RN-TM-0572

CLASSIFICATION CATEGORY	
UNCLASSIFIED	
<i>W.P. Gilles</i>	9-10-68
CLASSIFYING OFFICER	DATE

10 September 1968

NOTICE
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

AMENDED CALIBRATION DATA
FOR XE-2 GAMMA HEATING RATE SENSORS
(TE882 A, TE884 A, TE885 A, AND TE886 A)

Author:

R. G. Hoff
R. G. Hoff
Development Group
Instrumentation Section
Engineering Department

MASTER

Approved:

DISTRIBUTION OF THIS DOCUMENT UNLIMITED

W.P. Gilles
W. P. Gilles, Manager
Instrumentation Section
Engineering Department

I. L. Odgers
I. L. Odgers, Manager
Engineering Department
Nuclear Rocket Operations

J. J. Peterson
J. J. Peterson
NRO Technical Staff

FILE COPY
NRO

Technical Document Center

TECHNICAL DOCUMENT CENTER
Nuclear Rocket Operations

DOC. NO.

N-1591

AGCS 0120-48

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

The calibration data for the gamma heating rate sensors (AGC 1134280) mounted on the XE-2 pressure vessel forward flange were based on the assumption that the thermal conductivity of constantan is constant with respect to temperature. This assumption has since been found to be incorrect, and the following amended calibration equation corrects the variation with temperature.

$$\gamma = .001028 (T_1 - T_2) (1 + .000615 [T_1 + T_2])$$

where γ is gamma heating rate in watts/gram

T_1 is ball temperature in degree Rankine (TE882 A or TE884 A)

T_2 is base temperature in degree Rankine (TE885 A or TE886 A)

Gamma sensor data reduced with the amended equation will provide gamma heating rates within $\pm 10\%$ accuracy.

BACKGROUND INFORMATION

Figure 1 is a sketch of the gamma heating rate sensor. Essentially, the sensor consists of a polished solid copper ball of known mass, a tubular constantan thermal resistance, a copper base plate, two copper-constantan thermocouples, and a protective steel cover. The two thermocouples are connected so that one measures the base plate temperature and the other passes up through the thermal resistance tube to measure the ball temperature. The volume enclosed within the protective cover is evacuated to eliminate gas convective heat losses.

Operation proceeds as follows: Gamma absorption at a rate γ (watts/gram) to be determined in the copper ball of mass W (grams) develops heat H (watts) at a rate of,

$$(1) \quad H = \gamma \text{ (watts/gram)} \times W \text{ (grams)}$$

At steady state, flow of this heat from the ball across the thermal resistance R ($^{\circ}\text{R/watt}$) to the plate develops a temperature difference $T_B - T_P$ ($^{\circ}\text{R}$) between the ball and the plate of,

$$(2) \quad T_B - T_P = H \text{ (watts)} \times R \text{ (}^{\circ}\text{R/watt)}$$

By eliminating H between equations (1) and (2) and solving for γ ,

$$(3) \quad \gamma = \frac{T_B - T_P}{RW} \text{ (watts/gram)}$$

The product " RW " was first calculated using the physical dimensions of the components and published properties for the materials used. At that time, the only available information for constantan was for 20°C . This calculated " RW ", using the 20°C information for constantan, was then cross-checked against an experimentally determined " RW " derived from the time required by the sensor to respond to a step change in ambient temperature.

FIG. 1 GAMMA HEATING RATE SENSOR
(AGC 1134280)

This experiment consisted of immersing each sensor in boiling distilled water until uniformly heated, quickly plunging the sensor into a distilled water-ice mixture, and then measuring the time in seconds for the thermocouple in the ball to achieve 63.2% of its total voltage change. Because of nonlinearities in the copper/constantan voltage curve, this particular 63.2% change corresponds to a temperature of 563°R. The product "RW" is obtained from this information in the following way:

Using Newton's Law of Cooling,

$$(4) \quad c W \frac{dT}{dt} = - \frac{T - T_P}{R}$$

where "c" is the specific heat of copper (0.214 watt-sec/gm°R)

T is the variable ball temperature

T_P is the base plate temperature (492°R)

t is time in seconds

Separating variables and integrating both sides,

$$(5) \quad \int_{T_S}^{T_E} \frac{dT}{T - T_P} = - \int_0^t \frac{dt}{RcW}$$

where T_E is the 63.2% point (563°R)

T_S is the boiling point (672°R)

$$(6) \quad \ln (T - T_P) \Big|_{T_S}^{T_E} = \frac{-t}{RcW} \Big|_0^t$$

Substituting limits and simplifying,

$$(7) \quad \ln \frac{T_S - T_P}{T_E - T_P} = \frac{t}{RcW}$$

The product "RW" is thus,

$$(8) \quad RW = \frac{t}{c \ln \frac{T_S - T_P}{T_E - T_P}} \quad \frac{^{\circ}\text{R gm}}{\text{watt}}$$

Knowing t , c , T_E , T_P , and T_S , the product "RW" can be calculated by means independent of the physical dimensions and of the physical properties except for "c" which is well known. Of the two gamma heating rate sensors installed on XE-2, SN 000001 measured 116.0 seconds and SN 000004 measured 113.8 seconds. Substituting these figures in (8) gave "RW" products of 584 and 572 compared to 520 calculated from drawing dimensions and published material properties.

This difference in results between the two methods was within range of conceivable variations resulting from braze fillets, incomplete bonding, etc. Of the two methods, the time approach was considered the more reliable so was used to determine the "RW" term in the data reduction equation (3). Since this original calibration, inquiries were made to learn more about the thermal conductivity of constantan at elevated temperatures.

VARIABLE THERMAL CONDUCTIVITY

Information received from the Driver-Harris Company, manufacturers of the constantan used, indicates that constantan has a large change in thermal conductivity in the temperature range of interest. Table 1 is a listing of this information as received in degrees Centigrade and converted to degrees Rankine.

Figure 2 is a plot of the Rankine data. The curve is linear, hence has a general formula,

TABLE 1

THERMAL CONDUCTIVITY OF CONSTANTAN VERSUS TEMPERATURE

<u>Temperature</u> °C	<u>Conductivity</u> watts/cm °C	<u>Temperature</u> °R	<u>Conductivity</u> watts/cm °R
50	.194	592	.107
100	.212	672	.117
200	.251	852	.138
300	.289	1032	.159
400	.328	1212	.180
500	.367	1392	.202
600	.405	1572	.223
700	.463	1752	.255
800	.481	1932	.264
900	.520	2012	.286

FIG. 2 THERMAL CONDUCTIVITY OF CONSTANTAN

$$(9) \quad k = K (1 + \alpha T)$$

where k is thermal conductivity (watts/cm $^{\circ}\text{R}$)

T is temperature ($^{\circ}\text{R}$)

K and α are constants to be determined

K and α were evaluated by substituting into (9) the thermal conductivity and temperature information at two points, for example, 672°R and 1572°R , and then solving the resultant equations simultaneously for K and α . The solutions were found to be,

$$(10) \quad K = .0377 \text{ watts/cm } ^{\circ}\text{R}$$

$$(11) \quad \alpha = .00312/^{\circ}\text{R}$$

Substituting (10) and (11) in (9), the formula for the thermal conductivity of constantan becomes,

$$(12) \quad k = .0377 (1 + .00312 T)$$

Calculation of the ball temperature with the thermal conductivity of the constantan tube varying with temperature and shunted by the copper lead of the ball thermocouple proceeds as follows:

$$(13) \quad \Delta T = \frac{H \Delta x}{kA} = \frac{H \Delta x}{k_1 A_1 + A_2 K (1 + \alpha T)}$$

where k_1 is thermal conductivity of copper (2.17 watts/cm °R)

A_1 is cross-sectional area of copper thermocouple lead

$$\pi (.012" \times 2.54 \text{ cm/in})^2 = .00291 \text{ cm}^2$$

A_2 is cross-section of constantan tube and thermocouple lead

$$\pi (.302 \text{ cm})^2 - \pi (.240 \text{ cm})^2 + .00291 = 0.109 \text{ cm}^2$$

$$(14) \quad \Delta T = \frac{H \Delta x}{M (1 + NT)}$$

where $M = k_1 A_1 + A_2 K$

$$N = \frac{A_2 K \alpha}{k_1 A_1 + A_2 K}$$

Taking Δx sufficiently small, $\Delta T \rightarrow dT$ as $\Delta x \rightarrow dx$, hence,

$$(15) \quad dT = \frac{H dx}{M (1 + NT)}$$

Separating variables and integrating both sides,

$$(16) \quad \int_{T_P}^{T_B} (1 + NT) dT = \int_0^L \frac{H}{M} dx$$

$$(17) \quad \left[T + \frac{1}{2} NT^2 \right]_{T_P}^{T_B} = \left[\frac{Hx}{M} \right]_0^L$$

$$(18) \quad T_B + \frac{1}{2} NT_B^2 - T_P - \frac{1}{2} NT_P^2 = \frac{HL}{M}$$

Solving for H and simplifying,

$$(19) \quad H = \frac{M}{L} (T_B - T_P) \left(1 + \frac{1}{2} N [T_B + T_P] \right)$$

Substituting for H from equation (1) and solving for γ ,

$$(20) \quad \gamma = \frac{M}{WL} (T_B - T_P) (1 + \frac{1}{2} N (T_B + T_P))$$

The constants of equation (20) have the following values:

$$M = k_1 A_1 + A_2 K = 2.17 \times .00291 + .109 \times .0377 = .01042$$

$$N = \frac{A_2 K \alpha}{M} = \frac{.109 (.0377) (.00312)}{.01042} = .001230$$

$$W = 4/3 \pi R^3 \rho = 4/3 \pi (.648 \text{ cm})^3 (8.94 \text{ gm/cm}^3) = 10.14 \text{ gms}$$

$$L = 1.00 \text{ cm}$$

Substituting these values for the constants in equation (20), the calibration equation becomes,

$$(21) \quad \gamma = .001028 (T_B - T_P) (1 + .000615 (T_B + T_P))$$

CHECK

The constants M/WL and N appearing in equation (20) can be derived, as will later be evident, from time measurements made for two different percentages of the total change. Having measured the time at only one percentage change, the best check that can be made on the installed XE-2 sensors is to derive the equation for the time response, calculate the time based on dimensional and available information for M , W , L , and N , and then compare this calculated time with the measured time. This approach proceeds as follows:

Starting again with Newton's Law of Cooling,

$$(22) \quad cW \frac{dT}{dt} = - \frac{T - T_P}{R}$$

Substituting for R , using equation (14),

$$(23) \quad cW \frac{dT}{dt} = - (T - T_P) \frac{M}{L} (1 + NT^*)$$

where T^* is assumed to be the average temperature of the constantan tube, hence approximately $(T + T_P)/2$.

Substituting $(T + T_P)/2$ for T^* and separating variables,

$$(24) \quad \frac{dt}{(T - T_P) (T + T_P + \frac{2}{N})} = - \frac{MN}{2cWL} dt$$

$$(25) \quad \int_{T_S}^{T_E} \frac{dT}{(T - T_P) (T + T_P + \frac{2}{N})} = \int_0^t - \frac{MN}{2cWL} dt$$

Integrating with the aid of integral #51 of the CRC Handbook of Chemistry and Physics, 47th Edition, for the left side,

$$(26) \quad \left. \frac{1}{(-T_P) - (T_P + \frac{2}{N})} \ln \frac{T + T_P + \frac{2}{N}}{T - T_P} \right|_{T_S}^{T_E} = - \frac{MNt}{2cWL} \Big|_0^t$$

Substituting in the limits and simplifying,

$$(27) \quad \ln \frac{(T_S - T_P) (T_E + T_P + \frac{2}{N})}{(T_E - T_P) (T_S + T_P + \frac{2}{N})} = \frac{M (NT_P + 1) t}{cWL}$$

Solving for t ,

$$(28) \quad t = \frac{cWL}{M (NT_P + 1)} \ln \frac{(T_S - T_P) (T_E + T_P + \frac{2}{N})}{(T_E - T_P) (T_S + T_P + \frac{2}{N})}$$

Substituting in the values given earlier for the constants, t becomes,

$$(29) \quad t = \frac{(.214) (10.14) (1.00)}{(.01042) [(.00123) (492) + 1]} \ln \frac{(672 - 492) (563 + 492 + \frac{2}{.00123})}{(563 - 492) (672 + 492 + \frac{2}{.00123})}$$

$$(30) \quad t = 129 \ln 2.43$$

$$(31) \quad t = 114.5 \text{ seconds}$$

CONCLUSION

The calculated time of 114.5 seconds agrees within $\pm 2\%$ of the measured 116.0 and 113.8 seconds and agrees almost exactly with the average of these two time measurements. This close agreement indicates that the constants of the amended calibration equation (21) as applied to gamma heating rate sensors SN 000001 and SN 000004 are probably correct within $\pm 2\%$. This accuracy combined with the fact that the two temperatures involved in a gamma heating rate determination can be measured within $\pm 2\%$ indicates that the gamma heating rate sensors should meet the specified $\pm 10\%$ of full-scale accuracy requirement.

In future sensors of this type, the thermal resistance should be made, if possible, of a material which has little or no variation with temperature. Time measurements for check and calibration should be made at different percentages of the total change and in different temperature spans.