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APAE MEMO No. 97

EFFECT OF APPR-1 CONTROL ROD ACCELERATION AFTER SCRAM ON STARTUP ACCIDENT

Contract No.: AT(11-1)318

Issued: April 23, 1957

By

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CHAPTER I

Introduction

In the preoperational checkout of the APPR-1 it became apparent that the control rod acceleration after scram was lower than expected. It was expected that the control rod acceleration after scram would be 0.750 g(1) where g is the acceleration of gravity. However, measurements made at Ft. Belvoir indicated an acceleration of 0.4 g. The effect of this reduced acceleration on the safety of the APPR-1 is reported in this memo.

In the operation of the APPR-1 the most dangerous condition exists at cold startup. Therefore the behavior of the APPR-1 in a startup accident with a rod acceleration of 0.4 g after scram is considered.

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CHAPTER II

Startup Accident

2.1 Introduction.

A startup accident is defined as that condition resulting from continual rod withdrawal starting from source level. This accident can only be caused by numerous instrument failures (3 sec period scram, etc.) as well as operator error.

Normal startup procedure (1) involves pulling the two close packed eccentric rods to the fully withdrawn position and then withdrawing the five rod bank to a position of two inches or more from criticality. The reactor is then made critical by withdrawing the rods one at a time to the position expected for critical.

In this analysis it will be assumed that the two close packed rods are fully withdrawn and then the operator withdraws the five remaining rods as a bank continually. The reactor goes critical and prompt critical before it reaches the level scram set at 15 MW. Following the instrument delay the rods are inserted at 0.4 g.

2.2 Conditions at Startup.

The expected variation of reactivity in the APPR-1 at 68°F is reported in (3). The conditions of prime interest are startup, midlife and end of life where the reactivity in the core (after 60 hours shutdown)is 15.4%, 15.9%and 9%, 2.2 Conditions at Startup (Cont'd.)

respectively. The worth of the five rod bank has been measured as 19.5% (4). The startup accident will be investigated at beginning and end of life. The condition at midlife is very close to that at beginning of life. If the rod worth is invariant with core loading then the reactivity in the core with the five rod bank inserted is -4.1% and -10.5% at beginning and end of life, respectively. From Fig. 29 of (4) the reactor is expected to be critical at room temperature with the five rod bank withdrawn 3.7 and 7.2 inches at beginning and end of life, respectively. At these positions the five rod bank is worth 1.72 %/inch and 1.58 %/inch, respectively, as seen in Fig. 28 of (4). With a rod speed of 3 in. per min. reactivity is added to the core at critical at 0.086%/sec. and 0.079%/sec., respectively. The worth of the rods changes slightly during the time from sub-critical to level scram.

At beginning of life the shutdown flux in the APPR-1 has been estimated during the ZPE as 1×10^3 neutrons/cm^2-sec. Then if the flux at 10 MW at beginning of life is 1.5×10^{13} neutrons/cm^2-sec the ratio of the flux at the 15 MW level scram to the shutdown flux is 2.3×10^{10} (or $\log_{10} n/n_o = 10.36$).

At the end of life the shutdown flux should be 0.7×10^3 neutrons/cm^2-sec and the flux at 10 MW approximately 2.1×10^{13} neutrons/cm^2-sec. Therefore the ratio of the flux at the 15 MW level scram to the shutdown flux is 4.5×10^{10} (or $\log_{10} n/n_o = 10.65$).

2.3 Startup Condition Analyzed.

The solution of the reactor kinetic equations has been carried out for a large range of startup conditions as reported in (5). The following startup conditions

$\delta k/\mathrm{sec}$ = 0.1%/sec

δk_o = -0.10

ℓ^* = 2×10^{-5} sec

ℓ^* = prompt neutron lifetime

are conservative when compared with these expected in the APPR-1 either at beginning or end of life. Therefore the solution to the kinetic equations for these conditions is used to establish the reactor conditions at the 15 MW level scram.

The solution of the kinetic equations in (5) give the $\log_{10}$ of $n(t)/n_o$, the reciprocal period and reactivity added as a function of time. A value of $\log_{10}$ of 10.4 is selected to establish the time at 15 MW level scram.

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From Page 55 of WAPD-TM-1, the reactor conditions at the level scram are -

$\log_{10} \frac{n}{n_o} = 10.4$ when $t = 4.24$ seconds from beginning of transient shown.

When $t = 4.84$ seconds, $\frac{1}{\text{Period}} = 30$, Period $= 0.0333$ seconds.

When $t = 4.84$ seconds, $\delta k = 76.8 \times 10^{-4}$

$$\delta k \approx 77 \times 10^{-4}$$

CHAPTER III

REACTOR BEHAVIOR AFTER LEVEL SCRAM

3.1 Instrument Delay

The time between passing the 15 MW power level and the beginning of motion of the control rods is 50 milliseconds (6).

From Page 55 of WAPD-TM-1, $\frac{1}{T}$ and hence T does not change appreciably in 50 milliseconds. Then

$$\frac{P}{P_o} = e^{\frac{0.050}{0.030}} = e^{1.67} = 5.3$$

The reactor power when the rods begin to move is,therefore,(5.3) (15 MW) or 79.5 MW.

The control rod worth at the end of life was shown earlier to be 1.6%/inch. The control rod acceleration is assumed to be 0.4 g (6)

The introduction of negative reactivity by the control rods will be assumed to be linear rather than parabolic. The slope of the line will be determined by considering the average rod speed over the first 0.1 inch of travel.

$$s = \frac{1}{2} a t^2$$

$$t = \left(\frac{2s}{a}\right)^{\frac{1}{2}} = \left(\frac{(2)(0.1)}{(0.4)(32)(12)}\right)^{\frac{1}{2}} = 3.6 \times 10^{-2} \text{ sec}$$

$$v = \frac{s}{t} = \frac{0.1}{3.6 \times 10^{-2}} = 2.78 \text{ in/sec}$$

The negative reactivity insertion rate $= 2.78 \frac{\text{inch}}{\text{sec}} \times 1.6 \frac{\%}{\text{inch}} = 4.45 \frac{\%}{\text{sec}}$

3.2 Reactor Shutdown.

It is now necessary to solve the reactor kinetic equations for the initial conditions indicated below:

$$\text{Power} = 79.5 \text{ MW}$$

$$\frac{d}{dt} \text{ Power} = 2385 \text{ MW/sec}$$

$$\frac{\delta k_{eff}}{k_{eff}} = 77 \times 10^{-4}$$

$$\text{Reactivity insertion rate} = -4.45 \frac{\%}{\text{sec.}}$$

$$\text{Prompt neutron lifetime} = 2 \times 10^{-5} \text{ sec.}$$

Solutions to the reactor kinetic equations for certain conditions are presented in WAPD-13(7).

The solution in WAPD-13 which is applicable to the problem at hand is on Page 18, equation (18). This solution assumes only one group of delayed neutrons. The delayed neutron fraction is 0.0072 and the reciprocal of the decay constant is 10 seconds. Lumping the delayed neutrons considerably simplifies the mathematics without adverse effect on solution accuracy.

The solution is :

$$n(t) = e^{-t/\tau}\left[e^{-w^2/2}\left(C_1 + C_2\int_{w(o)}^{w(t)} e^{u^2/2}\,du\right)\right]$$

where: $w = \dfrac{At + F}{\sqrt{A}}$

$A = -\dfrac{\gamma}{\ell}$

$F = B - 2C/A$

$B = \dfrac{\beta - \alpha}{\ell} + \dfrac{1}{\tau}$

$C = -\dfrac{\gamma}{\tau\ell}$

γ = reactivity insertion rate = -0.0445/sec

$\alpha = \dfrac{\delta k_{eff}}{k_{eff}} = 77 \times 10^{-4}$

τ = reciprocal of delay neutron decay constant = 10 seconds

ℓ = prompt neutron lifetime = 2×10^{-5} sec

β = delayed neutron fraction = 72×10^{-4}

C_1 and C_2 are constants determined by initial conditions

The solution is valid if $\left(1 + \dfrac{\beta}{\gamma\tau}\right) \approx 1$

In the case at hand,

$$\left(1 + \frac{\beta}{\gamma\tau}\right) = 1 + \frac{72 \times 10^{-4}}{(-0.0445)(10)}$$

$$= 1 - 0.0162$$

$$= 0.9838$$

$$\approx 1$$

The constants and initial conditions will now be written in order to evaluate C_1 and C_2.

$n(o) = 1$ (arbitrarily)

$\frac{d}{dt} n(o) = 30 \quad (\frac{2385}{79.5} = 30)$

$A = 2225$

$B = -24.9$

$C = 222.5$

$F = -25.1$

$w = 47.2t - 0.532$

$$n(0) = [e^0]\left[e^{-\frac{[\omega(0)]^2}{2}}\right]\left[c_1 + c_2\int_{\omega(0)}^{\omega(0)} e^{\frac{u^2}{2}}\,du\right]$$

$$c_1 = e^{\frac{[\omega(0)]^2}{2}}\, n(0) = e^{0.1405} = 1.1502$$

$$\frac{dn(t)}{dt} = e^{-\frac{t}{\tau}}\left\{e^{-\frac{w^2}{2}} c_2 \frac{d}{dt}\int_{\omega(0)}^{\omega} e^{\frac{u^2}{2}}du + \left(c_1+c_2\int_{\omega(0)}^{\omega} e^{\frac{u^2}{2}}du\right)\left(e^{-\frac{w^2}{2}}\right)(-\omega)\left(\frac{d\omega}{dt}\right)\right\}$$

$$+\left(c_1+c_2\int_{\omega(0)}^{\omega}e^{\frac{u^2}{2}}du\right)\left(e^{-\frac{t}{\tau}}\right)\left(-\frac{1}{\tau}\right)\left(e^{-\frac{w^2}{2}}\right)$$

$$\frac{d}{dt}\int_{\omega(0)}^{\omega} e^{\frac{u^2}{2}}du = e^{\frac{\omega^2}{2}}\,\frac{d\omega}{dt}$$

$$\frac{dn(0)}{dt} = e^0\left\{e^{-\frac{[\omega(0)]^2}{2}}c_2\, e^{\frac{[\omega(0)]^2}{2}}\frac{d\omega}{dt} + \left(e^{\frac{[\omega(0)]^2}{2}} + c_2\int_{\omega(0)}^{\omega(0)} e^{\frac{u^2}{2}}du\right)e^{-\frac{[\omega(0)]^2}{2}}\right.$$

$$\left.(-\omega(0))\left(\frac{d\omega(0)}{dt}\right)\right\} + \left\{\left(c_1+c_2\int_{\omega(0)}^{\omega(0)}e^{\frac{u^2}{2}}du\right)(e^0)\left(-\frac{1}{\tau}\right)\left(e^{-\frac{[\omega(0)]}{2}}\right)\right\}$$

$\frac{dn(0)}{dt} = 30$ $c_2 = 0.106$

then

$$n(t) = e^{-t/10}\left[e^{-w^2/2}\left(1.1502 + 0.106\int_{-0.532}^{w} e^{u^2/2}\,du\right)\right]$$

where $w(t) = 47.2\,t - 0.532$

The function $e^{u^2/2}$ is not analytically integrable.

For the range $-1 < u < 1$ the function can be approximated by a series which converges rapidly.

$$e^{u^2/2} = 1 + \frac{u^2}{2} + \frac{u^4}{2^2 2!} + \frac{u^6}{2^3 3!} + \frac{u^8}{2^4 4!} + \text{----}$$

$$\int_b^a e^{u^2/2}\,du = u + \frac{u^3}{(3)(2)} + \frac{u^5}{(5)(2^2)(2!)} + \frac{u^7}{(7)(2^3)(3!)} + \text{----}\Bigg|_b^a$$

$$\approx u + 0.1667u^3 + 0.075u^5 + 0.00298u^7 \Bigg|_b^a$$

In all cases investigated the lower limit will be

$w(o) = -0.532$

$$\int_{-0.532}^{w} e^{u^2/2}\,du \approx u + 0.1667\,u^3 + 0.025u^5 + 0.00298u^7 \Bigg|_{-0.532}^{w}$$

$$\approx w + 0.1667\,w^3 + 0.025\,w^5 + 0.00298w^7 + 0.558$$

The equation for n(t) then becomes

$$n(t) = \frac{1.2093 + 0.106\,(w + 0.1667w^3 + 0.025w^5 + 0.00298\,w^7)}{e^{\frac{t}{10} + \frac{w^2}{2}}}$$

In the case of $w > 1$ the function $\int_{-0.532}^{w} e^{u^2/2}\,du$ was integrated graphically for values of w as great as 4. The results of the calculations are plotted in Figures 1 and 2.

CHAPTER IV

Conclusion.

Analysis of the most adverse startup accident in the APPR-1 indicated that at the 15 MW level scram the reactor is on a 33 ms. period. Rod insertion begins after a 50 ms. instrument delay, at which time the power is 80 MW. The maximum power reached with a rod acceleration of 0.4 g is 96.5 MW. The total energy released is about 5.8×10^3 BTU which is far below the 8×10^4 BTU which would result in melting at the center line of the fuel element (and probable core destruction.)

If the rod acceleration were 0.750 g the maximum power would be between 80 and 90 MW so that the reduced acceleration does not constitute any significant increase in energy release on a starting accident.

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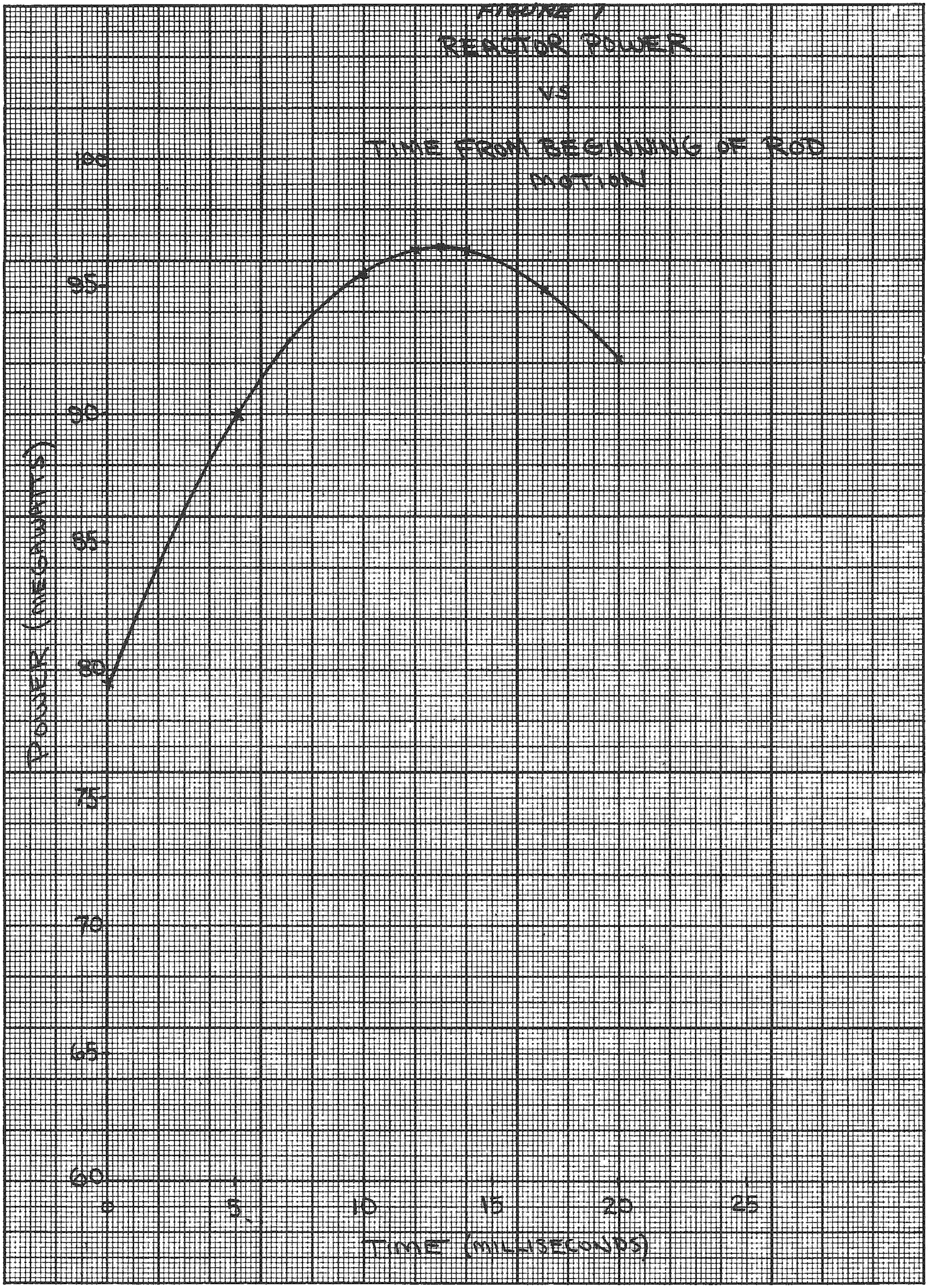

FIGURE 7
REACTOR POWER
VS
TIME FROM BEGINNING OF ROD MOTION

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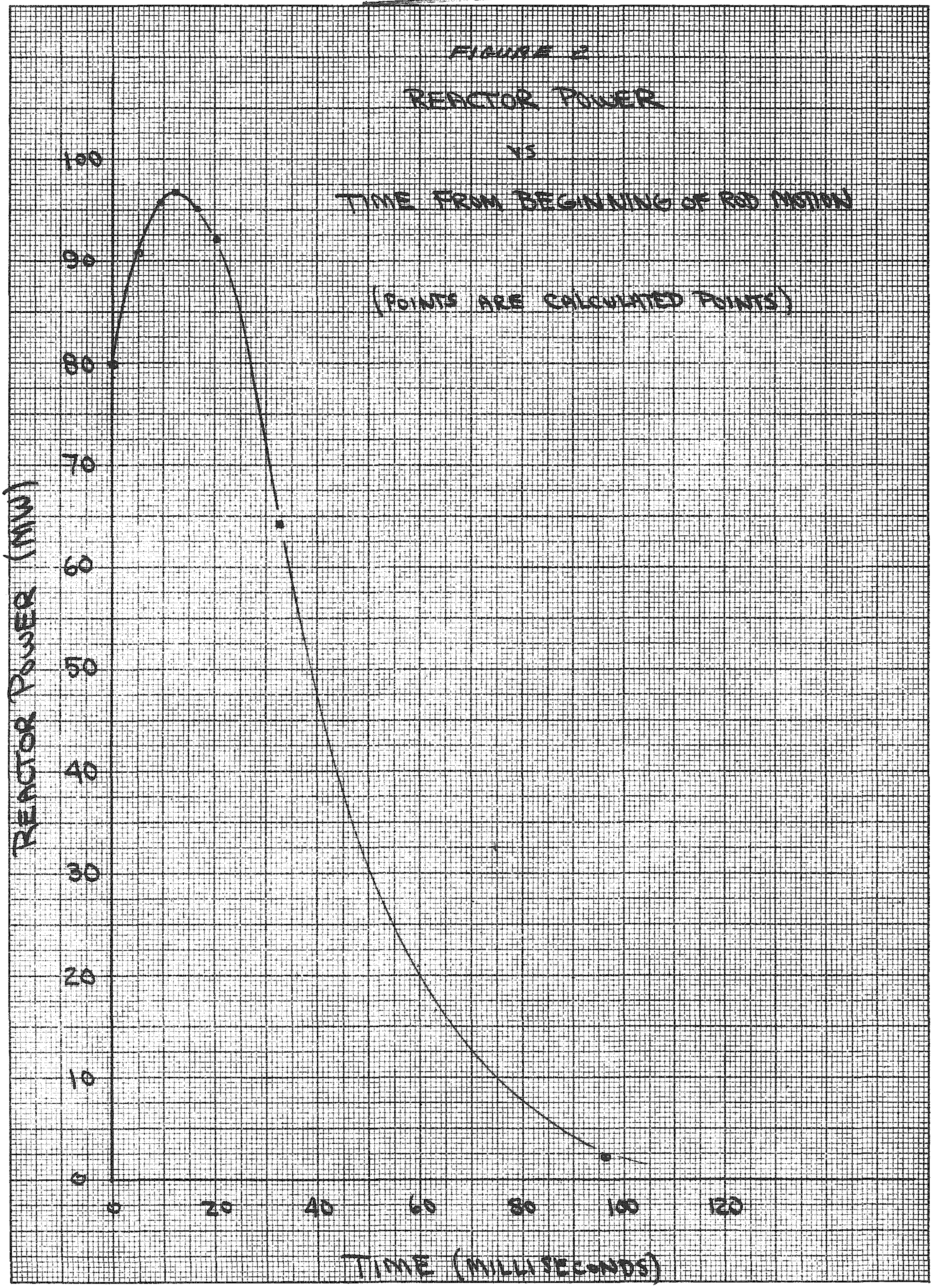

FIGURE 2

REACTOR POWER

VS

TIME FROM BEGINNING OF ROD MOTION

(POINTS ARE CALCULATED POINTS)