WHIP EFFECT IN A FALLING CHAIN

W. A. Heywood, H. Hurwitz, Jr., D. Z. Ryan

nd 45590

Knolls Atomic Power Laboratory,* Schenectady, New York

Abstract

A simple experiment is described for demonstrating the whip effect in a falling chain. The observed reduction in fall time due to the concentration of energy in the falling section agrees well with the assumption of small energy loss in the loop of the chain.

Text

The concentration of energy in the free end of a falling chain has been of interest because of its analogy to a possible mechanism of rupture of thin pressure vessels under certain transient pressure conditions. In this connection a simple experiment was undertaken to demonstrate the extent to which the energy concentration actually occurs. The experiment consisted of dropping one end of a chain of length $L = 39.75$ inches from a height L above the point where the other end was fastened and measuring the chain tension at the support point as a function of time by means of a strain gauge and Brush recorder (cf. Figure 1).

From the condition of momentum conservation it follows that the sum of the tensions T_1 and T_2 just above the loop on the supported and free sides of the loop must be equal to $(1/2)\rho V^2$ where V is the velocity of the falling part of the chain and ρ is the chain mass per unit length. If the tension T_2 on the free side of the loop were zero, no energy concentration would occur. The assumption that the tensions on the two sides of the loop are equal, on the other hand, leads to an added acceleration of the moving section of the chain which is just sufficient to concentrate all the available energy into this

*Operated by the General Electric Company for the United States Atomic Energy Commission under Contract W-31-109 Eng-52.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

section. That this latter assumption is reasonable can be seen by considering the situation in a frame of reference moving with the loop and ignoring the variation of chain velocity with time. A steady state condition then exists in which the chain tension has the constant value ρv^2 where v is the chain velocity as measured in the moving system. Regardless of the shape of the loop, the centrifugal force is just balanced by the net force per unit length of chain which arises from the tension. Since $v = 1/2 V$, the chain tension at the loop is $\frac{\rho v^2}{4}$ in accordance with the assumption of complete energy concentration. Although the existence of a nondissipative steady state solution makes plausible a high degree of energy concentration in the transient case, the theoretical evaluation of the energy loss which actually occurs due to abrupt changes in velocity of the chain loops and the excitation of oscillatory modes of chain vibration would require a detailed analysis of the transient case. In this analysis the actual geometry of the chain loop would have to be considered.

If the energy dissipation in the loop were complete so that no energy concentration occurred, the tension T at the support point would be given as a function of time by the relation

$$T = 3 \rho g L t^2 / t_0^2; \quad t_0 = 2 \sqrt{L/g} \quad (1)$$

On the other hand, if there is no dissipation so that all the energy is concentrated in the falling end of the chain

$$T = \rho g L \frac{x}{L} \left[1 + \frac{1 - x/2L}{1 - x/L} \right] \quad (2)$$

where x/L is given as a function of time by the implicit relationship

$$t = \sqrt{\frac{L}{g}} \left\{ 4E(\alpha, \phi) - 2F(\alpha, \phi) \right\} \quad (3)$$

$$\alpha = 45^\circ; \quad \phi = \sin^{-1} \sqrt{\frac{x}{L}}$$

Here E and F are elliptical integrals of the first and second kind.* When x/L is unity, equation (3) gives $t = t_c = .8471t_0$ so that the fall time of the chain is less than that of a freely falling body by 15.3 per cent.

Experimentally, the difference between the time of fall of the end of the chain (t_c) and the time of fall of a free body through the distance $2L$ (t_0) was found to be $.101 \pm .005$ seconds, in satisfactory agreement with the theoretical value .098 seconds. (This time difference was measured by dropping a weight attached to the strain gauge by a thread of length L simultaneously with the end of the chain.) In Figure 2 there is plotted the tension at the support as given by equation (1) and by equations (2) and (3) (solid lines). The experimental curve (dotted line) agrees with the curve for complete energy concentration within the accuracy of the experiment. Although accurate tension measurements were not made beyond the time when about three quarters of the chain had come to rest, measurements on a less sensitive scale indicated that the support tension rose to at least $30\rho gL$, i.e. ten times the maximum tension $3\rho gL$ which would exist with no energy concentration. It therefore appears that at least during the early stages of the fall, the energy dissipation in the loop was small so that most of the available energy was concentrated in the falling section of the chain.

*Jahnke and Emde, Tables of Functions, Dover publications, 1943; p 62 and ff.

4

FIGURE 1

FIGURE 2

657-5