variables would be insulation, spring pressure, and temperature. Despite the restrictions this appears to impose, each alternative of each variable is repeated many times in the presence of other combinations of variables. The effects of each variable can be singled out for closer scrutiny. As an example of the numerous comparisons represented, only two direct comparisons of iron versus molybdenum are made with all other variables equal. (Sets 1 vs 64 and 5 vs 60). Despite this fact, all other sets can be examined for the effects of iron and molybdenum alone, regardless of the presence of other variables. Additionally, it can be seen that iron versus molybdenum is compared thirty-two times, as are all other alternatives of any given variable.

One argument for "classical" experimentation, wherein variables are altered one at a time while all others are held constant, is that "If the variables other than the one under consideration are not held constant, their effects will interfere with the measurement of that one important variable". In propounding this argument, the presence of interactions between variables is admitted. These interactions are important. For example, in the present test, it might be said that it is unnecessary to compare iron hot junction electrodes at 1100° and with those at temperatures less than 1100° because that temperature induces little or no change in iron hot junction electrode performance. However, an effect might be induced in this combination when spring pressure is varied. In order to investigate this possibility (among others), this test matrix covers all possible interactions of this type, without degrading other test results, or adding to the expense or complexity by requiring additional modules.

The two remaining couples in each module repeat a portion of this experiment (referred to as replication) in order to establish an estimate of the error contained in the test data, and at the same time to provide a way of detecting module-to-module differences. This would further improve test validity.

APPENDIX H

HARDWARE DESIGN AND DEVELOPMENT

1.0 HARDWARE DESIGN AND DEVELOPMENT

Most of the hardware for SNAP-23A 10-couple modules has been designed. Much of this hardware is similar to that of the SNAP-27 10-couple module, with changes primarily in geometry. There were, however, several areas that required evaluation and design effort, as listed below:

- Thermoelectric couples
- Follower springs
- Center post-structural support
- Heater block and electrical heaters
- Electrical feedthrus
- Insulation materials

1.1 THERMOELECTRIC COUPLES

As proposed by the initial test matrix, two types of couples were to be used. These differ basically in material differences of the P-leg. Two types of N-legs will be combined with four types of P-legs, resulting in a total evaluation of 8 types of thermoelectric couples. Because instrumentation is attached to either the N- or P-leg cap, 16 types of thermoelectric couples were designed.

Couple Part No.	N-leg	P-leg
P ₁	A	A
P ₂	B	B
P ₃	A	C
P ₄	B	D
P ₉	A	B
P ₁₀	B	C
P ₁₁	A	D
P ₁₂	B	A

P₅ - P₈ identical to P₁ - P₄ except for instrumentation

P₁₃ - P₁₆ identical to P₉ - P₁₂ except for instrumentation

N-leg - A - Baseline fabrication
 B - Alternate fabrication

P-leg - A - Baseline fabrication - conventional
 B - Alternate fabrication - conventional
 C - Baseline fabrication - soft nose
 D - Alternate fabrication - soft nose

Geometry of these couples is the same as was used in the SNAP-23 converter thermoelectric couple. The 10-couple module will evaluate the relative performance of these thermoelectric couples.

1.2 FOLLOWER SPRINGS

The objective of this design is to provide follower springs with nominal electrode contact pressures of 150 psi or 250 psi on each thermoelectric leg when assembled in the 10-couple module.

The working length of the spring is determined using Figure H-1.

Figure H-1. Follower Spring Location

Cold frame:	1.849 ± 0.001
Spacing:	0.876 ± 0.001
Total:	2.725 ± 0.002

Subtracting from this value the sum of the piece part thicknesses or lengths between the spring and upper heater block, permits determination of the working length for the spring.

Boron insulator:	0.045 ± 0.001
Electrode:	0.040 ± 0.003
Button:	0.015 ± 0.002
Couple:	0.68 ± 0.02
Follower:	0.9385 ± 0.0015
Insulating washer:	0.014 ± 0.001
Support washer:	0.029 ± 0.001
Hole location:	0.060 ± 0.015
1/2 hole diameter:	0.025 ± 0.002
Total:	1.8465 ± 0.465

Working length $(2.725 \pm 0.002) - (1.8465 \pm 0.0465)$
 0.8785 ± 0.0485
 $0.830/0.927$

The spring force at this working length is determined from the leg areas and the pressure requirements.

Leg diameter:	0.187 ± 0.005
Max. surface area:	0.0290 in^2
Nom. surface area:	0.0275 in^2
Min. surface area:	0.0260 in^2
Nominal load 250 psi:	$250 \times 0.0275 = 6.87 \text{ lbs.}$
Nominal load 150 psi:	$150 \times 0.0275 = 4.12 \text{ lbs.}$

Material chosen for the springs is stainless steel wire, spring temper. The specification for Ni-Cr stainless spring wire, ASTM A313-63, shows minimum tensile strengths for wire, as follows:

0.032 diameter	277,000 psi
0.036 diameter	273,000 psi

Wahl recommends working stresses of 35 percent of tensile strength.*

$$0.35 \times 277,000 = 97,000$$

$$0.35 \times 273,000 = 95,500$$

The spring must work in a 0.218 inch maximum diameter hole and over a 0.08 inch maximum diameter protrusion on the follower.

Spring design – 250 psi – the uncorrected stress equation gives allowable load for a specified working stress.

$$P = \frac{\pi d^3 s}{8D}$$

where

- d = 0.032 wire diameter
- D = 0.203 coil diameter
- s = 97,000 working stress

$$p = \frac{\pi (0.032)^3 (97,000)}{8 (0.203)} = 6.15 \text{ lbs.}$$

Since the required load is somewhat higher, the stress will be higher. Decreasing the coil diameter or increasing the wire diameter will allow a greater load. Figure H-2 shows this variation.

*Wahl, Mechanical Springs. McGraw-Hill 1963, page 101.

Figure H-2. Spring Load as a Function of Wire Diameter and Outside Coil Diameter

The number of coils is found from the minimum working length minus 10 percent for spacing between the coils.

$$0.830 - 0.083 = 0.747$$

$$n = 0.747 / 0.032 = 24 \text{ coils maximum}$$

The free length of the spring is limited because an extra long spring may buckle as a column at relatively light loads. Sixteen coils will keep the free length under 1.500 inches. The spring rate becomes

$$I_c = \frac{P}{Y} = \frac{6.87}{0.608} = 11.3 \text{ lb/in.}$$

Rather than specifying tolerances for wire diameter, coil diameter, etc., the permissible load variation between the working length is specified.

$$1.487 - 0.927 = 0.560 \text{ minimum compression}$$

$$1.487 - 0.830 = 0.657 \text{ maximum compression}$$

$$0.560 \times 11.3 = 6.32 \text{ lbs. minimum}$$

$$0.657 \times 11.3 = 7.42 \text{ lbs. maximum}$$

$$\text{Spring load is } 6.87 \text{ lbs. } \pm 0.55 \text{ lbs.}$$

$$\text{or } 6.87 \text{ lbs. } \pm 8 \text{ percent}$$

The contact pressure as a result of this load variation will range between two values also.

$$7.42 / 0.026 = 285 \text{ psi maximum}$$

$$6.87 / 0.0275 = 250 \text{ psi nominal}$$

$$6.32 / 0.029 = 218 \text{ psi minimum}$$

$$\text{Contact pressure } 218 - 285 \text{ psi}$$

Spring design 150 psi – the spring is identical in size to the 250 psi spring except for free length because of the lower spring load required.

$$Y = \frac{8 PD^3 n}{G d^4}$$

$$Y = \frac{8 (4.12) (203)^3 n}{11.5 \times 10^6 (0.032)^4} = 0.0228 n$$

<u>n</u>	<u>Y</u>	<u>Free Length</u>
10	0.228	1.107
12	0.274	1.153
14	0.319	1.198
16	0.365	1.244
18	0.410	1.289
20	0.456	1.335
22	0.501	1.380
24	0.547	1.426

22 active coils are used.

Spring rate $4.12/0.501 = 8.2$ lb/in.

$1.380 - 0.927 = 0.453$ minimum compression

$1.380 - 0.830 = 0.550$ maximum compression

$0.453 \times 8.2 = 3.72$ lbs. minimum

$0.550 \times 8.2 = 4.52$ lbs. maximum

Spring load is 4.12 lbs. ± 0.40 lbs.

or 4.12 ± 10 percent

Contact pressure:

$$4.52/0.026 = 174 \text{ psi maximum}$$

$$4.12/0.0275 = 150 \text{ psi nominal}$$

$$3.72/0.029 = 128 \text{ psi minimum}$$

Range 128 - 174 psi

Because the working length lies in a range between 0.830/0.927 inch, the spring rate should be as small as possible to result in a smaller load variation and consequently a smaller contact pressure variation.

Deflection is given by the following:

$$Y = \frac{8 P D^3 n}{G d^4}$$

where

n = number of coils

D = coil diameter

d = wire diameter

G = shear modulus

P = spring load.

For the design under consideration

$$D = 0.203 \text{ inch}$$

$$d = 0.032 \text{ inch}$$

$$P = 6.87 \text{ lbs.}$$

$$G = 11.5 \times 10^6 \text{ psi}$$

$$Y = \frac{8 (6.87) (0.203)^3 n}{11.5 \times 10^6 (0.032)^4}$$

Substituting various values for $n < 21$, deflections and free length are tabulated.

<u>n</u>	<u>Y</u>	<u>Free Length</u>
14	0.695	1.574
16	0.608	1.487
18	0.540	1.419
20	0.486	1.365
21	0.462	1.341

The buckling criterion of springs with hinged ends is the ratio of free length-to-coil diameter. Buckling will not occur for ratios less than 2.7.* This problem is not acute, however, since the spring is guided by holes in the cold frame.

Allowing an extended length or 1/2 inch, the buckling criterion gives an extended length to coil diameter ratio of 2.5 inches, which should be stable for compression.

The maximum free length is then limited to 1.50 inches.

*Kern's Handbook, Design and Production Volume, Wiley 1961, pages 11-14

COMPRESSION SPRING SPECIFICATION
SPRING – FOLLOWER
150 PSI

MATERIAL:

STAINLESS STEEL WIRE

AISI – TYPE 302 SPRING TEMPER

WORKING CONDITIONS:

TO WORK IN 0.218 DIA. HOLE (MAX.)

TO WORK OVER 0.08 DIA. SHAFT (MIN.)

LOAD 4.12 LBS $\pm$ 0.40 LBS BETWEEN A LENGTH OF 0.920 AND 0.830 IN.

MAXIMUM SOLID HEIGHT 0.800 IN.

MAXIMUM FREE LENGTH 1.500 IN.

OPERATING TEMPERATURE 250° F

TYPE OF ENDS:

GROUND AND SQUARED

DESIGN DATA (REFERENCE):

WIRE DIA. 0.032 IN.

COIL OUTSIDE DIA. 0.203 IN.

RATE 8.0 LB/IN.

FREE LENGTH 1.394 IN.

NUMBER OF COILS 24 (22 ACTIVE)

COMPRESSION SPRING SPECIFICATION
SPRING – FOLLOWER
250 PSI

MATERIAL:

STAINLESS STEEL WIRE

AISI – TYPE 302 SPRING TEMPER

WORKING CONDITIONS:

TO WORK IN 0.218 DIA. HOLE (MAX.)

TO WORK OVER 0.08 DIA. SHAFT (MIN.)

LOAD 6.87 LBS $\pm$ 0.55 LBS BETWEEN A LENGTH OF 0.920 AND 0.830 IN.

MAXIMUM SOLID HEIGHT 0.800 IN.

MAXIMUM FREE LENGTH 1.500 IN.

OPERATING TEMPERATURE 250° F

TYPE OF ENDS:

GROUND AND SQUARED

DESIGN DATA (REFERENCE):

WIRE DIA. 0.032 IN.

COIL OUTSIDE DIA. 0.203 IN.

RATE 11.3 LB/IN.

FREE LENGTH 1.487 IN.

NUMBER OF COILS 18 (16 ACTIVE)

1.3 CENTER POST

The strength of the SNAP-27 center post is evaluated for use with 250 psi follower springs. Each spring exerts a nominal load of 6.87 lbs on the heater block. The total load is $20 \times 6.87 = 138$ lbs. The critical region is near the hot end, where area is a minimum (0.0154 in^2).

Creep strength of 304 stainless at 1150°F is about 10,000 psi*. (1 percent elongation 1000 hours) The stress at the minimum cross-section is S.

$$S = \frac{P}{A} = \frac{138}{0.0154} = 8970 \text{ psi}$$

With an 8 percent increase in P, the stress becomes

$$S = \frac{149}{0.0154} = 9700 \text{ psi}$$

which is very close to the creep strength limit.

The center post is increased in structural strength by using $1/4 - 28$ threads on each end. The minimum diameter becomes 0.190 inch. Increasing the strength in this manner does not increase heat conduction through the post from hot to cold side since the spacing part of the post does not increase in diameter.

The minimum diameter is 0.185 inch, minimum area is 0.0268 inch.

$$\text{Stress } 138/0.0268 = 5150 \text{ psi}$$

$$\text{Stress (8 percent higher load) } 149/0.0268 = 5550 \text{ psi}$$

$$\text{Design Factor } 10,000/5550 = 1.80$$

1.4 UPPER HEATER BLOCK AND ELECTRICAL HEATERS

To provide uniform heating to all thermoelectric elements, the elements are placed into four rows and two heaters are provided, one for each two rows. The following figure shows placement of the heaters in respect to the element rows and the probable temperature gradient in the heater block.

Temperature gradient calculations

$$g = \frac{2 \pi k L}{\ln \frac{r_2}{r_1}} \Delta T$$

$$g = 30 \text{ watts} = 102.5 \text{ Btu}$$

$$k = 10 \text{ Btu/hr } ^\circ \text{F ft}^2/\text{ft}$$

$$L = 1.5/12 \text{ ft}$$

$$2 \pi k L = 7.85$$

$$\Delta T = 102.5/7.85 \ln \frac{r_2}{r_1} = 13.1 \ln \frac{r_2}{r_1}$$

$$r_1 = 0.365/2 = 0.182$$

$\frac{r_2}{r_1}$	$\ln \frac{r_2}{r_1}$	ΔT
0.20	0.095	1F
0.30	0.218	3F
0.40	0.79	10F
0.50	1.01	13F
0.60	1.195	16F
0.70	1.38	18F

At 1150°F operating temperature, maximum heater watt density varies according to the fit or clearance between the hole and the heater. Allowable watt densities for the heaters chosen are:

165 watts/in² at 0.002 fit and 1150° F

60 watts/in² at 0.009 fit and 1150° F

Two heaters, 3/8-inch diameter x 2.00 inches long, are used. The thermal analysis made on the module indicates a minimum of 50-60 watts heater block power is required. If 30 watts per heater is delivered, the watt density is approximately 24 watts/in².

$$\text{Area } (2.00 - 50) \times 0.365 \times \pi = 1.26 \text{ in}^2$$

$$30/1.26 = 23.9 \text{ watts/in}^2$$

This is well below the allowable maximum for a 0.002 inch fit, which indicates that at normal operating conditions, long heater life is expected.

1.5 ELECTRICAL FEEDTHRU

Feedthrus are required that will provide leak-proof seals and electrical insulation for instrumentation and power leads. The power feedthrus used on the SNAP-27 10-couple modules were not recommended. It was also concluded that glass-fused-type headers would be susceptible to cracking at elevated temperature not only when the 10-couple module was operational but during fabrication and welding operations. The probable types of feedthru remaining were ceramic types or soft sealant types, as were used with SNAP-27 instrumentation.

Considering the time element and costs involved, the soft sealant feedthrus were chosen. The type of fitting is illustrated.

Transducer glands are primarily applicable in millivolt ranges and are used in high voltage applications. Power lead seals are available of the same general configuration, using teflon coated wire, but there are some indications that this seal leaks between the coating and the wire.

The design proposed for the power seals uses an oversize transducer gland with bare wire and a flexible teflon sleeve fed through the sealing gland. Sleeving eliminates the problem of voltage breakdown, but the problem of proper sealing still exists. This proposed method will be tested before it is actually used; alternative means of sealing the power leads have been provided for back-up in this area.

1.6 INSULATION MATERIALS

Two insulation materials are to be evaluated in the 10-couple modules for their comparative effects on thermoelectric performance. The thermal k value for one material is much higher than the other, as denoted in Figure H-3. To maintain these k values, the materials must be packed to a specified density. The following calculations determine the amounts of material to be packed into the 10-couple unit.

Volume to be packed with insulation

$$\text{Enclosure space } \pi \text{ DL} = \pi (5.79) (6.50) = 118 \text{ in}^3$$

Subtract volume of the following components

Cold Frame:	$\pi \text{ DL} = \pi (3.25) (1.85) = 18.9 \text{ in}^3$
Upper Heater Block:	$\pi \text{ DL} = \pi (2.25) (1) = 7.0 \text{ in}^3$
Lower Heater Block:	$\pi \text{ DL} = \pi (3.25) (1) = 10.2 \text{ in}^3$
Insulator:	$\pi \text{ DL} = \pi (2.25) (0.10) = 0.7 \text{ in}^3$
Thermal Elements:	$\pi \text{ DL} \times 20$
	$\pi (0.187) \times 20 \times 0.68 = 8.0 \text{ in}^3$
Insulation Retainer Void	$\pi [(5.79) - (5.63)] \times 3.50 = 1.8 \text{ in}^3$
Center Post	$\pi (0.20) \times 0.75 = 0.5 \text{ in}^3$
Total Void Volume	47.1 in ³
Packing Volume	118 - 47.1 = 70.9 in ³

A - MICROQUARTZ FELT 3M SPECIFICATION TPM-193
 B - MIN-K 1301 3M SPECIFICATION TPM 68

Figure H-3. Thermal K for Insulating Material Under Consideration

Min-K Units

These units also use shredded microquartz near the top and bottom plates which occupies a volume of 7.8 in^3 . The void volume is adjusted for this.

$$\text{Packing Volume} \qquad 70.9 - 7.8 = 61.1 \text{ in}^3$$

For Min-K to be packed to 11 lbs/ft^3 minimum density, 0.39 pounds of Min-K insulation is required.

$$11.0 \times \frac{1}{144} \times 12 = 0.00636 \text{ lb/in}^3$$

$$0.00636 \times 61.1 = 0.390 \text{ lbs Min-K}$$

or

$$0.390 \times 454 = 177 \text{ grams Min-K}$$

Microquartz Units

For density of $4/\text{ft}^3$, 0.12 pound of microquartz is required.

$$4 \times \frac{1}{144} \times 12 = 0.00172 \text{ lb/in}^3$$

$$0.00172 \times 70.9 = 0.122 \text{ lb microquartz}$$

or

$$0.122 \times 454 = 55 \text{ grams}$$

APPENDIX I

THERMAL ANALYSIS 10-COUPLE MODULE

1.1 OBJECTIVE

The objective of this thermal analysis was to provide design criteria for the 10-couple module so that proper hot and cold junction temperatures will be maintained.

Technical Approach

The SNAP-27 10-couple configuration was analyzed and compared to actual operating modules. This configuration was altered to accommodate the SNAP-23A design. Since the SNAP-23A design will use two types of thermal insulation, two sets of calculations are provided.

The 10-couple module was analyzed in four steps. The thermopile was considered first since the operating temperatures and heat flow are quickly determined. The hot side of the thermopile was then analyzed to determine heating requirements and heat losses. The cold side of the thermopile was similarly approached. The final step was to analyze the heat sink and iterate any calculations to determine a heat balance among the hot end, thermopile, cold end and heat sink.

1.2 ANALYSIS OF SNAP-27 10-COUPLE CONFIGURATION

SNAP-27 parameters of importance here are the hot junction temperature (1100°F) and cold junction temperature (525°F). Heat input per couple is 2.71 watts and heat rejection per couple is 2.86 watts.

1.2.1 Thermopile Analysis

Assuming a 20°F drop from the upper heater block to T_h and a 30°F drop from T_c to the cold frame, a heat balance is calculated.

Heat conducted through the insulation and the structural member is calculated to be 7.7 watts.

Conduction through Min-K insulation:

$$K = 0.020$$

$$L = 0.0654$$

$$A = 0.0356$$

$$R = \frac{L}{KA} = 92.0$$

$$q = \Delta T/R = (1120-495)/92.0 = 6.8 \text{ BTU} = 2.0 \text{ watts}$$

Conduction through center post:

$$K = 9.4$$

$$L = 0.0654$$

$$A = 0.000218$$

$$R = \frac{L}{KA} = 31.9$$

$$q = \Delta T/R = (1120-495)/31.9 = 19.6 \text{ BTU} = 5.7 \text{ watts}$$

$$\text{Total Loss } 5.7 + 2.0 = 7.7 \text{ watts}$$

Radial losses from the thermopile are estimated at 1.9 watts.

$$K = 0.020$$

$$L = 0.0654$$

$$\ln\left(\frac{r_2}{r_1}\right) = \ln\left(\frac{6.0}{2.75}\right) = \ln(2.18) = 7.8$$

$$R = \frac{\ln(r_2/r_1)}{2 \pi K L} = 95$$

$$\Delta T = 1/2 (1120 + 495) - 170 = 638$$

$$q = \Delta T/R = 638/95 = 6.7 \text{ BTU} = 1.9 \text{ watts}$$

A schematic diagram for thermopile heat balance is

10-couple rejection: $10 \times 2.71 = 27.1$ watts

Conduction through insulation and support: 7.7 watts

Total thermopile rejection: 34.8 watts

10-couple input: $10 \times 2.86 = 28.6$ watts

Conduction through insulation and support: 7.7 watts

Radial loss: 1.9 watts

Total thermopile input: 38.2 watts

1.2.2 Hot End Analysis

Assuming the upper heater block is at a uniform temperature of 1120°F, heat is lost to the cover and case which are assumed to be at a temperature of 170°F.

Axial heat losses to the cover:

Min-K insulation

$$K = 0.020$$

$$A = 0.0412$$

$$L = 0.0835$$

$$R_1 = \frac{L}{KA} = 101.5$$

Micro-quartz pad

$$K = 0.36$$

$$L = 0.18$$

$$A = 0.412$$

$$R_2 = 12.1$$

$$q = \Delta T / (R_1 + R_2) = \frac{1120 - 170}{113.5} = 8.35 \text{ BTU} = 2.5 \text{ watts}$$

Radial losses to the case

$$K = 0.020$$

$$L = 0.0835$$

$$\ln\left(\frac{r_2}{r_1}\right) = \ln \frac{6.0}{2.75} = 0.79$$

$$R = \ln\left(\frac{r_2}{r_1}\right) / 2\pi K L = 75.2$$

$$q = \Delta T / R = (1120 - 170) / 75.2 = 12.6 \text{ BTU} = 3.7 \text{ watts}$$

Estimated corner losses are 1/2 (axial and radial)

$$q = 1/2 (2.5 + 3.7) = 1/2 (6.2) = 3.1 \text{ watts}$$

Leakage from gas sample tube

$$L = 0.125$$

$$A = 0.0000595$$

$$K = 27$$

$$R = \frac{L}{KA} = 77.5$$

$$q = \Delta T / R = (1120 - 170) / 77.5 = 12.3 \text{ BTU} = 3.6 \text{ watts}$$

A schematic diagram for hot block heat balance is

Thermopile requires: 38.2 watts

Total losses are: 12.9 watts

Input to heater block: 51.1 watts

1.2.3 Cold End Analysis

Because the cold frame and lower heater block resistances are small and the heat flow is relatively small, the temperature of the cold frame and heater block is assumed constant at 495°F.

Cold Frame

$$K = 109$$

$$L = 0.125$$

$$A = 0.057$$

$$R = \frac{L}{KA} = 0.020$$

Lower Heater Block

$$K = 109$$

$$L = 0.0835$$

$$A = 0.069$$

$$R = \frac{L}{KA} = 0.011$$

Radial heat losses from the cold end are estimated

$$K = 0.020$$

$$L = 0.21$$

$$\ln \frac{r_2}{r_1} = \ln \frac{6.00}{3.62} = \ln (1.65) = 0.5$$

$$R = \frac{\ln(r_2/r_1)}{2\pi LK} = 19.0$$

$$q = \Delta T/R = (495-170)/19.0 = 17.1 \text{ BTU} = 5.0 \text{ watts}$$

A schematic diagram for heat balance of the cold end

Thermopile rejection: 35.8 watts

Radial losses: 5.0 watts

Cold block rejection: 30.8 watts

1.2.4 Heat Sink Analysis

Heat is dissipated by the cooling pedestal and by convection from the outer case and cover.

Convection coefficients and thermal resistances are first calculated.

Coefficient of convection from external surfaces at 190°F to air at 80°F*

$$N_u = 0.12 (G_r \cdot P_r)^{1/3}$$

where

N_u is Nusselt number

G_r is Grashof number

P_r is Prandtl number

$$G_r = \frac{gBL^3 \theta_s}{\nu^2} = \frac{4.18 \times 10^8 \cdot 1/540 \cdot (6.5/12)^3 \theta_s}{(0.715)^2} = 2.40 \times 10^5 \theta_s$$

$$\nu = \frac{\mu}{\rho} = \frac{0.05}{0.07} = 0.715$$

$$\theta_s = 190 - 80 = 110$$

$$G_r = 264 \times 10^5$$

$$P_r = \frac{\nu}{\alpha} = \frac{0.715}{0.95} = 0.755$$

$$N_u = 0.12 (2.64 \times 0.755 \times 10^7)^{1/3} = 32.5$$

$$N_u = \frac{hL}{K} \text{ or } h = \frac{K}{L} N_u = \frac{0.017}{0.54} (32.5)$$

$$h = 1.025$$

*Hilbert Schenk, Heat Transfer Engineering. New Jersey: Prentice-Hall 1959

The area of external surfaces is 1.688 ft^2

Vertical Can	0.85 ft^2
Cover	0.23 ft^2
Tube Cover	0.084 ft^2
Valve Bracket	0.103 ft^2
Bellows Valve and Tubing	0.06 ft^2
13 Fittings	0.214 ft^2
Exposed Bottom Plate	0.147 ft^2
Total	1.688 ft^2

Defining resistance as $\frac{1}{hA}$

$$R = \frac{1}{hA} = \frac{1}{1.025 (1.688)} = \frac{1}{1.73} = 0.58$$

Coefficient of convection assuming water at 80°F and a gradient of 10°F is calculated.

$$h = 0.37 (\Delta T)^{1/3} T_{\text{mean}} = 0.37 (10)^{1/3} 85 = 67.6$$

Area of cooling surface in pedestal

$$\begin{aligned} \pi DL &= \pi (0.25)(0.33) = 0.262 \text{ ft}^2 \\ \pi D^2/4 &= 0.785 (0.25)^2 = 0.49 \text{ ft}^2 \\ \hline \text{Total} &= 0.311 \text{ ft}^2 \end{aligned}$$

$$\text{Resistance} = \frac{1}{hA} = \frac{1}{67.6 (0.311)} = 0.475$$

Heat rejected from the cold block to the base must pass through an insulating system. The resistance of this system is calculated.

Insulation Pad

$$\begin{aligned} K &= 0.36 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{in} \\ L &= 0.75 \text{ in} \end{aligned}$$

$$A = 0.0676 \text{ ft}^2$$

$$R_A = \frac{L}{KA} = 30.8$$

Spacer Tube

$$K = 9.4 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{ft}$$

$$L = 0.0625 \text{ ft}$$

$$A = 0.00062 \text{ ft}^2$$

$$R_B = \frac{L}{KA} = 10.7$$

Cap Screw

$$K = 9.4 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{ft}$$

$$L = 0.0835 \text{ ft}$$

$$A = 0.000248 \text{ ft}^2$$

$$R_C = \frac{L}{KA} = 35.8/\text{inch}$$

The pads, tubes and screws are parallel resistances

$$\frac{1}{R_{E_Q}} = \frac{1}{R_A} + \frac{4}{R_B} + \frac{4}{0.75 R_C} = 0.555$$

$$R_{E_Q} = \frac{1}{0.555} = 1.80$$

Base resistance - axial

$$K = 9.4 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{ft}$$

$$L = 0.0625 \text{ ft}$$

$$\hat{A} = 0.071 \text{ ft}^2$$

$$R = \frac{L}{KA} = 0.094$$

Base resistance - radial

$$K = 9.4 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{ft}$$

$$L = 0.344/12 = 0.029 \text{ ft}$$

$$\ln \frac{r_2}{r_1} = \ln \frac{6.0}{3.62} = \ln 1.65 = 0.50$$

$$R = \ln \frac{r_2/r_1}{2\pi KL} = 0.292$$

Pedestal tube resistance

$$K = 9.4 \text{ BTU/hr } ^\circ\text{F ft}^2/\text{ft}$$

$$L = 0.33 \text{ ft}$$

$$A = 0.0094 \text{ ft}^2$$

$$R = \frac{L}{KA} = 3.72$$

The following schematic shows the heat sink and heat flow in the module

Resistances

R_2 - Insulation block

R_3 - Radial base

R_4 - Convection

R_5 - Pedestal tube

R_5' - Convection

The losses q_1 have been calculated for a case temperature of 170°F .

Hot Block	12.9 watts
Thermopile	1.9 watts
Cold Block	5.0 watts
Total	19.8 watts or 67.6 BTU

Substituting values for R in the circuit, values for T_B are assumed. When a heat balance in the R_4 leg is obtained, the value for q_2 is found,

T_B	q_2	q_5	q_3	T_C	q_4	$\Delta T_4/R_4$
150	182	37	135	111	202.6	53.5
200	156	63	93	172	160.6	158
225	142	76	66	206	133.6	218

$$\text{Balance when } q_4 = \Delta T_4/R_4 = q_3 + 67.6$$

Sink in equilibrium when $T_B = 200^\circ\text{F}$

$$q_2 = 156 \text{ BTU or } 45.5 \text{ watts}$$

$$T_C = 170^\circ\text{F}$$

$$T_W = 83^\circ\text{F}$$

Since q_2 is 45.5 watts for heat sink equilibrium and heat rejected from the cold block is 30.8 watts, the difference of $45.5 - 30.8 = 14.7$ watts must be supplied as cold block power.

1.2.5 Summary

The analysis basically coincides with operational modules. Hot block power was approximately 55-60 watts, cold block power was 10-20 watts. Temperatures of the case obtained were between 150-180°F and the pedestal temperature was between 70-90°F. More precise calculations can not be made because the convection coefficients cannot be accurately predicted particularly over the irregular surfaces of the 10-couple unit. The conclusion drawn, however, is that this analytical approach will determine sufficiently accurate values to allow redesigning the 10-couple module to the SNAP-23A configuration.

1.3 ANALYSIS OF SNAP-23A 10-COUPLE DESIGN #1

The previous analysis has shown the base temperature to be approximately 200°F, which is very close to that required for the SNAP-23 design. Eliminating the insulation block or spacer between the cold block and the module base should provide the proper cold junction temperature.

SNAP-23A parameters of importance are a T_h of 1100°F and a T_c of 210°F.

Heat input per couple is 3.0 watts.

Heat rejection per couple is 2.75 watts.

1.3.1 Thermopile Analysis

Assuming a 20°F drop from the upper heater block to T_h and a 30°F drop from T_c to the cold frame, a heat balance is calculated.

Heat conducted through the insulation and the structural member is calculated as 10.1 watts.

Conduction through insulation

$$K = 0.020$$

$$L = 0.073$$

$$A = 0.030$$

$$R = \frac{L}{KA} = 122$$

$$q = \Delta T/R = (1120-180)/122 = 7.7 \text{ BTU} = 2.3 \text{ watts}$$

Conduction through center post

$$K = 9.4$$

$$L = 0.073$$

$$A = 0.000218$$

$$R = \frac{L}{KA} = 35.6$$

$$q = \Delta T/R = (1120-180)/35.6 = 26.4 \text{ BTU} = 7.8 \text{ watts}$$

Estimated radial heat loss from the thermopile is 1.5 watts

$$K = 0.020$$

$$L = 0.073$$

$$\ln \frac{r_2}{r_1} = \ln \frac{6.00}{2.5} = \ln 2.4 = 0.875$$

$$R = \ln \frac{r_2}{r_1} \frac{1}{2\pi KL} = 95.5$$

$$\Delta T = 1/2 (1120 + 180) - 170 = 480$$

$$q = \Delta T/R = 480/95.5 = 5.0 \text{ BTU} = 1.5 \text{ watts}$$

A schematic for thermopile heat balance is

$$10\text{-couple rejection } 10 \times 2.75 = 27.5 \text{ watts}$$

$$\text{Conduction through insulation and structure} = 10.1 \text{ watts}$$

$$\text{Rejection from thermopile} = 37.6 \text{ watts}$$

10-couple input $10 \times 3.0 = 30.0$ watts

Conduction through insulation and structure = 10.1 watts

Radial loss = 1.5 watts

Total input to thermopile 42.6 watts.

1.3.2 Hot End Analysis

Assuming the upper heater block is at uniform temperature of 1120°F. Heat losses to the cover and case are assumed to be 170°F.

Axial losses from hot block

Min-K insulation

$$K = 0.020$$

$$A = 0.0276$$

$$L = 0.083$$

$$R_1 = \frac{L}{KA} = 150$$

Micro-quartz pad

$$K = 0.36$$

$$A = 0.0276$$

$$L = 0.18$$

$$R_2 = \frac{L}{KA} = 18.1$$

$$q = \Delta T / (R_1 + R_2) = (1120 - 170) / 168.1 = 5.65 \text{ BTU} = 1.65 \text{ watts}$$

Radial losses from hot block

$$K = 0.020$$

$$L = 0.083$$

$$\ln \frac{r_2}{r_1} = \ln \frac{6.00}{6.25} = \ln 2.67 = 0.981$$

$$R = \ln(r_2/r_1) / 2\pi KL = 93.5$$

$$q = \Delta T/R = (1120-170)/93.5 = 10.2 \text{ BTU} = 3.0 \text{ watts}$$

Estimated corner losses are 1/2 (axial and radial)

$$q = 1/2 (3.0 + 1.65) = 2.3 \text{ watts.}$$

Leakage from gas sample tube

$$K = 9.4$$

$$A = 0.0000595$$

$$L = 0.083$$

$$R = \frac{L}{KA} = 149$$

$$q = \Delta T/R = (1120-170)/149 = 6.4 \text{ BTU} = 1.9 \text{ watts.}$$

A schematic diagram for hot block heat balance:

Thermopile requires 42.6 watts

Total losses are 8.9 watts

Input required is 51.5 watts

1.3.3 Cold End Analysis

Since the cold frame and lower heater block have small thermal resistances, the cold frame and heater block are assumed to be at a uniform temperature of 180°F.

Radial losses from the thermopile are estimated:

$$K = 0.02$$

$$L = 0.234$$

$$\ln \frac{r_2}{r_1} = \ln \frac{6.00}{3.25} = \ln 1.85 = 0.62$$

$$R = \ln\left(\frac{r_2}{r_1}\right) / 2\pi KL = 21.1$$

$$q = (180-170)/21.1 = 0.47 \text{ BTU} = 0.2 \text{ watts.}$$

A schematic diagram for cold block heat balance is as follows:

Thermopile rejects 37.6 watts

Radial losses are 0.2 watts

Cold Block must reject 37.4 watts

1.3.4 Sink Analysis

Heat is dissipated by the cooling pedestal and by convection from the outer case and cover.

Convection coefficients and thermal resistances are the same as calculated for the SNAP-27 design.

The following is a schematic of the heat sink and heat flow in the module.

Resistances

- R_3 - Radial Base
- R_4 - Convection
- R_5 - Pedestal Tube
- R_5' - Convection
- R_2 - Axial Base

The losses q_1 have been calculated for a case temperature of 170°F

Hot Block	8.9 watts
Thermopile	1.5 watts
Cold Block	0.2 watts
Total	10.6 watts or 36.2 BTU.

Substituting values for R in the circuit, values for T_B are assumed. When a heat balance in the R_4 leg is obtained, the value for q_2 is found.

T_B	q_2	q_5	q_3	T_c	q_4	$\Delta T_4/R_4$
170	106	57	49	156	85.2	131
165	160	45	115	131	151.2	88
168	128	46	82	144	118.2	110
168.4	123	46	77	145.9	113.2	114

$$\text{Balance } q_4 = \Delta T/R_4 = q_3 + 36.2$$

Sink in equilibrium when $T_B = 168^\circ\text{F}$

$$q_2 = 123 \text{ BTU} = 36.0 \text{ watts}$$

$$T_c = 146^\circ\text{F}$$

$$T_w = 83^\circ\text{F}$$

Since q_2 is 36.0 watts for equilibrium of the heat sink, and heat rejected from the cold block is 37.4 watts, additional sink capacity of 1.4 watts is required. Before designing for extra capacity the second design is considered.

1.4 ANALYSIS OF SNAP-23A 10-COUPLE DESIGN #2

The basis change in this design is in thermal insulation. This merely requires adjusting figures for this change.

1.4.1 Thermopile Analysis

Center post losses	7.8 watts
Insulation losses $2.3 \times 0.06/0.02 =$	6.9 watts
Radial losses $1.5 \times 0.06/0.02 =$	4.5 watts

A schematic diagram for thermopile heat balance is

1.4.2 Hot Block Analysis

Axial losses $1.65 \times 0.06/0.02 =$	5.1 watts
Radial losses $3.0 \times 0.06/0.02 =$	9.0 watts
Corner losses $1/2 (14.1) =$	7.0 watts
Leakage gas sample tube	1.9 watts
Total losses	13.0 watts

A schematic diagram for hot block heat balance is

1.4.3 Cold End Analysis

Radial losses $0.2 \times 0.06/0.02 = 0.6$ watts

A schematic diagram for cold end heat balance is

1.4.4 Heat Sink Analysis

The value for q_1 changes

Hot Block 23.0 watts

Thermopile 4.5 watts

Cold Block 0.6 watts

Total Losses 28.1 watts or 96.0 BTU

The iterative heat balance calculation is made with this value for q_1 .

Since q_2 is equal to 26.4 watts for heat sink equilibrium, and heat rejected from the cold block is 41.6 watts, an additional 15.2 watts sink capacity is required.

T_B	q_2	q_5	q_3	T_c	q_4	$\Delta T_4/R_4$
170	106	47	59	153	155	126
171	96	48	48	157	144	133
172	85	48	37	161	133	140
171.5	90	48	42	159.5	138	137

Sink in equilibrium when $T_B = 172^\circ\text{F}$

$q_2 = 90 \text{ BTU} = 26.4 \text{ watts}$

$T_c = 160^\circ\text{F}$

$T_w = 82^\circ\text{F}$

1.5 ADDITIONAL HEAT SINK CAPACITY

The analyses for Designs #1 and #2 of the SNAP-23A 10-couple module indicate that increased heat sink capacity is required. For Design #1 this requirement is 1.4 watts and for Design #2 this requirement is 15.2 watts. It is proposed to provide this additional cooling by adding spines to the module base, extending them into pedestal cooling water.

Heat loss from a rod protruding from a heat source into a cooling medium is given by the following.*

$$q = \sqrt{KhAC} \frac{1 - pe^{-2ml}}{1 + pe^{-2ml}} \theta_o$$

where

$$p = \frac{km-h}{km+h}$$

$$m = \sqrt{\frac{hC}{KA}}$$

*Max Jacob, Heat Transfer, p. 215

For a 1/8" dia. x 2" stainless rod

$$K = 10 \text{ BTU/hr}^\circ\text{F ft}^2/\text{ft}$$

$$A = \pi D^2/4 = 8.5 \times 10^{-5} \text{ ft}^2$$

$$C = \pi D = 0.0327 \text{ ft}$$

The water convection coefficient has previously been determined for a 10°F gradient (water at 80°F).

$$h = 65 \text{ BTU/hr } ^\circ\text{F ft}^2$$

$$\sqrt{KhAC} = \sqrt{0.0018} = 0.042$$

$$m = \sqrt{\frac{hC}{KA}} = \sqrt{\frac{21.3}{8.5}} \times 10^3 = 50$$

$$p = \frac{km-h}{km+h} = \frac{500-65}{500+65} = \frac{435}{565} = 0.77$$

$$e^{-m2L} = e^{-16.7} = 5 \times 10^{-8}$$

$$q = 0.042 \left[\frac{1-0.77(5)10^{-8}}{1+0.77(5)10^{-8}} \right] \theta_o$$

$$q = 0.042 \theta_o$$

$$\theta_o = T_h - T_s = 180-80 = 100$$

$$q = 0.042 (100) = 4.2 \text{ BTU or 1.2 watts.}$$

Twelve of these rods are screwed into the base to provide an additional 14.4 watts of sink capacity.

Summary

Thermal analyses have shown that increased sink capacity is required. To provide this capacity, these spines were added to the module base.