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THE EFFECT OF GAMMA HEATING ON THE
APPR-1 PRESSURE SHELL

By
H. R. Kroeger
I. M. Neou
J. L. Meem

September 1956

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THE EFFECT OF GAMMA
HEATING ON THE APPR-1
PRESSURE SHELL

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Work performed under Contract No. AT(11-1)-318.

September 1956

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I SUMMARY AND CONCLUSIONS

For the reactor - thermal shield - pressure vessel configuration of APPR-1, the computed maximum stress in the pressure vessel is 16820 psi at an operating power of 10 Mw and a pressure of 1400 psig. The thermal stress portion of the above total stress is computed to be not greater than 2640 psi at a point even with the reactor midplane. This stress is due to a temperature difference in the pressure vessel wall of 13.8 F° which results from a computed equivalent energy incident on the cladding of 43550 Btu/ft³-hr. This value may be compared to corrected values of 45500 and 42400 Btu/ft³-hr found in tests at the LITR and the MTR respectively.

The precise computation of energy values for use in thermal stress analysis suffers by general comparison of methods used in shielding. In computing the required thickness of a biological shield one can make all assumptions conservative without greatly affecting the thickness. For example if the total dose is overestimated by a factor of 10 the required shield thickness would be only about 20% greater. On the other hand an over-estimate by a factor of 10 in thermal stress analysis might easily indicate a completely non-feasible design. It is believed that the present computations are somewhat conservative but not overly so. It is suggested that the APPR-1 critical assembly and the APPR-1 itself be

as thoroughly instrumented as feasible in order to check the degree of conservatism. This information should be extremely valuable in the construction of later models of APPR.

II INTRODUCTION

This report is the culmination of several months of detailed study of the problem of thermal stress in a reactor pressure vessel.

Initially a reasonable value of the total maximum stress in the APPR-1 was desired. Work toward this end uncovered numerous uncertainties and variations in the data and methods available for handling this problem. A second objective appeared naturally. The report should also be a "guide book" for the computation of pressure vessel stress in other reactor configurations. In addition, some examinations of "off design point" configurations are reported in the appendix. A review of these results should enable the designer to easily specify an optimum configuration.

In its present form, this study is the result of the contribution, and whole-hearted co-operation of Alco's reactor science section and ASTRA's technical groups. In addition, particular thanks are due John Haines of Alco and Dr. Willard Berggren, ASTRA associate, for their timely suggestions.

III THE GAMMA FLUX

The total radiation projected by an operating reactor is made up of neutrons, alpha and beta particles, and gamma rays. Because of the very short mean free paths associated with alpha and beta particles their energy (heat effect) may be assumed to be entirely within the core. The neutrons may be broadly separated into two groups, fast and thermal. The direct heating effect of neutrons at the pressure vessel will be negligible. The indirect effect, due to the production of capture and inelastic scatter gamma rays may be considerable and must be accounted for.

The gamma radiation from an operating reactor depends upon the relative number and energy of (1) fission gammas (2) capture gammas, (3) decay gammas from fission products, and (4) decay gammas from irradiated reactor materials. The gamma field outside of the reactor core depends upon the distance from the core, the material interposed between the core and the detector, and the neutron flux external to the reactor.

The prediction of a reactor gamma spectrum from basic physical data is a rather laborious job and the results may contain marked uncertainties.

For this analysis it was assumed that the spectrum (experimental) of the Bulk Shielding Reactor could be adequately corrected

to APPR-1 conditions. The gamma spectrum of the BSR has been measured at various distances from the reactor. It would be expected that the APPR-1 spectrum would be similar but with two exceptions:

(1) Because of the high fuel loading of the APPR-1, the flux per unit of power is less and the total number of capture gammas (2.23 mev) in the water would be much smaller.

(2) Although the capture gammas from iron and aluminum are comparable (7.63 and 7.724 mev) the APPR-1 contains a greater volume of iron, there are more iron nuclei per cm^2 , and each nucleus has a much higher cross section than aluminum. Hence an increased number of gammas at about 7.7 mev would be expected in the APPR-1 despite the lower flux.

A. THE PRIMARY GAMMA FLUX

The measured spectrum of the BSR at various distances from the core is shown in Figure 1.⁽¹⁾ These data were converted to four groups of gammas for ease in handling. These groups (1, 3, 5, and 7 mev nominal energy) are shown in Figure 2. Similarly the attenuation of the gamma dose (measured) from the BSR is shown in Figure 3. Since dose measurements are considered much more reliable than the spectrum measurements, the gamma groups shown in Figure 2 were normalized to agree with the dose data. (Figure 4). Conversion of gamma flux to dose made use of the factors⁽²⁾ in Figure 5.

The inside of the pressure vessel wall of the APPR-1 is 12.75 inches from the equivalent core surface. The normalized gamma flux at this distance from the BSR (and at 10 Mw) is given below:

Group (mev)	Gamma Flux (Photons/cm ² -sec)
1	1.46×10^{13}
3	3.20×10^{12}
5	5.85×10^{11}
7	1.98×10^{11}

The above spectrum is due to all gammas which arrive at 12.75 inches from the core surface. In order to find the contribution of the core gammas only, the capture gammas (2.2 mev) in the water must be deducted from Group 3.

Gamma Flux Spectrum of the Bulk Shielding Reactor

Figure 1

FIGURE 3

1. HXK 5-3-56

The capture gammas can be determined from the expression

$$I = \frac{\phi_0 C e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right] \quad (\text{see appendix})$$

$$\text{where } \phi_0 = 1.25 \times 10^{14} \quad (\text{Figure 6})$$

$$C = a = 0.0143 \quad (\text{at } 80^\circ\text{F})$$

$$\mu = 0.0458 \quad (\text{at } 80^\circ\text{F})$$

$$t = 12.75 \text{ in.} = 32.4 \text{ cm.}$$

From Figure 6,

$$e^{-Ax} = \frac{\phi}{\phi_0} = \frac{8.2 \times 10^{11}}{1.25 \times 10^{14}}$$

$$e^{-Ax} = .00655$$

$$Ax = 5.03$$

$$A = \frac{5.03}{32.4} = 0.155$$

$$I = \frac{1.25 \times 10^{14} \times 0.0143 \times e^{-1.48}}{2(0.155 - 0.046)} (1 - e^{-3.53})$$

$$I = \frac{1.25 \times 10^{12} \times 1.43}{0.218} \times 0.228(1 - 0.0293)$$

$$I = 1.81 \times 10^{12}$$

The core contribution in group 3 is then $(3.20 - 1.81) \times 10^{12}$
 $= 1.39 \times 10^{12}$ and the equivalent core spectrum at 12.75
inches is:

Group	Gamma Flux
1	1.46×10^{13}
3	1.39×10^{12}
5	5.85×10^{11}
7	1.98×10^{11}

Note that the above group 3 is somewhat conservative since
buildup was not considered.

The APPR-1 gamma flux would differ from that of the BSR for
several reasons: (1) the attenuation from the reactor core
is through hot rather than cold water, (2) the reactor area
the pressure vessel can "see" is somewhat larger, (3) the
power density is lower, (4) the self-absorption properties
of the core are greater, and (5) the gamma spectrum is
different due to the different materials on the core.

One method of allowing for the above differences is outlined
below.

1. The above BSR spectrum (core only) at 12.75 inches
from the core surface can be converted to a uniform
volume source of an equivalent cylindrical core.

2. Synthetic volume sources can be constructed for both BSR and APPR-1 type cores. The ratio of these values, which involve the power density and spectrum corrections, times the converted BSR volume source will provide an equivalent APPR-1 volume source normalized to measured gamma dose rates.

3. The gamma flux at the APPR-1 pressure vessel can be computed from the above volume source with due allowance for core size, self absorption, attenuation through the reflector and thermal shield, and buildup.

4. The equation and geometrical considerations for the transformation from a cylindrical volume source to an equivalent flux are given in Figure 7.

$$I(a) = \frac{BS_v R^2 F(\theta, b)}{2(a + z)} \quad (\text{from ref. (3), APEX 176 page 72})$$

where I is the gamma flux in photons/cm²-sec

S_v is the core volume source in photons/cm³-sec

R is the core radius in cm.

$$F(\theta, b) = \int_0^\theta e^{-b \sec \theta} d\theta \quad (\text{values from APEX 176})$$

$$b = \mu_w(a - t) + \mu_{ss} t + \mu_c z$$

a is the core-to-vessel distance in cm.

t is the thermal shield thickness in cm.

z is an effective distance due to self absorption
(from APEX 176 page 73 - a function of $\mu_c R$)

B is the build-up factor

FIGURE 7

STEP 1.

The equivalent volume source for the BSR is obtained from the expression:

$$S_v = \frac{2(a + z) I(12.75)}{B R^2 F(\theta, b)}$$

The BSR assembly considered (Loading 22A) contained a volume⁽⁴⁾ of 114,840 cm³ and was 61 cm in height. The squared equivalent radius (R²) is then $\frac{114840}{61} = 598 \text{ cm}^2$. The metal/water ratio of the BSR is 0.654 and the volume fraction of the constituents are H₂O : 0.605, AL : 0.393, and U : 0.002. The equivalent core absorption coefficient can be computed from the above volume fractions and operating densities. If the BSR operating temperature is 90°F:

$$\mu_c = 0.605 \times \frac{0.01602^*}{0.01610} \mu_w + 0.393 \mu_{Al} + 0.002 \mu_U$$

Group	$0.602 \mu_w^{(6)}$	$0.393 \mu_{Al}^{(6)}$	$0.002 \mu_U^{(6)}$	μ_c
1	0.0421	0.0656	0.0028	0.1105
3	0.0235	0.0370	0.0016	0.0621
5	0.0181	0.0299	0.0017	0.0497
7	0.0157	0.0268	0.0018	0.0443

The dimension z can be found from the quantity $\mu_c z^{**}$ which in turn is determined by $\mu_c R$.

* Reference (5).

** Reference (7).

Group	μ_{cR}	μ_{cz}	z	(a + z)
1	2.7	1.37	12.4	44.8
3	1.52	.84	13.5	45.9
5	1.21	.68	13.7	56.1
7	1.08	.62	14.0	46.4

The angle θ can be determined from the relationship

$$\tan \theta = \frac{h/2}{a + z}, \text{ where } h \text{ is the core height.}$$

Group	h/2	(a + z)	$\tan \theta$	θ
1	30.5	44.8	0.68	34°
3	30.5	45.9	0.665	34°
5	30.5	46.1	0.66	34°
7	30.5	46.4	0.66	34°

The function b is evaluated from the expression $b = \mu_{wa} + \mu_{cz}$.

Assuming the water outside the BSR core is at 80°F, the density

$$\text{factor } \frac{0.01602}{0.01608} = 0.995 \text{ is applicable.}$$

Group	$0.995 \mu_{wa}$	μ_{cz}	b	B
1	2.26	1.37	3.63	6.4
3	1.26	.84	2.10	2.45
5	.97	.68	1.65	1.81
7	.84	.62	1.46	1.61

Buildup factors for water and aluminum differ only slightly.

Thus the above value of B based on b and the factor for water will be just a little conservative (high).

The equivalent cylindrical volume source for the BSR may now be computed.

Group	$I_{(12.75)}$	B	$2(a + z)$	R^2	$F(\theta, b)$	$S_v(\text{BSR})$
1	1.46×10^{13}	6.4	89.6	598	0.0126	2.71×10^{13}
3	1.39×10^{12}	2.45	91.8	598	0.065	1.34×10^{12}
5	5.85×10^{11}	1.81	92.2	598	0.104	4.78×10^{11}
7	1.98×10^{11}	1.61	92.8	598	0.127	1.50×10^{11}

The above volume spectrum is in photons per $\text{cm}^3\text{-sec}$ at 10 Mw reactor power.

STEP 2

Synthetic (or theoretical) volume spectra can be built up of the fission gammas (proportional to power), capture gammas (proportional to thermal flux) and inelastic scatter gammas (proportional to fast flux) produced in the core.

The spectrum of fission gammas⁽⁸⁾ is:

Group	Gammas/Fission	Gammas/sec-kw
1	9.31	2.88×10^{14}
3	0.75	2.32×10^{13}
5	0.099	3.07×10^{12}
7	0.0183	5.67×10^{11}

The above spectrum includes prompt gammas, fission product gammas, and U^{235} capture gammas. All gammas of greater than 6 mev energy were lumped in group 7.

In addition to the above, the BSR spectrum includes contributions from the aluminum and the water. (9)

	Energy Group	Photons/sec-kw
Aluminum 28 decay	1	2.89×10^{12}
Water capture	3	8.63×10^{12}
Aluminum capture	3	3.66×10^{12}
" "	5	6.67×10^{11}
" "	7	1.0×10^{12}

The synthetic BSR volume spectrum (photons/cm³-sec) may be computed as below. (for 10 Mw reactor power) The BSR active core volume is 1.148×10^5 cm³

Group	Fission	Material	S _v (BSR synthetic)
1	2.51×10^{13}	0.025×10^{13}	2.54×10^{13}
3	2.02×10^{12}	1.07×10^{12}	3.09×10^{12}
5	2.67×10^{11}	$.58 \times 10^{11}$	3.25×10^{11}
7	4.94×10^{10}	8.71×10^{10}	1.36×10^{11}

For APPR-1, contributions from the stainless steel replace the contributions due to aluminum. In addition the contribution from the water will change due to a different water volume fraction and a different operating density.

For APPR-1 the effective core consists of 45 element each 3 x 3 x 22 inches. The core volume is 1.46×10^5 cm³. The metal/water ratio is 0.256.

The contribution of the various constituents in the stainless steel will be proportional to the average neutron flux and can be estimated from the expression:

$$I = \phi C$$

where ϕ_{th} is the average thermal flux in the core.

C is the unit gamma contribution = $\sum_a F_v F_\gamma$

$\sum_a$ is the macroscopic thermal absorption cross section of the solid constituent corrected for Maxwell-Boltzman distribution and for temperature

F_v is the volume fraction of the stainless steel constituent in the core

F_γ is the fraction of gammas emitted per neutron absorption.

Stainless steel is assumed to be 18% Cr, 8% Ni and 74% Fe by weight. For the APPR-1 the average thermal flux is 1.66×10^{13} n/cm²-sec (from Figure 8). The stainless steel occupies 18.7% of the core volume, the water 79.7% and the fuel 1.6%.

$$C_{Cr} = (0.20 \times 0.889 \times 0.76)(0.187 \times 0.18 \times \frac{791}{6.92}) F_\gamma$$

$$C_{Cr} = (0.135)(0.0384) F_\gamma$$

$$C_{Cr} = 0.00518 \times F_\gamma \text{ gammas/cm}^3\text{-sec}$$

Similarly

$$C_{Fe} = (0.21 \times 0.889 \times 0.76)(0.187 \times 0.74 \times \frac{7.91}{7.85}) F_\gamma$$

$$C_{Fe} = (0.142)(0.139) F_\gamma$$

$$C_{Fe} = 0.0197 \times F_\gamma$$

FIGURE 8

and

$$C_{Ni} = (0.41 \times 0.889 \times 0.76)(0.187 \times 0.08 \times \frac{7.91}{8.9}) F_{\gamma}$$

$$C_{Ni} = (0.277)(0.0133) F_{\gamma}$$

$$C_{Ni} = 0.00368 \times F_{\gamma}$$

The fraction of gammas resulting from one absorption is given below⁽¹⁰⁾ for the major constituents of stainless steel.

Group	Cr	Fe	Ni
1	0.45	0.05	--
3	0.14	0.17	0.07
5	0.15	0.23	0.22
7	0.78	0.61	0.87

The total capture gamma contribution of the stainless steel is tabulated below.

Group	Chromium	Iron	Nickel	Total ($\gamma/\text{cm}^3\text{-sec}$)
1	3.88×10^{10}	0.163×10^{11}	--	5.51×10^{10}
3	1.21 "	0.555 "	0.43×10^{10}	7.19×10^{10}
5	1.29 "	0.752 "	1.34 "	1.01×10^{11}
7	6.73 "	1.99 "	5.31 "	3.19×10^{11}

The contribution due to inelastic scattering of fast neutrons can be estimated in a similar manner. The effective fast flux is assumed to be the same as for the BSR⁽¹¹⁾, i.e.

1×10^{14} n/cm²-sec. The inelastic scattering cross sections

and resulting gamma energies for the stainless steel constituents are:

Material	Cross Section	Energy	Reference
Chromium	0.112 cm ⁻¹	1.4 MEV	(12), (13)
Iron	0.102 "	0.9 "	(13)
Nickel	0.073 "	1.45 "	(13), (14)

It is assumed that each scattering process results in one gamma photon.

$$I_{Cr} = (1 \times 10^{14})(0.112)(0.0384) = 0.43 \times 10^{12}$$

$$I_{Fe} = (1 \times 10^{14})(0.102)(0.139) = 1.42 \times 10^{12}$$

$$I_{Ni} = (1 \times 10^{14})(0.073)(0.0133) = \underline{0.097 \times 10^{12}}$$

$$\text{Total inelastic scatter gammas} = 1.947 \times 10^{12} \text{ } \gamma/\text{cm}^3\text{-sec}$$

The contribution due to the capture gammas from the water can be obtained from a comparison with BSR data.

$$I_W = \left(\frac{8.63 \times 10^{12} \times 10^4}{1.148 \times 10^5} \right) \left(\frac{\phi \text{ (APPR)}}{\phi \text{ (BSR)}} \right) \left(\frac{\rho \text{ (APPR)}}{\rho \text{ (BSR)}} \right) \left(\frac{M/W \text{ (APPR)}}{M/W \text{ (BSR)}} \right) \left(\frac{\sum a \text{ (APPR)}}{\sum a \text{ (BSR)}} \right)$$

$$I_W = 7.52 \times 10^{11} \left(\frac{1.66 \times 10^{13}}{1 \times 10^{14}} \right) \left(\frac{0.835}{0.996} \right) \left(\frac{1.654}{1.256} \right) \left(\frac{a \times 0.889 \times 0.76}{a \times 0.889 \times 1} \right)$$

$$I_W = 1.05 \times 10^{11} \text{ photons/cm}^3\text{-sec}$$

The APPR-1 synthetic volume spectrum may now be constructed as below: (based on 10 Mw and a core volume of $1.46 \times 10^5 \text{ cm}^3$).

Group	Fission	Inelastic Scatter	Capture Water	St.St.	S _v (APPR-1 synthetic)
1	1.97×10^{13}	0.195		0.006	2.17×10^{13}
3	1.52×10^{12}		0.105	0.072	1.70×10^{12}
5	2.10×10^{11}			1.01	3.11×10^{11}
7	3.88×10^{10}			31.9	35.78×10^{10}

The APPR-1 volume source normalized to BSR experimental results is computed below:

Group	S _v (BSR)	$\frac{\text{APPR-1 (syn)}}{\text{BSR (syn)}}$	S _v (APPR-1)
1	2.71×10^{13}	0.854	2.31×10^{13}
3	1.34×10^{12}	0.554	7.42×10^{11}
5	4.78×10^{11}	0.957	4.57×10^{11}
7	1.50×10^{11}	2.63	3.94×10^{11}

STEP 3

The above normalized volume source of the APPR-1 can be transposed to a gamma flux at the inside surface of the pressure shell by the expression (from Figure 7)

$$I_{12.75} = \frac{S_v B R^2 F(\theta, b)}{2(a + z)}$$

For APPR-1 the equivalent core radius is 28 cm. $R^2 = 784 \text{ cm}^2$. The volume fraction of the core constituents are $H_2O : 0.797$, St.St. : 0.187, and U : 0.016. The equivalent core absorption coefficient can be computed from the above volume fraction and from the operating densities.

$$\mu_c = 0.797 \frac{0.01602^*}{0.01920} \mu_w + 0.187 \mu_{ss} + 0.016 \mu_U$$

Group	$0.665 \mu_w$	$0.187 \mu_{ss}$	$0.016 \mu_U$	μ_c
1	0.0466	0.0916	0.0224	0.1606
3	0.0260	0.0529	0.0128	0.0917
5	0.0199	0.0455	0.0136	0.0790
7	0.0173	0.0434	0.0144	0.0751

The dimension z can be found from the quantity $\mu_c z^{(7)}$ which in turn is determined by $\mu_c R$

Group	$\mu_c R$	$\mu_c z$	z	(a + z)
1	4.50	1.96	12.2	44.6
3	2.56	1.30	14.2	46.6
5	2.21	1.16	14.7	47.1
7	2.10	1.12	14.9	47.3

The angle θ can be determined from the relationship

$$\tan \theta = \frac{h/2}{a + z} ; h/2 = 11 \text{ in} = 28 \text{ cm.}$$

*Reference (5).

Group	$\tan \theta$	θ
1	0.628	32°
3	0.601	31°
5	0.594	31°
7	0.592	31°

The function b is evaluated from the expression

$b = \mu_w(a - t) + \mu_c z + \mu_{sst}$. If the average temperature of the water outside the core is 432°F (at 1200 psia) the density factor is $\frac{0.01602}{0.01908} = 0.842$.

Group	$(0.842)\mu_w(a-t)$	$\mu_c z$	$(\mu_w a + \mu_c z)$	μ_{sst}	b
1	1.59	1.96	3.55	2.62	6.17
3	0.887	1.30	2.187	1.53	3.72
5	0.682	1.16	1.842	1.31	3.15
7	0.591	1.12	1.711	1.25	2.96

(t above is the thickness of the thermal shield (2") plus the flow separator (1/8") or 5.4 cm)

Group	$B_w(a-t + z)$	$B_{s.s.}$	B_{Total}
1	6.6	3.5	23.1
3	2.52	1.87	4.71
5	1.94	1.52	2.95
7	1.71	1.39	2.38

Group	S_v	B	R^2	$2(a + z)$	$F(\theta, b)$	$I_{12.75}$
1	2.31×10^{13}	23.1	784	89.4	0.00084	3.93×10^{12}
3	7.42×10^{11}	4.71	784	93.6	0.0109	3.29×10^{11}
5	4.57×10^{11}	2.95	784	94.4	0.0199	2.23×10^{11}
7	3.94×10^{11}	2.38	784	94.8	0.0240	1.86×10^{11}

B. THE SECONDARY GAMMA FLUX

The total gamma flux outside the core will include contributions from the capture of thermal neutrons and the inelastic scattering of fast neutrons. The thermal shield and flow separator (of stainless steel) effectively reduce the primary gammas but produce secondary gammas proportional to the neutron flux values. The pressure shell itself will produce gammas which in turn may be absorbed in the pressure vessel. In addition the neutrons absorbed in the water produce 2.2-mev capture gammas.

The secondary gammas are directly dependent on the magnitude and attenuation of the neutron fluxes. A reasonably precise exploration of flux values would involve considerable computation and would require experimental verification. This has been done for APPR-1 for the case of the thermal flux through an all-water reflector.

1. Thermal flux

The thermal flux from APPR-1 through an all-water reflector is shown in Figure 8. The flux at the outside surface of the thermal shield is estimated (from LID tank data⁽¹⁵⁾) to be $1/3$ of that for an all-water reflector. The effective thermal flux attenuation in the shield was assumed equal to e^{-Ax} , where $A = 1/L$ and L is the diffusion length in stainless steel. Figure 9 shows the details of the flux computation within the thermal shield.

$$A = \frac{1}{L} = \sqrt{3 \Sigma_s \Sigma_a}$$

for stainless steel at 480°F.

$$\Sigma_s = 0.93 \text{ cm}^{-1}$$

$$\Sigma_a = 0.170 \text{ cm}^{-1}$$

$$A = \frac{1}{L} = \sqrt{3 \times 0.93 \times 0.17} = \sqrt{0.474} = 0.689$$

For the outward-bound neutrons

$$\phi_1 = 2 \times 10^{13}$$

$$\phi_2 = 2 \times 10^{13} \times e^{-5.08 \times 0.689}$$

$$= 2 \times 10^{13} \times e^{-3.5} = 2 \times 10^{13} \times 0.0302 = 6.04 \times 10^{11}$$

$$\phi_2' = 3.3 \times 10^{12} - 6.04 \times 10^{11} = 2.7 \times 10^{12}$$

$$\phi_1' = 2.7 \times 0.0302 \times 10^{12} = 8.15 \times 10^{10}$$

The thermal flux level in the pressure vessel wall can be computed in a similar manner. However, it was assumed there was no source of thermal neutrons outside the pressure vessel and the total flux at any point is the incident flux (2×10^{12}) times e^{-Ax}

2. Fast flux

The general term "fast flux" is not necessarily a precise term. For purposes of reactor computation it may include all neutrons not thermal. If, however, the production of inelastic scatter gammas is of interest, the term should

refer only to those neutrons above about 0.1 mev. It is assumed in this analysis that the number of neutrons computed from the fast neutron dose curve (ORNL 11199) best represents the neutrons which can produce inelastic scatter gammas. The conversion factor used was 8.7 neutrons/cm²-sec equivalent to 1 MREP/HR. The BSR flux values were corrected for the higher water temperature. Figure 10 compares various estimates of "fast flux" which are not necessarily comparable. The attenuation of the fast neutrons by the thermal shield was based on the use of effective removal cross sections⁽¹⁶⁾ ($\sigma_R = 2$ barns)

$$\Sigma_R = 2 \times 10^{-24} \times \frac{0.602 \times 10^{24} \times 7.91}{56}$$

$$= 0.17 \text{ cm}^{-1}$$

$$\phi = \phi_0 e^{-\Sigma_R x}$$

$$\phi = 2.2 \times 10^{12} \times e^{-0.17 \times 5.08}$$

$$\phi = 2.2 \times 10^{12} \times e^{-0.863} = 2.2 \times 10^{12} \times 0.422$$

$$\phi = 9.3 \times 10^{11}$$

The above results, together with the thermal flux, are shown plotted in Figure 11. These estimated flux levels provide the magnitude and slope terms necessary to the computation of capture and inelastic scatter gammas.

FIGURE 11

HRK 6-26-56

3. Capture-gamma production

It can be shown (see appendix A-1) that the capture gamma rays leaving a slab of finite thickness are related to the incident neutron flux by the expression:

$$I = \frac{\phi_0 C e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

where I is the gamma flux at the slab surface
opposite the incident neutron flux

ϕ_0 is the incident neutron flux

C is the gamma ray contribution of the
material involved and relates the
absorption cross section, the fraction
of gammas produced per absorption, and
the volume fraction of the material
present.

A is the attenuating coefficient of the
neutrons in the material

μ is the linear absorption coefficient for
gammas

and t is the slab thickness in cm.

In order to determine the contribution to the heat generation in the pressure vessel the above gammas must be attenuated through interposed material.

a. Capture gammas from the flow separator.

The flow separator is a 0.125-inch thick stainless steel sheet immediately adjacent to the core. Because of the thinness of the slab (sheet) it is assumed that the neutron flux is constant ($\phi_0 = 2.6 \times 10^{13}$, $A = 0$)

$$I = \frac{2.6 \times 10^{13} \times C \times e^{-\mu t}}{2(-\mu)} (1 - e^{\mu t})$$

$$I = \frac{2.6 \times 10^{13} \times C}{2\mu} (1 - e^{-\mu t})$$

$$C = \sum_a \times F_v \times F_\gamma$$

$$C_{Cr} = (0.20 \times 0.889 \times 0.76)(0.18 \times \frac{7.91}{6.92}) F_\gamma = 0.0278 F_\gamma$$

$$C_{Fe} = (0.21 \times 0.899 \times 0.76)(0.74 \times \frac{7.91}{7.85}) F_\gamma = 0.106 F_\gamma$$

$$C_{Ni} = (0.41 \times 0.889 \times 0.76)(0.08 \times \frac{7.91}{8.9}) F_\gamma = 0.0197 F_\gamma$$

From page 29 F_γ is:

Group	Cr	Fe	Ni
1	0.45	0.05	--
3	0.14	0.17	0.07
5	0.15	0.23	0.22
7	0.78	0.61	0.87

and $C_{(total)}$ is:

Group	C_{Cr}	C_{Fe}	C_{Ni}	C_{total}
1	0.0125	0.0053	--	0.0178
3	0.0039	0.0180	0.0014	0.0233
5	0.0042	0.0244	0.0043	0.0329
7	0.0217	0.0647	0.0171	0.1035

Group	μ	μt	$e^{-\mu t}$	$(1 - e^{-\mu t})$	I
1	0.49	0.156	0.856	0.144	6.80×10^{10}
3	0.283	0.090	0.914	0.086	9.21×10^{10}
5	0.244	0.0775	0.925	0.075	1.31×10^{11}
7	0.232	0.0738	0.929	0.071	4.12×10^{11}

The above gammas will be attenuated by 10.625 inches of water and 2 inches of stainless steel (thermal shield). The attenuation due to the water can be obtained from BSR data corrected for the lower water density. From the conventional expression for attenuation:

$$\frac{I}{I_0} = f(R) B e^{-\mu x}$$

$$\text{and } \frac{I'}{I_0} = f(R) B' e^{-\mu' x}$$

$$\text{then } I' = I \frac{B' e^{-\mu' x}}{B e^{-\mu x}}$$

where I' results from attenuation through hot
(430°F) water

and μ' is the linear absorption coefficient in hot
water = 0.842 μ

From Figure 4 ($x = 13$ inches)

Group	I	μx	B	$e^{-\mu x}$	$\mu' x$	B'	$e^{-\mu' x}$	I'
1	0.052 I_0	2.30	4.2	0.100	1.95	3.7	0.142	0.065 I_0
3	0.100 I_0	1.29	1.9	0.275	1.09	1.75	0.336	0.113 I_0
5	0.112 I_0	1.00	1.53	0.368	0.85	1.49	0.427	0.127 I_0
7	0.127 I_0	0.85	1.4	0.427	0.72	1.37	0.487	0.142 I_0

The attenuation due to the stainless steel can be computed using
the conventional expression:

$$I/I_0 = f(R) \cdot B \cdot e^{-\mu x}$$

for the APPR-1 configuration

$$x = 5.08 \text{ cm.}$$

$$f(R) = \frac{6.5 + 11}{8.5 + 11}^2 = 0.806$$

Group	μ	μx	$e^{-\mu x}$	B	I/I_0
1	0.49	2.49	0.083	3.4	0.227
3	0.283	1.44	0.237	1.82	0.347
5	0.244	1.24	0.289	1.51	0.352
7	0.232	1.18	0.307	1.38	0.352

The above attenuation factors applied to the gammas produced by the flow separator are shown in Figure 12. Note that attenuating factors for other secondary gammas can be obtained graphically from this figure. The capture gammas from the flow separator which reach the pressure vessel are obtained graphically:

Group	$I_{12.75}$
1	1.67×10^9
3	5.5×10^9
5	8.7×10^9
7	3.0×10^{10}

b. Capture gammas from the thermal shield.

The APPR-1 thermal shield consists (for this analysis) of a 2-inch thick slab of stainless steel located 6.5 inches from the core surface. The capture gammas generated may be assumed to be the sum of those due to the two exponentials shown in Figure 9.

$$I = \frac{\phi_1 C e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

$$\phi_1 = 2 \times 10^{13} \quad (\text{Figure 9})$$

C = gamma contribution (from page 43)

t = 2 inches = 5.08 cm.

$$A = \frac{1}{L} = 0.689$$

$e^{-\mu t}$ - from page 44

FIGURE 12

Group	$e^{-\mu t}$	$(A - \mu)$	$t(A - \mu)$	$e^{-t(A - \mu)}$	$1 - e^{-t(A - \mu)}$	I
1	0.083	0.199	1.01	0.364	0.636	3.32×10^{10}
3	0.237	0.406	2.06	0.127	0.873	1.12×10^{11}
5	0.289	0.445	2.26	0.104	0.896	1.91×10^{11}
7	0.307	0.457	2.32	0.098	0.902	6.27×10^{11}

$$I' = \frac{\phi_1' C e^{-\mu t}}{2(A' - \mu)} \left[1 - e^{-t(A' - \mu)} \right]$$

$$\phi_1' = 8 \times 10^{10}$$

$$A' = -\frac{1}{L} = -0.689$$

Group	$e^{-\mu t}$	$(A' - \mu)$	$t(A' - \mu)$	$e^{-t(A' - \mu)}$	$(1 - e^{-t(A' - \mu)})$	I'
1	0.083	-1.179	-5.98	395	-394	1.40×10^{10}
3	0.237	-0.972	-4.94	140	-139	3.01×10^{10}
5	0.289	-0.933	-4.74	114	-113	4.67×10^{10}
7	0.307	-0.921	-4.68	108	-107	1.5×10^{11}

The capture gammas from the thermal shield reaching the pressure vessel may be computed using attenuation factors obtained from Figure 12.

Group	$I + I'$	Attenuation	$I_{12.75}$
1	4.72×10^{10}	119/2.85	1.97×10^{10}
3	1.42×10^{11}	5.2/10.5	7.03×10^{10}
5	2.38×10^{11}	8.6/16.9	1.21×10^{11}
7	7.77×10^{11}	3.0/5.6	4.16×10^{11}

c. Capture gammas from the pressure vessel.

The pressure vessel for APPR-1 consists of a 1/4-inch thick stainless steel cladding backed by 2-1/2 inches of pressure vessel steel. For this analysis the vessel wall will be considered all steel but only the outer 2-1/2 inches will be used in computing the stress.

The heating effect of gammas generated in the pressure vessel wall is a maximum inside the wall. An approximate and somewhat conservative approach would be to consider an equivalent source of gammas incident on the inside surface of the pressure vessel.

The gamma production at any point in the vessel wall can be computed from the general expression: (appendix A-1)

$$I = \frac{\phi_0 C}{2} \left[\frac{e^{-\mu x}}{A - \mu} \left[1 - e^{-x(A - \mu)} \right] + \frac{e^{\mu x}}{A + \mu} \left[e^{-x(A + \mu)} - e^{-t(A + \mu)} \right] \right]$$

Let $x = 0, 1, 2, 3, 4$, and t cm.

$$\phi_0 = 2.0 \times 10^{12} \quad (\text{from Figure 11})$$

$$t = 2-3/4 \text{ inches} = 7 \text{ cm}$$

$$C = (0.21 \times 0.889 \times 0.76) F_\gamma = 0.142 F_\gamma$$

C =	Group	F_γ	C
	1	0.05	0.0072
	3	0.17	0.0241
	5	0.23	0.0326
	7	0.61	0.0866

$$A = \frac{1}{L} = \sqrt{3 \Sigma_s \Sigma_a} = \sqrt{3 \times 0.93 \times 0.142} = 0.63$$

For $x = 0$

$$I = \frac{2 \times 10^{12}}{2} \times \frac{C}{A + \mu} \left[1 - e^{-7(0.63 + \mu)} \right]$$

where $t > 5$ the quantity in parenthesis approaches 1 and the equation (at $x = 0$) becomes:

$$I = \frac{\phi_0}{2} \times \frac{C}{A + \mu} = \frac{2 \times 10^{12}}{2} \times \frac{C}{A + \mu}$$

Group	μ	$A + \mu$	I_0
1	0.49	1.12	0.64×10^{10}
3	0.283	0.913	2.64×10^{10}
5	0.244	0.874	3.73×10^{10}
7	0.232	0.862	1.005×10^{11}

For $x = 1$

$$I = \frac{\phi_0 C}{2} \left[\frac{e^{-\mu}}{A - \mu} \left[1 - e^{-(A - \mu)} \right] + \frac{e^{\mu}}{A + \mu} \left[e^{-(A + \mu)} - e^{-7(A + \mu)} \right] \right]$$

Group	$e^{-\mu}$	$A - \mu$	$e^{-(A - \mu)} [1 - e^{-(A - \mu)}]$	$\frac{e^{-\mu}}{A - \mu} [1 - e^{-(A - \mu)}]$
1	0.613	0.14	0.869	0.131
3	0.754	0.347	0.707	0.293
5	0.783	0.386	0.680	0.320
7	0.793	0.398	0.672	0.328

Group	e^{μ}	$e^{-(A + \mu)}$	$\gamma(A + \mu)$	$e^{-7(A + \mu)}$	$\left[\right]$	$\frac{e^{\mu}}{A + \mu} \left[\right]$
1	1.632	0.326	7.83	0.0004	0.326	0.475
3	1.327	0.401	6.39	0.0017	0.399	0.580
5	1.276	0.417	6.11	0.0022	0.415	0.606
7	1.261	0.422	6.03	0.0025	0.419	0.613

Group	$\left[\right]$	$\frac{\phi_0 C}{2}$	I_1
1	1.049	0.70×10^{10}	0.73×10^{10}
3	1.216	2.41×10^{10}	2.93×10^{10}
5	1.255	3.26×10^{10}	4.09×10^{10}
7	1.266	8.66×10^{10}	1.095×10^{11}

Other values of x can be computed in a similar manner.

Figure 13 shows the distribution of capture gammas generated in the pressure vessel wall. The dotted lines represent the attenuation of equivalent incident gammas.

d. Capture gammas from the water.

The absorption of a thermal neutron by water (hydrogen) produces one gamma photon of about 2.2 mev energy. The water surrounding the APPR-1 core is exposed to a neutron flux which reaches a peak just outside the core. It is assumed here that the expression for gammas leaving a finite slab (from page 41) can be used, provided the value of "A" is properly determined. Figure 14 shows the computed thermal flux (dotted) from Figure 11 and the thermal flux pattern (solid) assumed for this calculation. The radiation reaching

the pressure shell can be predicted by using attenuation factors from Figure 15.

The data for the attenuation through water used in plotting Figure 15 was obtained by interpolation of the attenuation data for the flow separator gammas (page 44). The attenuation due to the thermal shield can be computed as on page 44 also.

$$I/I_0 = f(R) B e^{-\mu x}$$

$$I/I_0 = 0.806 \times 2.17 \times e^{-0.324 \times 5.08}$$

$$I/I_0 = 0.806 \times 2.17 \times 0.192 = 0.336$$

The thermal flux pattern determines the assumed slab thickness. The gammas from each slab can be computed from the expression

$$I = \frac{\phi_0 C e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

where $C = \Sigma_a$ for water at 432°F

and 1200 psi. From ALCO data,

$$\Sigma_a = 0.0126$$

$$\mu = 0.0465 \times 0.842 = 0.0391 \text{ (at 2.2 MEV)}$$

For slab number 1, $t = 1 \text{ inch} = 2.54 \text{ cm.}$

$$I_1 = \frac{2.6 \times 10^{13}}{2} \times \frac{0.0126 \times e^{-0.099}}{(A - 0.0391)} \left[1 - e^{-2.54(A-0.0391)} \right]$$

$$e^{-Ax} = \phi/\phi_0 = \frac{6.4 \times 10^{13}}{2.6 \times 10^{13}} = 2.46$$

$$Ax = -0.9$$

$$A = -0.0354$$

$$I_1 = 1.3 \times 10^{13} \times \frac{0.0126 \times 0.906}{(-0.0745)} (1 - e^{+0.189})$$

$$I_1 = -(1.99 \times 10^{12})(1 - 1.208) = 4.14 \times 10^{11}$$

At the pressure vessel:

$$I = 4.14 \times 10^{11} \times \frac{1}{16.7} = 2.48 \times 10^{10}$$

For slab number 2, $t = 2$ inches = 5.08 cm, $A = 0$

$$I_2 = \frac{6.4 \times 10^{13}}{2} \times \frac{0.0126 \times e^{-0.199}}{(-0.0391)} (1 - e^{+0.199})$$

$$I_2 = 3.2 \times 10^{13} \times \left(-\frac{0.0126 \times 0.820}{0.0391} \right) (1 - 1.22)$$

$$I_2 = 3.2 \times 10^{13} \times 0.264 \times 0.22 = 1.86 \times 10^{12}$$

At the pressure vessel

$$I = 1.86 \times 10^{12} \times \frac{1}{11.6} = 1.60 \times 10^{11}$$

For slab number 3, $t = 3.5$ inches = 8.9 cm

$$e^{-Ax} = \phi/\phi_0 = \frac{2 \times 10^{13}}{6.4 \times 10^{13}} = 0.313$$

$$Ax = 1.1615$$

$$A = 0.1305$$

$$I_3 = \frac{6.4 \times 10^{13}}{2} \times \frac{0.0126 \times e^{-0.348}}{0.0914} (1 - e^{-0.813})$$

$$I_3 = 3.2 \times 10^{13} \times \frac{0.0126 \times 0.706}{0.0914} (1 - 0.444)$$

$$I_3 = 3.2 \times 10^{13} \times 0.0973 \times 0.556 = 1.73 \times 10^{12}$$

At the pressure vessel

$$I = 1.73 \times 10^{12} \times \frac{1}{6.3} = 2.74 \times 10^{11}$$

For slab number 4, $t = 2.5$ inches = 6.35 cm, $A = 0$

$$I_4 = \frac{3.5 \times 10^{12}}{2} \times \frac{0.0126 \times e^{-0.248}}{(-0.0391)} (1 - e^{+0.248})$$

$$I_4 = 1.75 \times 10^{12} \times \frac{0.0126 \times 0.780}{(-0.0391)} (1 - 1.281)$$

$$I_4 = 1.75 \times 10^{12} \times 0.251 \times 0.281 = 1.23 \times 10^{11}$$

At the pressure vessel

$$I = 1.23 \times 10^{11} \times \frac{1}{1.36} = 9.04 \times 10^{10}$$

For slab number 5, $t = 1.75$ inches = 4.45 cm

$$e^{-Ax} = \phi/\phi_0 = \frac{2 \times 10^{12}}{3.5 \times 10^{12}} = 0.572$$

$$Ax = 0.559$$

$$A = 0.126$$

$$I_5 = \frac{3.5 \times 10^{12}}{2} \times \frac{0.0126 \times e^{-0.174}}{(0.126 - 0.0391)} (1 - e^{-0.387})$$

$$I_5 = 1.75 \times 10^{12} \times \frac{0.0126 \times 0.840}{0.0869} (1 - 0.679)$$

$$I_5 = 1.75 \times 10^{12} \times 0.122 \times 0.321 = 6.85 \times 10^{10}$$

I_5 = is the gamma flux from slab 5 at the pressure vessel.

The total incident gamma flux from water capture is the sum of the above.

$$I = (0.248 + 1.60 + 2.74 + 0.904 + 0.685) 10^{11}$$

$$I = 6.177 \times 10^{11} = 6.18 \times 10^{11}$$

4. Inelastic-scatter-gamma production

Gamma rays from the inelastic scattering of fast neutrons are produced in the flow separator, the thermal shield, and the pressure vessel. The resulting gamma flux at the inside surface of the pressure vessel can be estimated by the same methods used for the capture gammas.

a. Inelastic-scatter gammas from the flow separator.

$$I = \frac{\phi_0 C e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

$$\phi_0 = 1.55 \times 10^{13} \quad (\text{from Figure 11})$$

$$A = 0$$

$$t = 0.125 \text{ inches} = 0.318 \text{ cm}$$

$$C = \sum_a \times F_v \times F_\gamma$$

It is assumed that each scattering process results in one gamma photon, hence $F_\gamma = 1$. All gammas are in the 1 mev group (see page 30)

$$C_{Cr} = (0.112)(0.18 \times \frac{7.91}{6.92}) = 0.023$$

$$C_{Fe} = (0.102)(0.74 \times \frac{7.91}{7.85}) = 0.076$$

$$C_{Ni} = (0.073)(0.08 \times \frac{7.91}{8.9}) = \underline{0.0052}$$

$$C_{Total} = 0.1042$$

$$I = \frac{1.55 \times 10^{13} \times 0.1042 \times e^{-0.156}}{2(-0.49)} (1 - e^{+0.156})$$

$$I = 1.55 \times 10^{13} \times \frac{0.1042 \times 0.856}{(-0.98)} (1 - 1.169)$$

$$I = 1.55 \times 10^{13} \times 0.091 \times 0.169 = 2.38 \times 10^{11}$$

At the pressure vessel (attenuation from Figure 12)

$$I = 2.38 \times 10^{11} \times 1.18/47.5 = 5.91 \times 10^9$$

b. Inelastic-scatter gammas from the thermal shield.

For the thermal shield

$$\phi_0 = 2.2 \times 10^{12} \quad (\text{from Figure 12})$$

$$t = 2 \text{ inches} = 5.08 \text{ cm}$$

$$e^{-At} = \frac{9.4 \times 10^{11}}{2.2 \times 10^{12}} = 0.427 \quad (\text{from Figure 12})$$

$$At = 0.851$$

$$A = 0.1675$$

$$I = \frac{2.2 \times 10^{12}}{2} \times \frac{0.1042 \times e^{-2.49}}{(-0.3225)} (1 - e^{1.64})$$

$$I = 1.1 \times 10^{12} \times \frac{0.1042 \times 0.083}{(-0.3225)} (1 - 5.1552)$$

$$I = 1.1 \times 10^{12} \times 0.0268 \times 4.155 = 1.23 \times 10^{11}$$

At the pressure vessel wall

$$I = 1.23 \times 10^{11} \times 1.18/2.83 = 5.12 \times 10^{10}$$

c. Inelastic scatter gammas from the pressure vessel.

The inelastic scatter gammas generated in the pressure vessel wall are resolved as equivalent incident gammas as in section 3, c above.

$$\phi = 1.7 \times 10^{11} \quad (\text{from Figure 12})$$

$$A = 0.1675$$

$$t = 2.75 \text{ inches} = 7 \text{ cm.}$$

C is based on the contribution of iron only

$$= \Sigma_a(\text{Fe}) = 0.102$$

For $x = 0$

$$I = \frac{\phi}{2} \times \frac{C}{(A+\mu)} \left[(1 - e^{-t(A+\mu)}) \right] \approx \frac{\phi C}{2(A+\mu)}$$

$$I = \frac{1.7 \times 10^{11}}{2} \times \frac{0.102}{0.6575} = 1.32 \times 10^{10}$$

For $x = 1 \text{ cm.}$

$$I = \frac{\phi \cdot C}{2} \left[\frac{e^{-\mu x}}{A - \mu} \left[1 - e^{-x(A - \mu)} \right] + \frac{e^{\mu x}}{A + \mu} \left[e^{-x(A + \mu)} - e^{-t(A + \mu)} \right] \right]$$

$$I = \frac{1.7 \times 10^{11} \times 0.102}{2} \left[\frac{e^{-0.49}}{-0.3225} (1 - e^{0.3225}) + \frac{e^{0.49}}{0.6575} (e^{-0.6575} - e^{-4.6}) \right]$$

$$I = 8.67 \times 10^9 \left[\frac{0.613}{-0.3225} (1 - 1.381) + \frac{1.632}{0.6575} (0.518 - 0.01) \right]$$

$$I = 8.67 \times 10^9 (0.724 + 1.262)$$

$$I = 8.67 \times 10^9 \times 1.986 = 1.72 \times 10^{10}$$

Similar calculations for other values of x result in the relationship shown in Figure 16. As before, the dotted line indicates an equivalent incident gamma flux.

5. Recapitulation

It is assumed here that the total radiation heating effect in the pressure vessel wall will be due to the primary and secondary gammas generated by the operating reactor and estimated above. The following table summarizes these gammas in terms of flux $\times 10^{-11}$ incident on the inside of the pressure vessel wall and at the reactor mid plane.

Group (Energy)	1	2	5	7
<u>Primary (Core) Gammas</u>	39.3	3.29	2.23	1.86
<u>Secondary Gammas</u>				
Capture-Flow Separator	0.02	0.06	0.09	0.30
Capture-Thermal Shield	0.28	0.75	1.21	4.16
Capture-Pressure Vessel	0.10	0.44	0.64	1.74
Capture-Water	--	6.18	--	--
Capture-Total	0.40	7.43	1.94	6.10
In. Scatter-Flow Separator	0.06	--	--	--
In. Scatter-Thermal Shield	0.51	--	--	--
In. Scatter-Pressure Vessel	<u>0.61</u>	--	--	--
<u>In. Scatter-Total</u>	1.18			
Secondary Gammas-Total	1.58	7.43	1.94	6.10
Total Gammas	40.88	10.72	4.17	7.96

IV PRESSURE VESSEL TEMPERATURES

The APPR-1 pressure vessel consists essentially of a 48-inch inside-diameter cylinder welded of 2 1/2 inches of ASTM 212-B steel plate, clad on the inside with 1/4 inch of 304 stainless steel. The entire vessel is enclosed in 4 inches of thermal insulation.

Because of the insulation, the heat flow through the wall would be very small and only a minute temperature drop would occur across the pressure vessel wall if no gamma radiation were involved. If the wall were in an intense gamma field (normal reactor operation), heat would be generated in the vessel wall. Since the insulation limits the flow of heat outward, the wall temperature must rise until the radiation-generated heat is nearly all removed by the reactor coolant. The necessary temperature distribution for this condition indicates a maximum temperature very near the outer wall surface.

In general the temperatures (and resultant thermal stresses) which will exist in the pressure vessel wall depend upon the incident gamma flux, the wall thickness, and the method of cooling (or insulating) the wall. For this analysis the wall thickness and the method of cooling are fixed. The maximum temperature difference is then a function of the incident radiation only.

The volumetric heat release at the inside surface of the pressure vessel can be computed (for each energy group) from the expression:

$$Q_0 = I \times E \times \mu_e \times 1.55 \times 10^{-8}$$

where I is the incident gamma flux in photons/
cm²-sec.

E is the group energy in Mev

μ_e is the energy absorption (linear)
coefficient for the energy and material
involved.

1.55×10^{-8} is the conversion factor
mev/cm³-sec to Btu/ft³-hr

From the standpoint of shielding, the emergent gamma beam is of interest and the total linear absorption coefficient is used in attenuation calculation. However, for energy absorption in a shield or pressure shell, the use of the energy absorption (linear) coefficient is indicated. The numerical difference between the above two types of calculations can be accounted for by those gammas scattered out of the beam but not absorbed in the shield or shell.

The heat release (volumetric) at any point in the pressure shell may be computed from the expression. (17)

$$Q = Q_0 e^{-\mu_e x} f(R)$$

where x is the distance from the inside
 surface of the pressure vessel in cm.
 and $f(R) = \frac{R_0}{R_0 + t}$ and is the geometric factor

relating the cylindrical pressure shell
 to the derived slab conditions. (R_0 =
 inside radius and t = wall thickness)

At $x = 0$, $Q = Q_0 = I E \mu_e \times 1.55 \times 10^{-8}$

Group	I	μ_e	Q_0
1	4.09×10^{12}	0.205	13050 Btu/ft ³ -hr
3	1.07×10^{12}	0.176	8750 "
5	4.17×10^{11}	0.178	5750 "
7	7.96×10^{11}	0.185	16000 "
Total			43550 "

At $x = 1$ inch = 2.54 cm, $f(R) = \frac{23.75}{24.75} = 0.96$

Group	$\mu_e x$	$e^{-\mu_e x}$	Q_1
1	0.524	0.592	7420 Btu/ft ³ -hr
3	0.447	0.640	5370 "
5	0.452	0.636	3510 "
7	0.470	0.625	9600 "
Total			25900 "

At $x = 2$ inches = 5.08 cm,

$$f(R) = \frac{23.75}{25.75} = 0.923$$

Group	μ_{ex}	$e^{-\mu_{ex}}$	Q_2
1	1.048	0.351	4230 Btu/ft ³ -hr
3	0.894	0.409	3300 "
5	0.904	0.405	2150 "
7	0.940	0.391	5770 "
Total			15450 "

At $x = 2.75$ inches = 7 cm,

$$f(R) = \frac{23.75}{26.5} = 0.897$$

Group	μ_{ex}	$e^{-\mu_{ex}}$	$Q_{27.75}$
1	1.442	0.236	2760 Btu/ft ³ -hr
3	1.232	0.292	2290 "
5	1.246	0.288	1490 "
7	1.295	0.274	3930 "
Total			10470 "

The above results are shown graphically in Figure 17. It should be noted that the cladding material is effective in radiation attenuation but is not considered in computing the temperature and stress distributions.

For any one reactor, thermal shield, and pressure vessel configuration an effective absorption coefficient (A) can be estimated, based on the total volumetric heat generation in the vessel wall.

$$\frac{Q}{Q_0} = e^{-At}$$

$$e^{-At} = \frac{10470}{43550} = 0.2405$$

$$At = 1.425$$

(The dimensions of A depend on those chosen for t. For subsequent analyses A should be in terms of ft^{-1})

$$t = 2.75 \text{ inches} = 0.229 \text{ ft.}$$

$$A = \frac{1.425}{0.229} = 6.22 \text{ ft}^{-1}$$

The value of the effective A depends upon the relative contribution of the individual energy groups.

In the above work the tacit assumption that the entire pressure vessel is of a common material is satisfactory and saves considerable computation. However, in examining the temperature distribution in the pressure vessel, it is obvious that the lower value of thermal conductivity (k) in the stainless steel clad must be taken into account. Since the value of the thermal conductivity varies with temperature, a second-order effect is also introduced. The temperature distribution can be found from the expression (see appendix A-2)

$$T = \frac{Q_0}{k A^2} (1 - e^{-Ax} - xAe^{-At})$$

In order to approximate the operating temperature it is necessary to estimate the heat flow and resistance to heat flow. The quantity of heat (H) flowing across the inner surface of the vessel can be found from the expression (see Appendix A-3)

$$\begin{aligned}
 H &= \frac{Q_0}{A} (1 - e^{-At}) \\
 &= \frac{43500}{6.22} (1 - 0.2405) \\
 &= 5310 \text{ Btu/ft}^2\text{-hr}
 \end{aligned}$$

The temperature drop across the coolant film can be found from the expression:

$$H = h_f \Delta T$$

The film coefficient can be estimated from the expression: (18)

$$h_f \cong 170 (1 + T \times 10^{-2} - T^2 \times 10^{-5}) \frac{v^{0.8}}{D_e^{0.2}}$$

where T is the bulk water temperature

v is an average coolant velocity

and D_e is the equivalent hydraulic diameter.

The coolant flow is 4000 gpm and the annular cross section between the pressure shell and the thermal shield is

$$\frac{\pi}{4} \left[\left(\frac{47.5}{12} \right)^2 - \left(\frac{39}{12} \right)^2 \right] = \frac{5.1}{4} \pi$$

$$v \simeq 4000 \times 2.228 \times 10^{-3} \times \frac{4}{5.1\pi} = 2.225 \text{ ft/sec}$$

$$v^{0.8} \simeq 1.9$$

$$D_e \simeq \frac{4 \times 5.1\pi}{4} \times \frac{144}{\pi(47.5 + 39)}$$

$$\simeq \frac{5.1 \times 144}{86.5} = 8.5$$

$$D_e^{0.2} \simeq 1.535$$

$$T = 432 \text{ } ^\circ\text{F}$$

$$h_f \simeq 170(1 + 4.32 - 1.87) \frac{1.9}{1.54} = 725$$

$$\text{and } \Delta T = \frac{H}{h_f} = \frac{5310}{725} = 7.3 \text{ } ^\circ\text{F}$$

If a scale condition on the inside of the pressure vessel is assumed similar to that specified for the fuel elements, a temperature drop across the scale can be estimated from the expression

$$H \simeq \frac{k_s}{t_s} \Delta T$$

where k_s is the thermal conductivity
of the scale (= 1)

and t_s is the scale thickness
(= 0.01 inches = $\frac{0.01}{12}$ ft.)

$$T \simeq H \frac{t_s}{k_s} = 5310 \times \frac{0.01}{12} = 4.4 \text{ } ^\circ\text{F}$$

If the temperature drop across the clad is assumed to be 10 F° the average clad temperature is then:

$$T_c \simeq 432 + 7.3 + 4.4 + \frac{10}{2} \simeq 450 \text{ } ^\circ\text{F}$$

Similarly if the temperature drop across the pressure shell is 20 F° the average shell temperature will be:

$$T_{pv} \simeq 450 + \frac{20}{2} = 460 \text{ } ^\circ\text{F}$$

The variation of k with temperature is shown in Figure 18 for both stainless steel and carbon steel.

For the temperature distribution in the cladding:

$$Q_o = 43550 \text{ Btu/ft}^3\text{-hr}$$

$$A = 6.22 \text{ ft}^{-1}$$

$$k = 11.1 \quad (\text{at } 450 \text{ } ^\circ\text{F})$$

$$x = 0.125 \text{ inches} = 0.0104 \text{ ft}$$

$$\text{or } 0.25 \text{ inches} = 0.0208 \text{ ft.}$$

$$\begin{aligned} \Delta T_{0.125} &= \frac{Q_o}{k A^2} (1 - e^{-Ax} - xAe^{-At}) \\ &= \frac{43500}{11.1 \times 6.22^2} (1 - e^{-0.065} - 0.065 e^{-1.425}) \\ &= 101 (1 - 0.937 - 0.065 \times 0.2405) \end{aligned}$$

$$\Delta T_{0.125} = 101 (1 - 0.937 - 0.0156) = 101 \times 0.0474 = 4.8 \text{ F}^\circ$$

$$\text{and } \Delta T_{0.25} = 101 (1 - e^{-0.29} - 0.129 \times 0.2405)$$

$$\Delta T_{0.25} = 101 (1 - 0.879 - 0.031)$$

$$\Delta T_{0.25} = 101 \times 0.09 = 9.1 \text{ F}^\circ$$

For the temperature distribution in the pressure vessel proper:

$$Q_0 = 38000 \text{ Btu/ft}^3\text{-hr}$$

$$A = 6.22 \text{ ft}^{-1}$$

$$k = 26.4$$

$$x = 1, 1.75, \text{ and } 2.5 \text{ inches}$$

$$\text{or } x = 0.0832, 0.146, \text{ and } 0.208 \text{ feet}$$

$$\Delta T_1 = \frac{38000}{26.4 \times 6.22^2} (1 - e^{-0.517} - 0.517 e^{-1.293})$$

$$\Delta T_1 = 37.2 (1 - 0.596 - 0.517 \times 0.274)$$

$$\Delta T_1 = 37.2 (1 - 0.738) = 37.2 \times 0.272 = 7.4 \text{ F}^\circ$$

$$\text{Similarly } \Delta T_{1.75} = 37.2 (1 - e^{-0.908} - 0.908 \times 0.274)$$

$$\Delta T_{1.75} = 37.2 (1 - 0.403 - 0.249)$$

$$\Delta T_{1.75} = 37.2 \times 0.348 = 12.9 \text{ F}^\circ$$

$$\text{and } \Delta T_{2.5} = 37.2 (1 - e^{-1.293} - 1.293 \times 0.274)$$

$$\Delta T_{2.5} = 37.2 (1 - 0.274 - 0.354)$$

$$= 37.2 \times 0.372 = 13.8 \text{ F}^\circ$$

The above temperature distribution through the pressure vessel wall is shown graphically in Figure 19.

FIGURE 19

HRK 7-20-56

V PRESSURE VESSEL STRESSES

The total stress in the pressure vessel wall of the APPR-1 during operation is assumed to be the algebraic sum of the pressure stress and the thermal stress. Either the pressure or thermal stress may be considered in terms of three components, the axial stress, the radial stress, and the tangential stress. For both pressure and thermal stresses, the tangential stress at the inside surface is the greatest of the three components. In addition, the maximum thermal stress occurs in the pressure vessel at the reactor midplane. Thus the maximum total stress in the APPR-1 is the sum of the pressure and thermal tangential stresses at the reactor midplane. Sections III and IV give the incident energy and the temperature distribution in the pressure vessel wall at the reactor midplane.

A. THE TANGENTIAL PRESSURE STRESS

The APPR-1 pressure vessel is essentially a cylinder 47.5 inches inside diameter with a 2.75 inch thick wall. However the wall is a composite of 0.25 inches of stainless steel clad on the inside plus 2.5 inches of ASTM-212-B steel. Code practice indicates that the clad be ignored in computing the vessel stresses. Thus the effective inside diameter is 48 inches and the wall thickness is 2.5 inches.

The maximum tangential pressure stress can be computed from the expression (see Appendix A-4)

$$S_{TP} = p \left[\frac{a^2}{b^2 - a^2} \right] \left[\frac{b^2}{a^2} + 1 \right]$$

where S_{TP} is the tangential pressure stress

p is the design pressure (1400 psi)

a is the inside radius (24 inches)

and b is the outside radius (26.5 inches)

$$S_{TP} = 1400 \left[\frac{24^2}{26.5^2 - 24^2} \right] \left[\frac{26.5^2}{24^2} + 1 \right]$$

$$S_{TP} = 1400 \left[\frac{576}{702 - 576} \right] \left[\frac{702}{576} + 1 \right]$$

$$= 1400 \times 4.57 \times 2.218 = 14180 \text{ psi}$$

B. THE TANGENTIAL THERMAL STRESS

The thermal stress computation is somewhat more complicated.

The maximum tangential thermal stress can be computed from the expression (see Appendix A-4)

$$S_{TT} = \left(\frac{\alpha E}{1 - \nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2}{b^2 - a^2} \right) \left[a \left(c - \frac{1}{A} + e^{-Ac} \left(\frac{1}{A} - \frac{Ac^2}{2} \right) \right) \right. \\ \left. \left(+ \frac{c^2}{2} - \frac{1}{A^2} + e^{-Ac} \left(\frac{Ac + 1}{A^2} - \frac{Ac^3}{3} \right) \right) \right]$$

where α is the linear coefficient of expansion of

the vessel steel, 7.9×10^{-6} per $^{\circ}\text{F}$ at

464 $^{\circ}\text{F}$ (see Figure 20)

E is Young's modulus, 23.6×10^6 psi at

464 $^{\circ}\text{F}$ (see Figure 21)

FIGURE 20

YOUNG'S MODULUS FOR STEELS

(FROM "METAL DATA"
BY HOYT)

FIGURE 21

HRK - 4-6-56

ν is Poisson's ratio = 0.3

c is the wall thickness = 0.2083 ft

and other symbols are the same as before, except

a and b must be in feet to satisfy the expression

$$\frac{\infty E}{1 - \nu} = \frac{7.9 \times 23.6}{0.7} = 267.5$$

$$\frac{Q_0}{k A^2} = 37.2$$

$$\frac{2}{b^2 - a^2} = \frac{2}{4.875 - 4.000} = 2.286$$

$$e^{-Ac} = e^{-6.22 \times 0.2083} = e^{-1.296} = 0.2736$$

$$S_{TT} = 267.5 \times 37.2 \times 2.286 \left[2 \left[0.2083 - \frac{1}{6.22} \right. \right.$$

$$\left. + 0.2736 \left(\frac{1}{6.22} - \frac{6.22 \times 0.2083^2}{2} \right) \right]$$

$$+ \frac{0.2083^2}{2} - \frac{1}{6.22^2} + 0.2736 \left(\frac{6.22 \times 0.2083 + 1}{6.22^2} \right.$$

$$\left. - \frac{6.22 \times 0.2083^3}{3} \right) \left. \right]$$

$$\bullet S_{TT} = 22730 \left[2(0.2083 - 0.1608 + 0.2736(0.1608 - 0.1350)) \right. \\ \left. + (0.0217 - 0.0259 + 0.2736(0.0594 - 0.01875)) \right]$$

$$S_{TT} = 22730 \left[2(0.0475 + 0.2736 \times 0.0258) + (-0.0042 + 0.2736 \times 0.0406) \right]$$

$$S_{TT} = 22730 \left[2(0.0475 + 0.0071) + (0.0111 - 0.0042) \right]$$

$$S_{TT} = 22730 (2 \times 0.0546 + 0.0069)$$

$$S_{TT} = 22730 (0.1092 + 0.0069)$$

$$S_{TT} = 22730 \times 0.1161 = 2640 \text{ psi}$$

C. THE TOTAL TANGENTIAL STRESS

For this analysis it was assumed that the total maximum stress in the pressure vessel wall could be obtained by simply adding the tangential pressure and thermal stresses at the inside surface and at the reactor midplane.

$$S_T = 14180 + 2640 = 16820 \text{ psi}$$

A more detailed picture of the resultant stresses through the wall will be found in Appendix B.

APPENDIX

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VI APPENDIX

A. DERIVATIONS

1. Radiation from a non-uniform slab source with self absorption

A slab of material t cm. thick is exposed to a uniform neutron flux ϕ_0 . The neutron attenuation is assumed exponential:

$$\phi_x = \phi_0 e^{-Az}$$

The radiation resulting from the interaction of neutrons and matter at any point z is a function of the flux at z , the probability of an interaction, and the amount of radiation

per interaction.

$$S_z = \phi_z \sum Ff = \phi_0 e^{-Az} \sum Ff$$

where S_z is the radiation source at z
(particles/cm³-sec)

$\sum$ is the macroscopic cross section for
the interaction of interest

F is the number (or fraction) of radia-
tion particles per interaction

f is the volume fraction of the source
material present

The product $\sum Ff$ represents the radiation contribution due
to material properties and may be represented by the term C .

$$S_z = \phi_0 C e^{-Az}$$

For simplicity it is assumed that the radiation generated
at z is directed away from and normal to the dz plane element,
exactly one-half in each direction.

The radiation flux at some plane, x , due to interactions
occurring in dz is simply

$$dI = \frac{S_z}{2} e^{-\mu(x-z)} dz$$

where $e^{-\mu(x-z)}$ is an attenuating factor to
account for self absorption

If z were larger than x ,

$$dI = \frac{S_z}{2} e^{-\mu(z-x)} dz$$

The total radiation flux at x is due then, to the sum of the integrated sources from 0 to x and from x to t .

$$I_x = \int_0^x \frac{S_z}{2} e^{-\mu(x-z)} dz + \int_x^t \frac{S_z}{2} e^{-\mu(z-x)} dz$$

$$I_x = \frac{\phi_0 C}{2} \left(\int_0^x e^{-Az} e^{-\mu(x-z)} dz + \int_x^t e^{-Az} e^{-\mu(z-x)} dz \right)$$

$$\int_0^x e^{-Az} e^{-\mu(x-z)} dz = \int_0^x e^{-\mu x} + \mu z - Az dz$$

$$= e^{-\mu x} \int_0^x e^{-z(A-\mu)} dz$$

$$= e^{-\mu x} \left[-\frac{e^{-z(A-\mu)}}{(A-\mu)} \right]_0^x$$

$$= e^{-\mu x} \left[-\frac{e^{-x(A-\mu)}}{(A-\mu)} + \frac{1}{(A-\mu)} \right]$$

$$= \frac{e^{-\mu x}}{(A-\mu)} \left[1 - e^{-x(A-\mu)} \right]$$

Similarly

$$\int_x^t e^{-Az} e^{-\mu(z-x)} dz = \int_x^t e^{-\mu z + \mu x - Az} dz$$

$$= e^{\mu x} \int_x^t e^{-z(A + \mu)} dz$$

$$= e^{\mu x} \left[-\frac{e^{-z(A + \mu)}}{(A + \mu)} \right]_x^t$$

$$= e^{\mu x} \left[-\frac{e^{-t(A + \mu)}}{(A + \mu)} + \frac{e^{-x(A + \mu)}}{(A + \mu)} \right]$$

$$= \frac{e^{\mu x}}{(A + \mu)} \left[e^{-x(A + \mu)} - e^{-t(A + \mu)} \right]$$

And

$$I_x = \frac{\phi_0 C}{2} \left[\frac{e^{-\mu x}}{(A - \mu)} \left[1 - e^{-x(A - \mu)} \right] + \frac{e^{\mu x}}{(A + \mu)} \left[e^{-x(A + \mu)} - e^{-t(A + \mu)} \right] \right]$$

If the directed radiation at the slab surface is of interest, the above expression is simplified somewhat.

For $x = 0$

$$I_0 = \frac{\phi_0 C}{2} \left(\frac{1}{A + \mu} \right) \left(1 - e^{-t(A + \mu)} \right)$$

For $x = t$

$$I_t = \frac{\phi_0 C}{2} \frac{e^{-\mu t}}{A - \mu} (1 - e^{-t(A - \mu)})$$

If the source is uniform ($\phi_x = \phi_0$)

A must be equal to zero and

$$(I_t)_0 = \frac{\phi_0 C}{2} \left(\frac{e^{-\mu t}}{-\mu} \right) (1 - e^{-\mu t})$$

$$(I_t)_0 = \frac{\phi_0 C}{2} (1 - e^{-\mu t})$$

If, in addition, t becomes very large, the expression is effectively

$$(I_t)_0 = \frac{\phi_0 C}{2\mu} = \frac{\phi_x C}{2\mu} = \frac{S_x}{2\mu} = \frac{1}{2} S_x \lambda$$

This expression is identical to that derived by Blizard⁽¹⁹⁾

2. The temperature distribution in shields and pressure vessels due to an exponential heat source.

The differential equation for the conduction of heat in a cylinder wall is:

$$\frac{d^2 T}{dx^2} + \frac{1}{x} \frac{dT}{dx} = \frac{Q(x)}{k}$$

The solution of this equation is complicated but necessary where the outside radius (x) is large compared to the inside radius (x_0)

The APPR-1 pressure vessel has a large radius compared to the wall thickness (~ 10 to 1) and may thus be treated as a slab. For this geometry

$$\frac{d^2T}{dx^2} = \frac{Q(x)}{k}$$

If $Q(x) = Q_0 e^{-Ax}$ the general solution⁽¹⁷⁾ of the above equation is:

$$T = -\frac{Q_0}{kA^2} e^{-Ax} + C_1x + C_2$$

If T is treated as the differential temperature across the wall and if we assume the temperature reaches a maximum in the wall:

$$T = 0 \text{ when } x = 0$$

$$\text{and } \frac{dT}{dx} = 0 \text{ when } x = t$$

$$T = 0 = -\frac{Q_0}{kA^2} + C_2$$

$$C_2 = \frac{Q_0}{kA^2}$$

$$\frac{dT}{dx} = 0 = \frac{Q_0}{kA} e^{-At} + C_1$$

$$C_1 = -\frac{Q_0}{kA} e^{-At} = -\frac{Q_0}{kA^2} A e^{-At}$$

Thus

$$T = \frac{Q_0}{kA^2} (1 - e^{-Ax} - xAe^{-At})$$

An evaluation of the above expression requires knowledge of the point of inflection t .

For an insulated pressure vessel, essentially no heat will flow outward. And since heat generated in the vessel must all flow toward the center, the temperature will be a maximum and the temperature gradient will be zero at the outside surface of the pressure vessel. Thus $t = c$ (c = the pressure vessel wall thickness and

$$T = \frac{Q_0}{kA^2} (1 - e^{-Ax} - x A e^{-Ac})$$

For the case of a cylinder cooled on both sides (a thermal shield) it may be assumed that the temperature at both surfaces is the same.

$$T = 0 \text{ when } x = 0 \text{ or } x = c$$

For $x = 0$

$$T = 0 = - \frac{Q_0}{kA^2} + C_2$$

$$C_2 = \frac{Q_0}{kA^2}$$

For $x = c$

$$T = 0 = - \frac{Q_0}{kA^2} e^{-Ac} + C_1 c + \frac{Q_0}{kA^2}$$

$$C_1 = \frac{Q_0}{kA^2} \left(\frac{e^{-Ac} - 1}{c} \right)$$

$$T = \frac{Q_0}{kA^2} \left[1 - e^{-Ax} + \frac{x}{c} (e^{-Ac} - 1) \right]$$

3. Heat production in the pressure vessel wall.

The heat production at x is:

$$dH = Q_x dx$$

and since all heat generated must flow across the inside surface

$$H = \int_0^c Q_x dx$$

$$H = Q_o \int_0^c e^{-Ax} dx$$

$$= Q_o \left[-\frac{e^{-Ax}}{A} \right]_0^c$$

$$= \frac{Q_o}{A} \left(-e^{-Ac} + 1 \right)$$

$$H = \frac{Q_o}{A} (1 - e^{-Ac})$$

4. Pressure Vessel Stresses

The sketch below represents a section of the APPR-1 pressure vessel.

The symbols T, R, and Z represent the tangential, radial, and axial stress components, and will appear as subscripts. The internal and external pressures p_1 and p_2 constitute the loading on the shell. Where p_1 is much greater than p_2 the tangential pressure stress (S_{Tp}) will be a maximum

(tension) at the inside surface, decreasing slightly towards the outside surface. The radial pressure stress (S_{Rp}) will also be a maximum (compression = p_1) at the inside surface (for the case $p_1 > p_2$), decreasing to p_2 at the outside surface. The axial pressure stress (S_{Zp}) consists of the "piston" stress plus a contribution from the tangential and radial stresses.

The thermal stress picture will depend on the configuration and conditions assumed. For this analysis the temperature distribution of interest is in a radial direction and is assumed symmetrical about the cylinder axis. There is a temperature gradient in the axial direction but this does not result in stress since it is assumed that the vessel is free to move in this direction.

In addition to the conventional assumptions made in the basic reference⁽²⁰⁾ it is assumed here that, (a) the temperature profile is symmetrical about but independent of the axial coordinate, and (b) the elastic deformation, w , at various cross sections is independent of the axial coordinate. $\left(\frac{dw}{dz} = 0\right)$

The stress equations may be developed in the following manner. Consider a thin circular disk (assumption (a) above)

The equation of equilibrium is:

$$\frac{dS_R}{dr} = \frac{S_T - S_R}{r}$$

The equations of compatibility are:

$$\epsilon_R = \frac{du}{dr}$$

$$\epsilon_T = \frac{u}{r}$$

$$\text{and } \epsilon_Z = \frac{dW}{dz} = 0 \text{ (assumption (b) above)}$$

$$d\epsilon_T = d\left(\frac{u}{r}\right) = \frac{rdu - u dr}{r^2}$$

$$\frac{d\epsilon_T}{dr} = \frac{rdu}{r^2 dr} - \frac{u}{r^2} = \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2}$$

$$\frac{d\epsilon_T}{dr} = \frac{\epsilon_R - \epsilon_T}{r} \quad (2)$$

From Hooke's law:

$$\epsilon_R = \frac{1}{E} \left[S_R - \nu(S_T + S_Z) \right] + \alpha T \quad (3)$$

$$\epsilon_T = \frac{1}{E} \left[S_T - \nu(S_R + S_Z) \right] + \alpha T \quad (4)$$

$$\epsilon_Z = \frac{1}{E} \left[S_Z - \nu(S_T + S_R) \right] + \alpha T \quad (5)$$

Since $\epsilon_Z = 0$,

$$S_Z - \gamma(S_T + S_R) = -\alpha TE$$

$$S_Z = \gamma(S_T + S_R) - \alpha TE \quad (6)$$

Substituting (6) in (3) and (4) above

$$\begin{aligned} \epsilon_R &= \frac{1}{E} \left[(S_R - \gamma(S_T + \gamma S_T + \gamma S_R - \alpha TE)) \right] + \alpha T \\ &= \frac{1}{E} \left[S_R (1 - \gamma^2) - S_T \gamma (1 + \gamma) + \alpha TE (1 + \gamma) \right] \\ \epsilon_R &= \frac{1 + \gamma}{E} \left[S_R (1 - \gamma) - S_T \gamma + \alpha TE \right] \quad (7) \end{aligned}$$

$$\begin{aligned} \epsilon_T &= \frac{1}{E} \left[S_T - \gamma(S_R + \gamma S_T + \gamma S_R - \alpha TE) \right] + \alpha T \\ \epsilon_T &= \frac{1 + \gamma}{E} \left[S_T (1 - \gamma) - S_R \gamma + \alpha TE \right] \quad (8) \end{aligned}$$

$$\text{Defining } \triangle = \epsilon_R + \epsilon_T + \epsilon_Z \quad (9)$$

$$\triangle = \epsilon_R + \epsilon_T + 0$$

$$\triangle = \frac{1 + \gamma}{E} \left[S_R (1 - 2\gamma) + S_T (1 - 2\gamma) + \frac{2}{(1 - 2\gamma)} \alpha TE (1 - 2\gamma) \right]$$

$$\text{or } S_R + S_T = \frac{E \triangle}{(1 + \gamma)(1 - 2\gamma)} - \frac{2 \alpha TE}{1 - 2\gamma} \quad (10)$$

But from (7) above

$$\frac{\epsilon_R E}{1 + \gamma} = S_R (1 - \gamma) - S_T \gamma + \alpha TE$$

$$\text{and } S_T = \frac{1}{\gamma} \left[S_R (1 - \gamma) + \alpha TE - \frac{\epsilon_R E}{1 + \gamma} \right]$$

Substituting into (10)

$$S_R + S_R \frac{1-\nu}{\nu} = \frac{E \Delta}{(1+\nu)(1-2\nu)} + \frac{\epsilon_R E}{\nu(1+\nu)} - \frac{2\alpha TE}{1-2\nu} - \frac{\alpha TE}{\nu}$$

$$\frac{S_R}{\nu} = \frac{E}{(1+\nu)\nu} \left[\frac{\nu \Delta}{(1-2\nu)} + \epsilon_R \right] - \frac{\alpha TE}{\nu(1-2\nu)} (2\nu + 1 - 2\nu)$$

$$S_R = \frac{E}{1+\nu} \left(\epsilon_R + \frac{\nu}{1-2\nu} \Delta \right) - \frac{\alpha TE}{1-2\nu} \quad (11)$$

Similarly

$$S_T = \frac{E}{1+\nu} \left(\epsilon_T + \frac{\nu}{1-2\nu} \Delta \right) - \frac{\alpha TE}{1-2\nu} \quad (12)$$

$$\text{but } \frac{dS_R}{d\nu} = \frac{E}{1+\nu} \left[\frac{d\epsilon_R}{dr} + \frac{\nu}{1-2\nu} \frac{d\Delta}{dr} - \frac{(1+\nu)\alpha}{1-2\nu} \frac{dT}{dr} \right]$$

$$\text{and } \frac{S_T - S_R}{r} = \left[\frac{E}{1-2\nu} \left(\epsilon_T + \frac{\nu}{1-2\nu} \Delta - \epsilon_R - \frac{\nu}{1-2\nu} \Delta \right) - \frac{\alpha TE}{1-2\nu} + \frac{TE}{1-2\nu} \right] \frac{1}{r}$$

Substituting into equation (1)

$$\frac{d\epsilon_R}{dr} + \frac{\nu}{1-2\nu} \frac{d\Delta}{dr} - \frac{1+\nu}{1-2\nu} \alpha \frac{dT}{dr} = \frac{\epsilon_T - \epsilon_R}{r} \quad (13)$$

Since

$$\Delta = \epsilon_R + \epsilon_T$$

$$\frac{d\Delta}{dr} = \frac{d\epsilon_R}{dr} + \frac{d\epsilon_T}{dr}$$

But from equation (2)

$$\frac{d\epsilon_T}{dr} = \frac{\epsilon_R - \epsilon_T}{r}$$

Hence $\frac{d}{dr} \left(\frac{1}{r} \right) = \frac{d\epsilon_R}{dr} + \frac{\epsilon_R - \epsilon_T}{r}$ (14)

Substituting (14) into (13)

$$\begin{aligned} \frac{d\epsilon_R}{dr} + \frac{\gamma}{1-2\gamma} \frac{d\epsilon_R}{dr} + \frac{\gamma}{1-2\gamma} \frac{\epsilon_R - \epsilon_T}{r} - \frac{1+\gamma}{1-2\gamma} &\propto \frac{dT}{dr} \\ &= \frac{\epsilon_T - \epsilon_R}{r} \end{aligned}$$

$$\begin{aligned} \frac{1-2\gamma+\gamma}{1-2\gamma} \frac{d\epsilon_R}{dr} + \frac{1-2\gamma+\gamma}{1-2\gamma} \frac{\epsilon_R - \epsilon_T}{r} &= \frac{1+\gamma}{1-2\gamma} \propto \frac{dT}{dr} \\ \frac{d\epsilon_R}{dr} + \frac{\epsilon_R - \epsilon_T}{r} &= \frac{1+\gamma}{1-\gamma} \propto \frac{dT}{dr} \end{aligned} \quad (15)$$

Or in terms of the compatibility equations

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} = \frac{1+\gamma}{1-\gamma} \propto \frac{dT}{dr} \quad (16)$$

The above equation may be directly integrated when put in the form

$$\frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} (r u) \right] = \frac{1+\gamma}{1-\gamma} \propto \frac{dT}{dr}$$

$$\frac{1}{r} \frac{d}{dr} (ru) = \frac{1+\gamma}{1-\gamma} \propto T + C_1$$

$$\frac{d}{dr} (ru) = \frac{1 + \gamma}{1 - \gamma} \propto Tr + C_1 r$$

$$ru = \frac{1 + \gamma}{1 - \gamma} \propto \int_a^r Tr dr + C_1 r^2 + C_2$$

$$u = \frac{1 + \gamma}{1 - \gamma} \cdot \frac{1}{r} \propto \int_a^r Tr dr + C_1 r + \frac{C_2}{r} \quad (17)$$

$$\text{but } \epsilon_T = \frac{u}{r} = \frac{1 + \gamma}{1 - \gamma} \cdot \frac{1}{r^2} \propto \int_a^r Tr dr + C_1 + \frac{C_2}{r^2} \quad (18)$$

$$\epsilon_R = \frac{du}{dr} = \frac{1 + \gamma}{1 - \gamma} \propto \left[\frac{1}{r} \cdot Tr + \int_a^r Tr dr \cdot (-1) \frac{1}{r^2} \right]$$

$$+ C_1 - \frac{C_2}{r^2}$$

$$\epsilon_R = - \frac{1 + \gamma}{1 - \gamma} \propto \cdot \frac{1}{r^2} \int_a^r Tr dr + \frac{1 + \gamma}{1 - \gamma} \propto T$$

$$+ C_1 - \frac{C_2}{r^2} \quad (19)$$

$$\text{and } \triangle = \epsilon_R + \epsilon_T = \frac{1 + \gamma}{1 - \gamma} \propto T + 2C_1 \quad (20)$$

Substituting equations (19) and (20) into (11)

$$S_R = \frac{E}{1 + \gamma} \left[- \frac{1 + \gamma}{1 - \gamma} \propto \frac{1}{r^2} \int_a^r Tr dr + \frac{1 + \gamma}{1 - \gamma} \propto T + C_1 - \frac{C_2}{r^2} \right. \\ \left. + \frac{\gamma}{1 - 2\gamma} \left(\frac{1 + \gamma}{1 - \gamma} \propto T + 2C_1 \right) \right] = \frac{\propto TE}{1 - 2\gamma}$$

$$\begin{aligned}
S_R &= \frac{E}{1+\nu} \left[-\frac{1+\nu}{1-\nu} \cdot \frac{\alpha}{r^2} \int_a^r T r dr + \frac{1+\nu}{1-\nu} \alpha T + C_1 - \frac{C_2}{r^2} \right. \\
&\quad \left. + \frac{\nu}{1-2\nu} \cdot \frac{1+\nu}{1-\nu} \alpha T + \frac{\nu}{1-2\nu} \cdot 2C_1 - \frac{1+\nu}{1-\nu} \alpha T \right] \\
S_R &= \frac{E}{1+\nu} \left[-\frac{1+\nu}{1-\nu} \cdot \frac{\alpha}{r^2} \int_a^r T r dr + \frac{1+\nu}{1-\nu} \alpha T \left(1 + \frac{\nu}{1-2\nu} - \frac{1-\nu}{1-2\nu} \right) \right. \\
&\quad \left. + C_1 \left(1 + \frac{2\nu}{1-2\nu} \right) - \frac{C_2}{r^2} \right] \\
S_R &= \frac{E}{1+\nu} \left[-\frac{1+\nu}{1-\nu} \cdot \frac{\alpha}{r^2} \int_a^r T r dr + \frac{C_1}{1-2\nu} - \frac{C_2}{r^2} \right] \quad (21)
\end{aligned}$$

Similarly

$$S_T = \frac{E}{1+\nu} \left[\frac{1+\nu}{1-\nu} \cdot \frac{\alpha}{r^2} \int_a^r T r dr - \frac{1+\nu}{1-\nu} \alpha T + \frac{C_1}{1-2\nu} + \frac{C_2}{r^2} \right] \quad (22)$$

At both the inside and outside surfaces, the radial stress must just equal the pressure.

$$\text{Thus } S_R \text{ (at } r = a) = -p_1 \quad (23)$$

$$\text{and } S_R \text{ (at } r = b) = -p_2 \quad (24)$$

Substituting (23) and (24) in (21)

$$\frac{p_1 (1+\nu)}{E} = \frac{1+\nu}{1-\nu} \cdot \frac{\alpha}{a^2} \int_a^a T r dr - \frac{C_1}{1-2\nu} + \frac{C_2}{a^2} \quad (25)$$

$$\frac{p_2 (1 + \nu)}{E} = \frac{1 + \nu}{1 - \nu} \frac{\alpha}{b^2} \int_a^b T r dr - \frac{C_1}{1 - 2\nu} + \frac{C_2}{b^2} \quad (26)$$

Subtracting (26) from (25)

$$C_2 \left(\frac{1}{a^2} - \frac{1}{b^2} \right) = \frac{1 + \nu}{E} (p_1 - p_2) + \frac{1 + \nu}{1 - \nu} \frac{\alpha}{b^2} \int_a^b T r dr$$

$$C_2 = \left(\frac{a^2 b^2}{b^2 - a^2} \right) \left(\frac{1 + \nu}{E} \right) (p_1 - p_2) + \left(\frac{a^2}{b^2 - a^2} \right) \left(\frac{1 + \nu}{1 - \nu} \right) \alpha \int_a^b T r dr \quad (27)$$

$$C_1 = (1 - 2\nu) \left(- \frac{p_1 (1 + \nu)}{E} + \frac{C_2}{a^2} \right)$$

$$C_1 = (1 - 2\nu) \left[- \frac{p_1 (1 + \nu)}{E} + \frac{b^2}{b^2 - a^2} \left(\frac{1 + \nu}{E} \right) (p_1 - p_2) + \left(\frac{\alpha}{b^2 - a^2} \right) \left(\frac{1 + \nu}{1 - \nu} \right) \int_a^b T r dr \right]$$

$$C_1 = \frac{(1 - 2\nu)(1 + \nu)}{E} \left(\frac{p_1 a^2 - p_2 b^2}{b^2 - a^2} \right) + \left(\frac{1 + \nu}{1 - \nu} \right) \frac{1 - 2\nu}{b^2 - a^2} \alpha \int_a^b T r dr \quad (28)$$

Thus

$$\left(\frac{C_1}{1 - 2\nu} - \frac{C_2}{r^2} \right) = \frac{1 + \nu}{E} \left(\frac{p_1 a^2 - p_2 b^2}{b^2 - a^2} \right) + \left(\frac{1 + \nu}{1 - \nu} \right) \frac{\alpha}{b^2 - a^2} \int_a^b T r dr$$

$$- \frac{1}{r^2} \left(\frac{a^2 b^2}{b^2 - a^2} \right) \left(\frac{1 + \nu}{E} \right) (p_1 - p_2)$$

$$- \frac{1}{r^2} \left(\frac{a^2}{b^2 - a^2} \right) \left(\frac{1 + \nu}{1 - \nu} \right) \alpha \int_a^b T r dr$$

$$\begin{aligned}
\left(\frac{C_1}{1-2\nu} - \frac{C_2}{r^2} \right) &= \left(\frac{1+\nu}{1-\nu} \right) \left(1 - \frac{a^2}{r^2} \right) \left(\frac{\infty}{b^2-a^2} \right) \int_a^b T r dr \\
&+ \frac{1+\nu}{E} \cdot \frac{1}{b^2-a^2} \left[p_1 a^2 - p_2 b^2 - \frac{a^2 b^2}{r^2} (p_1 - p_2) \right] \\
\text{and } S_R &= \frac{E}{1+\nu} \left[- \frac{1+\nu}{1-\nu} \cdot \frac{\infty}{r^2} \int_a^r T r dr + \left(\frac{1+\nu}{1-\nu} \right) \left(1 - \frac{a^2}{r^2} \right) \left(\frac{\infty}{b^2-a^2} \right) \int_a^b T r dr \right. \\
&+ \left. \frac{1+\nu}{E} \cdot \frac{1}{b^2-a^2} \left(p_1 a^2 - p_2 b^2 - \frac{a^2 b^2}{r^2} (p_1 - p_2) \right) \right] \\
S_R &= \frac{\infty E}{1-\nu} \left[\left(1 - \frac{a^2}{r^2} \right) \left(\frac{1}{b^2-a^2} \right) \int_a^b T r dr - \frac{1}{r^2} \int_a^b T r dr \right] \\
&+ \frac{1}{b^2-a^2} \left[p_1 a^2 - p_2 b^2 - \frac{a^2 b^2}{r^2} (p_1 - p_2) \right] \quad (29)
\end{aligned}$$

Similarly

$$\begin{aligned}
\left(\frac{C_1}{1-2\nu} + \frac{C_2}{r^2} \right) &= \left(\frac{1+\nu}{1-\nu} \right) \left(1 + \frac{a^2}{r^2} \right) \left(\frac{\infty}{b^2-a^2} \right) \int_a^b T r dr \\
&+ \frac{1+\nu}{E} \cdot \frac{1}{b^2-a^2} \left[p_1 a^2 - p_2 b^2 + \frac{a^2 b^2}{r^2} (p_1 - p_2) \right] \\
\text{and } S_T &= \frac{\infty E}{1-\nu} \left[\left(1 + \frac{a^2}{r^2} \right) \left(\frac{1}{b^2-a^2} \right) \int_a^b T r dr + \frac{1}{r^2} \int_a^r T r dr - T \right] \\
&+ \frac{1}{b^2-a^2} \left[p_1 a^2 - p_2 b^2 + \frac{a^2 b^2}{r^2} (p_1 - p_2) \right] \quad (30)
\end{aligned}$$

By introducing the wall thickness rather than radius, by assuming $p_2 = 0$, and by separating the pressure from the thermal stresses, the above equations become

$$S_{RP} = p_1 \left(\frac{a^2}{b^2 - a^2} \right) \left[1 - \left(\frac{b}{a + x} \right)^2 \right] \quad (31)$$

$$S_{RT} = - \frac{\alpha E}{1 - \nu} \left[\left(\frac{1}{(a + x)^2} \right) \int_0^x T(a + x) dx - \left(1 - \left(\frac{a}{a + x} \right)^2 \right) \frac{1}{b^2 - a^2} \int_0^c T(a + x) dx \right] \quad (32)$$

$$S_{TP} = p_1 \left(\frac{a}{b^2 - a^2} \right) \left[1 + \left(\frac{b}{a + x} \right)^2 \right] \quad (33)$$

$$S_{TT} = \frac{\alpha E}{1 - \nu} \left[\frac{1}{(a + x)^2} \int_0^x T(a + x) dx + \left(1 + \left(\frac{a}{a + x} \right)^2 \right) \cdot \frac{1}{b^2 - a^2} \int_0^c T(a + x) dx - T \right] \quad (34)$$

Equation (6) represents the total axial stress developed in a cylinder if the ends were supported by external means. If the total force on the ends is to be supported by the cylinder a stress term which just counterbalances this force must be added to the axial stress:

$$\text{Total force} = \pi a^2 p_1$$

$$\text{Stressed area} = (\pi b^2 - \pi a^2)$$

$$\text{Equivalent stress} = \frac{\pi a^2 p_1}{\pi (b^2 - a^2)} = p_1 \left(\frac{a^2}{b^2 - a^2} \right)$$

$$\text{and } S_Z = \sqrt{(S_T + S_R)} = \alpha E T + p_1 \left(\frac{a^2}{b^2 - a^2} \right) \quad (35)$$

$$\text{or } S_{ZP} = (2\sqrt{\nu} + 1) p_1 \left(\frac{a^2}{b^2 - a^2} \right) \quad (36)$$

$$S_{ZT} = \frac{\alpha E}{1 - \nu} \left[\frac{2\sqrt{\nu}}{b^2 - a^2} \int_0^c T(a + x) dx - T \right] \quad (37)$$

$$\text{and } S_Z = S_{ZP} + S_{ZT} \quad (38)$$

It should be noted that the T used in the above derivation is a difference temperature. Obviously if the pressure vessel is at a uniform temperature no thermal stress will exist, regardless of the magnitude of the vessel temperature.

It is obvious from equations (32), (34) and (37) that the magnitude of the thermal stress depends not only on the magnitude of the temperature difference but also on the variation of this difference through the vessel wall.

From Appendix A-2:

$$T = \frac{Q_0}{k A^2} (1 - e^{-Ax} - x A e^{-Ac})$$

$$\text{let } \frac{Q_0}{k A^2} = M$$

$$\text{then } T(a + x) = M a(1 - e^{-Ax} - x A e^{-Ac}) + M (x - x e^{-Ax} - x^2 A e^{-Ac})$$

$$\text{and } \int_0^x T(a + x) dx = \left[M a \left(x - \frac{e^{-Ax}}{(-A)} - \frac{x^2}{2} A e^{-Ac} \right) \right.$$

$$\left. + M \left(\frac{x^2}{2} - \frac{e^{-Ax}}{A^2} (-Ax-1) - \frac{x^3}{3} A e^{-Ac} \right) \right]_0^x$$

$$\int_0^x T(a + x) dx = M \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2} A e^{-Ac} \right) + \frac{x^2}{2} + \frac{x e^{-Ax}}{A} \right.$$

$$\left. + \frac{e^{-Ax}}{A^2} - \frac{x^3}{3} A e^{-Ac} \right]_0^x$$

$$\int_0^x T(a + x) dx = M \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2} A e^{-Ac} - \frac{1}{A} \right) + \frac{x^2}{2} \right.$$

$$\left. + \frac{x e^{-Ax}}{A} + \frac{e^{-Ax}}{A^2} - \frac{x^3}{3} A e^{-Ac} - \frac{1}{A^2} \right]$$

(39)

Similarly

$$\begin{aligned}
 \int_0^c T(a+x)dx &= M \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-Ac} - \frac{1}{A} \right) + \frac{c^2}{2} \right. \\
 &\quad \left. + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-Ac} - \frac{1}{A^2} \right] \\
 &= M \left[a \left[c - \frac{1}{A} + e^{-Ac} \left(\frac{1}{A} - \frac{c^2 A}{2} \right) \right] + \frac{c^2}{2} - \frac{1}{A^2} \right. \\
 &\quad \left. + e^{-Ac} \left(\frac{c}{A} + \frac{1}{A^2} - \frac{c^3 A}{3} \right) \right] \quad (40)
 \end{aligned}$$

From the above

$$\begin{aligned}
 S_{RT} &= \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[- \frac{1}{(a+x)^2} \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2} A e^{-Ax} - \frac{1}{A} \right) + \frac{x^2}{2} \right. \right. \\
 &\quad \left. \left. + \frac{x e^{-Ax}}{A} + \frac{e^{-Ax}}{A^2} - \frac{x^3}{3} A e^{-Ax} - \frac{1}{A^2} \right] \right. \\
 &\quad \left. + \left(1 - \frac{a^2}{(a+x)^2} \right) \left(\frac{1}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-Ac} - \frac{1}{A} \right) + \frac{c^2}{2} \right. \right. \\
 &\quad \left. \left. + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-Ac} - \frac{1}{A^2} \right] \right] \quad (41)
 \end{aligned}$$

When $x = 0$ $\int_0^x T(a+x)dx = 0$ and $1 - \left(\frac{a}{a+x} \right)^2 = 0$

$$\text{When } x = c, \quad \frac{1}{(a+c)^2} = \left[1 - \left(\frac{a}{a+c} \right)^2 \right] \frac{1}{b^2 - a^2}$$

$$\text{since } b = a + c$$

$$\frac{1}{b^2} = \left[\frac{b^2 - a^2}{b^2} \right] \frac{1}{b^2 - a^2}$$

$$\text{Thus } S_{RT} = 0 \text{ when } x = 0 \text{ or } x = c$$

Similarly

$$\begin{aligned} S_{TT} = & \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[\frac{1}{(a+x)^2} \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2} A e^{-Ac} - \frac{1}{A} \right) \right. \right. \\ & + \frac{x^2}{2} + \frac{x e^{-Ax}}{A} + \frac{e^{-Ax}}{A^2} - \frac{x^3}{3} A e^{-Ac} - \frac{1}{A^2} \Big] \\ & + \left(1 + \frac{a^2}{(a+x)^2} \right) \left(\frac{1}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-Ac} - \frac{1}{A} \right) \right. \\ & + \frac{c^2}{2} + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-Ac} - \frac{1}{A^2} \Big] \\ & \left. - \left(1 - e^{-Ax} - x A e^{-Ac} \right) \right] \end{aligned} \quad (42)$$

When $x = 0$,

$$\begin{aligned} S_{TT} = & \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-Ac} - \frac{1}{A} \right) \right. \\ & + \frac{c^2}{2} + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-Ac} - \frac{1}{A^2} \Big] \end{aligned} \quad (43)$$

Also

$$S_{ZT} = \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[\frac{2\nu}{b^2-a^2} \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-Ac} - \frac{1}{A} \right) \right. \right. \\ \left. \left. + \frac{c^2}{2} + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-Ac} - \frac{1}{A^2} \right] \right. \\ \left. - (1 - e^{-Ax} - x A e^{-Ac}) \right] \quad (44)$$

When $x = 0$

$$S_{ZT} = \nu S_{TT}$$

It should be noted that equation (39) thru (44) above are based on the concept of zero heat flow across the outside pressure vessel surface. As such they are valid only for determining the stresses in a well insulated reactor pressure vessel.

5. Thermal shield stresses

A similar set of equations can be developed for the stress in a reactor member such as a thermal shield which is assumed cooled to the same temperature on both surfaces.

$$T = M \left[1 - e^{-Ax} - \frac{x}{c} (1 - e^{-Ac}) \right]$$

$$T(a+x) = Ma \left[1 - e^{-Ax} - \frac{x}{c} (1 - e^{-Ac}) \right] + M \left[x - x e^{-Ax} - \frac{x^2}{c} (1 - e^{-Ac}) \right]$$

$$\int_0^x T(a+x) dx = \left[Ma \left[x - \frac{e^{-Ax}}{(-A)} - \frac{x^2}{2c} (1 - e^{-Ac}) \right] + M \left[\frac{x^2}{2} - \frac{e^{-Ax}}{A^2} (-Ax-1) - \frac{x^3}{3c} (1 - e^{-Ac}) \right] \right]_0^x$$

$$\int_0^x T(a+x) dx = M \left[a \left[x + \frac{e^{-Ax}}{A} - \frac{x^2}{2c} (1 - e^{-Ac}) - \frac{1}{A} \right] + \frac{x^2}{2} + \frac{e^{-Ax}}{A^2} (Ax + 1) - \frac{x^3}{3c} (1 - e^{-Ac}) - \frac{1}{A^2} \right] \quad (45)$$

Similarly:

$$\int_0^c T(a+x) dx = M \left[a \left[c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right] + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^3}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \right] \quad (46)$$

And

$$\begin{aligned}
 S_{RT} = & \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[- \frac{1}{(a+x)^2} \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2c} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \right. \\
 & + \frac{x^2}{2} + \frac{e^{-Ax}}{A^2} (Ax + 1) - \frac{x^3}{3c} (1 - e^{-Ac}) - \frac{1}{A^2} \Big] \\
 & + \left(1 - \frac{a^2}{(a+x)^2} \right) \left(\frac{1}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \\
 & \left. \left. + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^3}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \right] \right] \quad (47)
 \end{aligned}$$

As before, $S_{RT} = 0$ when $x = 0$ or $x = c$

Likewise

$$\begin{aligned}
 S_{TT} = & \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[\frac{1}{(a+x)^2} \left[a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2c} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \right. \\
 & + \frac{x^2}{2} + \frac{e^{-Ax}}{A^2} (Ax + 1) - \frac{x^3}{3c} (1 - e^{-Ac}) - \frac{1}{A^2} \Big] \\
 & + \left(1 + \frac{a}{(a+x)^2} \right) \left(\frac{1}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \\
 & + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^3}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \Big] \\
 & \left. - \left(1 - e^{-Ax} - \frac{x}{c} (1 - e^{-Ac}) \right) \right] \quad (48)
 \end{aligned}$$

When $x = 0$ or when $x = c$

$$S_{TT} = \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \\ \left. + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^2}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \right] \quad (49)$$

Also

$$S_{ZT} = \left(\frac{\infty E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left[\left(\frac{2\nu}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right) \right. \right. \\ \left. + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^2}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \right] \\ \left. - \left(1 - e^{-Ax} - \frac{x}{c} (1 - e^{-Ac}) \right) \right] \quad (50)$$

When $x = 0$ or $x = c$

$$S_{ZT} = \nu S_{TT} \quad (51)$$

B. STRESS DETAILS

1. The pressure vessel

a. Pressure stresses

Using equations developed in appendix A-4 the pressure stresses as a function of position in the vessel wall can be computed.

$$S_{TP} = p_1 \left(\frac{a^2}{b^2 - a^2} \right) \left(1 + \left(\frac{b}{a+x} \right)^2 \right)$$

$$S_{RP} = p_1 \left(\frac{a^2}{b^2 - a^2} \right) \left(1 - \left(\frac{b}{a+x} \right)^2 \right)$$

$$\text{and } S_{ZP} = p_1 \left(\frac{a^2}{b^2 - a^2} \right) (2 \nu + 1)$$

For APPR-1

$$p_1 = 1400$$

$$a^2 = 576$$

$$b^2 = 702$$

$$p_1 \left(\frac{a^2}{b^2 - a^2} \right) = 1400 \left(\frac{576}{126} \right) = 6400$$

$$\nu = 0.3$$

and $S_{ZP} = 6400 \times 1.6 = 10240$ psi for all values of x .

For $x = 1$ in.

$$\left(\frac{b}{a+x}\right)^2 = \frac{702}{(24 + 1)^2} = \frac{702}{625} = 1.123$$

$$S_{TP} = 6400(1 + 1.123) = 13580 \text{ psi}$$

$$\text{and } S_{RP} = 6400 (1 - 1.123) = - 787 \text{ psi}$$

Similar results were obtained for other values of x and the results plotted in Figure B-1.

b. Thermal stresses

The thermal stresses through the pressure vessel wall may be computed using equations (41), (42) and (44). Since

the integral $\int_0^c T(a+x) dx$ is common to all equations

and does not vary with x the computation can be somewhat simplified. In addition the value of T can be read directly from Figure 19.

$$\begin{aligned} \int_0^c T(a+x) dx &= a\left(c + \frac{e^{-Ax}}{A} - \frac{c^2}{2} Ae^{-Ac} - \frac{1}{A}\right) + \frac{c^2}{2} + \frac{c e^{-Ac}}{A} \\ &+ \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} Ae^{-Ac} - \frac{1}{A^2} \end{aligned}$$

$$a = 2 \text{ ft}$$

$$c = 2.5 \text{ in.} = 0.2084 \text{ ft. } c^2 = 0.0434,$$

$$c^3 = 0.00905$$

$$A = 6.22 \text{ ft}^{-1} \quad A^2 = 38.7$$

$$Ac = 1.296$$

$$e^{-Ac} = 0.2736$$

$$\begin{aligned}
\int_0^c T(a+x)dx &= 2 \left(0.2084 + \frac{0.2736}{6.22} - \frac{0.0434 \times 6.22 \times 0.2736}{2} - \frac{1}{6.22} \right) \\
&\quad + \frac{0.0434}{2} + 0.2736 \left(\frac{0.2084}{6.22} + \frac{1}{38.7} - \frac{0.00905 \times 6.22}{3} \right) - \frac{1}{38.7} \\
&= 2(0.2084 + 0.044 - 0.0369 - 0.1608) + 0.0217 - 0.0258 \\
&\quad + 0.2736(0.0335 + 0.0258 - 0.0188) \\
&= 2 \times 0.0547 + 0.0217 - 0.0258 + 0.2736 \times 0.0405 \\
&= 0.1094 + 0.0217 - 0.0258 + 0.0111
\end{aligned}$$

$$\int_0^c T(a+x)dx = 0.1164$$

Other constants are also evaluated

$$\frac{\infty E}{1-\sqrt{\quad}} = 267.5 \quad (\text{from page 79})$$

$$\frac{Q_0}{k A^2} = 37.2 \quad (\text{from page 79})$$

$$\frac{1}{b^2 - a^2} = \frac{1}{4.875 - 4} = 1.143$$

$$\begin{aligned}
S_{TT} &= 9950 \left[\left(\frac{1}{a+x} \right)^2 \left(a \left(x + \frac{e^{-Ax}}{A} - \frac{x^2}{2} A e^{-Ac} - \frac{1}{A} \right) + \frac{x^2}{2} - \frac{1}{A^2} \right. \right. \\
&\quad \left. \left. + e^{-Ax} \left(\frac{x}{A} + \frac{1}{A^2} - \frac{x^3 A}{3} \right) + 0.133 \left(1 + \frac{a^2}{(a+x)^2} \right) - \frac{T}{37.2} \right] \\
&= 9950 \left[\left(\frac{1}{2+x} \right)^2 \left(2 \left(x + \frac{e^{-Ax}}{6.22} - 0.851 x^2 \right) + \frac{x^2}{2} - 0.3474 \right. \right. \\
&\quad \left. \left. + e^{-Ax} \left(\frac{x}{6.22} - 2.072 x^3 + 0.0258 \right) + 0.133 \left(1 + \left(\frac{2}{2+x} \right)^2 \right) - \frac{T}{37.2} \right]
\end{aligned}$$

For $x = 0.6 \text{ in.} = 0.05 \text{ ft.}$, $e^{-Ax} = e^{-0.311} = 0.7327$

$$\begin{aligned}
 S_{TT} &= 9950 \left[\frac{1}{4.2} \left(2(0.05 + \frac{0.7327}{6.22} - 0.851 \times 0.0025) + \frac{0.0025}{2} \right. \right. \\
 &\quad \left. \left. - 0.3474 + 0.7327 \left(\frac{0.05}{6.22} - 2.072 \times 0.000125 + 0.0258 \right) \right) \right. \\
 &\quad \left. + 0.133 \left(1 + \frac{4}{4.2} \right) - 0.1827 \right] \\
 &= 9950 \left[0.283 \left(2(0.5 + 0.1178 - 0.00213) + 0.00125 - 0.3474 \right. \right. \\
 &\quad \left. \left. + 0.7327(0.00804 - 0.00026 + 0.0258) \right) + 0.133 \times 1.952 - 0.1827 \right] \\
 &= 9950 \left[0.238(2 \times 0.1657 - 0.3461 + 0.7327 \times 0.0335) \right. \\
 &\quad \left. + 0.2597 - 0.1827 \right] \\
 &= 9950 (0.238 \times 0.0062 + 0.2597 - 0.1827) \\
 &= 9950 (0.0024 + 0.2597 - 0.1827)
 \end{aligned}$$

$$S_{TT} = 9950 \times 0.0785 = 790 \text{ psi}$$

$$\begin{aligned}
 S_{RT} &= 9950 - 0.0024 + 0.133(1 - 0.952) \\
 &= 9950 (-0.0024 + 0.133 \times 0.048) \\
 &= 9950 \times 0.0040 = 39.8 \text{ psi}
 \end{aligned}$$

$$\begin{aligned}
 S_{ZT} &= 9950 \frac{0.6}{0.875} \times 0.1164 - 0.1853 \\
 &= 9950 (0.0798 - 0.1853)
 \end{aligned}$$

$$S_{ZT} = 9950 \times (-0.1055) = -1050 \text{ psi}$$

Other values of x were similarly computed and the results shown in Figure B-2.

FIGURE B-2

The total stress in the pressure vessel wall is just the sum of the pressure and thermal stresses. Figure B-3 shows the total of each of the three stress components.

It is interesting to note that the average tangential thermal stress is just zero and the average total stress is therefore no greater than the average tangential pressure stress.

FIGURE B-3

2. The thermal shield

If we assume negligible pressure difference between the two sides of the thermal shield, the maximum stress in the shield is simply the tangential thermal stress at either surface.

$$S_{TT} = \left(\frac{\alpha E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2}{b^2 - a^2} \right) \left[a \left[c + \frac{e^{-Ac}}{A} - \frac{c}{2} (1 - e^{-Ac}) - \frac{1}{A} \right] \right. \\ \left. + \frac{c^2}{2} + \frac{e^{-Ac}}{A^2} (Ac + 1) - \frac{c^2}{3} (1 - e^{-Ac}) - \frac{1}{A^2} \right]$$

$$\frac{Q_0}{k A^2} = 487 \quad (\text{from page 137})$$

$$A = 6.82 \text{ ft}^{-1}$$

$$Ac = 1.135$$

$$(Ac + 1) = 2.135$$

$$e^{-Ac} = 0.321$$

$$(1 - e^{-Ac}) = 0.679$$

$$a = 1.46 \text{ ft}$$

$$a^2 = 2.125 \text{ ft}^2$$

$$b = 1.627 \text{ ft}$$

$$a^2 = 2.64 \text{ ft}^2$$

$$b^2 - a^2 = 0.515$$

$$\frac{2}{b^2 - a^2} = 3.88$$

$$c = 0.167 \text{ ft}$$

$$\alpha = 10.2 \times 10^{-6}$$

$$(\text{at } 500^\circ\text{F})$$

$$E = 25.3 \times 10^6$$

$$(\text{at } 500^\circ\text{F})$$

$$\frac{\alpha E}{1-\nu} = \frac{10.2 \times 25.3}{1 - 0.3} = 369$$

$$\begin{aligned}
S_{TT} &= 369 \times 487 \times 3.88 \left[1.46 \left(0.167 + \frac{0.321}{6.82} - \frac{0.167}{2} \times 0.679 - \frac{1}{6.82} \right) \right. \\
&\quad \left. + \frac{0.167^2}{2} + \frac{0.321}{6.822} \times 2.135 - \frac{0.167^2}{3} \times 0.679 - \frac{1}{6.82^2} \right] \\
&= 6.97 \times 10^5 \left[1.46(0.167 + 0.0471 - 0.0567 - 0.147) \right. \\
&\quad \left. + 0.0139 + 0.0147 - 0.0063 - 0.0215 \right]
\end{aligned}$$

$$S_{TT} = 6.97 \times 10^5 \times 1.12 \times 10^{-2} = 7.82 \times 10^3 = 7820 \text{ psi}$$

The variation of stress through the shield thickness will not be examined. A maximum compressive stress would be expected near the center, at the same depth as the maximum temperature.

C. HEAT GENERATION IN THE THERMAL SHIELD.

The calculation of the heat generation within the thermal shield cannot be simplified as was done for the pressure vessel. The total energy absorption depends on the incident energy from the inside, the incident energy from the outside, and the gammas generated in the thermal shield. The total absorption distribution across the thermal shield will also be required to compute the total heat generated.

1. Primary gammas

Primary gammas incident on the thermal shield can be computed from the expression:

$$I = \frac{S_v B R^2 F(\theta, b)}{2(a + z)}$$

$$a = 6.5 \text{ inches} = 16.5 \text{ cm.}$$

Group	$(a + z)$	$\tan \theta = \frac{28}{(a + z)}$	θ
1	28.7	0.97	44°
3	30.7	0.91	42°
5	31.2	0.90	42°
7	31.4	0.89	42°

Group	$0.842\mu_w(a-t)$	$\mu_c z$	Sum	$\mu_{ss}t$	b	B
1	0.95	1.96	2.91	0.16	3.07	5.50
3	0.53	1.30	1.83	0.09	1.92	2.34
5	0.41	1.16	1.57	0.08	1.65	1.85
7	0.35	1.12	1.47	0.07	1.54	1.64

Group	S_v	$2(a+z)$	B	R^2	$F(\theta, b)$	$I_{6.5}$
1	2.31×10^{13}	57.4	5.50	784	0.026	4.5×10^{13}
3	7.42×10^{11}	61.4	2.34	784	0.089	1.97×10^{12}
5	4.57×10^{11}	62.4	1.85	784	0.120	1.27×10^{12}
7	3.94×10^{11}	62.8	1.64	784	0.135	1.09×10^{12}

The distribution of the primary gammas across the thermal shield may be computed using the attenuations shown in Figure 12.

2. Secondary gammas from inside

a. Flow separator

The magnitude and distribution of capture gammas from the flow separator can be read directly from Figure 12. Inelastic scatter gammas can be determined using attenuation factors from Figure 12.

b. Water

The gammas due to water capture can be obtained from pages 55 and 56 attenuating to the thermal shield surface rather than the pressure vessel. Figure 15 shows the attenuation

through stainless steel

$$I_1 = 4.14 \times 10^{11} \times \frac{6.3}{16.6} = 1.57 \times 10^{11}$$

$$I_2 = 1.86 \times 10^{12} \times \frac{6.3}{11.7} = 1.0 \times 10^{12}$$

$$I_3 = 1.73 \times 10^{12} \times \frac{6.3}{6.3} = 1.73 \times 10^{12}$$

$$I_{\text{Total}} = 2.89 \times 10^{12}$$

Figure C-1 shows the total gammas reaching the thermal shield from the inside. These figures are tabulated below:

Energy Group	1	2	3	5	7
Primary	45×10^{12}	--	1.97	1.27	1.09
Flow Separator - (c)	0.02	--	.03	.05	.16
Flow Separator - (i.s.)	0.06	--	--	--	--
Water	--	2.89	--	--	--
Total	45.1×10^{12}	2.89	2.0	1.32	1.25

3. Secondary gammas from outside.

a. Water capture

Gamma radiation at the thermal shield from the two "slabs" of water between the shield and the pressure vessel can be computed from the expression (from Appendix A-1):

$$I = \frac{\phi_0 C}{2} \left(\frac{1}{A + \mu} \right) \left[1 - e^{-t(A + \mu)} \right]$$

The slab thicknesses and the flux patterns are those assumed in Figure 14. For the 2.5 inches nearest the thermal shield the contribution is identical with I_4 (page 56)

$$I = 1.23 \times 10^{11}$$

For the 1.75 inches next to the pressure vessel

$$A = 0.126$$

$$\phi = 3.5 \times 10^{12}$$

$$C = 0.0126$$

$$\mu = 0.0391$$

$$t = 1.75 \text{ in.} = 4.45 \text{ cm}$$

$$\text{Att. (thru 2.5 inch water)} = \frac{1.37}{2.1} \text{ (from Figure 15)}$$

$$I = \frac{3.5 \times 10^{12} \times 0.0126}{2} \left(\frac{1}{0.126 + 0.0391} \right) \left[1 - e^{-4.45(0.1651)} \right]$$

$$I = 1.34 \times 10^{11} (1 - 0.848) = 0.2 \times 10^{11}$$

$$I_{\text{Total}} = (1.23 + 0.20) \times 10^{11} = 1.43 \times 10^{11}$$

b. Pressure vessel gammas

The contribution of pressure vessel capture and inelastic scatter gammas can be obtained from Figures 13 and 16 with attenuation factors from Figure 12:

Group	$I_{12.75}$	Att.	$I_{8.5}$
1	1.32×10^{10} - i.s.	0.43	0.57×10^{10}
	0.64×10^{10}	0.43	0.27×10^{10}
3	2.65×10^{10}	0.495	1.31×10^{10}
5	3.7×10^{10}	0.51	1.9×10^{10}
7	1.0×10^{11}	0.52	5.2×10^{10}

Figure C-2 shows the total gammas reaching the thermal shield from the outside. These are tabulated below:

Energy Group	1	2.2	3	5	7
Water capture	--	1.43×10^{11}	--	--	--
Pressure vessel (i.s.)	0.06	--	--	--	--
Pressure vessel (c)	0.03	--	0.13	0.19	0.52

4. Secondary gammas generated in the thermal shield.

a. Capture gammas

Gammas generated in the thermal shield may be computed as the sum resulting from the two exponentials shown in Figure 9. The general expression for the gamma flux at any point in the shield is (from Appendix A-1):

$$I_x = \frac{\phi_0 C}{2} \left[\frac{e^{-\mu x}}{A - \mu} \left(1 - e^{-x(A - \mu)} \right) + \frac{e^{\mu x}}{(A + \mu)} \left(e^{-x(A + \mu)} - e^{-t(A + \mu)} \right) \right]$$

$$\phi_0 = 2 \times 10^{13}$$

$$A = 0.689$$

For $x = 0.4$ inches = 1.016 cm.

Group	μ	μx	$e^{\mu x}$	$e^{-\mu x}$	$(A - \mu)$	$(A + \mu)$
1	0.49	0.498	1.645	0.608	0.199	1.179
3	0.283	0.288	1.334	0.750	0.406	0.972
5	0.244	1.281	0.281	0.780	0.445	0.933
7	0.232	0.236	1.266	0.790	0.457	0.921

Group	$x(A-\mu)$	$x(A+\mu)$	$t(A+\mu)$	$e^{-x(A-\mu)}$	$e^{-(A+\mu)x}$	$e^{-t(A+\mu)}$
1	0.202	1.20	5.98	0.817	0.301	0.0025
3	0.412	0.987	4.94	0.662	0.373	0.0072
5	0.452	0.948	4.74	0.636	0.388	0.0087
7	0.464	0.935	4.68	0.629	0.393	0.0093

Group	$(1-e^{-x(A-\mu)})$	$(e^{-x(A+\mu)} - e^{-t(A+\mu)})$	$\left[\begin{array}{c} \end{array} \right]$
1	0.183	0.299	$0.559 + 0.417 = 0.976$
3	0.338	0.364	$0.624 + 0.501 = 1.125$
5	0.364	0.379	$0.638 + 0.520 = 1.158$
7	0.371	0.384	$0.641 + 0.527 = 1.168$

Group	$\phi_0 / 2$	c	$I_{6.9}$
1	1×10^{13}	0.0178	1.74×10^{11}
3	1×10^{13}	0.0233	2.62×10^{11}
5	1×10^{13}	0.0329	3.81×10^{11}
7	1×10^{13}	0.1035	1.21×10^{12}

Similar computation yields the following results.

Group	$I_{7.3}$	$I_{7.7}$	$I_{8.1}$	$I_{8.5}$ (from page 47)
1	1.45×10^{11}	1.08×10^{11}	7.45×10^{10}	4.72×10^{10}
3	2.36×10^{11}	1.98×10^{11}	1.56×10^{11}	1.19×10^{11}
5	3.5×10^{11}	2.97×10^{11}	2.43×10^{11}	1.91×10^{11}
7	1.12×10^{12}	9.55×10^{11}	7.88×10^{11}	6.27×10^{11}

For $x = 0$ a simplified expression can be used.

$$I_{6.5} = \frac{\phi_0 C}{2} \left(\frac{1}{A + \mu} \right) \left[1 - e^{-t(A + \mu)} \right]$$

Group	$\phi_0/2$	C	$\left[1 - e^{-t(A + \mu)} \right]$	$(A + \mu)$	$I_{6.5}$
1	1×10^{13}	0.0178	0.9975	1.179	1.51×10^{11}
3	1×10^{13}	0.0233	0.9928	0.972	2.38×10^{11}
5	1×10^{13}	0.0329	0.9913	0.933	3.5×10^{11}
7	1×10^{13}	0.1035	0.9907	0.921	1.11×10^{12}

The gammas due to the increasing exponential (Figure 9) can easily be obtained by using the "mirror-image" approach, i.e.:

$$I'_{6.5} = \frac{\phi'_0}{\phi_0} I_{8.5}$$

Group	ϕ'_0/ϕ_0	$I'_{6.5}$	$I'_{6.9}$	$I'_{7.3}$	$I'_{7.7}$	$I'_{8.1}$	$I'_{8.5}$
1	0.135	0.64×10^{10}	1.01	1.46	1.96	2.35	2.02
3	0.135	1.61×10^{10}	2.10	2.67	3.18	3.54	3.20
5	0.135	2.58×10^{10}	3.28	4.0	4.72	5.14	4.72
7	0.135	0.85×10^{11}	1.06	1.29	1.51	1.63	1.5

The total capture gammas due to the thermal shield are given by $I + I'$

Group	I + I' (6.5)	(6.9)	(7.3)	(7.7)	(8.1)	(8.5)
1	1.57×10^{11}	1.84	1.60	1.28	0.98	0.67
3	2.54×10^{11}	2.83	2.63	2.30	1.91	1.51
5	3.76×10^{11}	4.14	3.9	3.44	2.94	2.38
7	1.20×10^{12}	1.32	1.25	1.11	0.95	0.78

b. Inelastic scatter gammas

Inelastic scatter gammas may be computed much like the capture gammas. However, since there is essentially no source of fast neutrons outside the shield and since only group 1 gammas are produced, the computation is simplified.

For $x = 0$

$$I_{6.5} = \frac{\phi_0 C}{2} \left(\frac{1}{A + \mu} \right) \left[1 - e^{-t(A + \mu)} \right]$$

$$\phi_0 = 2.2 \times 10^{12} \quad (\text{from Figure 12})$$

$$A = 0.1675$$

$$\mu = 0.49$$

$$C = 0.1042 \quad (\text{from page 58})$$

$$t = 2 \text{ in.} = 5.08 \text{ cm.}$$

$$I_{6.5} = 1.1 \times 10^{12} \times 0.0125 \left[\frac{1}{0.6575} (1 - e^{-3.34}) \right]$$

$$I_{6.5} = 1.75 \times 10^{11} (1 - 0.035) = 1.69 \times 10^{11}$$

For $x = t$

$$I_{8.5} = 1.23 \times 10^{11}$$

(from page 59)

For other values of x :

$$I = \frac{\phi_0 C}{2} \left[\frac{e^{-\mu x}}{A - \mu} \left(1 - e^{-x(A - \mu)} \right) + \frac{e^{\mu x}}{(A + \mu)} \left(e^{-x(A + \mu)} - e^{-t(A + \mu)} \right) \right]$$

$x(\text{in})$	$x(\text{cm})$	μx	$e^{-\mu x}$	$e^{\mu x}$	$(A - \mu)$	$(A + \mu)$
0.4	1.016	0.498	0.608	1.645	-0.3225	0.6575
0.8	2.03	0.996	0.369	2.707		
1.2	3.05	1.495	0.224	4.459		
1.6	4.07	1.99	0.137	7.32		

$x(\text{in})$	$x(A - \mu)$	$x(A + \mu)$
0.4	-0.327	0.668
0.8	-0.655	1.335
1.2	-0.983	2.007
1.6	-1.312	2.68

x	$e^{-x(A - \mu)}$	$e^{-x(A + \mu)}$	$(1 - e^{-x(A - \mu)})$	$(e^{-x(A + \mu)} - e^{-t(A + \mu)})$
0.4	1.387	0.513	-0.387	0.478
0.8	1.925	0.263	-0.925	0.228
1.2	2.672	0.134	-1.672	0.099
1.6	3.714	0.069	-2.714	0.034

x	$\left[\right]$	$\phi_0/2$	C	I
0.4	$0.729 + 1.194 = 1.923$	1.1×10^{12}	0.1042	2.2×10^{11}
0.8	$1.058 + 0.938 = 1.996$			2.29×10^{11}
1.2	$1.161 + 0.671 = 1.832$			2.1×10^{11}
1.6	$1.153 + 0.378 = 1.531$			1.75×10^{11}

The above capture and inelastic scatter gammas generated in the thermal shield are shown graphically in Figure C-3.

Using Figures C-1, C-2 and C-3, the total gamma distribution (Figure C-4) and energy absorption distribution (Figure C-5) may be estimated.

FIGURE C-3

Energy Group	1	2.2	3	5	7	Total
$x = 0$						
Inside	45.1×10^{12}	2.89	2.0	1.32	1.25	
Outside	0.001	0.048	0.005	0.007	0.018	
Gen.	0.28	--	0.24	0.38	1.2	
Total	45.381×10^{12}	2.938	2.245	1.707	2.468	
Mev/cm ² -sec	45.4×10^{12}	6.47	6.73	8.53	17.3	
Mev/cm ³ -sec	9.35×10^{12}	1.15	1.18	1.52	3.2	16.40×10^{12}
Btu/ft ³ -hr						2.54×10^5

$x = 0.4$

Inside	33.4×10^{12}	2.32	1.62	1.08	1.02	
Outside	0.002	0.06	0.006	0.008	0.022	
Gen.	0.349	--	0.263	0.413	1.32	
Total	33.75×10^{12}	2.38	1.89	1.50	2.36	
Mev/cm ² -sec	33.75×10^{12}	5.24	5.67	7.5	16.5	
Mev/cm ³ -sec	6.95×10^{12}	0.93	1.0	1.33	3.05	13.26×10^{12}
Btu/ft ³ -hr						2.05×10^5

$x = 0.8$

Inside	24.8×10^{12}	1.87	1.31	0.875	0.825	
Outside	0.002	0.074	0.007	0.01	0.027	
Gen.	0.341	--	0.248	0.39	1.26	
Total	25.14×10^{12}	1.94	1.56	1.27	2.11	
Mev/cm ² -sec	25.14×10^{12}	4.27	4.68	6.35	14.77	
Mev/cm ³ -sec	5.18×10^{12}	0.76	0.82	1.13	2.73	10.62×10^{12}
Btu/ft ³ -hr						1.65×10^5

Group Energy	1	2.2	3	5	7	Total
$x = 1.2$						
Inside	18.4×10^{12}	1.5	1.05	0.71	0.67	
Outside	0.003	0.092	0.009	0.013	0.034	
Gen.	0.30	--	0.217	0.345	1.12	
Total	18.7×10^{12}	1.59	1.28	1.07	1.82	
Mev/cm ² -sec	18.7×10^{12}	3.5	3.84	5.35	12.7	
Mev/cm ³ -sec	3.85×10^{12}	0.62	0.68	0.95	2.35	8.45×10^{12}
Btu/ft ³ -hr						1.31×10^5

$x = 1.6$						
Inside	13.6×10^{12}	1.21	0.86	0.58	0.54	
Outside	0.004	0.0114	0.011	0.015	0.042	
Gen.	0.243	--	0.18	0.29	0.95	
Total	13.85×10^{12}	1.32	1.05	0.88	1.53	
Mev/cm ² -sec	13.85×10^{12}	2.90	3.15	4.40	10.70	
Mev/cm ³ -sec	2.85×10^{12}	0.52	0.55	0.78	1.98	6.68×10^{12}
Btu/ft ³ -hr						1.035×10^5

$x = 2.0$						
Inside	10.1×10^{12}	0.97	0.70	0.47	0.44	
Outside	0.006	0.142	0.013	0.019	0.052	
Gen.	0.169	--	0.143	0.24	0.78	
Total	10.27×10^{12}	1.11	0.86	0.73	1.27	
Mev/cm ² -sec	10.27×10^{12}	2.44	2.58	3.65	8.9	
Mev/cm ³ -sec	2.11×10^{12}	0.43	0.45	0.65	1.65	5.29×10^{12}
Btu/ft ³ -hr						8.19×10^4

5. Temperature Distribution in the Thermal Shield.

The thermal shield temperature profile can be computed from the expression:

$$T = \frac{Q_0}{k A^2} \left[1 - e^{-Ax} - \frac{x}{c} (1 - e^{-Ac}) \right]$$

where T is the temperature above the surface temperature

$$Q_0 = 2.55 \times 10^5 \text{ Btu/ft}^3\text{-hr}$$

$$c = 2 \text{ inches} = 0.167 \text{ ft}$$

$$e^{-Ac} = \frac{Q_0}{25.5} = \frac{8.19}{25.5} = 0.321$$

$$Ac = 1.135$$

$$A = \frac{1.135}{0.167} = 6.82 \text{ ft}^{-1}$$

$$k = 11.3 \text{ Btu/ft}^2\text{-hr-F}^\circ/\text{ft. (at } 490^\circ\text{F)}$$

$$\frac{Q_0}{k A^2} = \frac{2.55 \times 10^5}{11.3 \times 6.82^2} = 487$$

$$\left(\frac{1 - e^{-Ac}}{c} \right) = \frac{1 - 0.321}{0.167} = \frac{0.679}{0.167} = 4.07$$

$$T = 487 (1 - e^{-Ax} - 4.07 x)$$

x (in.)	x(ft)	Ax	e ^{-Ax}	4.07x	$\left[\frac{1 - e^{-Ac}}{c} \right]$	T
0.4	0.033	0.227	0.797	0.136	0.067	32.6
0.8	0.0667	0.455	0.634	0.271	0.095	46.3
1.2	0.1000	0.682	0.506	0.407	0.087	42.4
1.6	0.1333	0.908	0.403	0.542	0.055	26.8

The point of maximum temperature within the thermal shield can be found from the expression⁽¹⁷⁾

$$\begin{aligned} x(\max) &= -\frac{1}{A} \ln \left(\frac{1 - e^{-Ac}}{Ac} \right) \\ &= -\frac{1}{6.82} \ln \left(\frac{0.679}{1.135} \right) \\ &= -\frac{\ln 0.598}{6.82} = \frac{0.514}{6.82} = 0.075 \text{ ft} \end{aligned}$$

$$x(\max) = 0.90 \text{ inches}$$

The heat transferred through the thermal shield surfaces is obtained from the expressions:

$$H_i = \int_0^{0.075} Q(x) dx = \frac{Q_o}{A} \left(1 - e^{-0.075A} \right) \quad (\text{inward})$$

$$H_o = \int_{0.075}^{0.167} Q(x) dx = \frac{Q_o}{A} \left(e^{-0.075A} - e^{-0.167A} \right) \quad (\text{outward})$$

$$H_i = \frac{2.55 \times 10^5}{6.82} \left(1 - e^{0.511} \right) = 3.74 \times 10^4 (1 - 0.60)$$

$$= 1.5 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

$$H_o = 3.74 \times 10^4 (0.60 - e^{-1.14}) = 3.74 \times 10^4 (0.60 - 0.32)$$

$$H_o = 3.74 \times 10^4 \times 0.28 = 1.05 \times 10^4 \text{ Btu/ft}^2\text{-hr}$$

Using the value of h_f from page 70 the film temperature drop is:

$$T_f = \frac{15000}{725} = 20.7^\circ \text{F}^\circ$$

Similarly the temperature drop across the scale is:

$$T_s = H_1 \frac{t_s}{K_s} = 15000 \frac{0.01}{12} = 12.5^\circ \text{F}^\circ$$

It should be noted that the assumption of equal temperatures at both surfaces of the thermal shield is not completely compatible with the heat flows and resultant temperature drops indicated above. The computed temperature profile of the thermal shield is shown in Figure C-6.

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D. Overload Conditions

The possibility of operating the APPR-1 at greater than design power level has been considered. From the standpoint of allowable stress in the pressure vessel, several comments are in order.

1. The increase of the power plant load does not necessarily mean a proportional increase in reactor power.

Depending on turbine design, the increase in reactor power may be greater or less than the plant load increase.

2. Any increase in the reactor power of the APPR-1 must be accomplished by lowering the core inlet temperature.

This lower temperature applies also to the reflector water and results in a slightly greater gamma attenuation in the water.

3. Since the operating pressure remains at ~ 1200 psi regardless of load, the possibility exists of operating at a pressure vessel stress not exceeding the original design stress.

If we assume that the radiation energy incident on the pressure vessel is directly proportional to plant load then the thermal stress is also a direct function of load. For a constant total pressure vessel stress of 16820 psi the pressure stress must decrease as the load increases.

At an operating pressure of 1200 psi the maximum pressure stress is (from page 76):

$$S_{TP} = 1200 \times 4.57 \times 2.218 = 12180 \text{ psi}$$

If the primary system pressure never exceeded 1200 psi the allowable maximum thermal stress would be $16820 - 12180 = 4640$ psi. The equivalent plant load would then be:

$$\frac{4640}{2640} = 1.75 = 175\%$$

If a primary system pressure relief valve is assumed, the opening pressures necessary to limit the total pressure vessel stress can be simply computed. (see Figure D-1). Operation at 120% of power would require the primary system pressure to be not greater than ~ 1350 psi in order not to exceed design stresses.

FIGURE D-1

E. GAMMA HEATING MEASUREMENTS

1. MTR

C. H. Hogg reported in IDO-16093 the results of measurements of gamma heating in various positions near the MTR core. These results, published in September 1953, are in substantial agreement with an extrapolation of LITR data reported in Ley - 11-52A-M.

Direct comparison of Hogg's data to APPR-1 computed results is not meaningful. Major differences in the gamma spectra, the core geometry, and the interposed material are noted.

Figure E-1 is a plan of the MTR. All of Hogg's results were obtained for a 9 x 3 lattice except one reading for position VG-7 taken with a 5 x 5 lattice. The ratio of the two results, both with an aluminum sample, is $\frac{0.041}{0.091} = 0.45$. Since APPR-1 geometry approximates the 5 x 5 lattice it is assumed this ratio can be applied to all results.

A rough comparison of the gamma spectra from aluminum and stainless steel cores can be made from the data (at 12.75 inch from the core surface) on pages 20 and 34 .

PLAN OF MTR HEATING
MEASUREMENT LOCATIONS

FIGURE E-1

For aluminum

Group	Photons	Mev/cm ² -sec
1	1.46×10^{13}	14.6×10^{12}
3	1.39×10^{12}	4.17×10^{12}
5	5.85×10^{11}	2.92×10^{12}
7	1.98×10^{11}	1.39×10^{12}
Total	-----	23.08×10^{12}

For stainless steel

Group	Photons	Mev/cm ² -sec
1	3.93×10^{12}	3.93×10^{12}
3	3.29×10^{11}	$.99 \times 10^{12}$
5	2.23×10^{11}	1.12×10^{12}
7	1.86×10^{11}	1.30×10^{12}
Total	-----	7.34×10^{12}

The spectrum correction is then $\frac{7.34}{23.08} = 0.318$. Note that this correction includes the effect of size or power density and the 2 inch thick thermal shield.

The differences due to different materials between the core and the point of interest can be approximated as follows. The beryllium is approximately 1.4 times as effective as water in absorbing gammas of all energies. The graphite pebble zone is assumed equivalent to water due to the interstices. Some results (normalized to 10 Mw) may now be tabulated.

Position	Be Distance	Pebble Distance	1.4 x Be Distance	Effective Distance
A-33	9.5	---	13.3	13.3
A-41	16.0	---	22.5	22.5
VG-7	21.0	4.5	29.5	34
VG-11	21.0	4.5	29.5	34
VG-51	21.0	12	29.5	41.5

Run	q(w/gm @ 10 Mw)	Geometry	Spectrum	qo	Effective Distance
1	0.0137	1	0.318	0.0044	34
2	0.110	0.45	"	0.0157	22.5
3	0.0303	"	"	0.0043	34
4	0.366	"	"	0.0523	13.3
5	0.0280	"	"	0.0040	34
6	0.0276	"	"	0.0039	34
7	0.030	"	"	0.0043	34
8	0.0183	"	"	0.0026	34
9	0.027	"	"	0.0039	34
10	0.0197	"	"	0.0028	34
11	0.0012	"	"	0.0002	41.5

Figure E-2 is a plot of the above results.

At 12.75 inches the heating rate is 0.056 watts/gram. Since the pressure vessel density is 7.85 gm/cc the equivalent volumetric heat generation at the inside surface is:

$$Q_0 = 0.056 \times 7.85 \times 3.413 \times 2.832 \times 10^4$$

$$Q_0 = 42400 \text{ Btu/ft}^3\text{-hr}$$

The agreement between this result and the calculated value of 43550 Btu/ft³-hr is largely fortuitous.

FIGURE E-2

2. LITR

ORNL CF 53-3-276, "Nuclear Heating in the LITR" page 444 gives experimental results for the heating of stainless steel at various distances from the core. The following values are taken from DWC 20768A.

Distance from core (inches)	$q(w/gm)$ at 1.5 Mw
8	0.094
10	0.067
12	0.043
14	0.027
16	0.017
18	0.011

It is assumed the above values were obtained with air only between the detector and the core. The effect of water reflector and thermal shield must be approximated to obtain comparable results. Using basic results already available (the primary gamma spectrum at 12.75 inches, page 34 and the attenuations shown in Figure 12) it is possible to approximate the attenuation to be applied to the above results. Note that the geometric portion of the attenuation factor is already included in the observed results and must therefore be removed from the correction factor. The following values are in Mev/cm^2 -sec-10 Mw.

Group	$I_{12.75}$	Att _{Total}	I_0	Att _{ss}	I_2
1	3.93×10^{12}	40.8	16.0×10^{13}	0.226	36.0×10^{12}
3	$.99 \times 10^{12}$	17	1.7×10^{13}	0.35	6.0×10^{12}
5	1.1×10^{12}	15.6	1.8×10^{13}	0.355	6.3×10^{12}
7	1.3×10^{12}	14	<u>1.8×10^{13}</u>	0.355	<u>6.4×10^{12}</u>
Total	7.32×10^{12}		21.3×10^{13}		54.7×10^{12}

Assuming an equivalent 5 x 5 core array, the effective radius is 7.5 inches and the inverse geometric factor is $\left(\frac{7.5 + d}{7.5}\right)^2$

Then $I_0 = 21.3 \times 10^{13}$ Mev/cm²-sec

$$I_2 = 54.7 \times 10^{12} \times \left(\frac{9.5}{7.5}\right)^2 = 8.8 \times 10^{13}$$

$$I_{12.75} = 7.32 \times 10^{12} \times \left(\frac{20.25}{7.5}\right)^2 = 5.35 \times 10^{13}$$

The equivalent effective attenuations are then:

Distance	Attenuation
0	1
2	0.41
12.75	0.25

Figure E-3 is a plot of these attenuation or correction factors.

Comparable results are obtained using these factors and

normalizing Q to 10 Mw.

Core Distance (inches)	q	P/P ₀	Att	q ₀ (w/gm)
8	0.094	6.67	0.308	0.193
10	0.067	"	0.280	0.125
12	0.043	"	0.255	0.073
14	0.027	"	0.233	0.042
16	0.017	"	0.213	0.024
18	0.011	"	0.193	0.014

These results are plotted in Figure E-4. At 12.75 inches the heating rate is 0.060 watts/gram. For a pressure vessel material density of 7.85 gm/cc the equivalent volumetric heat generation at the inside surface is:

$$Q_0 = 0.06 \times 7.85 \times 3.413 \times 2.832 \times 10^4$$

$$Q_0 = 45500 \text{ Btu/ft}^3\text{-hr}$$

As in the previous section, it is felt that the agreement of the above value with the computed value (43550 Btu/ft³-hr) is largely fortuitous.

F. ALTERNATE CONFIGURATIONS

1. Pressure vessel diameter

The gamma heating at the inside surface of the cladding (47.5 inches I.D.) was found to be 43550 Btu/ft³-hr. This number would become greater as the diameter was reduced. Report 415 E 2.2 showed the equivalent gamma heating for a 40.5 inch pressure vessel to be 76700 Btu/ft³-hr. Assuming an exponential decrease with an increase in diameter, Figure F-1 shows the incident gamma energy at both the clad surface and the pressure vessel surface.

Stress variations with diameter may be computed for the maximum case (at the inside of the pressure vessel proper)

From page 76 the maximum pressure stress is

$$S_{TP} = 1400 \left(\frac{a}{b^2 - a^2} \right) \left(\frac{b^2}{a^2} + 1 \right)$$

where a is the inside radius

and b is the outside radius

Similarly, from pages 79 and 80 the maximum thermal stress

is

$$S_{TT} = Q_0 (267.5 \times 9.71) \left(\frac{2}{b^2 - a^2} \right) (5.46 a + 0.69) 10^{-6}$$
$$= \frac{Q_0}{b^2 - a^2} (5.46 a + 0.69) 5.2 \times 10^{-3}$$

where Q_0 is the incident energy on the pressure vessel

and a and b are as above, expressed in feet.

For a = 20 inches = 1.667 ft

$$S_{TP} = 1400 \left(\frac{2.78}{3.52-2.78} \right) \left(\frac{3.52}{2.78} + 1 \right)$$

$$= 1400 \times 3.77 \times 2.265 = 11940 \text{ psi}$$

$$S_{TT} = \left(\frac{70}{3.52 - 2.78} \right) (5.46 \times 1.667 + 0.69) \quad 5.2$$

$$= 94.5 \times 9.78 \times 5.2 = 4810 \text{ psi}$$

Similar results for other diameters were computed and plotted in Figure F-2. It should be noted that these results implicitly assume a constant gamma spectrum at the pressure vessel, regardless of diameter.

2. Pressure vessel thickness.

An analysis of the effect of different wall thickness on the pressure vessel stress can be made quite simply.

Since the inner radius remains constant, the maximum pressure stress is:

$$\begin{aligned} S_{TP} &= 1400 \left(\frac{4}{b^2 - 4} \right) \left(\frac{b^2}{4} + 1 \right) \\ &= 1400 \left(\frac{b^2 + 4}{b^2 - 4} \right) \end{aligned}$$

Similarly, the maximum thermal stress is:

$$\begin{aligned} S_{TT} &= 9950 \left(\frac{2}{b^2 - a^2} \right) \left[2(c - 0.161 + e^{-Ac}(0.161 - 3.11 c^2)) \right. \\ &\quad \left. + \frac{c^2}{2} - 0.0258 + e^{-Ac} (0.0258(Ac + 1) - 2.072 c^3) \right] \end{aligned}$$

For the thickness $c = 1.2$ inches $= 0.1$ ft

$$\begin{aligned} S_{TP} &= \left(\frac{4.41 + 4.00}{4.41 - 4.00} \right) \times 1400 \\ &= \frac{8.41}{0.41} \times 1400 = 28700 \text{ psi} \end{aligned}$$

$$\begin{aligned} S_{TT} &= 9950 \times \frac{2}{0.41} \left[2(0.1 - 0.161 + 0.537(0.161 - 0.0311)) \right. \\ &\quad \left. + 0.005 - 0.0258 + 0.537(0.0258 \times 1.622 - 0.0021) \right] \\ &= 48600 \left[2(-0.061 + 0.537 \times 0.130) - 0.0208 + 0.537 \times 0.0397 \right] \\ &= 48600 (2 \times 0.0068 + 0.0005) \\ &= 48600(0.0176 + 0.0005) = 880 \text{ psi} \end{aligned}$$

Similar results for other values of c are plotted in Figure F-3.

FIGURE F-3

3. Thermal shield diameter

Variations in the diameter of the thermal shield will not affect the primary gamma contribution at the pressure vessel but will have a considerable effect on the secondary gammas. As the diameter increases from the APPR-1 configuration the shield will be exposed to lower neutron fluxes, more of the water capture gammas will be attenuated by the shield, and the thermal neutron flux at the pressure vessel will be decreased.

The unchanged portion of the gamma flux reaching the pressure vessel is summarized below (in terms of flux $\times 10^{-11}$)

Group	1	3	5	7
Primary	39.3	3.29	2.23	1.86
Flow Separator	0.07	0.05	0.09	0.30
Water (Capture)	--	4.59	--	--
P. V. (In. Scat.)	0.61	--	--	--
Total	39.98	7.93	2.32	2.16

For the thermal shield positioned at 9 inches from the core (40 inches I. D.) the incident flux on the shield will be $\phi_T = 8.0 \times 10^{12}$ and $\phi_F = 8.4 \times 10^{11}$. Similarly the thermal neutron flux at the pressure vessel will be $\phi_T = 1.4 \times 10^{12}$ (see Figure F-4). The secondary gamma contribution will be reduced by the ratio of the fluxes. The water capture gamma will include an added contribution from the inside

of the shield and a decreased contribution from outside.

From page 52 :

$$\begin{aligned} I_4 &= \frac{2 \times 10^{13}}{2} \times \frac{0.0126 \times e^{-0.0391} \times 6.35}{0.0914} (1 - e^{-0.58}) \\ &= 0.138 \times 10^{13} \times 0.779 (1 - 0.56) \\ &= 4.73 \times 10^{11} \end{aligned}$$

At the pressure vessel

$$I = 4.73 \times 10^{11} \times \frac{1.56}{6.3} = 1.17 \times 10^{11}$$

The contribution of the water between the shield and vessel can be approximated from previous data (page 56) corrected for flux level and slab thickness

$$\begin{aligned} I &= 1.23 \times 10^{11} \times \frac{1.3 \times 10^{12}}{3.5 \times 10^{12}} \times \frac{1.75}{2.5} \\ I &= 3.2 \times 10^{10} \end{aligned}$$

For the thermal shield at 9 inches from the core the total incident radiation on the pressure vessel is summarized below in terms of flux $\times 10^{-11}$:

Group	1	3	5	7
Unchanged	39.98	7.93	2.32	2.16
Capture - T. S.	0.08	0.28	0.48	1.66
In. Scat. - T. S.	0.20	--	--	--
Capture - P. V.	--	0.03	0.05	0.12
Capture - Water	--	1.49	--	--
Total	40.26	9.73	2.85	3.94
Mev/cm ² -sec	40.26	29.19	14.25	27.58
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	8.29	5.14	2.54	5.10
Total				21.07
Btu/ft ³ -hr				32700

The thermal stress is directly proportional to the incident radiation.

$$S_{TT} = 2640 \times \frac{32700}{43550} = 1990 \text{ psi}$$

For the thermal shield positioned 10.75 inches from the core, the incident neutron flux can be determined from the dashed curves in Figure F-4. The incident flux on the thermal shield is $\phi_T = 4.1 \times 10^{12}$ and $\phi_F = 4.1 \times 10^{11}$. At the pressure vessel $\phi_T = 6.7 \times 10^{11}$.

The water capture gammas can be approximated by increasing the thickness of the contributing "slab" and attenuating

only through the thermal shield

$$I_4 = \frac{2 \times 10^{13}}{2} \times \frac{0.0126}{0.0914} \times e^{-0.0391 \times 10.8} (1 - e^{-0.986})$$

$$I_4 = 0.138 \times 10^{13} \times 0.655 (1 - 0.373)$$

$$I_4 = 0.138 \times 0.655 \times 0.627 \times 10^{13} = 5.65 \times 10^{11}$$

At the pressure vessel

$$I = 5.65 \times 10^{11} \times \frac{2.1}{6.3} = 1.88 \times 10^{11}$$

Summarizing in terms of flux $\times 10^{-11}$:

Group	1	3	5	7
Unchanged	39.98	7.93	2.32	2.16
Capture - T. S.	0.04	0.14	0.25	0.85
In. Scat. - T. S.	0.10	--	--	--
Capture - P. V.	--	0.01	0.02	0.06
Capture - Water	--	1.88	--	--
Total	40.12	9.96	2.59	3.07
Mev/cm ² -sec	40.12	29.88	12.95	21.49
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	8.26	5.26	2.30	3.98
Total				19.80
Btu/ft ³ -hr				30600

And the equivalent thermal stress is:

$$S_{TT} = 2640 \times \frac{30600}{43550} = 1860$$

Figure F-5 shows the effect of varying the thermal shield diameter only on the total pressure vessel stress.

4. Thermal shield thickness

A change in the thermal shield thickness affects the radiation reaching the pressure vessel in several ways. A thicker thermal shield, for example, will attenuate the primary gammas more and will change the secondary gamma contribution of both the shield and the water. Results for a 2 inch shield are available. Additional results at 0 and 4 inch thicknesses should be sufficient.

For no thermal shield the primary gammas at the pressure vessel can be found from the expression

$$I = \frac{S_v B R^2 F(\theta, b)}{2(a + z)}$$

where b and B differ from the values on page 33

(t = 1/8 in. = 0.318 cm)

Group	$\mu_w(a - t)$	μ_{cz}	$\mu_{ss}t$	b	B
1	1.89	1.96	0.154	4.004	7.7
3	1.055	1.30	0.090	2.445	2.7
5	.811	1.16	0.077	2.048	2.05
7	.703	1.12	0.074	1.897	1.8

Group	S_v	$2(a + z)$	B	R^2	$F(\theta, b)$	$I_{12.75}$
1	2.31×10^{13}	89.4	7.7	784	0.0078	1.22×10^{13}
3	7.42×10^{11}	93.6	2.7	784	0.032	5.37×10^{11}
5	4.57×10^{11}	94.4	2.05	784	0.063	4.9×10^{11}
7	3.94×10^{11}	94.8	1.8	784	0.073	4.28×10^{11}

The gammas from the flow separator can be estimated using the attenuation rate through water shown in Figure 12.

Group	$I_{12.75}$ (c)	$I_{12.75}$ (i.s.)
1	4.8×10^9	1.69×10^{10}
3	1.1×10^{10}	
5	1.8×10^{10}	
7	6.2×10^{10}	

The contribution from water capture can be determined as before (page 55) but using an attenuation factor due to water only (Figure 15)

For slab 1

$$I_{12.75} = 4.14 \times 10^{11} \times \frac{2.1}{16.7} = 5.21 \times 10^{10}$$

For slab 2

$$I_{12.75} = 1.86 \times 10^{12} \times \frac{2.1}{11.7} = 3.34 \times 10^{11}$$

Slab 3 can be computed as the remaining water of 9.75 inches thickness

$$I_{12.75} = \frac{\phi_0 C e^{-\mu t}}{2(A - \mu)} (1 - e^{-t(A - \mu)})$$

$$I_{12.75} = \frac{6.4 \times 10^{13}}{2} \times \frac{0.0126}{0.0914} \times e^{-0.0391 \times 24.8}$$

$$\times (1 - e^{-24.8 \times 0.0914})$$

$$I_{12.75} = 3.2 \times 10^{13} \times 0.138 \times 0.379 (1 - 0.0282)$$

$$I_{12.75} = 1.62 \times 10^{12}$$

The total water capture flux is then:

$$I = (0.52 + 3.34 + 16.2) \times 10^{11} = 20.1 \times 10^{11}$$

The neutron flux reaching the pressure vessel will be essentially the same as before and thus the secondary gammas generated in the vessel will not change.

Summarizing the gammas in terms of flux $\times 10^{-11}$:

Group	1	3	5	7
Primary	122.	5.37	4.9	4.28
Flow Sep.-Capt.	0.05	0.11	0.18	0.62
Flow Sep.-In. Scat.	0.17	--	--	--
Water-capture	--	20.1	--	--
Total	122.2	25.6	5.1	4.9
Mev/cm ² -sec	122.2	76.8	25.5	34.3
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	25.2	13.5	4.5	6.4
Total				49.6
Btu/ft ³ -hr				77000

A thermal shield 4 inches thick will considerably attenuate the low energy primary gammas but will enhance the higher energy secondary gammas. As before, the values of b, B, and t will change in computing the primary gammas (t = 4.125 inches = 10.5 cm.)

Group	$\mu_w(a - t)$	μ_{cz}	Sum	μ_{sst}	b
1	1.290	1.96	3.25	5.09	8.34
3	0.720	1.30	2.02	2.98	5.00
5	0.553	1.16	1.713	2.55	4.26
7	0.479	1.12	1.599	2.43	4.03

Group	B_w	B_{ss}	B_{Total}
1	4.8	7.0	33.6
3	2.38	2.8	6.67
5	1.88	2.1	3.94
7	1.65	1.81	2.98

Group	S_v	$2(a + z)$	B	R^2	$F(\theta, b)$	$I_{12.75}$
1	2.31×10^{13}	89.4	33.6	784	0.000087	5.92×10^{11}
3	7.42×10^{11}	93.6	6.67	784	0.0029	1.20×10^{11}
5	4.57×10^{11}	94.4	3.94	784	0.0062	9.3×10^{11}
7	3.94×10^{11}	94.8	2.98	784	0.008	7.8×10^{10}

The secondary gammas produced by the flow separator and the water between the core and the shield will be reduced by the ratio of attenuation in steel to that in water.

Flow Separator

Group	I_c	Total Att.	$I_{12.75}$
1 (i.s.)	2.4×10^{11}	0.0087	2.1×10^9
1 } 3 } 5 } 7 }	6.8×10^{10} 9.2×10^{10} 1.3×10^{11} 4.1×10^{11}	0.0087 0.030 0.032 0.035	5.9×10^8 2.8×10^9 4.2×10^9 1.4×10^{10}

Water Capture

Slab	I	Total Att.	$I_{12.75}$
1	4.14×10^{11}	0.029	1.2×10^{10}
2	1.86×10^{12}	0.041	7.6×10^{10}
3	1.73×10^{12}	0.076	1.3×10^{11}

The effect of the water between the shield and the pressure vessel can be computed from:

$$I = \frac{\phi_0 c}{2(-\mu)} e^{-\mu t} (1 - e^{\mu t})$$

$$\phi_0 = 2.8 \times 10^{12} \quad (\text{From Figure F-6})$$

$$t = 2.25 \text{ in.} = 5.72 \text{ cm}$$

$$\mu = 0.0391$$

$$I = 1.4 \times 10^{12} \times \frac{0.0126}{(-0.0391)} \times e^{-0.223} (1 - e^{0.223})$$

$$I = 4.5 \times 10^{11} \times 0.799 \times 0.25 = 0.9 \times 10^{11}$$

FIGURE F-6

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The total water capture gammas at the pressure vessel are:

$$I = (0.12 + 0.76 + 1.30 + 0.90) \times 10^{11}$$

$$= 3.08 \times 10^{11}$$

The secondary gammas due to the 4 inch shield can be computed as on page 45, using the flux levels indicated in Figure F-6.

$$I = \frac{\phi_1 c e^{-\mu t}}{2(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

$$\phi_1 = 2 \times 10^{13}$$

$$t = 10.16 \text{ cm}$$

$$A = 0.689$$

Group	μ	μt	$e^{-\mu t}$	$(A - \mu)$	$t(A - \mu)$	$e^{-t(A - \mu)}$
1	0.49	4.98	0.0069	0.199	2.02	0.1327
3	0.283	2.88	0.0561	0.406	4.12	0.0162
5	0.244	2.48	0.0837	0.445	4.52	0.0109
7	0.232	2.36	0.0944	0.457	4.64	0.0097

Group	$\left[1 - e^{-t(A - \mu)} \right]$	C	I
1	0.8673	0.0178	5.35×10^9
3	0.9838	0.0233	3.17×10^{10}
5	0.9891	0.0329	6.12×10^{10}
7	0.9903	0.1035	2.11×10^{11}

Similarly

$$I' = \frac{\phi_1' c e^{-\mu' t}}{a(A' - \mu')} \left[1 - e^{-t(A' - \mu')} \right]$$

$$\phi_1' = 2.22 \times 10^9$$

$$A' = -0.689$$

Group	$(A' - \mu)$	$t(A' - \mu)$	$e^{-t(A' - \mu)}$	I'
1	-1.179	-11.97	1.58×10^5	1.82×10^{10}
3	-0.972	-9.87	1.94×10^4	2.9×10^{10}
5	-0.933	-9.47	1.3×10^4	4.25×10^{10}
7	-0.921	-9.35	1.15×10^4	1.36×10^{11}

Group	$I + I'$	Att	$I_{12.75}$
1	2.36×10^{10}	0.64	1.51×10^{10}
3	6.07×10^{10}	0.69	4.19×10^{10}
5	1.04×10^{11}	0.70	7.3×10^{10}
7	3.47×10^{11}	0.71	2.46×10^{11}

The inelastic scatter gammas are computed from a single exponential expression:

$$\phi = 2.2 \times 10^{12}$$

$$c = 0.1042$$

$$A = 0.1675$$

$$\mu = 0.49$$

$$t = 10.16$$

$$I = \frac{2.2 \times 10^{12} \times 0.1042 \times e^{-4.98}}{2(0.1675 - 0.49)} (1 - e^{3.27})$$

$$I = 6.21 \times 10^{10}$$

$$I_{12.75} = 6.21 \times 10^{10} \times 0.64 = 3.98 \times 10^{10}$$

The secondary gamma contributions due to the pressure vessel will be in proportion to the incident flux.

Group	I	Flux Ratio	$I_{12.75}$
1 (i.s.)	6.1×10^{10}	1.6/1.7	5.74×10^{10}
1	1.0×10^{10}	2.7/2.0	1.35×10^{10}
3	4.4×10^{10}	"	5.93×10^{10}
5	6.4×10^{10}	"	8.63×10^{10}
7	1.74×10^{11}	"	2.35×10^{11}

Summarizing in terms of flux $\times 10^{-11}$

Group	1	3	5	7
Primary	5.92	1.20	0.93	0.78
Flow Sep.-Capt.	0.01	0.03	0.04	0.14
Flow Sep.-I.S.	0.02	--	--	--
Water Capt.	--	3.08	--	--
Thermal Sh.-Capt.	0.15	0.42	0.73	2.46
Thermal Sh.-I.S.	0.40	--	--	--
P.V. - Capt.	0.16	0.59	0.86	2.35
P.V. - I.S.	0.57	--	--	--
Total	7.23	5.32	2.56	5.73
Mev/cm ² -sec.	7.23	15.96	12.80	40.11
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	1.49	2.80	2.28	7.41
Total				13.98
Btu/ft ³ -hr				21600

If we assume the thermal stress is proportional to the incident heat energy, Figure F-7 shows the effect of thermal shield thickness on the thermal and total stress in the pressure vessel. Actually, because of the difference in the gamma spectra at the pressure vessel, the thermal stress for zero shielding will be slightly less than that shown ($A = 6.32 \text{ ft}^{-1}$). Conversely, for a 4 inch shield the thermal stress will be somewhat greater ($A = 6.1 \text{ ft}^{-1}$).

FIGURE F-7

5. Cooling the outside of the pressure vessel

Previous analyses of the pressure vessel thermal stress have assumed 100% efficient insulation on the outside. One means of reducing the maximum thermal stress for a given incident radiation is to lower the outside surface temperature of the pressure vessel. This could be done by deleting some or all of the insulation or by actually arranging a means of cooling the outside surface.

The optimum outside surface temperature would be one which provided equal total stresses at both the inside and outside surfaces. For APPR-1 the thermal stress at the outside surface should be that at the inside surface plus 1400 psi.

A temperature distribution which would exactly provide the above specifications could be found only with considerable difficulty. From page 88 the general expression for the temperature distribution is

$$T = \frac{Q_0}{k A^2} (1 - e^{-Ax - xA_e - At})$$

It is suggested that $t = 0.775$ inches $= 0.0646$ feet would provide a reasonably close temperature distribution.

For $x = t$

$$T = 37.2 \left[1 - e^{0.402} (1 + 0.402) \right]$$

$$T = 37.2 (1 - 0.669 \times 1.402)$$

$$T = 37.2 (1 - 0.938) = 37.2 \times 0.062 = 2.3 \text{ } ^\circ\text{F}$$

For $x = c = 0.208$ feet

$$T = 37.2 (1 - e^{-1.293} - 1.293 \times 0.669)$$

$$= 37.2 (1 - 0.274 - 0.866)$$

$$T = 37.2 \times (-0.140) = -5.21 \text{ F}^\circ$$

Figure F-8 shows an approximate temperature distribution for the above conditions.

From page 105 the thermal stress at the inside surface ($x = 0$) is:

$$S_{TT}(0) \left(\frac{\alpha E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2}{b^2 - a^2} \right) \left[a \left(c + \frac{e^{-Ac}}{A} - \frac{c^2}{2} A e^{-At} - \frac{1}{A} \right) \right. \\ \left. + \frac{c^2}{2} + \frac{c e^{-Ac}}{A} + \frac{e^{-Ac}}{A^2} - \frac{c^3}{3} A e^{-At} - \frac{1}{A^2} \right] \\ S_{TT}(0) = 9950 \left(\frac{2}{0.88} \right) \left[2(0.208 + \frac{0.274}{6.22} - 0.135 \times 0.669 - 0.161) \right. \\ \left. + 0.0216 + 0.0092 + 0.0054 - 0.0125 - 0.0258 \right] \\ = 22800 \left[2(0.208 + 0.0441 - 0.0903 - 0.161) \right. \\ \left. + 0.0362 - 0.0383 \right] \\ = 22800 (0.0020 + 0.0362 - 0.0383)$$

$$S_{TT}(0) = 22800 (-0.0001) = -2.3 \text{ psi}$$

Similarly at the outside surface ($x = c$)

$$S_{TT}(c) = S_{TT}(0) - \left(\frac{\alpha E}{1-\nu} \right) T \\ = -2.3 - 267.5 (-5.21)$$

$$S_{TT}(c) = -2.3 + 1393 = 1391 \text{ psi}$$

FIGURE F-8

Figure F-9 shows the approximate stress distribution for the above assumed conditions.

For $t = 0.775$ in the maximum amount of heat which must be removed from the outside surface can be computed from:

(appendix A-3)

$$H = Q_0 \int_t^c e^{-Ax} dx = \left[Q_0 \left(-\frac{e^{-Ax}}{A} \right) \right]_t^c$$

$$H = \frac{Q_0}{A} (e^{-At} - e^{-Ac})$$

$$H = \frac{38000}{6.22} (0.669 - 0.274)$$

$$H = \frac{38000}{6.22} \times 0.395 = 2410 \text{ Btu/ft}^2\text{-hr}$$

FIGURE F-9

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G. ALTERNATE MATERIALS

The choice of material for the APPR-1 pressure vessel and thermal shield was based on conventional and accepted engineering practice. Alternate materials were considered to determine the effect on total stress.

1. Pressure Vessel

The substitution of a higher strength alloy as a pressure vessel material could affect the design in several ways. For example consider a material with an allowable design stress for welded structures of 20000 psi. For the same wall thickness (2.5 inches) the allowable thermal stress would be $20000 - 14180 = 5,820$ psi. Figure F-7 indicates that no thermal shield would be required.

Conversely if the 2 inch thermal shield is retained the pressure vessel thickness could be reduced. Figure F-3 indicates a thickness of 2 inches would be adequate.

Note that the above comparison of materials assumes, implicitly, identical physical properties for the two steels. Actually (from appendix A-2 and A-4) the thermal stress is dependent on the thermal conductivity, the gamma absorption coefficient, the density, the coefficient of expansion, the modulus of elasticity, and Poisson's ratio. In addition the above properties vary somewhat with temperature. Thus a precise evaluation of a pressure vessel requires precise knowledge of the above physical properties and how they vary with temperature.

Stainless steel would seem to be an obvious choice as a high strength material. However a high expansion coefficient coupled with a lower thermal conductivity reduce its effectiveness in a gamma field. Aluminum has extremely advantageous conductivity and gamma absorption characteristics but is not considered a high strength material and is not suitable at temperatures above about 350°F.

Another alternate material possibility would be the loading of the pressure vessel material (and cladding) with boron to reduce the amount of secondary gammas generated in the vessel wall. Although this would drastically reduce the pressure vessel capture gammas the effect on total stress would not be great.

2. The Thermal Shield

A brief review of elements possibly suitable for thermal shields indicates that most are unsatisfactory because of cost (scarcity) or physical and temperature characteristics. If the ideal thermal shield material is that causing the greatest gamma attenuation while resulting in minimum secondary gamma production, several elements appear inherently better than iron. These are listed below in the order of an arbitrary figure of merit.

Bismuth	--	0.6
Silver	--	4.3
Lead	--	11.2

Niobium	--	28
Platinum	--	68
Molybdenum	--	80
Uranium (238)	--	90
Thallium	--	103
Iron	--	120
Chromium	--	235
Nickel	--	310

Cost wise only lead can compete with iron (or stainless steel). However lead is not very suitable as a structural material, particularly at elevated temperatures.

The addition of boron to the above materials should effectively reduce the capture gamma production by reducing the interior thermal flux. Obviously a significant reduction in the gamma heating effect can be attained this way only if the secondary gamma contribution is initially large.

a. The effect of boron in a stainless steel thermal shield.

In the APPR-1 about 25% of the gamma heating effect results from the secondary gammas produced in the thermal shield.

The effect of adding boron can be computed as on page 45 using different values of A. Using the Maxwell-Boltzman and temperature corrections the macroscopic absorption cross section of the thermal shield is:

$$\Sigma_a = \Sigma_a(B) f + \Sigma_a(S.S.) (1 - f)$$

where f is the volume fraction of boron

For f = 5%

$$\Sigma_a = 106 \times 0.05 + 0.173 \times 0.95$$

$$\Sigma_a = 5.28 + 0.164 = 5.44$$

$$\text{Similarly } \Sigma_s = 0.57 \times 0.05 + 0.93 \times 0.95$$

$$\Sigma_s = 0.03 + 0.88 = 0.91$$

$$\text{And } A = \frac{1}{L} = \sqrt{3 \Sigma_s \Sigma_a} = \sqrt{3 \times 0.91 \times 5.44} = \sqrt{14.85}$$

$$A = 3.85$$

$$\phi_1 = 2 \times 10^{13}$$

$$\phi_1' = 3.3 \times 10^{12} \times e^{-Ax} = 3.3 \times 10^{12} \times e^{-19.6}$$

$$\phi_1' = 3.3 \times 10^{12} \times 3.06 \times 10^{-9} = 1.01 \times 10^4$$

As on pages 45 and 47

Group	$e^{-\mu t}$	$(A - \mu)$	$t(A - \mu)$	
1	0.083	3.36	17.08	
3	0.237	3.57	18.12	For all groups
5	0.289	3.61	18.32	$(1 - e^{-t(A - \mu)})$ is
7	0.307	3.62	18.38	essentially unity.

Group	C	I
1	0.0178	4.42×10^9
3	0.0233	1.55×10^{10}
5	0.0329	2.64×10^{10}
7	0.1035	8.8×10^{10}

Also: $(A' = -3.85)$

Group	$(A' - \mu)$	$t(A' - \mu)$	$e^{-t(A' - \mu)}$	I'
1	-4.34	-22.1	3.96×10^9	6.74×10^9
3	-4.13	-21	1.32×10^9	8.83×10^9
5	-4.09	-20.8	1.08×10^9	1.26×10^{10}
7	-4.08	-20.7	9.74×10^8	3.8×10^{10}

Group	$I + I'$	Att.	$I_{12.75}$
1	1.12×10^{10}	0.417	4.62×10^9
3	2.43×10^{10}	0.495	1.20×10^{10}
5	3.90×10^{10}	0.509	1.98×10^{10}
7	1.26×10^{11}	0.535	6.73×10^{10}

From page 62 the unchanged gamma flux ($\times 10^{-11}$) is

Group	1	3	5	7
Unchanged	40.49	9.96	2.96	3.80
5% Boron T.S.	0.05	0.12	0.20	0.67
Total	40.54	10.08	3.16	4.47
Mev/cm ² -sec	40.54	30.24	15.80	31.29
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	8.35	5.31	2.81	5.79
Total				22.26
Btu/ft ³ -hr				34500

The thermal stress will be proportional to the incident energy

$$S_{TT} = 2640 \times \frac{34500}{43550} = 2090 \text{ psi.}$$

Other values were computed in a similar manner and the results are shown in Figure G-1. Figure G-2 shows the relation of thermal stress and B^{10} contained in the thermal shield.

The above results apply to the APPR-1 configuration. It should be noted that boron loaded thermal shields would be more effective for configurations featuring a greater proportion of thermal stresses.

b. The effect of a lead thermal shield

While lead is a somewhat more effective gamma shield than stainless steel it is not a noteable structural material. For use in the APPR-1 it should probably be canned in 1/8" stainless steel. Appendix F-4 gives the stress results for no thermal shielding. For a 1 inch lead shield clad with 1/8 inch stainless steel the effect can be computed as below.

The primary gammas can be estimated from the expression:
(from page 22)

$$I_{12.75} = \frac{S_v B R^2 F(\theta, b)}{2(a + z)}$$

$$t_{ss} = 3/8 \text{ inches} = 0.95 \text{ cm}$$

$$t_{pb} = 1 \text{ inch} = 2.54 \text{ cm}$$

FIGURE G-1

Using values from pages 33 and 34,

Group	$\mu_w(a-t)$	μ_{cz}	Sum	$\mu_{ss}t_{ss}$	$\mu_{Pb}t_{Pb}$	b
1	1.70	1.96	3.66	0.47	2.03	6.16
3	0.95	1.30	2.25	0.27	1.20	3.72
5	0.73	1.16	1.89	0.23	1.25	3.37
7	0.63	1.12	1.75	0.22	1.33	3.30

Group	B_w	B_{ss}	B_{Pb}	B_{Total}
1	6.9	1.15	1.68	17.5
3	2.6	1.1	1.42	4
5	1.95	1.1	1.28	2.7
7	1.73	1.1	1.21	2.3

Group	S_v	B	R^2	$2(a+z)$	$F(\theta, b)$	$I_{12.75}$
1	2.31×10^{13}	17.5	784	89.4	0.00084	2.98×10^{12}
3	7.42×10^{11}	4	784	93.6	0.0109	2.79×10^{11}
5	4.57×10^{11}	2.7	784	94.4	0.0157	1.53×10^{11}
7	3.94×10^{11}	2.3	784	94.8	0.0170	1.27×10^{11}

The secondary gammas can be estimated much as before. A major reduction is expected in gammas from the thermal shield.

Gammas from the flow separator will be attenuated by 11.375 inches of water, 0.25 inch of stainless steel, and 1 inch of lead. Water attenuation can be estimated from Figure 12.

The thermal shield attenuation can be found from the conventional expression

$$I/I_0 = f(R) B e^{-\mu x}$$

Group	f(R)	μx (S.S. + Pb)	$e^{-\mu x}$	B	I/I_0
1	0.87	2.50	0.0821	2.52	0.180
3	0.87	1.47	0.230	1.56	0.312
5	0.87	1.48	0.228	1.41	0.280
7	0.87	1.55	0.212	1.33	0.245

Group	$I_{0.125}$	Att _{T.S.}	Att _w	$I_{12.75}$
1	6.8×10^{10}	0.180	0.096	1.17×10^9
3	9.2×10^{10}	0.312	0.154	4.42×10^9
5	1.31×10^{11}	0.280	0.167	6.12×10^9
7	4.12×10^{11}	0.245	0.188	1.9×10^{10}

Other secondary gammas will depend on the flux levels in the reflector. Figure G-3 shows an estimate of flux levels associated with a 1 inch thick lead thermal shield.

The contributions of water capture gammas from the first 3 "slabs" will be identical with those on pages 55 and 56 except for the attenuation.

Slab	I_0	Att _{T.S.}	Att _w	$I_{12.75}$
1	4.14×10^{11}	0.29	0.155	1.86×10^{10}
2	1.86×10^{12}	0.29	0.220	1.19×10^{11}
3	1.73×10^{12}	0.29	0.380	1.91×10^{11}

FIGURE G-3

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The contribution from the water between the thermal shield and the pressure vessel can be computed from the expression (from page 53):

$$I = \frac{\phi c e^{-\mu t}}{A - \mu} \left[1 - e^{-t(A - \mu)} \right]$$

where $t = 5$ inches $= 12.7$ cm

and $\phi = 1.4 \times 10^{13}$ (Figure G-3)

$$I = \frac{1.4 \times 10^{13} \times 0.0126 \times e^{-0.496}}{2 (0.1305 - 0.0391)} (1 - e^{-1.16})$$

$$I = \frac{7 \times 10^{12} \times 0.0126 \times 0.609}{0.0914} (1 - 0.313)$$

$$I = 4.04 \times 10^{11}$$

The total water capture gammas at the pressure vessel would be :

$$I = (0.19 + 1.19 + 1.91 + 4.04) \times 10^{10}$$

$$I_{12.75} = 7.33 \times 10^{11}$$

The contribution of capture gamma from the thermal shield result primarily from the stainless steel can. An approximation can be easily obtained from the expression

$$I = \phi C t$$

for the inside steel $\phi = 2 \times 10^{13}$ and for the outside

$\phi = 1.4 \times 10^{13}$. Gammas from the inside steel will be attenuated by 1 in. lead, 1/8 in. steel and 5 in. of water. Outside steel gammas will be attenuated by 5 in. of water only.

INSIDE: $\phi = 2 \times 10^{13}$ t = 0.318 cm

Group	C	I	Att _{T.S.}	Att _w	I _{12.75}
1	0.0178	1.13×10^{11}	~ 0.230	0.34	8.8×10^9
3	0.0233	1.48×10^{11}	~ 0.33	0.43	2.1×10^{10}
5	0.0329	2.1×10^{11}	~ 0.30	0.45	2.8×10^{10}
7	0.1035	6.6×10^{11}	~ 0.26	0.48	8.3×10^{10}

OUTSIDE: $\phi = 1.4 \times 10^{13}$ t = 0.318 cm

Group	I	Att _w	I _{12.75}
1	7.92×10^{10}	0.34	2.69×10^{10}
3	1.04×10^{11}	0.43	4.46×10^{10}
5	1.46×10^{11}	0.45	6.6×10^{10}
7	4.67×10^{11}	0.48	2.24×10^{11}

For the lead, the gamma contribution is all in the 7 mev group - one photon per absorption.

$$I = \frac{\phi C e^{-\mu t}}{a(A - \mu)} \left[1 - e^{-t(A - \mu)} \right]$$

$$\phi \approx 2 \times 10^{13}$$

$$C_{Pb} = 0.007$$

$$\mu = 0.52$$

$$A = \frac{1}{L} = \sqrt{3 \Sigma_s \Sigma_a}$$

$$A = \sqrt{3 \times 0.26 \times 0.007} = \sqrt{0.0055} = 0.074$$

$$I = \frac{2 \times 10^{13} \times 0.007 \times e^{-1.32}}{2(0.074 - 0.52)} \left[1 - e^{-2.54(0.074-0.52)} \right]$$

$$I_{12.75} = 7 \times 10^{10} \times \frac{0.267}{(-0.446)} (1 - e^{1.13}) \times \text{Att.}$$

$$I_{12.75} = 7 \times 10^{10} \times 0.6 \times 2.1 \times 0.48 = 4.24 \times 10^{10}$$

The total gammas from the thermal shield are then:

Group	I_I	I_o	I_{Pb}	$I_{12.75}$
1	0.9×10^{10}	2.69×10^{10}	--	3.59×10^{10}
3	2.1×10^{10}	4.46×10^{10}	--	6.56×10^{10}
5	2.8×10^{10}	6.6×10^{10}	--	9.4×10^{10}
7	8.3×10^{10}	22.4×10^{10}	4.24×10^{10}	3.5×10^{11}

Capture gammas from the pressure shell are assumed proportional to the incident neutron flux.

Group	I_o	Flux Ratio	$I_{12.75}$
1	1.0×10^{10}	1.1	1.1×10^{10}
3	4.4×10^{10}	1.1	4.84×10^{10}
5	6.4×10^{10}	1.1	7.04×10^{10}
7	1.74×10^{11}	1.1	1.92×10^{11}

Inelastic scatter gammas are assumed comparable to those from stainless steel. The total effective incident radiation at the pressure vessel is summarized below in terms of flux $\times 10^{-11}$

Group (Energy)	1	2	3	4
Primary	29.8	2.79	1.53	1.27
Capt. - Flow Separator	0.01	0.04	0.06	0.19
Capt. - Thermal Shield	0.36	0.66	0.94	3.5
Capt. - Water	--	7.33	--	--
Capt. - Press. Vessel	0.11	0.48	0.70	1.92
Total Inelastic Scat.	1.8	--	--	--
Total	32.1	11.30	3.23	6.88
Mev/cm ² -sec	32.1	33.90	16.15	48.1
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	6.61	5.97	2.88	8.90
Total				24.36
Btu/ft ³ -hr				37600

The thermal stress is in proportion to the incident radiation energy.

$$S_{TT} = 2640 \times \frac{37600}{43550} = 2280 \text{ psi.}$$

A similar analysis for a thermal shield consisting of 2 inches of lead canned in 1/8 inch stainless steel gives a thermal stress of 1450 psi. Figure G-4 shows the effect of different thicknesses of lead thermal shields on the pressure vessel stresses. For comparison results from Figure F-7 are re-plotted here.

H. Alternate Methods

During the course of the work represented by this report it was evident that in many cases several alternate analytical methods were available. Some alternate methods and approaches are detailed below.

1. The use of 5 gamma groups.

The use of 4 gamma groups is usual in shielding work. Four group results have generally been adequate especially at the usual shield distances from the core. Near the core (within ~ 15 inches), the lower energy gammas definitely predominate and the 4 group method would tend to give conservative results. Figure H-1 shows the BSR spectrum with the former 1 MEV group (from Figure 2) divided into nominal 0.5 and 1.5 MEV groups. Figure H-2 shows the 5 groups normalized to the measured dose (from Figure 3).

The primary gamma flux at the APPR-1 pressure vessel inside surface can be determined using the method delineated on page 14. Note that the water capture gammas must be subtracted. A 5 group analysis of secondary gammas is not considered profitable since only minute corrections would appear. For a 5 group analysis the total incident energy is summarized below in terms of flux $\times 10^{-11}$.

Group	0.5	1.5	3	5	7
Primary	15.0	16.4	2.36	1.98	1.61
Mev/cm ² -sec	7.5	24.6	7.08	9.90	11.27
μ_e	0.240	0.188	0.176	0.178	0.185
Mev/cm ³ -sec	1.80	4.62	1.25	1.76	2.08
Total					11.51

Group	1	3	5	7
Secondary	1.39	7.37	1.94	6.10
Mev/cm ² -sec	1.39	22.11	9.70	42.70
μ_e	0.206	0.176	0.178	0.185
Mev/cm ³ -sec	0.29	3.89	1.73	7.90
Total				13.81

Total incident radiation (mev/cm³-sec) 25.32

Total incident radiation (Btu/ft³-sec) 39280

The approximate thermal stress computed using the above 5 gamma groups would be:

$$S_{TT} = 2640 \times \frac{39280}{43550} = 2380 \text{ psi}$$

The use of more gamma groups would also tend to reduce the thermal stress. For example the relatively large contribution from the water capture gammas would be computed at 2.5 mev instead of 3 mev for an energy reduction of about 17%. However added groups increase the computational labor considerably for relatively minor gains in overall accuracy. The 4 group method is considered conservative but not overly so.

2. Simplified stress formulas

a. Pressure stresses

Appendix B, 1 gives the exact method of computing the pressure stresses in a cylindrical pressure vessel. Where the radius is large compared to the wall thickness an approximate (and simplified) expression may be used

$$S_{TP} = p \frac{r}{c}$$

where p is the design pressure

r is the average radius

$$= \frac{a + b}{2}$$

and c is the wall thickness

For APPR-1

$$S_{TP} = 1400 \times \frac{25.25}{2.5} = 14150 \text{ psi.}$$

For preliminary estimates the use of the outside radius instead of the average radius will give conservative results

$$S_{TP} = p \frac{b}{c} = 1400 \times \frac{26.5}{2.5} = 14840 \text{ psi}$$

b. Thermal stresses

The expression for the tangential thermal stress (Equation 49 page 109) is somewhat involved and its solution is time consuming. For preliminary estimate the following method yields somewhat conservative (high) results.

From Equation 34 page 101 the expression for the maximum tangential stress (at the inside surface) is:

$$S_{TT} = \left(\frac{\alpha E}{1-\nu} \right) \left(\frac{2}{b^2 - a^2} \right) \int_0^c T(a+x) dx$$

$$\text{Assume } T = \frac{Q_0}{kA^2} x^{0.6}$$

$$\text{and } (a+x) = b$$

$$S_{TT} = \left(\frac{\alpha E}{1-\nu} \right) \left(\frac{Q_0}{kA^2} \right) \left(\frac{2b}{b^2 - a^2} \right) \left(\frac{c^{1.6}}{1.6} \right)$$

For APPR-1

$$\frac{\alpha E}{1-\nu} = 267.5$$

$$\frac{Q_0}{k A^2} = 37.2$$

$$a = 2$$

$$b = 2.208$$

$$c = 0.208$$

$$S_{TT} = 9950 \times \frac{4.42}{4.875-4} \times \frac{0.081}{1.6} = 2560 \text{ psi}$$

A better fit to the exact thermal stress curve (Figure F-3) can be obtained by empirically altering the last factor in the above expression

$$S_{TT} = \left(\frac{\alpha E}{1-\nu} \right) \left(\frac{Q_0}{k A^2} \right) \left(\frac{2b}{b^2 - a^2} \right) \left(\frac{c^{(1.8-c)}}{1.8-c} \right)$$

This expression quickly gives reasonable results for pressure vessel wall thicknesses to 6 inches. Figure H-3 compares the above method with the exact method.

I. General Radiation Levels Outside the Core.

Previous work was concerned only with maximum radiation effects at the reactor midplane. In considering the entire pressure vessel it is necessary to have an estimate of radiation energy levels as a function of distance from the reactor. Such an estimate can be obtained for the case of water only surrounding the core. However, if any reactor members (thermal shields) are interposed, a secondary gamma contribution will occur which may change the energy spectrum.

1. Radiation from a spherical reactor surrounded by water

a. Neutrons

Both fast and thermal neutron flux levels as a function of distance from the reactor are shown in Figure I-1. These plots were obtained by extending as straight lines the plots shown in Figures 8 and 10.

b. Primary gammas

Primary gammas are assumed here to include the secondary gammas generated in the flow separator. The attenuation of the primary gammas through water is assumed to be that of the flow separator gammas shown in Figure 12.

From page 62 the "primary" radiation, including attenuation by the thermal shield, at the pressure vessel (12.75 inches) is obtained. A correction factor (from Figure 12) removes the effect of the thermal shield.

Group	$I_p + I_{F.S.}$ (at 12.75)	C.F.	$I_{12.75}$
1	3.93×10^{12}	2.9	1.14×10^{13}
3	3.35×10^{11}	2.02	6.75×10^{11}
5	2.32×10^{11}	2.07	4.8×10^{11}
7	2.16×10^{11}	2.07	4.47×10^{11}

Figure I-2 shows the expected primary gammas as a function of distance from the core surface.

c. Secondary gammas.

Secondary gammas result from the capture of thermal neutrons and the inelastic scattering of fast neutrons.

The contribution due to the pressure shell will be proportional to the incident flux and will become negligible not far from the core. The following applicable data is from Figure 11, page 40 and Figure I-1

Core Distance (in.)	12.75 (with T.S.)	10	30
Thermal Flux	2×10^{12}	5.3×10^{12}	2.7×10^9
Proportional Factor (ϕ/ϕ_0)		2.65	1.35×10^{-3}
Fast Flux	1.7×10^{11}	5.6×10^{11}	2.1×10^8
Proportional Factor (ϕ/ϕ_0)		3.29	1.24×10^{-3}

Using the above factors the following gamma flux level were obtained.

Group	Core	Distance (inches)	
	10 "	30 "	
1 (i.s.)	2.01×10^{11}	7.54×10^7	
1 } (c)	0.26×10^{11}	1.35×10^7	
3 }	1.17×10^{11}	5.94×10^7	
5 }	1.7×10^{11}	8.63×10^7	
7 }	4.6×10^{11}	2.35×10^8	

The contribution due to water capture can be computed on the basis of attenuating the gammas from slabs 1 and 2 and adding a variable result from slab 3.

Distance	Slab 1	Slab 2	Slab 3	Total
1 inch	4.14×10^{11}	--	--	4.14×10^{11}
3 inches	2.88×10^{11}	1.86×10^{12}	--	2.15×10^{12}
6.5 "	1.14×10^{12}		1.73×10^{12}	2.87×10^{12}
10 "	6.04×10^{11}		1.66×10^{12}	2.26×10^{12}
20 "	1.03×10^{11}		9.57×10^{11}	1.06×10^{12}
30 "	1.75×10^{10}		3.6×10^{11}	3.78×10^{11}
40 "	2.98×10^9		1.35×10^{11}	1.38×10^{11}
50 "	5.06×10^8		4.84×10^{10}	4.89×10^{10}
60 "	8.6×10^7		1.83×10^{10}	1.84×10^{10}

Figure I-3 shows the estimated secondary radiation. It should be noted that the values shown presuppose a reactor member at the point of interest.

FIGURE I-3

By conservatively treating the water capture gammas as 3 Mev gammas, the total primary and secondary radiation may be summed and is shown in Figure I-4. By comparison with Figure I-3 it is evident that except near the core (< 12 inches) and except for the water capture gammas, the effect of the secondary gammas is negligible.

2. The effect of a thermal shield.

In general a thermal shield will effectively attenuate the incident gammas but will also produce secondary gammas proportional to the incident flux and the shield dimensions. Figure I-5 shows the approximate added attenuation of various thickness of thermal shielding over water of the same thickness.

Since the secondary gammas generated by the thermal shield will vary with distance from the core, they are plotted on the basis of unit flux in Figure I-6. The thermal shield gammas are also attenuated by water between the shield and pressure vessel. Water attenuation factors are shown in Figure I-7.

Using the above curves a good approximation of the net radiation incident on the pressure vessel can be obtained. For a particular core design a simpler although even more approximate method may be of interest. Briefly the radiation can be summed in terms of $\text{Mev/cm}^2\text{-sec}$ and empirical attenuation factors used. The combined effect of the thermal shield (attenuating incident gammas and generated secondary gammas)

FIGURE I-4

can be reported as a net attenuation. Figure I-8 shows the total unshielded radiation. Note that for APPR-1 no thermal shielding is required if the pressure vessel is 16 inches from the core and is not thicker than $2 \frac{3}{4}$ inches.

Where a thermal shield is located between the core and the pressure vessel, additional secondary gammas will be generated. Since the thermal shield will also reduce the primary radiation, an approximate net attenuation factor can be computed. These factors are shown in Figure I-9.

The thermal stress in the pressure vessel depends not only on the incident radiation but also on the vessel thickness.

Figure I-10 shows the relationship between thickness and stress.

The above three curves (Figures I-8, I-9, and I-10) are based on a number of approximations. They can be used effectively for quick determination of possible danger points in a design. If such points are found, a more thorough analysis is indicated to confirm the safety or suggest a remedy for the point (or area) of interest.

FIGURE 1-9

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