MUON PAIR CREATION VIA TWO PHOTON EXCHANGE

IN CHARGED PARTICLE COLLISIONS, II:

HADRON COLLISIONS

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September, 1971

The Enrico Fermi Institute

The University of Chicago, Chicago, Illinois 60637

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MUON PAIR CREATION VIA TWO PHOTON EXCHANGE

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ABSTRACT

Muon pair creation via two photon exchange in charged hadron collisions is discussed based on a numerical calculation as well as on an analytical formula. The form factor and also the large mass of hadrons make this cross section very small for a large invariant mass of the muon pair at low energies. However, at extremely high energies and for not so massive muon pairs the present process becomes comparable with other semi-leptonic processes. This process may be measurable at NAL energies. We also briefly discuss the q^2 -dependence of the missing mass spectrum in the ordinary deep inelastic process based on a muon-parton analogy.

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I. INTRODUCTION

In the previous study¹ we discussed in detail the q^2 -dependence of muon pair creation via two photon exchange in high energy electron scattering. We also established the validity of the "Weizsacher Williams approximation" for massive muon pair creation at large q^2 in the limited kinematical region.

In the present note we would like to discuss the muon pair creation via two photon exchange in charged hadron collisions. This problem has been discussed by Budnev et al.² They estimate the cross section at very large values. We show that the cross section is actually very small. This process is too small to be compared to the massive muon pair production experiment by Christensen et al.³ in the proton-uranium collision. At extremely high energies and for not so massive muon pairs, however, this process becomes important. We expect that this process may be measurable at NAL energies.

We discuss the effects of form factors of hadrons based on a numerical calculation. The proton energy in the above muon pair creation experiment is outside the region where a reliable analytical estimate can be made.

We also point out that the cross section of electron pair creation via two photon exchange is larger than that of muon pair creation by a factor of ~ 2 . On the other hand we do not expect any difference between muon pair and electron pair in the semi-leptonic process like vector meson dominance.⁴

We will also briefly discuss the ordinary deep inelastic process⁵ based on a muon parton analogy. In this case we identify the less massive photon in the two photon process to the gluon in the quark model. We discuss this problem based on an analytical formula derived in I. A similar consideration has been made by Bjorken⁶ when he discussed the transverse momentum distribution of final state hadrons.

II. MUON PAIR CREATION IN HADRON COLLISIONS

a. Introductory Remarks

Christensen et al. observed massive muon pairs in high energy proton-uranium scattering.³ They report a very large cross section for the process. The mass spectrum of the muon pair also exhibits an interesting behavior.

Budnev et al.² recently proposed a possible mechanism of this reaction. In particular they argue that the fourth order electromagnetic process (see Fig. 1)

$$p + p \rightarrow p + p + \mu + \bar{\mu} \quad (\text{II.1})$$

could be dominant for the large invariant mass of the muon pair. They estimate the cross section based on the asymptotic formula given by Landau and Lifshitz.^{2,7}

$$\frac{d\sigma}{d\Omega} \approx \frac{4\alpha^2}{\pi} \frac{\alpha^2}{s^2} (\ln s)^2 \ln\left(\frac{s}{2\Omega}\right) \left[\ln\left(\frac{s}{4\mu^2}\right) - 1\right] \quad (\text{II.2})$$

Based on this formula they estimate the cross section at

$$\frac{d\sigma}{d\Omega} \approx 6 \times 10^{-37} \text{ cm}^2/\text{BeV} \quad (\text{II.3})$$

per proton for the radiated mass of the muon pair at 4 BeV and $E = 30$ BeV.

The notation here is

s : invariant mass of the radiated muon pair

γ : Lorentz factor between the target and projectile particles

μ : muon mass

S : s-channel Mandelstam invariant, $S \equiv 2ME$

E : incident energy of the proton ≈ 30 BeV.

The value of $d\sigma/d\Omega$ in Eq. (II.2) is roughly an order of magnitude smaller than the experimental result.³ However the above authors² point out the possibility of the coherent process, namely the uranium nucleus acts as a whole. If the coherent process dominates the process the value in Eq. (II.2) is multiplied by a factor of $Z \sim 100$, the charge number of the uranium nucleus. Thus one can get an interesting explanation of the observed large cross section. Budnev et al.² also discuss that the above process combined with vector meson dominance may even explain the observed shape of the muon pair spectrum.³

On the other hand, the calculation of the invariant mass of the muon pair in a somewhat related process⁸

$$\gamma + (Z) \rightarrow \gamma + \mu + \bar{\mu} + (Z) \quad (\text{II.4})$$

(one of the photons in Fig. 1 is replaced by a "virtual neutral" weak boson) shows dominance of the very small invariant mass of the muon pair. The rapid decrease of the nuclear (or nucleon) form factors is responsible for the small invariant mass. Therefore, we expect a similar behavior for the process (II.1), although the presence of an extra propagator of the photon in Fig. 1 prevents the direct comparison of these two processes. We estimate the order of magnitude of the process (II.1) by taking into account nuclear (or nucleon) form factors correctly. The result shows a very small value for $d\sigma/d\Omega$ namely

$$\frac{d\sigma}{d\Omega} \leq 10^{-39} \text{ cm}^2/\text{BeV} \quad (\text{III.5})$$

per proton for $\sqrt{s} = 4$ BeV and for the incident energy of the proton at 30 BeV. The coherent process is negligibly small even compared with the result in (II.5); thus we show only the incoherent process in (II.5)

The small value of $d\sigma/d\sqrt{s}$ shown in (II.5) makes the process (II.1) practically useless at low energies. However the calculation illustrates the limitation of the Weizsacker-Williams approximation in a clear cut way.

b. Qualitative Estimate of the Cross Section

In the following we first show necessary qualitative modifications to the formula (II.2), and then we perform a more quantitative calculation of the cross section. The basic ingredient in our argument is kinematics. The minimum momentum transfer in the photon propagators in Fig. 1 is given by

$$|q^2|, |Q^2| \geq M^2 \frac{(\sqrt{s} + M)^2}{s(s - (\sqrt{s} + M)^2)} \quad (II.6)$$

where

$$p(P) + p(b) \rightarrow p(P') + p(b) + \mu\bar{\mu}$$

or

$$\gamma^*(q) + \gamma^*(Q) \rightarrow \mu\bar{\mu} \quad (II.7)$$

and M is the proton mass. This gives

$$|q^2|, |Q^2| \geq 0.3 \text{ (BeV)}^2 \quad (II.8)$$

for $\sqrt{s} = 4$ BeV and the incident energy of the proton at 30 BeV, i.e., $S = 2M \cdot E \approx 60$. For the coherent process, M is replaced by the mass of the uranium nucleus in Eq. (II.6), but Eq. (II.8) is still valid because of the simultaneous appearance of M in $S = 2M \cdot E$. Note that Eq. (II.8) shows the minimum momentum transfer. Due to other phase space constraints, the major contribution to the total cross section at this large $\sqrt{s}$ and small E comes from much larger values of q^2 and Q^2 , typically

$$|q^2| \sim |Q^2| \approx 1 \text{ (BeV)}^2 \quad (II.9)$$

On the other hand the cut-off mass of the coherent electromagnetic form factor of the uranium nucleus is given by⁹

$$m^2 \approx 0.2 \times (1.6 \times 10^{-13} \text{ cm})^2 \sim 0.014 \text{ (BeV)}^2 \quad (II.10)$$

Eq. (II.8) and Eq. (II.10) show that the coherent process is not effective for the massive muon pair production (the form factor suppresses the cross section by a factor of 10^{-5} , or more). Actually, the coherent process ceases the dominant role at $\sqrt{s} \sim 1$ BeV or less. The large momentum transfer expressed in Eq. (II.8) and the form factor eliminate the $(2 \log \gamma)^2$ enhancement factor in Eq. (II.2). Note that $2 \log \gamma = \log \gamma^2$ usually comes from the small virtual photon mass. The second factor in Eq. (II.2) is also modified due to this large minimum momentum transfer to

$$\ln\left(\frac{s}{\mu^2}\right) - 1 \rightarrow \ln\left(\frac{s^2}{|q^2||Q^2|}\right) - 1 \quad (II.11)$$

Furthermore the proton form factor reduces the incoherent cross section by a factor of ~ 10 .

In all these modifications reduce the cross section in Eq. (II.12) by a factor of

$$(2 \ln \gamma)^2 \frac{[\ln(\frac{q^2}{m^2}) - 1]}{[\ln(\frac{q^2}{10^4}) - 1]} \times \frac{1}{10} \sim 500 \quad (\text{II.12})$$

Thus we can understand the small incoherent cross section in Eq. (II.5).

c. Numerical Calculation at low Energies

In the following we show a more quantitative calculation. Our calculation is a sort of modified Weiszacker-William approximation. As usual we first go to the rest frame of the projectile particle (we call this the "projectile frame"). In this frame the nuclear current can be decomposed as

$$\begin{aligned} \langle P | J_\mu^{(v)} | P \rangle &= (P' + P)_\mu f(q^2) \\ &\approx f(q^2) \frac{\sqrt{-q^2}}{\sqrt{v^2 - q^2}} \left\{ \tilde{\epsilon}_\mu^{(0)}(q) + e^{-i\varphi} \epsilon_\mu^{(+)} + e^{i\varphi} \epsilon_\mu^{(-)} \right\} \\ &\quad \times \sqrt{2} \sqrt{E^2 - M^2 \frac{v^2}{(-q^2)}} \end{aligned} \quad (\text{II.13})$$

See Eq. (A-7) in Appendix of I. Eq. (II.13) is expected to be valid for

$$\frac{\sqrt{-q^2}}{v} \ll 1 \quad (\text{II.14})$$

This condition is met in the case of the massive muon pair creation. The momentum transfer q^2 is limited by the form factors whereas v is the same order of magnitude as the invariant mass squared of the massive muon pair. We use Eq. (II.13) in the following although this may slightly underestimate the result for the massive muon pair; for hadron collisions the relative velocity γ is not sufficiently large and the factor $(v^2 - q^2)/v^2$ is not small. We now get the primitive form of the WW approximation.

$$d\sigma \approx \frac{\alpha}{\pi} \frac{d\nu}{\nu} \frac{d\tilde{q}^2}{\tilde{q}^2} \left[1 - \frac{\nu^2}{(-\tilde{q}^2)/\gamma^2} \right] |f(q^2)|^2 d\sigma_Y(q^2, \nu) \quad (\text{II.15})$$

where $d\sigma_Y(q^2, \nu)$ is the cross section induced by the "equivalent" photons with a mass q^2 and energy ν . If one takes the limit $\gamma \rightarrow \infty$ and integrates dq^2/q^2 over an appropriate region, one recovers the familiar WW approximation for the scalar particle

$$d\sigma \approx \frac{\alpha}{\pi} \frac{d\nu}{\nu} \ln\left(\frac{\tilde{u}^2}{18^4 m_{\min}^4}\right) d\sigma_Y(\nu, \nu) \quad (\text{II.16})$$

where $\tilde{u}$ is the typical mass which appears in $d\sigma_Y$. As can be easily checked, $\log(\tilde{u}^2/q_{\min}^2) \sim \log(\gamma^2) = 2 \log \gamma$ for $\gamma \rightarrow \infty$. In our problem $\tilde{u}$ is the muon mass or the rho meson mass and $q_{\min}^2$ is larger than or comparable with $\tilde{u}^2$ for $E = 30$ BeV. See Eq. (II.8). Thus Eq. (II.16) is not valid. However Eq. (II.15) is expected to be valid as far as Eq. (II.14) is satisfied. In the case of proton-proton scattering, the Fermion current modifies Eq. (II.15) as

$$d\sigma = \frac{\alpha}{\pi} \frac{d\nu}{\nu} \frac{dQ^2}{Q^2} F(\nu, Q^2, E) d\sigma_f(Q^2, \nu) \quad (II.17)$$

where

$$F \equiv \frac{1}{4} \left[\frac{4M^2}{(-Q^2)} G_E(Q^2)^2 \mu_p^2 + G_M(Q^2)^2 (1 + \cosh^2 \eta) \right] \left(\frac{\nu}{E} \right)^2 \quad (II.18)$$

and

$$\cosh \eta \approx \frac{\sqrt{-Q^2}}{\sqrt{4M^2 - Q^2}} \left(\frac{2E - \nu}{\nu} \right) \quad (II.19)$$

See also Appendix of I.

The cross section $d\sigma_Y$ is given by

$$d\sigma_f(Q^2, \nu) = \frac{\alpha^3}{P(M\nu)^2} \frac{\Delta(\mu_p, \mu_p)}{\Delta} d\sigma \frac{dQ^2}{(Q^2)^2} \quad (II.20)$$

$$\times \left[\bar{M}^\mu_\mu G_1(Q^2) + \bar{M}^{\mu\nu} b_\mu b_\nu G_2(Q^2) \right]$$

where

$$G_1(Q^2) = \frac{Q^2}{4} \frac{\mu_p^2}{(1 - Q^2/0.71)^4} \quad (II.21)$$

$$G_2(Q^2) = \frac{(1 - Q^2\mu_p^2/4M^2)}{(1 - Q^2/4M^2)} \frac{1}{(1 - Q^2/0.71)^4}$$

with μ_p the magnetic moment of the proton. Other quantities are defined

in Eqs. (II.22) and (II.32) in I. Inside the matrix element $\bar{M}^\mu_\mu$ and $\bar{M}^{\mu\nu} b_\mu b_\nu$ we neglect Q^2 except in the logarithmic factor; this approximation is good in the present process where s is very large. After integration over $\int d\nu$, which can be done analytically, we have two dimensional integration $\int dq^2 dQ^2$ over the momentum transfers for a fixed invariant mass s . This integration has been done by a computer and we get the final result for $\sqrt{s} = 4$ BeV and $E = 30$ BeV

$$\frac{d\sigma_{incoh}}{d\sqrt{s}} \approx 2 \times 10^{-40} \text{ cm}^2/\text{BeV} \quad \text{per proton} \quad (II.22)$$

$$\frac{d\sigma_{coh}}{d\sqrt{s}} \approx 2 \times 10^{-43} \text{ cm}^2/\text{BeV} \quad \text{per proton} \quad (II.23)$$

where $d\sigma_{coh}$ denotes the coherent contribution.

As we expected, the fourth order electromagnetic process gives a very small result. The bremsstrahlung process will also give at most comparable values as in Eq. (II.22). Although the values in Eqs. (II.22) and (II.23) are not particularly accurate at this large momentum transfer region, it definitely shows that the process (II.1) is negligibly small at $E = 30$ BeV.

d. Cross Section at High Energies

It may be interesting to give a crude estimate at which energies the formula in Eq. (II.2) becomes applicable to the hadron collision. This is a purely kinematical problem. According to our calculation in Section II of I, the leading term in Eq. (II.2) alone is not sufficient, but we neglect this correction in the following rough estimate. The necessary kinematical condition would be (for proton-proton scattering)

$$m_p^2 \gg M^2 \frac{v^2}{s^2} \quad (II.24)$$

and

$$m_p^2 \gg M^2 \frac{s^2}{v^2} \quad (II.25)$$

where

v = invariant mass of the muon pair plus one of the final state protons

M = proton mass

m_p = rho-meson mass $\sim$ cut-off mass of the form factors

Eqs. (II.24) and (II.25) are the conditions such that we can regard the nucleon as a point-like particle. As a crude estimate one may take

$$s \gtrsim 20 v \quad \text{and} \quad v \gtrsim 20 s \quad (II.26)$$

i.e., the ratio of the left hand side to the right hand side is about 100 in Eqs. (II.24) and (II.25). This means that we should replace γ in Eq. (II.2) by " γ " = 10.

This is also the condition that the minimum momentum transfer given by Eq. (II.6) is about the same as the muon mass squared.

This estimate gives

$$s \gtrsim 400 s \quad (II.27)$$

or

$$\sqrt{s} \lesssim \frac{1}{20} \sqrt{s}$$

For example,

$$\sqrt{s} \approx 0.4 \text{ BeV for } E = 30 \text{ BeV}$$

$$1.5 \text{ BeV for } E = 500 \text{ BeV}$$

$$3 \text{ BeV for } 30 \text{ BeV}-30 \text{ BeV colliding beam.}$$

(II.28)

For the last case we have ($\gamma \approx 10$ in Eq. (II.2))

$$\frac{d\sigma}{ds} \sim \frac{4\alpha^2}{\pi} \frac{\alpha^2}{s^2} (\ln 10)^2 \ln\left(\frac{s}{2s}\right) \left[\ln\left(\frac{s}{\mu^2}\right) - 1 \right] \quad (II.29)$$

or

$$\frac{d\sigma}{ds} \sim 10^{-35} \text{ cm}^2/\text{BeV} \quad \text{at } \sqrt{s} = 3 \text{ BeV} \quad (II.30)$$

A numerical calculation based on Eq. (II.17) gives the following value for 30 BeV - 30 BeV proton collision at $\sqrt{s} = 3 \text{ BeV}$.

$$\frac{d\sigma}{ds} \approx 2 \times 10^{-35} \text{ cm}^2/\text{BeV} \quad (II.31)$$

This is comparable with the experimental result at $E = 30 \text{ BeV}$.³

Therefore if the cross section does not increase so much with energy due to some reasons, we may have some chance to observe this effect.

We also show the cross section in high energy proton-proton scattering in Fig. 2. As can be easily confirmed, Eq. (II.2) gives a fairly good estimate of the cross section for $\sqrt{s} \leq 1 \text{ BeV}$ at these high energies. The coherent cross section off a uranium target at 500 BeV is also shown in Fig. 3. Note that the coherent cross section is normalized to the cross section per proton. Those values provide a lower bound for muon pair creation based on the most reliable theory of all, namely QED for leptons and hadron elastic form factors for space-like momentum transfers.

The electron mass spectrum in proton-uranium scattering is shown in Fig. 4. The cross section is larger than that of muon pair creation by a factor of ~ 2 . This is expected based on Eq. (II.11). Note that $u = 0$ for the electron. The values of q^2 and Q^2 around m_π^2 give a dominant contribution to the cross section at $E_p = 500 \text{ BeV}$ and $\sqrt{s} \sim \text{a few BeV}$. Therefore

$$\ln \frac{s^2}{16^2 |14^2|} \sim 2 \ln \left(\frac{s}{m_\pi^2} \right) \quad (\text{II.31})$$

III. DISCUSSIONS

a. Two Photon Process

We would like to first discuss the experimental feasibility of lepton pair creation via two photon exchange in hadron collisions. The experimental result at $E_p = 30 \text{ BeV}$ ³ shows that the cross section is about $10^{-32} \text{ cm}^2/\text{BeV}$ at $\sqrt{s} = 1 \text{ BeV}$ and the mass spectrum rapidly decreases with increasing invariant mass. We expect that the value of the cross section at $\sqrt{s} \approx 1 \text{ BeV}$, namely $d\sigma/d\sqrt{s} \sim 10^{-32} \text{ cm}^2/\text{BeV}$ would not change so much when one increases the proton energy. The reason is as follows. The total inelastic cross section of p-p collisions at high energies is expected to be³

$$\sigma_{pp}(\text{inelastic}) \sim 20 \text{ mb} \quad (\text{III.1})$$

The single particle inclusive process to produce a virtual vector particle with a mass around 1 BeV may be lower than (III.1) by a factor of 10 or so. Next the decay probability of the vector meson into a lepton pair is proportional to α^2 , where α is the fine structure constant. We estimate this branching ratio at $\sim 10^{-5}$. Therefore we get a crude estimate (coherent diffractive processes may alter this estimate)

$$\frac{d\sigma}{d\sqrt{s}} \sim 20 \times \frac{1}{10} \times 10^{-5} \text{ mb/BeV} \sim 2 \times 10^{-32} \text{ cm}^2/\text{BeV} \quad (\text{III.2})$$

On the other hand Fig. 4, for example, shows that the mass spectrum of the electron pair for E_p at 500 BeV is exactly the same order of magnitude as the above estimate. Thus we expect that the electron pair experiment at NAL energies would measure the two photon process as well as the ordinary semi-leptonic process. Fig. 4 also shows that the coherent process decreases like

$$\frac{d\sigma}{ds} \propto \left(\frac{1}{\sqrt{s}}\right)^5 \quad (\text{III.3})$$

If the experimental result at 30 BeV in the low invariant mass region³ is valid even at 500 BeV, Eq. (III.3) indicates that the two-photon process can be important up to $\sqrt{s} \approx 3$ BeV. However the experimental tendency³ and also some of the models¹⁰ indicate that the mass spectrum rapidly increases with proton energy and may approach the "scaling limit" at high energies, namely,

$$\frac{d\sigma}{ds} \propto \left(\frac{1}{\sqrt{s}}\right)^3 \quad (\text{III.4})$$

In this latter case our process will be about 10% or so of the dominant semi-leptonic process at $\sqrt{s} \sim 3$ BeV. At much smaller values of s , e.g., $\sqrt{s} \sim 0.5$ BeV, the coherent electron pair spectrum in Fig. 4 gives

$$\frac{d\sigma}{ds} \approx 3 \times 10^{-31} \text{ cm}^2/\text{BeV} \quad \text{per proton} \quad (\text{III.5})$$

This value seems to be big enough to compete with the dominant semi-leptonic process.

For the muon pair process, Fig. 3 shows that the cross section is smaller than that of electron pair by a factor of ~ 2 . Therefore we expect that above consideration is also applicable to muon pair creation.

The characteristic "signal" of the two photon process compared with other background is that we have an energetic proton in the final state. The energies of three energetic final state particles sum to the incident particle energy (this is valid for incoherent as well as

coherent processes).

$$E_{\ell^+} + E_{\ell^-} + E_{p'} = E_p \quad (\text{III.6})$$

For the coherent process at 500-BeV and $\sqrt{s} \sim 3$ BeV, the typical values of those energies may look like

$$E_{p'} \sim 400 \text{ BeV} \quad \text{and} \quad (\text{III.7})$$

$$E_{\ell^+} + E_{\ell^-} \sim 100 \text{ BeV}$$

These values are estimated as follows. The momentum transfer to the nucleus is limited to the region

$$t \leq m_\pi^2 \quad (\text{III.8})$$

where m_π is the pion mass. The energy loss ν of the incident proton must satisfy the relation

$$\nu \cdot \sqrt{s} \geq \Delta \quad (\text{III.9})$$

This gives $\nu \sim 100$ BeV for $s = 10(\text{BeV})^2$ and $t = m_\pi^2$. For the incoherent process we expect that the final state proton is more energetic and the lepton pair is less energetic.

The momentum transfer of the proton in the incoherent process is distributed in the region

$$0.02 (\text{BeV})^2 \leq t \leq 0.5 (\text{BeV})^2 \quad (\text{III.10})$$

for $\sqrt{s} = 3$ BeV and $E_p = 500$ BeV. Eq. (III.10) immediately gives the transverse momentum distribution of the final state proton

$$p_{\perp} \simeq \sqrt{k} \simeq 0.15 \sim 0.7 \text{ BeV} \quad (\text{III.11})$$

It is interesting to recall that the decay branching ratio of a massive photon into a lepton pair does not depend on the final state leptons. Therefore the cross section for muon pair creation in the semi-leptonic process must be the same as that for electron pair creation. On the other hand the fourth order electromagnetic process contains a lepton propagator inside the amplitude (see Fig. 1). This propagator gives rise to different results for electrons and for muons. See also Section II.

For 30 BeV-30 BeV proton colliding beam experiment, Fig. 2 shows that the two photon process decreases like

$$\frac{d\sigma}{ds} \propto \left(\frac{1}{\sqrt{s}}\right)^3 \quad (\text{III.12})$$

for $\sqrt{s} \lesssim 5$ BeV. If the estimate given in Eq. (III.2) is correct, the lepton pair creation via two photon exchange will count about 10% of the dominant process up to $\sqrt{s} \sim 5$ BeV.

b. Semi-Leptonic Processes

Next we would like to discuss some of the implications of our calculation in I to the semi-leptonic processes.

We would first like to discuss the ordinary deep inelastic process⁵ based on our result in Section II of I. In this discussion, we identify the muon with the parton and also the nearly on-shell photon with the vector gluon (see Fig. 5). A similar consideration has been made

by Bjorken⁶ when he discussed the transverse momentum distribution of final state hadrons. The important property we want to discuss is the first factor of Eq. (II.76) in I, namely

$$\frac{d\sigma}{dt ds} \propto \frac{1}{s(s+t)^2} \quad (\text{III.13})$$

where s is the invariant mass of the muon pair. This shows that the invariant mass spectrum is almost uniformly distributed over the range

$$0 \leq s \leq t \quad (\text{III.14})$$

for fixed large $t = -Q^2$. In the ordinary deep inelastic process and in the identification of the muon with the parton, this invariant mass s may correspond to the missing mass when only the final state proton is observed at large ω . Eq. (II.14) suggests that the missing mass spectrum will spread over a very large range for fixed large photon mass Q^2 , eventually up to $-Q^2$. It is actually not so difficult to construct a model of scaling with this property at large ω based on generalized vector meson dominance.¹¹ Preliminary experimental data also indicate this tendency.¹² However, it is still to be checked by experiment whether this kind of tendency continues indefinitely for very large Q^2 .

This problem is also related to the transverse momentum distribution of the final state hadrons. Eq. (III.14) suggests that the transverse momentum of the secondaries with respect to the virtual photon would spread up to

$$p_{\perp} \lesssim \sqrt{-q^2} \quad (\text{III.15})$$

This was discussed by Bjorken⁶. However at high energies the lowest order calculation is certainly not sufficient. In particular, the Pomeron-like behavior of the photon-photon amplitude would require a diagram shown in Fig. 6 if one makes a QED analogy.¹³

Under these conditions, the situation will become more complicated. Actually, Bjorken⁶ indicates how to translate Eq. (III.15) into an actual hadron distribution for very large Q^2 . In any case, it is interesting to see whether Eqs. (III.14) and (III.15) are satisfied at large Q^2 in high energy muon scattering on the proton target.

Another interesting implication is obtained from Eq. (III.28) in I, namely

$$\frac{d\sigma}{ds dt} \propto \frac{1}{s(s+t)^2} \quad (\text{III.16})$$

It is interesting to see that the role of s and t is just exchanged in Eq. (III.13) and Eq. (III.16). The bremsstrahlung mechanics now shows that

$$0 < t \leq s \quad (\text{III.17})$$

for large fixed s . If a massive particle is emitted, the particle which emits it suffers a very large angle scattering. In the experiment of massive muon pair creation in hadron collisions one may check this property. If the muon pair is emitted via bremsstrahlung¹⁴, we expect that some of the final state hadrons are scattered into the far off-forward direction. On

the other hand, if the parton annihilation mechanism or the like is the dominant process we expect that the final state hadrons are scattered predominantly into the forward direction based on Eq. (III.13). If one only measures the muon pair, the transverse momentum distribution of the di-muon system may retain some information on this mechanism. The di-muon would be scattered into a wide range of angle via bremsstrahlung, whereas it would be scattered predominantly into the forward direction via parton annihilation. However, this check may not be decisive. The reason is that the two photon exchange process also prefers the similar configuration as bremsstrahlung when a massive particle is emitted at fixed incident energy. We discussed this tendency in Section II of paper I. This means that we should measure the muon pair at far above the threshold in high energy collisions to test the possible mechanism of pair creation.

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FIGURE CAPTIONS

Fig. 1 Muon pair creation via two space-like photon exchange in charged hadron scattering.

Fig. 2 The mass spectrum of the muon pair produced in proton-proton collision. The lowest curve corresponds to 500 BeV proton-proton scattering in the lab. frame.

Fig. 3 The mass spectrum of the muon pair produced in proton- U^{238} collision. The coherent cross section is normalized to the cross section per proton.

Fig. 4 The mass spectrum of the electron pair produced in proton- U^{238} collision. The coherent cross section is normalized to the cross section per proton.

Fig. 5 Parton-Antiparton creation in deep inelastic process.

Fig. 6 A QED diagram which simulates a Pomeron-like behavior of the photon-photon scattering amplitude.

Fig.1

Fig.5

Fig.6

Fig.2

Fig.3

Fig.4