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Laser-Plasma Interactions in the Scylla I-C

Experiment: Preliminary Analysis

by

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LASER-PLASMA INTERACTIONS IN THE SCYLLA I-C EXPERIMENT:

PRELIMINARY ANALYSIS

by

Thomas M. York and Kenneth F. McKenna

ABSTRACT

The interaction of high power laser beams with plasmas has recently received considerable theoretical and experimental attention. However, experimental determination of the laser-plasma interaction mechanisms in θ -pinch plasmas is limited. This work considers the interaction of a 3×10^{16} W/cm² peak flux CO₂ laser beam with the magnetically confined plasma generated in the Scylla I-C θ -pinch. For the specific constraints of the CO₂ laser and Scylla I-C, and with the θ -pinch fill pressure as a primary variable (100 mT - 1000 mT), the physical problems of plasma heating, laser light backscatter, and channeling (converging diffraction of the laser beam) are considered. Prediction of the θ -pinch plasma parameters, prior to laser heating, is developed using snowplow theory.

1. INTRODUCTION

The concept of using high power lasers to provide an independent source for heating a plasma to fusion temperatures in controlled thermonuclear reactors has been proposed.^{1,2} Efforts to proceed from concept to proof-of-concept experiments have been reported.^{3,4,5} Theoretical predictions of laser-plasma interactions have provided a basis for experimental evaluation of the physical interaction mechanisms⁶ and the critical ordering of mechanisms dominant in the different interaction regimes.⁷

The present work will key directly to efforts expended during the past year at LASL, which served to define and initiate a laser-plasma interaction experiment. Laser-plasma reactor concepts have been presented in LASL report form,^{8,9} and proposals for initial experiments are outlined in informal reports.¹⁰ In the LASL laser-plasma experiment, the CTR-3 Scylla I-C θ -pinch will be used in conjunction with a CO₂ cold cathode laser, supplied by the L-1 Group of the Laser Research and Technology Division, to study the interaction of axially directed long

wavelength laser light ($10.6 \mu\text{m}$) with the dense ($n_e \sim 10^{17} \text{ cm}^{-3}$) θ -pinch plasma column. A schematic drawing of the proposed experimental plan is shown in Fig. 1. Initially, the experimental investigation will be directed toward studying a) containment or diffraction of the laser beam as it traverses the meter-long plasma column, b) heating of the plasma column by laser energy absorption, and c)

Fig. 1. Schematic of Scylla I-C laser plasma experiment.

instability induced anomalous backscatter of the incident laser light. A detailed listing of possible areas of experimental investigation will be given below.

In this work, an ideal model of the Scylla I-C θ -pinch plasma creation and the CO_2 laser-plasma interaction, based on a well-defined θ -pinch plasma geometry, is developed in order to establish the expected behavior of the laser-heated plasma. Effects such as laser light backscatter and diffraction will be considered. Finally, effects of higher order plasma perturbations, which are not considered in detail but do deserve further evaluation, will be discussed.

While some studies^{1,2,9} indicate that higher plasma temperatures may be achieved by laser pre-heating followed by θ -pinch compression, the present work considers only the reverse sequence, i.e., compression to a magnetically confined plasma column followed by laser heating. This approach was adopted since the immediate interest in the present experiment is to study the laser-plasma interaction process, and the densities required for strong laser interaction are more readily obtained in the compressed plasma column.

II. TOPICAL AREAS FOR INVESTIGATION

The interaction of the CO_2 laser beam with the Scylla I-C θ -pinch plasma is analytically treated with the plasma in a right circular cylinder configuration and with the laser beam aligned along the cylinder axis. The presence or absence of convective plasma motion during and after laser heating forms a natural criterion with which to delineate the following areas of investigation:

A. Laser Heating of a Static Plasma

- i) Consequences of inverse bremsstrahlung absorption.
- ii) Thresholds for anomalous effects due to instabilities.
- iii) Effects of "bleaching" (variable absorption coefficient).
- iv) Species equilibration and thermal conduction effects (uniformity of heated plasma).
- v) Axial focusing or channeling due to radial density gradients.
- vi) Effects of axial density gradients in the plasma column.

B. Flow or Convection Effects During Plasma Heating

- i) Thresholds for local instabilities inducing convective instabilities.
- ii) Radiation induced convection.
- iii) Characteristic flow times in comparison with thermal equilibration times.
- iv) Generation of macroscopic fluid turbulence by microscopic instabilities.

C. Flow or Convection Effects After Plasma Heating but Before Stabilization in Magnetic Fields

- i) Flow and loss effects from ends of theta pinch.
- ii) Heated plasma and magnetic field interaction -- area waves.
- iii) Local surface heating, blow off.

III. SCYLLA I-C PARAMETERS

The geometry, energy storage capabilities, and electrical discharge characteristics of the Scylla I-C θ pinch have resulted from optimization studies intended to produce a dense plasma that will be compatible with the CO_2 laser system performance. The machine parameters, for operation at 40 kV main-bank voltage, are given in Table I.

IV. LASER PARAMETERS^{10,11}

A modification of the A-3 amplifier of the LASL CO_2 laser system was chosen as a satisfactory source for the 50-100 J requirement of the experiment. The laser characteristics are presented in Table II.

The 8-cm-diameter beam will be focused onto the end of the plasma column by one of two possible spherical reflecting mirrors. A mirror with radius of curvature, $R = 6$ m, will focus the beam to a 2-mm

TABLE I

SCYLLA I-C MACHINE PARAMETERS

Parameter	Value
Coil length	$L = 100$ cm
Coil bore	$D = 10.5$ cm
Discharge tube i.d.	$2b = 3.8$ cm
Bank capacity	$C_{TOT} = 97.2$ μ F
Source inductance	$L(S) = 5.4$ nH
1/4 Cycle Time	$\tau/4 = 2$ μ s
Coil Electric Field	$E(\text{coil i.d.}) = 0.8$ kV/cm @ 40 kV
Tube i.d. Electric Field	$E(\text{tube i.d.}) = 0.27$ kV/cm @ 40 kV
Coil Current	$I_{max} = 3.1$ MA @ 40 kV
Peak Magnetic Field	$B_o(\tau/4) = 35$ kG @ 40 kV

TABLE II
CO₂ LASER SYSTEM PARAMETERS

Parameter	Value
Optical aperture	: 50 cm ²
Beam diameter	: 8 cm
Laser length	: 100 cm
Short pulse output	: 80 ± 20 J in 60 ns (2 × 10 ⁹ W, peak)
Focusing mirror	: R = 6 m for post-pinch heat (in focus)
radius of curvature	: R = 8 m for laser preheat (out focus)
Focused beam diameter	: 2 mm
Laser power density	: 5 × 10 ¹⁰ W/cm ² (nominal average)

spot at the near end of the plasma column. This mirror is intended for plasma column heating experiments. A second mirror with a radius of curvature of 8 m will be available for laser heating of uncompressed plasma. The R = 3 m mirror would focus the beam at the far end of the plasma column. Only the behavior of the plasma under the influence of the near-end focus arrangement will be considered here.

V. PROPERTIES OF THE SCYLLA I-C Θ-PINCH PLASMA BEFORE LASER HEATING

Since the details of the laser-plasma interaction are dependent upon the plasma electron density and temperature, it is necessary to estimate these plasma parameters over the anticipated range of Θ-pinch initial-fill pressures. The model used to predict the plasma parameters is developed below.

A. Theta-Pinch Plasma Model

Theta-pinch implosion-phase studies carried out at LASL have concentrated on the imploding plasma behavior in discharge tubes of moderate diameter (20 cm) and moderate fill pressure (~ 10 mTorr). At 10-mTorr fill pressure, recent experiments¹² have shown that during the early stage of the implosion ($r_{\text{plasma}} \sim 7$ cm in the 10-cm-radius tube) the electron density is coincident with the magnetic piston field, a behavior indicative of "snowplow" dynamics. However, halfway through the implosion ($r_{\text{plasma}} \sim 5$ cm), a density front which leads the magnetic piston field is observed, indicating "bounce" model behavior.¹³ In an investigation¹⁴ of the Θ-pinch plasma implosion in a 10-cm-diameter discharge tube at higher fill pressures (50-200 mTorr hydrogen), separation of the electron density and magnetic piston field was not observed. The ion-ion mean free path in this case¹⁴ was about 0.025 cm, compatible with a collision-dominated sheath with a thickness

of about 1 cm. Accordingly, at high-fill pressures ($\gtrsim 50$ mTorr) and small discharge tube diameters ($\lesssim 10$ cm), it appears appropriate to consider the implosion as an event which is not characterized by a plane reflecting magnetic piston (bounce model), but rather an event which involves entrainment and continuous heating of the ions within the current sheath, i.e., a snowplow model. Since the basic interactions postulated in the snowplow model have not been delineated in other available references, the details of the development will be presented here.

Figure 2 presents the Θ-pinch magnetic field waveform, B(t). Specific times during the discharge, used in modeling the plasma behavior, are indicated on the figure. $\tau/4$ defines the time of peak magnetic field, B_0 , when the discharge current is crowbarred allowing gradual decay from the peak field. As will be seen, the pinch time, t_p (time for sheath implosion to $r = 0$) will occur with $t_p \ll \tau/4$. Thus B can be assumed to increase linearly over the time of the implosion.

B. Implosion and Plasma Column Formation Phase

Neglecting acceleration effects, the inelastic snowplow momentum balance across the imploding current sheath can be expressed as

$$\frac{B_s^2}{8\pi} = m_i n_o v_s^2, \quad (2)$$

where m_i is the mass of a deuterium ion, n_o is the number density of fill atoms, and B_s and v_s are the

Fig. 2. Θ-pinch magnetic field waveform. B_0 is the peak magnetic field, B_p the confining magnetic field at the termination of the implosion phase, and B_s the average field over the duration of the implosion.

average driving magnetic field and sheath velocity, respectively. It is assumed that the sheath implodes to the axis ($r=0$) in time, t_p , with the electrons in the imploding sheath remaining cold and the ions possessing equal kinetic and thermal energies.¹⁵ As a first approximation in determining the plasma state, the ion energy acquired during the implosion is assumed to be converted to ion thermal energy in the plasma column. Since gas pressure in the highly compressed column will be greater than magnetic pressure at t_p , the plasma will rapidly expand to an equilibrium position at R_{sh} (plasma boundary after shock heating and expansion). The expanding plasma performs work against the confining magnetic field B_p , and a temperature T_p and density n_p are obtained after the expansion to R_{sh} . The equilibrium energy balance at R_{sh} can be approximated as

$$\pi b^2 n_o^2 \cdot 2 \left(\frac{1}{2} m_i v_s^2 \right) \sim \pi b^2 n_o^2 \cdot \frac{3}{2} kT_p + \pi R_{sh}^2 \cdot \frac{B_p^2}{8\pi} \quad (3)$$

Assuming $\beta \sim 1$ (β is the ratio of plasma pressure to magnetic pressure), the equilibrium pressure balance at R_{sh} is given by

$$\frac{B_p^2}{8\pi} = n_p kT_p \quad (4)$$

and conservation of particles requires

$$n_o b^2 = n_p R_{sh}^2 \quad (5)$$

Combining (4) and (5) and substituting into (3), yields,

$$kT_p = \frac{2}{3} m_i v_s^2 \quad (6)$$

Then substituting (6) into (3) gives

$$R_{sh}^2 = b^2 \frac{n_o kT_p}{B_p^2 / 8\pi} \quad (7)$$

and from equation (5)

$$n_p = n_o \left(\frac{b}{R_{sh}} \right)^2 \quad (8)$$

With the specification of B_p and v_s , Eqs. (6)-(8) can be solved in closed form for T_p , R_{sh} , and n_p . B_p can be determined by noting that during the short time of the implosion phase,

$$B \approx B_o \sin \omega t \approx B_o \omega t \quad (9)$$

Then, in this model $B_s = B_o \omega t_p / 2$ is the average value of driving magnetic field over the duration of the implosion. The value of the confining magnetic field B_p at the equilibrium position R_{sh} is then $B(t=t_p) \approx B_o \omega t_p = 2B_s$. With B_s determined, the sheath velocity can be expressed from Eq. (2) as

$$v_s^2 = \frac{\left[\frac{B_o \omega t_p / 2}{8\pi m_i n_o} \right]^2}{\left(\frac{b}{t_p} \right)^2} \quad (10)$$

Using the previously noted Scylla I-C machine parameters, $B_o = 35$ kG, $\omega = \frac{2\pi}{\tau}$ and $b = 1.8$ cm yields

$$v_s = 5.16 \times 10^{10} n_o^{-1/4} \text{ cm/s} \quad (11)$$

and

$$t_p = 3.49 \times 10^{-11} n_o^{1/4} \text{ s} \quad (12)$$

This analytical formulation can be compared with the experimentally determined pinch times of the piston sheath obtained in the Scylla I-B 0-pinch.¹⁶ Both snowplow and bounce models of the implosion process predict a dependence of pinch time, t_p , on the initial fill density, n_o , and the initial azimuthal electric field, E_0 , which can be expressed as¹⁶

$$t_p \propto b n_o^{1/4} E_0^{-1/2} \quad (13)$$

Experimental results for Scylla I-B¹⁶ lead to the empirical relationship (for the 10-cm-radius discharge tube)

$$t_p = 9.0 \times 10^{-11} n_o^{1/4} E_0^{-1/2} \text{ s} \quad (14)$$

with E_0 in kV/cm. For the smaller radius of Scylla I-C (1.8 cm), the pinch time, with $E_0 = 0.27$ kV/cm, is predicted to be

$$t_p = 3.2 \times 10^{-11} n_o^{1/4} \text{ s} \quad (15)$$

within 10% of the analytical result obtained above [Eq. (12)].

The above model for the plasma implosion, column formation and expansion to the equilibrium position assumed for convenience that the electron population remained at its initial preionization temperature ~ 2 eV. In the plasma column, the higher energy ions will lose energy to the cold electrons as

a result of ion-electron collisions. The time for the ion energy to decrease by a factor of $1/e$ (37%), thus increasing the electron energy, is given for a deuterium plasma by¹⁷

$$\tau_{ie} = \frac{6.3 \times 10^8 T_e^{3/2}}{n_e \ln \Lambda} \text{ s}, \quad (16)$$

where T_e is in eV and n_e in cm^{-3} . For $T_e \sim 2$ eV and $\ln \Lambda \sim 6$ energy equilibration times between the ion and electron populations are only a few nanoseconds over the entire range of initial fill pressures considered (100-1000 mTorr). Accordingly, the ion and electron temperatures rapidly equalize so that $T_{ep} = T_{ip}$.

With fill pressure as the varied parameter, the plasma properties following shock heating and expansion to magnetic pressure equilibrium at B_p are given in Table III. R_{sh} has constant value of $b/\sqrt{10} = 0.57$ cm and $n_p = n_{ep} = n_{ip}$.

C. Adiabatic Compression Phase

Following the plasma implosion and expansion to an equilibrium radius R_{sh} with confining magnetic fields of strength B_p , the plasma is adiabatically compressed by the magnetic field increasing from B_p at $t = \tau_p$ to B_o at $t = \tau/4$. The conditions at the end of the compression will be denoted by the subscript f. For equilibrated electrons and ions the plasma properties after compression are given by¹⁸

$$R_f = R_{sh} \left(\frac{B_p}{B_o} \right)^{1/\gamma}, \quad (17)$$

$$T_f = T_p \left(\frac{R_{sh}}{R_f} \right)^{2(\gamma-1)}, \quad (18)$$

$$n_f = n_o \left(\frac{b}{R_f} \right)^2, \quad (19)$$

where $\gamma = 5/3$ for the collision dominated plasma,

$T_f = T_{ef} + T_{if}$ and $n_f = n_{ef} = n_{if}$.

TABLE III

PLASMA PARAMETERS FOLLOWING SHOCK HEATING

P_o , mTorr	$t_p = 1.5/v_s$, ns	$B_s = B_p/2$, kG	n_p , cm^{-3}	$T_p = T_{ep} = T_{ip}$, eV
100	519	4.4	7×10^6	27
500	477	6.6	5.5×10^7	12
1000	568	7.8	7×10^7	9

TABLE IV

PLASMA COLUMN PARAMETERS AT MAXIMUM COMPRESSION

P_o , mTorr	R_f , cm	n_f , cm^{-3}	$T_{ef} = T_{if}$, eV	E_f , J
100	.249	3.7×10^{17}	41	143
500	.316	1.1×10^{18}	13	230
1000	.351	1.8×10^{18}	8	283

The energy resident in the plasma column of length L at the termination of the adiabatic compression phase can be expressed as

$$E_f = \left(\frac{3}{2} n_f k T_f \right) (\pi R_f^2) L. \quad (20)$$

Assuming pressure balance ($\beta \approx 1$),

$$E_f = 2.3 \times 10^3 R_f^2 \text{ J} \quad (21)$$

for the Scylla I-C operating parameters.

With fill pressure as the varied parameter, the predicted (final) conditions of the plasma at the termination of the snowplow-adiabatic-compression processes, determined from Eqs. (17)-(21) using the values in Table III, are given in Table IV and presented graphically in Fig. 3.

Fig. 3. Plasma column parameters at the termination of the snowplow-adiabatic-compression process as a function of initial 0-pinch fill pressure.

The possibility of significant radiant energy loss on the time scale of the pinch and adiabatic compression events must be considered. Assuming free-free transitions of electrons (continuum radiation) to be the dominant radiant energy loss mechanism, the energy radiated per unit volume per unit time can be expressed as¹⁷

$$E_R = g \left(\frac{2\pi kT_{ef}}{3m_e} \right)^{1/2} \frac{2^5 \pi e^6}{3 h m_e c^3} Z^2 n_f^2 \quad (22)$$

or

$$E_R = 1.7 \times 10^{-32} n_f^2 T_{ef}^{1/2} \text{ W/cm}^3 \quad (23)$$

in the present case, with n_f in cm^{-3} and T_{ef} in eV. The greatest energy loss is found to occur at 1000 mTorr pressure where the loss rate is 10 J/ μ s or about 2% of the total plasma energy per μ s. At 100 mTorr fill, the plasma radiates about 0.2% of its energy per μ s. Such loss rates are sufficiently small over the time scale of the laser-plasma interaction to be neglected.

VI. IDEAL LASER-PLASMA INTERACTION

This work will first consider the response of the plasma under the influence of the expected dominant laser absorption mechanism, inverse bremsstrahlung, ignoring problems of laser light backscatter, and possible diffraction of the laser beam out of the plasma column. Thus, the results of such an analysis will provide an indication of the maximum perturbing effect of the laser heating. This information is crucial in the evaluation of possible diagnostic techniques and the identification of optimum θ -pinch operating conditions. Following this analysis, each of the effects (backscatter, etc.) which serve to reduce the local energy flux, and thus the heating effect, will then be evaluated.

A. Inverse Bremsstrahlung Absorption Length, ℓ_{ab}

In a fully ionized plasma, the energy of the incident laser light is absorbed through the process of inverse bremsstrahlung by which the electrons are accelerated in the electric fields of the focused light. The plasma ions are heated as a result of subsequent electron-ion collisions. The inverse bremsstrahlung absorption coefficient, K ,

has been presented by Johnston and Dawson¹⁹ and clarified by Kristiansen.²⁰ The absorption length, ℓ_{ab} , can be expressed, in terms of K , as

$$\frac{1}{K} = \ell_{ab} = 1.15 \times 10^{29} \frac{(kT_{ef})^{3/2}}{\lambda^2 Z n_f^2 \ell n \Lambda} \cdot \left(1 - \frac{\lambda^2}{\lambda_p^2} \right)^{1/2} \quad (24)$$

Here λ is the laser light wavelength, λ_p the wavelength of radiation at the plasma frequency, Z , the electronic charge number and $\Lambda = v_T / \omega P_{min}$

$\left(v_T = \left(\frac{kT_{ef}}{n_f} \right)^{1/2} \right)$, ω the wave frequency and $P_{min} = \frac{Ze^2}{3kT_{ef}}$. For $Z = 1$ and CO_2 laser light ($\lambda = 10.6 \times 10^{-4}$ cm), the above reduces to

$$\ell_{ab} = \frac{1.03 \times 10^{35} (kT_{ef})^{3/2}}{n_f^2 [1.5 \ell n(kT_{ef}) + 0.494]} \cdot \left(1 - \frac{n_{ef}}{10^{19}} \right)^{1/2} \text{ cm}, \quad (25)$$

for n_f in cm^{-3} and kT_{ef} in eV.

Substituting the values of the plasma column electron temperature, T_{ef} , and density, n_f , derived for the snowplow-adiabatic-compression model (Table IV) into equation (25), results in the absorption lengths presented in Table V.

B. Physical Interpretation of ℓ_{ab}

The physical problem under consideration is the attenuation of a laser beam, with a total incident flux density I (W/cm^2), upon passing through the plasma column where it loses an increment of flux density, $KI \cdot ds$, upon traversing a distance ds .

TABLE V

p_0 , mT	LASER ABSORPTION LENGTH		
	100	500	1000
ℓ_{ab} , cm	33	0.8	0.2

Ideally, neglecting scattering and emission, the distance rate of change of the incident flux density is given by,

$$\frac{dI}{ds} = -KI \quad (26)$$

For the case with the beam entering at the plasma-vacuum boundary, $S = 0$, the solution of the above equation takes the form

$$I(s) = I(0)e^{-\tau(s)} \text{ where } \tau(s) = \int_0^s K \cdot ds \quad (27)$$

This functional form represents an exponential decay of the flux density along the beam path. At one absorption length the beam flux density is reduced to $1/e$ of its initial value. For constant absorption coefficient, K , the above formulation allows interpretation of the absorption lengths presented in Table V. For a plasma with an absorption length, λ_{ab} , greater than the plasma extent, one could expect an almost linear decrease in absorption (and thus heating) to occur along the plasma column. For an absorption length smaller than the length of the plasma, beam energy would still be deposited at positions larger than the absorption length, but with significantly reduced intensity. This simple exponential reduction in flux density can be altered by a variation in the absorption coefficient, K , along the column due to the local plasma heating. Since $K \sim T_e^{-3/2}$, local laser heating of the electron plasma reduces the absorption coefficient (increasing λ_{ab}), thus allowing the laser beam to penetrate further into the plasma. This nonlinear absorption process is referred to as bleaching.

C. Inverse Bremsstrahlung Absorption with Nonlinear Absorption Coefficient

Laser-plasma interactions dominated by nonlinear inverse bremsstrahlung absorption (IBA) have been given a rather general treatment by Steinhauer and Ahlstrom.^{21,22} However, the strength of the physical arguments in the present case are best delineated in the work by Schappert,²³ which will be followed here. This analysis²³ assumes constant electron density within the interaction region with no electron-ion energy transfer and no electron thermal conduction to the surrounding plasma over the time scale of the laser pulse. The laser beam

is presumed to remain collimated at its initial incident diameter (2 mm in the present case) heating a cylindrical central core of the plasma column.

Nonlinear absorption is included in the analysis through the introduction of a variable inverse bremsstrahlung coefficient which, neglecting density variations, can be written as

$$\tilde{K} = K \left(\frac{T_{ef}}{T_e} \right)^{3/2} \text{ cm}^{-1} \quad (28)$$

K is the ideal IBA coefficient defined by Eq. (24), T_{ef} is the initial plasma column electron temperature (prior to the laser heating) and T_e is the local electron temperature which varies over the duration of the laser pulse. Substituting $\tilde{K}$ into the time dependent radiative transfer equation and integrating over the laser pulse length, the exact solution for the laser energy absorbed at a given axial position, $E(X)$, can be written as,²³

$$\left\{ \frac{1}{3}(1+Q)^{3/5} + (1+Q)^{1/5} + \frac{1}{2} \ln \left[\frac{(1+Q)^{1/5} - 1}{(1+Q)^{1/5} + 1} \right] \right\} \bigg|_{Q(0)}^{Q(X)} = -\frac{1}{2} KX; \quad (29)$$

$Q(X)$ and $Q(0)$ are the limits of integration. $Q(X)$ is a dimensionless parameter expressed in terms of the plasma state prior to laser heating,

$$Q(X) = \frac{5}{3} \frac{K E(X)}{n_f k T_{ef}} \quad (30)$$

$Q(0)$ can be considered as the ratio of the incident laser energy per unit area, $E(0)$, to the original plasma thermal energy is one absorption length. Eq. (29) can be reduced to

$$\frac{E(X)}{E(0)} = \begin{cases} e^{-KX} & \text{for } Q(0) < 1 \\ 1 - \frac{5}{3} \frac{KX}{Q(0)^{3/5}} & \text{for } Q(0) \gg \left(\frac{3}{5} KX\right)^{5/3} \end{cases} \quad (31a)$$

$$\frac{E(X)}{E(0)} = \begin{cases} e^{-KX} & \text{for } Q(0) < 1 \\ 1 - \frac{5}{3} \frac{KX}{Q(0)^{3/5}} & \text{for } Q(0) \gg \left(\frac{3}{5} KX\right)^{5/3} \end{cases} \quad (31b)$$

For low energy absorption (absorption length $>$ plasma extent), the constant absorption coefficient solution with exponential beam energy decay is obtained [see Eq. (27) and (31a)]. The nonexponential energy absorption, Eq. (31b), is indicative of bleaching along the column.

Fig. 4. Electron temperature profiles for nonlinear inverse bremsstrahlung absorption coefficient. The curves represent the maximum temperature ratios in the absence of all loss mechanisms.

Integrating the electron energy conservation equation over the laser beam pulse width and applying Eqs. (31a) and (31b), the laser-induced electron temperature, $T_{e\ell}(X)$, along the 2-mm-diameter heated plasma column core can be solved for as ²³

$$\frac{T_{e\ell}(X)}{T_{ef}} = \left[1 + Q(o)e^{-KX} \right]^{2/5}; \quad Q(o) < 1 \quad (32a)$$

$$\frac{T_{e\ell}(X)}{T_{ef}} = \left[1 + Q(o) - \frac{5}{3} Q(o) KX \right]^{2/5}; \quad Q(o) \gg \left(\frac{3}{5} KX \right)^{5/3}. \quad (32b)$$

The electron temperature profiles, for various fill pressures, are presented in Fig. 4.

For the high fill pressure cases the beam energy will be completely absorbed before reaching the far end of the plasma column, $X = 100$ cm. The limiting axial station, X_ℓ , where $E(X_\ell) = 0$ is given

$$X_\ell = \frac{3}{5} \frac{Q(o)^{3/5}}{K}; \quad Q(o) \gg \left(\frac{3}{5} KX \right)^{5/3}. \quad (33)$$

Values of X_ℓ , the initial electron temperature before heating, T_{ef} , and the electron temperatures after laser heating, $T_{e\ell}(X)$, for $X = 0$ and $X = X_\ell$ are given in Table VI as a function of fill pressure. When $X_\ell > 100$ cm (the plasma column length), $T_{e\ell}(X_\ell)$ is calculated at $X_\ell = 100$ cm.

A comparison of the laser absorption length, ℓ_{ab} , obtained for constant K and the length, $\tilde{\ell}_{ab}$, derived for the variable absorption coefficient, $\tilde{K}$, is presented in Fig. 5. As for ℓ_{ab} , the length $\tilde{\ell}_{ab}$ is defined as the axial position at which the laser beam energy is attenuated to $1/e$ of its initial value, i.e., $E(\tilde{\ell}_{ab}) = E(o)/e$. As can be seen from Fig. 5, nonlinear absorption significantly increases the laser beam penetration distance, the divergence between ℓ_{ab} and $\tilde{\ell}_{ab}$ increasing with fill pressure. The absorption lengths ℓ_{ab} and $\tilde{\ell}_{ab}$ actually represent two limiting cases. In the former, electron heating is neglected so that ℓ_{ab} is the smallest possible absorption length obtainable for a given set of initial plasma conditions. In the latter case, thermal heat conduction is neglected so that $\tilde{\ell}_{ab}$ is the maximum obtainable absorption length. As will be discussed below, the characteristic time for radial heat diffusion is less than the duration of the laser pulse. Accordingly, the rapid transfer of energy from the central core of heated plasma to the

TABLE VI

PLASMA PARAMETER FOLLOWING LASER HEATING

p_o , mTorr	$Q(o)$	T_{ef} , eV	X_ℓ , cm	ℓ_{ab} , cm	$T_{e\ell}(o)$, eV	$T_{e\ell}(X=100)$, eV
100	66.3	41	232	33	221	178
500	2.34×10^3	13	55	0.8	298	13
1,000	1.14×10^4	8.3	29	0.2	348	8.3

Fig. 5. Compression of laser absorption length, l_{ab} , for constant K , and absorption length, $\tilde{l}_{ab}$, for variable absorption coefficient K .

surrounding plasma will decrease the indicated absorption length, $\tilde{l}_{ab}$, together with the electron temperature, $T_{el}(X)$ (Fig. 4). The experimentally observed laser absorption length can therefore be expected to fall within the crosshatched region shown in Fig. 5, being greater than l_{ab} due to nonlinear absorption, but less than $\tilde{l}_{ab}$ as a result of radial heat diffusion.

VII. PLASMA RESPONSE TO LASER ENERGY ABSORPTION

In the preceding section, ideal electron heating in the 2-mm-diameter cylinder of plasma exposed to the laser beam (beam diam ≈ 2 mm) was analyzed. The laser induced particle interactions within this hot plasma core, the transfer of energy to the surrounding plasma, and the dynamic response of the entire plasma column will now be analyzed.

A. Electron-Electron Energy Transfer

As previously discussed, the absorbed laser energy directly heats the electrons through the inverse bremsstrahlung mechanism. The time, t_{ee} , for the heated electrons to attain a local Maxwellian energy distribution can be expressed as

$$t_{ee} = \frac{3.3 \times 10^5 T_{el}(x)^{3/2}}{n_e \ell n \Lambda} \text{ s}, \quad (34)$$

where T_{el} is in eV and n_e in cm^{-3} . Using the electron temperatures at the hot end of the plasma column, $T_{el}(o)$ (Table VI), $\ell n \Lambda \approx 10$, and $n_e = n_f$, the calculated equilibration time for the electron population $t_{ee} \sim 1$ ns over the entire range of initial fill pressures. Thus, the electrons will maintain a Maxwellian distribution even during the duration of the laser pulse (~ 50 ns).

Neglecting, momentarily, local heating of the ions in the plasma core, the primary process inducing electron temperature equalization of the hot core with the remainder of the plasma column is thermal conduction. Heat conduction basically results from electron thermal transport; since the velocity of electrons greatly exceeds that of the ions, due to the mass difference, and the plasma column is assumed to be magnetic-field free, only the electrons need be considered carriers of thermal energy. Neglecting density variations, the electron temperature equalization time is given by the time constant of the lowest order mode of heat conduction. The appropriate diffusion equation is

$$C_v \frac{\partial T}{\partial t} = \eta \nabla^2 T, \quad (35)$$

where C_v is the plasma specific heat, $C_v = \frac{3}{2}(n_{ef} + n_{if})k$, and η is the electron thermal conductivity. The characteristic thermal diffusion time can be expressed as

$$t_{th} = \frac{C_v}{\eta} L^2 = \frac{L^2}{D}, \quad (36)$$

where L is the characteristic length. With the electron thermal conductivity from Spitzer,¹⁷ the thermal diffusivity, in the absence of internal magnetic fields, can be written as

$$D = \frac{6.3 \times 10^{20}}{n_e \ell n \Lambda} T_{el}(X)^{5/2} \text{ cm}^2/\text{s}. \quad (37)$$

For local radial heat diffusion, $L^2 = (R_f - R_c)^2$ where R_c is the radius of the hot central core, $R_c = 0.1$ cm, and R_f is the plasma column boundary given in Table IV. Taking $T_{el}(X) = T_{el}(o)$, $\ell n \Lambda \approx 10$, and $n_e = n_f$, the radial diffusion time $t_{th}(R) \lesssim 1.0$ ns in all fill pressure cases. Accordingly, the input laser energy is transferred locally,

during the time of the laser pulse, to the entire plasma column electron population at a given axial position.

The electron temperature equilization process will have the effect of lowering the laser induced electron temperature $T_{e\ell}$ uniformly across the plasma column at a given axial station. The resulting temperature of the electron plasma, $T'_e(X)$, is determined from a local energy balance which can be expressed as,

$$\frac{3}{2} n_f k T_{e\ell}(X) \pi R_c^2 + \frac{3}{2} n_f k T_{ef} \pi (R_f^2 - R_c^2) = \frac{3}{2} n_f k T'_e(X) \pi R_f^2, \quad (38)$$

where T_{ef} , n_f and R_f are given in Table IV. The above equation assumes that the electron density remains constant over the nanosecond time scale of the energy transfer process. The energy balance equation can be reduced to

$$T'_e(X) = T_{ef} + [T_{e\ell}(X) - T_{ef}] \frac{R_c^2}{R_f^2}. \quad (39)$$

Substituting values from Table IV and VI, the resulting electron temperatures at $X = 0$ and $X = X_\ell$ are given in Table VII, together with the radial thermal diffusion times evaluated at $T_{e\ell}(0)$.

Also presented in this table is the time for axial electron temperature equalization, $t_{th}(z) = X_\ell^2/D$ where $X_\ell \leq 100$ cm (plasma column length = 100 cm), calculated at $T_e = T_{e\ell}(0)$. It should be noted $t_{th}(z)$ is several orders of magnitude greater than $t_{th}(R)$ in all cases. The electron distribution at the two extreme ends of the plasma column are effectively thermally insulated, and axial temperature equilization along the length of the plasma would not be expected to occur over the time scale of the experiment (~ 1 μ s).

TABLE VIII

ELECTRON-ION EQUIPARTITION TIME

p_0 , mTorr	$T'_e(0)$, eV	$t_{eq}(0)$, ns
100	70	100
500	42	16
1000	35	6

B. Electron-Ion Energy Equipartition

The electrons are heated directly by the inverse bremsstrahlung process and rapidly share their energy with the electrons in the local surrounding plasma. Ion heating results from subsequent electron-ion energy transfer collisions via the Coulomb interaction. The equipartition of energy between these two species is governed by the equation,

$$\frac{dT_i}{dt} = \frac{T'_e - T_{if}}{t_{eq}}, \quad (40)$$

where t_{eq} is Spitzer's¹⁷ equipartition time,

$$t_{eq} = \frac{6.3 \times 10^8 T'_e(X)^{3/2}}{n_e n_i \Lambda} \text{ s}. \quad (41)$$

Values for t_{eq} evaluated at $T'_e(0)$ and $n_e = n_f$ are given in Table VIII.

The electron-ion energy equipartition times are much larger than the radial electron thermal diffusion times, $t_{th}(R)$. Thus, the ion temperature lags the electron temperature on the time scale of the laser heating pulse. However, on the time scale of the experiment (~ 1 μ s) electron-ion equilibration is rapid enough (a few hundred ns) to affect plasma properties.

In the above calculations, it has been assumed that the plasma column does not expand before electron-ion thermalization. An estimate of the time scale for radial expansion of the plasma column boundary as a result of hot electron pressure can

TABLE VII

ELECTRON TEMPERATURE AFTER RADIAL HEAT DIFFUSION AND DIFFUSION TIMES

p_0 mTorr	T_{ef} , eV	$T'_e(0)$, eV	$T'_e(X_\ell)$, eV	$t_{th}(R)$, ns	$t_{th}(z)$, μ s
100	41	70	63	0.2	78
500	13	42	13	0.5	34
1000	8	36	8	0.9	12

TABLE IX
PLASMA EQUIPARTITION AND EXPANSION TIMES

p_o , mTorr	t_{eq} , ns	Δt , ns
100	100	10
500	16	21
1000	6	27

be obtained by equating the change of radial kinetic energy of expanding plasma to the work done by the electron pressure in expanding the plasma against the confining magnetic fields. When expressed per unit length, this gives the differential expression,

$$d\left\{\frac{1}{2} m_i n_f \pi R_f^2 \left(\frac{dR(X)}{dt}\right)^2\right\} = n_f k [T_e'(X) + T_{if}] 2\pi R(X) dR(X) - \frac{B_o^2}{8\pi} 2\pi R(X) dR(X), \quad (42)$$

where m_i is the ion particle mass and $R(X)$ is the outer boundary of the plasma at a given axial station. Assuming $T_e'(X)$ remains constant (and thus overestimating the expansion velocity) the above equation can be integrated and rearranged, yielding the characteristic expansion time,

$$\Delta t = \left\{ \frac{m_i}{2k} \cdot \frac{\Delta R^2}{[T_e'(X) - T_{ef}] \left[\frac{R(X)^2}{R_f^2} - 1 \right]} \right\}^{1/2}. \quad (43)$$

The only unknown variable in the above equation is the radius of the plasma boundary, $R(X)$. If the plasma expands adiabatically, the final radius will be independent of the expansion time and thus $R(X)$ can be determined from simple conservation considerations. This analysis is discussed below, but the results for $R(X)$ will be used here. Taking $T_e'(X) = T_e'(o)$ and $R(X) = R(o)$ (Table XI), the expansion time, Δt , and the electron-ion equipartition times, t_{eq} , are given in Table IX as a function of initial fill pressure.

It will be noted that the above Δt values are underestimated since $T_e'(X)$ decreases during the expansion. Comparing the expansion time, Δt , with the electron-ion equipartition times, t_{eq} , it can be observed that the assumption of ion-electron equilibration before expansion is reasonable only at the higher fill pressures. However, since the

higher fill pressure cases offer the most favorable absorption conditions for laser heating ($\tilde{\lambda}_{ab} < 100$ cm) the following analysis will maintain that energy equipartition precedes expansion.

Equipartition of energy between electrons and ions further reduces the local electron temperature to a value $T_e''(X)$ and increases the ion temperature to $T_i''(X)$. From energy balance,

$$\begin{aligned} \frac{3}{2} n_f k T_e''(X) + \frac{3}{2} n_f k T_{if} &= \frac{3}{2} n_f k [T_e'(X) + T_{if}] \\ + T_i''(X)] &= 3 n_f k T_e''(X), \end{aligned} \quad (44)$$

which reduces to

$$T_e''(X) = T_i''(X) = \frac{1}{2} [T_e'(X) + T_{if}], \quad (45)$$

where T_{if} is the ion temperature prior to laser heating (Table IV). The calculated values of $T_e''(X)$ at $X = 0$ and $X = X_k$ are shown in Table X with the initial electron temperature T_{ef} .

After equilibration of electron and ion temperatures the plasma column will have a uniform local temperature with $T_e''(X) = T_i''(X)$ and a gradient in temperature will exist between axial positions. This axial temperature gradient will be maintained for the duration of the experiment.

C. Plasma Column Expansion and New Equilibrium

State

The rapid increase of plasma temperature by laser heating produces a plasma-magnetic field pressure imbalance, since $B_o^2/8\pi < 2n_f k [T_i''(X)]$. The plasma must subsequently expand until pressure balance is again attained. Expansion is assumed to occur after laser energy absorption and temperature equalization between species, and be slow on the electron-ion equilibration time scale ($\sim 10^2$ ns), but fast on the time scale of external magnetic

TABLE X
ELECTRON TEMPERATURE AFTER ELECTRON-ION
ENERGY EQUIPARTITION

p_o , mTorr	$T_e''(o)$, eV	$T_e''(X_k)$, eV	$T_{ef} = T_{if}$, eV
100	56	52	41
500	28	15	13
1000	22	10	8

field variations ($\sim 1 \mu s$). The final state of the laser heated plasma is determined from the equation of pressure (assuming $\beta \approx 1$) and energy balance, and particle conservation:

$$\frac{B_0^2}{8\pi} = 2n_e(X)kT_e(X) \quad (46)$$

particle conservation,

$$n_e(X)\pi R^2(X) = n_f\pi R_f^2 \quad (47)$$

energy balance,

$$3n_e(X)kT_e(X)\pi R(X)^2 = 3n_fkT_f''(X)\pi R_f^2 - \pi[R(X)^2 - R_f^2] \frac{B_0^2}{8\pi}. \quad (48)$$

Here, $T_e(X) = T_1(X)$, $n_e(X)$ and $R(X)$ are the local plasma temperature, density and radius after expansion. $T_e''(X) = T_1''(X)$, n_f and R_f are the local plasma temperature, density, and radius immediately prior to expansion. The above equations can be combined and rewritten as,

$$T_e(X) = \frac{B_0^2}{8\pi} \frac{1}{2kn_e(X)} \quad (49)$$

$$n_e(X) = n_f \left[\frac{R_f}{R(X)} \right]^2 \quad \text{and} \quad (50)$$

$$R(X)^2 = \frac{2}{5} R_f^2 \left[1 + \frac{3}{2} \frac{T_e''(X)}{T_{ef}''} \right]. \quad (51)$$

Values of $T_e(X)$, $n_e(X)$ and $R(X)$ after expansion of the heated plasma column are given in Table XI.

D. Sequence of Plasma Events Following Laser Energy Absorption

The values presented in Table XI represent the final state of the plasma after laser heating, thermal equilibration, and plasma expansion. The sequence of events characterizing the plasma response to input laser energy is summarized below. It should be understood that such a charac-

terization is somewhat artificial, since each event is interdependent.

1. Laser input energy pulse of time duration t_k is absorbed by the electrons in the 0.2-cm-diameter central core of plasma.

1) Electrons in core thermalize with $t_{ee} < t_k$, increasing the electron temperature to $T_{ek}(X)$.

2. Electron thermal energy diffuses radially.

1) Electrons in core thermalize with the electrons in the surrounding plasma with $t_{th}(R) \approx t_{ee}$, decreasing the local electron temperature to $T_e'(X)$.

3. The electrons and ions thermally equilibrate.

1) Plasma temperature becomes radially uniform with $t_{eq} \gg t_{th}(R)$, decreasing the electron temperature and increasing the ion temperature until $T_e''(X) = T_1''(X)$.

4. The plasma column expands until pressure equilibrium is again obtained.

1) The outer boundary of the plasma expands with $\Delta t > t_{eq}$, uniformly decreasing both ion and electron temperatures to their final values, $T_e(X) = T_1(X)$.

The results presented above represent the idealized changes in the θ -pinch plasma that could be expected during the course of the Scylla I-C laser-heated plasma experiment. Specific aspects of interest are the axial variation of equilibrium radius, changes of electron and ion temperatures at one particular point, axial variation of temperatures, and changes in electron density following the laser heating, as well as the time history of these variables. The results presented above represent maximum changes in variables as the calculations have not included anomalous backscatter effects due to instabilities and beam divergence because of radial gradients in the electron density. Each of these effects will now be considered.

VIII. LASER LIGHT SCATTERING INSTABILITIES

The irradiation of plasma with laser light of sufficient power level can cause "laser driven instabilities" to occur which result in anomalous backscatter²⁴ or absorption²⁵ of the laser light. Anomalous absorption instabilities are effective for laser radiation near the plasma frequency or twice the plasma frequency. In the Scylla I-C experiment

TABLE XI
EQUILIBRIUM PLASMA PARAMETERS

P_0, mT	$T_e(o), \text{eV}$	$n_e(o), \text{cm}^{-3}$	$R(o), \text{cm}$	$T_e(X_k), \text{eV}$	$n_e(X_k), \text{cm}^{-3}$
100	49	3.1×10^{17}	.274	48	3.2×10^{17}
500	23	6.7×10^{17}	.406	15	1.0×10^{13}
1000	17	1.1×10^{18}	.495	10	1.9×10^{13}

$\omega_0 \ll \omega_{pe}$ (ω_0 is the CO_2 laser light frequency and ω_{pe} the plasma frequency) and thus anomalous absorption instabilities will not be important. However, laser light scattering instabilities, which occur in underdense plasmas, could result in the reflection of a significant fraction of the incident laser radiation. The theory²⁶ of such scattering processes indicates that the energy of the incident light wave is reflected by the plasma through the generation of a backward-traveling light wave and either a low frequency ion wave (stimulated Brillouin scattering instability) or an electron plasma wave (stimulated Raman scattering instability). In the laser heating of underdense magnetically confined plasmas of interest here, or laser-produced plasmas of higher density, a knowledge of the depth of laser penetration allowed by these instabilities and the amount of incident laser energy reflected (backscatter) is required.

Laser scattering by plasma instabilities is currently receiving a great deal of theoretical study.²⁶⁻³¹ Much work has been done on scattering instabilities in homogeneous plasmas²⁶ and more recent studies have treated inhomogeneous plasmas.²⁷ Thermal self-focusing instabilities (stimulated Rayleigh scattering) have also been theoretically identified.²⁸ However, experimental confirmation of the scattering instability mechanisms has not yet been established. With the moderate power levels ($\sim 3 \times 10^{10} \text{ W/cm}^2$) of the present experiment and the generally unresolved nature of the instability hierarchy, this work will concentrate upon identifying the power thresholds for excitation of instabilities and the backscatter scale lengths for Brillouin and Raman scattering instabilities, which are expected to be of primary importance.²⁹

A. Formulation of the Instability Problem

The set of coupled wave equations which describe the evolution of the scattering instabilities are obtained by combining the fluid equations for ions and electrons with Maxwell's equations. Non-linear theory includes the reaction of the back-scattered light wave and electrostatic plasma waves on the incident wave. It can be shown³⁰ that the solution of this set of equations is completely described in terms of three waves excited by the incident laser light. Two of the excited waves are

electromagnetic, the backscattered wave and a forward traveling nonresonant wave. The forward scattered "Stokes" wave, not being in resonance, can generally be neglected. The remaining excited wave is electrostatic, being either an ion wave (Brillouin scattering), or an electron plasma wave (Raman scattering).

When the combined fluid and Maxwell equations are Fourier analyzed in the spatial coordinate, the temporal evolution of the instabilities, their growth rates, and excitation power levels, can be obtained. The Fourier analyzed equations describe three coupled harmonic oscillators (the incident light wave, backscattered light wave, and an electrostatic wave) whose time behavior is different, depending on the magnitude of scattering instability growth rate, γ_0 , as compared to the frequency of the electrostatic wave, ω . The growth rate is found to be a function of the incident laser light power level.³⁰ For $\gamma_0 > \omega$, the "strong coupling" limit is obtained. In this regime the incident light wave determines the properties of the electrostatic wave, and the energy in the backscattered electromagnetic wave rapidly approaches that of the incident light wave. It will be noted that the introduction of nonlinear effects,³⁰ such as anomalous damping of the electrostatic wave, can significantly decrease the predicted amount of backscattered laser energy. The "weak coupling" limit, the general category of the present experiment, results when $\gamma_0 < \omega$. In this regime the incident light wave does not greatly affect the frequency of the electrostatic wave. The theory developed for weak coupling will be used in the following discussion.

When the combined fluid and Maxwell equations are Fourier analyzed in time, the spatial evolution of the laser-induced instabilities is obtained. From these resulting equations, the characteristic instability length scale, L_1 is determined.³⁰ L_1 describes the exponentiation distance of the backward scattered light wave obtained when damping effects on the electrostatic wave are neglected; it may also be thought of as the distance into the plasma at which total reflection of the incident laser light will occur. Computer simulations³⁰ indicate that the exclusion of damping effects is generally not valid. A dimensionless parameter, β ,

which describes the damping of the electrostatic ion or electron plasma waves, is thus introduced into the problem formulation. Then the factor βL_1 , which is dependent on the properties of the plasma and the incident laser power level, determines the penetration depth of the incident laser light. Strong damping of the electrostatic waves, $\beta \gg 1$, allows greater laser light penetration of the plasma than weak damping, $\beta \sim 1$. Thus for finite plasmas where the plasma length, ℓ , is $\ll 10\beta L_1$, no significant backscattering of the incident light is expected. However, if $\ell \sim 10\beta L_1$, backscatter instabilities will be important. $10\beta L_1$ can be shown³⁰ to be the penetration distance at which 50% of the incident laser light is reflected.

An additional length scale for laser light backscatter has been recently proposed by Liu, Rosenbluth, and White.²⁷ They consider the propagation of an electromagnetic "noise pulse" from a point far from the plasma boundary. As it propagates, the pulse grows at a rate, γ_0 , with a scale length, $c/2\gamma_0$, where c is the speed of light. If $10(c/2\gamma_0)$ is the smallest length involved in the laser-plasma interaction problem, then this effect dominates, and the scattering could be greatly enhanced. Although the theory dealing with the amplification and propagation of noise pulses is not well developed, and can be considered speculative at the present time, it is noted here for completeness.

B. Brillouin and Raman Scattering Instability

Thresholds

The Brillouin and Raman instability thresholds, growth rates, and scale lengths are dependent on the power level of the incident laser light and the temperature and density of the irradiated plasma. The power level dependence is written in terms of the electron velocity, v_0 , induced by the electric field of the incident light. For plane polarized light this velocity can be expressed as

$$v_0 = 25 \lambda_0 \sqrt{P} \text{ cm/s}, \quad (52)$$

where λ_0 is the laser wavelength in units of microns (10^{-4} cm) and P is the incident power density in units of W/cm^2 .

In the weak coupling regime, the Brillouin instability³⁰ occurs when $\omega_0 = \omega_s + \sqrt{c^2(k-k_0)^2 + \omega_{pe}^2}$ (obtained from balancing growth with damping); which

implies $k \approx 2k_0$, since $\omega_s \ll \omega_0$ and $\omega_{pe}^2 \ll c^2$. $(k-k_0)^2$. Here, $\omega_s = k \sqrt{T_e/m_i}$ is the ion wave frequency, k the wave number of the ion wave, k_0 the wave number of the incident light of frequency ω_0 , and ω_{pe} is the plasma frequency. The power threshold is then

$$\frac{v_0^2}{2} > 16 \frac{\gamma_p}{\omega_s} \frac{\gamma_-}{\omega_0} \frac{\omega_0^2}{\omega_{pe}^2}, \quad (53)$$

where $v_e = \sqrt{T_e/m_e}$ is the electron thermal velocity, γ_- is the collisional damping coefficient, $\gamma_- = v_{ei} \omega_{pe}^2 / 2\omega_0$ (v_{ei} is the electron-ion collision frequency), and γ_p is Landau damping rate. For a plasma where $T_e \approx T_i$, as in the present case, and $k\lambda_D < 1$ (λ_D is the Debye shielding distance), the ratio $\gamma_p/\omega_s \approx 0.43$.³² The instability growth rate is then given by,

$$\gamma_0 = \frac{k}{4} \frac{v_0 \omega_{pi}}{\sqrt{\omega_s \omega_0}} s^{-1}. \quad (54)$$

For Raman scattering,³⁰ the balancing of instability growth with damping occurs when $\omega_0 = \sqrt{\omega_{pe}^2 + 3k^2 v_e^2} + \sqrt{c^2(k-k_0)^2 + \omega_{pe}^2}$. Since $3k^2 v_e^2 \ll \omega_{pe}^2$, then

$$k = 2k_0 - k_0 \left(\frac{\omega_{pe}}{\omega_0} \right) \text{ cm}^{-1}, \quad (55)$$

where k in this case is the wave number of the electron plasma wave. The threshold power at this matching condition is then

$$\frac{v_0^2}{c^2} > 16 \frac{\gamma_p' \gamma_-}{\omega_k \omega_0} \frac{\omega_k^2 \omega_0 (\omega_0 - \omega_k)}{c^2 k^2 \omega_{pe}^2}, \quad (56)$$

where ω_k is the electrostatic wave frequency. With $k\lambda_D$ small, the Landau damping rate, γ_p , is replaced by $\gamma_p' = (\gamma_p + v_{ei})$ for the electron plasma wave,

$$\gamma_- = (v_{ei}/2) [\omega_{pe}^2 / (\omega_0 - \omega_k)^2] \text{ and } \omega_k^2 = \omega_{pe}^2 + 3k^2 v_e^2.$$

The instability growth rate is

$$\gamma_0 = -\frac{kv_0}{4} \frac{\omega_{pe}}{\sqrt{\omega_k (\omega_0 - \omega_k)}} s^{-1}. \quad (57)$$

From the above expressions, the threshold power levels required for excitation of the Brillouin and Raman scattering instabilities and the instability

growth rates, can be calculated for the previously determined plasma conditions presented in Table IV. The power levels and growth rates for instabilities are given in Table XII as a function of initial fill pressure.

The CO₂ laser beam power level in the Scylla I-C experiment will be approximately 3×10^{10} W/cm². At this power level, it can be seen from Table XII that the Brillouin scattering instability may be excited over the entire range of initial fill pressures, with the possible exception of the 1000 mTorr fill case. The Raman instability would not be expected to occur at fill pressures much above 100 mTorr.

C. Brillouin and Raman Scattering Instability

Length Scales

For both Brillouin and Raman instabilities the primary length scale L_1 , which is derived by neglecting damping effects, can be expressed as³⁰

$$L_1 = 4 \sqrt{\gamma} \frac{c}{v_o} \lambda_D \left(\frac{k}{k_o} \right)^{1/2} \text{ cm.} \quad (58)$$

For Brillouin scattering, $\gamma = 1$ and $k \approx 2k_o$. In the Raman case $\gamma = 3$ and $k \approx 2k_o$ as presented in Section VIII B.

Damping of the electrostatic wave, which is indicated by the dimensionless parameter β , is given for Brillouin scattering as

$$\beta = 4 \frac{\gamma_p}{\omega_s} k \lambda_D \frac{c}{v_o} \left(\frac{k}{k_o} \right)^{1/2} \quad (59)$$

and Raman scattering by

$$\beta = \frac{4}{\sqrt{3}} \frac{\gamma_p}{\omega_{pe}} \frac{1}{k \lambda_D} \frac{c}{v_o} \left(\frac{k}{k_o} \right)^{1/2}. \quad (60)$$

For the CO₂ laser beam with incident power level of 3×10^{10} W/cm², the laser light penetration distance, $10\beta L_1$, at which theory predicts 50% backscatter of the incident light, is given in Table XIII for both Brillouin and Raman scattering instabilities. The inverse bremsstrahlung absorption length, ℓ_{ab} , and the scale length proposed to indicate amplification, $10(c/2\gamma_o)$, are also presented.

For both Brillouin and Raman scattering the absorption length, $\ell_{ab} \lesssim 10\beta L_1$. For cases where $\ell_{ab} \ll 10\beta L_1$, instability scattering theory cannot be applied and backscatter may or may not be observed. For $\ell_{ab} \approx 10\beta L_1$ local plasma heating effects, which increase the required power level for instability, must be considered. In such a case, the local increase of electron temperature could shut off the instability and thus the observation of scattering is uncertain.

IX. LASER BEAM REFRACTION-CHANNELING AND SELF-FOCUSING

In the theoretical study of laser heating of confined plasmas, it has been proposed that gradients within the plasma would refract the laser beam towards regions of lower density³³ (higher index of refraction). If the incident laser light is directed along the cylindrical plasma column axis, as in the present case, then a density minimum on axis ($dn/dr > 0$) should result in channeling of the beam down the column axis, and possibly beam self-focusing. Conversely, the laser beam would be expected to refract away from the axis, and thus out of the plasma column, if the plasma density has an on-axis maximum. Experimentally, it has been established^{3,34} that channeling of the laser beam does result when the plasma has a density minimum on

TABLE XII

INSTABILITY POWER THRESHOLDS AND GROWTH RATES

P_o , mTorr	<u>Brillouin Instability</u>		<u>Raman Instability</u>	
	P , W/cm ²	γ_o , s ⁻¹	P , W/cm ²	γ_o , s ⁻¹
100	6.1×10^9	2.6×10^{10}	2.1×10^{10}	6.1×10^{10}
500	2.4×10^{10}	3.9×10^{10}	8.0×10^{12}	7.7×10^{10}
1000	4.0×10^{10}	8.5×10^{10}	1.2×10^{14}	7.8×10^{10}

TABLE XIII

INSTABILITY LENGTH SCALES

p_0 , mTorr	Brillouin Scattering			Raman Scattering	
	$10\delta L_1$, cm	$10 \frac{c}{2\gamma_0}$, cm	l_{ab} , cm	$10\delta L_1$, cm	$\frac{c}{2\gamma_0}$, cm
100	39	6	31	60	2.5
500	4.8	2.5	0.9	372	1.9
1000	1.0	1.3	0.2	1402	1.9

axis, and for low power density laser beams ($\sim 10^7$ W/cm²), refraction of the beam out of the plasma has been observed^{3,34} for the case of an on-axis density maximum. However, Hoffman³⁵ has found that a laser beam of sufficient power density ($\sim 10^{10}$ W/cm²) can create and maintain an on-axis density minimum (and thus channel) even if a density maximum is initially present on axis of the confined plasma column. It would thus appear that a laser power density "self-channeling" threshold exists, above which the laser beam will "drill" its own density minimum through the plasma regardless of the on-axis density conditions. It might be expected that such a threshold would coincide with the initiation of the "bleaching" phenomena discussed earlier. It should be noted that in Hoffman's experiment,^{5,35} the plasma column length was an order of magnitude smaller (10 cm) than the column produced in Scylla I-C (100 cm). Accordingly, the investigation of beam channeling in the present experiment, and the possible identification of the self-channeling threshold, is of significant interest.

Considering the experimental evidence above, it appears realistic, in the present experiment, to presume that the incident 3×10^{10} W/cm² laser beam will sufficiently heat the plasma to create and maintain a density minimum on axis over duration of the laser pulse. With this assumption, an estimate of the laser generated density gradient and the associated beam refraction and self-focusing effects can be made.

A. Laser-Generated Density Gradient

In order for the incident laser beam to channel down the plasma column axis, a density minimum must be created during the early stages ($t < t_d$) of

laser pulse. The maximum attainable local electron temperature of the 2-mm-diameter heated plasma core at the end of the pulse duration is $T_{el}(X)$, neglecting thermal heat conduction, electron-ion equipartition and plasma column expansion. Although electron-ion temperature equilibration can be neglected for times $t < t_d$, electron heat conduction is quite rapid ($t_{th}(R) \sim 1$ ns) and this process will determine the maximum electron temperature at any given time during the laser heating pulse. As an order of magnitude estimate, the transient electron temperature of the heated plasma core will be assumed to approach the temperature obtained as a result of thermal heat conduction, $T'_e(X)$, (Table VII).

Before laser heating, the plasma-magnetic field pressure balance can be written as,

$$n_f k(T_{ef} + T_{if}) = \frac{B_0^2}{8\pi}, \quad (61)$$

for $\mu \sim 1$. During the laser heating pulse, the electron temperature in this model is assumed to increase from T_{ef} to $T'_e(X)$, locally expanding the heated plasma to a mean density n'_e . The resulting pressure balance between the heated core and the surrounding plasma (or equivalently, the external field) can be expressed as

$$n'_e k[T'_e(X) + T_{if}] = \frac{B_0^2}{8\pi}. \quad (62)$$

Combining equations (61) and (62), the average density of the heated plasma is,

$$n'_e = \frac{n_f}{\left[\frac{T'_e(X)}{T_{ef}} + \frac{1}{2} \right]}. \quad (63)$$

TABLE XIV

LASER INDUCED DENSITY GRADIENTS

P_0 , mTorr	n'_e/n_f	dn_e/dr , cm^{-4}
100	0.74	9.6×10^{17}
500	0.47	5.8×10^{18}
1000	0.36	1.2×10^{19}

Since the laser heating is spread over the radius of the laser beam, R_c (~ 1 mm), an estimate of the density gradient induced by local heating can be taken as

$$\frac{dn}{dr} \sim \frac{\Delta n}{\Delta R} \approx \frac{n_f - n'_e}{R_c}. \quad (64)$$

Based on the parameters of the present experiment, with $T'_e(X) = T'_e(0)$, the numerical values shown on Table XIV are determined.

B. Channeling and Beam Focusing with $dn/dr > 0$.

The refraction of the laser beam within the confined plasma column can be determined from calculated light ray trajectories. The expression for the ray path, $r(X)$, at any axial position X within the plasma can be derived from the implicit ray equation³³

$$X = (1 - \zeta_0)^{1/2} \int_{r_0}^r \frac{dr}{(\zeta_0 - \zeta)^{1/2}}, \quad (65)$$

where $r_0 = r(X=0)$ defines the entry radius of a specific ray, $\zeta_0 = \zeta(r_0)$ and $\zeta(r) = n_e(r)/n_{ec}$ is the ratio of plasma density to the critical density (at n_{ec} the laser frequency equals the plasma frequency).

Assuming a linear increase in electron density with radius over the heated plasma core, and designating the minimum density on axis as n_m , the density profile can be written as,

$$\zeta(r) = \zeta_m + (d\zeta/dr)r, \quad r \lesssim R_c, \quad (66)$$

where $\zeta_m = n_m/n_{ec}$ is the value of ζ on axis ($r=0$), and $d\zeta/dr = (1/n_{ec})dn_e/dr$. From conservation of line density, it can be shown that

$$n_m = n' - \frac{2}{3} R_c \frac{dn_e}{dr}, \quad (67)$$

where n' is given by Eq. (63). When the expression for the linear density profile, $\zeta(r)$, is substituted

into Eq. (65) and the integration carried out, the ray trajectory equation,

$$r(X) = r_0 - \frac{1}{2(1 - \zeta_0)} \left(\frac{d\zeta}{dr} \right) X^2, \quad (68)$$

is obtained. This expression gives the radial location of any ray entering the plasma column at $0 < r_0 \lesssim R_c$ as a function of axial position, X .

Considering the ray entering the plasma at $r_0 \approx R_c$ where $\zeta_0 \approx \zeta(r_0) \approx n_f/n_{ec}$, the ray equation becomes

$$r(X) = R_c - \frac{1}{2(n_{ec} - n_f)} \left(\frac{dn_e}{dr} \right) X^2. \quad (69)$$

For CO_2 laser radiation $n_{ec} \approx 10^{19} \text{ cm}^{-3}$, significantly larger than the greatest density expected in the present experiment ($\lesssim 10^{18} \text{ cm}^{-3}$). Accordingly, with $dn_e/dr > 0$ converging diffraction of the incident laser light is indicated in all cases since $r(X) < R_c$ for all $X > 0$. Further, Eq. (69) predicts that for sufficiently large X , point-focusing [$r(X) \rightarrow 0$] of the channeling laser beam will occur. The axial position, ℓ_{fp} , at which all of the internal laser beam is focused to a point can be obtained by setting $r(X) = 0$ in Eq. (69) and solving for X ($= \ell_{fp}$). (It should be noted that total focusing of the laser beam will not occur since, in any real plasma, the density gradient (dn_e/dr) goes to zero as $r \rightarrow 0$). Using the values of dn_e/dr given in Table XIV, it can be seen that $\ell_{fp} \lesssim 2$ cm for all fill pressures. With $\ell_{fp} < \ell_{ab}$, in all cases, local "hot spots," beam filamentation and the necessarily inherent nonlinearities associated with intense heating of very small plasma volumes, might be expected to occur. Such phenomena could represent a serious problem for the present 100-cm-long plasma column heating experiment and for future fusion applications. These effects will be investigated in detail in the present experiment.

X. THETA-PINCH END EFFECTS

The results presented above neglect the effects of convective loss of plasma from the ends of the theta pinch, as well as axial plasma energy losses. Both of these effects could have a pronounced influence on the expected laser-plasma interaction through alteration of the local density and temperature profiles and related factors, such as the

ordering of characteristic times. Although the plasma end loss time ($\tau \sim L/v_1 \approx 10 \mu s$ at 100 eV) is much larger than the laser pulse time (~ 50 ns), the possibility of local "blow off" at the hot end of the plasma column is highly possible in the present experiment. This effect could significantly alter existing theories³⁶ for end loss involving area wave perturbations of the magnetic confinement equilibrium. Clearly, this problem requires investigation as it relates to the present experiment and future fusion applications.

XI. SUMMARY

The present analysis has attempted to provide a preliminary indication of the results that can be expected in the proposed Scylla I-C laser-plasma interaction experiment. Throughout the analysis, it has been assumed that the incident laser beam will create and maintain the appropriate on-axis density conditions required for channeling. If self-channeling is not obtained, simple modification of the Scylla I-C theta pinch can be carried out, which would result in the creation of favorable on-axis density conditions necessary for trapping the laser light. In either case, the basic analytical procedures used in this work can be applied to estimate the results of the laser-plasma interaction.

Using snowplow theory as the relevant high-density model with $\beta \approx 1$, Scylla I-C plasmas in the 10^{17} to 10^{18} range with corresponding temperatures in the 50-to 10-eV range, are predicted. (Somewhat lower plasma column densities might be observed in the actual experiment since β can be expected to be less than unity.) For this range of plasma densities and temperatures, nonlinear absorption of the CO_2 laser light is indicated. This bleaching process results in absorption lengths significantly greater than predicted for classical inverse bremsstrahlung absorption with constant absorption coefficients. Such an effect should be easily observed in the Scylla I-C experiment.

The plasma response to laser energy absorption has been treated in a step-wise fashion, considering interparticle energy transfer mechanisms, heat conduction, and bulk motion of the heated plasma. The analysis indicates that a laser induced plasma column temperature increase of about 10 eV will be

obtained after the transient energy transfer processes are completed and the heated plasma has reached a new equilibrium state. This temperature increment is nearly independent of the initial θ -pinch fill pressure. Significantly higher temperature increases, on the order of 100 eV, can be expected during the time of the heating laser pulse (~ 50 ns).

Analysis of the power density thresholds for the laser light scattering instabilities shows that the Brillouin instability threshold is exceeded over most of the Scylla I-C operating range. Accordingly, strong backscatter of the incident laser light could be expected, based on the calculated threshold levels. However, since the characteristic backscatter length scale $10\beta L$ is generally $\sim \lambda_{ab}$, the classical inverse bremsstrahlung absorption length, the observation of instability induced backscatter is uncertain.

The creation of an on-axis density minimum has been analyzed, assuming that local laser heating of the central core of irradiated plasma results in a local plasma expansion that is rapid on the time scale of the laser pulse. The diffraction of the laser beam within this laser-induced density gradient results in channeling of the incident laser light. Analysis of the laser light ray trajectories indicates that point-focusing of the channeling laser beam will occur, even in the presence of a very small, favorable density gradient ($dn/dr > 0$). Such beam focusing could result in local "hot spots" and beam filamentation, which would represent a serious problem in attempting to uniformly heat the 100-cm-long theta-pinch plasma column.

REFERENCES

1. A. Hertzberg, "Review of Controlled Fusion Research Using Laser Heating," AIAA Paper No. 73-258, January 1973.
2. I. C. Steinhauer and P. H. Rose, "Some Reactor Concepts Based on the Laser-Heated Solenoid," MSNW Report No. 74-1903-1, April 1974.
3. N. A. Amherd and G. C. Vlases, " CO_2 Laser Heating of a Small θ -Pinch," Proc. of 2nd APS Topical Conf. on Pulsed High- β Plasmas, Paper G4, July 1972.
4. R. Decoste et al., "Transverse Heating of a Cold Dense Helium Plasma by a Pulsed CO_2 Laser Beam," J. Appl. Phys. 45, 1127 (1974).

5. A. L. Hoffman, "Strong Axial Laser Heating of a O -Pinch Plasma," *Appl. Phys. Lett.* 23, 693 (1974).
6. G. M. Molen, "CO₂ Laser Refraction and Heating Effects on a Magnetized Plasma Column," *Tech. Report No. 3*, Texas Tech. Univ., Dec. 1973.
7. J. F. Drake et al., "Parametric Instabilities of Electromagnetic Waves in Plasmas," *Phys. Fluids* 17, 778 (1974).
8. W. R. Ellis and G. A. Sawyer, "Scaling Laws for the Linear Theta Pinch, I," LA-5434-MS, Oct. 1973.
9. G. T. Schappert, "Some Aspects of Heating with Laser Pulses," personal communication.
10. K. Boyer et al., "Laser Heating of Magnetic Plasmas," personal communication.
11. W. H. Reichelt et al., "Gigawatt Pulses From a 3 Atmosphere CO₂ Electron-Beam-Controlled Laser," personal communication.
12. K. F. McKenna, R. Kristal, K. S. Thomas, "Measurements of Plasma Density Distribution and Current-Sheath Structure on the Implosion Phase of a Theta-Pinch Discharge," *Phys. Rev. Lett.* 32, 410 (1974).
13. F. L. Ribe, Ed., "Proposed Experiments on Heating, Staging, and Stabilization of Theta Pinches," LA-5026-P (Revised), Feb. 1973.
14. J. Benford, "Rotation During the Implosion of a Theta Pinch," *Phys. Fluids* 15, 435 (1972).
15. J. E. Hammel et al., "Implosion Heating Experiment," LA-5656-PR (1974).
16. K. F. McKenna et al., *CTR-3 Quarterly Report*, July-Sept. 1974.
17. L. Spitzer, *Physics of Fully Ionized Gases*, 2nd Ed., Interscience Pub., New York (1962).
18. S. C. Burnett et al., "A Reference Theta-Pinch Reactor (RTPR)," LA-5121-MS, Dec. 1972.
19. T. W. Johnston and J. M. Dawson, "Correct Values for High-Frequency Power Absorption by Inverse Bremsstrahlung in Plasmas," *Phys. Fluids* 16, 722 (1973).
20. M. Kristiansen, Texas Tech, Univ., Lubbock, Texas, personal communication.
21. L. C. Steinhauer and H. G. Ahlstrom, "One-Dimensional Laser Heating of a Stationary Plasma," *Phys. Fluids* 14, 81 (1971).
22. L. C. Steinhauer and H. G. Ahlstrom, "Bleaching Wave Phenomena," *AIAA Journal* 10, 429 (1972).
23. G. T. Schappert, "Some Aspects of Laser Heating with Laser Pulses," personal communication.
24. K. Buchl et al. in *Laser Interaction and Related Plasma Phenomena*, Plenum Press, New York, 1971, H. J. Schwarz and H. Hora, Eds.
25. E. Fabre and C. Stenz, "CO₂ Laser-Beam Absorption by a Dense Plasma," *Phys. Rev. Lett.* 32, 823 (1974).
26. D. W. Forslund, J. M. Kindel, and E. L. Lindman, "Nonlinear Behavior of Stimulated Brillouin and Raman Scattering in Laser-Irradiated Plasmas," *Phys. Rev. Lett.* 30, 739 (1973).
27. C. S. Liu, M. N. Rosenbluth, and R. B. White, "Parameter Scattering Instabilities in Inhomogeneous Plasmas," *Phys. Rev. Lett.* 31, 697 (1973).
28. F. W. Perkins and E. J. Valeo, "Thermal Self-Focusing of EM Waves in Plasmas," *Phys. Rev. Lett.* 32, 1234 (1974).
29. J. M. Kindel, "Stimulated Backscatter in Laser-Heated Pinches," *Bull. Am. Phys. Soc.* 18, 1294 (1973).
30. D. W. Forslund, J. M. Kindel, and E. L. Lindman, "The Theory of Stimulated Scattering Processes in Laser Irradiated Plasmas, I," submitted to *Phys. of Fluids*.
31. D. W. Forslund, J. M. Kindel, and E. L. Lindman, "Stimulated Scattering Processes in Laser Irradiated Plasmas, II," submitted to *Phys. of Fluids*.
32. B. D. Fried and F. W. Gould, "Longitudinal Ion Oscillations in a Hot Plasma," *Phys. Fluids* 4, 139 (1961).
33. L. C. Steinhauer and H. G. Ahlstrom, "Propagation of Coherent Radiation in a Cylindrical Plasma Column," *Phys. Fluids* 14, 1109 (1971).
34. G. M. Molen M. Kristiansen, and M. O. Hagler, "CO₂ Laser Beam Refraction in a Linear Discharge Plasma," *Appl. Phys. Lett.* 23, 601 (1973).
35. A. L. Hoffman, Mathematical Sciences Northwest, Inc., Seattle, Wash., personal communication.
36. J. P. Freidberg and H. Weitzner, "Endloss from a Linear O -Pinch," submitted to *Nuclear Fusion*.