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Thermal Stresses in Externally Heated and Cooled Cylinders

Introduction

Quite often it is convenient to supply heat to pressurizers and circulating systems by strapping strip heaters to the outside of pipes and vessels containing the fluid. The temperature gradients in the container wall will, of course, cause thermal stresses. This report is in response to a request from the Development Section to determine the maximum allowable heat flow across such container walls.

The problem consists of an investigation of both steady-state and unsteady-state conditions. It has been assumed that in both cases the temperature in the circumferential and longitudinal directions is uniform, gradients existing only in the radial direction. This assumption is valid since the heating elements generally are not in direct contact with the container wall. The geometry of the containers considered is cylindrical.

Stress Equations

The stresses of interest are those at the inside and outside surfaces of the container wall. On these surfaces the radial stresses caused by thermal gradients must be zero. Also on these surfaces the thermal gradient stresses in the circumferential and longitudinal directions are equal away from any discontinuities. Therefore, the necessary equations in terms of the temperature gradients are as follows:⁽¹⁾

$$\sigma_a = \frac{\alpha E}{1 - \nu} \left(\frac{2}{b^2 - a^2} \int_a^b T_r dr - T_a \right) \dots (1)$$

$$\sigma_b = \frac{\alpha E}{1 - \nu} \left(\frac{2}{b^2 - a^2} \int_a^b T r \, dr - T_b \right) \dots (2)$$

where

- σ_a = circumferential stress at $r = a$
- σ_b = circumferential stress at $r = b$
- r = radius
- a = inside radius
- b = outside radius
- T = temperature at r in wall
- T_a = temperature at $r = a$
- T_b = temperature at $r = b$
- α = coefficient of thermal expansion
- ν = Poisson's ratio
- E = Modulus of elasticity

Steady State Equations

For the steady heat flow through a cylindrical wall the temperature distribution is as follows:

$$T = \frac{q'}{2 \pi k} \ln \frac{r}{a} \dots (3)$$

where

- T = Difference in temperature between inside surface and some point r in the wall
- k = Thermal conductivity
- q' = Heat flow across wall per unit length of pipe

Substitution of equation (3) into equations (1) and (2) leads to the following:

$$\left[\frac{q' \propto E}{(1 - \gamma) k \sigma_a} \right]_{r=a} = 4 \pi \left[\frac{\left(\frac{b}{a}\right)^2 - 1}{2 \left(\frac{b}{a}\right)^2 \ln \frac{b}{a} - \left(\frac{b}{a}\right)^2 + 1} \right] \dots (4)$$

$$\left[\frac{q' \propto E}{(1 - \gamma) k \sigma_b} \right]_{r=b} = 4 \pi \left[\frac{\left(\frac{b}{a}\right)^2 - 1}{2 \ln \frac{b}{a} - \left(\frac{b}{a}\right)^2 + 1} \right] \dots (5)$$

These equations are represented by the curves in Fig. 1.

Unsteady-State Equations

To simplify the mathematical manipulations involved slab geometry will be used for the unsteady-state conduction equations. In most cases of interest this is justifiable.

The general equation for conduction is

$$\frac{\partial \rho C_p T}{\partial t} = G + k \Delta^2 T \dots (6)$$

Using rectangular coordinates with conduction in only one direction, no internal heat generation, and average values of ρC_p , equation (6) reduces to

$$\frac{k}{\rho C_p} \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \dots (7)$$

where

ρ = density

C_p = specific heat

t = time

For the unsteady-state condition there are, of course, numerous boundary conditions that could be used, depending on how the heat is applied to the wall. The following boundary conditions were assumed:

$$1. T(x = L, t) = T_0 t$$

$$2. T(x = 0, t) = 0$$

where

L = Thickness of slab or pipe wall

T_0 = Temperature rise per unit of time

$\neq f(t)$

The first boundary condition is conservative because in a practical case, such as when strip heaters are used on the outside of a pipe, the rate of temperature rise of the surface falls off with time because of the resistant film at the surface and because of the temperature characteristics of the heating elements. However, for short periods of heating relative to the time it takes for the heating elements to reach their maximum temperature, and for small values of film resistance the first boundary condition should be reasonable. The second boundary condition will also be adequate for short periods of time.

The solution to equation (7), using boundary conditions (1) and (2), is⁽²⁾

$$T(x, t) = T_0 \pi^2 \lambda \left[\frac{x}{6L} \left(\frac{x^2}{L^2} - 1 \right) + \frac{x}{L} \frac{t}{\lambda \pi^2} + \frac{2}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} e^{-\frac{n^2 t}{\lambda}} \sin \frac{n \pi x}{L} \right] \dots (8)$$

$$\text{where } \lambda = \frac{L^2}{\pi^2 K}$$

$$K = \frac{k}{\rho C_p} = \text{thermal diffusivity}$$

The following substitution is made in order that equation (8) can be used in equations (1) and (2):

$$x = r - a$$

$$L = b - a$$

This substitution in equation (8) gives

$$T(r,t) = T_0 \left\{ \pi^2 \lambda \frac{\left(\frac{r}{a} - 1\right)}{6\left(\frac{b}{a} - 1\right)} \left[\frac{\left(\frac{r}{a} - 1\right)^2}{\left(\frac{b}{a} - 1\right)^2} - 1 \right] + \frac{\left(\frac{r}{a} - 1\right)}{\left(\frac{b}{a} - 1\right)} t \right. \\ \left. + \frac{2\lambda}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} e^{-\frac{n^2 t}{\lambda}} \sin n\pi \frac{\left(\frac{r}{a} - 1\right)}{\left(\frac{b}{a} - 1\right)} \right\} \quad \dots\dots (9)$$

$$\text{where } \lambda = \frac{a^2 \left(\frac{b}{a} - 1\right)^2}{\pi^2 K}$$

λ is not dimensionless in "a" and "b", and, therefore neither will the final equations for stresses.

Equation (9) now must be substituted into equations (1) and (2).

From boundary condition (2) and equations (1) and (2)

$$\frac{\sigma_a(1-\nu)}{\alpha E} = \frac{2}{a^2 \left[\left(\frac{b}{a}\right)^2 - 1\right]} \int_a^b T_r dr \quad \dots\dots (10)$$

$$\frac{\sigma_b(1-\nu)}{\alpha E} = \frac{\sigma_a(1-\nu)}{\alpha E} - T_b \quad \dots\dots (11)$$

Therefore, only the integral in equation (10) and T_b need to be determined.

$$\text{Let } \frac{\frac{r}{a} - 1}{\frac{b}{a} - 1} = R = \frac{r - a}{b - a}$$

$$\text{Then } r = (b-a) R + a$$

$$dr = (b-a) dR$$

$$\text{When } r = a, R = 0$$

$$r = b, R = 1$$

$$\therefore \int_a^b T r dr = \int_0^1 T [(b-a) R + a] (b-a) dR$$

$$= T_0 \frac{\pi^2 \lambda}{6} (b-a)^2 \int_0^1 (R^4 - R^2) dR$$

$$+ \frac{T_0 \pi^2 \lambda}{6} (b-a) a \int_0^1 (R^3 - R) dR$$

$$+ T_0 t (b-a)^2 \int_0^1 R^2 dR$$

$$+ T_0 t (b-a) a \int_0^1 R dR$$

$$+ \frac{T_0 2 \lambda}{\pi} (b-a)^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} e^{-\frac{n^2 t}{\lambda}} \frac{1}{(n \pi)^2} \int_0^1 (n \pi R) \sin n \pi R d(n \pi R)$$

$$\begin{aligned}
& + T_0 \frac{2\lambda}{\pi} (b-a) a \sum_{n=1}^{\infty} (-1)^{n-1} e^{-\frac{n^2 t}{\lambda}} \frac{1}{n\pi} \int_0^1 \sin n\pi R d(n\pi R) \\
& = T_0 \frac{\pi^2 \lambda}{6} \left(\frac{b}{a} - 1 \right)^2 a^2 \left(\frac{1}{5} - \frac{1}{3} \right) \\
& + T_0 \frac{\pi^2 \lambda}{6} \left(\frac{b}{a} - 1 \right) a^2 \left(\frac{1}{4} - \frac{1}{2} \right) \\
& + T_0 t \left(\frac{b}{a} - 1 \right)^2 a^2 \left(\frac{1}{3} \right) \\
& + T_0 t \left(\frac{b}{a} - 1 \right) a^2 \left(\frac{1}{2} \right) \\
& + T_0 \frac{2\lambda}{\pi} \left(\frac{b}{a} - 1 \right)^2 a^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} e^{-\frac{n^2 t}{\lambda}} \frac{1}{(n\pi)} (-\cos n\pi) \\
& + T_0 \frac{2\lambda}{\pi} \left(\frac{b}{a} - 1 \right) a^2 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3} e^{-\frac{n^2 t}{\lambda}} \frac{1}{n\pi} (-\cos n\pi + 1) \\
& = T_0 a^2 \left(\frac{b}{a} - 1 \right) \left\{ -\frac{\pi^2 \lambda}{45} \left(\frac{b}{a} + \frac{7}{8} \right) + \frac{t}{6} \left(2 \frac{b}{a} + 1 \right) \right. \\
& \left. - \frac{2\lambda}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4} e^{-\frac{n^2 t}{\lambda}} \left[(-1)^n \frac{b}{a} - 1 \right] \right\} \dots\dots (12)
\end{aligned}$$

$$T_b(t) = T_0 t \dots\dots (13)$$

Substitution of equations (12) and (13) into equations (10) and (11) gives

$$\frac{\sigma_a (1 - \nu)}{\alpha E T_o} = 2 \frac{\left(\frac{b}{a} - 1\right)}{\left[\left(\frac{b}{a}\right)^2 - 1\right]} \left\{ - \frac{\pi^2 \lambda}{45} \left(\frac{b}{a} + \frac{7}{8}\right) + \frac{t}{6} \left(2 \frac{b}{a} + 1\right) \right. \\ \left. - \frac{2\lambda}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^4} e^{-n^2 t/\lambda} \left[(-1)^n \frac{b}{a} - 1 \right] \right\} \dots\dots (14)$$

$$\frac{\sigma_b (1 - \nu)}{\alpha E T_o} = \frac{\sigma_a (1 - \nu)}{\alpha E T_o} - t \dots\dots (15)$$

General Discussion of Equations

The sign of the stress (tension = +, compression = -) is important in the above equations because often a high compressive stress is not as serious as an equally high tensile stress. The signs are also important, of course, for cases of super position. In equations (1) through (15) it has been assumed that heat is being added to the container from the outside. For this case the outside surface is the hottest. This means that the outside surface will be in compression and the inside surface in tension. Therefore, in equation (4) σ_a will be positive and equal to the numerical difference between the allowable working stress in tension and the positive pressure stress at $r = a$. On the outside of the pipe the thermal stress (-) and the pressure stress (+) tend to cancel each other so that equation (5) need not be considered for this particular case.

If heat is removed from the container through the wall, the equations

are still valid. Only the signs of the stresses will be different.

The unsteady-state case can be discussed qualitatively by observing Figs. 2, 3, and 4. Figure 2 shows the spacial temperature distribution through the wall of the pipe when the outside surface temperature is raised one $^{\circ}\text{F}$ with three different values of T_0 . Figure 3 shows a comparison of transient and steady-state stresses. Figure 4 gives the temperature differential across the wall that will produce the given allowable stress when a particular value of T_0 is being used. The curves were obtained from Figure 3.

Since thermal stress is proportional to the difference between the area averaged temperature across the wall and the temperature at the point under consideration it is seen from Fig. 2 that as the heating rate, T_0 , is increased the stress on the heated surface increases and the stress on the opposite surface decreases. As $T_0 \rightarrow \infty$, $\sigma_a \rightarrow 0$ and $\sigma_b \rightarrow \text{constant}$. The value of the constant can be readily obtained from equation (11).

$$\sigma_b = -\frac{\alpha E}{1-2} T_b \quad \dots\dots (16)$$

$$T_0 = \infty$$

This condition is also indicated by curve B in Fig. 4. The allowable ΔT reaches a minimum limit as T_0 is increased. The existence of the relation in equation (16) provides a very convenient and conservative means for estimating an allowable ΔT when T_0 is not known.

The behavior of curve A in Fig. 4 is explained by the fact that for a given ΔT σ_a decreases rapidly with an increasing T_0 . This follows from the discussion based on Fig. 2.

It is interesting to compare the case of supplying heat to the outside and the case of removing heat from the outside. During the steady-state, there is no significant difference. However, during the unsteady-state, the case

of cooling is more serious because the thermal stress and pressure stress are both positive. When the outside is heated the negative thermal stress tends to cancel the positive pressure stress. In both bases the thermal stresses on the inside are less than during the steady-state and so are of little concern.

Example Problem

The problem that initiated this study was one of applying heat to the HRT mockup through three and one-half inch high pressure HRT piping. The following quantities were used in the calculations:

$$b = 2.000$$

$$a = 1.530$$

$$\alpha = 9.8 \times 10^{-6} \text{ in/in/}^{\circ}\text{F}$$

$$E = 25 \times 10^6 \text{ psi}$$

$$k = 140 \text{ Btu in/hr}^{\circ}\text{F ft}^2$$

$$\nu = .3$$

$$K = .005 \text{ in}^2/\text{sec}$$

1. Assume the system to be at 2000 psi and 600°F.

Pressure stress at inside surface (circumferential),

$$\begin{aligned} \sigma_P (r = a) &= P \left[\frac{\left(\frac{b}{a}\right)^2 + 1}{\left(\frac{b}{a}\right)^2 - 1} \right] \\ &= 2000 \text{ psi} \left[\frac{2.709}{.709} \right] = + 7,640 \text{ psi} \end{aligned}$$

$$\sigma_P (r = b) = P \left[\frac{2}{\left(\frac{b}{a}\right)^2 - 1} \right]$$

$$= 2000 \text{ psi} \left[\frac{2}{.709} \right] = + 5,640 \text{ psi}$$

The allowable stress in tension is 14,850 psi.

$$\therefore \sigma_a \leq 14,850 - 7,640 = + 7,210 \text{ psi}$$

$$\sigma_b \leq 14,850 - 5,640 = + 9,210 \text{ psi}$$

The allowable stress in compression for stainless steel 347 at this temperature could be about 35,000 psi since the summation of thermal forces across the wall must be zero.

From Fig. 1

$$\frac{q' \propto E}{h(1 - \nu)\sigma_a} = 44.2$$

$$\therefore q' = \frac{140 \text{ Btu in/ft}^2 \text{ hr } ^\circ\text{F} (1 - .3) 7,210 \text{ lb/in}^2 \times \text{ft/12 in.}}{9.8 \times 10^{-6} \text{ in/in/}^\circ\text{F} \times 25 \times 10^6 \text{ lb/in}^2} \times 44.2$$

$$= 10,620 \text{ Btu/hr ft} \times \frac{\text{Kw}}{3413 \text{ Btu/hr}} = 3.11 \text{ Kw/ft}$$

2. For the unsteady-state several conditions could be assumed. In the case of a pressurizer the system might be at atmospheric conditions when the heaters are turned on. Under these conditions the limiting stress would be the 35,000 psi compressive stress on the outside, if rapid heating rates are used. Figs. 2, 3, and 4 have been plotted for the example problem. Curve B in Fig. 4 has been plotted for a maximum thermal stress at $r = b$ of -35,000 psi. From this curve the allowable ΔT across the wall can be estimated for a given T_o .

3. Assume unsteady-state, 2000 psi, and 600°F.

Curve A in Fig. 4 has been plotted for a maximum thermal stress at $r = a$ of + 7,210 psi. In this case this curve is not really of interest because as seen in Fig. 3 the transient thermal stress on the inner wall is less than for the steady-state.

The problem of removing heat from the outside surface can be treated in a similar manner. It is evident from the treatment of the above problem that transient heating is in most cases less severe than the steady-state. However, in general this will not be the case when the container is cooled since the positive thermal stress and positive pressure stress on the outside surface are additive.

/mh

References

1. Theory of Elasticity, Timoshenko and Goodier, 2nd ed., McGraw-Hill Book Co., Inc., 1951, p. 413.
2. "Proceedings of the General Discussion on Heat Transfer:", The American Society of Mechanical Engineers, 11th - 13th September, 1951, p. 253.

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Fig. 1. Allowable steady-state heat flow/unit length, across cylinder wall for a specified maximum circumferential thermal stress.

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Fig. 2. Temperature distribution for different T_0 and $T_0 t = 1$.

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Fig. 3. Circumferential thermal stress at inner and outer surface of cylinder heated on outer surface.

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Fig. 4. Maximum allowable temp. difference across cylinder wall for a specified T_0 and allowable circumferential stress.