INTRINSIC FLUCTUATION EFFECTS AND CERTAIN OTHER
ASPECTS OF SUPERCONDUCTING WEAK LINKS

by

Walter Harvey Henkels

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INTRINSIC FLUCTUATION EFFECTS AND CERTAIN OTHER ASPECTS
OF SUPERCONDUCTING WEAK LINKS

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Recent theoretical developments have led to evermore refined theories for the Josephson effects in a variety of superconducting "weak link" structures. One of the major refinements has been the inclusion of the effects of intrinsic thermal fluctuations upon Josephson-like behavior. Some other developments include the investigation of mechanisms for the origin of the Josephson effects in non-tunneling structures, and equivalent circuit representations. We have made experimental investigations which encompass both of these areas, with special emphasis placed upon fluctuation effects, in particular, the effect of fluctuations upon the driven a.c. Josephson effect has been studied in considerable detail.

The weak links studied included niobium point contacts, tin "whisker" crystals, and tin thin film bridges, the latter fabricated via an optical photoresist technique. The experiments were conducted in a temperature regime which extended to within a few mK of T_c , the regime in which fluctuation effects are enhanced (and thus more experimentally accessible). The data consisted of relatively high resolution (1nV) low noise I-V characteristics obtained with an apparatus which incorporated ($\pm 10\mu K$) temperature control with wide-band r.f. transmission, in a highly shielded environment.

Experimental r.f.-induced step profiles (the driven a.c. Josephson

effect) and zero voltage step profiles (the d.c. Josephson effect) were compared in detail with the relevant intrinsic fluctuation theories (due to Ambegaokar, Halperin, and Stephen) via a one parameter fit, using the respective no-fluctuation step amplitude as the (natural) fitting parameter. We were able to distinguish between external noise effects and intrinsic effects and effectively excluded external noise; thus we maintain a high degree of confidence that the effects reported are indeed intrinsic to the weak links themselves. The agreement between theory and experiment was found to be very good in both the driven and the d.c. effects over a wide range of experimental parameters and the single-parameter fits yielded step heights which corresponded reasonably well to the d.c. critical currents. These results indicate that the conceptually simple theory in which thermal fluctuations are introduced as a rapidly fluctuating (Langevin) force and a tilted periodic potential is introduced by the coupling energy (d.c. effect) or by phase locking to incident radiation (driven a.c. effect) provides a rather good description of the intrinsic fluctuation phenomena in superconducting weak links of negligible capacitance. The broad applicability of the theoretical concepts is indicated by the consistency between the d.c. and driven effect results, and by the wide range of parameters over which convincing agreement between theory and experiment was obtained.

By making some observations in regimes where fluctuation effects were diminished, evidence was found that slight deviations that were observed in the fluctuation experiments could be attributed to deviations from the no-fluctuation model used as the starting point in the fluctuation theories.

Supplemental considerations related to junction modeling were also

pursued to a certain degree and in some cases, led to new results or confirmations of recently discovered results of other researchers. These observations included I-V comparisons, r.f. step amplitude vs. voltage dependences, r.f. step threshold frequencies, and the Dayem-Wyatt effect.

1. INTRODUCTORY REMARKS

By now the Josephson effects ^(1,2) are very familiar to most solid state physicists, and due to their striking properties and seemingly very useful applications Josephson-type devices have been studied quite extensively. This extensive study has led to the development of evermore refined theories for these effects in real (i.e. not particularly ideal) structures which exhibit the Josephson effects. One of the major recent theoretical refinements has been the inclusion of the effects of intrinsic thermal fluctuations upon Josephson-like behavior. Another major development has been the search for mechanisms for the origin of the Josephson effects in structures physically quite different from the original superconductor-insulator-superconductor thin film sandwich in which superconductive tunneling was originally discovered. A third area of development has been the quest for adequate lumped equivalent circuits with which to model Josephson devices. This thesis relates experimental work touching upon all three of these areas, however the major thrust here is in the area of intrinsic fluctuations, in particular, the effect of fluctuations on the so-called driven a.c. Josephson effect has been studied in considerable detail. The driven a.c. Josephson effect has been utilized by Parker et al. ⁽³⁾ in very significant measurements of fundamental constants. Prior to this work there was really no detailed quantitative comparison of fluctuation effects in the driven a.c. effect with the theory. This comparison has been done here and results have been compared

with intrinsic fluctuation effects in the d.c. Josephson effect also. Very good agreement between theory and experiment was found in both cases, and slight deviations were shown to be attributable to deficiencies in the no-fluctuation model; as opposed to shortcomings in the theoretical treatment of the fluctuations. Supplemental considerations related to junction modeling were pursued to a certain degree and in some instances led to new results or confirmations of newly discovered results of other researchers; these results are also reported in this thesis.

A considerable amount of introductory material is contained in the next chapter, chapter 2, along with a phenomenological presentation of the relevant theories. A significant part of the experimental problem in a study such as this is the sample preparation; this is discussed in chapter 3. Chapter 4 contains a description of the basic experimental techniques used to gather the data. Detailed comparisons of theory and experiment on the effect of intrinsic fluctuations are given in chapter 5. In chapter 6 are discussed some supplemental considerations related to deviations from ideal Josephson behavior. Finally, in chapter 7 a closing statement is given.

2. THEORY AND REVIEW

2.1 Introduction

Certain conductors and semiconductors whose temperature is lowered below a material-dependent critical temperature, T_c , exhibit the phenomenon of superconductivity.^(4,5) The superconducting state is characterized by two fundamental observable properties. First, the electrical resistance is zero in a superconductor and secondly, in a weak magnetic field, magnetic flux is completely expelled from the interior of a superconductor. The field at a superconducting surface dies off exponentially in a distance called the penetration depth, denoted λ . Both of these fundamental properties are derivable from the microscopic, or BCS,⁽⁶⁾ theory of superconductivity which asserts that the superconducting state is a consequence of a weakly attractive interaction which couples the conduction electrons of a normal material into bound electron pairs (Cooper pairs⁽⁷⁾). Minimization of the free energy of a superconductor requires that the microscopic quantum mechanical wave functions of the pairs all have the same phase (the pair phases are then said to be "locked" to each other). The net macroscopic result of this coherence of pair phases is an ordered state which can be phenomenologically described by a single macroscopic wave function, the so-called order parameter, ψ ^(8,9). The order parameter is complex valued, having a magnitude, ψ ,

the square of which is the density of superconducting electrons, $|\psi|^2 = n_s$, and a phase γ , i.e. $\psi = \psi e^{i\gamma}$. In a bulk singly-connected superconductor the phase is constant because the "locking" of individual microscopic pair phases extends over macroscopic distances. Certain peculiar effects, known as the Josephson effects arise (10,11) when this phase locking is degraded or weakened in some manner.

Historically the first type of device studied which displayed the Josephson effects were superconducting tunnel junctions (Figure 2.1(a)), which are basically thin film sandwiches consisting of a thin film superconducting electrode, a thin insulating layer of thickness t (usually the native oxide of the electrode), and a superconducting film counterelectrode. For large insulator thicknesses ($t \gtrsim 100\text{\AA}$) the electron pairs in the electrode are essentially totally decoupled from the pairs in the counterelectrode. However, for very small insulator thicknesses, on the order of $50\text{\AA}$ or less, the pairs on either side of the insulator do couple together, resulting in the possibility for electron pairs to tunnel through the insulating barrier. That is, the two superconducting films are then weakly coupled via overlap of the pair wave functions within the insulating barrier. Josephson has shown that the free energy of such a system has a phase dependent part, $E_\phi = - \frac{\hbar I_1}{2e} \cos \phi$ where $\phi = \gamma_2 - \gamma_1$ is the phase difference across the barrier (γ_2 is the phase in one electrode, γ_1 the phase in the other) and I_1 is

Figure 2.1 Common types of superconducting weak links.

the maximum supercurrent (pair current) which can be sustained within the barrier, and depends microscopically upon the pair-tunneling probabilities (matrix elements). E_ϕ is called the "coupling energy"⁽¹²⁾ and represents the decrease of system free energy acquired as a result of the coherent coupling of the phases of the electrode and counterelectrode superconductors. A larger I_1 (critical supercurrent) corresponds to a "stronger" coupling. A supercurrent may exist through the insulator with the presence of a coherent phase difference across it, whereas in the interior of bulk superconductors phase differences do not exist. This total phenomenon is called the Josephson effect. There exist many other sorts of interconnections (besides thin insulating layers) which serve to couple together (however weakly or strongly) the phases of otherwise-independent bulk superconductors. A few of the more common types are depicted in Figure 2.1 along with the insulator-tunnel junction described above, (and commonly abbreviated S-I-S for superconductor-insulator-superconductor). Structures of this sort which exhibit a weak coupling between bulk superconductors are commonly termed superconducting "weak links".

In general as the coupling energy is decreased the probability for a fluctuation of the phase difference across a weak link increases. The phase fluctuation may be triggered by a variety of sources, such as thermal fluctuations, external electrical noise, shot noise internal to the pair current, acoustic noise, etc. Supposing that all noise sources external to a weak link

or junction could be eliminated, there would still remain the intrinsic fluctuations in the weak link current due to the fundamental "graininess" of the thermal environment and of the internal charge flow. The primary intent of this thesis is to study the quantitative decoupling of superconductors induced by intrinsic fluctuations internal to the interconnecting weak links. As a secondary objective we wish to study how practical weak links might be modeled and how they differ from the basic model of the S-I-S junction, which has been studied quite extensively and is thus quite well described quantitatively by existing theories (both phenomenological and microscopic). The intent of this chapter is to provide a theoretical background upon which our experiments are based. In section 2.2 we review the phenomenological Josephson equations and their implications. In section 2.3 we present an approximate practical first order model for weak links, which serves as the springboard for the remaining two sections. The bulk of the theory we present is concerned with intrinsic fluctuation effects in superconducting weak links and is found in section 2.4. The intrinsic fluctuation theories we present are essentially due to Ambegaokar and Halperin,⁽¹³⁾ Stephen,⁽¹⁴⁾ Scalapino and Rogovin,⁽¹⁵⁾ and Stratonovich⁽¹⁶⁾ with some extensions and modifications by Lee,⁽¹⁷⁾ and Waldram, Pippard and Clarke.⁽¹⁸⁾ In the last section, 2.5, we very briefly review some refined models for weak links

and theories which attempt to explain some of the unique features of specific weak links. In all of this we take the viewpoint of a phenomenological review, with ample references to more detailed and more rigorous works. Throughout we use rationalized MKS units.

2.2 The Josephson Effects--Simplest Model

The most extensively studied, and as a result the best understood weak link is the superconductor-insulator-superconductor (SIS) tunnel junction. The quantitative description of the SIS tunnel junction serves as the basis for the understanding of other types of weak link structures, so for this reason we review briefly the equations which describe this type of junction.

The basic problem which Josephson solved was to derive the relationship between the pair tunneling current and the gauge invariant phase difference across an insulating barrier. His fundamental results may be expressed phenomenologically by three equations. The first of these,

$$j_s = j_1 \sin \phi \quad (2.1)$$

relates the supercurrent density, j_s , to the (gauge invariant) phase difference, ϕ , across the barrier. This result is a consequence of the phase dependence of the coupling energy density,

$$F_\phi = -\frac{\hbar}{2e} j_1 \cos \phi \quad (2.2)$$

and the result

$$j_s = 2e \frac{dN}{dt} = 2e \left(\frac{1}{\hbar} \frac{\partial F}{\partial \phi} \right) \quad (2.3)$$

where the latter result derives from the circumstance that ϕ and N , the pair number density, are canonically conjugate variables. (19) The maximum supercurrent density, j_1 , is strongly temperature dependent (20) for $T \rightarrow T_c$ for which case $j_1 \sim (1 - T/T_c)$. j_1 is also slightly sensitive to the frequency of applied radiation, (21) but this dependence is of practical importance only near the gap frequency (the so-called Riedel (22) singularity).

The second fundamental equation,

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar} \quad (2.4)$$

displays the time dependence of the phase difference (2) and shows that if the phases on either side of the barrier are not locked to each other (i.e. $\gamma_1 - \gamma_2 \neq \text{constant}$), there can result a potential difference across the barrier.

The third (and last) fundamental equation shows that the spatial dependence of ϕ is a function of the magnetic flux threading the barrier,

$$\nabla \phi = \frac{2ed}{\hbar} \mu_0 (\vec{H} \times \hat{n}) \quad (2.5)$$

where $\vec{H}$ is the field within the barrier, $\hat{n}$ is a unit vector perpendicular to the plane of the barrier, and $d = t + \lambda_1 + \lambda_2$ is the axial depth of total penetration of the magnetic field within the barrier (λ_1, λ_2 are the penetration depths in superconductors 1 and 2 respectively.)

Equations (2.1), (2.4), and (2.5) may be combined with

Maxwell's equations (valid in the insulator) to yield⁽²⁾

$$v^2 \phi - \frac{1}{2} \frac{\partial^2 \phi}{\partial t^2} = \frac{\sin \phi}{\lambda_j^2} \quad (2.6)$$

where

$$\lambda_j^2 = \frac{\phi_0}{2\pi j_1 \mu_0 d} \quad \text{and } v = \frac{c}{\sqrt{\epsilon_r (d/t)}} \quad (2.7)$$

with $\phi_0 = \frac{h}{2e}$ the flux quantum, c the speed of light in vacuum and λ_j is the Josephson penetration depth, which plays the same role in the barrier region that λ plays in a bulk superconductor. We see from eq. (2.6) that for zero applied magnetic field and for the crosssectional dimensions of the junction $\lesssim \lambda_j$ there will be negligible transverse coordinate dependence in ϕ , hence the supercurrent density will be uniform. It is to this regime (uniform supercurrent density) which we now restrict the rest of our considerations; equations (2.5) and (2.6) have been included for completeness only.

Single particle tunneling is also possible in SIS junctions. In a superconductor the bound state is separated (lower) in energy from the normal electron (or quasi-particle) continuum by a temperature dependent energy gap, $2e\Delta(T)$, equal to the binding energy of a Cooper pair. At small but finite temperatures and for zero or small voltages ($\ll 2\Delta$) the probability for single particle tunneling through the barrier is small because the availability of states into which to tunnel is small, due to the forbidden energy gaps on either side of the barrier. However, for junction bias voltages $\gtrsim 2\Delta$, the Fermi levels of the

electrode and counterelectrode are offset sufficiently that now an abundance of states are available into which to tunnel. Hence for temperatures much lower than T_c , the expected (and observed) ⁽²³⁾ single particle tunneling current density, $j_n(V)$, is small until $V = 2\Delta$ at which point it rises sharply. The observable result is a non-linear single-particle conductance, $\sigma_0(V)$. The total tunneling current density (Cooper pairs and single-particle) is given by,

$$j_T = j_1 \sin \phi + \{ \sigma_0(V) + \sigma_1(V) \cos \phi \} V \quad (2.8)$$

The third term here is due to a small interaction between single particles and pairs, ⁽²⁾ and is usually neglected, whereas the second term, $\sigma_0(V) V = j_n(V)$ is the single particle tunneling current mentioned above. The precise form of $j_n(V)$ can be found in references (24) and (25).

The observable consequences of Josephson's equations have been conveniently labeled the d.c., a.c., and driven a.c. Josephson effects, though in essence they all have the same origin, equations (2.1) and (2.4). First, consider an SIS junction biased by a low-source-resistance d.c. source (nearly ideal voltage source) as shown in Figure 2.2(a).

The so-called d.c. Josephson effect for a superconducting weak link is the existence of a (zero voltage) supercurrent, or pair current, (through the link) brought about by the phase locking ($\phi = \text{constant}$) of the two "bulk" superconductors which the link interconnects. For the SIS junction, as current is

Figure 2.2 The Josephson effects in voltage-biased SIS tunnel junctions.

first applied, pairs tunnel through the barrier and zero voltage exists across the junction. The zero voltage state persists until the current reaches the critical value I_1 , at which point it becomes no longer energetically favorable for the phases of the superconducting electrodes to remain locked. Subsequently $\frac{\partial \phi}{\partial t} \neq 0$, a voltage appears across the junction, and the junction current is determined (cf. eq. 2.8) by

$$I_T = I_1 \sin \phi(t) + \frac{V}{R_N(V)} \quad (2.9)$$

where $R_N(V)$ is the non-linear single-particle tunneling resistance, which we will assume to be much greater than the source resistance R_S . In such case the voltage across the junction is externally required to be V_0 and is purely d.c. From equation (2.4) we determine the phase $\phi(t)$

$$\begin{aligned} \frac{\partial \phi}{\partial t} &= \frac{2eV_0}{\hbar} \\ \phi(t) &= \frac{2eV_0}{\hbar} t + \phi(0) \end{aligned}$$

and thus the supercurrent contribution to the total current is purely sinusoidal and given by (eq. 2.1)

$$\begin{aligned} I_S &= I_1 \sin(\omega_0 t + \phi(0)) \\ \text{where } \omega_0 &= \frac{2eV_0}{\hbar} = \frac{2\pi V_0}{\phi_0} \end{aligned} \quad (2.10a)$$

Thus we see that the net result of a d.c. voltage bias, V_0 , is an a.c. supercurrent of frequency proportional to the applied bias

voltage. This effect is called the a.c. Josephson effect. Hence we see that in the presence of a voltage bias, even though the average (or d.c.) supercurrent is zero ($\bar{I}_s = 0$), there still exists a supercurrent, but it is an a.c. current i.e. the junction is an oscillator. Also the existence of an a.c. supercurrent implies that the junction emits radiation of frequency

$\frac{\omega_0}{2\pi} = \frac{V_0}{\Phi_0}$. Microscopically this corresponds to pair tunneling with emission or absorption of photons, such that energy is conserved. (2)

In the "Josephson frequency relation", eq. (2.10a), the proportionality gives the following correspondence between voltage and frequency,

$$1\mu V \leftrightarrow 483.6 \text{ MHz} \quad (2.10b)$$

With a d.c. voltage bias, the existence of both pair and single particle tunneling in an SIS junction results in the d.c. current-voltage characteristic depicted schematically in Figure 2.2(b). Since (for the voltage bias case) $\bar{I}_s = 0$ the d.c. current for finite voltages is simply the single particle current $I_n(V) = \frac{V}{R_N(V)}$ and reflects the increase in availability of vacant quasi-particle tunneling sites for $V = 2\Delta(T)$. For $V > 2\Delta(T)$ the quasi-particle resistance is approximately ohmic with value R_N (typically $0.1-100\Omega$). As we have indicated in the figure the quasi-particle resistance approaches a linear characteristic as T approaches T_c and also reflects the temperature dependence of the energy gap $\Delta(T)$.

The driven a.c. effect derives its name from and results from

the application of an a.c. electromagnetic field to a junction biased at a fixed d.c. voltage. Again we assume for now that the a.c. source impedance is small enough that the source may be considered to be a voltage source. In such case we have, using eq. (2.4)

$$V = V_0 + V_{rf} \cos \omega_{rf} t = \frac{\hbar}{2e} \frac{d\phi}{dt}$$

$$\phi = \frac{2e}{\hbar} V_0 t + \frac{2e}{\hbar} \frac{V_{rf}}{\omega_{rf}} \sin(\omega_{rf} t) + \phi(0)$$

and then using eq. (2.1),

$$I_s(t) = I_1 \sin(\omega_0 t + \frac{2eV_{rf}}{\hbar\omega_{rf}} \sin \omega_{rf} t + \phi(0))$$

Expansion in a Fourier series gives,

$$I_s(t) = I_1 \sum_{n=-\infty}^{\infty} (-1)^n J_n \left(\frac{2eV_{rf}}{\hbar\omega_{rf}} \right) \sin[(\omega_0 - n\omega_{rf})t + \phi(0)] \quad (2.11)$$

where the J_n are Bessel functions. The time averaged supercurrent $\bar{I}_s$ is zero except for d.c. voltages such that $\omega_0 = n\omega_{rf}$ which is equivalent to

$$V_0^{(n)} = n \frac{\hbar\omega_{rf}}{2e} \quad (2.12)$$

For these special voltages, $V_0^{(n)}$, the phase of the Josephson oscillator is locked onto the phase of the external oscillator. There is one degree of freedom yet available, $\phi(0)$, whose value is determined by the external d.c. supply. Variation of the

external supply will result in a finite range of $\bar{I}_s$ over which the voltage is constant. These spikes in $\bar{I}_s$ have total amplitude $\bar{I}_s^{\max} - \bar{I}_s^{\min} \equiv 2j_2(n)$, with

$$j_2(n) = I_1 \left| J_n \left(\frac{2eV_{\text{rf}}}{\hbar\omega_{\text{rf}}} \right) \right| \quad (2.13)$$

Note that this effect is very much like the d.c. effect, where a single "spike" existed at zero voltage. That is, the driven a.c. effect has features related to both the d.c. and a.c. Josephson effects. For a d.c. voltage source the effect would manifest itself as a series of spikes spaced equally with an interval of $\frac{\hbar\omega_{\text{rf}}}{2e}$, as shown in Figure 2.2(c). In a real SIS junction $I_n(V)$ increases considerably when radiation is applied, due to photon-assisted single-particle tunneling. (26) The driven a.c. effect is of significant practical importance for experiments involving very accurate determinations of the fundamental constant e/h , (1) and for r.f. detection purposes. (27)

The sandwich-structure of SIS junctions resembles considerably a resonant cavity structure, and thus lends itself to self excitation via the a.c. Josephson current. (28) Such cavity modes are of much less significance in the other types of weak link structures shown in Figure 2.1.

2.3 A More Realistic Model - Impedance Considerations

The Josephson effects have been observed by many authors, however, except for a few instances, the experimental arrangement

are slightly different than the simple voltage source model depicted in Figure 2.2.

A first obvious modification to the simple model discussed in the previous section is the inclusion of the inherent junction capacitance, C , associated with the stripline structure of the SIS junction. Capacitance values for SIS junctions range as high as a few hundred pf.

Another important realization is the effect of the relative source impedances (both a.c. and d.c.) upon junction behavior. Consider first the case with no external a.c. sources. With a perfect d.c. voltage bias ($R_S \ll R_N(V)$) the oscillating supercurrent "sees" essentially zero impedance, i.e. the effect of the quasi-particle resistance upon the supercurrent is negligible (i.e. zero "feedback"). However, if we replace the voltage source by a d.c. current source supply (R_S much greater than the effective junction impedance), the impedance that the junction sees will no longer be zero, but will be the parallel combination of the capacitance and the normal (quasi-particle) state resistance. In such case there ensues a certain amount of feedback which results in a distortion of the (time dependent) supercurrent. A Fourier decomposition of the distorted supercurrent gives an average (d.c.) component and harmonics, in addition to the fundamental Josephson frequency component (which in the voltage-source or no-feedback case is the only component). These latter statements are quantified below. For the ideal d.c. current-source biased junction the circuit model appears as in Figure 2.3(a) and the circuit equation is,

CURRENT BIASED SIS JUNCTION

Figure 2.3 The Josephson effects in current-biased SIS tunnel junctions.

$$I = C \frac{dV}{dt} + \frac{V}{R_n(V)} + I_1 \sin \phi \quad (2.14)$$

with the voltage given as usual by eq. (2.4),

$$V = \frac{\hbar}{2e} \frac{d\phi}{dt} \quad (2.15)$$

Substitution of (2.15) into (2.14) gives

$$\frac{I}{I_1} = \frac{\hbar C}{2eI_1} \frac{d^2\phi}{dt^2} + \frac{\hbar}{2eI_1 R_n(\phi)} \frac{d\phi}{dt} + \sin \phi \quad (2.16)$$

Numerical solutions to (2.16) have been given by McCumber⁽²⁹⁾ and Stewart⁽³⁰⁾ for the case where $R_n(V) \approx R_N = \text{constant}$. (For an SIS junction this is a good approximation when T is near T_c .) They show that in general for $I < I_1$ two stable solutions exist, consequently giving rise to hysteresis in the d.c. current-voltage characteristics. The parameters which determine the degree of hysteresis may be lumped into one dimensionless parameter β .

$$\beta = \frac{2\pi I_1 R_N^2 C}{\phi_0} \quad (2.17)$$

Scott has obtained similar solutions, but used the actual theoretical non-linear form for $R_n(V)$.⁽³¹⁾ Both Scott and Hansma et al.⁽³²⁾ have obtained good agreement with the solutions of eq. (2.16). The d.c. I-V of a current-source-biased junction is depicted in Fig. 2.3(b). (Compare with voltage-biased case, Fig. 2.2(b)).

In practice one need not know details of the actual tunneling mechanisms in an SIS junction in order to describe the total (time dependent) junction current as the sum of a supercurrent

term and a normal (or dissipative) current term, in analogy to the two-fluid model of superconductivity.⁽³³⁾ This has basically been the approach taken in order to obtain a first order understanding of the behavior of other types of superconducting weak links (such as depicted in Fig. 2.1). In fact, this model appears to have a reasonably firm foundation, based upon the Ginsberg-Landau equations, for superconductor-normal-superconductor (SNS) junctions and point contacts.

The capacitance associated with weak link structures other than SIS junctions is typically very small, being of the order of 1 pF for an SNS junction⁽³⁴⁾ and even smaller for thin film weak links,⁽³⁵⁾ and as a result in practice the "damping parameter", β , can be quite small ($\ll 1$). In such case the first term in (2.16) can be neglected, resulting in

$$\frac{I}{I_1} - \sin \phi = \frac{\hbar}{2eI_1 R_N} \frac{d\phi}{dt} \quad (2.18)$$

For $I < I_1$ the only steady-state solution to (2.18) is $\phi = \text{constant}$, which once again displays the phase locking phenomena which accompanies bias currents less than the critical supercurrent. For $I > I_1$ (2.18) can be solved analytically (cf. appendix 2.A) to give expressions for the dc voltage, $\bar{V}$,

$$\bar{V} = I_1 R_N \left[\left(\frac{I}{I_1} \right)^2 - 1 \right]^{1/2} ; \quad I > I_1 \quad (2.19)$$

the (time dependent) phase,

$$\phi(t) = \sin^{-1} \left(\frac{I_1 + I \sin \omega_o t}{I + I_1 \sin \omega_o t} \right) \quad (2.20)$$

$$\text{with } \omega_0 = \frac{2e}{\hbar} I_1 R_N \left[\left(\frac{I}{I_1} \right)^2 - 1 \right]^{1/2} = \frac{2e\bar{V}}{\hbar} = \frac{2\pi\bar{V}}{\Phi_0}$$

the supercurrent,

$$I_s(t) = I_1 \frac{(I_1 + I \sin \omega_0 t)}{(I + I_1 \sin \omega_0 t)} \quad (2.21)$$

or the time dependent voltage $V(t)$,

$$V(t) = \frac{R_N [I^2 - I_1^2]}{I + I_1 \sin \omega_0 t} \quad (2.22)$$

Thus we see that the d.c. current-voltage relationship is hyperbolic and the supercurrent is no longer a simple sinusoid of frequency equal to the Josephson frequency ω_0 , but instead is a periodic function with fundamental frequency ω_0 , and harmonics given by the Fourier expansion, (36)

$$I_s(t) = \left(I - \frac{\bar{V}}{R_N} \right) - \frac{2\bar{V}}{R_N} \sum_{n=1}^{\infty} \left(\frac{I - \sqrt{I^2 - I_1^2}}{I_1} \right)^n \cos(n \omega_0 t) \quad (2.23)$$

Also note that the supercurrent in this case also has an average (or d.c.) component. In the limit $I \gg I_1$ the supercurrent tends toward a nearly perfect sinusoid, with zero average value. The d.c. I-V of any current-source-biased weak link for which this model, Fig. 2.4(a), is a good approximation should look as drawn in Figure 2.4(b).

Warman and Blackburn have numerically solved the corresponding model with finite source resistances. (37) As one might expect, the calculated I-V's show a negative resistance region near $\bar{V} = 0$, and indeed this negative resistance has been observed in cases where

CURRENT BIASED WEAK LINK $\beta \ll 1$

Figure 2.4 The Josephson effects in a highly damped weak link.

the junction resistance $R_n(V)$ is comparable to the effective source resistance.⁽³⁸⁾ (This situation is most readily achieved with SIS tunnel junctions where the quasi-particle resistance $R_n(V)$ can be quite large ($>1000\Omega$) for $V < 2\Delta$). Typical normal state resistance values, R_N , of the weak links depicted in Fig. 2.1 (other than the SIS tunnel junction) are $\sim 10^{-7}\Omega$ for the SNS junction, $\sim .005 - 1\Omega$ for the thin film and proximity effect weak links, and $\sim 1-10\Omega$ for point contacts, hence most practical d.c. supplies will appear as constant current sources rather than constant voltage sources, unless of course one intentionally shunts the weak link with a resistance less than R_N .⁽³⁹⁾ Thus in most circumstances negative resistance regions aren't to be expected in weak link I-V's.

Thus far we have only considered the effects of a high impedance d.c. source, we now turn to some a.c. source considerations which are of pertinence in the observation of the driven a.c. Josephson effect. One can get a rough idea of the a.c. impedance of a weak link when $I < I_1$ by considering the supercurrent to be represented by a non-linear inductor⁽⁴⁰⁾ since,

$$\frac{dI_s}{dt} = I_1 (\cos \phi) \frac{d\phi}{dt} = I_1 \sqrt{1 - \sin^2 \phi} \frac{2eV(t)}{\hbar} = \frac{1}{L_J} V(t) \quad (2.24)$$

$$\text{and thus, using } \frac{I}{I_1} = \sin \phi, \quad L_J = \frac{\frac{\phi_0}{2\pi}}{\sqrt{I_1^2 - I^2}}$$

For practical weak link experiments invariably the reactance $X_L = \omega L_J$

is of the same order of magnitude as R_N , e.g. for the point contact used in these experiments $I_1 \sim 1 \mu A$, $L_J \sim \frac{\phi_0}{2\pi I_1} = 3.3 \times 10^{-10} h$, $\omega \sim 2\pi \times 10^9$, yielding $\omega L_J \sim 2.1 \Omega$ whereas $R_N \sim 1.5 \Omega$. Furthermore, recently Auracher and Van Duzer have calculated the exact r.f. impedances for finite d.c. voltage and r.f. currents, for a wide range of experimental conditions, ⁽⁴¹⁾ using the $C = 0$ model discussed above. Again, their results indicate total r.f. impedances of the order of R_N . In the case of an SIS tunnel junction the reactance of the capacitance $X_C = \frac{1}{\omega C}$ should also be considered. In typical experiments, $C \sim 200 \text{ pf}$, $\omega \sim 6 \times 10^{10}$, $X_C \sim 0.1$. Thus, the point is that, in most instances at least, the r.f. (or a.c.) impedance of all types of weak link structures is small, $\lesssim 1 \Omega$, quite a bit smaller than the output impedances of practical a.c. generators and the impedance of free space. For these reasons then, it is more appropriate in circuit models to represent the a.c. source as an ideal current source rather than as a voltage source.

For a highly damped weak link ($C \approx 0$ or $\beta \ll 1$) biased by both a d.c. current source and an r.f. current source (a.c. source), the circuit model is as in Fig. 2.4(c) and the circuit equation is,*

$$I + I_{rf} \cos \omega_{rf} t = \frac{V}{R_N} + I_1 \sin \phi \quad (2.25)$$

or

$$\frac{d\phi}{dt} = \frac{2eR_N}{\hbar} [I + I_{rf} \cos \omega_{rf} t - I_1 \sin \phi]$$

In general this equation cannot be solved analytically, however,

*Use of either $I_{rf} \cos \omega_{rf} t$ or $I_{rf} \sin \omega_{rf} t$ for the r.f. source yields the same results.

can still be shown that phase locking to the applied r.f. takes place.

In the limit, $I_{rf} \gg I_1$ and $I \gg I_1$, the zeroeth order equation is

$$\frac{d\phi}{dt} = \frac{2eR_N}{\hbar} [I + I_{rf} \cos \omega_{rf} t] \text{ which gives a zeroeth}$$

order solution

$$\phi(t) = \frac{2eR_N}{\hbar} \left[It + \frac{I_{rf}}{\omega_{rf}} \sin \omega_{rf} t \right] + \phi(0) \quad (2.26)$$

Substitution of (2.26) into (2.25) gives the first order equation,

$$\frac{d\phi}{dt} = \frac{2eR_N}{\hbar} \left\{ I + I_{rf} \cos \omega_{rf} t - I_1 \sin \left(\frac{2eR_N}{\hbar} \left[It + \frac{I_{rf}}{\omega_{rf}} \sin \omega_{rf} t \right] + \phi(0) \right) \right\} \quad (2.27)$$

Expanding the last term,

$$\frac{d\phi}{dt} = \frac{2eR_N}{\hbar} \left\{ I + I_{rf} \cos \omega_{rf} t - I_1 \sum_{n=-\infty}^{\infty} (-1)^n J_n \left(\frac{2eI_{rf}R_N}{\hbar \omega_{rf}} \right) \sin (\omega_0 t - n\omega_{rf} t + \phi(0)) \right\} \quad (2.28)$$

where $\omega_0 = \frac{2eIR_N}{\hbar}$; $I_1 \ll I_{rf}$, I

The time averaged (d.c.) supercurrent is,

$$\bar{I}_s(t) = \overline{I - \frac{V(t)}{R_N}} = I - \frac{\hbar}{2eR_N} \overline{\frac{d\phi}{dt}} \quad (2.29)$$

Equations (2.29) and (2.28) display the same phase locking for the driven a.c. effect and the same (Bessel function) amplitude dependence of the constant voltage regions as was obtained in the voltage-biased case (cf. eqs. 2.11-2.13). Specifically, the time averaged (d.c.) supercurrent is zero except when $\omega_0 = n\omega_{rf}$.

In the d.c. current source biased junction one would not expect to see current "spikes" (as in Fig. 2.2(c)) but rather one would expect steps of constant voltage, and indeed this is what is observed.⁽⁴²⁾ Figure 2.3(c) depicts the driven a.c. effect for an SIS junction in the case where both d.c. and a.c. sources are good approximations to ideal current sources. Though (for clarity) it has not been shown, the steps may show hysteresis just as does the zero voltage step (cf. Lee⁽¹⁷⁾), when the capacitance is not negligible.

Returning now to the highly damped weak links ($C \approx 0$), Aslamazov and Larkin have shown that in the small I_{rf} limit the first order equation of the d.c. current-voltage characteristic is⁽³⁶⁾

$$I = \left[\frac{V^2}{R_N^2} + I_1^2 \right]^{1/2} + \frac{I_{rf} I_1}{\left[\frac{V^2}{R_N^2} + I_1^2 \right]^{1/2}} \times \frac{\sin(\omega_0 t - \omega_{rf} t + \phi(0))}{(I_{rf} \ll I_1, I)} \quad (2.30)$$

The first term is just the hyperbolic solution which obtains for no r.f. (cf. eq. 2.19), whereas the second term represents the first order perturbation due to the r.f. The second term is zero unless $\omega_0 = \omega_{rf}$, so once again the phase locking is evident in this limit also.

Waldram, Pippard, and Clarke have provided a description of phase locking in the general case,⁽¹⁸⁾ (any I_1 or I_{rf} in eq. 2.25). They show that solutions of eq.(2.25) which are exactly periodic in $\omega_{rf} t$, with n periods of $\sin \phi$ corresponding to m periods of $\cos \omega_{rf} t$,

correspond to points all of the same voltage, $\frac{n}{m} \frac{\hbar \omega_{rf}}{2e}$ but correspond to different d.c. currents I . Using a couple of approximations which do not affect the essential nature of the outcome they find that,

$$\frac{d\theta_o}{dt} = \frac{2eR_N}{\hbar} [(I - I_c) - j_2 \sin \theta_o] \quad (2.31)$$

where $\theta_o \equiv \phi(t) - \omega_{rf}t$, I_c denotes the current at the center of the constant voltage regions (constant voltage "step") and $2j_2$ is the total current amplitude of the step. Note that equation (2.31), for the case here of a driven Josephson oscillator, is identical in form to the circuit equation of the d.c. effect (eq. 2.18). The solution for θ_o follows exactly as the solution for ϕ in the d.c. case. The solution of (2.31) is,

$$\theta_o = \sin^{-1} \frac{\Delta I}{j_2} \quad \text{for } \Delta I < j_2 \quad (2.32)$$

which says that $\phi = \omega_{rf}t + \text{constant}$, thus displaying the phase locking. Figure 2.4(d) depicts the predicted d.c. I-V for this driven a.c. Josephson effect in a low-capacitance weak link.

As we have said, (2.31) is only an approximate equation. Waldram et al. have pointed out that though the exact form of the right hand side of (2.31) changes periodically, it still has the same average period and j_2 value. In other words, though the exact form cannot be written as in (2.31) the physical distinctions between the exact and approximate forms will be small. The exact model equation, eq. (2.25), has been solved numerically by several people ^(18,43,44) including ourselves (cf. chapter 6). One feature

that arises distinct from the ideal-voltage source solution is the prediction of "subharmonic" steps, which may be attributed to mixing of harmonics of the Josephson a.c. current with the r.f. induced a.c. current.

The point we have been trying to make in this section is that the behavior of superconducting weak links is quite dependent upon the connection circuit impedances. Figures 2.2 and 2.3 compare the predicted d.c. current-voltage characteristics of an SIS tunnel junction for the ideal voltage source and ideal current source extremes. A comparison of Figures 2.3 and 2.4 displays the differences to be expected between an SIS junction and low-capacitance weak links (with constant R_N). We use the model we have discussed in this section as the starting point in our comparisons with experiment; the intrinsic fluctuation theories for superconducting weak links employ this model as the no-fluctuation framework from which deviations (due to fluctuations) are predicted.

2.4 Intrinsic Fluctuations in Superconducting Weak Links

2.4.1 Introduction

Due to the fundamental graininess of thermal and electrical environments, fluctuations in the supercurrents and normal currents, (be they tunneling currents or what have you), are inevitable. These current fluctuations will tend to disorder the long range phase correlations inherent in superconductors. Normally the phase coherence in superconductors is so predominant that the energy barriers opposing fluctuations are enormous, and as a resu

observable effects of phase fluctuations in bulk superconductors are very small (but indeed they have been observed - cf. Gollub et al.⁽⁴⁵⁾). By reducing the effective dimensions of a superconductor the energy barriers are decreased (for a fixed energy density), and indeed fluctuation effects in these cases are more readily observable (for example see the experimental review by Parks⁽⁴⁶⁾, and a thesis by Warburton⁽⁴⁷⁾). In a superconducting weak link the probability for a phase fluctuation across it is expected to be governed by the relative strength of the coupling energy E_ϕ , which for an SIS junction is $E_\phi = -\frac{\hbar I_1}{2e} \cos \phi$ (cf. eq. 2.2). From the form of E_ϕ we see that for $\phi = 0$ the barrier to phase fluctuations is greatest, whereas the barrier decreases as the supercurrent approaches the critical value I_1 , ($\phi \rightarrow \pi/2$).

The problem of phase fluctuations in weak links has been found to be analagous to the problem of the Brownian motion of a particle and as such the phase fluctuation problem has been mapped onto this earlier problem as much as possible. In the analogy, phase fluctuations correspond to displacement fluctuations and the weak link capacitance corresponds to the mass of the particle. The equivalent model of the circuit and weak link used in the formulation of the theories is the d.c. current source model discussed in the previous section, with $R_n(V) = R_N$, (As we have said before this latter approximation appears to be quite reasonable for all weak links except the SIS tunnel junction where it

is a good approximation only for T close to T_c , which is the region of interest in fluctuation studies, since I_1 and hence the energy barriers are smallest in that regime). At the starting point the weak link capacitance, C is included, however we will be mainly dealing here with the $C \rightarrow 0$ limiting case of the theories. Also, we mainly discuss the driven a.c. effect and d.c. effect problems, since this material is most relevant to our experimental observations (chapter 5). An excellent reference for the material discussed in this section is a collection of lecture notes given by V. Ambegaokar⁽⁴⁸⁾. A nice reference for background material is MacDonald⁽⁴⁹⁾.

Before preceeding with more quantitative aspects, let us briefly summarize the qualitative effects of intrinsic phase fluctuations upon the three basic Josephson effects. The qualitative effects are depicted in Figure 2.5. The solid lines indicate the Josephson effects in the absence of phase fluctuations. In the d.c. effect and in the driven a.c. effect the "steps" are no longer perfectly vertical but instead show a broadening and rounding (or tilting). In the a.c. effect the resultant degradation is a broadening of the linewidth of emitted radiation (or degradation of the spectral purity of internal a.c. supercurrents). The theories we discuss below are concerned with the quantitative predictions for the degree of phase-fluctuation-degradation. In particular, the theories predict the shapes of the steps and the linewidth of the radiation.

2.4.2 Background - The Langevin equations for Brownian motion

In the case of Brownian motion, the problem is to describe the motion of a heavy particle subject to the buffeting of a sea of much

Figure 2.5 Qualitative effects of thermal fluctuations in the d.c., a.c., and driven a.c. Josephson effects.

EFFECT OF THERMAL FLUCTUATIONS IN THE THREE JOSEPHSON EFFECTS

— "Highly Damped Junction" — $\beta = \frac{2\pi C R_N^2 I_1}{\Phi_0} \ll 1$

lighter particles. The equation of motion of the heavy particle (of mass M) is (in one dimension, for simplicity),

$$M \frac{d^2 x}{dt^2} = F(t)$$

In a "Langevin" treatment of this problem one assumes that the force $F(t)$ may be written as the sum of the average force due to the long-term interaction with the environment i.e. the viscous damping, and a very rapidly fluctuating force $L(t)$ which accounts for the very frequent individual collisions with the lighter particles,

$$F(t) = - \eta M \frac{dx}{dt} + L(t)$$

The resultant equation of motion is,

$$M \frac{d^2 x}{dt^2} + \eta M \frac{dx}{dt} = L(t) \quad (2.33)$$

The assumption is that $L(t)$ varies on a time scale T^* much shorter than the time scale η^{-1} of the damping force. For time averages over a period of time $t \gg T^*$

$$\overline{L(t)} = 0 \quad (2.34)$$

$$\overline{L(t_1)L(t_2)} = 2D \delta(t_1 - t_2)$$

The latter equation expresses that individual collisions occur extremely rapidly and thus have only a very short correlation time in comparison with η^{-1} .

A formal solution of (2.33) subject to the assumptions (2.34) leads to the following results in limiting cases,

$$\begin{aligned}\overline{x^2} &= V_0^2 t^2 & \text{for } t \ll \eta^{-1} \\ \overline{x^2} &= 2Dt & \text{for } t \gg \eta^{-1}\end{aligned}\quad (2.35)$$

The latter equation displays (irreversible) diffusion behavior with diffusion constant D , (whereas the former equation displays the reversible mechanical motion associated with times on the scale of the individual correlations).

Equation 2.33 is the Langevin equation for a free particle. For a particle subject to a force derivable from a potential $U(x)$ the appropriate equation which replaces (2.33) is

$$M \frac{d^2 x}{dt^2} + \eta M \frac{dx}{dt} + \frac{\partial U}{\partial x} = L(t) \quad (2.36)$$

In the special case where the observation time is $\gg \eta^{-1}$ and $\frac{\partial U(x)}{\partial x}$ varies slowly over the thermal mean free path $\eta^{-1} \left(\frac{kT}{M} \right)^{1/2}$ the velocity of the particle $\frac{dx}{dt}$ will have reached its equilibrium value, in which case the first term in (2.36) would be negligible, leaving

$$\frac{dx}{dt} = \frac{1}{\eta M} \left\{ - \frac{\partial U}{\partial x} + L(t) \right\} \quad (2.37)$$

This case may also be called the highly damped or large viscosity case, i.e. $\eta \gg 1$ ((however the whole treatment breaks down if the condition $T^* \ll \eta^{-1}$ is not maintained). The equations describing the intrinsic phase fluctuation behavior of superconducting weak links will be shown to be identical to the preceeding equations. Their solution is conveniently treated via the method of the

Fokker-Planck equation, described briefly in section 2.4.6.

2.4.3 Junction Current Fluctuation Spectrum; Linewidth in the A. C. Effect

The mean-square current fluctuation of a current I_T (or the same-time current-current correlation function) and the current power spectrum $P_{I_T}(\omega)$ are related by

$$\overline{I_T^2} = \int_0^\infty P_{I_T}(\omega) d\omega \quad (2.38)$$

Hence the current fluctuations are determined once the power spectrum for fluctuations of the quasiparticle and pair current is known. The pair current and quasiparticle current power spectrums have been calculated for an SIS tunnel junction by Stephen⁽²⁴⁾, and Rogovin and Scalapino^(15,50) respectively. The calculations are based upon the microscopic theory of an SIS tunnel junction. The most general results are,

$$P_{I_P}(\omega) = \frac{2e}{\pi} I_P(V) \coth \frac{eV}{k_B T} \quad (2.39)$$

and

$$P_{I_{qp}}(\omega) = \frac{e}{2\pi} \left\{ I_{qp}\left(V + \frac{\hbar\omega}{e}\right) \coth \frac{(eV + \hbar\omega)}{2k_B T} + I_{qp}\left(V - \frac{\hbar\omega}{e}\right) \coth \frac{(eV - \hbar\omega)}{2k_B T} \right\} \quad (2.40)$$

where P_{I_P} is the pair spectrum, $P_{I_{qp}}$ is the quasiparticle spectrum and $I_P(V)$ and $I_{qp}(V)$ are the pair and quasiparticle currents respectively at the bias voltage V . The quasiparticle spectrum has two limiting cases,

$$P_{I_{qp}}(\omega) = \frac{e}{2\pi} \left[I_{qp}\left(\frac{\hbar\omega}{e}\right) - I_{qp}\left(-\frac{\hbar\omega}{e}\right) \right] \coth \frac{\hbar\omega}{2k_B T} \quad \text{for } V < \frac{\hbar\omega}{e} \quad (2.41(a))$$

$$\frac{e}{\pi} I_{qp}(V) \coth \frac{eV}{2k_B T} \quad \text{for } V > \frac{\hbar\omega}{e} \quad (2.41(b))$$

In the limit, $\hbar\omega, eV \ll k_B T$

the power spectrum formulae reduce to

$$P_{I_p}(\omega) = \frac{1}{\pi} I_p \frac{2k_B T}{V} \quad (2.42(a))$$

$$\frac{1}{\pi} \frac{2k_B T}{R(\omega)} \quad \text{for } V < \frac{\hbar\omega}{e} \quad (2.42(b))$$

$$P_{I_{qp}}(\omega) = \frac{1}{\pi} I_{qp} \frac{2k_B T}{V} \quad \text{for } V > \frac{\hbar\omega}{e} \quad (2.42(c))$$

$$\text{with } R(\omega) = -\frac{e}{2\hbar\omega} \left[I_{qp}\left(\frac{\hbar\omega}{e}\right) - I_{qp}\left(-\frac{\hbar\omega}{e}\right) \right] \quad (2.43)$$

The formula for $R(\omega)$ applies to SIS tunnel junctions,⁽⁵¹⁾ and for $T \rightarrow T_c$, $R(\omega) \rightarrow \frac{I_{qp}}{V}^{-1}$. We assume that this latter form holds for most weak links, then for the total power spectrum in these circumstances we get,

$$P_{I_T}(\omega) = \frac{1}{\pi} (I_p + I_{qp}) \frac{2k_B T}{V} \quad (\hbar\omega, eV \ll k_B T) \quad (2.44)$$

Vernon and Kanter compared eq. (2.44) with experiments on point contacts and they found reasonable agreement.⁽¹²²⁾

The voltage fluctuations are related to the current fluctuations by

$$\overline{V^2} = \frac{R_D^2 \overline{I_T^2}}{1 + \omega^2 \tau^2} \quad (2.45)$$

where R_D is the dynamic resistance of the junction, $\left(\frac{dI}{dV}\right)^{-1}$, and

T is the time constant associated with the capacitance and quasiparticle resistance. Using $\overline{V^2}$ Dahm et al. (15) have calculated the linewidth of emitted radiation (a.c. Josephson effect) for the voltage-biased junction. They obtain,

$$\Delta\nu = \frac{4\pi R_D^2 k_B T}{V} \left\{ \frac{2e}{h} \right\}^2 (I_{qp} + I_p) \quad (2.46)$$

Stephen obtained the same result by considering the Langevin equation for the system, (24,52) analagous to eq. (2.33) for the case of Brownian motion of a free particle. He finds that in the long time limit $\overline{\phi^2} = Dt$ analagous to diffusion behavior in Brownian motion. Thus in the a.c. Josephson effect the phase diffuses freely, and this phase diffusion leads to the linewidth of emitted radiation.

2.4.4 Formulation of the Langevin Equations for the Driven A. C. and D. C. Josephson Effects

We have seen in sections 2.2 and 2.3 that when r.f. radiation is applied to a superconducting weak link, steps of constant voltage are induced in the d.c. current-voltage characteristic. This phenomenon results due to a locking of the phase of the Josephson oscillations onto the phase of the external generator. Thus in such case the phase suffers a loss of a degree of freedom and we expect that phase fluctuations would be greatly reduced over the phase fluctuations in the regular a.c. effect. The phase fluctuations are in this case analagous to the Brownian motion of a particle subjected to a potential barrier set up by the phase locking between the Josephson oscillator and the r.f. generator.

Stephen was the first to formulate these ideas quantitatively for the driven a.c. Josephson effect.⁽¹⁴⁾ Stephen derives the phase locking equation, eq. (2.31), but there appears to be a contradiction in his original model where he assumes a perfect voltage source for the r.f. or a.c., but an ideal current source for the d.c. In order to obtain the phase locking equation he assumes the "weak coupling approximation", which amounts to the assumption $\frac{2e}{\hbar} \frac{V_{rf}}{\omega_{rf}} \ll 1$, where V_{rf} is the r.f. voltage within the junction. We believe that the correct model should include the r.f. source as an a.c. source, as discussed in section 2.3, in which case the phase locking equation may be obtained via the arguments of Waldram et al. for any r.f. level (i.e. no assumptions about I_{rf} or V_{rf} are necessary). However, as we have seen in the discussions surrounding equations (2.26) - (2.30), in the limiting case of small I_1 the distinction between the current-source results and voltage-source results fades, and in the limit of small I_{rf} the correct result is a small step superimposed upon an otherwise unperturbed background, which is the final result that Stephen obtains. Hence we may expect that the essential physical points of Stephen's derivation are basically correct, but that his results are extendable to a larger range of r.f. voltage (due to the fact that in the a.c. current source approach the same phase locking equation is obtained with no restrictions on I_{rf}). To extend Stephen's resultant formula for the d.c. current-voltage characteristic of a step to higher r.f. levels it is necessary to include the effect of the r.f. upon the total d.c.

I-V, as the a.c. current source model provides. In practical terms this means a reinterpretation of Stephen's parameters R and J as corresponding to the extrapolated dynamic resistance (not the step slope) and total d.c. current at the step center rather than simply as the dynamic resistance and total d.c. current (at the step voltage) in the absence of the r.f. (this latter interpretation is Stephen's interpretation and is correct for very small r.f. levels, in which cases the background I-V is perturbed very little). With all of these considerations in mind we present a brief account of the essential points of Stephen's formulation.

After applying a weak coupling approximation and the rotating approximation, (which amounts to neglecting rapidly varying components), to the supercurrent in an SIS tunnel junction Stephen obtains the following equation for charge conservation for the model depicted in Fig. 2.6(a).

$$C \frac{dV}{dt} = I - j_2 \sin(\phi - \omega_{rf}t) - J_s(V) - J_n(V) + G(t) \quad (2.47)$$

where C is the junction capacitance, $J_n(V)$ is the normal (quasi-particle) current, $G(t)$ is a rapidly fluctuating current source representing the fluctuations in the junction currents, and $J_s(V) + j_2 \sin(\phi - \omega_{rf}t)$ represents the total supercurrent, where the latter term is the resultant contribution due to the applied radiation of frequency $\nu_{rf} = \frac{\omega_{rf}}{2\pi}$ and j_2 is a function of the strength of the r.f. field. Next the phase is assumed to vary

Figure 2.6 (a) Model used in Stephen's formulation.
 (b) A stylized step--with and without fluctuations.

approximately with the frequency ω_{rf} ,

$$\phi(t) = \omega_{rf}t + \theta_0(t) \quad (2.48)$$

where $\theta_0(t)$ is a slowly varying correction, i.e. $\frac{d\theta_0}{dt} \ll \omega_{rf}$.

Then

$$V(t) = \frac{\hbar}{2e} \frac{d\phi}{dt} = \frac{\hbar\omega_{rf}}{2e} + \frac{\hbar}{2e} \dot{\theta}_0 = V_0 + \frac{\hbar}{2e} \dot{\theta}_0$$

where V_0 is the d.c. component, and $\frac{\hbar}{2e} \dot{\theta}_0 \ll V_0$. Expanding $J_s(V)$ and $J_n(V)$ in the small parameter $\dot{\theta}_0$ one gets,

$$\begin{aligned} J_s(V) &\approx J_s(V_0) + \frac{\hbar}{2eR_s} \dot{\theta}_0 \\ J_n(V) &\approx J_n(V_0) + \frac{\hbar}{2eR_n} \dot{\theta}_0 \end{aligned} \quad (2.49)$$

where $R_s = \left[\frac{\partial J_s}{\partial V} \right]_{V_0}^{-1}$ is the dynamic resistance for the supercurrent. Substitution of (2.49) and (2.48) into (2.47) yields

$$\frac{\hbar C}{2e} \ddot{\theta}_0 + \frac{\hbar}{2eR} \dot{\theta}_0 - (\Delta I - j_2 \sin \theta_0) = G(t) \quad (2.50)$$

where $\Delta I = I - (J_s(V_0) + J_n(V_0))$ and $\frac{1}{R} = \frac{1}{R_s} + \frac{1}{R_n}$: Note that

eq. (2.50) is identical in form to the equation describing the Brownian motion of a particle subject to a potential barrier, eq. (2.36). In the highly damped regime (or $C \rightarrow 0$), this simplifies to

$$\frac{\hbar}{2eR} \dot{\theta}_0 = \Delta I - j_2 \sin \theta_0 + G(t) \quad (2.51)$$

$$\text{or} \quad \dot{\theta}_0 = \frac{2eR}{\hbar} \left[- \frac{\partial U(\theta_0)}{\partial \theta_0} + G(t) \right] \quad (2.52)$$

$$\text{where} \quad U(\theta_0) = - (\Delta I \theta_0 + j_2 \cos \theta_0) \quad (2.53)$$

in analogy to the corresponding highly damped case for Brownian motion, eq. (2.37). Equation (2.51) is the basic phase locking equation (cf. eq. 2.31) which has already been discussed, with the addition of a rapidly fluctuating term $G(t)$. In the absence of fluctuations, $G(t) \equiv 0$, the solution of (2.51) is

$$\begin{aligned} \theta_0 &= \sin^{-1} \frac{\Delta I}{j_2} && \text{for } |\Delta I| < j_2 \\ \dot{\theta}_0 &= \pm \frac{2e}{\hbar} j_2 R \left[\left\{ \frac{\Delta I}{j_2} \right\}^2 - 1 \right]^{1/2} && \text{for } |\Delta I| > j_2 \\ &&& + \text{periodic terms} \end{aligned} \quad (2.54)$$

The time independent solution ($|\Delta I| < j_2$) corresponds to a constant voltage step of d.c. voltage $\bar{V} = \frac{\hbar \dot{\phi}}{2e} = \frac{\hbar}{2e} \omega_{rf} = V_0$ and total amplitude $2j_2$. For $|\Delta I| > j_2$, $\bar{V} = V_0 + j_2 R \left[\left\{ \frac{\Delta I}{j_2} \right\}^2 - 1 \right]^{1/2}$ in precise analogy to the no-fluctuation d.c. Josephson effect for this model (cf. eq. 2.19). This no-noise (no-fluctuation) solution for a step is depicted by the thin-lined curve in Fig. 2.6(b), where $\Delta V \equiv \bar{V} - V_0$.

In reality, of course, there are fluctuations; $U(\theta_0)$ (eq. 2.53) is the potential that phase fluctuations encounter. Figure 2.7 illustrates this potential for a case where $0 < \Delta I < j_2$. In the mechanical analogy the peaks in $U(\theta_0)$ are the barriers which the particle must overcome if its position is to change. In the phase locked condition, ($\theta_0 = \text{constant}$), the particle is trapped in one of the potential wells, where its motion is confined to small oscillations (plasma oscillations^(2,53)). At finite temperatures, however, there exists a finite probability for the particle to hop over a barrier; this hopping from well to

Finite $T \Rightarrow \bar{\dot{\theta}}_0 > 0$

$$\frac{\hbar}{2e} \bar{\dot{\theta}}_0 = \frac{\hbar}{2e} (\bar{\dot{\phi}} - \omega_{rf})$$

or $\Delta V = \bar{V} - V_0$

Where $\Delta V \equiv \frac{\hbar}{2e} \bar{\dot{\theta}}_0$

Note $\Delta V = \Delta V(\Delta I, j_2)$

Barrier Tilt

Barrier Height

Figure 2.7 The potential opposing fluctuations--relationship to the noise voltage, ΔV .

well corresponds to phase fluctuations. Since the potential is tilted (i.e. the barrier on the left is larger than the one on the right) the net long term result due to fluctuations will be a drift of the particle down to the right, (cf. Fig. 2.7) which corresponds to a net phase diffusion. This net phase diffusion gives rise to a small d.c. voltage ΔV as indicated in Figure 2.7,

$$\Delta V = \frac{\hbar}{2e} \bar{\dot{\theta}}_0 \quad (2.55)$$

ΔV , then, corresponds to the departure of the step voltage from the voltage at the step's center, as indicated in Fig. 2.6(b) where the heavy solid line corresponds to the step profile in the presence of fluctuations, ΔI and ΔV being the coordinates of the step relative to its center. We note that for $\Delta I = 0$ (step center) the potential has no tilt and thus $\bar{\dot{\theta}}_0 = 0$ which means that the step center voltage is unaffected by the presence of fluctuations. As the d.c. supply current is increased (ΔI increased) the average slope (tilt) of the potential increases, increasing the net phase diffusion and thus resulting in larger departures ΔV from the step center voltage. ΔV also depends of course upon j_2 , the amplitude of a barrier. For $\Delta I > j_2$ the potential is monotonic and as a result the phase locking is destroyed. The same arguments apply for negative ΔI , thus for a fixed R (cf. Fig. 2.6(b)) the step is symmetric;

for a voltage dependent R , $R = R(V)$, the step will be appropriately distorted, however its center will always remain fixed.

We have pointed out before that in general (arbitrary r.f. level) eq. (2.53) for the potential is an approximation. Waldram et al. point out that the actual potential periodically changes its shape somewhat but that the average period and the depth of wells is the same as for the sinusoidal potential (2.53). Since the probability for a fluctuation is exponentially dependent upon the barrier height, and rather insensitive to the exact shape of the potential, it is expected that the fluctuation rounding of a step will not be drastically modified by this refinement. Similarly we do not expect any drastic differences in the fluctuation mechanism due to refinements of the no-fluctuation circuit model.

To calculate the precise step profile, equation (2.51) has been solved by using the method of the Fokker-Planck equation. Before turning to that we should point out that the problem of intrinsic fluctuations in the d.c. Josephson effect (no external oscillators) is entirely identical to the problem for the driven a.c. effect -- in fact, the d.c. problem was formulated first and solved first by Ambegaokar and Halperin,⁽¹³⁾ and Ivanchenko and Zil'berman⁽⁵⁴⁾. The appropriate equation for the general case (finite C) is eq. 2.16 modified by the addition of a fluctuating current term $I(t)$,

$$\frac{\hbar C}{2e} \ddot{\phi} + \frac{\hbar}{2eR_N} \dot{\phi} - (I - I_1 \sin \phi) = I(t) \quad (2.56)$$

and in the highly damped limit,

$$\dot{\phi} = \frac{2eR_N}{\hbar} \left[-\frac{\partial U(\phi)}{\partial \phi} + I(t) \right] \quad (2.57)$$

with

$$U(\phi) = - (I\phi + I_1 \cos \phi) \quad (2.58)$$

We note that these equations are identical to those for the driven a.c. effect (eqs. 2.50 - 2.53) with $\theta_0 = \phi - \omega_{rf}t$ replaced by ϕ ; hence we expect the shapes of the zero voltage step of the d.c. effect, and the r.f. induced steps of the driven a.c. effect to be the same. However, for the d.c. effect the form of the potential, eq. (2.56) is exact for the no-fluctuation model being used, whereas in the driven a.c. effect this form is only an approximation, but a very good approximation as far as fluctuations are concerned.

2.4.5 The Fokker-Planck Solution

In order to simplify the problem the assumption is made that,

$$\hbar\omega, eV \ll k_B T \quad (2.59)$$

which holds in most experimental situations. In such case the total power spectrum of fluctuations in the pair and quasi-particle currents is given by eq. (2.44), $P_{I_T}(\omega) = \frac{1}{\pi}(I_p + I_{qp}) \frac{2k_B T}{V}$.

By the Wiener-Khintchine theorem⁽⁴⁹⁾ correlation functions and power spectrums are the Fourier transforms of each other, thus the correlation function of the fluctuating current $G(t)$ in the driven a.c. effect case is,

$$\overline{G(t_1)G(t_2)} = \int_0^\infty P_{I_T}(\omega) d\omega$$

$$= \frac{(I_p + I_{qp}) (2k_B T)}{V} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t_1 - t_2)} d\omega$$

or

$$\overline{G(t_1)G(t_2)} = \frac{(I_p + I_{qp}) (2k_B T)}{V} \delta(t_1 - t_2) \quad (2.60)$$

In the d.c. effect case $V=0$, so we must use eqs. (2.41(a)) and (2.42(b)), with $R(\omega) = R_N$. The result is,

$$\overline{L(t_1)L(t_2)} = \frac{2k_B T}{R_N} \delta(t_1 - t_2) \quad (2.61)$$

Equations (2.60) and (2.61) have the form of "thermal noise" or "Johnson noise". Thus far we have neglected any noise contributions from external sources such as the d.c. current supply resistance R_e . The power spectrum of thermal noise from this source is $\frac{2k_B T}{\pi R_e}$ but since $R_e \gg R_N$ for a current supply the current fluctuation contribution from the current source is negligible. Also it should be pointed out that in practical experiments the noise spectrum of the source resistor is considerably attenuated above a certain critical frequency determined by the transmission properties of the current input lines. This critical frequency may be many orders of magnitude smaller than the Josephson frequencies encountered in the experiments $\omega = \frac{2eV_0}{\hbar}$ (483.6MHz $\leftrightarrow$ 1 μ V), in which case it is not correct to consider the (low frequency) external noise in the same context as the intrinsic thermal noise (i.e. not correct to use an effective noise source with spectrum $\frac{2k_B T^*}{R^*}$ with $\frac{1}{R^*} = \frac{1}{R_e} + \frac{1}{R_N}$ and

$$T^* = \frac{T R_e + T_e R_N}{R_N + R_e} \quad (T_e = \text{temperature of external resistor}).$$

As an alternative to solving the general Langevin equation for Brownian motion, eq. (2.36), one may consider the time development of the distribution function $P(x,p,t)$ for an ensemble of systems, where $P(x,p,t)dx dp$ is the probability of finding the displacement and momentum of a system to be between x and $x + dx$, and p and $p + dp$ respectively, at time t ,⁽⁴⁹⁾ where $p = M\dot{x}$. Hopefully after a sufficient passage of time the distribution function undergoes no further change (i.e. is said to become stationary). We outline here the steps in the derivation of the equation describing the time evolution of $P(x,p,t)$, (the Fokker-Planck equation). For a more detailed account see Ambegaokar⁽⁴⁸⁾ or references 49, 55 & 56. Over the time interval $T^* \ll \Delta t \ll \eta^{-1}$

$$P(x,p,t+\Delta t) = \int P(x-\Delta x, p-\Delta p, t) w(\Delta x, \Delta p; x-\Delta x, p-\Delta p) d(\Delta x) d(\Delta p) \quad (2.62)$$

where $w(\Delta x, \Delta p; x-\Delta x, p-\Delta p)$ is the probability for a transition from the state $(x-\Delta x, p-\Delta p)$ to (x,p) during the time interval Δt . For Brownian motion one assumes that the transition probabilities have a sharp peak about $\Delta x, \Delta p \approx 0$. This seems reasonable since the macroscopic system is assumed to be undergoing very small random molecular collisions on a time scale much smaller than the macroscopic system itself. From the Langevin equation, eq. (2.36), one may obtain the relationship between Δx , Δp and Δt ,

$$\Delta p = -\left(\eta p + \frac{\partial U}{\partial x}\right) \Delta t + \int_t^{t+\Delta t} L(t) dt \quad (2.63)$$

$$\Delta x = \frac{1}{M} p \Delta t$$

with $(T^* \ll \Delta t \ll \eta^{-1})$

The Fokker-Planck equation is obtained by expanding the left and right hand sides of eq. (2.62) in the small variables

Δx , Δp , Δt and then equating terms of order Δt recognizing from (2.63) and (2.34) that $\overline{(\Delta x)^2} \sim (\Delta t)^2$, $\overline{(\Delta p)^2} \sim \Delta t$, and $\overline{(\Delta p \Delta x)} \sim (\Delta t)^2$.

The result is

$$\frac{\partial P}{\partial t} = - \frac{\partial}{\partial x} \left(P \frac{\overline{\Delta x}}{\Delta t} \right) - \frac{\partial}{\partial p} \left(P \frac{\overline{\Delta p}}{\Delta t} \right) + \frac{1}{2} \frac{\partial^2}{\partial p^2} \left(P \frac{\overline{(\Delta p)^2}}{\Delta t} \right) \quad (2.64)$$

where the averages are given by

$$\overline{f(\Delta x, \Delta p)} = \int f(\Delta x, \Delta p) w(\Delta x, \Delta p; x, p) .$$

However $\frac{\overline{\Delta x}}{\Delta t}$, $\frac{\overline{\Delta p}}{\Delta t}$, and $\frac{\overline{(\Delta p)^2}}{\Delta t}$ may be found from eq. (2.63), for example,

$$\overline{(\Delta p)^2} = \left(\eta p + \frac{\partial U}{\partial x} \right)^2 (\Delta t)^2 - \left(\eta p + \frac{\partial U}{\partial x} \right) \Delta t \int L(t) dt + \int L(t) L(t') dt dt'$$

then to first order in Δt , using the properties of $L(t)$ (eq. 2.34)

$$\overline{(\Delta p)^2} = \int L(t) L(t') dt dt' = 2D \int_t^{t+\Delta t} dt = 2D\Delta t$$

and so

$$\frac{\overline{(\Delta p)^2}}{\Delta t} = 2D; \text{ similarly } \frac{\overline{\Delta x}}{\Delta t} = \frac{p}{M} \text{ and } \frac{\overline{\Delta p}}{\Delta t} = - \left(\eta p + \frac{\partial U}{\partial x} \right) .$$

Thus finally (2.64) may be written,

$$\frac{\partial P}{\partial t} = - \frac{\partial}{\partial x} \left(P \frac{p}{M} \right) + \frac{\partial}{\partial p} \left(P \left(\eta p + \frac{\partial U}{\partial x} \right) \right) + \frac{\partial^2}{\partial p^2} [DP] \quad (2.65)$$

This then is the full Fokker-Planck equation in the general case, (finite damping and a potential included).

Kramers has shown that for the highly damped case, where one expects momentum equilibrium to set in the quickest, equation

(2.65) can be reduced further by integration over p ⁽⁵⁷⁾, giving

$$\frac{\partial P(x,t)}{\partial t} = - \frac{\partial G}{\partial x} \quad (2.66)$$

$$\text{with } G \equiv \left[\left(- \frac{\partial U}{\partial x} \right) \frac{P}{M\eta} - \frac{D}{(M\eta)^2} \frac{\partial P}{\partial x} \right] \quad (2.67)$$

The condition whereby this equation (the Smoluchowski equation) is valid is given in section 2.4.6.

One seeks a stationary (or steady-state) solution to (2.66), i.e. $\frac{\partial P}{\partial t} = 0$, which implies that G is independent of x . For the particular problem we wish to solve the potential has the form

$$U(x) = -(\Omega x + \Omega_0 \cos x) \quad (2.68)$$

and thus $\frac{\partial U}{\partial x}$ is periodic in x with period 2π , hence we also require $P(x) = P(x + 2\pi)$. Using these properties along with the normalization

$$\frac{1}{2\pi} \int_0^{2\pi} P(x) dx = 1 \quad (2.69)$$

the average value of $\dot{x}$ is (making use of the Langevin eq. (2.37)),

$$\begin{aligned} \overline{\dot{x}} &= \frac{1}{\eta M} \overline{\left(- \frac{\partial U}{\partial x} \right)} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{\eta M} \left(- \frac{\partial U}{\partial x} \right) P(x) dx \\ &= \frac{1}{2\pi} \int_0^{2\pi} \left\{ G + \frac{D}{(M\eta)^2} \frac{\partial P}{\partial x} \right\} dx \\ &= G + \frac{D}{2\pi (M\eta)^2} [P(2\pi) - P(0)] \end{aligned} \quad (2.70)$$

that is

$$\overline{\dot{x}} = G$$

The solution of (2.66)-(2.69) has been obtained by Stratonovich⁽¹⁶⁾, and Ambegaokar and Halperin⁽¹³⁾. The solution for G can be put into the following form, as shown in appendix 2.B:

$$G = \frac{D}{2(M\eta)^2} \sinh \left\{ \frac{M\eta\pi\Omega}{D} \right\} \left[\int_0^{\frac{\pi}{2}} \cosh \left\{ \frac{2M\eta\Omega}{D} y \right\} I_0 \left\{ \frac{2M\eta\Omega}{D} \cos y \right\} dy \right]^{-1} \quad (2.71)$$

This solution to the Brownian motion problem may be directly applied to the problems of intrinsic fluctuations in the d.c. Josephson effect and the driven a.c. Josephson effect for highly damped superconducting weak links, using the appropriate correspondences. For the driven a.c. effect (compare eqs. (2.50) vs. (2.36); (2.53) vs. (2.68); (2.60) vs. (2.34)),

$$M \leftrightarrow \frac{\hbar C}{2e}$$

$$\eta M \leftrightarrow \frac{\hbar}{2eR}$$

$$\eta^{-1} \leftrightarrow RC$$

$$D \leftrightarrow \frac{k_B T (I_p + I_{qp})}{V}$$

$$\Omega \leftrightarrow \Delta I$$

$$\Omega_0 \leftrightarrow j_2$$

$$\dot{\bar{x}} \leftrightarrow \dot{\bar{\theta}}_0 = \frac{2e}{\hbar} \Delta V$$

Therefore, in Stephen's notation, the d.c. I-V profile of a step, see Fig. 2.6(b) is

$$\Delta V = \frac{j_2 R}{2a_2} \sinh(\pi a_1) \left[\int_0^{\pi/2} \cosh(2a_1 y) I_0(2a_2 \cos y) dy \right]^{-1} \quad (2.72)$$

with
$$a_2 \equiv \frac{\hbar j_2}{2ek_B T} \times \frac{V_0}{RJ}$$

$$a_1 \equiv \frac{\hbar \Delta I}{2ek_B T} \times \frac{V_0}{RJ} \quad (2.72) \text{ cont.}$$

The dimensionless parameter a_2 indicates a measure of the barrier height, or coupling energy $\frac{\hbar j_2}{2e}$, relative to the thermal energy $k_B T$; a_1 provides a measure of the tilt of the potential that is the driving force for uncoupling of the phase locking.

Similarly, in the notation of Ambegaokar and Halperin one obtains for the d.c. I-V in the d.c. Josephson effect,

$$\bar{V} = \frac{I_1 R_N}{\gamma} \sinh \frac{\pi}{2} \gamma x \left[\int_0^{\pi/2} \cosh(\gamma x y) I_0(\gamma \cos y) dy \right]^{-1} \quad (2.73)$$

with
$$\gamma \equiv \frac{\hbar I_1}{ek_B T}; \quad x \equiv \frac{I}{I_1}$$

This form of the theoretical result is equivalent to, but somewhat more simplified than the form presented in the original paper of Ambegaokar and Halperin (eq. 9), which is

$$\bar{V} = \frac{4\pi I_1 R_N}{\gamma} \left\{ (e^{\pi \gamma x} - 1)^{-1} \left(\int_0^{2\pi} d\theta f(\theta) \right) \left(\int_0^{2\pi} d\theta' \frac{1}{f(\theta')} \right) + \int_0^{2\pi} d\theta \int_0^{2\pi} d\theta' \frac{f(\theta)}{f(\theta')} \right\}^{-1} \quad (2.73')$$

with $f(\theta) \equiv \exp [-U(\theta)/T]^* = \exp [\frac{\gamma}{2}(x\theta + \cos \theta)]$.

A limiting case of some utility is,

$$\lim_{I \rightarrow 0} \frac{\bar{V}}{IR_N} = [I_0(\frac{\gamma}{2})]^{-2} \quad (2.74)$$

* $U(\theta)$ here as defined in their (AH) paper (ref. 13).

and
$$\lim_{\Delta I \rightarrow 0} \frac{\Delta V}{(\Delta I) R} = [I_O(a_2)]^{-2}$$

and hence the slope of a step at its center is

$$\frac{1}{R_c} = \frac{[I_O(\frac{\gamma}{2})]^2}{R_N} \quad (\text{d.c.}) \quad (2.75)$$

or
$$\frac{[I_O(a_2)]^2}{R} \quad (\text{driven a.c.})$$

In the limit $\gamma \rightarrow \infty$, or $a_2 \rightarrow \infty$ (2.72) and (2.73) reduce to the standard no-fluctuation result (eqs. (2.19) & (2.54)).

2.4.6 The Approximations

In the theory just outlined there are two explicit assumptions. First it is assumed that one is considering low enough voltages such that $eV \ll k_B T$. In such case the spectrum of the pair and quasiparticle current fluctuations reduces to a flat spectrum or thermal noise spectrum. For any other spectrum the fluctuation current-current correlation function must again be calculated as indicated at the beginning of the previous section,

$$G(t_1)G(t_2) = \int_0^\infty P_{IT}(\omega) d\omega$$

and in general the highly theoretical question of whether Fokker-Planck techniques are indeed applicable must be reconsidered.

The second, and major approximation made has been the use of the Smoluchowski equation instead of the full two dimensional Fokker-Planck equation. We have already mentioned that this procedure is valid for an extremely highly damped system.

Kramers investigated this problem and came up with a criterion

for which the simplification is valid,⁽⁵⁷⁾ which in terms of junction parameters is,

$$\Omega \equiv R_N C \left(\frac{2eI_1}{\hbar C} \right)^{1/2} \ll 1$$

P. Lee has recently reinvestigated Kramers arguments and has shown that the proper criterion is⁽¹⁷⁾

$$\Omega^2 \ll 1 \quad (2.76)$$

For finite Ω , the full Fokker-Planck equation, eq. (2.65) must be considered. Ivanchenko and Zil'berman have obtained an approximate analytic solution for the highly underdamped case ($\beta \gg 1$) in the limit $\gamma \gg 1$, $x < 1$. Similarly Biswas and Jha⁽⁵⁸⁾ have obtained an analytic approximation for finite damping in the same $\gamma \gg 1$, $x < 1$ limit. They obtain,

$$\begin{aligned} \frac{\bar{V}}{I_1 R_N} &= \frac{1}{\beta C} \left\{ [1 + 4\beta(1-x^2)^{1/2}]^{1/2} - 1 \right\} \\ &\times \exp \left\{ -\gamma[(1-x^2)^{1/2} + x \sin^{-1} x] \right\} \sinh\left(\frac{\pi}{2}\gamma x\right) \\ (\gamma \gg 1; \quad x \equiv \frac{I}{I_1} < 1) \end{aligned} \quad (2.77)$$

Finally one last note: Kurkijärvi and Ambegaokar have solved the finite damping case numerically for general γ and x using Monte Carlo techniques.⁽⁵⁹⁾ Qualitatively they find that the I-V curves for finite damping cases approach the straight line $\bar{V} = I R_N$ more rapidly than the curve for infinite damping.

2.5 Contemporary Theories and Refined Models

The Josephson effects as observed in SIS tunnel junctions are well understood in terms of the microscopic theory of quasi-particle and Cooper pair tunneling through an insulating barrier. In recent years much theoretical and experimental interest has been devoted toward the understanding of the physical mechanisms which lead to Josephson-like behavior in SNS junctions, superconducting point contacts, and thin film bridges -- particularly the latter since they are the simplest and most rugged of the weak link structures (hence conceivably the most useful). We briefly mention here some of the recent refinements to the somewhat oversimplified model of section 2.3.

It seems that recent work in this area might be categorized to answer the following three (general) questions:

(1) Can Josephson behavior in short narrow superconducting weak links be derived from and explained by the general phenomenological and/or microscopic theories of superconductivity? In a bulk superconductor the supercurrent density is proportional to the gradient of the phase, $j_2 \sim \nabla \gamma$; how does one go from this to the periodic relationship $j_s \sim \sin \phi$ which seems characteristic of most weak links, (60,61) at least in certain regimes of temperature, coherence length, electron mean free path, etc. Baratoff, Blackburn, and Schwartz (BBS) provide a derivation of $j_s = j_1 \sin \phi$ for weak links based upon the one-dimensional Ginsberg-Landau

equations.⁽⁶²⁾ In their model the weak link is "weak" only insofar as its critical current is lower than that of the adjacent electrodes. They find that the resultant critical current of the weak link in this environment is much larger than the bulk value for an equivalent isolated section with the same material properties. This enhanced critical current is thus due to the presence of the "bulk" superconductors on each side of the weak link. Their derivation appears to be slightly more general than the earlier derivation of Aslamazov and Larkin,⁽³⁶⁾ in that the latter authors approximate the spatial dependence of the order parameter (near the link) by a harmonic function independent of the current, whereas (BBS) include non-linear effects. P. V. Christiansen et al. provide a similar derivation also based upon the one-dimensional G-L theory, in particular they show that for T near enough to T_c the supercurrent density has the same temperature dependence as an SIS tunnel junction,⁽⁶³⁾ i.e. $j_s \sim (1-T/T_c)$. It should also be pointed out deGennes recognized very early the sinusoidal $j_s(\phi)$ relation for the SNS geometry based upon the G-L equation and representation of the barrier by effective boundary conditions on the order parameter.⁽⁶⁴⁾

(2) Can a more accurate working model for a weak link be proposed? Should the simple two fluid model of McCumber, Stewart, and Aslamazov and Larkin, $(\frac{V}{R} + j_1 \sin \phi)$, be supplanted by a modified version? Can a real weak link be represented sufficiently accurately by a lumped equivalent circuit? It was first pointed out by Notarys and Mercereau⁽⁶⁵⁾ that the simple resistor-shunted-

supercurrent model did not sufficiently account for the observed d.c. I-V for $I \gg I_1$ in their proximity-effect bridges. In the simple model $\bar{I}_s \rightarrow 0$ as I becomes large with respect to I_1 , whereas it was observed that $\bar{I}_s \rightarrow \frac{I_1}{2}$ for large currents. To account for this behavior a relaxation oscillation model for $I_s(t)$ was proposed⁽⁶⁶⁾ and this model eventually evolved as the basis for an intensive theoretical investigation of time-dependent superconductivity by Rieger, Scalapino, and Mercereau.⁽⁶⁷⁾ This latter theory is based upon one-dimensional time-dependent Ginsberg-Landau equations (similar to the TDGL theory of Gorkov and Eliashberg for gapless superconductors⁽⁶⁸⁾). Essential to their model is the concept of "phase-slip" occurring when the amplitude of the order parameter is fully depressed. The basic idea is that in the finite voltage state the supercurrent accelerates until it reaches a critical value at which time a phase slip of 2π occurs, after which the order parameter relaxes and the supercurrent falls. The resultant supercurrent $I_s(t)$ has a finite d.c. component, $\frac{I_1}{2}$ for $I \gg I_1$. This process repeats itself at a repetition rate equal to the Josephson frequency. During the time the order parameter is zero there is dissipation of energy, so that the total energy dissipation is $\frac{V^2}{R_N} + \bar{I}_s V$. Deaver and Pierce have pointed out that the "excess supercurrent", $\frac{I_1}{2}$, can be effectively accounted for by the inclusion of an inductance, L , in series with the supercurrent term $I_1 \sin \phi$, in the supercurrent branch of the simple equivalent circuit of McCumber etc.⁽⁶⁹⁾ The I-V relation is then given by $\bar{V} = \frac{I_1 R}{\ln(1 - I_1/I)}$ for $I > I_1$ and $L > \frac{\Phi_0}{2\pi I_1}$. The

inductance here is the "kinetic" inductance of the supercurrent⁽⁷⁰⁾ plus any magnetic inductance. The net result of this inductance is a skewing of the $I_s(t)$ relationship, and for $L > \frac{\phi_0}{2\pi I_1}$, $I_s(t)$

is reentrant--- which results in an instability leading to an effective relaxation oscillation form for $I_s(t)$, similar to the proposal of Notarys et al. This is illustrated in Figure 2.8(a). This inductance model is based upon a discussion of weak links with external shunt inductances, given by Sullivan et al.⁽⁷¹⁾

Quite recently Notarys, Yu, and Mercereau have proposed an additional term to the supercurrent based upon an extension of the Aslamazov-Larkin formulation which allows the phase to be a function of position.⁽⁷²⁾ (Aslamazov-Larkin consider only the order parameter amplitude to vary (cf. ref. 36)). The result is,

$$j_s = j_1 \sin \phi + BV_Y \{1 + \cos \phi\}.$$

The term proportional to the gradient of the phase is important only in cases of large supercurrent densities, which is usually the case for thin film bridges. The presence of this extra term appears to explain the so-called Wyatt⁽⁷³⁾ or Dayem⁽⁷⁴⁾ effect which quite characteristically is usually only seen in thin film bridges but has also been seen by us in one-dimensional whisker crystals (cf. chapter 6).

(3) The third general question concerns the transition from Josephson-like behavior to bulk-like behavior. Experimentally it appears that in general a broad spectrum of behavior is observable in weak links. Relevant questions then are, under what conditions does phase coherence exist in a weak link? Is an unambiguous $j(\phi)$

Figure 2.8 (a) Relaxation-oscillation model ala Deaver & Pierce.
 (b) Range of applicability of contemporary theories.

(a) Deaver-Pierce Model

(b) Weak Link Behavior (@ Finite Voltages)
 Versus Relative Dimensions

relationship really necessary for phase coherent effects? Can one general equivalent circuit be proposed which will hold in all situations (obviously, an engineer's dream)? The theoretical answer to the latter two questions appears at present to be no.

Song and Rochlin have clearly demonstrated that the temperature dependence of the critical current in thin film bridges is strongly dependent upon the effective length (relative to the coherence length) of the bridges.⁽⁷⁵⁾ For short effective lengths $I_1 \sim (1 - T/T_c)$, i.e. Josephson junction behavior, whereas as the effective length was increased the temperature dependence changed toward that of a bulk superconductor, $I_1 \sim (1 - (T/T_c)^2)^{3/2} (1 + (T/T_c)^2)^2$. These results are consistent with the calculations of Baratoff et al. who show that pure sinusoidal behavior of $j(\phi)$ maintains only in the very weak, very short limit. In this regard we should point out that Ishii⁽⁷⁶⁾, and Bardeen and Johnson⁽⁷⁷⁾ have shown (based upon microscopic considerations) that the $j(\phi)$ relation for an SNS junction tends toward a sawtooth waveform as $T \rightarrow 0$ (whereas $j(\phi) = j_1 \sin \phi$ is rapidly approached for higher temperatures). Additionally they show that the temperature dependence of I_1 for an SNS junction is exponential, in agreement with the experimental results of Clarke,⁽⁷⁸⁾ and Shepherd.⁽⁷⁹⁾

As early as 1964 Anderson and Dayem proposed a moving vortex model for thin film bridge behavior⁽⁸⁰⁾ (cf. next paragraph). In their model, at finite voltages vortices transverse the bridge and move in coherent fashion, which results in an effective supercurrent dissipation (as discussed by Rieger et al.), and which allows synchronization of the vortex motion with an external

a.c. field, resulting in the driven a.c. Josephson effect.

Anderson subsequently pointed out the effective equivalence of the Josephson supercurrent and vortex points of view.⁽⁸¹⁾ Rieger et al. point out that their one-dimensional phase slip model is essentially the one dimensional limit of the vortex motion model. The moving vortex model is the analog of the vortex solution to the two dimensional SIS tunnel junction, and the two dimensional SNS junction which has been investigated by Waldram, Pippard, and Clarke.⁽¹⁸⁾

Likharev has provided a nice review of the current status in the understanding of general thin film bridge behavior, for the situation in which the films display type II superconductor behavior.⁽⁸²⁾ This situation results owing to a small residual mean free path in a film, resulting in $\lambda > \xi$. The destruction of the purely superconductive state in such a film of width $w \gg \xi$ under the influence of the field of a current is accompanied by the entry of quantized vortices into the film (with axes perpendicular to the film)⁽⁸³⁾. The core of a vortex has radius ξ , but has a current sheath of radius $\lambda_{\perp} = \lambda \coth \frac{t}{2\lambda}$ where t is the film thickness and λ is the bulk penetration depth⁽⁸⁴⁾. For a bridge whose length l is much greater than $\lambda_{\perp}$ many rows of vortices enter the film; subsequent motion of the rows of vortices are then most likely random (rather than coherent with respect to each other) due to film inhomogeneities, and in such case one would not expect Josephson-like effects to occur. This random motion of vortices has been observed experimentally⁽⁸⁵⁾. It would

seem then that for the existence of inherent coherence effects it is necessary for $\ell \lesssim \lambda_{\perp}$ (a vortex radius), whereas synchronization via an external a.c. field might result over a slightly wider range of ℓ (weak link length). Note that this criterion allows $\lambda_{\perp} \sim \ell \gg \xi$ for type II-like films whereas $\ell \sim \lambda_{\perp} \lesssim \xi$ for type I-like films ("clean" films, having large electron mean free path). Indeed coherence effects have been observed experimentally by Janocko et al.⁽⁸⁶⁾ in very "dirty" films with $\lambda_{\perp} \gtrsim \ell \gg \lambda \gg \xi$.

These ideas are summarized in Fig. 2.8(b) which is essentially similar to Fig. 1 of Likharev.⁽⁸²⁾ This figure shows the regions of applicability for some of the various theories with respect to the bridge dimensions; length plotted vertically and width horizontally. For $\ell, w \ll \xi$ pure Josephson behavior results and the current density depends uniquely on (and in fact is a sinusoidal function of) the phase difference across the link i.e. $j = j_1 \sin \phi$. As w is increased one must extend the theory to two dimensions. For $w \gtrsim \xi$ and $\ell \gg \lambda_{\perp}$ (the upper horizontal strip) one expects a dynamic vortex state of a random nature, having bulk-like properties. In the intermediate region a coherent vortex state exists, or in the case of narrow bridges $w \lesssim \xi$, the one dimensional limit of the vortex state. Likharev himself has investigated quantitatively the region indicated by the triangle $\xi \ll \ell \ll \lambda_{\perp}$, $\ell \ll w \ll \lambda_{\perp}$. He contends that in the absence of any external field the actual vortex motion consists of the opposite motion of pairs of opposite-polarity vortices with creation of pairs of vortices at the film edges and their mutual annihilation at the center of the film. Likharev's analysis shows that phase coherent

effects are quite evident in his considered regime, but quantitatively the effects differ from the Josephson effects for very small structures. For example, the constant voltage steps (harmonic and subharmonic) induced (by r.f.) in the d.c. I-V do not oscillate with changing r.f. amplitude. Also the power radiated by a bridge decreases sharply as the bridge length is increased. Likharev concludes that the presence of an unambiguous relation between the current density, j , and the phase difference, ϕ is not a necessary condition for the realization of Josephson-like behavior in superconducting weak links. In view of these results one might conclude that it is difficult to model this regime with a single lumped equivalent circuit; Gregers-Hansen et al. have indeed encountered such difficulties in their analog computer simulations. (87)

In conclusion, it appears that no one equivalent circuit model will hold under all conditions of ℓ , w , mean free path, temperature, etc. for superconducting weak links. It is not entirely clear how all of the contemporary theories fit together, (if indeed they do fit together), but most of the current ideas show many similarities. In particular, in the regime $\ell, w \ll \xi$ it is fairly clear both theoretically and experimentally that pure Josephson-like behavior is observed, $j_s = j_1 \sin \phi$, and a circuit model similar to Fig. 2.8(a) may be sufficiently accurate if j_s is generalized to include the effects of high current densities.

2.A Appendix: Solutions to Equation (2.18)

The equation is,

$$\frac{d\phi}{dt} = A(j - \sin \phi) \quad \text{where } j \equiv \frac{I}{I_1} \text{ and } A \equiv \frac{2e}{\hbar} I_1 R_N$$

The indefinite integral is,

$$\int \frac{d\phi}{j - \sin \phi} = A \int dt \quad (2.A.1)$$

Let $\mu = I_1 \sin \phi$,

$$I_1 \int \frac{d\mu}{\sqrt{I_1^2 - \mu^2} (I - \mu)} = A \int dt$$

which gives,

$$\frac{I_1}{\sqrt{I^2 - I_1^2}} \sin^{-1} \left[\frac{I_1^2 - I\mu}{(\mu - I) I_1} \right] \bigg|_{\mu=\mu(0)}^{\mu=\mu(t)=I_S(t)} = At$$

or

$$\left[\frac{I_1^2 - I\mu}{(\mu - I) I_1} \right]_{\mu(0)}^{I_S(t)} = \sin^{-1} (\sqrt{j^2 - 1} At)$$

Let us arbitrarily define $\mu(0) = \frac{I_1^2}{I}$, then

$$\frac{j I_S(t) - I_1}{I - I_S(t)} = \sin (\sqrt{j^2 - 1} At) \quad (2.A.2)$$

We note that the integrand in (2.A.1) is periodic, with period 2π .

Consider integration over one period.

$$\int_0^{2\pi} \frac{d\phi}{j - \sin \phi} = A \int_0^{\tau} dt$$

which yields,

$$\frac{2\pi}{\sqrt{j^2-1}} = A\tau \quad (2.A.3)$$

The period τ is related to the average voltage $\bar{V}$ by the Josephson relation, $\tau = \frac{2\pi}{\omega_0} = \frac{h}{2e\bar{V}}$ thus from (2.A.3) we get

$$\bar{V} = A \frac{\hbar}{2e} \sqrt{j^2-1} = I_1 R_N \left(\left\{ \frac{I}{I_1} \right\}^2 - 1 \right)^{1/2} \quad (2.A.4)$$

Combining (2.A.2) and (2.A.4), and solving for $I_s(t)$ we get the result,

$$I_s(t) = \frac{I_1(I_1 + I \sin \omega_0 t)}{(I + I_1 \sin \omega_0 t)} \quad (2.A.5)$$

We may get the phase by

$$\phi(t) = \sin^{-1} \frac{I_s(t)}{I_1} = \sin^{-1} \left(\frac{I_1 + I \sin \omega_0 t}{I + I_1 \sin \omega_0 t} \right) \quad (2.A.6)$$

The time dependent voltage $V(t)$ may be obtained from (2.A.5) in conjunction with $\frac{V(t)}{R_N} = [I - I_s(t)]$

$$\begin{aligned} \frac{V(t)}{R_N} &= \frac{I[I + I_1 \sin \omega_0 t] - I_1[I_1 + I \sin \omega_0 t]}{[I + I_1 \sin \omega_0 t]} \\ V(t) &= \frac{R_N [I^2 - I_1^2]}{I + I_1 \sin \omega_0 t} \end{aligned} \quad (2.A.7)$$

As an alternative approach, the indefinite integral (2.A.1) may be integrated directly, to yield

$$\tan \frac{\phi(t)}{2} = \frac{I_1}{I} + \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \tan \beta t \quad (2.A.8)$$

where we have set $\phi(0) = 2 \tan^{-1} \frac{I_1}{I}$ and $\beta \equiv \frac{\sqrt{\left\{ \frac{I}{I_1} \right\}^2 - 1}}{2} \frac{2eI_1 R_N}{\hbar}$

After differentiation (with respect to time) and a little

trigonometry and algebra,

$$\dot{\phi}(t) = 2\beta \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} [\cos^2 \beta t (\tan^2 \frac{\phi}{2} + 1)]^{-1} \quad (2.A.9)$$

whereupon ϕ may be eliminated from (2.A.9) by substitution of (2.A.8),

$$\dot{\phi}(t) = 2\beta \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \frac{\sec^2 \beta t}{1 + \left(\frac{I_1}{I} + \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \tan \beta t \right)^2} \quad (2.A.10)$$

From (2.A.10) we may get the time dependent voltage $V(t)$ directly or we can integrate over one period, $\frac{\pi}{\beta}$, in order to find the average voltage,

$$\bar{\dot{\phi}}(t) = 2\beta \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \frac{\beta}{\pi} \int_{-\frac{\pi}{2\beta}}^{+\frac{\pi}{2\beta}} \left[\frac{\sec^2 \beta t}{1 + \left\{ \frac{I_1}{I} + \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \tan \beta t \right\}^2} \right] dt$$

Let $u = \frac{I_1}{I} + \sqrt{1 - \left\{ \frac{I_1}{I} \right\}^2} \tan \beta t$, then

$$\bar{\dot{\phi}} = \frac{2\beta}{\pi} \int_{-\infty}^{\infty} \frac{du}{1+u^2} = \frac{2\beta}{\pi} \tan^{-1} u \Big|_{-\infty}^{\infty} = 2\beta$$

hence,

$$\bar{V} = \frac{\hbar \bar{\dot{\phi}}}{2e} = I_1 R_N \sqrt{\left\{ \frac{I}{I_1} \right\}^2 - 1} \quad \text{as in (2.A.4).}$$

We note that in (2.A.8) then, $\beta = \frac{\omega_0}{2}$ where ω_0 is the Josephson frequency, $\omega_0 = \frac{2e\bar{V}}{\hbar}$.

2.B Appendix: Solution of the Smoluchowski Equation

For completeness we outline a solution given by Stratonovich⁽¹⁶⁾. From equations (2.67) and (2.68) we have,

$$G = \frac{D}{(M\eta)^2} \left\{ (\Omega - \Omega_0 \sin x) \frac{M\eta}{D} P - \frac{\partial P}{\partial x} \right\} = \text{constant}$$

Define $\xi = \frac{\Omega M\eta}{D}$, $\xi_0 \equiv \Omega_0 \frac{M\eta}{D}$,

$$G = \frac{D}{(M\eta)^2} \left\{ (\xi - \xi_0 \sin x) P - \frac{\partial P}{\partial x} \right\} = \text{constant} \quad (2.B.1)$$

Then P satisfies,

$$\frac{d^2 P}{dx^2} - \frac{d}{dx} [(\xi - \xi_0 \sin x) P] = 0 \quad (2.B.2)$$

The general solution of (2.B.2) is,

$$P(x) = C_1 e^{\xi x + \xi_0 \cos x} \int_{C_2}^x e^{-\xi \psi - \xi_0 \cos \psi} d\psi$$

The property $P(x+2\pi) = P(x)$ implies $C_2 = \infty$. $P(x)$ can then be written,

$$P(x) = \frac{2\pi}{N} e^{\xi x + \xi_0 \cos x} \int_x^{x+2\pi} e^{-\xi \psi - \xi_0 \cos \psi} d\psi \quad (2.B.3)$$

$$\text{with } N \equiv -2\pi \frac{(1 - e^{-2\pi \xi})}{C_1}$$

We use the normalization $\frac{1}{2\pi} \int_0^{2\pi} P(x) dx = 1$, then

$$N = \int_0^{2\pi} e^{\xi x + \xi_0 \cos x} \int_0^{x+2\pi} e^{-\xi \psi - \xi_0 \cos \psi} d\psi dx$$

Changing variables, $\chi = \psi - x$,

$$N = \int_0^{2\pi} \int_0^{2\pi} e^{-\xi \chi + 2\xi_0 \sin \frac{\chi}{2} \sin \{x + \frac{\chi}{2}\}} d\chi dx$$

$$N = 2\pi \int_0^{2\pi} e^{-\xi x} I_0(2\xi_0 \sin \frac{x}{2}) dx$$

where I_0 is a modified Bessel function of zero order which in integral form may be written,

$$I_0(z) = \frac{1}{\pi} \int_0^{\pi} e^{\pm z \cos \theta} d\theta = \frac{1}{\pi} \int_0^{\pi} \cosh(z \cos \theta) d\theta$$

Changing variables again,

$$y = \begin{cases} \frac{1}{2}(\pi - x) & 0 < x < \pi \\ \frac{1}{2}(x - \pi) & \pi < x < 2\pi \end{cases}$$

$$N = 8\pi e^{-\xi \pi} \times \int_0^{\pi/2} \cosh(2\xi y) I_0(2\xi_0 \cos y) dy \quad (2.B.4)$$

To determine G from $P(x)$ we make use of the following formula (written here for reference purposes):

$$\text{if } F(t) = \int_{a(t)}^{b(t)} \phi(x, t) dx \quad \text{with } a, b \text{ differentiable and } \phi, \frac{\partial \phi}{\partial t} \text{ continuous in } x \text{ \& } t.$$

$$\frac{dF}{dt} = \left[\int_{a(t)}^{b(t)} \frac{\partial \phi}{\partial t} dx \right] + \phi(b, t) \frac{db}{dt} - \phi(a, t) \frac{da}{dt}$$

Then, using (2.B.3)

$$\frac{\partial P}{\partial x} = \frac{2\pi}{N} e^{\xi x + \xi_0 \cos x} \left\{ (e^{-2\pi\xi} - 1) e^{-\xi x - \xi_0 \cos x} + (\xi - \xi_0 \sin x) \times \int_x^{x+2\pi} e^{-\xi \psi - \xi_0 \cos \psi} d\psi \right\}$$

$$\frac{\partial P}{\partial x} = \frac{2\pi}{N}(e^{-2\pi\xi}-1) + (\xi-\xi_0 \sin x) P(x)$$

and from (2.B.1),

$$G = \frac{D}{(M\eta)^2} \frac{2\pi}{N} (1-e^{-2\pi\xi})$$

or finally, using (2.B.4)

$$G = \frac{D}{2(M\eta)^2} \sinh(\xi\pi) \left[\int_0^{\pi/2} \cosh(2\xi y) I_0(2\xi_0 \cos y) dy \right]^{-1}$$

(2.B.5)

3. SAMPLES AND THEIR PREPARATION

3.1 Introduction

The regime of interest for these experiments (cf. chapters 2 and 5) was that regime in which two "bulk" superconductors are weakly coupled - in the sense that a phase difference may exist across the interconnection, (whereas for a singly-connected bulk superconductor the order parameter phase is constant throughout). Interaction with the phase of the superconductor, via applied electromagnetic fields, and the effects of fluctuation energies $\sim k_B T$ could only be expected to be observable for relatively small coupling energies. Since the spatial extent of correlation of the electrons in a Cooper pair is given by $\xi(T)$, one might expect coherence to break down, and hence to obtain a weak coupling between two bulk superconductors via an interconnection of dimension $\lesssim \xi(T)$, whereas for interconnections of dimensions $\gg \xi(T)$ one might expect the superconducting behavior of the link to approach that of the adjacent bulk superconductors. Hence in these experiments we desired at least one dimension, a , of the "weak link" to be less than the temperature dependent coherence length,

$$\text{i.e. } a \lesssim \xi(T) \quad (3.1)$$

Secondly, it was desirable to study cases which are theoretically tractable. In this respect it is of great advantage to study geometries in which the supercurrent density is uniform

across the sample cross section, since this reduces the problem to one-dimensional. The magnetic field and current within a superconductor are known to vary spatially over a distance $\sim \lambda(T)$. Hence a one-dimensional situation is obtained if the largest cross sectional dimension, b , is less than $\sim 2\lambda(T)$, twice the temperature dependent penetration depth.

$$\frac{b}{2} \lesssim \lambda(T) \quad (3.2)$$

Thus equations 3.1 and 3.2 were the basic criteria for one or more of the dimensions of the samples that were studied. For typical superconductors dimensions $\lesssim$ a few microns are implied. General problems encountered with the preparation of devices of such small dimensions are more or less obvious: (1) fabrication techniques had to be devised; (2) techniques for establishing electrical contact to the devices had to be devised; (3) susceptibility to thermal and electrical shocks had to be considered; (4) detailed inspection required the use of electron microscopy and scanning electron microscopy; and (5) caution in handling had to be exercised.

During the course of these experiments four types of "weak links" were studied to varying degrees. These included niobium point contacts, filamentary tin single crystals ("whisker" crystals), tin thin film bridges of wedge geometry and "improved" tin thin film bridges of rectangular geometry fabricated via a photoresist technique.

3.2 Point Contacts

Mechanically and thermally stable niobium point contacts were fabricated for these experiments following Buhrman, Lukens, Strait and Webb⁽⁸⁸⁾. Figures 3.1(a) and (b) show a schematic and photograph respectively of such a point contact device. The actual weak link between the two bulk niobium superconductors, labeled A and B, is formed at the contact of a sharpened 000-120 niobium screw with the B plate. (The screw was sharpened to a $\sim 1\text{-}10\mu$ diameter point by machining followed by sanding with #600 emery paper.) For the devices used in these experiments a thermal oxide was grown upon the B plate by heating it on a hot plate, in air, for ~ 5 minutes. (The screws were not heated.) The device is rendered mechanically and thermally stable by the C-clamp construction, employing boron silicate glass (Corning optical code 8260 or 8270), whose thermal contraction closely matches niobium, as the insulating spacers.

As used in these experiments such a device was repeatedly cycled (from room temperature to 4.2K) and stored at room temperature in a helium atmosphere for a period of 3 months with no discernable change in critical current or normal state resistance, R_N . The same device stored at room temperature in air showed a factor of 4 increase in critical current and a 13% decrease in R_N after a period of one year. Thus, with proper storage (liquid nitrogen temperature or dry inert atmosphere) these devices maintain their characteristics quite well.

(b)

Figure 3.1 (a) Schematic of a niobium point contact device.
(b) Photograph of an actual device.

3.3 Tin Whisker Crystals

Filamentary Sn and 2% In-Sn single crystals of diameter $\sim 0.3\text{-}5\mu$ and lengths $\sim 50\mu\text{-}0.5\text{mm}$ were grown via stress activation⁽⁸⁹⁾ and handled in the same manner as documented in detail by R. J. Warburton⁽⁴⁷⁾. Figure 3.2 shows a scanning electron microscope (SEM) photograph of such a tin whisker resting upon a #42 copper wire. Note the uniformity and nearly circular cross section.

The major problem in working with these whisker crystals was that of making electrical contacts without locally damaging them at the contact points. Several techniques were devised, only a few of which were consistently successful. Only those useful techniques not already described by Warburton will be described here.

In general, contact damage manifested itself as two types of structure in the otherwise smooth (low current) voltage vs. temperature characteristic of the whisker (an example of a "good" characteristic is shown in the next chapter, Figure 4.12): (i) A generally broadened characteristic with structure throughout, and/or (ii) an otherwise good characteristic with a long "tail" at the low temperature side of the characteristic. Structure of the first type could be largely eliminated: (1) by insuring that the expansion coefficient of the substrate was greater or equal to that of the whisker (hence insuring that the whisker wouldn't be strained); (2) by suspending the whisker above the substrate; and (3) by checking the whisker under an optical microscope after completion of contacts - the whisker should appear of uniform color and be free of spots

Figure 3.2 Scanning electron micrograph of a tin whisker lying across a #42 copper wire.

X4500

X20,000

and particles; invariably there was a correlation between physical appearance and performance. The second type of structure could be attributed to local damage or proximity effects at the points of contact themselves, in effect creating weak spots with lower critical currents than the rest of the whisker. This sort of structure was the more subtle, but could be controlled sufficiently after much practice in making the contacts.

The following was a consistently successful method for obtaining a four point contact to a whisker and refers to Figure 3.3, which shows a schematic and the final device. Steps (2) - (6) below were carried out under a microscope set at $\sim 40\times$.

- (1) A small hole ($\sim 400\mu$ diameter) and grooves (to provide isolated terminals) are cut in copper clad breadboard.
- (2) Using a very small homemade soldering iron, two $\sim 10\mu$ diameter silver whiskers (made via a vapor reaction technique) are coated with Wood's metal by drawing the whiskers (with #5 tweezers) through a molten drop on the tip of the iron.
- (3) The coated Ag whiskers are laid across the hole and attached to the copper via Dupont #4817 silver paint.
- (4) A tin whisker is laid (manipulated with a needle) across the hole and upon the coated Ag whiskers. The ends of the tin whisker are fastened down with Dupont #7713 paint.
- (5) A dab of Supersafe flux, diluted 3:1 with water, is applied at the points of contact of the tin and silver whiskers using a tweezer and a single cotton fiber.
- (6) Heat is immediately applied via a standard soldering iron touched to the copper adjacent to an end of the Ag whisker. In the microscope one observes the Wood's metal liquify in

Figure 3.3

- (a) A whisker in four-point contact arrangement.
(hole diameter 0.4 mm ; Ag whisker diameter 10μ)

- (b) Photograph of an actual device--length between inner contacts is $\sim 330\mu$.

a wave initiating near the soldering iron. As soon as the wave hits and wets the Sn whisker the heat is removed.

Figure 3.3(b) shows a good 1μ diameter whisker contacted using this procedure. The voltage contacts in this case were Wood's metal, which was superconducting at the measuring temperatures used ($\leq 4.2\text{K}$).

A partially successful method of obtaining normal metal voltage contacts was devised by making use of a mixture of silver paints consisting of ~ 5 parts (by volume) of Dupont #7713 and ~ 2 parts of #4817. After sufficient drying time (depending upon the volume of paint mixed) very fine lines of paint $\sim 5\text{--}15\mu$ wide could be made by touching the drop of paint with a needle and drawing it out with a smooth motion. This technique was generally useful in other instances also, where thin conducting lines were desirable.

3.4 Thin Film Bridges - Wedge Geometry

Dayem and Anderson were the first to observe Josephson-like effects in thin film constrictions⁽⁸⁰⁾. A schematic of such a device is shown in Figure 3.4(a). Dayem's devices were made via evaporation over two sharpened (by lapping) V-shaped strips which had to be carefully aligned⁽⁷⁴⁾. With such masks widths, w , of $2\text{--}6\mu$ were obtained.

We have devised a technique for making similar devices utilizing a formvar stencil, thus eliminating the need for sharpened strips, aligning etc. With this technique widths as small as 0.5μ could be obtained, though $w = 1\text{--}3\mu$ was more typical. The

Figure 3.4 Dayem bridge schematic, fabrication process, and actual device.
(In (c) the constriction is 1μ)

technique consisted essentially of three steps - stencil preparation, metal evaporation, and stripping of the stencil. The technique, which is described below, is outlined schematically in Figure 3.4(b).

The substrates used were 0.75" diameter $\times$ 0.040" thick single crystal, optically flat sapphire windows obtained from Hugo P. Keller, Inc. Single crystal sapphire was chosen for its outstanding low temperature thermal conductivity (relative to glass or silicon for example). The substrates were coated with a thin film of formvar by dipping them into a 0.5% (by weight) formvar-chloroform solution and drying under a warm lamp for approximately one hour. Typically, this provided a formvar film 4700 $\text{\AA}$ thick. Formvar films much thinner or thicker than this were difficult to work with. Other coatings tried included 0.1-5% formvar-chloroform solutions, 0.1-1% collodium-methanol solutions, and 2-8% cellulose acetate-acetone solutions, none of which worked nearly as well as the 0.5% formvar solution.

Next (Fig. 3.4(b)(ii)) a razor blade - micromanipulator setup was used to cut an X in the formvar film. This operation was observed through a Cooke, Troughton, and Simms microscope at 52.5x. This microscope was especially convenient for observing the cuts because it has a glass specimen table, thus providing a transparent background. The cutting setup consisted of a homemade spring loaded razor blade mount which was attached to a Narishige micromanipulator, which provided X, Y, Z control. For cutting, a double edged stainless steel razor blade was attached and weighed down with 1-3 gram weights. Cutting was accomplished

by slowly pulling the blade (via the fine control) through the formvar at a cutting angle of approximately 5° . This caused the formvar to be pushed aside, revealing lines as thin as 0.5μ . Other cutting devices tried included single edged razor blades, scribes made from sapphire or glass, and sapphire whisker-platelets, none of which worked as well as the double edged blade; in particular, the scribes tended to tear the formvar rather than cut it.

Using a single edged razor blade and a pair of #5 tweezers two opposite sections of the X were peeled off (by hand, under the microscope), resulting in a formvar stencil, cf. Fig. 3.4(b)(iii). This was the most crucial step in the process and required caution and practice. After peeling off two sections any residue or dust was cleaned off by spraying with ethyl alcohol and blowing dry with aerosol clean air. Initial cleaning of the substrates consisted only of cleaning with lens tissue. For this technique to work nicely a compromise had to be reached between the degree of cleaning and the ease with which the formvar would peel off.

Following stencil completion, tin was evaporated over the entire surface (Fig. 3.4(b)(iv)). Subsequently the tin coated stencil was stripped off by floating it off in chloroform (in some cases ultrasonic agitation was also employed), resulting in the final structure, Fig. 3.4(b)(v). The thickness of the evaporated metal was $\leq 1500\text{\AA}$, hence much thinner than the formvar film ($4700\text{\AA}$) - such a condition was necessary in order to insure that any metal on the edges of the mask was sufficiently weakened so that a clean break and hence a smooth edge was obtained when

the stencil was floated off.

A photograph of an actual device made in this manner is shown in Figure 3.4(c). As many as five such devices could be easily put onto one substrate. The method used for isolation of individual devices and for making electrical contacts was identical to that described in section 3.5.3.

3.5 Thin Film Bridges via a Photoresist Technique

3.5.1 Motivation

During the course of experimentation with thin film bridges of wedge geometry it was observed that an undesirable resistive background appeared in the current-voltage characteristics before coherence effects disappeared, indicating perhaps that the measurement was no longer a four point measurement. Such a situation could arise because the superconducting-to-normal phase change in the (two dimensional) electrodes would be expected to proceed outward from the weak link along the wedge, as depicted schematically in Figure 3.5(a). If such were the mechanism one would expect this situation to be largely rectifiable by using a rectangular geometry as in Figure 3.5(b). This was the primary reason for choosing to study also bridges of rectangular geometry. The technique developed for fabrication of these devices of rectangular geometry had several additional advantages over the formvar-razor blade technique used in making wedge shaped bridges:

- (1) Bridge geometry could be controlled precisely, in particular, bridges of any length could be made. This fact was significant in view of the result that very little success was

Figure 3.5 The phase transition in rectangular versus wedge geometry bridges.

(a) Transition of Link Muddled by Transition of Electrodes

(b) Transition of Link Much More Distinct from Electrodes

achieved in attempting to control the length of whisker crystals for lengths $\leq 75\mu$.

- (2) The substrates could be more thoroughly cleaned - no compromises in cleanliness had to be conceded, in contrast to the formvar process.
- (3) The process was much quicker and results more reproducible than the formvar-razor technique.

3.5.2 The General Technique

With the microminiaturization of the electronics industry has come the development of high resolution light sensitive chemicals called photoresists⁽⁹⁰⁾. Thin photoresist films possess the property that regions exposed to ultraviolet light undergo a chemical decomposition (positive resists) or polymerization (negative resists), resulting in exposed and unexposed regions differing greatly in their solubility in certain solutions. Hence a photoresist stencil can be fabricated by projecting a pattern (via UV light) onto a photoresist film, and subsequently dissolving away exposed or unexposed regions. (Those resists for which exposed regions dissolve are called positive, those for which unexposed regions dissolve are called negative.)

Using commercially available photoresists having an advertized resolution of $\leq 0.5\mu$ we⁽⁹¹⁾ have arranged an inexpensive setup for fabrication of thin film bridges of width 1μ and lengths ranging from $1-250\mu$. The general technique eventually used by us is schematically depicted in Figure 3.6 and consisted of the following steps.

Figure 3.6 The general photoresist technique used.

- (a) The (sapphire) substrates were first thoroughly cleaned.
- (b) Next the substrates were coated with a thin photoresist film.
- (c) The photoresist was exposed by projecting through a chromium master mask in contact with the photoresist.
- (d) The exposed film was developed, leaving a photoresist stencil of the image of the master.
- (e) The superconducting film was deposited via evaporation.
- (f) Stripping of the photoresist stencil then left the desired superconductive metal pattern.

3.5.3 Details of the Technique

Positive vs. Negative:

Due to the availability of both positive and negative resists and due to the fact that either a negative or a positive image could be projected onto the photoresist film (depending upon the master mask), four possible methods are available - positive resist with positive master mask, negative resist with negative master, negative resist with positive master, and positive resist with negative master. The technique depicted in Figure 3.6(a)-(f) consists of the last permutation. The first two methods require the superconductive film to be evaporated prior to resist coating and involve a subsequent etching, using the photoresist as a protective coating to prevent etching where the superconductive pattern is desired. We preferred to try the latter two methods, for which etching is not necessary. A comparison of the masks and stencil for these methods is shown in Figure 3.7. Our experience has shown that the negative resist-positive mask approach gave considerably more problems in stripping of the stencil after

Figure 3.7 Comparison of positive vs. negative stenciling technique.

(a) POSITIVE RESIST - NEGATIVE MASTER MASK

(b) NEGATIVE RESIST - POSITIVE MASTER MASK

evaporation of the superconductor. We attribute this problem to the way in which the light diffracts and scatters about the edge of the chromium master mask, resulting in opposite slopes on the edges of the photoresist stencil for the positive resist vs. the negative resist stencil, making it difficult to properly strip the negative resist stencil without damaging the 1μ lines. On the other hand, stripping of the positive resist stencil was quite simple, hence the positive resist-negative master mask technique was the one chosen for actual use; from here on only this method will be discussed.

Master Mask:

The chromium master mask was made (via photographic reduction) according to our specifications by Qualitron Corporation (Danbury, Connecticut). A chromium mask has the advantage of being much more durable than a photographic emulsion. The mask itself consisted of six 1μ wide channels of lengths 1, 3, 8, 25, 75 and 250μ with a 120μ spacing between adjacent channels (cf. figures further on).

Environment:

Because the production of 1μ lines is very much susceptible to dust particles of comparable magnitude and because the photoresists were sensitive to the UV end of the visible spectrum, care was taken to provide a suitable environment. All steps of the process up to and including the development step were carried out in a freshly painted (for cleanliness) darkroom ($\sim 9' \times 14'$) illuminated by two 60 watt yellow incandescent lamps. This room was used exclusively for photoresist work. Whenever

possible the samples were handled and stored in two Edcraft clean hoods (Model 342-BRL-1). For work with the positive resist no special attention was paid toward controlling the temperature or humidity of the room. These simple precautions proved adequate for our purposes.

Substrate Cleaning:

Substrates were prepared and cleaned for positive photoresist coating according to the following procedure:

- (1) Gross contamination was first removed by cleaning with lens tissue, after which substrates were loaded into a glass basket.
- (2) The basket was then lowered into a slowly boiling concentrated detergent solution (3:1 by volume, deionized distilled water (DIDW): Fisher biodegradable detergent).
- (3) After 10-15 minutes the beaker containing detergent solution and basket was transferred into a small ultrasonic tub containing hot water. The ultrasonic was run for 5-10 minutes.
- (4) The basket was then transferred to a beaker of DIDW for a 5 minute rinse soak. This step was repeated with fresh DIDW.
- (5) Each substrate was then sprayed individually with DIDW.
- (6) Next the basket was placed into a beaker of trichloroethylene and ultrasoniced for 5-10 minutes.
- (7) There followed a soak in isopropyl alcohol for 2-5 minutes after which the basket was transferred to a clean hood where each substrate was individually rinsed with isopropyl alcohol.

- (8) The substrates were transferred to clean petri dishes and baked out at 180-200°C for 15 minutes.
- (9) Following bakeout the substrates were either immediately coated or stored (for no longer than a day) in a dessicator inside the clean hood, as it is important for the sake of good adhesion to keep the surfaces dry before coating with photoresist.

Following this procedure substrates were observed to have ~ 30 micron-sized particles per square millimeter ($1000\mu \times 1000\mu$) which for our purposes was quite adequate. Figure 3.8 shows photomicrographs of substrates before and after cleaning in this manner.

Coating:

The positive photoresist used was Shipley AZ-1350, used full strength. The resist was filtered during application by filtering with a syringe fitted with a Sweeny adapter (Millipore Corporation). The filters used were Millipore Corporation solvinert having 0.25μ pore sizes. A uniform photoresist film was obtained by applying ~ 12 drops of (filtered) resist to the substrate ($3/4$ " diameter) followed by a 10 second spin at 3000 rpm (via a homemade vacuum holder-motor arrangement). The film thickness depends primarily upon the motor acceleration. For our motor 3000 rpm final speed resulted in a thickness of $4950\text{\AA} \pm 150\text{\AA}$ which was independent of total spin time for times greater than the time required to reach full speed, ~ 5 seconds for our motor. The films obtained were free of pinholes and were sufficiently uniform to insure uniform contact with the master

Figure 3.8 Substrates before and after cleaning (scale--total horizontal range, 232μ).

BEFORE

AFTER

mask. Drying of the films was insured by subsequent baking at ~70°C for 15 minutes.

Exposure, Development and Rinse:

The UV light source used for exposure was an unfiltered 100 watt PEK high pressure mercury vapor lamp housed in a homemade casing. Best resolution requires that the light source be incoherent and the beam well collimated. To meet the latter need the light source was made into a good approximation of a point source by provision of a pinhole aperture in the housing.

The optimum exposure and development times, light source to sample distance, and light intensity have to be determined experimentally for any particular setup and lamp. We chose to use relatively long exposure times in order to improve the relative accuracy in exposures. For our setup typical parameter values were $1-1\frac{1}{2}$ minutes exposure, $4-7\frac{1}{2}$ minutes development, 30 seconds rinse, 30 cm source to sample distance and 105-115 watts lamp power consumption.

A properly exposed and developed photoresist stencil for 8 and 75 micron bridges is shown in Figure 3.9(A). In determining the optimum exposure and development times the following practical observations seemed to hold true:

- (1) Exposure time above a critical threshold (empirical - cf. (iii) below) determined the width of the channels in the stencil.
- (2) For any given exposure time above the threshold an optimum development time could be found.
 - (i) If the development time was too large (overdevelopment)

Figure 3.9 Photographs of good and bad stencils (120μ between channels).
(a) Properly exposed and developed photoresist stencil.
(b) Overdeveloped example. (c) Underdeveloped example.

the developer started eating into the remaining mask at the corners. Such a case is shown in in Figure 3.9(B).

- (ii) If development time was too small (underdevelopment) the channels were not clean and/or a residual film remained in areas which were supposed to be clean. An example of this case is shown in Figure 3.9(C).
- (iii) If for all development times (for the fixed exposure time) development was uneven then the film was underexposed; underdevelopment, case (ii), tends to be uniform over the whole film (as in Fig. 3.9(C)) whereas underexposure affects the longest and narrowest bridges first.

(3) Extreme overexposure resulted in very wide channels.

The films were developed in an ultrasonically agitated 1:1 (by volume) solution of Shipley AZ-Developer and DIDW. A subsequent water (DIDW) rinse was necessary to stop development. The substrates were then dried with filtered (1 μ pore size) dry nitrogen.

With this process photoresist stencils could be produced which had edge uniformities as good as the master mask. Figure 3.10 shows a comparison of photoresist stencils versus the chromium master. Once proper exposure and development times had been determined, yield on substrates having 4-6 good bridges (out of a possible 6) was ~60%; almost all substrates had at least one useable bridge stencil. The yield could probably have been considerably improved with use of a better regulated and more

Figure 3.10 Comparison of the photoresist stencil with the chromium master mask. Distances between adjacent channels are 120μ . Lengths of the channels in upper photographs are (from left to right) 8μ (off the photo), 75μ , 1μ , 250μ , 3μ , and 25μ (off the photo). The bottom photographs show closeups of the 3μ and 25μ channels.

sophisticated lamp-power-supply arrangement.

Optical examination of contamination, stencils, and masks was done on a Reichert Zetopan microscope equipped with polarization, dark field and phase contrast options - all of which were very convenient and used quite frequently in this work.

Undesired remnants of the photoresist could be removed from the substrate by swabbing them with acetone. This was a convenient way of providing for long electrodes to which contacts could later be made.

Evaporation and Stripping:

Following completion of the photoresist masks vapor deposition of the superconductor was done. Thicknesses ranged from 400-1800^oÅ and materials used were Al, Sn and In. (Some evaporation and film details are described in section 3.5.4)

The metal-clad photoresist was then stripped off by spraying (Sprayon Jet-Pak) the substrates for ~15 seconds with acetone. Figure 3.11 shows some scanning electron microscope closeups of sections of finished devices. The edge uniformity was limited only by the uniformity of the master mask.

Final Contacts:

No provisions were made in the chromium original for mutual electrical isolation of the bridges. This was easily accomplished after evaporation and stripping by scribing lines via a rigid razor blade-micromanipulator setup. Positioning of the blade was greatly aided by painting the blade black near the cutting edge. With the rigid arrangement the blade could be positioned to ~2μ,

Figure 3.11 Scanning electron micrographs of sections of finished bridges.

scribed lines were typically 5-10 μ wide; the cutting angle used was $\sim$ 5 degrees.

Contact to the superconducting film was made via indium strips soldered (with a rubbing motion) near the edges of the substrates. This was done while the sapphire substrates were heated on a hotplate. (Hotplate heating of glass substrates was unnecessary because of their relatively poor thermal conductivity.) Indium contact pads were then cut, with an X-acto knife, from the indium strips. Any defects in the indium to metal film contact were patched via a relatively good adhering silver paint (GC Electronics No. 21-1/Walsco No. 36). Copper leads were then soldered onto the indium pads. These leads terminated onto a 12 pin Augat plug (integrated circuit) and 4 "relia tac" connectors, thus providing for four independent four-point measurements. Electrical continuity of the final devices was checked at room temperature via a 0.25 μ A current.

3.5.4 Evaporation and Film Structure

Conventional resistive heating apparatus and procedures were used for evaporations⁽⁹²⁾. Pressures used ranged from 5×10^{-7} mm to 2×10^{-5} mm, deposited thicknesses ranged from 400 to 2000 $\text{\AA}$, evaporation rates ranged from $\sim 2 \text{\AA}/\text{sec}$ to $\sim 100 \text{\AA}/\text{sec}$, and the materials used were 99.999% pure Al, In and Sn (though the measurements reported in chapters 5 and 6 were made for Sn only.) The aluminum was cleaned prior to evaporation via a 30 second dip in 75% phosphoric, 25% nitric acid solution at 80°C. The indium was cleaned via a 1-2 minute wash in concentrated HCl (at room temperature) and the tin was cleaned in hot concentrated HCl for

10-15 minutes.

During evaporation film thicknesses and growth rates were coarsely monitored and controlled via a quartz crystal oscillator controller. For precise post evaporation thickness measurements an optical interferometer (Watson) was used in conjunction with a precision microdensitometer. Photomicrographs of the interference patterns were taken with Polaroid 46L transparency film, these transparencies were then scanned via a Joyce, Loeb1 & Co. Ltd. Model MK III C microdensitometer, which converted the interferogram into an amplified analog recording. Using this technique film thickness could be measured to an accuracy of $\pm 20\text{\AA}$.

The photoresist process described in previous pages was developed first using aluminum as the evaporant, at which time no problems were encountered during evaporation or stripping. However when tin film bridges were initially made in the same manner certain peculiarities arose. In particular:

- (1) In some cases, the short bridges were electrically continuous, whereas the longer bridges were not, even though under an optical microscope all bridges looked faultless.
- (2) In certain instances, immediately after evaporation a tin film was electrically continuous, but two or three months later (having been stored in air at room temperature) the same films were not electrically continuous.

In order to gain insight into these problems electron microscopic examinations of the tin films were done. Figures 3.12(a) and (b) show scanning electron micrographs of a $1640\text{\AA}$ tin film

which demonstrates the cause of (1). Photo (a) shows an 8μ long bridge. Note the large (relative to bridge width) grain sizes and the channel-like holes between some adjacent grains. This bridge was electrically continuous. Figure 3.12(b) shows a segment (at greater magnification than (a)) of a 75μ bridge on the same substrate. Note the two $\sim 540\text{\AA}$ wide channels which transverse the bridge and consequently render it discontinuous. For the long bridges the probability of a destructive channel transverseing the bridge is greater than that for a short bridge.

Figure 3.13 shows an electron micrograph (not SEM) of a different film $1250\text{\AA}$ thick evaporated under similar conditions and stored in air at room temperature for 3 months. Though this film was electrically continuous after evaporation it was discontinuous 3 months later, when the photographs were taken. What is observed are large grain sizes, $\sim 0.25\text{--}1.0\mu$ across and a complicated labyrinth structure of channel-like holes. Because the phenomenon observed is a function of time it seems probable that the effect is the result of migration of atoms driven by forces due to strains rendered during evaporation. Man'kovskii^Y et al.⁽⁹³⁾ have reported film structures similar to Fig. 3.13. This "labyrinth" structure occurs for tin films of thickness near the critical thickness (that thickness for which the film just becomes continuous). They also report a grain size increase and critical thickness increase as the substrate condensation temperature is increased. In our early evaporation setup the substrates were supported only by glass, hence considerable heating of the substrates

Figure 3.12 Bridge appearance due to evaporation onto heated substrates.
(a) 8μ long bridge @ 9900x . (b) section of 75μ bridge
@ 49,000x displaying "breaks".

(a)

(b)

X 5920 **X 14,800** **X 29,600** **X 74,000**

Figure 3.13 Tin film structure for thickness close to the critical thickness.

could have ensued during evaporation, thus resulting in a relatively large critical thickness.

Subsequent tin evaporation tests were done with a cooled copper substrate holder, the temperature of which was monitored via a copper-constantan thermocouple. Figures 3.14 and 3.15 show the tin film structures for films of approximately equal thicknesses ($695\text{--}780\text{\AA}$) deposited at -140°C , -50°C and $+25^{\circ}\text{C}$. The evaporation parameters for these films are summarized in the table of Figure 3.16. Electron diffraction patterns for all samples were identical and indeed identified the deposited material as pure tin. Examination of these electron micrographs reveals at least three results: (i) grain sizes increase as substrate temperature is increased; (ii) the presence of channel-like holes increases as substrate temperature is increased (indicating that the critical thickness increases as substrate temperature is increased); and (iii) the presence of channel-like holes increases for slower growth rates (critical thickness increases for slower growth rates). These observations are in agreement with Man'kovskii^y et al.

Hence in fabricating tin thin film bridges a degree of caution must be observed to ensure that they display electrical continuity. It is our experience that for best stripping of the photoresist mask a slight heating of the substrates is favorable (rather than cooling the substrate, which is favorable for "good" films). Alternatively, we believe the best solution is to use a

Figure 3.14 Tin films evaporated at various pressures and substrate temperatures (cf. Fig. 3.16). Magnifications are 12,000x and 120,000x respectively for the left and right side photos.

Figure 3.15 Continuation of Fig. 3.14 (cf. Fig. 3.16).

<u>Temperature</u>	<u>Sample No.</u>	<u>Thickness ($\text{\AA}$)</u>	<u>Rate ($\text{\AA}$)</u>	<u>Pressure (mm)</u>
-140°C	142	700	3.3	5×10^{-7}
	137	735	36.0	5×10^{-7}
- 50°C	111	700	32	1×10^{-6}
	138	750	104	1×10^{-6}
Room T.	110	694	3.2	4×10^{-7}
	113	780	36.0	7×10^{-7}

Figure 3.16 Evaporation parameters for the tin films shown in Figs. 3.14 & 3.15.

thicker photoresist film than prescribed in 3.5.3 (resolution of the positive resist does not degrade with increasing thickness) accompanied by a room temperature condensation temperature, using an evaporation rate $\sim 20\text{\AA}/\text{sec}$.

3.5.5 Summary

With a minimum of apparatus and expense we have succeeded in making $\sim 1\mu$ wide lines, of good uniformity, with lengths 1-250 μ . The technique was demonstrated to work well for Sn, Al and In; however in the experiments (cf. chapters 5 and 6) data was taken on Sn samples only. For 1μ wide tin samples the one-dimensional criteria of section 3.1 is met for $T_c - T < 20\text{mK}$, using pure bulk tin formulae for $\lambda(T)$ (a conservative estimate). Hence these samples provided a reasonable range of experimental accessibility to the one-dimensional regime.

Combination of a photoresist technique with electron beam exposure has been demonstrated by N. Broers.⁽⁹⁴⁾ This technique provides the ultimate in microminiaturization. It would appear then that should superconducting Josephson devices ever become of significant practical technological importance, minute thin film bridges would be the simplest and most economical to fabricate. Such potential points to the need then for a thorough physical understanding of the devices themselves.

4. EXPERIMENTAL APPARATUS AND TECHNIQUE

4.1 Introduction

From chapter 2 one can ascertain that very much of the physics of superconducting weak links is accessible for study through detailed analyses of their d.c. current-voltage (I-V) characteristics and resistive transitions. The experimental objective of this research was to exploit this fact in order to study fluctuations in superconducting weak links and to investigate sources of deviation between non-ideal (but quite useful) superconducting weak links which display the Josephson effects (such as point contacts and small thin film bridges) and the theoretical "ideal" Josephson junction (which is approximated quite well by the S-I-S tunnel junction). These studies involve careful measurements of such subtleties as the fluctuation rounding of structure in the I-V characteristics, and small quantitative differences between qualitatively similar data. With these thoughts in mind an apparatus for measuring current-voltage (or derivative of voltage with respect to current) characteristics and/or resistive transitions was constructed with special care taken to insure a high degree of isolation of the superconducting weak links from extraneous electromagnetic and thermal interference.

Because intrinsic fluctuation effects are quite small for large (relative to $k_B T$) coupling energies it is most convenient to make measurements for temperatures near T_c (critical temperature) where intrinsic fluctuation effects are enhanced (due to relative

smaller coupling energies). Hence one of the foremost requirements of the experimental apparatus was for it to have sufficiently good temperature control to insure reproducible results for temperature close to the critical temperature. A second major requirement was to have a means of beaming electromagnetic radiation onto the weak links without disturbing temperature control. It was desirable to vary the frequency of this radiation considerably, in order for example to vary the voltage at which radiation-induced steps appear in the I-V characteristics, hence a broadband transmission was implied. A third important requirement was for the various measuring circuits to be effectively shielded and grounded in order to reduce extraneous noise down to the intrinsic limitation imposed by the voltage measuring instrument.

4.2 General Construction, Shielding and Filtering

As implied above, the major components of the apparatus were a temperature measurement and control circuit, an electromagnetic wave generation and coupling circuit, and a current-voltage measurement circuit. In designing the apparatus these three primary systems were kept mutually isolated as far as was practical.

In Figure 4.1 is shown a diagram of that portion of the apparatus or "hangdown" which gets immersed in liquid helium. All materials used were free of ferromagnetic impurities. All tubing to room temperature was 304 stainless steel. Four copper radiation

Figure 4.1 The low temperature apparatus.

shields were spaced at 4" intervals along the tubes. The two 3/8" OD 20 mil (wall) tubes provided the support for a brass plate to which everything else was attached. Twisted pairs of current-voltage leads and thermometer and heater leads threaded the center of separate 1/8" OD s.s. tubes, the tubes providing shielding and mechanical protection.

The two remaining 1/8" OD s.s. tubes were control rods for the two miniature Grayhill switches (model 8A36-02-2-5N) which were used on the I-V leads and enabled running several samples at once with only a single pair of I-V leads going to room temperature. This was desirable in order to reduce the heat leak and external electrical noise. The switches had to be thoroughly cleaned of their (freezable) lubrication and bearing prior to first use at low temperatures, but once cleaned they operated well if free of moisture (or other fluids) when cooled.

A box adjacent to the switches contained low temperature low pass filters which could be switched in or out of the I-V circuit. A schematic of the filter arrangement and components is shown in Figure 4.2(a). The cutoff frequencies for the I and V lead filters were ~300Hz and ~10kHz respectively. It was necessary to use inductors in the voltage lead filtering because the d.c. electronic voltmeter (described later) prefers to look into a low impedance. The capacitance of the Cornell Dubilier mylar capacitors dropped to ~90% of their room temperature value when immersed in liquid nitrogen. A few other types of capacitors tested showed less degradation at low temperatures but for the capacitance values desired were either unavailable

Figure 4.2 (a) The low temperature low pass filters. (b) Details of the coaxial seals (epoxy--Stycast 2850FT or 2850GT base mixtures).

or physically too large. The leads, switches, and filtering were enclosed entirely within copper shielding.

Suspended beneath the brass plate were two vacuum tight cans which provided thermal isolation as well as additional shielding for the sample, which was positioned inside the inner can. Vacuum seals for the (removable) cans were provided by indium O-ring seals made by squeezing 30 mil diameter indium wire between two 45° V-grooves (one on each flange) whose total volume was 70% of that of the indium wire. The inner can (made of copper) could be pumped out, filled with helium exchange gas, and sealed off via a 1/8" OD copper pinchoff tube. The outer can, made of brass, could be pumped out via the 3/8" s.s. tube leading to the top of the apparatus.

The central 3/8" s.s. tube provided a channel and shielding for a 0.141" OD 50Ω semi-rigid coaxial line (Uniform Tubes Corp.) whose function was to transmit r.f. or microwave radiation to the inner can. Holes in the 3/8" tube allowed the coax to be heat sunk to the helium bath. Both conductors of the coax were stainless steel (rather than a metal of higher electrical and thermal conductivity) consequently minimizing this source of heat leak at the sacrifice of a greater r.f. attenuation. Because of the high sensitivity of superconducting weak links to radiation this latter fact was not a significant disadvantage in these experiments.

Because the coax itself was not leaktight, and to prevent extrusion of the teflon dielectric due to differential thermal

contraction, the ends of the coax were sealed with epoxy also. Details of the bottom coax seals are shown in Figure 4.2(L). All leads into the cans were sealed via (Emerson and Cummings) Stycast 2850 FT epoxy feedthrus.

The sample holder was machined from OFHC copper and designed to be removable. It was clamped into a rigid position in either of two orientations (90° apart) when the two halves of the inner can were fastened together. The sample itself was electrically insulated from the sample holder by 2 mil thick mylar foil, but good thermal contact was provided by a thin coating of Apiezon M grease.

A thermometer, also coated with M grease, was fit snugly into a hole in the sample block 0.1" (2540 μ) directly below the sample. This provided for rapid attainment of thermal equilibrium between the sample and thermometer, and shielded the thermometer from electromagnetic radiation incident upon the sample, since at the frequencies used, 5MHz - 10GHz, the block had a skin depth $\leq 12\mu - 0.3\mu$.⁽⁹⁵⁾ The thermometer leads were enclosed in a double layer of copper braid, so that there was a continuous shield between the thermometer and sample, the purpose of which was to reduce any coupling between thermometer and sample. The heater coil (used for temperature control - described later) was wound non-inductively around the outside of the inner can, again well shielded from the sample. That the thermometer and heater shielding was sufficient was born out by the fact that careful comparisons of noise rounding data with the thermometer and heater separately on and off showed no differences.

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During an experiment the "hangdown" was inserted into a 4" ID glass liquid helium dewar (capacity ~6 liters) which fit into a 7" ID glass liquid nitrogen dewar. The dewars were surrounded by three concentric cylindrical mu-metal shields which provided shielding from the earth's magnetic field and 60 cycle pickup. The helium level (obstructed from view) was indicated by monitoring the resistance of three 330 Ω 1/10 watt Allen-Bradley carbon resistors positioned at various depths along the hangdown. The hangdown length (~36"), the dewars, and the mu-metal shields were designed so that the samples were positioned in the best shielded region approximately 1/3 way up from the bottom of the shields. The d.c. magnetic field at this point was $\leq 10^{-5}$ G after degaussing the inner shield. For further shielding of low frequencies a lead foil bag was put over the bottom of the apparatus before each run. (Flux trapping was not troublesome for these experiments because of the small cross sectional areas of the weak links used.) The entire apparatus with accompanying instrumentation was placed inside a double walled copper screened room which effectively shielded fields above the threshold frequency, ~100KHz, with ~100db attenuation. In addition, the dewars and mu-metal shields were enclosed in copper screening for further redundancy. With the system described experimental run times readily exceeded 24 hours with a single helium transfer.

4.3 I-V Circuit

Direct current was supplied by either of two high impedance current sources, depicted schematically in Figure 4.3. High source

Figure 4.3 The current sources. (a) Earlier version. (b) Low noise version.

impedances insured that noise currents due to the current supplies were insignificant relative to the intrinsic noise currents in the superconducting links, i.e. $(k_B T/R)_{\text{source}} \ll (k_B T/R_N)_{\text{weak link}}$ where R_N is the normal state resistance of the weak link.

The supply in Figure 4.3(a) was powered by a shielded 12 volt storage battery and had a current range of 0-1 amp with high output impedance for the upper ranges maintained by a transistor output. For this supply the current was monitored by a Keithley 602 electrometer, the output of which could be used to drive the current axis of an X-Y recorder.

The supply in Figure 4.3(b) was powered by a mercury battery and has a range of 0-13.5 mA. For this supply there was no electronic monitoring of the current, thus eliminating a possible source of external noise; also, the potentiometer had 25 turns (rather than the conventional 10) in order to give finer current resolution during a sweep. The current was monitored by mechanically monitoring the number of turns of the 25 turn potentiometer. A plastic non-slip pulley coupled the shaft of the 25 turn potentiometer with another (electrically separate) potentiometer which was used to drive the I-axis of an X-Y recorder. The monitor circuit contained an offset, variable from 0-100%, which was very convenient for offsetting in order to amplify structure at any place along an I-V curve. A calibration of monitor output versus supply current was done prior to each run.

The current potentiometers (both sources) were swept using a reversible 6-speed motor having a speed range of 0.47 - 15rpm. Use of a potentiometer, instead of a switch, to turn the motor on greatly eliminated annoying switching transients.

Figure 4.4 is a schematic of the entire I-V measurement circuit, with details of the switch arrangement shown. Single twisted pairs of #34 copper I-leads and mu-metal shielded (for reduction of 60 cycle pickup) #30 copper V-leads from room temperature (top of "hangdown") terminated onto the two 2-pole, 2-deck, 5 position/deck switches in liquid helium. For the wiring arrangement shown four terminal measurements could be done on 4 samples, with filters (cf. Fig. 4.2(a)) either in or out. To prevent possible blowout of the samples due to large transient currents caution had to be taken to discharge filter capacitors before switching from sample to sample.

To help reduce the heat leak into the inner can 0.005" diameter brass wires, ~12" long, were segmented into the 16 I-V leads which connected the switches to the samples. Inside the inner can 12 of the I-V leads terminated onto a 12 pin Augat integrated circuit plug, the other 4 I-V leads terminated on Amphenol "relia tac" connectors. (In cases where only a single sample, e.g. a point contact, was run the relia tac connectors were used whereas for multiple samples all leads were used.)

Voltages were measured with a Keithley 148 nanovoltmeter. Optimum performance of this instrument on the most sensitive (10nV f.s.) scale requires voltage lead resistance be kept $\lesssim 10\Omega$. This criterion was met both with filters in and out of

Figure 4.4 Schematic of the I-V measurement circuit with the switch arrangement displayed.

the circuit. To prevent offset voltages due to thermoelectric effects the voltage leads were continuous copper (no room temperature solder joints) except for a low thermal solder joint to the nanovoltmeter plug. Typical resulting offset voltages were $\leq 50\text{nV}$.

The entire I-V circuit was grounded at one point only, the low input terminal of the nanovoltmeter. This eliminated ground loops, to which small samples are quite sensitive. With this arrangement no differences could be observed between a.c. or battery operation of the nanovoltmeter.

The output of the nanovoltmeter was used to drive the V-axis of a Hewlett Packard 7000A X-Y recorder. The Keithley 148 output has an 8 Hz component which is bothersome on the most sensitive ranges. This 8 Hz noise was eliminated by using the low pass Butterworth mode of a Kron-Hite 3322 filter where in most cases the cutoff was set at 5.4 Hz.

Maximum sweep speed for an I-V trace was limited by the bandwidth of the nanovoltmeter ($\sim$ few Hz for small deflections) and the Kron-Hite filter. Too fast a sweep was indicated in the I-V trace by hysteresis, which could be eliminated simply by slowing the sweep speed. Typically a full scale I-V trace (positive voltages only) took a total time of 2 minutes and 30 seconds to trace both up (increasing current) and down (decreasing current). All I-V's were traced both up and down, in order to check for drifts. Thermal equilibrium and good temperature control are the important factors in eliminating drifts. Typically, drifts in the voltage could be maintained to be

$\lesssim \ln V$ in a 2 minute interval. The differential accuracy of the nanovoltmeter was measured to be $\sim 0.1\%$ (by comparison of spacings of very steep r.f. induced steps, known to be equally spaced⁽³⁾). This accuracy was comparable to the differential accuracy of the current measurement. To assure that the absolute voltage accuracy ($\sim 1\%$) was not degraded by the Kron-Hite filter, this filter was calibrated prior to a run by comparison of I-V's taken with and without the filter in place.

In a few isolated instances the I-V circuit was modified. Figure 4.5(a) shows a modified circuit to measure dV/dI vs I. For one experiment involving looking for low frequency induced steps in one-dimensional whisker crystals, a circuit and apparatus employing a superconducting quantum interferometer to measure voltages was employed. Figure 4.5(b) shows a schematic of this circuit - full details of the apparatus are described by Lukens, Warburton and Webb⁽⁹⁶⁾.

4.4 Temperature Measurement and Control

Motivation for measuring and controlling the sample temperatures close to T_C was due to several factors. For one, fluctuation effects are enhanced (and hence more susceptible to measurement) near T_C . This can be seen, for example, by looking at the predicted form of the dynamic resistance, R_C , at the origin (zero current and voltage) in the d.c. Josephson effect.

$$R_C = \frac{R}{I_0^2 (\gamma/2)} \quad \text{where} \quad \gamma \equiv \frac{\hbar I_1(T)}{ek_B T}$$

Figure 4.5 (a) dV/dI vs. I measurement circuit. (b) Schematic of use of SQUID to measure a whisker I - V .

is the ratio of the coupling energy of the junction to the thermal energy $k_B T$ (cf. chapter 2 for rest of notation). For large γ (equivalent to a large barrier for fluctuations) $I_0(\gamma/2) \sim e^{\gamma/2}$ hence for large $I_1(T)$, R_C becomes very small, i.e. the I-V curve at the onset is very sharp. Since near T_C (for a perfect tunnel junction) ⁽²⁰⁾

$$I_1(T) = \frac{\pi}{4eR} \frac{\Delta^2(T)}{k_B T_C} \approx (6.3 \times 10^{-4}) \frac{(T_C - T)}{R}, \quad \{\text{MKS}\}$$

γ can be reduced by making $T_C - T$ small. Appreciable tilting of the onset should be observed for example if $R/R_C = 2$, which implies that $\gamma = 3.1$ and $T_C - T \approx 0.25\text{mK}$ for a 0.63Ω tunnel junction.

Another reason for controlling close to T_C was that it was desirable to maintain a "one dimensional" regime, for the purposes of comparison with reasonably tractable theories. By "one dimensional" it is meant that widths and thicknesses of the weak links studied are $\lesssim \lambda(T)$, $\xi(T)$. For typical type I superconductors this regime requires that $T_C - T$ be of the order of a few millidegrees.

A third reason for controlling near T_C was for the purpose of studying resistive transitions of the weak links. The transition widths of pure single crystal tin whiskers have been shown by Warburton and Webb ⁽⁹⁷⁾ to be as small as 10^{-4}K . In the course of this research it was desired to have the capability of studying these structures under the influence of r.f. radiation.

Figure 4.6 shows a schematic of the temperature measurement and control circuit. Temperature was measured by sensing the

Figure 4.6 Schematic of the temperature measurement and control circuit.

resistance of a germanium resistance thermometer (Solitron) which was measured by an a.c. Wheatstone bridge employing phase sensitive detection with a P.A.R. HR-8 lock-in amplifier with type C preamp. The standard resistors were $1.5K \pm 0.1\%$ and were chosen to approximately match the resistance of the thermometer at 4.2K. A Dekapacitor variable capacitor was used to balance out any phase shifts due to stray lead capacitance and transformer phase shift. An operating frequency of 1000Hz was chosen in order for operation to be at the minimum of the noise contour for the type C preamp for thermometer resistances corresponding to temperatures between 3.7K ($\sim T_c$ for Sn) and 4.2K. So that changes in lead resistances due to the constant drop of the helium level balanced each other out, three leads (rather than two) connected the thermometer to room temperature circuitry (shown in Fig. 4.6). The thermometer was calibrated against a standard calibrated thermometer attached, in place of the sample, to the sample block. The subsequent differential resolution was $\sim 1-2\mu K$ and at 3.8K the sensitivity was $1mK/\Omega \pm 1.7\%$. The thermometer was calibrated and operated at a power dissipation level of $\sim 1-2 \times 10^{-7} W$. The absolute accuracy of the thermometer was $\pm 5mK$.

Control of the temperature of the helium bath (outside the outer can) to 0.1mK was maintained by controlling the helium vapor pressure via a manostat connected to a 30 gallon pressure ballast tank. The helium was pumped below the γ -point in order to exploit the higher thermal conductivity of superfluid helium,

hence assuring more rapid approach to thermal equilibrium when temperature changes were made. The ballast was necessary to prevent drifts due to temperature changes in the outer chamber of the manostat. With ballast in place drift was less than $40\mu\text{K}$ over a 20 minute period.

The temperature of the inner can (sample chamber) was raised above the temperature of the outside bath by power dissipated in a non-inductively wound 700Ω evanohm heater coil wrapped around the top of the inner can. Thus the temperature was controlled by balancing the heat loss to the helium bath (cold leak) against the heat input from the heater.

The copper inner can was isolated from the helium bath heat sink by suspending it in vacuum ($\sim 10^{-5}\text{mm}$) via the coax and three $5/16"$ long 4-40 nylon screws. With this design the major source of "cold leak" was the coax, and was thus localized to the center of the top of the symmetrical inner can. With outside bath at $\sim 1.8\text{K}$ and sample chamber at 3.9K this cold leak was ~ 0.35 milliwatts. Typical power levels of the radiation coming down the coax were $\leq 0.05\text{mW}$, hence much less than the cold leak, and could be compensated for by reducing the bias current to the heater. Thermal equilibrium within the inner can was maintained by helium exchange gas which was sealed in at room temperature via a $1/8"$ OD copper pinchoff tube. (Lack of thermal equilibrium is evidenced by presence of hysteresis in I-V's or resistive transitions.) The exchange gas pressure at room temperature was 100-200mm. Using a low temperature specific heat value of $10^{-4}\text{J/gm K-degree}^{(98)}$ and a low temperature thermal conductivity value

of $4\text{W/cm K-degree}^{(99)}$ the thermal time constant ($R_T C$ where $R_T \equiv$ thermal resistance, $C \equiv$ heat capacity) of the copper inner can (mass $\sim 370\text{gm}$) was estimated to be ~ 0.026 sec.

Control of the inner can temperature was accomplished through negative feedback of the a.c. bridge offset (from null) to the heater power supply. Feedback from the bridge was added to the heater bias current via a Nexus SA-2 operational amplifier, the output of which was amplified by a Nexus BL-100 booster. The op. amps were battery powered in order to avoid the introduction of line frequency noise. With this supply the maximum power which could be supplied was $\sim 0.2\text{W}$ which was sufficient to reach temperatures in excess of 20K . The minimum temperature which could be attained (limited by dewars, apparatus, and pumps) was $\sim 1.5\text{K}$.

The response of the electronic circuitry was set by the time constant of the HR-8. Optimum control was obtained using a time constant of 0.03 sec. Faster times than this had no effect since the response was in such cases limited by the thermal time constant. Slower time constants gradually degraded control. The bridge was operated with enough sensitivity to be able to detect changes in temperature as small as $\sim 1\mu\text{K}$. The actual temperature was controlled (using a 0.03 sec. t.c.) to $\leq 5\text{--}10\mu\text{K}$ over periods of an hour.

Operation of the bridge and control system was simple; changes in temperature could be effected quickly and efficiently. The usual procedures used for large ($> 0.15\text{K}$) desired temperature changes was (1) first, without feedback, the bridge resistance

was set to the desired value (corresponding to the desired temperature) using a low sensitivity scale on the HR-8, (2) second, heater bias was varied until the bridge was balanced as the sensitivity was successively increased to the final desired setting, (3) third, feedback was connected and any phase shift was balanced out with the variable capacitor. For small ($\leq 50\text{mK}$) changes in temperature all one need do was change the bridge to the desired resistance and adjust the heater bias slightly.

4.5 Microwave Circuits

One of the objectives of these experiments was to study the interaction of external radiation with the a.c. Josephson current over a wide range of frequencies. Thus in order not to be restricted to a narrow band of frequencies (and a correspondingly narrow band of voltages, via $h\nu = 2eV$) 50Ω coaxial lines and components were used for transmission instead of waveguide.

A system was assembled to enable transmission over a 0-10GHz bandwidth. The upper frequency limit was imposed by the coupling factor (between coax and sample) and by the attenuation ($\geq 1\text{db/ft}$ @ 10GHz) of the 0.141" OD all stainless steel semirigid coax used to transmit the energy from the top of the apparatus to the sample chamber. For study of the driven Josephson effect a practical lower frequency limit of 500KHz was set by the $\sim 1\text{nV}$ resolution of the nanovoltmeter ($1\mu\text{V} \leftrightarrow 483.6\text{ MHz via } h\nu$ ev

In most cases the radiation was coupled to the sample via

a $3/4"$ x $1/2"$ loop spaced $1/4"$ above the sample. Orientation of the sample was such as to provide that the r.f. electric field was approximately parallel to the supercurrent direction (see Figure 4.7(a)). Estimates of the voltage coupling efficiency could be obtained from measurements of the r.f. power dependence of step amplitudes in the driven Josephson effect. This efficiency varied greatly with frequency and with how well the total line was tuned, or matched, to the generator, but approximately was in the range 10^{-2} - 10^{-4} for most measurements. This coupling could have been improved by using a cavity but again this implies very narrow bandwidth. (The method used for optimizing the coupling is described below.) In some cases for the lower frequencies used (≤ 10 MHz) the a.c. was fed directly down the d.c. current leads in order to improve coupling at these frequencies - see Figure 4.7(b).

Figure 4.8(a) is a composite schematic which shows the basic r.f. circuit. At times slight and insignificant alterations of this circuit were made according to the frequency being used or the apparatus available. Figures 4.8(b) and (c) show circuits which were used for demonstrating mixing and detection.

Throughout the course of the experiments several standard continuous wave generators (50Ω output impedance) were used, varying in frequency from 5 MHz - 10 GHz. For one isolated experiment involving use of a superconducting femtovoltmeter to search for steps in whiskers, low frequencies were used, 1-100 KHz. Typical residual f.m. noise at 3.4 GHz was 20 KHz as measured with a

(a) Standard Coupling Loop

(b) Low Frequency Coupling

Figure 4.7 Methods of coupling r.f. into the sample.

(a) BASIC CIRCUIT

(b) MIXING CIRCUIT

(c) DETECTION CIRCUIT

Figure 4.8 Microwave circuits used.

Tektronix 1L20 spectrum analyzer. For a Josephson device this frequency spread corresponds to a d.c. voltage spread of ~ 40 picovolts, hence well below the resolution of the Keithley 148 nanovoltmeter used for almost all d.c. voltage measurements. A further discussion of the effects of generator noise is contained in chapter 5.

Maximum output powers of the generators were in the range 10-100mW and were quite sufficient due to the high sensitivity of the weak links to radiation. As can be seen from Figure 5.8(a) power sent to the sample holder was monitored by three means, an accurate (to 0.4%) continuously variable attenuator, a Boonton microwattmeter, and a diode detector. This seeming redundancy was provided because the r.f. power measurement was the least accurate of the measurements performed. Prior to an experiment the diode detector was calibrated, at several frequencies, every one or two db over the 60db range of the attenuator. Calibration was done with a matched 50Ω system using the microwattmeter to provide a measure of absolute power. The resulting 50-60 points were then fit, via computer, to a polynomial of degree 12 using a least squares fit; this polynomial was then used as the calibration curve for the detector. The net result was that relative power resolution was better than 0.1db. Though the diode was calibrated absolutely it could only be used for relative measurements in the actual circuit because the mismatch at the antenna loop implied a reflection; the diode measures power propagation in both directions.

A stub stretcher tuner, placed between the diode detector

and the sample holder coax, was used to optimize the match between the 50 Ω line and the antenna coupling loop. Minimum reflection was desirable so that for a given generator the maximum range of power could be applied to the device. Because it was desired to match only roughly over a very wide range of admittances, a stub stretcher tuner was chosen instead of a more precise tuner. It was found that the best and most convenient method of tuning was to use the superconducting weak link itself as a monitor - in short, the stub stretcher was tuned to give maximum depression of the zero voltage current (for a fixed generator power small enough so as to be on the first lobe of the zero voltage current vs. r.f. power curve; c.f. chapter 6). It was found that tuning changed slightly with the variable attenuator setting, but that this change was insignificant at the frequencies used. In cases where a tuner was unavailable the line could be tuned roughly by varying the generator frequency until a maximum I_1 depression was observed. (For example this was done for one of the generators in the mixing experiment, the tuner being used for the other generator, cf. Fig. 4.8(b)) Though the actual coupling factor was not very important for these experiments, it was quite important that this coupling be constant for any particular set of measurements involving r.f. power as a parameter, hence it was necessary for all connections to be tight and rigid - for this reason type N connectors were used on all components.

In certain cases where a direct external measurement of frequency was desired the Tektronix spectrum analyzer was used

in place of the power meter (cf. Fig 4.8(a)). However, in practice it was much more convenient and accurate to measure frequency using the separation of induced steps in the weak link I-V's, via $v = 2eV_n/nh$. With the latter method frequency could be determined quite simply to an accuracy of $\sim 0.1\%$ (the differential accuracy of the nanovoltmeter).

Most measurements were performed with such relatively small incident r.f. powers, typically a few microwatts, versus a typical heater bias of 0.35 milliwatts, that additional heating due to the r.f. was negligible. However in a few rare instances where relatively large r.f. powers were used slight heating could be observed. In the worst cases this was $\sim 2\text{mK}$ and could easily be compensated for by lowering the heater bias. This compensation is correct only if no temperature gradients exist between sample and thermometer. If the germanium thermometer were not well shielded it might be possible for it to heat up more than the sample. In such case, lowering the heater bias in order to bring the thermometer reading back up to its initial value (no r.f. value) would result in a sample temperature slightly lower than the thermometer reading, which would result in a larger zero voltage current, $I_1(T)$, for the weak link. Such a possibility was checked carefully utilizing particular situations where the r.f. power gave rise to heating but had little or no effect upon I_1 . (Such particular situations arose for example while studying the Dayem-Wyatt effect at frequencies much higher or much lower than the optimum, cf. chapter 6.) No disparities in the weak link I_1 's were observed and thus it was

concluded that the sample and thermometer were indeed always at the same temperature, i.e. the thermometer was sufficiently shielded from the r.f.

4.6 Examples of Raw Data

The raw data derived from the apparatus and samples consisted essentially of recorder tracings of the following combinations of d.c. current (I), d.c. voltage (V), and temperature (T)--with temperature, r.f. power (P) or d.c. current as parameters:

- (1) I vs. V; T-fixed; P-parameter
- (2) I vs. V; T-parameter; P=0
- (3) V vs. T; I and/or P-parameters
- (4) (a) dV/dI vs. I
(b) r.f. detection

Figure 4.9 is an example of data of the first type taken on a niobium point contact junction. The curves in the background are relatively low resolution d.c. I-V curves with incident r.f. power as the parameter varied (the zero of the V-axis is shifted for each successive curve for clarity). Gross parameters such as dynamic resistances and current-voltage coordinates of structure were obtained from these curves. Details of structure such as the steps shown in Fig. 4.9 could be further amplified by (1) stopping the sweep at the base of the structure, (2) increasing the gain of the nanovoltmeter and/or X-Y recorder, (3) making use of the d.c. offsets in the current monitor and nanovoltmeter to center the

Figure 4.9 Example of raw data: d.c. I-V curves of a point contact with applied r.f. power as parameter (horizontal zero point shifted for clarity).

amplified trace, and (4) slowing the motor sweep speed. In the figure the curves in the foreground are the amplified versions of the step indicated by the arrow. The curves were always traced in both direction (increasing and decreasing current) in order to check reproducibility and drift. (Thermal equilibrium was indicated by lack of hysteresis.)

Figure 4.10 is another example of the same type of data as Fig. 4.9 (this time for a tin thin film bridge, different frequency and temperature), but shows the ultimate noise limits obtained. The noise and drift are $\leq \text{InVp-p}$, which is the rated short circuit specification of the nanovoltmeter, hence no observable extra noise is being introduced into the trace of the I-V, temperature, or r.f. circuits.

Figure 4.11 is an example of data of the second type, I vs V; T-parameter, with no incident r.f. Again the V-axis zero is shifted for clarity and the sample is a point contact. Temperature increments for this data are $\sim 0.1\text{K}$. The time required to take the six curves was roughly 20 minutes.

Figure 4.12 is an example of a V vs. T curve with current held fixed (type (3) listed above). In this case the sample was a tin whisker. For this sort of data temperature was swept up and down by increasing and decreasing heater bias current.

Figure 4.13 is an example of an I-V plotted for comparison purposes along with the corresponding I vs dV/dI curve (type (4b) above). This sort of data provided another method of searching for structure, though it was not used often in the experiments described in chapters 5 and 6.

Figure 4.10 An example of data similar to Fig. 4.9, but for a tin film bridge, under optimum experimental noise conditions.

Figure 4.11 D.C. I - V 's of a point contact, with temperature as parameter. Full scale horizontal-- $11.75\mu V$, vertical-- $9.91\mu A$.

Resistive Transition of a Tin Whisker

$I = 1 \mu A$

Figure 4.12 An example of a resistive transition, or voltage vs. temperature (V-T) curve.

Figure 4.13 An I - V shown with the corresponding I vs. dV/dI curve. [Point Contact]

4.7 Summary

In summary, a low noise system with the following salient characteristics was assembled for very careful measurements of small level d.c. current-voltage characteristics and resistive transitions. The system employed precise temperature control in conjunction with the application of c.w. radio frequency and microwave radiation:

- (1) r.f. frequency range 0-10GHz
- (2) r.f. power range 0-100mW
- (3) d.c. current source range 0-500mA
- (4) d.c. voltage resolution 1nV
- (5) temperature range 1.5K - 20K
- (6) temperature control $\pm 5-10\mu K$
- (7) sample capacity 4 thin film bridges, switchable
or
4 whisker crystals
or
1 point contact
- (8) switchable low pass low temperature filters

5. RESULTS AND ANALYSIS - FLUCTUATIONS

5.1 Introduction

The preceding chapters have dealt with the theory and with the practical considerations related to making measurements on certain types of superconducting weak links.

The primary topic discussed in chapter 2 was the theoretical description of the effects of intrinsic thermal fluctuations on superconducting weak links. To reiterate, theorists have modeled this problem by forming a Fokker-Planck equation for the distribution function of the phase difference across the link in precise analogy to the problem of a particle subject to the buffeting of a Langevin force (Brownian motion), with the analogy of particle displacement being the superconducting phase difference. Thermal fluctuations were shown to lead to phase fluctuations. In the a.c. Josephson effect the phase fluctuates freely (no potential barriers) and leads to a finite linewidth of emitted radiation. In the d.c. and driven a.c. Josephson effects the phase fluctuations are subject to a tilted periodic potential imposed by coupling energies. A predicted observable manifestation of fluctuations in these effects is a finite width and rounding of "constant voltage steps". The theories take readily tractable forms only in the limit of uniform current distribution and negligible weak link capacitance. (However the finite capacitance cases have been attacked -- see chapter 2 for references.)

Chapters 3 and 4 described details of the weak link samples (point contact and thin film bridges) and the measurement technique. Because our micron-sized samples had intrinsically low capacitance and because their dimensions were less than $\lambda(T)$ and $\xi(T)$ (thus implying uniform current distribution), we expected that the theories could be experimentally tested. Initial d.c. effect results, as measured by Anderson and Goldman in superconducting tunnel junctions, suggested problems with external noise and non-negligible junction capacitance (comparison was made to the $C \rightarrow 0$ limit theory). Simmonds and Parker alleviated these problems in subsequent measurements on thin film bridges and obtained results in good agreement with the d.c. effect theory. However early measurements by Parker on the a.c. effect linewidth suggested a factor of 2 discrepancy with theory (experimental wider). We therefore undertook to explore the scope of the theory in the d.c. effect and in the driven a.c. effect, where the d.c. and a.c. effects meet.

In this chapter are reported the experimental results and analysis of careful measurements concerned exclusively with testing the appropriate versions of the theory for intrinsic fluctuations effects in the d.c. effect (Ambegaokar and Halperin -- abbreviated AH below), and in the driven a.c. effect (Stephen). Other results, such as the r.f. dependence of step amplitudes, etc. will be found in chapter 6. In the course of demonstrating that the observed effects were truly intrinsic to the weak links

themselves we have examined the effects of relatively low frequency externally applied noise.

Section 5.2 contains a reiteration of the essential physical ideas, along with the basic theoretical prediction and other considerations which are generally applicable to the rest of the chapter. The bulk of the chapter is contained in section 5.3 which presents the results on point contacts, and is essentially an expanded version of the previously reported results of Henkels and Webb.⁽¹⁰⁰⁾ These measurements were extended to thin film bridges made via the photoresist technique (cf. chapter 3). Results in these bridges and comparison with the point contact results are presented in section 5.4. Section 5.5 contains conclusions and a review of the current status of experimental-theoretical comparison for the Langevin treatment of intrinsic fluctuations in superconducting weak links. The appendix to this chapter provides some of the details of the computer fitting as done with APL.

Chronologically the first experiments of this sort which we did involved thin film bridges of wedge geometry. As was pointed out in chapter 3, these bridges displayed a small but annoying resistive background for small critical currents. Initial order of magnitude comparisons were made with Stephen's theory by subtracting off the resistive backgrounds. The results of this analysis were quite encouraging and paved the way for the subsequent experiments on point contacts and bridges of rectangular geometry.

5.2 General Considerations

The basic phenomena investigated in this chapter are the d.c. Josephson effect and the driven a.c. Josephson effect. Most simply stated the d.c. effect is simply the existence of a supercurrent through a "weak" interconnection between two "bulk" superconductors. The interconnection is weak in the sense that it supports a supercurrent even though a coherent phase difference may exist across it, whereas in the interior of bulk superconductors the phase is constant. In effect then the interconnection couples together the phases of the two bulk superconductors. Fluctuations of the phase difference across the interconnection are restricted by the coupling energy. The "weaker" the interconnection the smaller its critical current I_1 , the smaller the coupling energy $\frac{\hbar I_1}{2e}$, and consequently the greater the phase fluctuations. Figure 5.1 displays the d.c. effect for high and low critical currents, I_1 , for one of our point contacts. Note that for the smaller I_1 value (smaller coupling energy) the change from zero voltage to the normal state is much less sharp than for the larger I_1 (larger coupling energy).

The Josephson equations (chapter 2) show that when a finite d.c. voltage exists across a Josephson device (e.g. after the critical current I_1 has been exceeded) an a.c. supercurrent exists within the device. Hence a Josephson device biased above I_1 is essentially an oscillator. In the absence of external constraints the phase of the oscillator is free to fluctuate at the will of thermal fluctuations. However, if an external oscillator

Figure 5.1 Examples of the d.c. Josephson effect in a Nb point contact for large and small critical currents.

of frequency ν_{rf} is applied to the device (in the form of r.f. or microwave radiation for example) a degree of freedom is lost; the net result is a locking of the phase of the Josephson oscillator onto the phase of the external oscillator. Due to a mixing of the Josephson a.c. current and the externally induced a.c. current of frequency ν_{rf} , zero frequency sidebands ensue; these manifest themselves as steps of nearly constant voltage in the d.c. current-voltage characteristic of the device at integral multiples of the voltage $h\nu_{rf}/2e$. According to theory, intrinsic fluctuations in the quasiparticle and pair currents in a Josephson device degrade this phase locking, leading to observable tilting and rounding of the steps. However, similar to the d.c. effect, the phase fluctuations encounter an energy barrier (imposed by the external oscillator). This effective coupling energy is $\frac{\hbar j_2}{2e}$ where j_2 is half the total amplitude of a step. Since j_2 is directly related to and of the same order of magnitude as I_1 , (e.g. see eq. 5.4 below), then just as in the d.c. effect, enhancement of fluctuations should occur for relatively small critical currents. Figure 5.2(a) displays the so-called driven a.c. effect in a tin thin film bridge of wedge geometry for a relatively large critical current. The frequency of applied radiation here is $\sim 900\text{MHz}$ corresponding to a fundamental voltage $1.86\mu\text{V}$. The resultant d.c. I-V displays a sequence of equally spaced steps at multiples of this voltage. Note how sharp and vertical the steps are for this case. In case (b) is shown the same effect in a rectangular-geometry thin film bridge at 110MHz . In this case the critical current is much smaller than it was for case (a),

Figure 5.2 Examples of the driven a.c. Josephson effect in Sn thin film bridges for large and small critical currents.

and the steps are now much more rounded and tilted.

For these two effects the theoretical output is the d.c. I-V characteristic of the fluctuation-softened step structure (in the d.c. effect there is just one step, the "zero-voltage step"). In the sections that follow the experimental step profiles are compared in detail with the theoretical profiles. The theoretical prediction for both effects can be cast into the following useful form (cf. chapter 2 for details), which gives the reduced theoretical voltage as a function of the reduced current, for a negligible capacitance uniform current Josephson device.

$$v = \frac{\sinh \frac{\pi \alpha x}{2}}{\alpha} \left[\int_0^{\pi/2} \cosh(\alpha x y) I_0(\alpha \cos y) dy \right]^{-1} \quad (5.1(a))$$

I_0 here is a modified Bessel function, hence for practical purposes the integral is a double integral. The following table summarizes the corresponding notations of AH and Stephen with the notations of the above equation.

<u>Equation 5.1(a)</u>	<u>Ambegaokar & Halperin</u>	<u>Stephen</u>
v	$v \equiv \frac{\bar{V}}{I_1 R}$	$\frac{\Delta V}{j_2 R}$
x	$x \equiv \frac{I}{I_1}$	$\frac{\Delta I}{j_2} \quad (5.1(b))$
α	$\gamma \equiv \frac{\hbar I_1}{ek_B T}$	$2a_2 \equiv \frac{\hbar j_2}{ek_B T} \frac{V_0}{RJ}$

For the d.c. effect R is the normal state resistance of the Josephson device, I_1 is the temperature dependent critical current

in the absence of fluctuations, T is the device temperature, and $\bar{V}$ and I are respectively the d.c. voltage and current of the device. For the driven a.c. effect R is the dynamic resistance of the device (of section 5.3.2 for more details on R), $2j_2$ is the temperature dependent step amplitude in the absence of fluctuations, T is the device temperature, and ΔV and ΔI respectively express the voltage and current along the step measured relative to the step center. In addition J and V_0 are respectively the current and voltage at the step center. The crucial parameters here are the dimensionless parameters $\frac{\gamma}{2}$ and a_2 , the ratio of coupling energies to the thermal energy $k_B T$. These dimensionless parameters provide a measure of the barrier height in the potential seen by the phase, and hence determine the degree of degradation of structure in the device d.c. I-V characteristic.

At a step center (or the origin, in the d.c. effect) $x \rightarrow 0$ and eq. 5.1 reduces to the form,

$$\lim_{x \rightarrow 0} \frac{v}{x} = \frac{1}{I_0^2 \left(\frac{\alpha}{2} \right)} \quad (5.2(a))$$

or in terms of resistances,

$$R_c = \frac{R}{I_0^2 \left(\frac{\alpha}{2} \right)} \quad (5.2(b))$$

where R_c is the dynamic resistance at the step center (or at $V=0$, in the d.c. effect). At low temperatures a_2 can become very large and the step extremely sharp, due to the temperature dependence of j_2 and I_1 . For example, Clarke's experiments⁽¹⁰¹⁾ on h/e suggest values of $R_c \lesssim 10^{-14} \Omega$ for $a_2 \approx 300$. However, broader

step profiles that are more accessible to accurate measurement can be obtained by selecting temperature and r.f. power, (recall j_2 depends upon r.f. power as well as temperature, cf. chapters 2 and 6), such that a_2 is relatively small. This was the technique used in these experiments to obtain access to the regime in which intrinsic fluctuation effects could be investigated in detail.

5.3 Results on Point Contacts

5.3.1 Motivation; Range of Parameters

The initial calculations of Ambegaokar, Halperin and Stephen were done with superconducting tunnel junctions (S-I-S) in mind. Their relatively simple model included a capacitor, representing the capacitance of the parallel-plate type structure; a supercurrent term $I_1 \sin \phi$, representing the tunneling of Cooper pairs; and a resistor, representing normal single quasiparticle tunneling; and a random noise current source with a flat infinite spectrum, representing fluctuations in the quasiparticle and pair currents. The theoretical results derived from this model should remain valid for any device which can be approximated by a similar model.

In most general terms the model consists simply of a capacitive current, an oscillatory (with respect to phase) supercurrent, and a normal current. This model does not explicitly involve the mechanism whereby the supercurrent and normal currents are produced, for example tunneling in an S-I-S junction. Experimental

evidence suggests that such a model may indeed be a fairly reasonable approximation for such weak link structures as point contacts and short narrow thin film bridges (see chapters 2 and 6 for a somewhat more extensive discussion of models).

With these considerations in mind our first experiments involved investigation of the driven a.c. effect in thin film bridges of wedge geometry. Though the results were encouraging as mentioned previously, there appeared to be a small non-linear background resistance. It was hypothesized that this background was most probably due to an imperfect 4-point measurement due to normal regions adjacent to the actual weak link. (cf. chapter 3) It was desirable to investigate other types of weak links (1) in order to see if our hypothesis was correct and (2) in order to compare theoretical and experimental step profiles in detail.

We turned first to point contacts. These devices seemed particularly well suited to these experiments for several reasons: (1) they are intrinsically low capacitance devices and hence could be compared quantitatively with the tractable form of the theory, (2) the devices could be designed to be quite thermally and electrically stable, thus their characteristics would be reproducible over the course of many measurements, (3) the devices were readily available, (4) for the particular devices which we used the variation of critical current, I_1 , with temperature was sufficient to provide a reasonably wide range of γ 's and a_2 's (i.e. a wide range of fluctuation barrier height

and (5) the value for I_1 at 4.2K could be easily adjusted to a desired value, prior to locking the point in place.

Two point contacts were investigated in detail. The 4.2K I_1 values were set at $\sim 0.4\mu\text{A}$ and the normal state resistances were 1.46Ω and 1.275Ω at this temperature. In our d.c. effect measurements T was varied from 3.368K - 4.402K and controlled to $\pm 10\mu\text{K}$; the corresponding I_1 variation was $\sim 0.2\mu\text{A}$ - $4.0\mu\text{A}$. γ for these measurements ranged from ~ 2.5 - 57.3 .

For most of the data on the driven a.c. effect j_2 ranged between 0.2 and $2.1\mu\text{A}$, however in a couple isolated cases j_2 was varied up to $\sim 3.3\mu\text{A}$. The temperature range here was 3.061-4.004K and also controlled to $\pm 10\mu\text{K}$. Dynamic resistances varied from 1.3 - 2.2Ω with a few cases as high as 3.0Ω . In most cases the fundamental step ($n=1$) was the primary one examined, however data was taken on $n=2,3,4$ steps also and in one isolated case an $n=11$ step was analyzed. The frequency of radiation applied to the point contact ranged from 290MHz - 3.14GHz, at power levels varied by a factor of 100. In most cases studied the resultant a_2 range was 1.1-4.5 (recall this would correspond to a γ range of 2.2-9.0). Again in a couple isolated cases, a_2 was increased, ranging up to ~ 17.6 .

In order to get an estimate of the capacitance of the point contact we made use of the fact that for large enough I_1 the capacitance will result in hysteresis in the d.c. I-V.⁽²⁹⁾ It should be pointed out that heating while in the normal state will

also produce hysteresis (cf. chapter 6), hence in interpreting hysteresis as due to capacitance one must be certain that heat flow away from the weak link is sufficient to insure that I_1 doesn't change (decrease) due to heating. Since the thermal resistance of the bulk niobium electrodes on point contacts is much smaller than the thermal resistance of the thin film electrodes on thin film bridges, heating effects are more prominent in the bridges than in the point contacts. Indeed we observed no hysteresis whatsoever in the point contact I-V's down to our lowest measurement temperature, $\sim 1.65\text{K}$. At this temperature $I_1 \approx 73.0\mu\text{A}$ and our amplified current resolution was $\sim 0.03\mu\text{A}$ giving a lower bound of 0.9996 for α_c (McCumber's notation for the ratio of critical current while sweeping current down to the critical current while sweeping current up). From McCumber's results (Fig. 3) we can obtain a rough upper bound for $\beta_c \equiv \frac{2eI_1CR^2}{\hbar}$; we estimate $\beta_c \lesssim 0.1$ and

$$C = \frac{\hbar\beta_c}{2eI_1R^2} \lesssim \frac{0.1\phi_0}{2\pi I_1R^2} = 0.21 \times 10^{-12} \text{f} \quad (5.3)$$

That is, for the capacitance of our point contacts we estimate 0.2pf as an upper bound. This is the value that is used in section 5.3.6 in examinations of how well these point contacts satisfy the approximations of the theory.

5.3.2 Method of Analysis

Detailed d.c. I-V profiles of r.f.-induced steps and the d.c. effect were measured via methods described in chapter 4. The usual procedure was to obtain a low resolution total (i.e.

including the origin) I-V and then to offset and amplify the step. These amplified steps were recorded on a ten inch scale with scale factors ranging from 0.01-0.50 μ V/inch. An example of the raw data for the driven a.c. effect was given in Figure 4.9.

From the low resolution curves the parameters R, J and V_0 were determined to better than 1%. (In some cases, e.g. for steps in the region of rapidly changing R, the uncertainty in R was $\sim \pm 5\%$). As pointed out in section 2.4.4 in chapter 2, a reinterpretation of Stephen's parameters R and J is required for r.f. power levels which significantly affect the shape of the entire I-V curve. According to Stephen's paper⁽¹⁴⁾ R and J are to be the values of dynamic resistance and current at the step voltage, in the absence of applied radiation. In that paper the implicit assumption is made that the step center current is unaffected by the presence of radiation (this is the weak coupling assumption, $\frac{2eV_{rf}}{\hbar\omega_{rf}} < 1$, cf. section 2.4.4). In actual experiment the step center current and dynamic resistance vary due to the fact that the amplitudes of the zero voltage step and all the other steps vary with r.f. power. Whereas Stephen's interpretation of R and J is correct for small r.f. levels, the fact that a good approximation to the same phase locking equation (which serves as the starting point in the fluctuation treatment) can be obtained for any r.f. current level (Waldram et al.⁽¹⁸⁾) allows the more general interpretation of R and J as the extrapolated dynamic resistance (not to be confused with the slope of

the step itself, defined R_c in eq. 5.2(b)) and current at the actual step center. This is the interpretation which we used. For relatively small steps the actual step center differs very little from the corresponding point on the I-V with no radiation present; several of the steps analyzed fitted into this classification. For those instances, in our observations, where the step center was significantly ($\geq 10\%$) lowered, R was significantly ($\geq 10\%$) raised and consequently the product RJ remained nearly constant. Hence for our data the practical distinctions between our more general interpretation of R and J and Stephen's interpretation were not very great.

The high resolution curves provided I-V coordinates of experimental points that were used for detailed comparison with theory. For these high resolution curves the step centers were located by a tracing-and-inverting technique, which worked quite well for reasonably symmetrical steps. The next stage in analysis was computer fitting of the basic prediction, equation 5.1(a), to this digitized data.

Recall that in equations 5.1(a) and 5.1(b) the parameters I_1 and j_2 are respectively the critical current and the critical step amplitude in the absence of fluctuations. However since we are studying the regime where fluctuations are not unimportant I_1 and j_2 are not directly determinable. In fact I_1 and j_2 are the only parameters not precisely determinable from experiment, hence for the d.c. effect experiment I_1 is the "natural" fitting parameter, whereas in the driven a.c. experiment j_2 is the natural fitting parameter. For each step or

zero voltage step the theoretical curve, eq. 5.1(a) was fit to the data in a one parameter least squares fit, employing either I_1 or j_2 as the only fitting parameter. All of the computer fitting was done employing the powerful and compact interactive language APL. This language was particularly suitable to our purposes because of the nearly instantaneous response, thus enabling very rapid debugging and immediate testing of new ideas-- for example the effect of variation of a parameter over a range of uncertainty could rapidly be ascertained. For the integrations, the Gaussian integration technique was used because for a fixed number of integration intervals this method was found to be very much more accurate than some more familiar techniques, such as Simpson's rule. Also, Gaussian integration was well suited to the structure of the APL language-- specifically, no looping was required. Both the AH form of the theory (cf. chapter 2 eq. 2.73') and eq. 5.1(a) (or eq. 2.73) were used for the d.c. effect; the results agreed but of course the more compact form, eq. 5.1(a), was more quickly calculated and required fewer integration intervals for comparable accuracy. The number of integration intervals was chosen such that the result of an integration was always accurate to better than 0.05% and usually was accurate to <0.01%. For this accuracy the AH form (eq. 2.73') required

$$N = \begin{cases} 12 & \text{for } \gamma \lesssim 8 \\ 16 & 8 < \gamma \lesssim 20 \\ 20 & 20 < \gamma < 50 \end{cases}$$

where $N \equiv$ the number of integration intervals. For the forms 5.1(a) and 5.1(b) the following values were used.

$$N = \begin{cases} 10 & \alpha \lesssim 8 \\ 12 & 8 < \alpha \lesssim 12 \\ 16 & 12 < \alpha \lesssim 30 \end{cases}$$

For the modified Bessel function I_0 , the following form was used.

$$I_0(z) = \frac{1}{\pi} \int_0^{\pi} \cosh(z \cos \theta) d\theta \quad (5.4)$$

The least squares fitting was done automatically with I_1 , or j_2 being the parameter incremented. More details of the fitting technique and the APL programs are contained in appendix 5.A. In order to compare the quality of fits having different γ 's or a_2 's it was necessary to define a reasonable criterion for comparison. Of three criteria used, one seemed to be the most consistently reasonable. This criterion was the root-mean-square deviation normalized by the inherent scale factor $j_2 R$ or $I_1 R$, and divided by the data scale factor ($\mu V/\text{inch}$). (The last factor enables quick comparison of quality of fits for different data scale factors). Of course in all comparisons the experimental voltage uncertainty, $\sim 1-2 \times 10^{-3} \mu V_{p-p}$, must be considered.

5.3.3 Results for the Driven Josephson Oscillator

We discuss here the results of observations on a superconducting point contact designed to provide a quantitative test of the corresponding theory (driven a.c. effect), due to Stephen, for the shape of induced steps in the current-voltage characteristic. We have measured many complete step profiles and have found quite

good agreement with theoretical profiles (eq. 5.1(a)) in a one-parameter fit, employing j_2 as the fitting parameter. We should like to emphasize that the temperature used was the actual temperature of the device as should be the case for thermal fluctuations intrinsic to the device itself. Good agreement was seen to exist between the fitted j_2 's (step heights in the absence of fluctuations) and values obtained by extrapolating the regions adjacent to the step. For all results shown in this section low temperature low pass filters were used on current and voltage leads (cf. chapter 4). The effect of removing these filters is described in section 5.3.5.

Figure 5.3 displays a fluctuation rounded step profile at three different voltage sensitivities. The noisy curves are tracings of the actual data; the smooth dashed curves are the theoretical fits and are labeled with the corresponding bestfit values of the fitting parameter. The fitted curves are so nearly coincident that the theoretical curves are barely distinguishable from the data. Another observation is that the bestfit j_2 varies $<0.8\%$ as the voltage sensitivity is increased by a factor of 5. This is one manner of expressing the good agreement between theory and experiment for both small and high (relatively speaking) voltages along the step.

Further representative results are displayed in Figure 5.4. In the upper left corner, Fig. 5.4(a), is shown a low resolution current-voltage characteristic with sufficient r.f. (3.14GHz)

$T = 3.494^\circ\text{K}$

$\nu = 3.14\text{GHz}$

$n = 1$

Figure 5.3 An r.f.-induced step at three different voltage sensitivities with theoretical fits superimposed (dashed lines). [Point Contact]

Figure 5.4 Representative results for a single step as r.f. power was varied. (a) Typical I-V tracing with sufficient rf (3.14 GHz) power applied to the junction to induce many steps. (b) Development of a step as rf power is first increased (relative powers shown) for $T=3.801$ K, $\nu_{rf}=3.14$ GHz, and step number $n=1$. Data are represented as actual tracings and as points (cf. text). Smooth solid lines are one-parameter (j_2) fits to data using Eq. 5.1. Relevant parameters: Curve A (entire step), $j_2=0.3450$ μA , $a_2=1.986$; (step center) $j_2=0.3353$ μA , $a_2=1.930$. Curve B, $j_2=0.4588$ μA , $a_2=2.642$. Curve C, $j_2=0.6621$ μA , $a_2=3.814$. (c) Lower-resolution tracings of same steps as in (b), along with theoretical curves for the corresponding cases with no fluctuations $[\Delta V(0)]$, using the fitted j_2 values. [Point Contact]

power applied to the point contact to induce a sequence of many steps. The set of curves in the center, Fig. 5.4(b) shows the development of a step as r.f. power is first increased. In Fig. 5.4(b) case (B) displays tracings of the actual data recorded going both up and down a step in a time span of ~ 3 minutes. The bottom half of case (A) is also a tracing (up only). For clarity in comparison with theory curves, the rest of the data are displayed as points. The smooth solid lines are one-parameter fits of the theory to the data, again using only j_2 as the fitting parameter. (Values of T , V_0 , R , and J were independently measured for each step.) For case (A), two theoretical curves are shown as a measure of the uncertainty of the fit; the longer one is the least-squares fit to the entire data curve, the shorter one is the best fit to points near the step center. These results indicate quite good agreement between theory and experiment, the slight imperfections in the fits being just barely resolvable. In Fig. 5.4(c) we have plotted, along with lower-resolution tracings of the steps, the corresponding theoretical curves (McCumber-Stewart model, cf. chapter 2) for the hypothetical case of no fluctuations using the fitted values of j_2 . Note that the fitted j_2 values are quite sensible, being consistent with reasonable extrapolations for the no-fluctuation limit.

Figure 5.5 is a magnified replot of Fig. 5.4(b)-case (A),

Figure 5.5 The typical slight deviation of theory from experiment (magnified replot of Fig. 5.4(b) - case (A)). [Point Contact]

showing the slight imperfection of a typical fit by comparison of the best fit to the entire step versus the best fit to the step center. The corresponding fitting parameters differ by approximately 3%, which was typical for most steps. On the "best" fits this difference was on the order of 0.8% and in no case was it more than 5%. We conclude that in most cases the difference between experimental and theoretical profiles is within the experimental instrumental noise; however, a very slight systematic deviation from the theory is suggested by plots such as Fig. 5.5.

In the upper half of Figure 5.6 we have again shown low resolution tracings of steps along with the hypothetical no fluctuation curves using the bestfit j_2 's (similar to Fig. 5.4(c) but different data). For both small and large steps the fitted j_2 's were reasonable. The bottom half of the figure displays the correct anticipated behavior of the inverse ratio of the fitted j_2 to an extrapolated step amplitude (fluctuations present) j_e , defined as in the figure. For a fixed temperature $j_2 \rightarrow j_e$ for large steps (i.e. fluctuations become less prominent). Also, due to the temperature dependence of I_1 and j_2 , steps of the same amplitude but at different temperatures correspond to different a_2 's (hence to different degrees of rounding). To obtain a given a_2 at two different temperatures a smaller step corresponds to the lower temperature.

We have examined in detail many different step profiles over the range of parameters given in section 5.3.1. We have varied

Figure 5.6 Fluctuation rounded steps superimposed with no-fluctuation steps using the fitted values for J_2 (cf. text for description of bottom plot). [Point Contact]

a_2 (which determines the sharpness of a step) by varying temperature, T , r.f. power, r.f. frequency, ν_{rf} , and step number, n . Within our ranges of investigations we found no systematic deviations in the quality of the fits for variations of j_2 , a_2 , T , ν_{rf} , n , and r.f. power (see discussion pertaining to Fig. 5.9 regarding determination of r.f. power). Figure 5.7 displays the upper half of four different steps whose parameters are listed in the adjacent table. The crosses are experimental data points and the solid lines are the corresponding theoretical fits. Because it is confusing to plot many results in this manner, the results of parameter variations for several steps is summarized in a slightly different format in Figure. 5.8. The circles and triangles are points taken from the upper half of experimental step profiles and normalized with the step height, j_2 (cf. eqs. 5.1(b) for notation); the solid lines are again the best theoretical fits to these points. For comparison purposes the simple model result for the case of no fluctuations is also plotted. This semilogarithmic plot provides a sensitive display of the fit of theory to experiment and demonstrates good agreement over the entire range of parameters tabulated in the upper left of the figure. Curve (2) corresponds to case (C) of Fig. 5.4: curves (3), (4), (9) and (11) correspond respectively to steps (A), (B), (C), and (D) of Fig. 5.7. For the points along (7) and (8), curve (8) is the best fit to the bottom two points (i.e. the step center) and (7) the best fit to all points

Curve	j_2	a_2	T	ν_{rf}	n	$x(\mu V)$	$y(\mu A)$
(A)	1.1792	3.225	3.152	3.14GHz	1	0.05	0.197
(B)	0.7106	2.811	3.340	3.14GHz	1	0.10	0.197
(C)	0.2208	1.271	4.004	3.14GHz	1	0.10	0.0985
(D)	0.2608	1.117	3.340	943MHz	4	0.10	0.197

Figure 5.7 Comparison of theoretical fits (solid lines) and experiment (crosses) for the upper halves of four different steps, with relevant parameters tabulated. [Point Contact]

Figure 5.8 Semilog plot of normalized current and voltage for several steps (upper halves only) for various parameter variations. Circles and triangles are data points. Solid curves are again best theoretical fits to the data points. Error bars correspond to the experimental voltage uncertainty of ± 1.5 nV. [Point Contact]

(the entire step). Curves (9) and (10) (upward and downward pointing triangles respectively) are examples of steps of very nearly the same height but frequencies near opposite ends of the range studied. Only the step in curve (6) shows a significant deviation from theory. This sort of deviation occurred at relatively low frequencies in cases where closely spaced steps of comparable magnitudes overlapped to a degree; that is, when the (rounded) voltage range of one step extended into the voltage range of an adjacent step of comparable amplitude. Curve (6) is an example of such a situation, whereas curve (10) is a step of the same frequency and approximately the same r.f. power but for a case where the adjacent steps were of much smaller amplitude. This mutual overlap distortion was particularly evident for $n=1$ steps when the zero voltage step was depressed (by the r.f.) very nearly to zero.

In their derivation of the Langevin equation for the driven a.c. effect P. Lee⁽¹⁷⁾, and Waldram et al.⁽¹⁸⁾ did not employ the explicit assumption $\frac{2eV_{rf}}{h\nu_{rf}} \ll 1$, where $V_{rf} \equiv$ the r.f. voltage within the junction, that appeared in Stephen's derivation. (The assumption does not relate to the dynamics of the fluctuations but rather to the mechanism whereby an r.f. contribution is included in the Langevin equation). Therefore we investigated a range of r.f. powers in order to determine if this assumption of Stephen was necessary. As a means of obtaining rough estimates of $\frac{2eV_{rf}}{h\nu_{rf}}$ we made use of the experimental observation that for small critical currents the step amplitude - r.f. voltage relationship is reasonably approximated by the phenomenological formula⁽¹⁰²⁾

$$j_2(n) = I_1(T) \left| J_n \left(\frac{2eV_{rf}}{h\nu_{rf}} \right) \right| \quad (5.4)$$

where $n \equiv$ step number and J_n is the n^{th} order Bessel function.

(Explicit comparisons of our experimental data with this equation are found in chapter 6 and in section 5.3.4 of this chapter.)

Our specific method of estimating $\frac{2eV_{rf}}{h\nu_{rf}}$ goes as follows, for $n=1$:

Let $V_{rf} \equiv$ r.f. voltage in junction for a step in question and $V_{rf}^{\text{max}} \equiv$ r.f. voltage corresponding to the first maximum step.

$$2e \frac{V_{rf}}{h\nu_{rf}} = \frac{2eV_{rf}^{\text{max}}}{h\nu_{rf}} \times \frac{V_{rf}}{V_{rf}^{\text{max}}}$$

Assuming eq. 5.4 holds, $\frac{2eV_{rf}^{\text{max}}}{h\nu_{rf}} = 1.84$, the argument corresponding to the first maximum of the first order Bessel function, J_1 . Also $\frac{V_{rf}}{V_{rf}^{\text{max}}} = \left(\frac{P_{rf}}{P_{rf}^{\text{max}}} \right)^{1/2}$ where the latter quantity can be taken

either as the ratio of actual powers in the junction, or as the ratio of the applied powers as measured outside the apparatus.

Hence,

$$\left(\frac{2eV_{rf}}{h\nu_{rf}} \right)_{n=1} = 1.84 \left(\frac{P_{rf}}{P_{rf}^{\text{max}}} \right)^{1/2} \quad (5.5)$$

where $P_{rf} =$ externally applied r.f. power for the step in question, and $P_{rf}^{\text{max}} =$ external power corresponding to the first maximum step amplitude.

Figure 5.9 displays a comparison of a step for two vastly different r.f. power levels. For $T = 3.340\text{K}$ and $\nu_{rf} = 3.14\text{GHz}$, Fig. 5.9(a) displays the succession of low resolution current-voltage characteristics obtained as the r.f. level was monotonically

Figure 5.9 (a) Succession of I-V curves for increasing r.f. power.
(b) Comparison between theoretical fit (solid line) and experiment for a step at two vastly different r.f. power levels (steps A and B of (a)). [Point Contact]

increased. For clarity the voltage axis origin was shifted to the right after each successive curve.) The oscillatory behavior of the $n=1$ and zero voltage steps is clearly evident. The steps (A) and (B) are of nearly equal amplitude, but correspond to r.f. power levels respectively less than, and greater than P_{rf}^{max} , the power required to produce the maximum step. Theoretical fits were done for the amplified versions of (A) and (B); the results are plotted in semilogarithmic format in Figure 5.9(b). It is observed that the fits to the theory for $\frac{2eV_{rf}}{h\nu_{rf}} = 0.6$ and 4.3 are both equally good.

Another comparison showed equally good fits for $\frac{2eV_{rf}}{h\nu_{rf}} = 0.45$ and 4.5 , which corresponds to a power ratio of 100. The values for $\frac{2eV_{rf}}{h\nu_{rf}}$ corresponding to the data of Fig. 5.8 are fairly uniformly distributed between these extremes (0.45-4.5). Hence our results seem to confirm that the assumption $\frac{2eV_{rf}}{h\nu_{rf}} \ll 1$ is not necessary for good agreement between experimental and theoretical profiles.

In all of our studies we observed that the shapes of the $n=0$ steps (with r.f. applied) were identical to the shapes of the rest of the r.f. induced steps, and to the shape of the zero voltage step in the absence of radiation. This observation is certainly to be expected, and is due to the fact that the d.c. effect and driven a.c. effect problems are nearly identical. (This point is discussed further in section 5.3.4 and chapter 2.)

We would at this point like to discuss briefly one minor point, the effect of variation of more than one parameter.

Let us first write equation 5.1(a) in another form,

$$\Delta V = \underbrace{\frac{ek_B}{\hbar}}_X \cdot \underbrace{\frac{R^2 JT}{V_0}}_Y \cdot \sinh(\pi a_1) \cdot \underbrace{[f(a_1, a_2)]}_Z$$

where $a_1 = \frac{\hbar \Delta I}{2ek_B T} \frac{V_0}{RJ}$, $a_2 = \frac{\hbar j_2}{2ek_B T} \frac{V_0}{RJ}$, f is a definite integral and the rest of the parameters are defined in the discussion following eq. 5.1(a). Variation of j_2 (as was standard fitting procedure) affects only a_2 and hence only factor Z . Variation of R affects all three factors non-linearly. The effects of variation of J or T will be identical, affecting X linearly and the other two factors non-linearly. Thus in general multiple parameter fits will involve a complex series of compensations between all three factors X , Y , and Z , whereas a single parameter fit with j_2 as parameter involves only factor Z and hence provides the severest test of the theory. For a few of our experimental profiles we artificially varied two or three of the parameters, (R , J , T , and j_2), at a time by $\lesssim 10\%$. We found that in most cases the difference between a single j_2 fit and a three parameter fit was not significant, within the resolution of the data.

Finally, one last minor point: In Stephen's formulation (which we followed), $\alpha = 2a_2$ is proportional to the product $j_2 \times \frac{V_0}{RJ}$, whereas in the work of Lee and the work of Waldram et al. we

do not find the factor $\frac{V_0}{RJ}$. In most practical cases (step far from the origin) $\frac{V_0}{RJ} \approx 1$ so that the consequences of this factor are unimportant. In a couple of instances where $\frac{V_0}{RJ} \lesssim 0.6$ we compared fits with and without the $\frac{V_0}{RJ}$ factor. In both cases the fits which included the $\frac{V_0}{RJ}$ factor were much better (with R interpreted as the dynamic resistance (but not R_c of eq. 5.2(b)), and thus it appears to be necessary.

5.3.4 Results for the D. C. Josephson Effect

The close correspondence between the theory of intrinsic fluctuations in the d.c. Josephson effect and the driven a.c. effect suggests that agreement with the theory in one case should be accompanied by agreement in the other. Accordingly, in order to confirm or deny this hypothesis, we have measured high resolution point-contact I-V characteristics, with no applied radiation, in order to determine the fluctuation rounding of the d.c. Josephson effect in our case. Indeed we found close agreement with the corresponding theory using only I_1 , the critical current without fluctuations, as the fitting parameter (analogous to j_2 in the driven effect). As in the driven effect, the values of the fitting parameter were quite consistent with no-fluctuation extrapolations. The results in this section correspond to comparisons with the theoretical description of intrinsic fluctuations put forth by Ambegaokar and Halperin (cf. eqs. 5.1(a) and 5.1(b)); discussion of the effect of external noise is contained in section 5.3.5.

The fluctuation rounding in the d.c. effect is clearly displayed in the current-voltage curves in Figure 5.10. The effect is here displayed at three different sensitivities, increasing from right to left. (For clarity the voltage axis zero was shifted to the left after each tracing). For this figure the continuous lines are the actual data, traced up and down, and the solid circles with corresponding fitting parameter I_1 are the theoretical fits. As in the driven effect, the coincidence between theory and experiment is nearly perfect. Observe that the fitting parameter, I_1 , changes by less than 1.2% as the sensitivity is increased, hence pointing out the high quality of the fit for both relatively low and high voltages.

The next figure, Figure 5.11, displays the data and theory as the sample temperature (and consequently critical current) was varied. The data again consists of d.c. I-V characteristics but for this set the sensitivity is fixed and T is the parameter. This time the data is displayed as points taken from actual tracings and the theoretical fits are the solid curves. The fitting parameters (I_1 's) and the corresponding γ 's are tabulated at the right of the curves. Again, the agreement between experiment and theory is good. For $T = 4.122$ two fits are displayed, the upper is the best fit to all the data points, the lower (designated by the arrow) the bestfit to points near the origin. The fitted I_1 's for these two fits differ by $\sim 1.8\%$, again similar to the analogous analysis for the steps in the driven effect (Fig. 5.5).

The range of γ 's in the figures thus far corresponds roughly

Figure 5.10 Fluctuation rounding in the d.c. effect for three voltage sensitivities. The discrete points are the theoretical fits corresponding to the indicated fitting parameter (I_1) values. [Point Contact]

Figure 5.11 Fluctuation theory (solid line) vs. experiment in the d.c. Josephson effect -- temperature as parameter. [Point Contact -- $R_N = 1.45\Omega$]

to the range of a_2 's of the preceding section. Within these ranges we observed no significant differences between the quality of fits in the d.c. effect and the quality of fits in the driven a.c. effect. The almost-unnoticeable deviations between theory and experiment, as expressed by comparing fits of the total steps vs. step centers, were of the same magnitude in both d.c. and driven effects; the trend in both cases was that fits for points near the step center (or origin) yielded slightly lower fitting parameter, (I_1 or j_2), than fits to points from the total steps.

Direct connection between the d.c. and driven a.c. data can be made through equation 5.4, which relates the (no-fluctuation) step height of the n^{th} step to the (no-fluctuation) critical current for no radiation present, I_1 . This relation was checked for the maximum $n=1$ step amplitudes at three different temperatures where data for I_1 and J_2^{max} were available. According to eq. 5.4 for $n=1$,

$$\frac{j_2^{\text{max}}}{I_1(T)} = |J_1^{\text{max}}\left(\frac{2eV_{\text{rf}}}{h\nu_{\text{rf}}}\right)| = 0.58 \quad (5.6)$$

where J_1 is a Bessel function and the superscript denotes the first maximum value. The following table summarizes experimental comparison with eq. 5.6:

$T(K)$	$I_1(\mu A)$	$j_2^{\text{max}}(\mu A)$	$\frac{j_2^{\text{max}}}{I_1}$	% dev. from eq. 5.6	$P_{\text{rf}}^{\text{max}}(\mu\text{watt})$
4.004	.699	.411	0.59	+1.7	6.6
3.801	1.22	.661	0.54	-6.9	4.2
3.494	2.92	1.604	0.55	-5.2	6.7

X

The agreement with equation (5.6) is within experimental error, which is chiefly determined by the uncertainty in the determination of $j_2^{\max}$. In addition we observe that $P_{\text{rf}}^{\max}$, the applied power corresponding to the maximum step height, is independent of temperature, again in agreement with the prediction of eq. (5.6). (The fact that $P_{\text{rf}}^{\max}$ is lower for the $T = 3.801$ data most probably indicates that the step has not quite reached a maximum, hence would explain why $j_2^{\max} = .661$ is slightly on the low side.) Thus we conclude that the agreement between d.c. and driven a.c. effects is good and variation of j_2 with T , V_{rf} (P_{rf}), and I_1 was predictable; we saw no indication of anomalies in this data.

Figure 5.12 displays the d.c. effect fits for the cases of larger γ 's than discussed above. The solid lines are again the theoretical fits to the experimental data points (solid circles). We observe that even for the largest γ , 57.3, only a very slight deviation is noticeable. In the driven a.c. experiments the large a_2 regime corresponding to this has not been studied, although it would be accessible using a superconducting quantum voltmeter if a motive arose to pursue the small deviations. We believe that the deviations (slightly) more apparent for larger γ 's have the same character and source as the slight deviation observed for smaller γ 's (and a_2 's). Another manner in which we observed this deviation was to examine the results if we let R vary (as well as I_1). What we found was that as γ increased the best fit value of R decreased. Specifically, as the

Figure 5.12 Extension of Fig. 5.11 to larger γ 's ($R_N = 1.46 \Omega$ -- Point Contact)

experimental temperature was decreased from 4.26K to 3.68K we found that the best fit R value decreased by 5.6%. As one final method of expressing the same deviation, we tried fits in which the parameter T was artificially varied (as well as I_1). We found that best fits corresponded to an increase in T of $\sim 4.5\%$, corresponding to an $\sim 3.7\%$ decrease in γ . Both of these results (variation of R and T) are merely other ways of expressing the same slight deviation as expressed by comparing fits to the origins vs. fits to the total profiles. For more on this R variation see chapter 6.

5.3.5 The Effect of External Noise

We have attributed the observed fluctuation rounding reported in the previous two sections as due to intrinsic sources--fluctuations in the quasiparticle and pair currents within the junction (point contact) itself. Because these intrinsic effects are small, and because noise introduced by external sources (e.g. room temperature generators) could also trigger phase fluctuations, it is reasonable to ask whether all external noise has been effectively excluded relative to the intrinsic noise. In this section we will first attempt to establish that the effects we have been observing are indeed intrinsic. Secondly we identify the effect of external noise and demonstrate that it is suitably accounted for simply as a modulation of the d.c. I-V. We observed similar phenomena in both the d.c. and driven Josephson effects, so for brevity we consider here only the d.c. effect. Also we found that the conclusions reached for point contacts also held

for our measurements on thin film bridges (described in the next major section), hence we include here only data for the point contacts.

It was emphasized in chapter 4 that numerous painstaking precautions were taken in order to effectively isolate and shield the junctions. The presence or absence of thermometer and heater was observed to have no effect whatsoever upon the detailed I-V's. Lee has treated the effect of finite bandwidth of the external oscillator (in the driven effect); in our case (and in most cases conceivable) this effect was entirely negligible. An upper bound estimate for the thermal radiation transmitted via the coaxial line from room temperature showed this source of noise to be negligible also. The magnetic and electromagnetic shielding was highly redundant--the only remaining possible source of external noise were the I-V leads into which low temperature low pass filters could be inserted during a set of measurements. The intrinsic noise theories assume that the frequency spectrum of the fluctuations is flat and of nearly infinite bandwidth (cf. chapter 2). Vernon and Kanter⁽¹⁰³⁾, and Kose and Sullivan⁽¹⁰⁴⁾ have pointed out that the response of a biased junction to low frequency external noise (low frequency with respect to the Josephson frequency), is different from the theoretical predictions for thermal excitations (for which case the spectrum bandwidth extends to frequencies much greater than the Josephson frequencies encountered) thus it is not really proper to assign an "effective noise temperature" as several authors have done. Although one may obtain a better fit by insertion of some effective temperature, this

temperature may not correspond to the actual temperature of the low frequency external noise source. If the external noise were of flat bandwidth extending to frequencies greater than the Josephson frequencies encountered, then the assignment of a noise temperature would be correct, i.e. the theoretical Langevin analysis would apply in this case to both the intrinsic source and the external source.

We assume that any external current noise transmitted via our twisted pair of I-V leads has an upper frequency cutoff much lower than most of the Josephson frequencies encountered (recall $1\mu\text{V} \leftrightarrow 483.6\text{MHz}$). The introduction of such low frequency noise into the circuit should simply have the effect of modulating the instantaneous I-V characteristic,⁽¹⁰³⁾ subsequently leading to a change in the average (d.c.) voltage as measured by our very low bandwidth ($\sim 2\text{Hz}$), nanovoltmeter. Vernon and Kanter have pointed out that for small γ a Gaussian average (due, for example, to low frequency noise currents) of the no-fluctuation I-V of a junction leads to a rounding qualitatively similar to that predicted by the intrinsic noise theory. Following Vernon and Kanter, for a low frequency noise current of Gaussian amplitude distribution about a d.c. supply current, I , the average (or d.c.) voltage V is given by,

$$\frac{V}{I_1 R} = \frac{\int_{-\infty}^{\infty} f\left(\frac{I}{I_1}\right) \exp\left(-\frac{(I-I)^2}{2I_n^2}\right) dI}{\int_{-\infty}^{\infty} \exp\left(-\frac{(I-I)^2}{2I_n^2}\right) dI} \quad (5.7)$$

where I_n is the RMS noise-current amplitude and where $f(\frac{I}{I_1})$ is the simple model (cf. chapter 2) theoretical I-V in the absence of all fluctuations,

$$f\left(\frac{I}{I_1}\right) = \begin{cases} 0 & \text{for } \left|\frac{I}{I_1}\right| < 1 \\ +\left(\left\{\frac{I}{I_1}\right\}^2 - 1\right)^{1/2} & \text{for } \frac{I}{I_1} > 1 \\ -\left(\left\{\frac{I}{I_1}\right\}^2 - 1\right)^{1/2} & \text{for } \frac{I}{I_1} < -1 \end{cases} \quad (5.8)$$

Substitution of (5.8) into (5.7) yields,

$$\frac{V}{I_1 R} = \frac{1}{(2\pi)^{1/2}} \frac{1}{I_n} \left\{ \int_{I_1}^{\infty} \left(\left\{\frac{I}{I_1}\right\}^2 - 1\right)^{1/2} \exp\left(-\frac{(I-I)^2}{2I_n^2}\right) dI \right. \\ \left. - \int_{-\infty}^{-I_1} \left(\left\{\frac{I}{I_1}\right\}^2 - 1\right)^{1/2} \exp\left(-\frac{(I-I)^2}{2I_n^2}\right) dI \right\} \quad (5.9)$$

In the following table we compare the root-mean-square deviations (RMSD) of theoretical one parameter (I_1) fits using the intrinsic noise prediction of Ambegaokar and Halperin (eq. 5.1(a)) with the RMSD for two parameter (I_1 and I_n) fits using eq. 5.9, for the same experimental data: (cf. appendix 5.A for fitting programs etc.)

<u>y</u>	Intrinsic (eq. 5.1(a))	Low Freq. (eq. 5.9)
4.56	0.71	8.5
7.65	6.5	10.2
17.7	6.4	17.8
23.5	15.6	14.2

Thus we see that except for the largest γ the intrinsic noise theory gave quantitatively better fits despite the fact that only one parameter was varied, whereas for the low frequency noise fits two independent parameters were varied. The data for the $\gamma = 17.7$ case is displayed as curve (1) in Figure 5.13(A). The continuous line is experiment, the dotted line is the AH intrinsic noise theory fit, and the dashed line is the low frequency noise (LFN) fit.

All data presented thus far was taken with our low-temperature low-pass filters inserted in the I and V leads. Curves (a) and (b) of Figure 5.14 show the outcome of removing the filters. Curve (a), (identical to curve (1) of Fig. 5.13), is the I-V profile with the filter in place whereas curve (b) is the profile without filtering. Note that the effect of removing the filters is quite significant. In all cases the LFN theory fit the no-filter curves much better than the AH theory did; whereas as pointed out above, the AH theory fit the low-temperature-filter curves better than the LFN. The implication here is that the filters are indeed fulfilling their intent and that they were definitely necessary for our experimental environment.

As a further test of this implication we did an experiment in which low frequency current noise was purposely injected into current supply leads. A schematic of the experimental arrangement is shown in Fig. 5.13(B). The output spectrum of the noise generator was flat from 10Hz to 5MHz, and the amplitude probability

Figure 5.13 (A) - D.C. I-V curves with and without the application of external noise -- with the low temperature filters in place. In curve (1) the solid line is the experimental curve with no applied noise current whereas the dotted line is the theoretical fit of the intrinsic fluctuation theory ($T=4.19\text{K}$, $I_1=1.5513\text{ }\mu\text{A}$, $\gamma=17.65$, $R_N=1.23\Omega$) and the dashed line is the fit of the low frequency noise theory ($R_N=1.27\Omega$, $T=4.19\text{K}$, $I_1=1.470$, $I_2=0.3354\text{ }\mu\text{A}$). The parameter for the experimental curves (2) - $n(4)$ is a relative measure of the applied noise current. [Point Contact]

(B) Circuit used to apply the low frequency (10 Hz - 5 MHz) external noise current.

LOW TEMPERATURE FILTER
IN PLACE

(A)

(B)

Figure 5.14 (a) and (b): D.C. I-V curves with no applied external noise current, showing the effect of removal of low temperature filters. The solid lines of (b) - (e) are experimental d.c. I-V curves with and without the application of external noise -- with no filters in place. (The experimental relative applied noise current labels the curves, as in Fig. 5.13(A).) The dotted lines are fits (fitting parameter I_N , the noise current) of the low frequency noise theory to the data --- in all cases $I_1 = 1.45 \mu\text{A}$ and $T = 4.19\text{K}$. [Point Contact]

NO FILTERS
(except (a))

(a) (No Applied Noise) FILTER IN

(b) (No Applied Noise) FILTER OUT
 $I_N = 0.4814$

$I_N = 1.0945$

(c) $1 \times 10^\circ$

(d) $3 \times 10^\circ$ $I_N = 2.9439$

(e) $6 \times 10^\circ$ $I_N = 4.3363$

$0.2 \mu A$
 $0.2 \mu V$

distribution was Gaussian. In Figure 5.13(A), curves (2)-(4) are the experimental I-V profiles observed as the noise generator voltage was increased for the case where the low temperature filters were in place. In Figure 5.14 the continuous curves (c)-(e) are the profiles observed for the case where no filters were used. In both figures the numbers labeling the curves are relative values of the applied noise voltage. Two major observations are clear: (1) The effect of the applied noise upon the filter-in-place curves is identical to the effect of removing the filters, and (2) the minimum relative applied current noise amplitude which will cause a noticeable degradation for the filter-in-place curves is at least 2.5×10^4 times greater than the noise level which will produce a corresponding degradation for the no-filter profiles. These observations confirm that the filters were indeed necessary, and that they are indeed quite effective in eliminating low frequency external noise currents.

Finally we checked to see if eq. 5.9 provided a reasonably accurate description for the cases where external noise was applied and known to be present. In Fig. 5.14 we fitted via I_1 and I_n the LNF description (eq. 5.9) first to case (b), for which the noise generator was off. Using the value of I_1 obtained for (b) we then fit the rest of the curves, (c)-(e), in a one parameter fit using I_n as the only variable. The fitted results are indicated by the dotted lines which are

labeled with the fitted I_n values (actually labeled I_N in the figure). Observe that these fits are better than the fit of this theory to the no-noise, filter-in-place data--curve(1) Fig. 5.13. Note that the proportion of applied noise currents for (c), (d), and (e) is 1:3:6 whereas the ratio of fitting parameter, I_N , for these curves is 1:2.7:4.0. The agreement is reasonable in light of the neglect of intrinsic noise and the fact that the originally flat noise spectrum is probably considerably distorted by the input coupling capacitors and by the current leads.

In summary, (1) we have identified the effect of external low frequency noise and have shown that it can be understood in terms of simple modulation of the I-V characteristics; (2) we have demonstrated the effectiveness of our filtering technique and have shown that for sensitive intrinsic noise measurements, filtering is necessary; and (3) the sum total of all evidence endorses our assertion that fluctuation effects reported in sections 5.3.3 and 5.3.4 on the driven and d.c. Josephson effects have indeed been interpreted correctly as due to sources intrinsic to the junctions themselves.

5.3.6 Approximations; Deviations

The theory, eq. 5.1(a), with which we compared our results corresponds to the zero capacitance limit. Lee⁽¹⁷⁾ has reexamined the criterion for small C which in the work of Ambegaokar, Halperin and Stephen was based upon an early investigation of the Fokker-Planck equation done by Kramers⁽⁵⁷⁾. Lee indicates that the proper

criterion to reach the low-junction-capacitance limit is,

$$\Omega^2 \equiv (RC)^2 2eI_1/\hbar C \ll 1 \quad (\text{d.c. effect})$$

or

$$\Omega_1^2 \equiv (RC)^2 2ej_2/\hbar C \ll 1 \quad (\text{driven a.c. effect}) \quad (5.10)$$

whereas the criteria mentioned by AH was $\Omega \ll 1$. As pointed out by AH, for a superconducting tunnel junction (S-I-S), Ω is independent of the junction area, and depends only upon j_1 (the maximum supercurrent density) and T , hence Ω cannot be made smaller by simply using smaller area (and hence smaller capacitance) tunnel junctions.

Stephen's corresponding criteria were,

$$\Delta V \ll \left\{ \frac{k_B T}{C} \right\}^{1/2}$$

$$\frac{2eRC}{\hbar} \left\{ \frac{k_B T}{C} \right\}^{1/2} \ll 1 \quad (5.11)$$

These criteria are easily satisfied for our point contact data:

$R = 1.46$, $C < 0.2 \text{ pF}$, $I_1 \leq 4 \mu\text{A}$ ($j_2 < I_1$), $T \geq 3\text{K}$, and $\Delta V \sim 0.5 \mu\text{V}$ which gives

$$\Omega \lesssim .07$$

$$\Omega^2 \lesssim .005$$

$$\Delta V \left\{ \frac{C}{k_B T} \right\}^{1/2} \lesssim .035$$

$$\frac{2eRC}{\hbar} \left\{ \frac{k_B T}{C} \right\}^{1/2} \lesssim .013$$

Lee has shown that Ω^2 (or Ω_1^2) is the appropriate expansion parameter for correction terms due to finite capacitance, hence we see that expected corrections for our case would be $< 0.5\%$.

Thus the $G \rightarrow 0$ limit is quite appropriate for our point contacts.

Another important assumption in the Langevin and Fokker-Planck treatments is that the fluctuation source have an infinite bandwidth flat spectrum (Johnson or thermal noise.) This is a well founded assumption as long as the voltages encountered are small enough such that $eV \ll k_B T$. For our lowest T , 3K, and our largest V_0 , 7.8 μ V, we have $\frac{eV}{k_B T} < .03$.

Two further assumptions in the theory are that (1) the junction area is sufficiently small such that the current density is uniform and (2) that R is independent of the voltage. Though it is difficult to directly determine what the actual area of contact is in a point contact, we estimate it to be less than or on the order of one square micron, (the point diameter before contact is $\sim 10\mu$). Critical current versus temperature data (cf. appendix 6.B of the next chapter) suggests that our point contacts resemble SNS junctions. In such case we roughly estimate a Josephson penetration depth from $\lambda_j = (\Phi_0 / 2\pi\mu_0 j_1 d)^{1/2}$ where j_1 is the maximum supercurrent density and $d = t + 2\lambda_{Nb}$ where t = the N-layer thickness and λ_{Nb} = the penetration depth in the niobium. If we take a crude estimate $d = 1\mu$ and let W = the contact area diameter, then the requirement $\lambda_j > W$ gives $0.042 \text{ Amp} > j_1 W^2 \geq I_1$. Our I_1 range was much smaller than 42mA, hence we expect that the current density was sufficiently uniform over the entire range of the experiments. For the d.c. effect the assumption that R is constant (i.e. not a function of voltage) is a good one for a point contact. For the driven

a.c. effect, this assumption is not perfectly true for widely rounded steps in regions where the dynamic resistance is rapidly changing with the voltage (i.e. the region near zero voltage.) In fact one does observe asymmetry in steps in this region which is not present in steps in regions of the I-V where the dynamic resistance is rather linear. This asymmetry is readily accounted for by the voltage dependence of R.

As we showed in section 5.3.3 good theoretical fits to experiment were obtained throughout our r.f. power range, $0.45 \lesssim \frac{2eV_{rf}}{h\nu_{rf}} \lesssim 4.5$. This would appear to imply that the "weak coupling" assumption of Stephen (cf. chapter 2) was not necessary; indeed, Lee and Waldram et al. make no such assumption in their formulations of the theory.

We have emphasized that the only observed systematic deviations between theory and experiment were very slight, and were similar in both the d.c. effect and the driven a.c. effect (cf. discussion in sections 5.3.3 and 5.3.4). The source of these slight deviations may lie within the simple model for the junction in the absence of fluctuations and thus have no connection with the fluctuation mechanisms and formulations at all. That is, the no-noise model has been derived with tunnel junctions in mind, where the mechanisms of the normal current and supercurrent are well understood microscopically, in terms of quantum mechanical coefficients for pair tunneling and quasiparticle tunneling. Though this model appears to be quite accurate for tunnel junctions it may be less accurate for

point contacts and bridges, for which a microscopic understanding of normal current and supercurrent mechanisms is not nearly so well defined. Indeed in the next chapter we show that for both point contacts and thin film bridges there are deviations from the hyperbolic I-V relation (with temperature independent R) predicted by the simple no-noise tunnel junction model (T near T_c), $(V/I_1 R) = [(I/I_1)^2 - 1]^{1/2}$. For bridges there appears to be an "excess supercurrent" at voltages relatively far from zero, whereas in both types of samples the observed R_N appears to decrease with decreasing temperature (cf. next chapter). Our observation that the best fit R (in the d.c. case--section 5.3.4) was smaller for larger γ is compatible with these observed distortions of the no-noise hyperbola. Refinements of the no-fluctuation model for weak links are considered in the next chapter.

5.3.7 Summary-Point Contacts

In summary, we have measured and analyzed in detail many step profiles of the driven and d.c. Josephson effects in low-capacitance point contact junctions over a wide range of parameters in a regime where intrinsic thermal fluctuation rounding was substantial and external noise was effectively excluded. We have found that the experimental data is well accounted for by the intrinsic fluctuation theories (Ambegaokar & Halperin, Stephen, and the extensions of Lee, and Waldram et al.) within the range of the theoretical approximations in the low capacitance limit, and

within the accuracy of the data. Single-parameter fits yielded step heights j_2 corresponding to the d.c. critical currents I_1 and good fits were obtained using temperatures equal to the actual experimental temperatures. We observed only slight departures from theory, barely within the experimental resolution, which we believe are associated with departures from the no-fluctuation model and thus not directly related to the physical ideas concerned with the fluctuation phenomena.

5.4 Results in Thin Film Bridges of Rectangular Geometry

We have extended the analysis of the preceding sections to thin film bridges of rectangular geometry, fabricated by the photoresist technique detailed in chapter 3. It was of interest to compare results in these bridges with the point contact results and with our earliest results on bridges of wedge geometry where the presence of a non-linear resistive background hindered detailed quantitative analysis (cf. discussions in section 3.5.1, 5.3.1). In addition these results provide a direct basis for comparison of our bridge results and our point contact results with the published d.c.-effect results of Anderson and Goldman on tunnel junctions and Simmonds and Parker's corresponding d.c.-effect results on thin film bridges. The method of analysis and the measurement techniques were the same as described earlier for the point contact, hence the descriptions in this section will be brief.

The results reported here are for tin bridges $\sim 1\mu$ wide with nominal lengths of 1.5, 3, or 8 microns. Normal state resis-

tance ranged from 0.07 to 0.30Ω . For the d.c. effect measurements temperature was varied between 3.770K and 3.826K , which provided for a range of I_1 , $0.3\mu\text{A} \leq I_1 \leq 8.0\mu\text{A}$, with a resultant γ range $3.8 \leq \gamma \leq 102$. In the driven a.c. effect measurements a comparable temperature range was used and $0.27 \leq j_2 \leq 2.1\mu\text{A}$, with a dynamic resistance range 0.046 - 0.26Ω . In most cases only the $n=1$ step was analyzed in detail, over a total frequency range from 35MHz to 8.81GHz , considerably extended over the 0.29 - 3.14GHz range used for the point contacts. The dimensionless parameter a_2 ranged between 1.3 and 13.4 .

Abrupt switching and hysteresis was observed in the bridge I-V's for large critical currents. We interpret this hysteresis as due to Joule heating (while in the normal state--cf. chapter 6 for further discussion) rather than due to capacitance, thus the method used to estimate the capacitance for the point contacts could not be used here. Rough estimates based upon the geometry yield very small values ($\sim 10^{-15}$ farad) for capacitance. In any case, the capacitances of the bridges would not be expected to be any larger than the point contact value ($< 2 \times 10^{-13}$ farads). Consequently the zero-capacitance-limit criterion $\Omega^2 \ll 1$ was easily satisfied.

For our highest frequency, 8.8GHz , and the corresponding temperature $T = 3.76\text{K}$ we have $\frac{eV}{k_B T} = .056$ so that the approximation $eV \ll k_B T$ maintains over our entire voltage range.

Representative data for the driven a.c. effect at 110MHz in a 1.5μ long bridge is shown in Figure 5.15. In this figure the

Figure 5.15 Representative data for the driven a.c. effect at 110 MHz in a 1.5μ long Sn thin film bridge. The discrete points are the theoretical fits. For (A) $j_2=0.8427$, $a_2=3.947$, and $I_x=0.4 \mu\text{A}$; for (B) $j_2=0.4307$, $a_2=2.266$, and $I_x=0.2 \mu\text{A}$; for (C) $j_2^x=0.2649$, $a_2=1.657$, and $I_x=0.2 \mu\text{A}$. [Sn bridge]

continuous lines are actual data traced both up and down the steps and the solid circles are the theoretical fits. Also shown in the figure are the corresponding values of $\frac{2eV_{rf}}{h\nu_{rf}}$.

Just as for the point contacts, we found no degradation of the fits which could be correlated to increasing $\frac{2eV_{rf}}{h\nu_{rf}}$ (i.e. increasing r.f. power), within our experimental range of analysis ($\frac{2eV_{rf}}{h\nu_{rf}} \lesssim 10$).

Steps (A) and (B) correspond to the $n=1$ step at two different r.f. powers, whereas (C) is the $n=3$ step at a third (larger) power. The agreement between theory and experiment is very good in all three cases and the corresponding values of the fitting parameter j_2 are quite reasonable.

In Figure 5.16 we show some results for various frequencies and a_2 values for two different bridge lengths. The plot is a semilogarithmic plot similar to the corresponding analysis done for point contacts, Fig. 5.8. The error bars correspond to ± 1.5 nV, the estimated uncertainty in the data. The error bars appear larger here than in Fig. 5.8 because the dynamic resistances of the bridges were considerably smaller than those for the point contacts, so for a given $\frac{\Delta V}{j_2 R}$ value ΔV is much smaller, and hence its uncertainty relatively larger, for the bridges. The solid circles are the experimental points taken from the step profiles and the theoretical fits are the solid lines. Again, the agreement between theory and experiment is well within the resolution of the data for all of the parameter ranges displayed in the table. Incidentally, curve (2) corresponds to curve (A) of Fig. 5.15; curves

Figure 5.16 Semilog plot comparison of theory (solid lines) vs. experiment (discrete points) for r.f.-induced steps in two different tin bridges. (Similar to Fig. 5.8 for the point contacts.)

(1) and (4) correspond to the extremes in frequency range for the 1.5μ bridges (35MHz-2GHz), whereas curves (3) and (5) represent the frequency range considered for the 8.2μ bridges (3GHz-8.8GHz). The actual thresholds for the appearance of steps occurred at lower frequencies--these threshold data are considered in the next chapter. In our analysis we found no significant systematic distinctions between the 1.5 , 3 and 8μ bridges and between the low and high frequency extremes.

In general we found both the driven a.c. and the d.c. effect fitting results to be of comparable quality for the same γ and $2a_2$ range. Figure 5.17 displays the actual raw data for the d.c. rounding in an 8.2μ bridge with $\gamma = 3.885$. The resistive background observed in the wedge geometry bridges was no longer present, thus lending credence to our hypothesis for its existence (cf. 3.5.1, 5.3.1). Again the agreement between the experimental data (continuous line) and the theoretical fit (discrete points) is very good. Figure 5.18 is a normalized plot for the same bridge with γ as parameter. Curve (5) corresponds to the data of Fig. 5.17; the error bars again denote $\pm 1.5\text{nV}$. This time the theory curves are the smooth solid lines and the experimental points, normalized by I_1 , are discrete. In general the agreement is close though a small deviation is evident, particularly at the larger γ values. The small discrepancy is of the same nature (though slightly greater) as that previously described for the point contacts, namely the experimental "knee" is

Figure 5.17 Representative d.c. Josephson effect in a tin bridge --- continuous line = experiment, discrete points = theoretical fit.

Figure 5.18 Representative plots of theoretical fits (solid lines) to normalized d.c. I-V curves (no r.f.) in a tin bridge with $I_1(T)$ (or γ) as parameter.

not quite as sharp as the theoretical knee. Fits to all of the experimental points yielded I_1 values typically 3% larger than the values obtained via fitting only to points near the origin; in the worst cases, such as for curve (3), this difference was 6%. These observations were equally true for the shorter bridges also. Just as for the point contacts, the fits could be improved by decreasing R (artificially, in the theoretical fits) as γ increased. In particular for a 1.5μ bridge the best-fit value of R decreased by $\sim 15\%$ as γ increased from 20 to 100. This observation is compatible with the observed deviation of the total non-fluctuation-rounded- current-voltage characteristic from the simple model hyperbola--cf. chapter 6 results.

In summary an analysis paralleling that for point contacts was done for rectangular tin thin film bridges of length $1.5-8.2\mu$. The resistive background problem that we observed in our wedge-geometry bridges was found to have been alleviated. In general we found that fits of the theory of intrinsic noise rounding to the experimental results were good though not quite as good as the corresponding fits for the point contacts. In particular the small deviations, either expressed as differences in the fitting parameter for fits to the total steps vs. points near the centers, or expressed as a variation of the best-fit R , were percentage-wise approximately a factor of two smaller for the point contacts.

Nevertheless the nature of the small deviations in both the bridges and point contacts was the same, and thus the source is

believed to be the same. The previously stated observations and conclusions for point contacts with regard to independence of the results to significant r.f. power variation were also found true for the bridges. No significant differences were observed in the quality of fits between the d.c. and driven effects or between the individual bridges (1.5, 3 or 8μ lengths). Finally, no significant alterations in the fits attributable to the r.f. frequency were found over the full extent of the frequency range used, 35MHz - 8.8GHz (a factor of 250 in frequency).

5.5 Summary & Conclusions

In brief, we have carefully measured in detail (1nV resolution) the current-voltage characteristics of two types of low-capacitance superconducting weak links, niobium point contacts and tin thin film bridges, in a regime where the effects of intrinsic fluctuations in the normal current and supercurrent were predicted to be substantial. We have studied both the driven a.c. Josephson effect and the d.c. Josephson effect, for which the theory takes on identical forms, and the predicted observation is an intrinsic rounding of the otherwise-vertical step structure in the current-voltage characteristics normally associated with these effects. We have been able to distinguish between external noise effects and intrinsic effects and have effectively excluded external noise, and thus we maintain a high degree of confidence that the effects we have reported are indeed intrinsic to the weak links themselves.

The data has been compared with the intrinsic fluctuation theories of Ambegaokar, Halperin, and Stephen using only a single parameter fit. In the d.c. effect the "natural" fitting parameter used was the no-fluctuation critical current, I_1 , whereas in the driven effect the corresponding parameter was the no-fluctuation step amplitude j_2 . The agreement between theory and experiment was found to be very good in both the driven and the d.c. effects, and the single-parameter fits yielded step heights j_2 which corresponded reasonably well to the d.c. critical currents I_1 .

These results indicate that the conceptually simple theory in which thermal fluctuations are introduced as a rapidly fluctuating (Langevin) force and a tilted periodic potential is introduced by the coupling energy (d.c. effect) or by phase locking to incident radiation (driven a.c. effect) provides a rather good description of the intrinsic fluctuation phenomena in superconducting weak links of negligible capacitance. The broad applicability of the theoretical concepts is indicated by the consistency between the d.c. and driven effect results, and by the wide range of parameters over which convincing agreement between theory and experiment was obtained.

Our bridge results on the d.c. effect confirm the measurements of Simmonds and Parker on similar bridges⁽³⁵⁾, including the small discrepancy in the shape of the I-V characteristic that they observed near its point of maximum curvature, which was particularly evident for large coupling parameters $\gamma (\equiv \frac{\hbar I_1}{ek_B T})$. We observed this

small discrepancy to be relatively larger in the bridges than in the point contacts. We attribute these deviations as due to slight inadequacies in the no-fluctuation model itself, not to the Langevin, Fokker-Planck treatment of the fluctuations. Evidence of deviation of real weak link characteristics from the perfect hyperbola current-voltage characteristic of the simple model is exhibited in the next chapter.

In our initial report of the point contact results we conjectured that the consistency of our results for both the d.c. and driven a.c. effects suggested that the factor-of-2 discrepancy reported by Dahm et al. for the linewidth of the Josephson radiation (regular a.c. effect) might be due to experimental error or to a special aspect of the a.c. effect problem. Dahm et al. suggested that their lack of agreement might be due to a violation of a weak coupling approximation of the theory. Since that time, Vernet and Adde have obtained linewidth results (on niobium point contacts) which agree much better with the intrinsic noise theory for the regular a.c. effect. (105) They report that in their case the internal r.f. field was weak, however no quantitative estimates were given. In light of the fact that we see no apparent differences between weak and strong coupling for the driven a.c. effect we suggest that it might be interesting to determine more explicitly whether weak coupling is indeed a necessary approximation for the regular a.c. effect theory. If so, then this would appear to be an aspect of the theory special to the radiation-emission problem. Aside from this (perhaps minor) point, the general theory of intrinsic fluctuations in superconducting weak links has been completely tested in the low capacitance

limit and appears to be basically correct. On the other hand, to date no detailed experimental-theoretical comparisons have been made in the corresponding finite capacitance theory,⁽⁵⁹⁾ although it might be possible to use the data of Anderson and Goldman for such a comparison.*

*Anderson and Goldman were the first to compare tunnel junctions to the theory of Ambegaokar and Halperin.⁽¹⁰⁶⁾ Though they found qualitative agreement, they found significantly large quantitative discrepancies, due to

- (1) violation of the $C=0$ approximation and/or
- (2) the presence of significant external noise in the experiments.

If source (2) could be ruled out, then their data could in principle be compared with the finite damping calculations of Ambegaokar and Kurkijärvi.

5.A Appendix: Computer Fitting, Integration and Programs

General Fitting Procedure

The fitting procedure consisted of a one-parameter least squares fit. The relevant sections of the experimental current-voltage characteristics were digitized and stored in an APL workspace as two vectors, the current I , and the (experimental) voltage VE . Letting the theoretical voltage, corresponding to the same I vector, be VT , then the mean square deviation between the theory and experiment is given by $MSD = (LS)^{1/2}$ where $LS \equiv \sum_{\text{indices}} (VE_i - VT_i)^2$.

In order to determine the best fit of the theory to experiment the fitting parameter (whatever variable it might be) was varied until the quantity LS was minimized.

Integral Evaluation

In all cases in this chapter the theoretical predictions are represented in terms of definite integrals (or quadratures). The theoretical voltage (VT in the above paragraph) was evaluated by the so-called Gaussian integration procedure, which may be expressed as the formula,

$$\int_{-1/2}^{1/2} f(u) du = R_1 f(u_1) + R_2 f(u_2) + \dots + R_n f(u_n) \equiv \sum_{i=1}^n R_i f(u_i) \quad (5.A.1)$$

This scheme was chosen because fewer numbers of points are required (n smaller), relative to some more common techniques, in order to achieve a specified accuracy. The u_i are the zeroes of the n^{th}

Legendre polynomial $P_n(u)$. Procedures for obtaining the R_i and tables of the u_i and R_i can be found in many books on numerical analysis (e.g. Pennington, reference 107). In Figure 5.A.1 these values are tabulated for $n=10$, 16 and 20, which were sufficient for most of our purposes (cf. section 5.3.2). In practice these values only need be stored once in an APL workspace; in the specific calculations (performed according to the functions listed later below) the variables containing these values were UJ and RJ, hence before any calculation was done n was chosen and UJ and RJ specified, e.g. with $n=16$, UJ $\leftarrow$ U16 and RJ $\leftarrow$ R16. In order to use the algorithm of eq. (5.A.1) for a general quadrature of the form $\int_a^b \Psi(x) dx$ a simple change of variables is required,

$$x = (b-a)u + \frac{(a+b)}{2}$$

$$\Psi(x) = \Psi\left[(b-a)u + \frac{(a+b)}{2}\right] \equiv f(u).$$

Subsequently,

$$\int_a^b \Psi(x) dx = (b-a) \sum_{i=1}^n R_i \Psi\left[(b-a)u_i + \frac{(a+b)}{2}\right] \quad (5.A.2)$$

Specifics of Comparison with Stephen (Driven A. C. Effect)

The APL functions designed to perform the least squares fit to the formulation of Stephen are shown in Figure 5.A.2. The theoretical voltage is calculated via the three functions GVT, INT, and BES, where the latter function calculates the modified Bessel function $I_0(z)$. Before performing a calculation one must store the dynamic resistance, step center voltage and current, and temperature

Figure 5.A.1 Tabulation of the weighting coefficients for the Gaussian integration procedure.

<i>U10</i>	<i>R10</i>
-0.4869532643	0.03333567215
-0.4325316833	0.07472567458
-0.3397047841	0.1095431813
-0.2166976971	0.1346333597
-0.07443716949	0.1477621124
0.07443716949	0.1477621124
0.2166976971	0.1346333597
0.3397047841	0.1095431813
0.4325316833	0.07472567458
0.4869532643	0.03333567215

<i>U16</i>	<i>R16</i>
-0.4947004675	0.01357622971
-0.4722875115	0.03112676197
-0.4328156012	0.04757925584
-0.3777022042	0.06231448563
-0.3089381222	0.07479799441
-0.2290083888	0.0845782597
-0.1408017754	0.09130170752
-0.04750625492	0.09472530523
0.04750625492	0.09472530523
0.1408017754	0.09130170752
0.2290083888	0.0845782597
0.3089381222	0.07479799441
0.3777022042	0.06231448563
0.4328156012	0.04757925584
0.4722875115	0.03112676197
0.4947004675	0.01357622971

<i>U20</i>	<i>R20</i>
-0.4965642996	0.00880700357
-0.4819859636	0.0203007149
-0.4561172141	0.03133602417
-0.4195584859	0.04163837079
-0.3731659532	0.05096505991
-0.3180268404	0.05909726598
-0.255433501	0.06584431922
-0.1868530444	0.07104805466
-0.1138929256	0.07458649324
-0.03826326057	0.07637669357
0.03826326057	0.07637669357
0.1138929256	0.07458649324
0.1868530444	0.07104805466
0.255433501	0.06584431922
0.3180268404	0.05909726598
0.3731659532	0.05096505991
0.4195584859	0.04163837079
0.4561172141	0.03133602417
0.4819859636	0.0203007149
0.4965642996	0.00880700357

```

                                VBES[[]]V
                                V CQ+BES 2
                                [1] Z+,Z
                                [2] CQ+ + / (((pZ), (pUJ)) pRJ) * 60Z * . * 200UJ
                                V
                                V GVT[[]]V
                                V GT+GVT PP;XW
                                [1] PP+,PP
                                [2] UJ+UJ+0.5
                                [3] A2+J2*(XW+(23.84*V0)+R*X*T)
                                [4] A1+PP*XW
                                [5] GT+(R*J2+A2*2)*(500A1)+A2 INT A1
                                [6] UJ+UJ-0.5
                                V

                                VCOMPFIT[[]]V
                                V COMPFIT;TERM;S;M
                                [1] TIME
                                [2] 'ENTER INDICES'
                                [3] INDICES+[]
                                [4] 'INITIAL J2'
                                [5] J2+[]
                                [6] LS+ + / (VE[INDICES]-GVT I[INDICES])*2
                                [7] 'ENTER TERM,'
                                [8] TERM+[]
                                [9] 'ΔJ2'
                                [10] S+[]
                                [11] M+ 1
                                [12] M+M+1
                                [13] +((J2 COMPARE(J2+S))=2)/12
                                [14] +(M=0)/16
                                [15] +((J2 COMPARE(J2-S))=2)/15
                                [16] +(TERM≤S+S+10)/11
                                [17] ''
                                [18] FITNUM; ' T(OHMS)=';INFO[1]; ' FREQ.=';INFO[2]; ' STEP NO.=';INFO[3]; ' SENS=';INFO[4]; ' J2(NOISE)=';INFO[
                                S]
                                [19] ''
                                [20] 'J2=';J2; ' LS=';LS; ' R=';R; ' J=';J; ' T=';T; ' V0=';V0; ' INDICES=';INDICES
                                [21] DEGREE
                                [22] TIME
                                V

                                VINT[[]]V
                                V GR+A2 INT A1;UY
                                [1] A1+,A1
                                [2] UY+OUJ+2
                                [3] GR+ + / (((pA1), (pUJ)) pRJ) * (60Z * A1 * . * UY) * (((pA1), (pUJ)) pBES 2 * A2 * 20UY) * 0 + 2
                                V

                                VCOMPARE[[]]V
                                V Q1Q+A COMPARE B;BU;INDEX;W
                                [1] BU+A,B
                                [2] J2+B
                                [3] J2+BU[INDEX+W\LS+[]/W+LS,(+/(VE[INDICES]-GVT I[INDICES])*2)]
                                [4] Q1Q+INDEX
                                V

```

Figure 5.A.2 APL listing of functions used in fitting the fluctuation theory in the driven a.c. Josephson effect.

in the global variables R, V0, J, and T respectively. (Also UJ and RJ must be defined, as indicated in the previous paragraph.) The units required are ohms, microvolts, microamps, and degrees Kelvin. (These units apply to I, VE, and the theoretical voltage also.) To simply calculate the theoretical voltage for a given vector of current points I, one then evaluates GVT I.

In order to do a complete fit to a set of experimental points VE (corresponding to I) one initiates the procedure by typing START and entering information as called for. The fitting parameter incrementation and reiterations are performed by the function COMPFIT (called by START). In response to the command 'ENTER INDICES' one enters the indices of the experimental points one wishes to compare with theory (i.e. the indices of VE). The fitting parameter in this case is J2 (one half the no-fluctuation step height); $\Delta J2$ is the initial increment chosen and must be a power of 10, eq. 0.1. The variable TERM specifies how far one wishes to carry out the fitting parameter and again must be a power of 10, e.g. .0001. At the heart of the fitting algorithm is the two-argument function COMPARE. This function compares LS (cf. first paragraph) for the two arguments (which are the current value of the fitting parameter and its incremented value) and returns as a result an index number, 1 if the current fitting parameter gives a smaller LS value, and 2 if the new fitting parameter gives the smaller LS. -- in the latter case J2 and LS are assigned the new values. A flowchart of the essential operation of COMPFIT is shown in Fig. 5.A.3; in this chart M is an internal

Figure 5.A.3 Flowchart of the fitting procedure for the driven a.c. effect.

counter and S is the current value of the increment.

The functions DEGREE and TIME (called by COMPFIT) are auxiliary functions and are not listed here. DEGREE was used merely to give a measure of the degree of the fit of theory to experiment according to various criteria (cf. section 5.3.2). TIME simply indicated the CPU time used for the calculation; TIME is included in the function listings of appendix 6.C.

Specifics of Comparison with Ambegaokar and Halperin (D.C. Effect)

The fitting procedure used for comparison with Ambegaokar and Halperin was essentially identical to that outlined above for the driven a.c. effect, since the theoretical equations have the same form in the driven a.c. and d.c. effects. In particular the functions on the left of Figure 5.A.4 in combination with INT and BES of Figure 5.A.2 constitute the required set of functions analogous to the driven a.c. effect. In particular BFIT is analogous to START and COMPFIT combined, and AHVT is analogous to GVT. The only (trivial) differences are that R and T must be stored prior to the calculation and all indices of VE are used.

It has already been pointed out in the main text that the form which we preferred to use was significantly simpler than the form of the theory given in the initial paper of Ambegaokar and Halperin. In our calculations we actually used both forms of the quadrature. At the right of Figure 5.A.4 is given one particularly useful version of the function (AHVT) used to evaluate equation (9) (in unnormalized form) of Ambegaokar and Halperin. The function UPHI is the potential $U(\phi)$ and the function IPHI is the indefini

```

      VAHVT[0]V
      V GT-AHVT PP;XW
[1] PP+,PP
[2] UJ+UJ+0.5
[3] A2+I1*(XW+(23.84*UJ)+R*X*T)
[4] A1+PP*XW
[5] GT+(R*I1+A2*2)*(500A1)+A2 INT A1
[6] UJ+UJ-0.5
      V

```

```

      VBFIT[0]V
      V BFIT;TERM;S;M
[1] TIME
[2] 'CHECK R,T'
[3] V0+R
[4] J+1
[5] 'ENTER I1'
[6] I1+0
[7] LS+/(VE-AHVT I)*2
[8] 'ENTER TERM,'
[9] TERM=0
[10] 'ENTER ΔI1'
[11] S=L
[12] L4:M+1
[13] L1:M+M+1
[14] +((I1 COMPARE(I1+S))=2)/L1
[15] +(M=0)/L2
[16] L3:+((I1 COMPARE(I1-S))=2)/L3
[17] L2:+(TERM≤S+S+10)/L4
[18] JSD+LS+0 I
[19] GAMMA+2*A2
[20] 'GAMMA=';GAMMA;' T=';T;' R=';R;' I1=';I1;' LS=';LS;' MEAN SQ. DEV.=';MSD
[21] TIME
      V

```

```

      VCOMPARE[0]V
      V QQ+A COMPARE B;W;BU;INDEX
[1] BU+A,B
[2] I1+B
[3] I1+BU[INDEX+W\LS+[/W+LS,(+/(VE-AHVT I)*2)]
[4] QQ+INDEX
      V

```

```

      VAHVT[0]V
      V RAT-AHVT PP;FUJ;GUJ;HUJ;L1;RES;L3;K;IT;TERM1
[1] IT+PP+,PP+K+0
[2] RES+10
[3] GAMMA+I1*47.7+T
[4] PP+IT[X+K+1]
[5] FUJ+*(UPHI(L1+01+2*UJ))-AZZA
[6] TERM1+(*-AZZA)*(+(-*-AZZA)+*(OGAMMA*PP+I1)-AZZA))*(02*+/RJ+FUJ)*(02*+/RJ+FUJ)
[7] L3+(((02)-L1)*.XUJ)+((02)+L1)*.X((0UJ)0.5)
[8] HUJ+*(-UPHI L3)+AZZA
[9] GUJ+((02)-L1)*+/(((0RJ),(0RJ)0RJ)*HUJ
[10] RES+RES,I1*R*04+GAMMA*(TERM1+02*+/RJ+FUJ+GUJ)
[11] +(K<0IT)/4
[12] PP+IT
[13] RAT+RES
      V

```

```

      VUPHI[0]V
      V XX+UPHI TU
[1] +2+00TU
[2] +0*00(XX+GAMMA*0.5*(IPHI TU)+PP*UJ+I1)
[3] +0*00(XX+GAMMA*0.5*((0PP),(0UJ)0IPHI TU)+PP*.XUJ+I1)
[4] +0*00(XX+GAMMA*0.5*((0PP),(0UJ),(0UJ)0IPHI TU)+PP*.XUJ+I1)
      V

```

```

      VIPHI[0]V
      V YY+IPHI Y
[1] YY+20Y
      V

```

Figure 5.A.4 APL listing of functions used to perform fits of the theory in the d.c. effect.

integral of the normalized current-phase relation, in this case $\cos \phi$ (corresponding to $I_s = I_1 \sin \phi$). To experiment with different current phase relations one simply modifies IPHI.

Functions for Comparison with the Low Frequency Noise Theory
(ala Vernon and Kanter)

The fitting algorithm used here was again analogous to that described above, the functions are listed in Figure 5.A.5. In this case VFIT corresponds to START and COMPFIT, VERNON corresponds to GVT, and VCO corresponds to COMPARE. The fitting parameter automatically incremented was I_1 , the no-noise critical current; the external noise parameter IN was incremented manually. The function VTAB (called in line (19) of VFIT) was just an auxiliary function used to tabulate the results, and is not included here. The integration limits are given by the variables AF and BF (internal to VERNON) evaluated in lines (5), (6), (8) and (9) of VERNON. The upper and lower cutoffs to the integration were set at $\pm A \times IN$. We found that sufficiently accurate (better than 0.2% accuracy in all points) and consistent results were obtained with $A=5$ and $n=20$. However for the fits of Figs. 5.13 and 5.14, $A=10$ and $n=32$ were used; $U32$ and $R32$ are tabulated in Fig. 5.A.5. We should note in passing that we found that our calculations disagreed somewhat with the numerical results published by Vernon and Kanter (though we do not disagree with their conclusions). Since we rechecked our results with many choices of AF , BF , and integration intervals (n) and reached the same conclusion, we suggest that their numerical calculations are slightly in err.

```

      VVERNON[[]]V
V PT+VERNON WW;AF;BF;JJ;F;LL
[1] JJ1+JJ2+10
[2] LL+0
[3] WW+,WW
[4] FOX:F+WW[LL+LL+1]
[5] AF+I1
[6] BF+A*IN
[7] JJ1+JJ1,(BF-AF)*+/RJ*DGN
[8] AF+-A*IN
[9] BF+-I1
[10] JJ2+JJ2,(BF-AF)*+/RJ*DGN
[11] +(LL<0WW)/FOX
[12] PT+I1*R*(JJ1-JJ2)+IN*(02)*0.5
V

      VVFIT[[]]V
V VFIT;TERM;S;M;RATIO
[1] TIME
[2] 'CK R,I1'
[3] 'ENT. IN'
[4] IN+[]
[5] LS+/(VE-VERNON I)*2
[6] 'ENT. TERM.'
[7] TERM+[]
[8] 'ENTER ΔIN'
[9] S+[]
[10] M+1
[11] M+M+1
[12] +((IN VCO(IN+S))=2)/11
[13] +(M=0)/15
[14] +((IN VCO(IN-S))=2)/14
[15] +(TERM≤S+S+10)/10
[16] RATIO+IN+I1
[17] ' I1=';I1;' R=';R;' IN=';IN;' L.SQ.=';LS;' IN/I1=';RATIO
[18] TIME
[19] VTAB
V

      VVCO[[]]V
V RV+AA VCO BB;W;RU;INDEX
[1] BU+AA,BB
[2] IN+BB
[3] IN+BU[INDEX+W\LS+[/W+LS,(+/(VE-VERNON I)*2)]
[4] RV+INDEX
V

      VDGN[[]]V
V GQ+DGN;X;ZC;L
[1] X+((AF+BF)÷2)+(BF-AF)*UJ
[2] ZC+*(-0.5*((X-F)*2)÷(IN*2))
[3] L+(-1+(X+I1)*2)*0.5
[4] GQ+ZC*L
V

```

	U32	R32
	-0.4986319309	0.003509305005
	-0.4928057558	0.008137197365
	-0.4823811278	0.01269603265
	-0.467453038	0.01713693146
	-0.4481605779	0.02141794901
	-0.4246838069	0.02549902963
	-0.397241898	0.02934204674
	-0.3660910594	0.03291111139
	-0.3315221335	0.03617289705
	-0.2938578786	0.03909694789
	-0.2534499545	0.04165596211
	-0.2106756381	0.0438260465
	-0.1659343011	0.04558693935
	-0.1196436811	0.04692219954
	-0.07223598079	0.04781936004
	-0.02415383284	0.04827004426
	0.02415383284	0.04827004426
	0.07223598079	0.04781936004
	0.1196436811	0.04692219954
	0.1659343011	0.04558693935
	0.2106756381	0.0438260465
	0.2534499545	0.04165596211
	0.2938578786	0.03909694789
	0.3315221335	0.03617289705
	0.3660910594	0.03291111139
	0.397241898	0.02934204674
	0.4246838069	0.02549902963
	0.4481605779	0.02141794901
	0.467453038	0.01713693146
	0.4823811278	0.01269603265
	0.4928057558	0.008137197365
	0.4986319309	0.003509305005

Figure 5.A.5 APL listing of functions used to fit the low frequency noise modulation formulation.

6. OTHER RESULTS

6.1 Introduction

It was found that the data taken in the course of our fluctuation-oriented experiments also yielded additional information on topics of current interest in the burgeoning study of superconducting weak links. Our well-shielded and well-controlled apparatus provided a very suitable environment for studying some weak-link phenomena at temperatures very close to T_c , where an abundance of literature data is noticeably lacking. We also felt it was of interest to study a variety of weak-link structures all within the same experimental environment.

The results contained in this chapter are presented by necessity in a somewhat open-ended and qualitative fashion. Though in some instances detailed quantitative analysis was undertaken and in some cases heretofore unreported new results are indicated, the chief intent of this chapter is to serve as a supplement to the previous chapter. Not all of our observations are reported here, including cursory demonstrations of detection and mixing with niobium point contacts, and observations of subharmonic steps in tin thin film bridges (subharmonics were not observed by us in either our point contacts or our tin whiskers).

A reasonable question is whether our tin film bridges consist essentially of one weak link or several weak links in series. We feel that for all practical purposes a bridge effectively constitutes a single structure. This opinion is based upon the smooth continuous nature of the resistive transitions of these bridges,

as can be observed in Figure 6.18 (for the present refer only to the "no-r.f." data). The total transition consists of two parts, the bridge transition (starting at the lower temperature), and the transition of the film electrodes (the smaller rise near the end of the total transition). Were the bridge composed of several sections of differing critical currents, a significant amount of structure would be evident on these curves (such structure is quite evident for "damaged" whiskers crystals).

6.2 Current - Voltage Characteristics -- No Applied Fields

In the preceding chapter study of fluctuation rounding of step structures involved relatively small voltage ranges centered at the step centers. In this section we examine the overall shapes of the entire I-V curves for the various types of samples with no applied fields. In all cases the supply was a current source and critical current values in our regime of study were in almost all cases $< 100 \mu A$, (i.e., relatively low).

We compare the data with two distinct negligible - capacitance no - fluctuation models discussed in chapter 2, namely: (1) the simple resistor - supercurrent model, which predicts the hyperbolic I-V for $I \geq I_1$,

$$V = R(I^2 - I_1^2)^{1/2} \quad I \geq I_1 \quad (6.1(a))$$

and (2) the relaxation oscillation model, or inductance model which (using the Deaver - Pierce formulation) predicts,

$$V = \frac{-I_1 R}{\ln[1 - \frac{I_1}{I}]} \quad I \geq I_1 \quad (6.1(b))$$

For convenience we shall refer to these models as model I and model II in the rest of this section. The two forms (6.1(a)) and (6.1(b)) differ quantitatively in both the small and large voltage limits. For small voltages, $V/I_1 R \ll 1$, the differential conductances for the respective models are,

$$\frac{dI}{dV} \approx \frac{V}{I_1 R^2} \quad (6.2(a))$$

and

$$\frac{dI}{dV} \approx \frac{I_1^2 R}{V^2} \exp - \frac{I_1 R}{V} \quad (V/I_1 R \ll 1) \quad (6.2(b))$$

Thus in the region near the critical current the relaxation model I-V is the flatter. In the opposite extreme, $V/I_1 R \gg 1$, the average supercurrents $\bar{I}_s = I - \bar{V}/R$, in the respective models are,

$$\bar{I}_s \approx \frac{1}{2} I_1 \frac{I_1 R}{V} \quad (6.3(a))$$

$$\bar{I}_s \approx \frac{1}{2} I_1 \quad (I_1 R/V \ll 1) \quad (6.3(b))$$

Thus in the large voltage limit model II displays a constant average supercurrent whereas in the simpler model, model I, the average supercurrent diminishes as V^{-1} . (Eventually of course for large enough currents the relaxation model must breakdown and $\bar{I}_s \rightarrow 0$). In both cases

$$\frac{dI}{dV} \rightarrow \frac{1}{R} \quad \text{for} \quad \frac{I_1 R}{V} \ll 1.$$

For very small critical currents, I_1 , we know from the previous

chapter that intrinsic thermal fluctuations lead to a depression of I_1 and distortion of the low voltage region. For this regime, in the following analysis, we used a one parameter fit to AH in order to obtain I_1 , just as in the previous chapter. For increasingly larger I_1 the relative fluctuation effects decrease and consequently I_1 may be readily determined by a simple extrapolation of the data. The only other experimental parameter in the eqs. (6.1(a)) and (6.1(b)) is the normal state resistance R , which may be determined as the slope of the I-V for $I_1 \rightarrow 0$. The experimental I_1 values were varied by varying temperature.

6.2.1 Point Contacts

In Figure 6.1 are shown some point contact I-V's for the small I_1 regime (V-origins shifted for clarity). The predictions of models I and II are shown as the dashed and dotted lines respectively. The insert, (b'), shows the results of fitting to AH using in one case a very small total voltage range, ($V \leq 1.8 \mu V$), ($I_1 = 1.546$ case) and in the other case using the entire voltage range shown in (b), ($I_1 = 1.63$ case). The resultant I_1 's differ by $\sim 5\%$, thus indicating relatively good consistency between the low and high voltage region of (b) with respect to the no - fluctuation limit of AH, which is model I. Figure 6.2 shows the same point contact for larger critical currents (lower temperatures). In plotting the predictions of models I and II the same R value (determined as stated above) was used for all of the curves of Figs. 6.1 and 6.2.

Before proceeding, we point out two items which we will

Nb POINT CONTACT

$$R_N = 1.273 \Omega$$

--- MODEL I

..... MODEL II

— EXPERIMENT

Figure 6.1 Point contact static I-V curves for the small I_1 regime (cf. text).

Figure 6.2 I-V's for the same point contact as in Fig. 6.1 but for larger I_1 (lower T).

Nb POINT CONTACT

$R_N = 1.273$

--- MODEL I
 MODEL II
 — EXPERIMENT

not be concerned with here. First, as can be seen in Figure 6.2 there is a structure at about 12 μ V. The relative amplitude of this structure was observed to first increase and then decrease as I_1 was increased, however the voltage position of the structure was invariant. The frequency corresponding to this voltage is ~ 5.8 GHz which corresponded very closely to the estimated lowest cavity mode of our cylindrical inner copper can -- thus we believe this structure to be the cavity induced resonance seen by several authors. (108) The second point we wish to make here is that, except for the tin whiskers, all of our sample I-V's are eventually (for low enough temperature) affected by the electrodes. The beginning of the transition of the electrodes is indicated by the bending of the I-V toward the right in the large current region, as can be observed in Fig. 6.2(d).

In Figs. 6.1 and 6.2, whereas model I appears to fit the low I_1 cases better (after accounting for intrinsic fluctuation effects), model II appears to fit the large I_1 cases better -- however, model II does not give very satisfactory agreement in the low voltage region. Moreover, there are indications in both figures that better agreement with eqs. 6.1(a) and 6.1(b) would be obtained by allowing R to decrease as I_1 increases. (This phenomenon was first pointed out in chapter 5.) Following up on this clue we fit models I and II to the data, allowing R to be a fitting parameter. The result was that model I gave a much better fit than model II; in Figure 6.3 we replot the data (of Fig. 6.1(a)-(c) and Fig. 6.2(a)-(c)) and the predictions of model I using the best-fit R values. The degree of agreement in this latter fitting is so

Figure 6.3 The point contact data of Figs. 5.1 and 5.2 replotted along with best fits allowing R_N to vary.

convincing that we believe the temperature dependence of R may indeed be real. A plot of the best-fit R vs. T in Figure 6.4 shows that the relationship is remarkably linear. Though we make no assertions regarding the origin of this decreasing resistance (the bulk resistivity of normal metals varies as T^5 at low temperatures, (109) hence this does not appear to be the source), one might reason that for a very short weak link, length $< \xi(T)$, the resistance would be determined by the "effective" length of the weak link as determined by $\xi(T)$.

Thus we conclude that point contact I-V characteristics are well described by the hyperbolic relationship 6.1(a), with the additional provision that the normal state resistance decrease slightly with temperature. This hyperbolic behavior in point contacts is in agreement with the observations of Gubankov et al. (110) (the most complete systematic study we know of) in the regime where heating effects and vortex state modes are not present. This observation, together with the observed $I_1(T)$ relationship (cf. appendix 6.B) show that our point contacts behave very much like S-N-S junctions. (78,79) Exceptions to the above conclusion appear only at the lowest temperatures (largest I_1 's) investigated where the relaxation oscillation model may be a better approximation, e.g. see Fig. 6.2(d); this would be expected to occur when $L > \frac{\phi_0}{2\pi I_1}$ where L is the inductance of the weak link, and ϕ_0 is the flux quantum. (111) As the temperature was lowered still further (I_1 increased) eventually a point was reached where abrupt switching from the zero-voltage state to the voltage state was

Figure 6.4 Plot of best-fit R_N vs. T for the data of Fig. 6.3.

observed -- this appeared to be a heating effect, which will be discussed below in connection with the identical phenomenon in thin film bridges.

6.2.2 Thin Film Bridges

In this section we display I-V data, analogous to the preceding section, on the same thin film bridges discussed in the previous chapter. The wide range I-V characteristics of our thin film bridges of wedge geometry were very similar to the data to be shown here for the 1.5μ long bridge of rectangular geometry. Throughout the following discussion it should be kept in mind that the shorter bridges were wider than the longer ones (cf. appendix 6.A).

Figure 6.5 displays I-V data for an 8μ long bridge. All of the curves could be fit (varying R) quite well with eq. 6.1(b) (model II). For curve (e) we show the best fit (varying R) for both models. Except for possibly the smallest I_1 case (where fluctuation rounding obscures things very greatly), model II fit much better than model I; this is in contrast to the point contact data of the previous pages. Just as in the point contact we found that the best-fit R value decreased slightly (and linearly) with temperature.

In Figure 6.6 some data for a 3μ long bridge is displayed. There are two points of interest here, (1) first, for the lower I_1 the data fits model II fairly well, (2) whereas, for larger I_1 one begins to observe deviation, as though the "excess" super-current ($\bar{I}_s = I - \bar{V}/R$) were greater than $\frac{I_1}{2}$. This sort of deviation

Figure 6.5 D.C. I-V data and theoretical (no fluctuation) model fits (temperature or I_1 as parameter) for an 8μ long $\times$ 1.1μ wide tin thin film bridge.

TIN BRIDGE
 LENGTH $\sim 8 \mu$
 WIDTH $\sim 1.1 \mu$

Figure 6.6 D.C. I-V data and theoretical (no fluctuation) model fits for a 3μ long x 1.5μ wide tin thin film bridge.

TIN BRIDGE

LENGTH $\sim 3\mu$

WIDTH $\sim 1.5\mu$

(a) $x = 0.3\mu V$
 $y = 3\mu A$

(b) $x = 2\mu V$
 $y = 20\mu A$

is even more prominent in the shortest (1.5μ long) bridge -- see Figure 6.7. Again for the lowest I_1 's the data is fit by model II fairly well, whereas as I_1 is increased eventually deviations are observed, and these deviations increase as I_1 is increased further. The I_1 value at which the deviation first appears is smaller for the 1.5μ bridge than for the 3μ bridge.

We hypothesize that this deviation may be due to impending two-dimensionality of the electrode regions immediately adjacent to the weakest part of the link. As the temperature is decreased the coherence length and penetration length decrease and so the effective relative length of the weakest part of the link shrinks, however the slightly wider adjacent regions near the ends of the link are still "weaker" than the very wide sections of the electrodes further out --- effectively then the case would be like three links in series, the two outer ones being stronger (slightly higher critical current) than the inner one. (More accurately one could consider a link whose critical current varies continuously as a function of position along the length of the link). As a consequence the transition of the center link represents only a fraction of the total transition and so in an I-V one observes an average supercurrent $> \frac{I_1}{2}$. Naturally this effect would be more prominent the shorter, wider, and more non-uniform the link; this is indeed the case here (cf. dimensions in appendix 6.A and Fig. 3.11). In essence then we are observing here the effects of non-abrupt transition (in the geometry) from the weak link to the bulk two-dimensional electrodes; were the rectangular geometry perfect one would expect to see only the weak link properties in the I-V.

Figure 6.7 D.C. I-V data and theoretical (no fluctuation) model fits for a 1.5μ long x 2.7μ wide tin thin film bridge.

The hypothesis of the preceding paragraph is supported by the data of Figure 6.8 which shows an I-V of a "short" through an S-I-S tunnel junction.* (i.e., this is an instance of a "bad" tunnel junction). The length of this superconducting shorting section is just the insulation thickness, $\sim 50\text{\AA}$, i.e. very short. Then from the normal state resistance obtained from the I-V and using the resistivity of the electrodes we estimate the diameter of the link to be $\sim 1890\text{-}4000\text{\AA}$. The electrodes are thick ($>\lambda$) and the total area of the original tunnel junction was $\sim 1.3 \times 10^{-8} \text{ m}^2$, which is approximately a factor of 2×10^5 greater than the superconducting short crosssectional area, hence the discontinuity from the weak link to the electrodes is very abrupt. We estimate the damping parameter β to be $\sim .06$, hence we still expect a highly damped model to hold. In Fig. 6.8 (c) the data of (a) is replotted with model II overlaid -- as can be seen the agreement is quite good. Hence for this very short but very abrupt weak link the relaxation oscillator model predicts the I-V quite well; we assert that if we could have made the electrode-to-link interconnection just as abruptly in our 1.5μ long bridges they too would follow the predictions of model II over a much wider range of temperature (or critical current).

In our estimation, for the 1.5 bridge, the point of departure from strictly model II behavior occurs at the temperature where $\text{width}/\xi(T) = .97$ and $\text{width}/\lambda(T) = 3.5$. We conclude that in our observation of progressively changing I-V behavior

*Data courtesy of IBM

Figure 6.8 D.C. I-V of a "short" through an SIS tunnel junction.
Scales: (a) 2mA and 10 μ V per large division; (b) 5mA
and 50 μ V per large division; (c) same as (a).

(A) **(B)** **(C)**

(from the 8 to 1.5 μ bridges) we have indeed isolated a geometrical effect, but this effect is unrelated to the length (the tunnel junction "short" was very short but displayed an I-V similar to our 8 μ bridge); rather the effect may be due to a trend toward bulk two-dimensional behavior in the wider sections of the bridges near their ends. Though the theoretical work of Likharev on wide structures does not specifically apply (Likharev considers type II, ours appear to be type I) our observations show similarities to some of his results.

As I_1 was increased eventually a point was reached where abrupt switching out of the superconducting state was observed. Further increases in I_1 led in addition to hysteresis. This phenomenon was common to both the tin film bridges and the point contacts, although no observable hysteresis was observed for the point contacts, down to the low temperature obtainable. If one assumes that the observed hysteresis in the bridges is due to finite capacitance ala McCumber & Stewart, one obtains a capacitance value on the order of 130 pf which is several orders of magnitude larger than calculated values based on the geometry, thus this explanation of the hysteresis appears very dubious. We rather believe that the hysteresis is a Joule heating effect, similar to that observed by Skocpol et al. (112) and Gubankov et al. (110). The idea here is that for very large critical currents, when a small section of the weak link goes normal, Joule heating in this section is sufficient to induce a transition in adjoining sections,

i.e. the effect is a thermal propagation effect.⁽¹¹³⁾ Gubankov et al. derive a modification to model I (eq. 6.1(a)) assuming that this self-heating effect is pertinent; this is

$$V^2 = I^2 R^2 - [I_1^\dagger R - \frac{V}{(I/I_t)}]^2 \quad (6.4)$$

where $I_1^\dagger$ denotes the critical current in the absence of heating (which is just the upper critical current in the hysteresis loop); and I_t is the thermal current, determined by material thermal properties. The characteristics derived from eq. (6.4) have negative resistance regions near the critical current, for sufficiently large I_t , -- hence with a current source one will observe switching and hysteresis.

A corresponding hysteresis occurs of course in the resistive transitions of the bridges (V-T curves), an example of which is shown in Figure 6.9 (vertical zeroes shifted, for clarity). For this data we make a very rough calculation to determine if the above ideas are plausible. For a given V-T curve let T_1 be the temperature of switching as temperature is swept up, and T_2 be the other case. At T_1 the device temperature, T_d , has not had a chance to rise, so $T_d = T_1$. On the other hand self-heating is occurring for $T = T_2$ and we have the same critical current, so that $T_d = T_2 + (\Delta T)_{\text{heat}}$. Therefore the temperature rise due to heating is roughly $(\Delta T)_{\text{heat}} = T_1 - T_2$. But we expect $(\Delta T)_{\text{heat}} \sim I_1^2 R$. Therefore

Figure 6.9 Resistive transition (voltage vs. temperature) for an 8μ long $\times$ 1μ wide tin bridge for two relatively large critical currents.

(1) 1 UNIT = $5\mu\text{V}$ $I = 50\mu\text{A}$

(2) 1 UNIT = $10\mu\text{V}$ $I = 100\mu\text{A}$

T SCALE -- 1 UNIT $\approx 26\text{mK}$

for the two curves of the figure we expect $\frac{(T_1 - T_2)_1}{(T_1 - T_2)_2} \sim \frac{(I_1^2)_1}{(I_1^2)_2}$.

From the data of Fig. 6.9 we get $\frac{(T_1 - T_2)_1}{(T_1 - T_2)_2} = 0.29$, and

$\frac{(I_1^2)_1}{(I_1^2)_2} = 0.25$. Considering the crudeness of the calculation the

agreement is reasonable. The onset of switching and hysteresis occurred at a lower I_1 level for the longer bridges than the shorter ones -- this fact is also compatible with the heating model, since for a shorter link the heat is dissipated into the electrodes more rapidly. A similar heating effect hysteresis was observed in some of our Wyatt effect measurements (cf. section 6.3.3).

6.2.3 Whiskers

For comparison with the above short links we show in Figure 6.10 an I-V and a V-T curve of a "good"* Sn whisker crystal. The whisker is 1.5μ in diameter and 325μ long. Along with the I-V is shown a line of slope $\frac{1}{R}$. Note that the I-V displays the same sort of "excess" supercurrent characteristic of model II. However, this I-V appears to differ from those of short bridges for voltages near zero where for the same I_1 there appears to be considerably more fluctuation rounding than in the bridges. This observation is consistent with the suggestions of Rieger et al. that for narrow ($W < \xi$) long structures ($\ell \gg \xi$) there exist a multitude of possible points

*according to the criterion of Warburton (thesis)

Figure 6.10 D.C. I-V curve and V-T curve for a good tin whisker crystal.

for phase slip to occur, consequently the resulting phase slip process is quite noisy.⁽⁶⁷⁾ Fluctuations in long filamentary superconductors have been the subject of extensive experimental and theoretical study in recent years (cf. the thesis of R. J. Warburton, for example).⁽⁴⁷⁾

6.2.4 Summary

In the low critical current regime we find that the highly damped resistor-shunted-supercurrent model (McCumber; Stewart; Waldram et al.; Aslamazov-Larkin) quite adequately describes the static I-V of point contacts. This result and the rapid $I_1(T)$ dependence shown in appendix 6.B indicate that the conduction mechanism in our point contacts (cf. chapter 3 for fabrication details) may be very similar to those in S-N-S tunnel junctions. On the other hand for our weak links in which the interconnecting region between the electrodes was definitely a superconductor (i.e. our thin film bridges), the I-V's are adequately describable by the relaxation oscillation model (Notarys & Mercereau; Rieger et al.; Deaver & Pierce). Both of the above statements must be qualified, however, to allow that $R(T)$ decrease slightly with decreasing temperature -- to our knowledge this observation has not been previously pointed out in the literature.

As the temperature is lowered geometrical effects play an ever increasingly significant role particularly the more non-uniform the width of the link. The distributed critical current dependence across a non-abrupt weak link (equivalently -- smeared

out boundary conditions) manifests itself as an excess supercurrent in excess of $I_1/2$ (the expected value in model II). As the temperature is lowered still further (increasing I_1), self-heating effects predominate.

We suggest that most all of the available high critical current data on thin film bridges shows evidence of geometrical and heating effects, though they are not always recognized as such. Obviously, in order to reliably predict the properties of devices made with weak links it is desirable to have at one's fingertips a high fidelity model of the weak link which applies over a wide temperature range. For single-material thin film weak links made by current techniques this appears not to be the case, because of the geometrical and heating effects, and the fact that if low enough currents are used to avoid these effects, intrinsic fluctuation effects subsequently become significant. Geometrical effects could probably be eliminated by making the transition from weak link to electrode more abrupt, for example by the use of electron-beam resist techniques, or as Notarys and Mercereau have shown, by the use of a normal metal overlay of well defined geometry (proximity effect bridge). In order to eliminate heating effects it would appear advantageous to make the link as short as possible and to provide for very low resistance thermal conduction paths away from the link. We suggest that a weak link consisting of a "1-dimensional" short through a thin insulating film separating two 3-dimensional electrode films provides one means of

eliminating both geometrical and heating effects and at the same time gives a structure compatible with striplines*, (striplines being a very sensible way to couple into thin film weak links). By varying the insulator thickness one could vary the length of the links and thus make comparisons to the length dependent contemporary theories all of which assume rather ideal boundary conditions which may not hold for the more conventional thin film structures.**

*It appears that several Russian authors have studied devices of this nature e.g. see ref. 114.

**One possible technique for producing controllable diameters might be to use electron beam exposure to form a pinhole (or several pinholes) in a thin photoresist insulating layer, followed by evaporation of a thick counterelectrode.

6.3 Some R. F. Properties

6.3.1 Thresholds for R. F. - Induced Steps

The criterion commonly used to predict the low frequency threshold, v_t , for r.f.-induced steps in a Josephson junction is^(34,66) $v_t \sim (\Delta v)_{\text{thermal}}$ where $(\Delta v)_{\text{thermal}}$ is the thermal noise generated linewidth of the a.c. Josephson oscillations, (cf. eq. 2.46),

$$(\Delta v)_{\text{thermal}} = \frac{4\pi k_B T_d^2}{\Phi_0^2 R_s} \quad (6.5)$$

with R_d and R_s the dynamic and static resistances respectively, at the bias point.

The basic system parameter occurring in equations describing the synchronization of a Josephson oscillator to an external oscillator is the frequency $\frac{2eR_d j_2}{h}$ (cf. eq. 2.51), which may be thought of as a "synchronization bandwidth".⁽¹⁶⁾ The scale of the step amplitude $2j_2$ is set by the critical current I_1 (cf. chapter 2 and the next section). Thus we write the synchronization bandwidth $(\Delta v)_{\text{synch}}$,

$$(\Delta v)_{\text{synch}} = \frac{a I_1 R_d}{\Phi_0} \quad (6.6)$$

where a is a dimensionless parameter < 1 .

We assert that for any given experimental situation it is the larger of (6.5) and (6.6) which determines the low frequency

threshold. Minimum bandwidths are obtained for $R_d, R_s \rightarrow R_N$, the normal state resistance. In this regime

$$(\Delta v)_{\text{thermal}} \rightarrow \frac{4\pi k_B T R_N}{\phi_0^2} \quad (6.7(a))$$

$$(\Delta v)_{\text{synch}} \rightarrow \frac{a I_1 R_N}{\phi_0} \quad (6.7(b))$$

Thermal fluctuation effects are substantial when $\frac{\hbar I_1}{2e} \sim k_B T$; if we substitute this value of I_1 into (6.7(b)) we find

$$(\Delta v)_{\text{synch}} \rightarrow \frac{a 2\pi k_B T R_N}{\phi_0^2} \quad (I_1 \rightarrow 0) \quad (6.8)$$

which is similar to (6.7(a)). Hence in the determination of the absolute lowest threshold frequency possible for a given junction eqs. (6.5) and (6.6) yield approximately the same result, whereas (6.6) also yields information about relative frequency thresholds as a function of the critical current. Thus the criterion

$$v_t \sim (\Delta v)_{\text{synch}} = \frac{a I_1 R_N}{\phi_0} \quad (6.9)$$

is more general than (but compatible with) the absolute minimum threshold given by $v_t \sim (\Delta v)_{\text{thermal}}$.

Experimentally we observed that for large critical current values no low frequency steps could be induced in the region of large R_d (near $V \rightarrow 0$) until the critical current was sufficiently

suppressed (shown in next section). We also observed that after I_1 suppression the threshold frequency then observable, ν_t , was indeed an increasing function of I_1 , as indicated by eq. (6.9). In Figure 6.11 we display ν_t vs. $I_1 R_N$ for three tin bridges and a niobium point contact. One can observe that to a rough approximation the expected criterion (6.9) is followed. The large amount of scatter in the data is due to the inherent subjective nature of any criterion used in the experimental determination of ν_t . In our (rough) measurements we pinpointed ν_t down using discrete frequencies; the bars on the data indicate that ν_t lies somewhere between the endpoints. The slope of the solid line gives an empirical value of $a=0.11$ for the parameter of eq. (6.9). With this value of a then, to within about a factor of 2, eq. (6.9) correctly predicts the experimentally observed low frequency threshold values for our data.

According to the arguments of Rieger et al. and Likharev the phase slip process for a very long narrow link is expected to be quite noisy, and therefore one expects phase coherence across the entire length to eventually disappear as the length, L , is increased. For a type I superconductor (which is what our films were, according to the estimates of appendix 6.A) the coherence breakdown is expected to occur for $L \sim k \xi(T)$ where k is on the order of unity or slightly greater. For a type II superconductor a corresponding breakdown is expected for $L \sim k \lambda_L(T)$, where $\lambda_L \equiv \lambda \coth \frac{d}{2\lambda}$ with d the film thickness⁽⁸⁴⁾. The presence of an external synchronizing field might be expected to reduce the randomness of the phase slips somewhat and therefore yield a somewhat larger

Figure 6.11 Threshold frequency for r.f.-induced steps as a function of the product $I_1 R_N$ for three Sn bridges and a Nb point contact.

value of k . With this as motivation we attempted to observe r.f.-induced steps in our tin whisker crystals. An extensive effort was made to see this effect in pure Sn and 2%In-Sn filamentary single crystals over a frequency range 250-960 MHz, a critical current range 0.01-50 μ A, and length range 100-500 μ . The result was that in all instances where the whiskers were definitely undamaged, as evidenced by structureless no-r.f. V-T characteristics and I-V characteristics, similar to Fig. 6.10, no steps were observed. (In certain instances where steps were observed, these were attributable to weak spots in a damaged or imperfect whisker.) In one instance a search was made into a much smaller voltage regime, where Lukens et al.⁽⁹⁶⁾ most convincingly tested the fluctuation theory⁽¹¹⁵⁾ for long one-dimensional superconductors. Using a superconducting voltmeter⁽¹¹⁶⁾ a search was made over the frequency range 1-50 KHz and critical current range 5 nA-17 μ A. As in the higher voltage investigation, no r.f.-induced structure was discerned. The $\frac{L}{\xi(T)}$ values for these whisker experiments were $\gg 10$, whereas for the lowest temperature we tried for an 8 μ thin film bridge, (and saw steps), $\frac{L}{\xi(T)}$ was ~ 7.5 .

6.3.2 Amplitude vs. R. F. Voltage Dependence of R. F.-Induced Steps

For an r.f. voltage-biased junction the expected amplitude - r.f. voltage dependence for the n^{th} step is expected to be given by the n^{th} Bessel function, i.e. equation (2.13),

$$j_2(n) = I_1 \left| J_n \left(\frac{2eV_{\text{rf}}}{\hbar\omega_{\text{rf}}} \right) \right| \quad (6.10)$$

This same Bessel function behavior is also expected to be approximated for an r.f. current-biased junction in the limiting case $I_1 \ll I_{rf}$. (cf. discussion surrounding eq. (2.28)). In Figure 6.12 the predictions of eq. (6.10) are displayed along with the experimental data for a 1.5μ long Sn bridge irradiated with 110MHz; I_1 in this case is $\sim 1\mu A$. The amplitudes are normalized at one point, the $n=1$ first maximum, the r.f. voltages are also normalized at one point, the $n=1$ first zero. Similar data for a point contact, with $I_1 \sim 2.3\mu A$ and $\nu_{rf} \sim 4GHz$, are shown in Figure 6.13. The agreement with eq. (6.10) particularly with respect to the position of the zeros is reasonable. Correction for intrinsic noise rounding (not done here) would tend to increase the relative amplitudes of the smaller lobes thus improving the amplitude agreement slightly.

It was pointed out in chapter 2 that for most weak links we expect that the r.f. source approximates a current source, hence one may expect some deviations from eq. (6.10). The first experimentally observed deviations attributed to this origin were reported by Gregers-Hansen et al. (117). These sort of deviations were also observed by us independently at nearly the same time; we include some of our results here for comparison. Figure 6.14 shows data with $\nu_{rf} = 270MHz$ for a 1.5μ bridge (same bridge as in Fig. 6.12) taken at four different I_1 levels (varied by varying temperature), I_1 increased from (a)-(d). Two principle deviations

Figure 6.12 R.f.-step amplitude vs. r.f. voltage dependence for a tin bridge with small I_1 (with step number indicated). Solid line---Bessel function; discrete points---experimental data.

Figure 6.13 R.f.-step amplitude vs. r.f. voltage dependence for a niobium point contact with small I_1 . Again the solid lines are Bessel functions and the discrete points experimental data.

Figure 6.14 R.f.-step amplitude vs. r.f. voltage dependence for a tin bridge with I_1 as parameter (cf. text).

TIN BRIDGE
(1μ)

270 MHz

(a)
 $I_1 = 3.34\mu A$

(b)
 $I_1 = 14.7$

(c)
 $I_1 = 32.9$

(d)
 $I_1 = 86.1$

are observed as I_1 increases: (1) A broadening of the $n=0$ first lobe is observed, with subsequent delay of the appearance of well defined higher order steps ($n \geq 1$), (note the change in horizontal scales from (a)-(d)), and (2) again for the $n=0$ step, the relative ratio of the second maximum to the first maximum decreases sharply. It appears that once the large critical supercurrent is suppressed, quasi-Bessel function behavior is restored.

The equation describing a highly damped junction biased by both r.f. and d.c. current sources is e.g. (2.25), which can be rewritten in a normalized form as,

$$\frac{d\phi}{dX} = A + A_{rf} \cos X - A_1 \sin \phi \quad (6.11)$$

$$\text{with } A \equiv \frac{2eR}{\hbar\omega_{rf}} I, \quad A_{rf} \equiv \frac{2eR}{\hbar\omega_{rf}} I_{rf}, \quad A_1 \equiv \frac{2eR}{\hbar\omega_{rf}} I_1, \quad X \equiv \omega_{rf} t,$$

and where the simple resistor-supercurrent model is used for the junction (model I of previous section). Several authors have solved this equation numerically in order to obtain the theoretical step amplitude-r.f. voltage dependence. ^(18,43,44) In most cases solutions were obtained using analog computers. Since an analog computer was not available to us we have solved (6.11) using a digital computer, again using the APL interactive programming language. We used the 4th order Runge-Kutta integration technique and found that 50 integration points per 2π interval (in X) was sufficiently

accurate for our purposes. The APL functions are listed in appendix 6.C.

Our calculation proceeds as follows for a given radiation frequency and critical current:

- (1) An r.f. current value, I_{rf} , is chosen (A_{rf}).
- (2) Next, a d.c. current value, I , is chosen (A).
- (3) Eq. (6.11) is then solved for $\phi(X)$. Then $I_s(X) = I_1 \sin \phi$
- (4) The supercurrent $I_s(t)$ is then time averaged, yielding $\bar{I}_s$.
Use is made of the fact, that when the junction is biased on a step $I_s(X)$ has the same periodicity as the r.f. current i.e. 2π .
Kose and Fack have pointed out that substeps occur when $I_s(X)$ has larger periods than 2π .
- (5) The d.c. voltage is then given by $\bar{V} = (I - \bar{I}_s)R$.
- (6) One then returns to step (2) and reiterates until the range of current is found for which $\bar{V} = \text{constant}$. In the notation of eq. (6.11) $\bar{V} = \frac{\hbar\omega_{rf}}{2e} (A - A_1 \overline{\sin \phi})$, i.e. the step number is $q = A - A_1 \overline{\sin \phi(X)}$.
(q rational)
- (7) One then choses another I_{rf} value and starts all over again,

In the general calculation one does not know the initial phase, consequently one practical precaution necessary is to allow sufficient time for initial transients to die out. We found that over most of our I_{rf} range the transients had disappeared by the time $X=2\pi$ (one r.f. oscillation) hence we did most of our averaging over $X=2\pi-4\pi$ or $X=4\pi-6\pi$. Figure 6.15 displays some examples

PARAMETERS:

$R = .066$, $FREQ. = 270.1 \text{ MHz}$, $I_1 = 32.9 \mu\text{A}$

(a) and (b) -- $I = 3.13 \mu\text{A}$, $I_{rf} = 40 \mu\text{A}$

(c) -- $I = 10 \mu\text{A}$, $I_{rf} = 55 \mu\text{A}$

Figure 6.15 Representative theoretical integrated waveforms related to the theoretical determination of step amplitude vs. r.f. voltage for an r.f. current source.

of the integrated waveforms. Fig. 6.15(a) shows the phase vs. $\omega_{\text{rf}}t$ and (b) the resultant time dependent supercurrent I_s for the parameter values indicated. The phase clearly displays the initial transient mentioned above. Note that I_s in this case has a period 4π ; this is an example of the supercurrent corresponding to a bias on the $q = 1/2$ substep. Fig. 6.15(c) displays $I_s(\omega_{\text{rf}}t)$ for a different set of parameters, in this case the period is 2π and $q=1$.

This procedure is rather tedious and time consuming, consequently we made no effort to best-fit all of our data in detail. Figure 6.16 which shows the theoretical result for parameter values close to those of the experimental case of Fig. 6.14(c), $n=0$ is representative of our results (for the moment disregard the $L \neq 0$ case). For comparison the dashed curve is the zeroeth order Bessel function J_0 . Comparing Fig. 6.16 with Fig. 6.14(c), $n=0$ we see that the r.f. current source model goes a long way toward explaining the experimental results. Gregers-Hansen and Levinson,⁽¹¹⁸⁾ and Taur and Richards⁽¹¹⁹⁾ have reported a similar conclusion. In general, however there appears to exist a few remaining discrepancies. First, there is considerable rounding of the experimental curve at regions near the zeroes and lobe peaks --- this is quite possibly due to noise, although this is most likely not the source of the rounding of the first $n=0$ lobe in Fig. 6.14(c). Secondly, there experimentally appears to be more of a decrease in succeeding lobe maxima than the theory predicts. Thirdly, there appears to exist high frequency anomalies, one

Figure 6.16 Theoretical $n=0$ step amplitude vs. r.f. voltage for an r.f. current source ($L=0$ ---solid lines, $L=8.5\text{ph}$ ---discrete points) with indicated parameter values. This calculation should be compared with Fig. 6.14 (c), $n=0$. For comparison is shown the zeroeth order Bessel function.

notable anomaly being the Dayem-Wyatt effect (cf. section 6.3.3).

In view of the evidence of section 6.2 that for $I_{rf} = 0$ our bridges appear to be better described by an inductive model with $LI_1 \gg \frac{\phi_0}{2\pi}$ (shown by Deaver and Pierce to approximate the relaxation oscillation model) it seemed to us reasonable to consider the inclusion of a self-inductance in the r.f. current-biased case also (cf. the equivalent circuit in Fig. 2.8(a)). In this case the appropriate equation which replaces eq. (6.11) is,

$$\frac{d\phi}{dX} = \frac{A + A_{rf} \cos X - A_L \sin \phi}{1 + A_L \cos \phi} \quad (6.12)$$

where $A_L = \frac{2eLI_1}{\hbar} = \frac{2\pi LI_1}{\phi_0}$. We have solved this to calculate

the step amplitude dependence for a case where $L < \frac{\phi_0}{2\pi I_1}$; in the case tried we found the resultant modification to the $L=0$ model very small -- this case, for which $\frac{2\pi LI_1}{\phi_0} = 0.85$ is plotted along

with the $L=0$ case in Fig. 6.16. (Fig. 6.15 (a) and (b) also correspond to this same LI_1 value.)

The real regime of interest is $LI_1 > \frac{\phi_0}{2\pi I_1}$, for which the solution $I_s(\omega_{rf}t)$ becomes reentrant. Because of this reentrant property we have not pursued this approach any further on a digital computer, since it would appear much less complicated to do the calculation with an analog computer. We suggest that it still would be interesting to pursue this model systematically over a large range of I_{rf} , I_1 , and ω_{rf} for $\frac{2\pi LI_1}{\phi_0} \gg 1$ in order to discover if

any observable r.f. properties of superconducting weak links are indeed attributable to the self-inductance of the link. Of course any effects due to heating or two dimensional geometrical distortions would certainly not be expected to follow this simple model, since in the former case heat currents would need to be included and in the latter case ϕ would depend upon transverse position. It may be the case that these latter effects were significant in certain reported cases where fitting to simple lumped models over a wide temperature range proved inconclusive. (87)

6.3.3 Observations of the Dayem-Wyatt Effect

Some time ago Wyatt⁽⁷³⁾, and Dayem and Wiegand⁽⁷⁴⁾ independently investigated a peculiar r.f. effect in which the r.f. appeared to increase the critical current or critical temperature of short thin film weak links. Until very recently a satisfactory theory for the origin of this phenomena has been lacking. Recently however Notarys, Yu, and Mercereau have included high current density effects in the Aslamazov-Larkin formulation of the Josephson effect and the theoretical implications appear to explain the Dayem-Wyatt effect in their data on proximity effect bridges. (72)

During the course of our experiments on r.f.-induced steps, by increasing the frequency, we have also taken systematic data on this effect. We include this data here, without at this time comparing with the theory of Notarys et al. (or any other theory) in the hope that it might nevertheless shed some light on this topic

and be useful in comparison with the results of others. We believe that our observations include more frequency dependence data very close to T_c than any currently reported in the literature.

The basic phenomenon is illustrated in Figure 6.17(a), which shows the effect on the I-V curve of applying various amounts of 3GHz radiation to an 8 μ tin bridge. With increasing r.f. power the critical current, I_1 , first increases, reaches a maximum, and then decreases. Fig. 6.17(b) displays the extrapolated critical current vs. r.f. voltage for a 3 μ bridge; the shape of this curve is fairly representative for our data. The effect was observed in all of our thin film bridges but not in our point contacts.

In (6.17(a)) it can be seen that the effect of the r.f. is more than just to increase I_1 ; in effect, the entire transition from the superconducting state to the normal state sharpens, consequently reducing the "excess supercurrent" discussed in section 6.2. In fact with a sufficient r.f. power eventually one can determine R_N quite easily. We found that the values of R_N obtained in this manner were temperature dependent and consistent with $R_N(T)$ found as a fitting parameter in section 6.2. This provides additional evidence that the slight temperature dependence of R_N is indeed real.

One may also conveniently observe this effect via the resistive transition, as shown in Figure 6.18. For clarity in this figure only the no-r.f. curves and the curves of maximum T_c increase are

Figure 6.17 (a) The Dayem-Wyatt effect for a tin bridge as observed in d.c. I-V curves. (b) Representative zero voltage step amplitude vs. r.f. voltage for conditions similar to (a).

Figure 6.18 The D-W effect in a tin bridge as observed via resistive transition curves (cf. text).

plotted. It can be observed in this data that the r.f. only affects the weak link, not the film electrodes, as indicated by the stability of the electrode transition (the small, higher temperature transition).

We have also observed the Dayem-Wyatt effect in our long (100-500 μ) pure tin whisker crystals. Figure 6.19 displays the effect as seen in resistive transition curves for $\nu_{rf}=960\text{MHz}$. To our knowledge this is the first report of the existence of the Dayem-Wyatt effect in very long structures. It is interesting to note that we saw this effect in whiskers but we did not see r.f. induced steps (cf. section 6.3.1). Also we did not see this effect in any of our 2% In-Sn whiskers, though our measurements were not sufficiently systematic to rule out that the effect might occur in the doped whiskers in a markedly different regime of frequencies, critical currents, etc.

Some of our results for thin film bridges are plotted in Figures 6.20, 6.21, and 6.22. In all of the plots ΔI_c is defined as the maximum shift of the critical current and I_c^{max} is the actual value of the critical current at that maximum. In Fig. 6.20 we plot (in two ways) the magnitude of the effect versus frequency whereas in the other plots critical current and temperature are the abscissas. Several points are clear from these plots: (1) The threshold frequency for the D-W effect increases as $T \rightarrow T_c$ (or $I_c \rightarrow 0$). (2) For a given I_c the threshold frequency decreases as the bridge is lengthened. This

Figure 6.19 The D-W effect observed in the resistive transition of a long ($>200\mu$) thin ($\sim 1\mu$) tin whisker crystal.

— no r.f.
 ----- with r.f. (960MHz)

Figure 6.20 Frequency dependence of the D-W effect for our tin bridges-- zero freq. I_c as parameter; the key is common to Figs. 6.20-6.22.

Figure 6.21 Critical current dependence of the D-W effect for our tin bridges.

Figure 6.22 Temperature dependence of the D-W effect for our tin bridges.

observation is compatible with our observation on whiskers where the threshold frequencies were ~ 400 MHz. (3) The effect is generally enhanced as the frequency is increased. (4) For a fixed frequency the relative effect first increases as temperature is increased, reaches a maximum, and then decreases (more rapidly than was the rate for the increasing regime). The total temperature span for this occurrence is within ~ 100 mK of T_c . In all cases where both the driven Josephson effect and the Dayem-Wyatt effect were observable in a particular weak link, the threshold of frequency for the driven effect was substantially lower than the threshold for the D-W effect.

The general r.f.-sensitive behavior of a weak link is observed to consist of a few distinct phases, dependent upon the proximity to T_c . (1) Very close to T_c the threshold frequency for the D-W effect is much greater than the threshold for the driven a.c. effect, and the critical current vs. r.f. voltage dependence very closely approximates the ideal zeroth order Bessel function. (2) From the standpoint of a fixed frequency greater than the intrinsic thermal limit given by eq. (6.6) or (6.7(a)), (i) very close to T_c Bessel function behavior is dominant, (ii) farther from T_c the Dayem-Wyatt effect threshold is reached, (for the given frequency), and so over a certain temperature range the D-W effect is dominant, (iii) eventually, at much lower temperatures both effects are seen to gradually disappear. (3) For a fixed temperature the D-W effect increases with frequency; below the D-W threshold quasi-Bessel function-type behavior is observed, provided the driven a

effect threshold frequency is exceeded.

6.4 Summary

In brief, we have experimentally observed the current-voltage behavior of various sorts of superconducting weak links, with and without applied electromagnetic fields. Our observations have extended into the experimentally difficult regime very close to T_c .

We have found that, in the regime where heating effects and geometry effects are negligible, the static I-V curves of our point contact could be quite accurately described by the highly damped limit of the simple two fluid representation (McCumber; Stewart; Aslamazov-Larkin) whereas our thin film weak links were more accurately described by the addition of a self-inductance to the model (Deaver-Pierce), which then yields an I-V similar in shape to that found in the phase slip theory or relaxation oscillation model (Rieger, Scalapino, and Mercereau; Kirschman, Notarys, and Mercereau). However, it was found that the effective normal state resistance of our weak links (both point contacts and thin film bridges) decreased with decreasing temperature. This decrease in R_N was also confirmed in our observation of the Dayem-Wyatt effect. As far as we know, this observation has not been reported elsewhere; we believe its origin to be related to the temperature dependent coherence length of the superconductors.

We have identified distortions from the ideal models mentioned above, as due to geometrical and heating effects. The

trend toward two-dimensionality as temperature was lowered is observed sooner in the wider less uniform bridges and is manifested in I-V curves as an ~~excess~~ supercurrent above the phase slip model value of $I_1/2$. Self-heating manifests itself as abrupt switching and hysteresis for large critical currents.

The temperature dependence of the critical current in our point contacts was observed to be exponential in nature, somewhat similar to that observed for "clean" S-N-S junctions. This result, along with the fact that our point contacts and S-N-S junctions both appear to be well described by the same lumped circuit model, suggests that the actual physical mechanisms for the origin of the Josephson effects in these two types of structures may be quite similar.

The threshold frequency for the driven a.c. Josephson effect was found to depend approximately linearly upon critical current, over our range of investigation. This dependence appears to correspond well with the "synchronization bandwidth". For a particular sample, the lowest possible frequency threshold is obtained for $I_1 \rightarrow 0$ and is then given by the thermal noise generated bandwidth of the a.c. Josephson current.

The step amplitude-r.f. voltage dependence of our weak links was found to be in reasonably good accord with interpretation of the r.f. source as a current source, confirming the observations of Gregers-Hansen et al. We have found that introduction of the weak link self-inductance, L , into the calculations chang

the step amplitude dependence only slightly for $L < \frac{\Phi_0}{2\pi I_1}$.

Though we have not done so, we suggest that it would be very interesting to extend these calculations to the relaxation oscillation regime of $L \gg \frac{\Phi_0}{2\pi I_1}$. For $I_1 \rightarrow 0$ the r.f. current source and r.f. voltage source theoretical formulations merge; indeed in this limit we observed the experimental step amplitude dependence to approach the expected pure Bessel function behavior.

We have experimentally characterized the Dayem-Wyatt effect for our weak links. We find that the threshold frequency for this effect is many times higher than the threshold for the driven a.c. effect. Over our range of investigation we always saw the effect in thin film bridges but never saw it in point contacts. An extensive search for the driven a.c. Josephson effect in long one-dimensional tin whisker crystals (~ 100 - 500μ long; ~ 0.5 - 1.5μ wide) showed no driven a.c. effect (in undamaged whiskers) but did reveal the existence of the Dayem-Wyatt effect in these structures. This observation of the D-W effect in such long structures is new, as far as we know. The effect appeared to be suppressed by the addition of indium as an impurity to the tin whiskers.

Finally, for a fixed frequency, let us characterize the r.f. behavior of our thin film bridges (the only structures we had which showed both the driven a.c. and D-W effects) as a function of temperature.

- (i) Sufficiently close to T_c the r.f. has no affect. Intrinsic thermal fluctuations dominate.

- (ii) As the temperature is lowered (if the frequency is greater than the linewidth of the a.c. Josephson oscillations) eventually r.f. induced steps are observed; at first the step amplitude vs. r.f. voltage dependences are Bessel functions.
- (iii) Further decrease in temperature results in the distortions of the Bessel function r.f. step behavior, characteristic of an r.f. current source.
- (iv) Eventually a temperature is reached where the Dayem-Wyatt effect threshold frequency has decreased below the r.f. frequency, and as a result the D-W effect is observed.
- (v) Further temperature decrease enhances the D-W effect up to a point, and then the effect of the r.f. gradually diminishes.

6.A Appendix: Some Estimated Thin Film Parameters

From tabulated room temperature values, for pure tin,
 $\rho_{R.T.} \approx 1.15 \times 10^{-5} \Omega\text{cm.}$ and $\rho_{R.T.} \ell_{R.T.} = \frac{mv_F}{ne^2} \approx 10^{-11} \Omega\text{cm.}^2$

(where $\rho \equiv$ resistivity and $\ell \equiv$ electronic mean free path), then
 $\ell_{R.T.} \approx 87\text{\AA}$. For the data shown in this chapter $\frac{\rho_{R.T.}}{\rho_{4.2}} = 24$ hence

the low temperature mean free path is $\ell_{4.2} = \ell_{R.T.} \frac{\rho_{R.T.}}{\rho_{4.2}} \approx 2100\text{\AA}$.

The film thickness here was $1640\text{\AA}$, hence the actual mean free path is a little smaller than $2100\text{\AA}$, although we use this value for the estimates below. The films here showed indications of some of the labyrinth structure discussed in chapter 3; however $\ell_{4.2} <$ labyrinth structure size $<$ bridge width hence we expect that the low temperature resistivity is independent of the film structure. Weak link transition temperatures, T_c , were obtained by extrapolation of $I_1(T)$ data for $I_1 \rightarrow 0$. For T near T_c the bulk penetration depth and coherence length are given by,

$$\begin{aligned} \lambda(T) &= \frac{\lambda_L(0)}{\sqrt{2}} \frac{1}{(1-T/T_c)^{1/2}} \frac{1}{\chi^{1/2}(.88\xi_0/\ell)} \equiv \frac{\lambda(0)}{(1-t)^{1/2}} \\ \xi(T) &= 0.74\xi_0 \frac{\chi^{1/2}(.88\xi_0/\ell)}{(1-T/T_c)^{1/2}} \equiv \frac{\xi(0)}{(1-t)^{1/2}} \end{aligned} \quad (6.A.1)$$

where $\chi(.88\frac{\xi_0}{\ell})$ is the Gorkov impurity function.⁽¹²⁰⁾ In the following tabulation $\lambda_L(0) = 355\text{\AA}$ and $\xi_0 = 2300\text{\AA}$ are used for pure tin.

Length (μ)	Width (μ)	d ($\text{\AA}$)	T_c	$\xi(0)$ ($\text{\AA}$)	$\lambda(0)$ ($\text{\AA}$)
1.48	2.7	1640	3.830	1260	338
3.27	1.5	1640	3.826	1260	338
8.19	1.09	1640	3.792	1260	338

According to these estimates the tin films here are type I superconductors.

6.B Appendix: $I_1(T)$ for the Point Contacts

The temperature dependence of the critical current for our data was observed to most closely resemble the experimental behavior observed in some "clean" S-N-S thin film sandwiches.⁽⁷⁹⁾ Our data is similar to that recorded in a thesis of S. F. Strait⁽¹²¹⁾ in which "dirty" (i.e. oxidized) point contact surfaces were compared to "clean" (chemically etched --- minimum exposure to air). Of Strait's results ours resembles the "dirty" contacts. In Figure 6.B.1(a) $I_1(T)$ is shown for the total T range measured. The inset, Figure 6.B.1(b) shows in a semilog plot the region near T_c ; the lower curve is the extrapolated ("eyeball") value, whereas the upper curve is a plot of $I_1(T)$ obtained as a result of fitting the fluctuation rounded characteristics to the theory of Ambegaokar and Halperin (cf. chapter 5).

Figure 6.B.1 (a) Critical current vs. temperature for a Nb point contact.
 (b) Same data near T_c in a semilog plot (cf. text).

6.C Appendix: APL Programs for Determination of Step Amplitude vs. R.F. Voltage

The functions listed here pertain to the general case where a weak link self inductance has been included. To get the zero inductance case one simply enters zero, (0), when the inductance value is asked for ('INDUCTANCE VALUE-L'). To initialize the calculation one calls the function GENERAL and enters parameter values when asked (W is the R. F. frequency, I0 the critical current, etc.). $I_s(X)$ is calculated and then one is asked for parameters relating to averaging $I_s(X)$. The first time through one has to call VOLT2 to initiate the averaging. The output is then given as the area under $I_s(X)$ and as the d.c. voltage. To reiterate with a different d.c. current or r.f. current value one then simply types GE. All of the functions listed below are related to this calculation except for TIME which outputs the CPU time used for a particular iteration. The quantity $\frac{2\pi}{\phi_0}$ must be stored prior to the calculation in the variable EHBAR in units compatible with the inductance; zero should be initially stored in TIMER.


```

      VAREA2[[]]V
V R←AREA2:P:A1:A2:A3
[1] C←G
[2] INDEX←AX1(02)
[3] P←10G[INDEX].G[INDEX+(1(ρAX)-INDEX)]
[4] 'ΔX=';ΔX
[5] A1←+/ΔX×P[1(ρP)-1]
[6] A2←+/ΔX×P[1+(ρP)-1]
[7] A3←(A1+A2)÷2
[8] 'A1=';A1;' A2=';A2;' AVG.AREA=';A3
[9] R←A3
V
      VGALL[[]]V
V ΔX GALL XF;XX;Z;O:P;Q;S;T;U
[1] AX←MX+10
[2] RT←AX+AX.X
[3] JX←MΔ.C
[4] AX←X+ΔΔ
[5] Z←B+(BRF×(20XX))
[6] O←(Z[1]-B0×(10C))÷(1+L×20C)
[7] C←C+(O+(2×(P+Q))+(Z[4]-B0×(10U))÷(1+L×20U+C+Q+(Z[3]-B0×(10T))÷(1+L×20T+C+0.5×P+(Z[2]-B0×(10S))÷(1+L×20S+C+0.5×O))))÷6
[8] X←((XF+ΔX)×X+X+ΔX)/RT
[9] C←((ρAX),1)ρHX
V
      VTIME[[]]V
V Z←TIME;T
[1] Z←60 60 60 T(T+I21)-TIMER
[2] TIMER←T
V
      VVOLT2[[]]V
V SS←VOLT2
[1] SS←(W÷EHBAR)×Δ-Δ0×AREA2÷02
V
      VGALL2[[]]V
V GALL2
[1] 'INIT. VALUE OF X'
[2] X←[]
[3] 'INITIAL VALUE OF Y(X)'
[4] C←[]
[5] CONTINUE:'WHAT IS ΔX?'
[6] ΔX←[]
[7] 'WHAT IS THE FINAL VALUE OF X TO BE?'
[8] XF←[]
[9] B0←ΔX×Δ0
[10] B←ΔX×Δ
[11] BRF←ΔX×ΔRF
[12] ΔΔ←0,(ΔX÷2),(ΔX÷2),ΔX
[13] 'INDUCTANCE VALUE-L'
[14] L←EPBAR×I0×[]
[15] ΔX GALL XF
V
      VGE[[]]V
V GE
[1] 'NEW I'
[2] A←(EHBAR×R×I+[])÷W
[3] 'NEW IRF'
[4] ARF←(EHBAR×R×IRF+[])÷W
[5] B←ΔX×Δ
[6] BRF←ΔX×ΔRF
[7] X←0
[8] C←(10(I÷I0))
[9] ΔX GALL XF
[10] VOLT2
[11] TIME
V
      VINPUT[[]]V
V BG←INPJT;K0
[1] 'ENTER R'
[2] K0←EHBAR×R+[]
[3] 'ENTER W'
[4] K0←K0÷W+[]
[5] 'ENTER I'
[6] Δ←K0×I+[]
[7] 'ENTER I0'
[8] Δ0←K0×I0+[]
[9] 'ENTER IRF'
[10] ΔRF←K0×IRF+[]
V
      VGENERAL[[]]V
V GENERAL
[1] INPUT
[2] GALL2
V

```

Figure 6.C.1 APL functions used for the determination of step amplitude versus r.f. voltage.

7. CONCLUDING REMARKS

We believe that the most significant scientific contribution of these experiments has been the demonstration of a convincingly high degree of fidelity between the theory and experimental observations of intrinsic fluctuation effects in superconducting weak links, as manifested in static current-voltage characteristics. Good agreement has been found for fluctuation effects in both the driven a.c. Josephson effect and the d.c. Josephson effect, in both niobium point contacts and tin thin film bridges, over a wide range of experimental parameters. By making some observations in regimes where fluctuation effects were diminished it was found that slight deviations that were observed in the fluctuation experiments could be attributed to deviations from the no-fluctuation model used as the starting point in the fluctuation theories. The nature of these deviations and the theoretical implications of the agreement between theory and experiment have been expounded upon in the chapter summaries.

With confirmations that the intrinsic fluctuation theory in superconducting weak links appears to be in good order, (although the small damping and non-uniform current distribution regimes haven't been explored experimentally in much detail), theoretical and experimental emphasis has shifted toward areas outlined in the last section of chapter 2; it is hoped that some of our latter observations and suggestions (chapter 6) though somewhat loose-ended, will furnish some helpful clues to this endeavor.

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