GRAPHIC DISPLAY
OF THREE-DIMENSIONAL SURFACES
USER'S MANUAL

Battelle

Pacific Northwest Laboratories
Richland, Washington 99352

1972

Prepared for the U.S. Atomic Energy
Commission under Contract AT(45-1):1830

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PACIFIC NORTHWEST LABORATORY
operated by
BATTELLE
for the
U.S. ATOMIC ENERGY COMMISSION
Under Contract AT(45-1)-1830

Printed in the United States of America
Available from
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy \$5.45; Microfiche \$1.45

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by

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Water and Land Resources Department

1972

This report is based on work sponsored by the
Atlantic Richfield Hanford Company under United
States Atomic Energy Commission Contract AT(45-1)-2130

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SUMMARY

This document describes a program written to output three-dimensional surfaces onto a cathode ray tube (CRT) display. Through the use of orthogonal transformations surfaces can be rotated to provide more complete visualization of the physical characteristics of data being analyzed.

The form of the data to be displayed may be a matrix of values representing a surface or a series of lines representing the edges of a three-dimensional object. Minor additions to the program allow displays of surfaces described by a mathematical function.

CONTENTS

	<u>Page</u>
SUMMARY.	iii
FIGURES.	iv
INTRODUCTION	1
PROGRAM OPERATION.	2
LINE MODE.	2
ARRAY MODE	3
REFERENCES	7
ACKNOWLEDGMENTS.	7
APPENDIX A - MATHEMATICAL CONCEPT.	A-1
APPENDIX B - PROGRAM LOAD MAP.	B-1
APPENDIX C - DISPLAY KEYBOARD.	C-1
APPENDIX D - LINE DATA	D-1
APPENDIX E - ARRAY DATA.	E-1

FIGURES

Figure

1 Line Data Stereo Pair	3
2 Array Data Stereo Pair	4
3 Southeast View of Hanford Reservation Groundwater Surface	4
4 Northwest View of Hanford Reservation Groundwater Surface	5
A-1 One Rotation of Axis.	A-1
A-2 Two Rotations of Axis	A-2
A-3 Top View of System Showing Parameters	A-6
A-4 Side View of System Showing Parameters.	A-8

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INTRODUCTION

The major objective of the Radionuclides in Soils Program conducted by Battelle-Northwest for the Atlantic Richfield Hanford Company has been the development of a groundwater management system for the Hanford Reservation. The primary segments of this management system are mathematical models which can predict transient groundwater and contaminant movement.

Secondary segments perform various related tasks, such as input, editing, and visual display of data relating to the groundwater system. This document describes a program designed to visually output this data to produce a graphic three-dimensional effect.

PERSPC is a machine language program written for the PDP-9 digital computer with a UNIVAC 1558 display. This program searches data, each point of which has three components representing three dimensions. A mathematical transformation is performed for each of these points. The results are converted to display commands and output to the CRT, which yields a visual display. Use of all points results in a display with the appearance of a three-dimensional surface. If the figure pairs in the following pages of this report are viewed as stereo pairs, this three-dimensional effect can be visualized. Using this program, three-dimensional displays of Hanford groundwater data (Figures 3A-B and 4A-B) have been produced which may be helpful in understanding the surface configurations.

This report also describes the mathematical concepts (Appendix A) and operation of the PERSPC program. Appendices B, C, D, and E describe loading procedures, keyboard operation and types of data used.

PROGRAM OPERATION

At present the data and the program are loaded at the same time. Setup procedures for the data are described later in this section.

The program PERSPC BIN is transferred to disc unit 2 from the DEC tape labeled PERSPECTIVE. The data is also transferred to disc unit 2 under the name DATA BIN. Both of these must have previously been assembled using the DOS (disc operating system) system program HYMAC. The program is then loaded from disc unit 2. Appendix B shows the load map after loading with DDT.

Commands to program operation are initiated with the CRT keyboard.

At program initiation the message PERSPECTIVE ROTATION appears at the top of the display screen.

When the key labeled SOM is depressed the data will be displayed as in either Figure 1 or Figure 3, depending upon whether it is in array or line mode. The displayed figure can be rotated using the up, down, left or right keys. These keys add or subtract a small amount from the two angles of rotation and the program recalculates the transformation for display. To obtain the view as soon by the other eye the AT key (@) is depressed and the alternate view is displayed. Appendix C illustrates the display keyboard, showing these keys and their functions.

The program can utilize two different forms of data input, lines or arrays.

LINE MODE

Appendix D contains a sample of line data input in which five lines are used; there are 5, 6, 2, 2, and 2 points, respectively, in the lines. The line tagged COORD begins the coordinate data. Each data point is a set of three coordinates (X1, X2, X3). Each point in each line is written in consecutive order. Figure 1 is the stereo pair resulting from this line data.

A

B

FIGURE 1. Line Data Stereo Pair

ARRAY MODE

Appendix D is a sample of data input for a dimensional array 6 by 6. The line tagged ARRAY is the start of the data, which is essentially the X2 coordinate position. This data is loaded two values per word to conserve storage. The storage method for an even number per line in a 2 x 2 matrix is:

```
X2(1,1),X2(1,2)
X2(2,1),X2(2,1)
```

An odd number (3 x 3) matrix looks like this:

```
X2(1,1),X2(1,2)
X2(1,3),X2(2,1)
X2(2,2),X2(2,3)
X2(3,1),X2(3,2)
X2(3,3), 0
```

Figure 2 is the stereo pair resulting from the array data from Appendix E.

A

B

FIGURE 2. Array Data Stereo Pair

A

B

FIGURE 3. Southeast View of Hanford Reservation Groundwater Surface

A

B

FIGURE 4. Northwest View of Hanford Reservation
Groundwater Surface

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ACKNOWLEDGMENTS

This research was sponsored by the Advanced Technology Development Section, Research and Development Department, Atlantic Richfield Hanford Company. The authors are grateful for the assistance and direction provided by Dr. M. D. Veatch, Staff Hydrologist; D. J. Brown, Staff Geologist; and R. E. Isaacson, Manager of the Advanced Technology Development Section of Atlantic Richfield Hanford Company.

The efforts of Leila Counts, who provided editorial assistance, and Jan Greenwell, Loretta Howard and Velva Harris, who typed the many drafts and final reports, are also greatly appreciated.

APPENDIX A

MATHEMATICAL CONCEPT

APPENDIX A

MATHEMATICAL CONCEPT

This section describes the mathematics involved in the transformation of axes required for viewing an object from different angles.

Figures A-1 and A-2 show the rotation from one coordinate system to another. The rotation of coordinates as used here is defined from matrix theory.

FIGURE A-1. One Rotation of Axis

FIGURE A-2. Two Rotations of Axis

Let $X=AY$ represent an orthogonal transformation of coordinates in which the columns of matrix A are given by the direction cosines of the new coordinate system with respect to the old coordinate system. The unit vectors of the y -coordinate system are $\{\cos\theta, 0, -\sin\theta\}$, $\{0, 1, 0\}$ and $\{\sin\theta, c, \cos\theta\}$ in the X coordinate system. Placing these vectors as the columns of matrix A results in:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

where

$$A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

From matrix theory the equation $X=AY$ may be rewritten as $Y=A^T X$, where

$$A^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

therefore

$$Y=A^T X = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \cdot X \quad (1)$$

In Equation 1 the X coordinate system is rotated at an angle of θ in the (X_1, X_3) plane to a new coordinate system defined by Y(y_1, y_2, y_3). A new coordinate system Z is now defined, which is a rotation of the Y coordinate system by an angle of ϕ in the (Y_2, Y_3) plane. This is accomplished in the same way as the preceding transformation and the rotation is shown in Figure 2.

The new unit vectors of the Z coordinate system are the unit vectors $\{1, 0, 0\}$, $\{0, \cos\phi, -\sin\phi\}$ and $\{0, \sin\phi, \cos\phi\}$ in the Y coordinate system. The transformation will be represented by:

$$Y = B \cdot Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \cdot Z$$

Now:

$$Z = B^T Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \cdot Y \quad (2)$$

Z is defined in terms of X using equations 1 and 2

$$Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \cdot X$$

using matrix multiplication

$$Z = UX = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ -\sin\theta\sin\phi & \cos\phi & -\cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\phi & \cos\theta\cos\phi \end{bmatrix} \cdot X \quad (2.1)$$

which is the final form of the equation.

This is a true orthogonal transformation, as will be proved. From matrix theory² a real matrix U is orthogonal if and only if the transpose of the matrix is equal to the inverse of the matrix or $U^T = U^{-1}$. By transposing rows and columns of matrix U the transpose of U is obtained:

$$U^T = \begin{bmatrix} \cos\theta & -\sin\theta\sin\phi & \sin\theta\cos\phi \\ 0 & \cos\phi & \sin\phi \\ -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (3)$$

the inverse of U satisfies

$$U \cdot U^{-1} = I \quad (4)$$

I is the unit matrix of order 3 represented by

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (5)$$

let

$$U^{-1} = A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad (6)$$

Using Equations 5 and 6 in Equation 4 gives:

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ -\sin\theta\sin\phi & \cos\phi & -\cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\phi & \cos\theta\cos\phi \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Using matrix multiplication and solving the resulting simultaneous equations for the a_{ij} components gives:

$$U^{-1} = A = \begin{bmatrix} \cos\theta & -\sin\theta\cos\phi & \sin\theta\cos\phi \\ 0 & \cos\phi & \sin\phi \\ -\sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (8)$$

Equation 8 is equal to Equation 3; therefore the matrix U is orthogonal, and the transformation equation is

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ -\sin\theta\sin\phi & \cos\phi & -\cos\theta\sin\phi \\ \sin\theta\cos\phi & \sin\phi & \cos\theta\cos\phi \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (9)$$

where x_1, x_2, x_3 are the vectors in the original X coordinate system and z_1, z_2, z_3 are the corresponding vectors in the Z coordinate system with a rotation of θ and ϕ .

To provide stereo views, two projection planes represent the object, each as seen by one eye of the viewer. Two methods may be used to accomplish this: one allows the viewer to remain a constant distance from the centroid of the object and the other allows the viewer to remain a constant distance from the coordinate axis. Due to the shorter calculation times involved the second method was selected for use here. With this method the object increases or decreases in size depending upon the view angle.

Figure A-3 shows an object in the z_1, z_3 plane, as viewed along the z_3 line.

FIGURE A-3. Top View of System Showing Parameters

The variables shown in Figures A-3 and A-4 are defined as

$z_3 \text{ max}$ = maximum z_3 value of object

SL = distance from origin of system to observer

(z_1, z_2, z_3) = any point in the z coordinate system

S_1 = z_1 distance of point (z_1, z_2, z_3) on plane 1 as seen from left eye

S_2 = z_1 distance on plane 2 as seen from right eye

S_3 = z_2 distance of point on plane 1 and 2

ID = distance between eyes

D = distance from observer to planes 1 and 2

Allowing the distance SL to remain constant assures that the viewer is always positioned on a sphere of constant radius about the coordinate axis. SL is defined as the maximum value in the X coordinate system times some constant K; i.e., $SL = K(x_1, x_2, x_3)_{\text{Max}}$. S_1 and S_2 are defined in the following manner:

$$\frac{S_1}{D} = \frac{(ID/2 - S_1 + z_1)}{SL - D - z_3}$$

$$S_1 = \frac{D * (ID/2 + z_1)}{SL - z_3} \quad (10)$$

$$\frac{-S_2}{D} = \frac{(ID/2 - z_1 + S_2)}{SL - D - z_3}$$

$$S_2 = \frac{-D * (ID/2 - z_1)}{SL - z_3} \quad (11)$$

S_3 may be defined as the distance to the projection planes in the z_2 direction from the z_1, z_3 plane.

FIGURE A-4. Side View of System Showing Parameters

$$\frac{S_3}{D} = \frac{z_2}{SL - z_3} \quad (12)$$

$$S_3 = \frac{D * z_2}{SL - z_3}$$

In the z_1, z_3 plane the projection planes are adjusted to the right using the minimum z_1 value.

$$S_{1 \text{ Min}} = \frac{(ID/2 + z_{1 \text{ Min}}) * D}{SL - z_3 z_1} \quad (13)$$

$$S_{2 \text{ Min}} + \frac{-(ID/2 + z_{1 \text{ Min}}) * D}{SL - z_3 z_1} \quad (14)$$

where $z_3 z_1$ is the z_3 value for $z_1 \text{ Min}$.

Also in the z_2, z_3 plane the projection planes are adjusted upward using the minimum z_2 value.

$$S_3^{\text{Min}} = \frac{z_2^{\text{Min}} * D}{SL - z_3 z_2} \quad (15)$$

where $z_3 z_2$ is the z_3 value for z_2^{Min} .

Subtracting Equations 13, 14 and 15 from 10, 11 and 12, respectively, results in:

$$S_1 = \frac{(ID/2 + z_1) * D}{SL - z_3} - S_1^{\text{Min}} \quad (16)$$

$$S_2 = \frac{(ID/2 - z_1) * D}{SL - z_3} - S_2^{\text{Min}} \quad (17)$$

$$S_3 = \frac{z_2 * D}{SL - z_3} - S_3^{\text{Min}} \quad (18)$$

When this is done the projections are more closely oriented to the center of the display, preventing display overflow.

Redefining Equations 16, 17 and 18 using the orthogonal transformation of Equation 9 results in:

$$S_1 = \frac{(ID/2 + (x_1 \cos \theta - x_3 \sin \theta)) * (SL - z_3^{\text{Max}})}{(SL - (x_1 \sin \theta \cos \phi + x_2 \sin \phi + x_3 \cos \theta \cos \phi))} - S_1^{\text{Min}} \quad (19)$$

$$S_2 = \frac{-(ID/2 - (x_1 \cos \theta - x_3 \sin \theta)) * (SL - z_3^{\text{Max}})}{(SL - (x_1 \sin \theta \cos \phi + x_2 \sin \phi + x_3 \cos \theta \cos \phi))} - S_2^{\text{Min}} \quad (20)$$

$$S_3 = \frac{(-x_1 \sin \theta \sin \phi + x_2 \cos \phi - x_3 \cos \theta \sin \phi) * (SL - z_3^{\text{Max}})}{(SL - (x_1 \sin \theta \cos \phi + x_2 \sin \phi + x_3 \cos \theta \cos \phi))} - S_3^{\text{Min}} \quad (21)$$

which is the final form of the equation.

All points in the X coordinate system are transformed in this manner and projected onto planes 1 and 2 (shown in Figures A-3 and A-4).

APPENDIX B

PROGRAM LOAD MAP

APPENDIX B

PROGRAM LOAD MAP¹

\$ DDT

LOADER V4A
>PERSPC, DATA
PERSPC 33320
DATA 33267
HMP2. 33242
HSQR. 33223
HSQRT. 33071

DDT V4A

>↑Q
DKAD 15000

DDT V4A

>↑C

\$ ENTER 15000 2 PERSPC DMP
\$ G 2 PERSPC

DDT V4A

>•

APPENDIX C

DISPLAY KEYBOARD

DISPLAY KEYBOARD

APPENDIX D

LINE DATA

APPENDIX D

LINE DATA

.TITLE LINE DATA FOR PERSPC
.GLOBL DATA,TYPE,SCALE,INC,MAXZ,MAXX

DATA	COORD	/STARTING ADRS FOR COORDINATES OF POINTS
TYPE	1	/ ALWAYS 1 FOR LINE TYPE CALC.
SCALE	0	/ USED TO ADJUST VALUES TO MAX. OF 500.
INC	0	/ NOT USED IN LINE MODE.
MAXZ	5	/ NO. OF LINES
MAXX	.+1	/ STARTING ADDRESS FOR THIS DATA
	5	/ NO. OF PTS IN EA. LINE

	6	
	2	
	2	
	2	
	.DEC	
COORD	0	
	0	
	0	
	500	
	0	400
	0	0
	500	0
	400	0
	0	
	0	500
	400	0
	0	0
	0	500
	0	0
	0	400
	0	500
	0	400
	0	0
	400	500
	500	400
	0	400
	400	0
	500	400
	400	0
	400	0
	0	400
	400	400
	400	.OCT
	0	.END
	0	

APPENDIX E

ARRAY DATA

APPENDIX E

ARRAY DATA¹

.TITLE ARRAY DATA FOR PERSPC
.GLOBL DATA,TYPE,SCALE,INC,MAXZ,MAXX

DATA	ARRAY	/ STARTING ADDRESS FOR ARRAY DATA
TYPE	0	/ ALWAYS 0 FOR ARRAY TYPE DATA
SCALE	0	/ USED TO ADJUST SCALE
INC	144	/ INCREMENTAL STEP SIZE BETWEEN NODES
MAXZ	6	/ NODAL DIMENSION IN X3 DIRECTION
MAXX	.+1	
	6	/ NODAL DIMENSION IN X1 DIRECTION
ARRAY	000000	/ BEGINNING OF X2 DATA.
	000000	/ 2 VALUES PER LINE
	000000	
	000144	
	144144	
	144000	
	000144	
	310310	
	144000	
	000144	
	310310	
	144000	
	000144	
	144144	
	144000	
	000000	
	000000	
	000000	
	.END	

>