PROPULSION DIVISION

ENGINEERING METHODS

KINETIC, DIVERGENCE AND BOUNDARY LAYER
PERFORMANCE LOSS CHARTS

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KINETIC, DIVERGENCE AND BOUNDARY LAYER

PERFORMANCE LOSS CHARTS

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T H R U S T

C H A M B E R

E N G I N E E R I N G

Liquid Rocket Operations

Sacramento, California

FOREWORD

The divergence and boundary layer performance loss charts presented herein were developed under Dynamic Science P. O. 4447. The kinetic loss charts and publication of this report were accomplished under an Aerojet 1968 Methods Development Program (W. O. P406-14-964).

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This report provides the rocket engine performance analyst with design charts for predicting performance losses resulting from kinetic, divergence, and boundary layer effects. The kinetic performance loss accounts for the effect of finite reaction rates which do not permit equilibrium compositions to be maintained during the combustion gas expansion process. The divergence performance loss accounts for the two-dimensional inviscid flow effects which occur within the nozzle. These effects include: (1) nonplanar sonic surface which influences both the mass flowrate and thrust and (2) nonaxially directed gas flow at the nozzle exit which influences the resultant axial momentum (thrust). The boundary layer performance loss results from viscous and heat transfer effects which influence the local velocity and density of the expansion gas mixture, and are generally confined to a thin layer along the thrust chamber wall. As a result of these effects, the total momentum and mass flux of the system are altered from their respective inviscid adiabatic flow values, thus affecting both the performance and effective nozzle cross sectional area.

The performance loss charts presented in this report are intended for use with preliminary design and proposal efforts and cost or time limited analyses. These charts are also suitable for parametric analyses for design optimization. Under other conditions, where best available accuracy is desired, specific performance loss calculations should be made. The following computer programs are available for determining these performance losses:

E 23401	ODK - Kinetic Performance Loss
10036	Divergence Performance Loss
E25204	TBL - Boundary Layer Performance Loss
E23402	TDK - Kinetic and Divergence Losses (Available sometime during Fiscal 1969)

II KINETIC PERFORMANCE LOSS CHARTS

This section presents charts for use in estimating the kinetic performance loss occurring with liquid propellant rocket thrust chambers. The performance losses were computed using the one-dimensional kinetic computer program (ODK) No. E23401. This program computes the one-dimensional performance of a reacting gas expanding adiabatically and isentropically. The reaction rate constants used for each specific reaction are the same as those recommended by the ICRPG Performance Standardization Working Group*.

Several design and operating parameters affect the kinetic performance loss. The most significant of these parameters are as follows:

1.	Propellant combination	}	Combustion Product Composition and Temperature
2.	Mixture Ratio (O/F)		
3.	Chamber Pressure (p_o)	}	Combustion Product Density
4.	Expansion Area Ratio		
5.	Thrust Level (F)	}	Combustion Product Stay Time
6.	Throat Radius (R_t)		
7.	Nozzle Shape		

The kinetic losses for three propellant combinations are included in detail in this section. These are, (1) $N_2O_4/A-50$ at O/F's from 1.6 to 2.8, (2) F_2/H_2 at O/F's from 4 to 16, and (3) F_2/N_2H_4 at O/F's from 1.3 to 2.8. Chamber pressures from 10 to 3000 psia, expansion area ratios from 1 to 100, thrust levels from 25 to 100,000 lb_f , and throat radii from .66 to 5.95 in. are included. Tables of calculated specific impulse values for the three propellants are included at the end of this section.

The kinetic performance losses for $N_2O_4/A-50$ are presented in Figures 1 - 14, kinetic losses for F_2/N_2H_4 are included in Figures 15 - 27, and

* These rate constants are listed in Table C-2 of the "ICRPG Liquid Propellant Thrust Chamber Performance Evaluation Manual", CPIA Publication No. 178, Aerojet Report TCER 9642:0085

the F_2/H_2 losses are presented in Figures 28 - 40. A detailed listing of the kinetic loss curves is given on pages 7 and 8. Briefly, Figures 1 - 10, 15 - 24, and 28 - 37 present the kinetic loss as a function of mixture ratio (O/F) and expansion ratio (ϵ) for $N_2O_4/A-50$, F_2/N_2H_4 , and F_2/H_2 , respectively, at various chamber pressures (p_o) and throat radii (R_t). These sets of curves are the basic charts from which estimates of kinetic performance losses can be made. Evaluation at specific operating conditions (p_o , ϵ , R_t) can be made through appropriate cross plots of these curves. A description of the interpolation and/or extrapolation of these data with respect to mixture ratio, expansion ratio, chamber pressure, thrust level, throat radius, expansion angle, and propellant combination follows:

1. Mixture Ratio

The typical effect of mixture ratio on the kinetic performance loss is shown in the loss charts described above. Generally, the kinetic loss is greatest at or near the stoichiometric mixture ratio where the maximum combustion temperature is achieved. Higher combustion temperatures result in greater dissociation and thus can lead to higher kinetic losses.

2. Expansion Ratio

Increasing expansion ratio results in higher kinetic losses through further departure from equilibrium. This effect is shown in Figure 41 for several liquid propellant combinations. This effect can also be determined by interpolation of the basic loss charts presented in this section.

3. Chamber Pressure

The kinetic performance loss will increase with decreasing chamber pressure since a reduction in density of the combustion products increases the time interval between molecular collisions and thereby reduces the rate of

chemical energy conversion. This typical effect is shown in Figures 11, 25, and 38 for $N_2O_4/A-50$, F_2/N_2H_4 , and F_2/H_2 , respectively. Note that chamber pressure has a slightly greater effect on the kinetic loss for a constant throat radius (variable thrust) than for a constant thrust condition (variable throat radius).

4. Thrust Level and Throat Radius

Kinetic losses increase with decreasing thrust and/or throat size because of decreasing gas residence time. This typical effect is shown in Figures 12 & 13, 26 & 27, and 39 & 40 for the three basic propellants included in this study. Evaluation of specific design conditions can be made through interpolation or extrapolation of the basic loss charts described earlier.

5. Expansion Angle

The performance loss included in all the kinetic loss charts in this report were computed for a conical nozzle having a 20° half-angle (α) expansion. Changing the nozzle contour by changing the expansion angle or shape (e.g. Bell or Rao nozzles) could affect the kinetic loss by changing the gas residence time. Long nozzles (lower expansion angle to the same area ratio) will have lower kinetic losses than short nozzles. A typical example of this effect is shown in Figure 14 ($N_2O_4/A-50$, $O/F = 2.0$, $p_o = 100$ psia, $R_t = 1.33$ in.). These results show that the ratio of the kinetic loss for a particular expansion angle to the kinetic loss for a 20° expansion angle varies from .88 to 1.04 as the expansion angle increases from 10° to 30° . This effect, in this case, is independent of area ratio. This data can be used to correct the kinetic performance loss estimates when an expansion angle of other than 20° is required. An equivalent conical nozzle half-angle* can be used when estimating the kinetic loss

* An equivalent conical nozzle half-angle is that angle formed between the nozzle axis and a straight line drawn from the end point of the throat circular arc to the end point of the nozzle.

of a Rao or Bell contour nozzle.

6. Propellant Combination

A comparison of the kinetic loss of several typical propellant combinations is shown in Figure 41. This data shows that the propellant combination can have a significant influence on the kinetic loss particularly at high area ratios. The variation of the kinetic loss with propellant combinations is difficult to generalize because of the large variability in the composition of the resultant combustion products. In addition, the operating mixture ratio will also influence the composition and combustion temperature thus further complicating the solution.

The generalization of kinetic losses with propellant combination is beyond the scope of this effort. However, inspection of the data presented in Figure 41 can supply some insight into this problem. For example, in general, increasing combustion temperature tends to result in higher kinetic losses (in terms of $\text{lb}_f\text{-sec}/\text{lb}_m$). Therefore, high energy propellants (e.g. F_2/H_2) would be expected to have greater kinetic losses than low energy propellants (e.g. IRFNA/UDMH). Also, certain groups of propellants appear to have similar kinetic loss trends with area ratio. Inspection of Figure 41 shows that curves 1, 2, 3, 4, 5, 8, 9 and 12, 10, 11, and 13, and 6 and 7 form three distinct groups of similar shaped curves. The first group is comprised of the earth storable and fluorine/amine propellants. The second group contains the space storable propellants FLOX/ B_2H_6 , FLOX/ CH_4 , and F_2/H_2 . Finally, the third group contains the cryogenic propellants LOX/RP-1 and LOX/ H_2 .

Therefore, extrapolation of the basic kinetic loss data included in this report to other propellants can be made by using the basic loss data of the propellant which is included in the same class as the propellant under evaluation. For example, for earth storable propellants use the $\text{N}_2\text{O}_4/\text{A-50}$ data, for fluorine/amine propellants use the $\text{F}_2/\text{N}_2\text{H}_4$ data, and for space storable propellants use the F_2/H_2 data*. Differences within a class of propellants can then be approximated by use of the comparative data presented in Figure 41, or any other data suitable for such a correlation.

Extension of the kinetic loss data to other propellants will, of course, result in a larger uncertainty in the predicted loss. However, the extrapolation of a known kinetic loss to other operating conditions (e.g. extrapolation with area ratio, chamber pressure, thrust level, etc.) can be accomplished with a reasonable degree of certainty through use of the included loss charts.

* Due to funding limitations detailed charts of cryogenic propellants were not developed.

KINETIC PERFORMANCE LOSS

FIGURE LIST

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-----F ₂ /N ₂ H ₄ -----		
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$$-F_2H_2$$

KINETIC PERFORMANCE LOSS VERSUS MIXTURE RATIO

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 ($p_0 = 10 \text{ psia}$, $F \sim 1000 \text{ lb}_f$)

Figure 17. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 ($p_o = 100 \text{ psia}$, $F \sim 25 \text{ lb}_f$)

Figure 18. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 ($p_0 = 100 \text{ psia}$, $F \sim 1000 \text{ lb}_f$)

Figure 19. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 ($p_0 = 100 \text{ psia}$, $F \sim 10,000 \text{ lb}_f$)

Figure 20. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 ($p_o = 500 \text{ psia}$, $F \sim 1000 \text{ lb}_f$)

Figure 21. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 $(p_o = 500 \text{ psia}, F \sim 10,000 \text{ lb}_f)$

Figure 22. F_2/N_2H_4 Kinetic Performance Loss Versus Mixture Ratio
 ($p_0 = 500 \text{ psia}$, $F \sim 100,000 \text{ lb}_f$)

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 ($p_o = 3000 \text{ psia}$, $F \sim 10,000 \text{ lb}_f$)

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Figure 26. $\text{F}_2/\text{N}_2\text{H}_4$ Kinetic Loss Versus Thrust Level

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 $(p_o = 10 \text{ psia}, F \sim 25 \text{ lb}_f)$

Figure 29. F_2/H_2 Kinetic Loss Versus Mixture Ratio
 $(p_o = 10 \text{ psia}, F \sim 1000 \text{ lb}_f)$

Figure 30. F_2/H_2 Kinetic Loss Versus Mixture Ratio
 ($p_o = 100 \text{ psia}$, $F \sim 25 \text{ lb}_f$)

Figure 31. F_2/H_2 Kinetic Loss Versus Mixture Ratio
 ($p_o = 100 \text{ psia}$, $F \sim 1000 \text{ lb}_f$)

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Figure 34. F_2/H_2 Kinetic Loss Versus Mixture Ratio
 $(p_o = 500 \text{ psia}, F \sim 10,000 \text{ lb}_f)$

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 ($p_o = 500 \text{ psia}$, $F \sim 100,000 \text{ lb}_f$)

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 ($P_o = 3000 \text{ psia}$, $F \sim 10,000 \text{ lb}_f$)

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 ($p_o = 3000 \text{ psia}$, $F \sim 100,000 \text{ lb}_f$)

Figure 38. F_2/H_2 Kinetic Loss Versus Chamber Pressure

Figure 39. F_2/H_2 Kinetic Loss Versus Thrust Level

Figure 40. F_2/H_2 Kinetic Loss Versus Throat Radius

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TABLE 1

CALCULATED SPECIFIC IMPULSE FOR

$$\text{N}_2\text{O} / \text{A-50} \quad \text{AT} \quad p_o = 10 \text{ psia}$$

$$\text{4}$$
ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		214.6	290.1	308.4	322.4	333.4	338.8	344.6
2.0		208.9	285.1	305.6	322.3	335.7	342.2	349.4
2.4		202.8	277.0	297.0	313.5	327.2	333.9	341.4
2.8		197.4	269.0	287.9	303.1	315.2	321.2	327.6

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(ODK)}$)
$$F \approx 25 \text{ lb}_f \quad (R_t = .665)$$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		210.8	275.3	289.8	300.8	309.1	312.9	317.0
2.0		205.0	268.4	282.8	293.8	302.1	306.1	310.3
2.4		199.1	260.8	275.0	285.7	293.9	297.8	302.0
2.8		193.7	253.8	267.5	278.0	286.0	289.8	293.9

$$F \approx 1000 \text{ lb}_f \quad (R_t = 4.20)$$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		211.8	277.2	292.0	303.2	311.7	315.7	320.0
2.0		206.0	270.3	285.1	296.4	305.0	309.1	313.5
2.8		194.5	255.3	269.3	280.0	288.3	292.2	296.4

TABLE 2. CALCULATED SPECIFIC IMPULSE FOR

$$\text{N}_2\text{O}_4/\text{A-50} \quad \text{AT} \quad p_o = 100 \text{ psia}$$
ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp} \text{ (ODE)}$)

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		219.6	293.8	311.2	324.6	335.2	340.3	345.9
2.0		214.6	291.4	311.3	327.0	339.4	345.5	352.2
2.4		208.2	283.0	302.6	318.5	331.3	337.6	344.5
2.8		202.2	274.0	292.3	306.7	318.1	323.6	329.7

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp} \text{ (ODK)}$)
$$F \approx 25 \text{ lb}_f \quad (R_t = 0.21 \text{ in.})$$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		217.7	285.5	301.0	312.8	321.8	326.1	330.7
2.0		212.5	279.6	295.2	307.2	316.4	320.9	325.6
2.4		206.2	271.5	286.6	298.3	307.4	311.7	316.4
2.8		200.2	263.3	278.0	289.3	298.0	302.2	306.7

$$F \approx 1000 \text{ lb}_f \quad (R_t = 1.33 \text{ in.})$$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		218.7	288.2	304.2	316.4	325.8	330.3	335.0
2.0		213.6	283.0	299.3	311.8	321.7	326.4	331.5
2.4		207.3	274.6	290.4	302.7	312.2	316.9	321.8
2.8		201.2	266.0	281.2	292.9	302.1	306.5	311.3

$$F \approx 10,000 \text{ lb}_f \quad (R_t = 4.20)$$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		219.1	290.0	306.3	318.8	328.5	333.1	338.1
2.0		214.1	285.5	302.4	315.4	325.7	330.7	336.1
2.4		207.8	277.0	293.4	306.1	316.1	320.9	326.2
2.8		201.7	268.1	283.8	295.9	305.4	310.0	314.9

TABLE 3.

CALCULATED SPECIFIC IMPULSE FOR

 $\text{N}_2\text{O}_4/\text{A-50}$

AT

 $p_o = 500 \text{ psia}$ ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(\text{ODE})}$)

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		222.4	295.5	312.5	325.7	336.0	341.0	346.5
2.0		218.3	295.1	314.4	329.4	341.4	347.3	353.7
2.4		211.7	286.6	305.9	321.2	333.5	339.5	346.1
2.8		205.2	276.8	294.5	308.5	319.5	324.9	330.7

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(\text{ODK})}$) $F \approx 1000 \text{ lb}_f$ $(R_t = 0.595 \text{ in.})$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		222.1	293.3	309.8	322.4	332.2	336.8	341.8
2.0		217.9	290.9	308.2	321.7	332.3	337.4	343.0
2.4		211.4	282.3	299.1	312.3	322.7	327.7	333.2
2.8		204.8	272.6	288.7	301.1	310.9	315.6	320.7

 $F \approx 10K \text{ lb}_f$ $(R_t = 1.88 \text{ in.})$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		222.2	294.1	310.7	323.4	333.3	338.0	343.1
2.0		218.1	292.4	310.2	324.1	335.0	340.4	346.1
2.8		205.0	274.0	290.4	303.2	313.2	318.1	323.4

 $F \approx 100K \text{ lb}_f$ $(R_t = 5.95 \text{ in.})$

O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		222.3	294.5	311.2	324.1	334.0	338.8	343.9
2.0		218.2	293.4	311.6	325.8	337.1	342.6	348.5
2.8		205.1	275.0	291.8	304.8	315.0	320.0	325.5

TABLE 4.

CALCULATED SPECIFIC IMPULSE FOR
 $\text{N}_2\text{O}_4/\text{A-50}$ AT $p_o = 3000$ psia

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE (I_{sp} (ODE))								
O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		224.6	296.7	313.4	326.4	336.5	341.5	346.9
2.0		221.9	298.2	316.9	331.4	343.0	348.7	354.9
2.4		215.2	289.9	308.7	323.5	335.3	341.2	347.5
2.8		208.0	279.1	296.4	310.0	320.7	325.9	331.6

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE (I_{spODK})								
F $\approx$ 10K $R_t = .77$ in.								
O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		224.4	296.0	312.6	325.2	335.1	339.8	344.8
2.0		221.7	296.9	314.9	329.0	340.1	345.6	351.5
2.4		215.0	288.7	306.6	320.6	331.8	337.2	343.2
2.8		207.9	277.7	294.4	307.4	317.7	322.7	328.1

F $\approx$ 100K $R_t = 2.43$								
O/F	$\epsilon =$	1	5	10	20	40	60	100
1.6		224.5	296.1	312.7	325.4	335.3	340.0	345.1
2.0		221.8	297.3	315.5	329.7	340.9	346.4	352.3
2.8		207.9	278.1	295.0	308.0	318.4	323.5	328.9

TABLE 5.

CALCULATED SPECIFIC IMPULSE FOR

$$\text{F}_2/\text{N}_2\text{H}_4 \quad \text{AT} \quad p_o = 10 \text{ psia}$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp}(\text{ODE})$)

O/F	1	5	10	20	40	60	100
1.3	258.8	347.1	369.0	385.5	397.8	403.4	409.3
1.8	262.3	353.7	376.9	395.5	410.2	417.2	424.4
2.3	259.9	353.2	378.1	398.6	415.4	423.5	432.1
2.8	255.4	346.3	369.3	386.1	398.3	403.8	409.6

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp}(\text{ODK})$)

$$\text{F} \sim 25 \text{ lb}_f \quad (R_t = .665)$$

O/F	1	5	10	20	40	60	100
1.3	253.5	322.6	336.3	345.9	352.8	355.8	358.9
1.8	256.2	326.0	339.8	349.5	356.3	359.4	362.5
2.3	252.7	320.5	333.7	342.9	349.4	352.3	355.2
2.8	247.8	313.1	325.7	334.4	340.5	343.2	345.9

$$\text{F} \sim 1000 \text{ lb}_f \quad (R_t = 4.20)$$

O/F	1	5	10	20	40	60	100
1.3	255.5	326.8	341.1	351.1	358.3	361.5	364.8
1.8	258.6	331.9	346.7	357.1	364.5	367.8	371.2
2.8	250.2	317.7	330.8	339.9	346.3	349.2	352.1

TABLE 6.

CALCULATED SPECIFIC IMPULSE FOR

$$\frac{F}{2} \frac{N}{2} \frac{H}{4} \quad \text{AT} \quad p_o = 100 \text{ psia}$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp} \text{ (ODE)}$)

O/F	1	5	10	20	40	60	100
1.3	267.1	354.9	375.3	390.2	401.3	406.4	411.6
1.8	271.3	363.1	385.5	402.8	415.9	422.0	428.3
2.3	269.8	364.3	388.7	408.1	423.1	430.2	437.6
2.8	265.0	355.9	376.5	391.3	402.1	407.1	412.3

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp} \text{ (ODK)}$)

$$F \sim 25 \text{ lb}_f \quad (R_t = 0.21 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	264.9	340.8	356.3	367.4	375.5	379.1	382.9
1.8	268.9	347.0	363.2	374.8	383.1	386.9	390.8
2.3	266.6	342.8	358.3	369.4	377.3	380.9	384.5
2.8	261.5	334.3	348.8	359.1	366.4	369.7	373.1

$$F \sim 1000 \text{ lb}_f \quad (R_t = 1.33 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	266.3	346.6	363.4	375.5	384.3	388.3	392.4
1.8	270.6	354.5	372.5	385.4	394.9	399.2	403.6
2.3	268.7	351.5	369.0	381.6	390.8	394.9	399.2
2.8	263.7	342.3	358.4	369.9	378.2	381.9	385.8

$$F \sim 10,000 \text{ lb}_f \quad (R_t = 4.20)$$

O/F	1	5	10	20	40	60	100
1.3	266.8	349.8	367.5	380.3	389.7	393.9	398.4
1.8	271.0	358.2	377.2	391.0	401.1	405.8	410.5
2.3	269.3	356.7	375.7	389.5	399.7	404.3	409.1
2.8	264.5	347.4	364.8	377.3	386.4	390.5	394.7

TABLE 7.

CALCULATED SPECIFIC IMPULSE FOR

$$\text{F}_2/\text{N}_2\text{H}_4 \quad \text{AT} \quad p_o = 500$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	1	5	10	20	40	60	100
1.3	272.3	359.1	378.4	392.6	403.0	407.8	412.8
1.8	277.4	368.9	390.4	406.6	418.8	424.4	430.2
2.3	276.6	371.5	395.2	413.4	427.2	433.6	440.4
2.8	271.7	361.0	380.1	393.9	404.1	408.8	414.0

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(ODK)}$)

$$F \sim 1000 \text{ lb}_f \quad (R_t = 0.595 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	272.1	356.2	374.2	387.3	397.0	401.4	406.0
1.8	277.2	365.8	385.2	399.5	410.0	414.9	419.9
2.3	276.4	366.5	386.6	401.3	412.3	417.3	422.5
2.8	271.3	356.6	374.7	387.8	397.4	401.8	406.2

$$F \sim 10K \text{ lb}_f \quad (R_t = 1.88 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	272.2	357.5	376.0	389.4	399.4	403.9	408.7
1.8	277.3	367.3	387.4	402.2	413.2	418.3	423.5
2.8	271.5	358.9	377.7	391.3	401.3	405.8	410.5

$$F \sim 100K \text{ lb}_g \quad (R_t = 5.95 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	272.2	358.2	377.0	390.7	400.9	405.6	410.4
1.8	277.4	368.0	388.7	403.9	415.3	420.6	426.1
2.8	271.6	360.1	379.2	393.0	403.1	407.8	412.6

TABLE 8

CALCULATED SPECIFIC IMPULSE FOR

$$\frac{F}{2} \frac{N}{2} \frac{H}{4} \quad \text{AT} \quad p_o = 3000 \text{ psia}$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	1	5	10	20	40	60	100
1.3	277.1	362.3	380.8	394.3	404.3	408.9	413.7
1.8	283.6	374.1	394.5	409.7	421.1	426.3	431.8
2.3	283.9	378.5	401.0	417.9	430.6	436.6	442.8
2.8	278.4	364.9	382.9	396.0	405.7	410.4	415.7

ONE-DIMENSIONAL KINETIC SPECIFIC ($I_{sp(ODK)}$)

$$F \sim 10K \quad (R_t = .77 \text{ in.})$$

O/F	1	5	10	20	40	60	100
1.3	277.1	361.9	380.2	393.6	403.5	408.1	412.8
1.8	283.5	373.5	393.6	408.5	419.6	424.8	430.2
2.3	283.8	377.7	400.0	415.9	428.2	433.9	439.9
2.8	278.2	364.5	382.5	395.6	405.2	409.6	414.1

$$F \sim 100K \quad (R_t = 2.43)$$

O/F	1	5	10	20	40	60	100
1.3	277.1	361.9	380.3	393.8	403.8	408.3	413.1
1.8	283.6	373.5	393.8	408.9	420.1	425.3	430.7
2.8	278.3	364.4	382.5	395.5	405.1	409.5	414.1

TABLE 9.

CALCULATED SPECIFIC IMPULSE FOR

$$\text{H}_2/\text{F}_2 \quad \text{AT} \quad p_o = 10 \text{ psia}$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	1	5	10	20	40	60	100
4.0	328.5	429.1	449.2	463.5	473.8	478.5	483.3
8.0	306.4	412.6	439.5	460.4	476.1	483.3	490.7
12.0	296.7	400.2	427.1	449.2	467.5	476.6	486.4
16.0	287.5	390.8	418.3	441.0	460.0	469.7	480.5

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(ODK)}$)

$$F \sim 25 \text{ lb}_f \quad (R_t = .665)$$

O/F	1	5	10	20	40	60	100
4.0	322.0	409.3	426.7	438.4	447.1	451.0	455.0
8.0	299.6	380.1	395.6	406.3	413.9	417.3	420.7
12.0	290.2	368.3	383.2	393.6	400.8	403.9	407.1
16.0	280.0	354.1	368.2	377.8	384.6	387.6	390.5

$$F \sim 1000 \text{ lb}_f \quad (R_t = 4.20)$$

O/F	1	5	10	20	40	60	100
4.0	323.4	411.6	429.0	441.2	450.0	453.9	458.0
8.0	302.3	385.8	402.1	413.5	421.6	425.2	428.8
16.0	283.2	361.8	377.0	387.5	394.9	398.1	401.4

TABLE 10.

CALCULATED SPECIFIC IMPULSE FOR

 H_2/F_2 AT $p_o = 100$ psiaONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	1	5	10	20	40	60	100
4.0	333.9	431.4	450.9	464.7	474.6	479.2	483.8
8.0	317.2	423.4	448.8	467.5	481.3	487.6	494.2
12.0	307.8	412.6	439.1	460.3	477.0	484.9	493.2
16.0	299.0	403.9	431.2	453.2	471.2	480.2	489.7

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(ODK)}$) $F \sim 25 \text{ lb}_f$ ($R_t = 0.21 \text{ in.}$)

O/F	1	5	10	20	40	60	100
4.0	331.0	422.4	440.5	453.4	462.7	466.9	471.2
8.0	314.7	404.5	422.7	435.5	444.6	448.8	453.0
12.0	305.5	394.6	412.8	425.6	434.7	438.8	442.9
16.0	296.3	382.4	399.9	412.3	421.0	424.9	428.9

 $F \sim 1000 \text{ lb}_f$ ($R_t = 1.33 \text{ in.}$)

O/F	1	5	10	20	40	60	100
4.0	332.3	425.3	443.9	457.0	466.5	470.8	475.3
8.0	316.5	412.6	432.6	446.9	457.2	461.8	466.6
12.0	307.2	403.6	424.1	438.7	449.3	454.1	459.0
16.0	298.3	393.6	414.2	429.0	439.7	444.5	449.4

 $F \sim 10,000 \text{ lb}_f$ ($R_t = 4.20 \text{ in.}$)

O/F	1	5	10	20	40	60	100
4.0	333.1	427.3	446.2	459.6	469.2	473.6	478.2
8.0	316.9	417.0	438.2	453.5	464.6	469.7	474.8
12.0	307.6	407.7	429.7	445.6	457.2	462.5	467.9
16.0	298.8	398.7	421.3	437.8	450.0	455.5	461.2

TABLE 11.

CALCULATED SPECIFIC IMPULSE FOR

O/F	$\frac{H}{2} / \frac{F}{2}$ AT $p_o = 500 \text{ psia}$ ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)						
	1	5	10	20	40	60	100
4.0	336.2	432.2	451.5	465.1	474.9	479.4	484.0
8.0	324.2	429.5	453.5	471.0	483.9	489.8	495.9
12.0	315.5	420.7	446.6	466.8	482.0	489.2	496.6
16.0	307.2	412.9	439.7	461.0	477.9	486.1	494.6

O/F	$F \sim 1000 \text{ lb}_f$ ($R_t = 0.595 \text{ in.}$) ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_{sp(ODK)}$)						
	1	5	10	20	40	60	100
4.0	335.7	430.4	449.4	463.0	472.7	477.2	481.7
8.0	324.1	425.7	447.6	463.4	474.9	480.1	485.6
12.0	315.3	417.7	440.5	457.2	469.6	475.2	481.0
16.0	307.0	409.7	433.3	450.9	463.9	469.9	476.1

O/F	$F \sim 10K \text{ lb}_f$ ($R_t = 1.88 \text{ in.}$)						
	1	5	10	20	40	60	100
4.0	335.9	431.2	450.4	463.9	473.8	478.2	482.8
8.0	324.2	427.5	450.0	466.4	478.4	483.9	489.5
16.0	307.1	411.5	436.4	455.0	469.0	475.5	482.2

O/F	$F \sim 100K \text{ lb}_f$ ($R_t = 5.95 \text{ in.}$)						
	1	5	10	20	40	60	100
4.0	336.0	431.7	450.9	464.5	474.4	478.8	483.4
8.0	324.2	428.5	451.6	468.4	480.7	486.3	492.2
16.0	307.2	412.3	438.0	457.6	472.4	479.2	486.4

TABLE 12.

CALCULATED SPECIFIC IMPULSE FOR

$$H_2/F_2$$

$$AT p_o = 3000 \text{ psia}$$

ONE-DIMENSIONAL EQUILIBRIUM SPECIFIC IMPULSE ($I_{sp(ODE)}$)

O/F	1	5	10	20	40	60	100
4.0	337.6	432.7	451.8	465.4	475.1	479.6	484.1
8.0	330.9	434.5	457.2	473.8	485.9	491.5	497.2
12.0	323.6	428.6	453.4	472.0	486.1	492.6	499.4
16.0	308.6	411.1	435.7	454.3	468.4	475.0	481.9

ONE-DIMENSIONAL KINETIC SPECIFIC IMPULSE ($I_s (ODK)$)

$$F \sim 10K$$

$$(R_t = .77 \text{ in.})$$

O/F	1	5	10	20	40	60	100
4.0	337.6	432.5	451.6	465.2	475.0	479.4	484.0
8.0	330.9	433.9	456.4	472.7	484.8	490.2	495.9
12.0	323.6	428.0	452.3	470.3	483.8	490.1	496.6
16.0	308.6	410.7	434.7	452.7	466.2	472.4	479.0

$$F \sim 100K$$

$$(R_t = 2.43 \text{ in.})$$

O/F	1	5	10	20	40	60	100
4.0	337.6	432.6	451.6	465.2	475.0	479.5	484.1
8.0	331.0	433.9	456.5	473.0	485.1	490.6	496.4
16.0	308.6	410.7	435.0	453.3	467.0	473.4	480.2

III DIVERGENCE PERFORMANCE LOSS CHARTS

This section contains seven charts which can be used to approximate the divergence efficiency of conical or Rao contoured nozzles. The divergence efficiency ($\mathcal{N}_{\text{DIV}}$) is defined as the ratio of two-dimensional specific impulse to one-dimensional specific impulse,

$$\mathcal{N}_{\text{DIV}} = \frac{I_{\text{sp}} (2\text{D})}{I_{\text{sp}} (1\text{D})} \quad \gamma = \text{constant} \quad (\text{Eq. 1})$$

Thus, $\mathcal{N}_{\text{DIV}}$ includes both the effects of nonaxially directed momentum at the nozzle exit and the throat curvature effect on the discharge coefficient. The divergence efficiencies presented in the charts of this appendix were computed on the basis of constant gamma flow. The two-dimensional specific impulse was computed using a constant gamma method of characteristics computer program. The transonic solution used for the conical nozzles was that of Hall's as modified by Kliegel which is applicable for wall radii of curvature less than one. The transonic solution used for the Rao contoured nozzles was that of Sauer and an upstream wall radius of curvature (RR) of two was used for these calculations. The one-dimensional constant gamma performance was computed using standard relationships.

A. Conical Nozzles

The divergence efficiencies for conical nozzles having normalized throat wall radii of curvature of 1.0 are presented in Figures 42 and 43. These figures present $\mathcal{N}_{\text{DIV}}$ for gamma of 1.20 and 1.32, respectively, as a function of area ratio and expansion angle. Data for nozzles having expansion area ratios from 1.1 to 150, and expansion half angles of 10, 15, 20, 25, and 30° are included.

The effect of the wall radius of curvature (RWT) on η_{DIV} of low area ratio nozzles was investigated and typical results are included in Figure 44. Typically, a 0.3% change in η_{DIV} can result in low area ratio nozzles as RWT is varied from 0.5 to 5.0.

B. Rao Contoured Nozzles

The divergence efficiencies for Rao contoured nozzles are presented in Figures 45 through 48. Charts are included for γ 's of 1.20, 1.26, 1.32, and 1.37. The charts present η_{DIV} as a function of area ratio for various ambient to stagnation chamber pressure ratios and nozzle lengths. Specifying two of these three parameters uniquely defines a Rao nozzle contour and its corresponding η_{DIV} . For a given length, as the ambient pressure decreases the area ratio of a Rao nozzle increases. Correspondingly, for a given p_a/p_o , as the area ratio of a Rao nozzle increases so does its length. Note also, that any single variation in any of these parameters will result in a change in η_{DIV} .

To find the η_{DIV} of an existing Rao nozzle, the area ratio and length should always be used. If, in this case, the actual p_a/p_o ratio does not correspond to the value read from the chart then the existing nozzle does not result in an optimum expansion. For a given p_a/p_o ratio and either a given length (X/R_t) or area ratio, only one Rao optimum nozzle will exist. Therefore, to design an optimum expansion nozzle using these charts, one must specify the p_a/p_o ratio and either the length or the area ratio.

DIVERGENCE PERFORMANCE LOSS

FIGURE LIST

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$\gamma = 1.2$
 RWT = 1.0
 CONICAL NOZZLES

Figure 42. Divergence Efficiency for Conical Nozzles at a $\gamma = 1.20$

$\gamma = 1.32$
 RWT = 1.0
 CONICAL NOZZLES

Figure 43. Divergence Efficiency for Conical Nozzles at a $\gamma = 1.32$

Figure 44. Typical Effect of the Throat Wall Radius of Curvature on the Divergence Efficiency of Conical Nozzles

Figure 45. Divergence Efficiency for Rao Contoured Nozzles at a $\gamma = 1.20$

Figure 46. Divergence Efficiency for Rao Contoured Nozzles at a $\gamma = 1.26$

Figure 47. Divergence Efficiency for Rao Contoured Nozzles at a $\gamma = 1.32$

Figure 48. Divergence Efficiency for Rao Contoured Nozzles at a $\gamma = 1.37$

Charts for estimating turbulent boundary layer momentum thickness (θ) and the shape factors (δ^*/θ) are presented in this section. These parameters are shown as a function of the exit Mach number for various thrust chamber wall temperature ratios ($T_w/T_o = 0.2, 0.5, \text{ and } 0.8$), gammas (1.20 and 1.32), and nozzle inlet conditions. These charts were made using information generated by the ICRPG TBL computer program.

The boundary layer momentum and displacement thicknesses are key parameters in determining the boundary layer performance loss. The thrust decrement due to viscous and heat transfer effects along the thrust chamber wall is

$$\Delta F_{BL} = (2\pi r_e p_o (\rho_e U_e^2 / p_o) \theta_e \cos \alpha_e) \left[1 - (p/p_o)_e (\delta^*/\theta)_e \right] \quad (\text{Eq. 2})$$

where, for constant γ and one-dimensional expansions,

$$(p/p_o)_e = 1/(\gamma M^2)_e \quad (\text{Eq. 3})$$

and

$$(\rho_e U_e^2 / p_o) = \frac{(\gamma M^2)_e}{\left[1 + \frac{\gamma-1}{2} M^2 \right]_e} \frac{\gamma}{\gamma-1} \quad (\text{Eq. 4})$$

The boundary layer performance loss is calculated by taking the ratio of the boundary layer thrust decrement to the total mass flowrate, i.e.

$$I_{sp(BL)} = \frac{F_{BL}}{\dot{m}_T} \quad (\text{Eq. 5})$$

The following procedure may be used to determine the appropriate θ and δ^*/θ parameters from the charts presented in this section. First, the one-dimensional exit Mach number for the specific system under evaluation must be found. Using the appropriate γ and expansion area ratio (ϵ) the ID Mach number can be found from isentropic gas tables or by satisfying the following equation:

$$\epsilon = \frac{1}{M_{1D,e}} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_{1D,e}^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (\text{Eq. 6})$$

Next, this Mach number (M_{1D}) should be corrected for nozzle curvature flow effects. Correction factors $[M_{2D}/M_{1D}]$ for conical nozzles are given in Figure 49, while correction factors for Rao contoured nozzles are given in Figure 50. Thus, the two-dimensional Mach number can be computed by multiplying the M_{1D} by the correction factor from Figure 49 or 50, i.e.,

$$M_{2D} = M_{1D} [M_{2D}/M_{1D}] \quad (\text{Eq. 7})$$

Using this two-dimensional Mach number the approximate values for $\theta/r [Re^*]^{1/5} (\sin \alpha)^{1/2}$ and δ^*/θ can be selected from Figures 51, 52, 53, and/or 54. Figures 51 and 52 present $\theta/r [Re^*]^{1/5} (\sin \alpha)^{1/2}$ for $\gamma = 1.20$ and 1.32, respectively, as a function of M_{2D} . Included in these charts are three wall temperature-to-stagnation temperature ratios: 0.2, 0.5, and 0.8. The Reynold's number to the one-fifth power ($(Re^*)^{1/5}$) evaluated at the nozzle throat is used as a scaling factor to account for the effects of thrust level and chamber pressure. The angle term $(\sin \alpha)^{1/2}$, where α is the conical nozzle exit half-angle, is used to account for the two-dimensional contouring effects. For Rao contoured nozzles, an effective wall angle (α) has been determined and is shown in Figure 55. This effective angle must be used for Rao nozzles in order to determine the proper value of θ from Figures 51 or 52. The value of θ from these charts (hereby noted as $\theta_{ref.}$) can thus be found as follows:

$$\theta_{ref} = \left[(Re^*)^{1/5} (\sin \alpha)^{1/2} \frac{\theta}{r} \right]_{\text{chart}} (r_e) \left[1/(Re^*)^{1/5} \right] \frac{1}{(\sin \alpha)^{1/2}} \quad (\text{Eq. 8})$$

where:

$$Re^* = \frac{24 p_o g_c r_t}{\mu_{c*}}$$

and

$$\begin{aligned}
 p_o &= \text{Stagnation pressure (lb}_f\text{/in.}^2\text{)} \\
 g_c &= 32.174 \text{ (lb}_m\text{/lb}_f\text{) (ft/sec}^2\text{)} \\
 r_t &= \text{Throat radius (in.)} \\
 \mu &= \text{Viscosity (lb}_m\text{/ft-sec)} \\
 c^* &= \text{Characteristic Velocity (ft/sec)} \\
 r_e &= \text{Exit radius}
 \end{aligned}$$

The information presented in Figures 51 and 52 was calculated using a standard nozzle inlet geometry consisting of a 15-degree conical inlet angle (α_i), a 2.5:1 contraction area ratio (EC), and a cylindrical combustion chamber length-to-throat diameter ratio ($\ell_{chb.}/D^*$) of 2.0. These parameters will affect the resulting exit momentum thickness as shown in Figures 56 and 57. In these figures, the various nozzle inlet geometries are noted as inlet angle/contraction ratio/chamber length-to-throat diameter ratio ($\alpha_i/EC/\ell_{chb.}/D^*$). For example the standard inlet configuration described above is noted as 15/2.5/2. Figures 56 and 57 show the ratio $\theta/\theta_{ref.}$ for various inlet angles (5, 15, and 30°), contraction area ratios (1.5, 2.5, and 5), and $\ell_{chb.}/D^*$ (0, 2, and 10), for a cooled ($T_w/T_o = 0.2$) and uncooled ($T_w/T_o = 0.8$) nozzle wall, respectively. Using these two figures,* the effect of nozzle inlet geometry can be accounted for, i.e.,

$$\theta = \theta_{ref.} \left[\theta/\theta_{ref.} \right] \quad (\text{Eq. 9})$$

Figures 53 and 54 present δ^*/θ as a function of exit Mach number for $Y = 1.20$ and 1.32, respectively. Included in these figures are wall temperature ratios of 0.2, 0.5, and 0.8. Using the Mach number of Equation 7 and

* The information shown in Figures 56 and 57 was computed for a $Y = 1.20$ only. However, the trends shown will be approximately the same for all Y . When using Figure 56 or 57 for Y 's other than 1.20, one should use the Mach number which corresponds to the area ratio of the nozzle under consideration evaluated by use of Equation 6 for $Y = 1.20$.

the appropriate γ and T_w/T_o , the appropriate value of δ^*/δ can be determined. This value, together with δ of Equation 9 are then used in Equation 2 to calculate the boundary layer thrust decrement.

BOUNDARY LAYER PERFORMANCE LOSS

FIGURE LIST

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Figure 49. Mach Number Correction Factor for Conical Nozzles

Figure 50. Mach Number Correction Factor for Rao Contoured Nozzles

Figure 51. Growth of the Boundary Layer Momentum Thickness for $\gamma = 1.20$

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Figure 53. Boundary Layer Shape Factor (δ^*/θ) for $\gamma = 1.20$

Figure 54. Boundary Layer Shape Factor (δ^*/θ) for $\gamma = 1.32$

Figure 55. The Effective Nozzle Wall Angle for Rao Contoured Nozzle At Various Wall Temperature Ratios

Figure 56. Effect of Nozzle Inlet Conditions on the Momentum Thickness for a Cooled Nozzle Wall

Figure 57. Effect of Nozzle Inlet Conditions on the Momentum Thickness for an Uncooled Nozzle Wall

Figure 58. Effect of Nozzle Inlet Conditions on the Momentum Thickness at the Throat for Cooled and Uncooled Nozzle Walls