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ACCELERATOR DEPARTMENT
Informal Report

THE WAKE FIELD PRODUCED BY A PARTICLE IN THE PRESENCE OF CONDUCTIVE
PLATES: A vehicle for the excitation of the coupled betatron-
synchrotron instabilities

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Abstract

The interaction of a single particle betatron oscillation with conductive plates are here analyzed. First, Fourier analysis of the field source is performed, then the usual approach to determine the plate response is used by treating the plates as transmission lines. Finally, with a process of anti-transformation the total response to the oscillation of a single particle is constructed.

The wake field decays over a distance which is of the order of magnitude of the length of the plates themselves. Nevertheless, the shape of the wake field depends on the terminating independence. Several cases, for instance matched, open and shorted plates are considered.

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I.

During the past years coherent instabilities of intense bunched beam have been observed in several particle circular accelerators or storage rings. These instabilities are said to be coherent because the mass center of the beam executes some radial or vertical oscillations. The amplitude of these oscillations grows exponentially, above some intensities, with the effect of increasing excessively the transverse beam size, and also of losing the beam itself.

The causes of the instabilities are various, but certainly they are all to be indebted to the electromagnetic interactions between the beam and the surrounding media (vacuum chamber wall equipment). We wish here to turn our attention to the instabilities observed in ADONE⁽¹⁾, the 1.5 GeV electron-positron storage ring in Frascati. These instabilities seemed to be caused by the presence of conductive plates inside the vacuum chamber (clearing electrodes, deflecting electrodes, ...). Really, the removal of these plates allowed an increase of the beam current to be stored⁽¹⁾.

The mechanism of the instabilities can be explained by the following "head-tail effect" model⁽²⁾. A particle, placed inside a bunch, executes on the same time betatron and synchrotron oscillations. It produces a field backward to the motion of the particle and thus acting only on the subsequent particles. This field depends on the distance from the particle, source of the field, and on the electric properties of the surrounding media. (In our case, the vacuum chamber wall with the conductive plates insertion.) Each

particle is at the same time the object and the subject of the field inside the bunch. The field is enhanced by the transverse betatron oscillation, but the (complex) amount of the field seen by a particle depends on the position of the particle with respect to the others of the bunch. This position changes because of the phase oscillation of the particle.

Because of its asymmetry, the field is practically zero at the "head" of the bunch and the largest at the "tail" of the bunch. On the other hand the same particle produces identical fields when it is at the "head" and when at the "tail", but in the former case the field affects all the particles of the bunch, and it is ineffective in the latter one.

We have deeply investigated the wake field of a particle in the presence of conducting plates^(3,4). The analytical expression of the field has been used in a coupled betatron-synchrotron dynamics investigation to explain the instability observed in ADONE⁽²⁾.

2.

A particle moves along the equilibrium rectified orbit (z) with equation of motion.

$$z = vt + \sigma(t) \quad (1a)$$

v , velocity of the bunch;

σ , particle linear phase, with respect to the synchronous particle.

The particle executes also a transverse oscillation (y),

$$y = \zeta e^{i\nu t} \quad (1b)$$

ζ , amplitude of the oscillation;

ν , a quantity very close to $2\pi \times$ betatron oscillation frequency.

We associate to our particle an electric dipole per unit length,

$$p(z, t) = e\zeta e^{i\nu t} \delta(z - vt - \sigma) \quad (2)$$

and the transform in the (k, ω) space

$$\tilde{p}(k, \omega) = \frac{e\zeta}{2\pi} e^{ik\alpha} \delta(\nu + k\nu - \omega) \quad (3)$$

such that

$$p(z, t) = \int p(k, \omega) e^{-i(kz - \omega t)} dk d\omega.$$

In equations (2) and (3), e is the particle charge.

We take (3) as the source of the field produced in the presence of conductive plates of length L , terminated at both the ends by impedances Z_1 and Z_2 , and with characteristic impedance

$$Z_0 = 1/c C$$

c , light velocity

C , capacitance per unit length (see the Figure).

The transformed dipole (3) induces on the plate a charge λ and a current I per unit length to be evaluated neglecting (in first approximation) the presence of the plate itself.

The charges and the currents moving on the plate surface produce a vector $A_1 \equiv (0, 0, A_1)$ and a scalar V_1 potential distribution governed by the usual transmission line equations, in the form as they have been used by L.J. Laslett⁽⁵⁾.

3.

Solving the differential equation for the scalar potential in the space, V ,

$$\Delta^2 V - \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = 0,$$

and the equivalent for the vector potential in the space, $A \equiv (0, 0, A)$, and taking the solutions V, A that satisfy

$$\left. \begin{aligned} V &= V_1 \\ A &= A_1 \end{aligned} \right\} \text{ at the conductive plate surface}$$

and

$$V = A = 0 \quad \} \text{ at the vacuum chamber wall,}$$

we may construct the force per unit charge

$$\tilde{F} = V - \beta A, \quad \beta = v/c$$

acting in the proximity of the beam axis (z) and directed along the transverse oscillation axis (y).

By a process of anti-transforming we obtain, then, the total force produced by the particle

$$\begin{aligned} F(z, t) &= \iint \tilde{F}(z, k, \omega) e^{-i(kz - \omega t)} dk d\omega \\ &= \iint g(z, k, \omega) \tilde{p}(k, \omega) e^{-i(kz - \omega t)} dk d\omega \end{aligned} \quad (4)$$

The form-factor $g(z, k, \omega)^{(3,4)}$ describes the coupling between the transformed dipole $\tilde{p}$ and the transmission line structure formed by the plates distributed around the accelerator circumference and the vacuum chamber wall. This factor has a periodic z -dependence, with period equal to one revolution around the accelerator circumference. We have, then, replaced the form factor $g(z, k, \omega)$ with its mean value $g(k, \omega)$ over a revolution.

From (3) we have, hence,

$$F(\tau, t) = \frac{e\epsilon}{2\pi} e^{i\omega t} \int g(k, \nu + kv) e^{ik\tau} dk \quad (5)$$

where τ is the distance (positive) from the particle producing the field.

4.

The form factor g has been evaluated for several configurations^(3,4), and it shows always a series of poles of the first order which can be subdivided in two groups:

- (1) The first ones describe the resonance when the signal induced by the dipole $\tilde{p}$ on the plate travels at the light velocity, i.e.

$$\left| \frac{\omega}{k} \right| = c \quad (6)$$

(ii) The poles of the second group are placed around those (complex) frequencies such that our transmission line structure, with the actual termination impedances, resonates. These frequencies are given by solving the following equation in $\theta = \omega l / 2c$,

$$\theta = i \theta^* - \frac{\pi}{2} s \quad (7a)$$

where

$$s = 0, 1, 2, 3, \dots,$$

and

$$\tanh(2\theta^*) = \frac{Z_1(\theta) + Z_2(\theta)}{Z_0^2 + Z_1(\theta) \cdot Z_2(\theta)} \quad Z_0 = \quad (7b)$$

The last resonances disappear when the plate is tuned at its characteristic impedance Z_0 at least one of the two ends.

Using the usual residual method we have

$$F(\tau, t) = 2\pi i \sum_s R_s \quad (8)$$

where R_s is the residual of $g(k, v + kv)$ at the s -th pole. Thus, we can say that the force F is given by the sum of the values of the form factor g in the proximity of all the resonance frequencies given by (6) and (7).

5.

It results that for a plate perfectly matched at both ends ($Z_1 = Z_2 = Z_0$) we have:

$$F(\tau, t) = \pi \frac{A}{c} e^{-i \frac{v\tau}{(1+\beta)c}} \left\{ \beta - i \frac{v}{c} \frac{(1+\beta) - \tau}{1+\beta} \right\}, \quad 0 < \tau < 2\ell\beta \quad (9)$$

$= 0, \text{ elsewhere.}$

In this case, eq. (9) can be derived from eq. (8) only taking into account the poles of the first group (6), since the poles of the second group are all confined at the infinite and they do not give any contribution. The fact that the force is zero outside the mentioned interval can be explained by the total absorption of the induced signal at the first reflection.

A is the following geometric factor

$$A = \frac{ev}{2\pi} \left(\frac{M\ell}{2\pi R} \right) \left(\frac{\sin^2 \delta}{\pi^2} \right) \frac{8}{b^2 c}$$

where M is the number of (identical) plates, 2δ the angular aperture of a plate, b the vacuum chamber radius and $2\pi R$ the machine circumference.

6.

In the more general case where (7a) and (7b) yield to no divergent solutions of $\emptyset$, the contribution of the poles of the first group can be neglected and the sum at the r.h. side of eq(8) can be evaluated taking into account only the poles of the second group.

Considering for the moment only resistive terminations (Z_1 and Z_2 , real and positive) we have, as a result, that the tail of the wake field grows longer with a decreasing of the amplitude. This fact can be easily explained reminding that the signal induced on the plate, not terminated at its characteristic impedance, can decompose in a series of stationary waves only partially absorbed at each reflection at the ends of the plate. As we go further from the tuning of the plate, the less is the amount of absorption of the signal at the ends.

In the extreme case of shorted and floating plates the induced signal is always fully reflected at the ends and never absorbed. The tail of the wake field extends infinitely, but the amplitude reaches its minimum.

All the cases of resistive terminations can be condensed in the following expression for the force:

$$F(\tau, t) = 4\pi \frac{A}{L} \left\{ \beta \frac{Z_1 + Z_2}{Z_0} \sin 2i\theta^* + i \frac{\sqrt{L}}{2cZ} \beta \cos 2i\theta^* + \right. \\ \left. i \frac{\sqrt{L}}{2cZ} \frac{Z_1 Z_2}{\beta Z_0^2} \cos 2i\theta^* \right\} \left[a_1 (2\beta \ln \tau - \tau) + a_2 \right] e^{-i \frac{\sqrt{L}}{c} \frac{2\ln \tau + \tau}{1 + \beta}} e^{-4\theta^* n} \quad (10)$$

with

$$a_1 = -i \frac{\sqrt{L}}{\beta(1 + \beta)} \frac{e^{-i \frac{\sqrt{L}}{(1 + \beta)c} - 2\theta^*}}{\sin 2(i\theta^* - \frac{\sqrt{L}}{2(1 + \beta)c})},$$

$$a_2 = \frac{\sqrt{L}}{1 + \beta} \frac{2}{\sin^2 2(i\theta^* - \frac{\sqrt{L}}{2(1 + \beta)c})} + \frac{e^{-i \frac{\sqrt{L}}{(1 + \beta)c} - 2\theta^*}}{\sin 2(i\theta^* - \frac{\sqrt{L}}{2(1 + \beta)c})}$$

Eq. (10) applies for $2\beta \ln \tau < \tau < 2\beta \ln(\tau + 1)$, with $n = 0, 1, 2, \dots$

7.

Also the cases with reactive termination impedances (Z_1 and Z_2 complex) have been investigated⁽⁶⁾. When Z_1 and Z_2 resonates at some frequencies, i.e., when they have poles which are not necessarily only of the first order, the summation at r.h. side of eq. (8) must be extended also to the residuals in these poles.

In particular it had been shown that a RLC series circuit has no poles, and when the impedance of the RLC series circuit is very small or very large, with respect to the characteristic impedance Z_0 , at any resonance frequencies given by (7), we can still make use of eq. (10) respectively with Z_1 and Z_2 close to zero or very large.

8.

In conclusion we can say:

It is not possible to find a case where the wake field vanishes identically. The extreme cases are the following

(1) The floating and the shorted plates excited by an oscillating particle produce a field with a tail extending infinitely and with the lowest amplitude. This field acts certainly on the particles immediately subsequent, but its strong effect is given by summing the residual fields found after several revolutions.

(2) A particle oscillating in the presence of matched plates produces a wake field which acts only in a short range, and its effect is already reduced to zero after a revolution. Nevertheless the amplitude of the field is the largest, and its effect can be comparable to that of a small wake field but acting at long range, as in the case (1), after several revolutions.

In the "head-tail" effect one is interested only in the component at "short range" of the wake field; and it had been shown (2) that just the case (2) mentioned above of tuned plates can give rise to instability of this kind; in fact the amplitude of the field is the maximum.

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Vacuum chamber wall

z-axis

conductive plate

l

LC

Z_1

Z_2

terminations