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Corrections to ORNL-CF-59-6-90

Page 23, Line 10: The instantaneous reactor period should be defined as

$$\frac{1}{\frac{d [\ln (m)]}{dt}}$$

Page 57, Fig. 6.1: The lowest curve should be designated as 10^{13} .

Page 87, Equation above
Eq. 10.18:

$$\alpha_i = 1 - \frac{\tau_2}{\tau_2 + \tau_1}$$

Page 89, Eq. 3.5:

$$\frac{dm}{dt} = [K(1-\beta) - 1] \frac{m}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_{Ti}$$

Page 89, Eq. 10.21

$$\frac{dm}{dt} = [K(1-\beta) - 1] \frac{m}{\ell} + \sum_{i=1}^{i=m} \lambda_i a_i C_{Ti}$$

Page 91, Equation at
bottom of page:

$$K \approx 1 + \beta \frac{\tau_2}{\tau_\ell}$$

Page 123, Fig. 11.24: The output of amplifier 15 should be designated as + (k-1)

Page 124, Line 12: The statement should read: ----- The servo multiplier takes the value of + (k-1) from amplifier 15 -----

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FOREWORD

This course on reactor controls is for the purpose of acquainting the student with some of the elementary considerations involved in setting up a control and safety system for a nuclear reactor.

This material is the outgrowth of the notes prepared for the lectures on Reactor Controls given in the Oak Ridge School of Reactor Technology. The lectures given by T. E. Cole in 1953 - 1955 are the basis of portions of the subject matter on reactor kinetics. Many suggestions and ideas contributed by E. R. Mann are included throughout almost every section. In addition, several members of the Oak Ridge National Laboratory, and of the Reactor Controls Department in particular, have provided information which has been included in these lecture notes.

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1. REACTOR WITH NO DELAYED NEUTRONS AND NO SOURCE

1.1 Kinetic Equation with no Delayed Neutrons

The derivation of the reactor kinetic equations will begin with the simple case of a reactor that has no delayed neutrons and no independent source of neutrons. The symbol n will represent the number of neutrons existing within the reactor at the present instant of time. These neutrons are being lost from the reactor system by both absorption and leakage. The loss rate by absorption is proportional to the product of the neutron flux and the macroscopic absorption cross section at each particular energy level. The flux is proportional to the number of neutrons existing at each energy level. If the neutron energy spectrum remains constant in shape even though the total neutron population changes, the total loss rate by absorption is proportional to the neutron population. A similar argument will lead to the conclusion that the loss rate by leakage is also proportional to the neutron population. Therefore, the total loss rate by both absorption and leakage is proportional to the total neutron population and will be written as λn . Accordingly, λ must have the dimensions of reciprocal time.

The value of λ depends upon the geometry of the system and the cross sections of the materials used in constructing the system. It is essential that the energy spectrum of the neutrons must be constant in shape in order that λ be a constant. Thus, the number of neutrons existing in any selected energy range must increase and decrease in proportion to the total number of neutrons within the reactor system.

The symbol k represents the multiplication of the reactor, and will be defined as the average number of neutrons released per neutron lost. Each time a neutron is lost, on the average k neutrons are released. The neutrons are lost at all energy levels, and are released at fission energies as a result of absorptions in the fuel which produce fissions. On the average, a neutron will spend a certain amount of time within each particular energy range, from fission energies to thermal energies. The sum of these time intervals will be the average life span of a neutron within the reactor. This average life span will be identified by the symbol ℓ . The time intervals within each energy range are constant as long as the shape of the energy spectrum remains constant. This shape and hence the average neutron

lifetime ℓ will remain constant for multiplications near unity.

Since the loss rate is λn neutrons per second, the release rate is $k\lambda n$ neutrons per second, from the definition of k .

The kinetic equation may be stated in the following manner:

$$\left[\begin{array}{c} \text{rate of} \\ \text{increase} \end{array} \right] = \left[\begin{array}{c} \text{rate of} \\ \text{release} \end{array} \right] - \left[\begin{array}{c} \text{rate of} \\ \text{loss} \end{array} \right]$$

Thus,

$$\frac{dn}{dt} = k\lambda n - \lambda n \quad 1.1$$

This equation may be solved for n directly.

$$\ln(n) = (k-1)\lambda t + C$$

$$\text{At } t=0, n = n_0, \text{ and } C = \ln(n_0)$$

$$\text{Thus, } \ln \frac{n}{n_0} = (k-1)\lambda t$$

$$\text{or, } n = n_0 e^{(k-1)\lambda t} \quad 1.2$$

Note that n is the only variable with time. The value of k is constant.

The definitions of ℓ and λ show that there must be a definite relationship between ℓ and λ . This relationship will be determined by means of Eq. 1.2. There are n_0 neutrons within the reactor at the instant t is zero. The rate of loss of this particularly identified group of neutrons at any succeeding instant of time will be directly proportional to the number of this particular group still in existence at the succeeding instant of time. The loss of this identified group will continue independently of the production of other neutrons by fission. In other words, the loss of this group continues as if k were zero. Under these conditions, the population of the identified group of neutrons is obtained directly from Eq. 1.2 by setting k equal to zero.

Then, $n = n_0 e^{-\lambda t}$

The number of these neutrons still in existence at any instant of time will follow the curve drawn in Fig. 1.1. The average life span of the n_0 neutrons is the average neutron lifetime ℓ . This average is obtained by determining the area under the curve and dividing this area by n_0 , in the same manner that any other average is obtained. Therefore,

$$\begin{aligned}\ell &= \frac{1}{n_0} (\text{area, in neutron-sec.}) \\ &= \frac{1}{n_0} \int_0^{\infty} n_0 e^{-\lambda t} dt \\ &= \frac{1}{n_0} \left[-\frac{1}{\lambda} n_0 e^{-\lambda t} \Big|_0^{\infty} \right] \\ &= \frac{1}{n_0} \left[-\frac{n_0}{\lambda} (0-1) \right]\end{aligned}$$

or, $\ell = \frac{1}{\lambda}$. 1.3

Another approach to the concept of the average lifetime is to consider a pool containing n gallons of water. The water is being drained from the pool at a constant rate and is being replenished at the same constant rate. These rates are defined as λn gallons per hour. In a time interval of $\frac{n}{\lambda n}$ hours, n gallons of water will have passed through the pool. The average time spent by the water in the pool is this time interval $\frac{n}{\lambda n}$; thus, the average lifetime is $\frac{1}{\lambda}$.

By the use of Eq. 1.3 and the definitions of k , n , λ , and ℓ , the rate of neutron release may be written as $\frac{k n}{\ell}$ neutrons per second, and the rate of loss as $\frac{n}{\ell}$ neutrons per second. On the average, each time a neutron is lost, k neutrons are released at fission energy, and the average time lapse between the original fission and the succeeding fission is ℓ seconds. Therefore, ℓ may be taken as the average time between successive generations, including the slowing down time, the thermal diffusion time, and the effects of fast fission, leakage at all energies, and absorption at all energies. Refer to Glasstone and Edlund, pages 290, 203, and 184.

A graphical representation of the loss and gain of neutron population is described in Fig. 1.2. The total rate of loss of neutrons by leakage and absorption is $\frac{n}{\ell}$; the symbol $\frac{n}{\ell} (1 - \frac{k}{\lambda})$ represents the rate of loss by the sum of leakage and absorptions not producing fissions. Therefore,

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Fig. 1.1. Neutron Average Lifetime.

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Fig. 1.2. Neutron Population Loss and Gain, with no Delayed Neutrons.

the rate of loss by absorptions which cause fissions is $\frac{kn}{\ell}$. Since each fission releases ν neutrons on the average, the rate of release of neutrons is $\frac{kn}{\ell}$.

Then Eq. 1.1 may be rewritten with the aid of Eq. 1.3:

$$\frac{dn}{dt} = \frac{kn}{\ell} - \frac{n}{\ell} \quad 1.4$$

The solution to Eq. 1.4 is similar to that of Eq. 1.1.

$$n = n_0 e^{\left(\frac{k-1}{\ell}\right)t} \quad 1.5$$

The quantity $(k-1)$ is the excess multiplication and will be written as δk .

For reactors to be discussed, the neutron lifetime will be in the range from 10^{-5} sec to 10^{-3} sec. In a graphite-natural uranium system, $\ell \cong 10^{-3}$ sec, and in a light water-enriched uranium system, $\ell \cong 10^{-4}$ sec.

Consider an example:

$$\begin{aligned} \text{Take } \ell &= 10^{-4} \text{ seconds and } k = 1.001 \\ n &= n_0 e^{\left(\frac{k-1}{\ell}\right)t} = n_0 e^{\frac{\delta k}{\ell}t} \\ &= n_0 e^{10t} \end{aligned}$$

In a time interval of one second the level would increase by a factor of about 10^4 . This is rather startling; the significant point to note is that $\frac{k-1}{\ell}$ is the determining factor for this case, and $\frac{\ell}{k-1} = \frac{\ell}{\delta k}$ is the time required for the neutron population to increase by a factor of e . This time for the neutron level to change by a factor of e is defined as the reactor period. For this example, the period is 0.1 sec.

1.2 Multiplication of a Thermal Reactor

The following procedure will show that the previous definition of k is consistent with the customary definition of multiplication of a thermal reactor based upon the geometry and cross sections. Refer to Glasstone and Edlund, pages 84 and 200 through 203. For a thermal reactor, the following expression defines k , based upon the "four factor" formula.

$$k = \eta f \epsilon p P_s P_{th}$$

1.6

Where,

$$\eta = \frac{\nu \Sigma_{ff}}{\Sigma_{af}}$$

$$f = \frac{\Sigma_{af}}{\Sigma_a}$$

ν = neutrons released per fission.

Σ_{ff} = macroscopic fission cross section of the fuel in cm^{-1} .

Σ_{af} = macroscopic absorption cross section of the fuel in cm^{-1} .

Σ_a = mean value of the macroscopic absorption cross section of the reactor in cm^{-1} .

ϵ = fast fission factor.

p = resonance escape probability.

$P_s = e^{-B^2 \tau}$ = non-leakage probability during slowing down.

$P_{th} = \frac{1}{1 + L^2 B^2}$ = non-leakage probability of the thermal neutrons, the fraction of the neutrons that are thermalized which are absorbed as thermal neutrons.

L = diffusion length of thermal neutrons in the reactor in cm.

B^2 = buckling in cm^{-2} , the lowest eigenvalue of the wave equation for the critical reactor, subject to the condition that the flux shall go to zero at the extrapolated boundary.

τ = Fermi age in cm^2 .

The expression for k in Eq. 1.6 may be rewritten:

$$\begin{aligned} k &= \left(\frac{\nu \Sigma_{ff}}{\Sigma_{af}} \right) \left(\frac{\Sigma_{af}}{\Sigma_a} \right) \epsilon p P_s P_{th} \\ &= \frac{\nu \Sigma_{ff}}{\Sigma_a} \epsilon p P_s P_{th} \end{aligned}$$

1.7

Multiply denominator and numerator of Eq. 1.7 by $V\phi$.

Where,

V = volume of reactor in cm^3

ϕ = average thermal flux in neutrons per $\text{cm}^2\text{-sec}$.

Then,

$$K = \frac{\nu V \phi \Sigma_{ff} \epsilon p \beta_s P_{th}}{V \phi \Sigma_a} \quad 1.8$$

or,

$$K = \frac{\nu V \phi \Sigma_{ff} \epsilon p \beta_s}{\frac{V \phi \Sigma_a}{P_{th}}} \quad 1.9$$

$\nu V \phi \Sigma_{ff} \epsilon p \beta_s$ = total rate of release of thermal neutrons in neutrons per sec.

$V \phi \Sigma_a$ = total rate of absorption of thermal neutrons in neutrons per sec.

Now, since $P_{th} = \frac{\text{thermal absorption}}{\text{thermal absorption} + \text{thermal leakage}}$,

$\frac{V \phi \Sigma_a}{P_{th}}$ = total rate of loss of thermal neutrons by both absorption and leakage in neutrons per sec.

Therefore, by Eq. 1.9,

$$K = \frac{\text{rate of release of thermal neutrons}}{\text{rate of loss of thermal neutrons}}$$

On the basis that the neutron energy spectrum of the reactor remains constant,

$\frac{\text{rate of release of thermal neutrons}}{\text{rate of loss of thermal neutrons}}$

= $\frac{\text{total rate of release of neutrons inside reactor}}{\text{total rate of loss of neutrons inside reactor}}$

= neutrons released per neutron lost.

Therefore, the definitions of K are consistent.

If thermal neutrons only are considered,

$$\begin{aligned}\frac{dn_{th}}{dt} &= \left[\text{rate of production of thermal neutrons} \right] - \left[\text{rate of loss of thermal neutrons} \right] \\ &= \nu V \phi \Sigma_{ff} \epsilon p P_s - \frac{V \phi \Sigma_a}{P_{th}} \\ &= \left(\frac{\nu V \phi \Sigma_{ff} \epsilon p P_s P_{th}}{V \phi \Sigma_a} - 1 \right) \frac{V \phi \Sigma_a}{P_{th}}\end{aligned}$$

Make use of the definition of k in Eq. 1.8.

$$\frac{dn_{th}}{dt} = (k-1) \frac{V \phi \Sigma_a}{P_{th}} \quad 1.10$$

Now, $\phi = \frac{n_{th}}{V} v$

Where n_{th} total number of thermal neutrons in reactor.

v average velocity of thermal neutrons

$$\begin{aligned}\text{Thus, } \frac{V \phi \Sigma_a}{P_{th}} &= v \frac{n_{th}}{V} v \Sigma_a (1 + L^2 B^2) \\ &= n_{th} v \Sigma_a (1 + L^2 B^2)\end{aligned}$$

$$\text{Now, } l_{th} = \frac{1}{v \Sigma_a (1 + L^2 B^2)} = \text{life of a thermal neutron in seconds in a bare reactor.}$$

Then,

$$\frac{V \phi \Sigma_a}{P_{th}} = \frac{n_{th}}{l_{th}} \quad 1.11$$

Therefore,

$$\frac{dn_{th}}{dt} = (k-1) \frac{n_{th}}{l_{th}} \quad 1.12$$

Compare Eq. 1.12 to Eq. 1.4. Note that the derivation of Eq. 1.10 considered that thermal neutrons were released at the instant of fission, consequently the life span does not include any slowing down time. The effects of resonance capture and fast leakage were included, although the time between fission energies and thermal energies was neglected. This error is small for thermal reactors.

2. NEUTRON ENERGIES

The energy level at which the neutron population is taken will be discussed next. The prime consideration from the standpoint of reactor control is nuclear power (fissions per second, or neutrons released per second). The derivation of the kinetic equations being described are applicable to the actual multi-velocity reactor. Consider that the neutrons are released at fission energy and are lost at all energies. The fission energy itself will cover a range of energies. The value of N is the total number of neutrons within the reactor covering the range of all energy levels. The only requirement is that the spectrum of neutron energies for the reactor does not change in shape and that the average neutron lifetime remains constant. Thus, the number of neutrons within any particular energy range is directly proportional to the rate of neutron release. Therefore, the total number of neutrons in existence is directly proportional to the nuclear power.

The shape of the spectrum of neutron energies as well as the average neutron lifetime will vary slightly during a transient. In fact, the neutron spectrum during a transient must be somewhat different from that in steady state. For example, a thermal reactor on a positive excursion will contain a slightly larger fraction of high energy neutrons, not yet slowed down, as compared to the case of the same reactor at steady state. In addition, the removal of a neutron absorber having a high absorption for thermal neutrons and a low absorption of fast neutrons will affect both the spectrum and the lifetime. The energy spectrum and average lifetime are affected to some extent by changes in fuel loadings, by fuel burn-up, and by fission product poisoning. In the case of very fast transients, the spatial distribution of neutrons will vary in shape. All of the described changes are small provided K is near unity.

If desired, the value of N may be considered to be the neutron population within a particular unit volume of the reactor. The spatial distribution of neutron population is assumed to remain constant in shape.

Actually, N is often taken as the population of neutrons within a particular range of energies. For example, the neutron detecting instruments used in reactor control systems may be sensitive to thermal neutrons only. In any case, ℓ should be taken as the total lifetime, not the time spent within a particular energy range.

3. REACTOR WITH DELAYED NEUTRONS AND NO SOURCE

3.1 Delayed Neutrons

In the previous discussion, all of the neutrons were assumed to be produced at the instant of fission. A very large fraction of the neutrons do originate at the moment of fission and are called prompt neutrons. However, a small portion of the neutrons are released by the decay of certain radioactive nuclei which were formed by the fissioning of the fuel. A neutron emitted through this decay process is called a delayed-neutron because its release is delayed for a time interval after fission occurs. Although the total fractional yield of the delayed neutrons is less than 1% of all those released, this small fraction is extremely important in reactor control.

The mean life of the precursor of a delayed neutron will determine the time interval between a fission and the subsequent release of a delayed neutron. The beta-emitters bromine-87 and iodine-137, have been identified as precursors of the two delayed neutron groups having the longest delay, and beta-emitters bromine-89 and tellurium-136 probably are the precursors of the next two longest delay groups. A summary of data from several sources^{1,2} are presented in Table 3.1. A given isotope will be the precursor of a particular delayed neutron group regardless of the type of nuclear fuel.

3.2 Average Neutron Lifetime Approximation

The kinetic behavior of a reactor can be approximately determined at multiplications very near to unity by using an average neutron lifetime, including the effect of delayed neutrons, in Eq. 1.5. This weighted mean lifetime can be obtained from the following expression in which m delayed neutron groups are considered:

$$\bar{l} = (1 - \beta) l + \beta_1 (t_1 + l) + \beta_2 (t_2 + l) + \cdots + \beta_m (t_m + l)$$

¹Glasstone, S., Principles of Nuclear Reactor Engineering, Van Nostrand, 1955, Table 2.11, Page 113.

²Reactor Handbook, Physics, USAEC and McGraw-Hill, 1955, Tables 1.2.9 and 1.2.10, Page 109, and Table 2.1.5, Page 624.

TABLE 3.1 DELAYED NEUTRONS FROM FISSION BY THERMAL NEUTRONS

Delay Group	Precursor	Precursor Mean Life t_1 , sec	Precursor Decay Constant λ_1 , sec ⁻¹	Yield, β_i , Per 10 ⁴ Neutrons		
				U ²³⁵	U ²³³	Pu ²³⁹
1	Br ⁸⁷	80.2	0.0125	2.5	1.8	1.2
2	I ¹³⁷	31.7	0.0315	16.6	6.2	9.4
3	Br ⁸⁹	6.51	0.153	21.3	8.6	11.2
4	Te ¹³⁶	2.19	0.456	24.1	5.8	10.5
5		0.62	1.61	8.5	1.8	4.0
6		0.07	14.3	2.5		
Total Yield, β , Per 10 ⁴ Neutrons				75.5	24.2	36.3

where:

$\bar{l}$ = average neutron lifetime, including the effect of delayed neutrons, sec.

l = average neutron lifetime from release to loss, sec.

β = summation of the fractional yield of all the precursors of delayed neutrons.

$\beta_1, \beta_2, \dots, \beta_m$ = fractional yield of each group of precursors of delayed neutrons.

$t_1, t_2, \dots, t_m$ = mean life of each group of precursors of delayed neutrons, sec.

$1 - \beta$ = fractional yield of prompt neutrons.

The equation is solved for $\bar{l}$ as follows:

$$\bar{l} = l - \beta l + \beta_1 t_1 + \beta_2 t_2 + \dots + \beta_m t_m + (\beta_1 + \beta_2 + \dots + \beta_m) l$$

Now, $\beta_1 + \beta_2 + \dots + \beta_m = \beta$

And, $\beta_1 t_1 + \beta_2 t_2 + \dots + \beta_m t_m = \sum_{i=1}^{i=m} \beta_i t_i$

Where, β_i = fractional yield of the i th group of precursors of delayed neutrons.

t_i = mean life of the i th group of precursors of delayed neutrons, sec.

Thus, $\bar{l} = l - \beta l + \sum_{i=1}^{i=m} \beta_i t_i + \beta l$

Or,

$$\bar{l} = l + \sum_{i=1}^{i=m} \beta_i t_i \quad 3.1$$

Now, $\lambda_i = \frac{1}{t_i}$ = decay rate per precursor in precursors per sec.

Therefore, Eq. 3.1 can be written as follows:

$$\bar{l} = l + \sum_{i=1}^{i=m} \frac{\beta_i}{\lambda_i} \quad 3.2$$

It should be noted that the lifetime from release to loss has been taken as constant for both the prompt and delayed neutrons.

From Table 3.1, $\bar{\ell} \cong 0.1$ sec for uranium - 235. Consider the previous example in which the excess multiplication was 0.001.

$$\begin{aligned} n &= n_0 e^{\frac{\delta k}{\bar{\ell}} t} \\ &= n_0 e^{0.01 t} \end{aligned}$$

Then, the reactor period is 100 sec.

This is a decided improvement in terms of being able to control the reactor when a disturbance is encountered.

The preceding approximation will yield reasonable results only if δk is small compared to β .

3.3 Kinetic Equations With Delayed Neutrons

The kinetic equations with delayed neutrons included will now be derived. The procedure will be the same as for Eq. 1.4; however, the delayed neutrons must be subtracted and added at the proper time.

The derivation begins with the following statement:

$$\left[\begin{array}{c} \text{rate of} \\ \text{increase} \end{array} \right] = \left[\begin{array}{c} \text{actual rate} \\ \text{of release} \end{array} \right] - \left[\begin{array}{c} \text{actual rate} \\ \text{of loss} \end{array} \right]$$

This expression is the same as for the case of no delayed neutrons, except for the word "actual" to distinguish between neutrons and precursors of delayed neutrons. The symbol n will represent the actual number of neutrons in existence inside the reactor system; thus, $\frac{n}{\bar{\ell}}$ is the actual rate of loss as before. The neutrons represented by n come from two sources:

- (1) prompt neutrons directly from fissions
- (2) delayed neutrons from the decay of precursors

Although the decay of precursors releases delayed neutrons at energies considerably below that of the energies of the prompt neutrons, the energies of the delayed neutrons will be taken as being in the fission energy range.

The multiplication will now be defined as

$$k = \frac{\text{rate of release of potential neutrons}}{\text{rate of loss of actual neutrons}}$$

The word "potential" means that these particular neutrons include both the prompt group released at fission and the delayed groups which will be released later upon the decay of precursors that were formed at fission. From the

definitions of k , n , and ℓ , the rate of release of potential neutrons is $\frac{kn}{\ell}$, as before.

The rate of storage of neutrons in precursors formed by fissions is $\frac{kn\beta}{\ell}$, which is the total rate of production of precursors. The symbol β represents the summation of the yields of all the precursors of delayed neutrons.

In addition, the total rate of release of delayed neutrons by the decay of m groups of precursors is $\sum_{i=1}^{i=m} \lambda_i C_i$. The value of C_i is the number of

precursors of the i_{th} group of delayed neutrons in existence at the present time, λ_i is the decay rate per precursor of the i_{th} group of precursors in precursors per second. Then, $\lambda_i C_i$ is the rate of release of the i_{th} group of delayed neutrons.

The loss and release of neutrons are graphically described by Fig. 3.1. The loss rate of neutrons by all causes is $\frac{n}{\ell}$, the loss rate by leakage and absorptions not leading to fission is $\frac{n}{\ell} (1 - \frac{k}{\nu})$, leaving the fission rate at $\frac{kn}{\nu\ell}$ fissions per second. Thus, the sum of the release rates of prompt neutrons and precursors of delayed neutrons is $\frac{kn}{\ell}$. Precursors are being formed at the rate $\frac{kn\beta}{\ell}$, with the i_{th} group being produced at the rate of $\frac{kn\beta_i}{\ell}$. There are C_i precursors of the i_{th} group in the reactor. The total production rate of delayed neutrons by the radioactive decay of precursors is $\sum_{i=1}^{i=m} \lambda_i C_i$. Prompt neutrons are being released at the rate $\frac{kn}{\ell} (1 - \beta)$. Therefore, the total rate of neutron release is

$$\frac{kn}{\ell} (1 - \beta) + \sum_{i=1}^{i=m} \lambda_i C_i$$

The following equation may next be stated:

$$\left[\begin{array}{c} \text{actual rate} \\ \text{of release} \end{array} \right] = \left[\begin{array}{c} \text{rate of release of} \\ \text{potential neutrons} \end{array} \right] - \left[\begin{array}{c} \text{rate of storage of} \\ \text{neutrons in precursors} \end{array} \right] + \left[\begin{array}{c} \text{rate of release of} \\ \text{delayed neutrons} \end{array} \right]$$

$$= \frac{kn}{\ell} - \frac{kn\beta}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_i$$

3.3

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Fig. 3.1 Neutron Population Loss and Gain, with Delayed Neutrons.

The complete differential equation may be stated:

$$\begin{aligned} \left[\begin{array}{c} \text{rate of} \\ \text{increase} \end{array} \right] &= \left[\begin{array}{c} \text{rate of release of} \\ \text{potential neutrons} \end{array} \right] - \left[\begin{array}{c} \text{rate of storage of} \\ \text{neutrons in precursors} \end{array} \right] \\ &+ \left[\begin{array}{c} \text{rate of release of} \\ \text{delayed neutrons} \end{array} \right] - \left[\begin{array}{c} \text{actual rate} \\ \text{of loss} \end{array} \right] \end{aligned}$$

$$\frac{dn}{dt} = \frac{kn}{\ell} - \frac{kn\beta}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_i - \frac{n}{\ell} \quad 3.4$$

or,

$$\frac{dn}{dt} = \frac{n}{\ell} \left[k(1-\beta) - 1 \right] + \sum_{i=1}^{i=m} \lambda_i C_i \quad 3.5$$

The differential equations for the delayed neutron precursors are also necessary. The rate of formation of precursors of the i th group by fissions is $\frac{kn\beta_i}{\ell}$, in which β_i is the yield of the i th group of precursors.

The rate of decay of these precursors is $\lambda_i C_i$. The differential equation for the precursors of each group may be stated:

$$\left[\begin{array}{c} \text{rate of} \\ \text{increase} \end{array} \right] = \left[\begin{array}{c} \text{rate of} \\ \text{formation} \end{array} \right] - \left[\begin{array}{c} \text{rate of} \\ \text{decay} \end{array} \right]$$

$$\frac{dC_i}{dt} = \frac{kn\beta_i}{\ell} - \lambda_i C_i \quad 3.6$$

There will be m equations of this type, one equation for each delayed neutron group.

The kinetic equations will be restated for emphasis:

$$\frac{dn}{dt} = \frac{n}{\ell} \left[k(1-\beta) - 1 \right] + \sum_{i=1}^{i=m} \lambda_i C_i \quad 3.5$$

$$\frac{dC_i}{dt} = \frac{kn\beta_i}{\ell} - \lambda_i C_i \quad 3.6$$

3.4 Solution of the Kinetic Equations

In general, both t and K are independent variables, and C_i and n are the dependent variables in Eqs. 3.5 and 3.6. Under these conditions, Eq. 3.6 represents m linear first-order equations, but Eq. 3.5 is non-linear since it contains both $\frac{dn}{dt}$ and the product of n and K . The solution of these interrelated equations, one being non-linear, can best be carried out on a computing machine and will not be discussed in this section. Refer to Section 11 on analog computing for an example of one method of solving these equations when K is a variable.

When K is a constant, Eq. 3.5 as well as Eq. 3.6 are linear and the solutions are of the general form illustrated below:

$$\begin{aligned} n &= A_0 e^{\omega_0 t} + A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + \dots + A_m e^{\omega_m t} \\ C_1 &= C_{10} e^{\omega_0 t} + C_{11} e^{\omega_1 t} + C_{12} e^{\omega_2 t} + \dots + C_{1m} e^{\omega_m t} \\ C_2 &= C_{20} e^{\omega_0 t} + C_{21} e^{\omega_1 t} + C_{22} e^{\omega_2 t} + \dots + C_{2m} e^{\omega_m t} \\ &\vdots \\ C_m &= C_{m0} e^{\omega_0 t} + C_{m1} e^{\omega_1 t} + C_{m2} e^{\omega_2 t} + \dots + C_{mm} e^{\omega_m t} \end{aligned}$$

These equations are usually written in the more abbreviated manner of Eqs. 3.7 and 3.8.

$$n = \sum_{j=0}^{j=m} A_j e^{\omega_j t} \quad 3.7$$

$$\text{and, } C_i = \sum_{j=0}^{j=m} C_{ij} e^{\omega_j t} \quad 3.8$$

The constants in the exponents, represented by ω_j are dependent entirely on the value of k under the assumption that β_i , λ_i and ℓ do not vary. The coefficients A_j and C_{ij} are dependent on the initial conditions.

The equations that provide the means of evaluating the various values of ω_j , A_j , and C_{ij} will be derived. The equation for ω_j will be derived first. Substitute Eq's. 3.7 and 3.8 into Eq. 3.6.

$$\sum_{j=0}^{j=m} \omega_j C_{ij} e^{\omega_j t} = \frac{k\beta_i}{\ell} \sum_{j=0}^{j=m} A_j e^{\omega_j t} - \lambda_i \sum_{j=0}^{j=m} C_{ij} e^{\omega_j t}$$

This equality must hold for each individual value of j . Then, divide by $e^{\omega_j t}$, and collect terms:

$$C_{ij} (\omega_j + \lambda_i) = \frac{k A_j \beta_i}{\ell}$$

or,

$$C_{ij} = \frac{k A_j \beta_i}{\ell (\omega_j + \lambda_i)} \quad 3.9$$

Substitute Eq's. 3.7 and 3.8 into Eq. 3.5

$$\sum_{j=0}^{j=m} \omega_j A_j e^{\omega_j t} = \frac{[k(1-\beta)-1]}{\ell} \sum_{j=0}^{j=m} A_j e^{\omega_j t} + \sum_{i=1}^{i=m} \lambda_i \sum_{j=0}^{j=m} C_{ij} e^{\omega_j t}$$

This equality must also hold for each individual value of j .

Divide by $e^{\omega_j t}$;

$$\omega_j A_j = \frac{A_j}{\ell} [k(1-\beta)-1] + \sum_{i=1}^{i=m} \lambda_i C_{ij} \quad 3.10$$

Substitute Eq. 3.9 into Eq. 3.10, then divide by A_j and multiply by ℓ .

$$\begin{aligned} \ell \omega_j &= [k(1-\beta)-1] + k \sum_{i=1}^{i=m} \frac{\lambda_i \beta_i}{\omega_j + \lambda_i} \\ &= (k-1) - k\beta + k \sum_{i=1}^{i=m} \frac{\lambda_i \beta_i}{\omega_j + \lambda_i} \end{aligned}$$

Now,
$$\beta = \sum_{i=1}^{i=m} \beta_i$$

So:
$$\ell \omega_j = (k-1) - k \sum_{i=1}^{i=m} \beta_i + k \sum_{i=1}^{i=m} \frac{\lambda_i \beta_i}{\omega_j + \lambda_i} \quad 3.11$$

Write $-k \sum_{i=1}^{i=m} \beta_i$ as $-k \sum_{i=1}^{i=m} \frac{\omega_j \beta_i + \lambda_i \beta_i}{\omega_j + \lambda_i}$

in order to obtain a common denominator, and substitute this expression into Eq. 3.11.

Then,

$$\begin{aligned} \ell \omega_j &= (k-1) - k \sum_{i=1}^{i=m} \frac{\omega_j \beta_i + \lambda_i \beta_i}{\omega_j + \lambda_i} + k \sum_{i=1}^{i=m} \frac{\lambda_i \beta_i}{\omega_j + \lambda_i} \\ &= (k-1) - k \sum_{i=1}^{i=m} \frac{\omega_j \beta_i}{\omega_j + \lambda_i} \end{aligned}$$

Divide by k , and collect terms.

$$\frac{k-1}{k} = \frac{\ell \omega_j}{k} + \sum_{i=1}^{i=m} \frac{\omega_j \beta_i}{\omega_j + \lambda_i} \quad 3.12$$

The reactivity, represented by ρ , is defined:

$$\rho = \frac{k-1}{k} = \frac{\delta k}{k} \quad 3.13$$

Then,

$$\rho = \frac{\ell \omega_j}{k} + \sum_{i=1}^{i=m} \frac{\omega_j \beta_i}{\omega_j + \lambda_i} \quad 3.14$$

The units of ω_j are per second, the quantity T_j has the units of seconds.

Use, $T_j = \frac{1}{\omega_j}$

Then,

$$\rho = \frac{\ell}{T_j k} + \sum_{i=1}^{i=m} \frac{\beta_i}{1 + \lambda_i T_j} \quad 3.15$$

Eq. 3.13 can be rearranged as follows:

$$k = \frac{1}{1 - \rho} \quad 3.16$$

When Eq. 3.16 is substituted into Eq. 3.12

$$\rho = \ell \omega_j (1 - \rho) + \sum_{i=1}^{i=m} \frac{\omega_j \beta_i}{\omega_j + \lambda_i}$$

This equation can be solved for ρ :

$$\rho = \frac{\ell \omega_j}{1 + \ell \omega_j} + \frac{1}{1 + \ell \omega_j} \sum_{i=1}^{i=m} \frac{\omega_j \beta_i}{\omega_j + \lambda_i} \quad 3.17$$

Either Eqs. 3.12, 3.14, 3.15, or 3.17 may be referred to as the "inhour formula" (Glasstone and Edlund, pages 298, 301, and 302).

There are $(m+1)$ numerical values of ω_j which will satisfy the inhour formula. A graphical solution of Eq. 3.17 can be obtained by considering ω_j as the continuous variable ω and plotting the curve in Figure 3.2. As ω becomes large in either the positive or negative sense, ρ becomes asymptotic to +1. Also, as ω approaches any of the values of $-\lambda_i$, the curve undergoes a discontinuity as shown. One other discontinuity in ρ occurs as ω approaches $-\frac{1}{\ell}$. The values of λ_i range from 0.0125 sec^{-1} for λ_1 , to 14.3 sec^{-1} for λ_6 , from Table 3.1 for uranium - 235. The quantity $\frac{1}{\ell}$ will have a value which probably lies between 10^3 and 10^5 sec^{-1} . Thus, Figure 3.2 is intended to be qualitative only. Operating reactors will have a range of ρ probably not greatly exceeding $(1 + \beta)$ or being smaller than -1. The numerical limits of ρ are +1 for $k = +\infty$, and $-\infty$ for $k = 0$.

After the graph of Figure 3.2 is drawn, the $(m+1)$ values of ω_j can be determined for a given value of ρ . A suggested method of identifying the various values of ω_j is shown in Figure 3.2, in which ω_0 has the possibility of covering the range between $-\lambda_1$ and positive infinity. Note that ω_0 is positive when the reactor multiplication exceeds unity, ω_0 is zero when

Fig. 3.2. General Graph of the Inhour Formula.

the multiplication is exactly unity, and ω_0 is negative when the multiplication is less than unity. All of the other values of ω_j are negative for all values of k .

The reciprocal of ω_0 is defined as the stable period T_0 , and the reciprocals of $\omega_1, \omega_2, \dots, \omega_m$, are defined as transient periods. At reactor multiplications greater than unity but less than $(1 + \beta)$, the value of the neutron lifetime has only a small effect on the stable period. Approximate values of the stable period for multiplications greater than unity may be obtained from the curve in Fig. 3.3.

The instantaneous reactor period, T , is defined as $\frac{d[\ln(n)]}{dt}$, which is really $\frac{n}{\frac{dn}{dt}}$. (Refer to Glasstone, article 4.29, page 233, and to the footnote, page 364).

$$T = \frac{n}{\frac{dn}{dt}} \quad 3.18$$

The instantaneous period is also graphically illustrated in Figure 3.4.

For increasing values of time, the value of n of Eq. 3.7 will asymptotically approach that given by Eqs. 3.19 or 3.20 because of the successively larger negative values of $\omega_1, \omega_2, \dots, \omega_m$.

$$n = A_0 e^{\omega_0 t} \quad 3.19$$

or,

$$n = A_0 e^{\frac{t}{T_0}} \quad 3.20$$

At a sufficiently large magnitude of time, Eq. 3.19 is a reasonable approximation of Eq. 3.7. Then, from Eqs. 3.18 and 3.20:

$$T = \frac{A_0 e^{\frac{t}{T_0}}}{\frac{1}{T_0} (A_0 e^{\frac{t}{T_0}})} = T_0$$

The instantaneous period, T , will asymptotically approach T_0 , and T_0 may be called the asymptotic period as well as the stable period.

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Fig. 3.3. Positive Stable Reactor Period.

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Fig. 3.4. Instantaneous Reactor Period.

Next, the initial conditions will be used to derive the equations for A_j .
Substitute Eq. 3.9 into Eq. 3.8.

$$C_i = \frac{k B_i}{l} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} e^{\omega_j t} \quad 3.21$$

Now, at the instant of the change in k , $t = 0$, and $C_i = C_{i0}$, then

$$C_{i0} = \frac{k B_i}{l} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} \quad 3.22$$

Assume steady state up to $t = 0$, then, from Eq. 3.6, when $\frac{dC_i}{dt} = 0$, $k = 1$, and $n = n_0$

Thus,

$$0 = \frac{n_0 B_i}{l} - \lambda_i C_{i0}$$

or,

$$C_{i0} = \frac{B_i}{\lambda_i l} n_0 \quad 3.23$$

Set Eq. 3.22 equal to Eq. 3.23 and solve for n_0 .

$$n_0 = k \sum_{j=0}^{j=m} \frac{\lambda_i A_j}{\omega_j + \lambda_i} \quad 3.24$$

Also, Eq. 3.7 may be written for $t = 0$.

$$n_0 = \sum_{j=0}^{j=m} A_j \quad 3.25$$

Eq. 3.25 represents one equation having $(m+1)$ terms of A_j . Eq. 3.24 represents m independent equations; one equation for each value of λ_i from

$i = 1$ through $i = m$; each equation contains $(m+1)$ terms of A_j and $(m+1)$ terms of ω_j . In the case of five delayed neutrons groups, Eq. 3.25 becomes:

$$n_0 = A_0 + A_1 + A_2 + A_3 + A_4 + A_5$$

The five equations represented by Eq. 3.24 are as follows:

$$\frac{n_0}{k} = \frac{\lambda_1 A_0}{\omega_0 + \lambda_1} + \frac{\lambda_1 A_1}{\omega_1 + \lambda_1} + \text{-----} + \frac{\lambda_1 A_5}{\omega_5 + \lambda_1}$$

$$\frac{n_0}{k} = \frac{\lambda_2 A_0}{\omega_0 + \lambda_2} + \frac{\lambda_2 A_1}{\omega_1 + \lambda_2} + \text{-----} + \frac{\lambda_2 A_5}{\omega_5 + \lambda_2}$$

$$\frac{n_0}{k} = \frac{\lambda_5 A_0}{\omega_0 + \lambda_5} + \frac{\lambda_5 A_1}{\omega_1 + \lambda_5} + \text{-----} + \frac{\lambda_5 A_5}{\omega_5 + \lambda_5}$$

There are a total of six equations here, with six terms in each equation. Once the six values of ω_0 through ω_5 are determined from Eq's. 3.12, 3.14, or 3.17, the six values of A_0 through A_5 may be determined from the six simultaneous equations represented by Eq's. 3.24 and 3.25. (also see Schultz, page 31). Then Eq. 3.7 may be written:

$$n = A_0 e^{\omega_0 t} + A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} + A_3 e^{\omega_3 t} + A_4 e^{\omega_4 t} + A_5 e^{\omega_5 t}$$

The values of C_{ij} may be obtained from Eq. 3.9. For five delayed neutron groups, there are five values of C_i , with six values of C_{ij} in each group.

Next, consider the case where the reactor has been at steady state; the reactivity is made slightly positive (or negative), then changed again after a certain time interval.

For $t < 0$, the reactivity is zero, the multiplication is unity

For $0 \leq t < t_1$, the reactivity is ρ , the multiplication is k .
(the unprimed symbols will be used during this time interval)

For $t \geq t_1$, the reactivity is ρ' , the multiplication is k' .
(the primed symbols will be used during this time interval)

The general expressions in Eq's. 3.7 and 3.8 are applicable to these new conditions for $t \geq t_1$. The inhour formula, Eq. 3.12, 3.14, or 3.18 can be used with the new values ρ' or k' to determine the new ω_j' to use in Eq.'s 3.7 or 3.8. The new values A_j can be determined by using the new boundary conditions as follows:

For $t \geq t_1$, from Eq. 3.21,

$$C_i' = \frac{k' B_i}{l} \sum_{j=0}^{j=m} \frac{A_j'}{\omega_j' + \lambda_i} e^{\omega_j'(t-t_1)} \quad 3.26$$

For $t \geq t_1$, from Eq. 3.7,

$$n' = \sum_{j=0}^{j=m} A_j' e^{\omega_j'(t-t_1)} \quad 3.27$$

Now, at $t = t_1$, Eq. 3.21 equals Eq. 3.26 and $t - t_1 = 0$, and,

$$\frac{k B_i}{l} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} e^{\omega_j t_1} = \frac{k' B_i}{l} \sum_{j=0}^{j=m} \frac{A_j'}{\omega_j' + \lambda_i}$$

Then,

$$\sum_{j=0}^{j=m} \frac{A_j'}{\omega_j' + \lambda_i} = \frac{k}{k'} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} e^{\omega_j t_1} \quad 3.28$$

Also, at $t = t_1$, Eq. 3.7 equals Eq. 3.27 and $t - t_1 = 0$.

Then

$$\sum_{j=0}^{j=m} A_j' = \sum_{j=0}^{j=m} A_j e^{\omega_j t_1} \quad 3.29$$

Eq. 3.29 represents one equation with $(m + 1)$ terms of A_j' and of A_j . Eq. 3.28 represents m equations, one for each value of λ_i , each equation containing $(m + 1)$ terms of A_j' and of A_j and $(m + 1)$ terms of ω_j and of ω_j' .

There are no general rules for the signs and values of A_j . Each solution will depend upon the initial conditions of the reactor. However, the following graphical process, based upon experimental data, will yield a limited knowledge of the signs and magnitudes of A_j .

The known experimental behavior of a reactor after a step increase in multiplication, with the reactor originally at steady state, is illustrated in Fig. 3.5. After sufficient time, the dominant term in Eq. 3.7 is $A_0 e^{\omega_0 t}$ and the approximate values of A_0 and ω_0 can be graphically obtained from the curve. In this case, both A_0 and ω_0 are positive numbers.

Now,

$$n_0 = A_0 + A_1 + A_2 + \dots + A_m$$

Since $A_0 > n_0$ then

$$[A_1 + A_2 + \dots + A_m] < 0$$

requiring that at least one, if not all these A's be negative.

When $|n - A_0 e^{\omega_0 t}|$ is plotted as in Fig. 3.6, the "peeling off"

process may be used to determine the approximate values of $A_1, A_2, \dots, A_m$ as well as $\omega_1, \omega_2, \dots, \omega_m$. All of these ω 's and A's are negative since the slope of the curves will not reverse sign.

For a negative ρ , n will decrease as illustrated in Fig. 3.7; again from experimental evidence. Obviously, ω_0 is negative. Here, $n_0 > A_0$, and

$$[A_1 + A_2 + \dots + A_m] > 0$$

The peeling off process will show that all the A's are positive because there will be no reversal of the slope of the "peeled off" curves. In summary, for the reactor being initially at steady-state:

For a positive ρ :

A_0 and ω_0 are positive

A_1 through A_m are negative

ω_1 through ω_m are negative

For a negative ρ :

A_0 through A_m are positive

ω_0 through ω_m are negative.

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Fig. 3.5. Step Increase in Multiplication.

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Fig. 3.6. Graphical Determination of Transient Terms.

Fig. 3.7. Step Decrease in Multiplication

Fig. 3.8. Power Change for Positive and Negative Reactivities.

The approximate dynamic behavior of a reactor under various circumstances has been computed and sketched in Figs. 3.8 and 3.9, and 3.10. The reactor was assumed to be at steady state preceding the step change in multiplication in each case. In Fig. 3.10, the multiplication was reduced to unity 10 sec. after the initial step increase. Only one group of delayed neutrons was considered in these calculations, β was taken as 0.0075, λ as 0.08 sec^{-1} , and ℓ as 10^{-3} sec .

The reactor power, or fission rate, was assumed to be proportional to the neutron population in these curves. This is an approximation, however, it is quite common to refer to the reactor neutron flux and reactor power as being directly proportional.

3.5 Initial Variation in Neutron Population

For small step increases in K , from the steady state, the neutron population will jump as if there were no delayed neutrons; then as the neutrons from the decay of precursors no longer correspond to the new level, the increase slows, and then rises exponentially on a stable period determined largely by the delayed neutrons. A study of Eq. 3.5 will reveal the situation. The approximate height of the transient rise may be obtained by assuming that the rate of rise is so rapid that the rate of decay of precursors does not change during the transient. Rewrite Eq. 3.5:

$$\frac{dn}{dt} = \frac{n}{\ell} [k-1] - \frac{kn\beta}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_i$$

Under steady state conditions, $\frac{dn}{dt} = 0$, $k=1$, and $n=n_0$. Then

$$\frac{n_0\beta}{\ell} = \sum_{i=1}^{i=m} \lambda_i C_{i0}$$

where n and C_{i0} are the conditions at the beginning of the transient. Assume that the rate of decay of delayed neutron precursors remains constant at this initial value. Then,

$$\frac{dn}{dt} = \frac{n}{\ell} [k-1] - \frac{kn\beta}{\ell} + \frac{n_0\beta}{\ell}$$

Fig. 3.9. Power Change for Positive and Negative Reactivities.

Fig. 3.10. Power Change for Positive and Zero Reactivities.

or,
$$\frac{dn}{dt} = \frac{n}{\ell} [(k-1) - k\beta] + \frac{n_0\beta}{\ell} \quad 3.30$$

For the case of a small increase in k , such that $k\beta > (k-1)$, $[(k-1) - k\beta]$ is negative. As n increases, the quantity

$\frac{n}{\ell} [(k-1) - k\beta] + \frac{n_0\beta}{\ell}$ becomes smaller and $\frac{dn}{dt}$ will be zero at $n = n_1$. The initial jump will be defined as $\frac{n_1}{n_0}$.

At this point,

$$0 = \frac{n_1}{\ell} [(k-1) - k\beta] + \frac{n_0\beta}{\ell}$$

and

$$\frac{n_1}{n_0} = \frac{-\beta}{k-1-k\beta} = \frac{\beta}{k\beta-(k-1)} = \frac{\beta}{k\beta-\delta k} \quad 3.31$$

A useful approximation of Eq. 3.31 is given by Eq. 3.32

$$\begin{aligned} \frac{n_1}{n_0} &\cong \frac{\beta}{\beta - \delta k} \\ &\cong \frac{\beta}{\beta - \rho} \end{aligned} \quad 3.32$$

Introducing a k of 1.001 produces an $\frac{n_1}{n_0} = 1.15$.

Note that this approximate ratio is also an approximation for A_0 (Glasstone, Eqn. (4.25.1) page 231).

The foregoing approximation of the size of the jump also applies to the case of a reduction in k ; $k < 1$. Here, $[(k-1) - k\beta]$ is again negative and $\frac{dn}{dt}$ has its largest negative value at $t = 0$.

In other words,

$$\left| \frac{n}{\ell} [(k-1) - k\beta] \right| > \left| \frac{n_0\beta}{\ell} \right| \quad \text{at } t=0$$

As n falls, a point will be reached at $n = n_1$ such that

$$\left| \frac{n_1}{\ell} [(k-1) - k\beta] \right| = \left| \frac{n_0 \beta}{\ell} \right|$$

At this point $\frac{dn}{dt} = 0$, and the ratio of $\frac{n_1}{n_0}$ is again given by Eq. 3.31.

The initial rate of change of neutron population may also be determined. At the instant of a step change in k from the steady state, the rate of decay of precursors is $\frac{n_0 \beta}{\ell}$, and Eq. 3.30 may be rewritten with n_0 substituted for n .

$$\frac{dn_0}{dt} = \frac{n_0}{\ell} [(k-1) - k\beta] + \frac{n_0 \beta}{\ell}$$

Note that this equation contains no approximations.

Rearranging,

$$\frac{dn_0}{dt} = n_0 \frac{[(k-1)(1-\beta)]}{\ell} \quad 3.33$$

Or, approximately;

$$\frac{dn_0}{dt} \cong n_0 \frac{\delta k}{\ell} \cong n_0 \frac{\rho}{\ell} \quad 3.34$$

This is approximately the initial rate of change of $n_0 \frac{k-1}{\ell}$ from Eq. 1.5 with the delayed neutrons neglected.

The solution of Eq. 3.30 is a simple exponential equation having a value of n_0 at $t=0$, and with an asymptotic value of n_1 , given by Eq. 3.31.

3.6 Approximate Values of the Stable Period for Small and Large Reactivities

An approximate expression for ω_0 at small reactivities will be derived. When the stable period is of the order of 800 sec, either positive or negative, $|\omega_0| \ll |\lambda_i|$. Thus, all of the quantities $(\omega_0 + \lambda_i)$ may be written as λ_i . Substitute this approximation into Eq. 3.12.

$$\frac{k-1}{k} = \frac{\ell \omega_0}{k} + \sum_{i=1}^{i=m} \frac{\omega_0 \beta_i}{\lambda_i}$$

Or,

$$\frac{k-1}{k} = \omega_0 \left[\frac{\ell}{k} + \sum_{i=1}^{i=m} \frac{\beta_i}{\lambda_i} \right]$$

Since k is nearly unity, a further approximation is possible.

$$k-1 = \omega_0 \left[l + \sum \frac{\beta_i}{\lambda_i} \right] \quad 3.35$$

Now,

$$\bar{l} = l + \sum_{i=1}^{i=m} \frac{\beta_i}{\lambda_i} \quad 3.2$$

Substitute Eq. 3.2 into 3.35, and solve for ω_0 .

$$\omega_0 = \frac{k-1}{\bar{l}} \quad 3.36$$

Eq. 3.36 agrees with the simple assumption made in Section 3.1, using Eq. 1.5 and Eq. 3.2.

Whenever a reactor has a large negative reactivity, the stable period approaches -80.2 sec, as indicated by Fig. 3.2 and Table 3.1. Actually, this approach is quite rapid, and a value of k as large as 0.97 will produce a negative stable period shorter than 81 sec in a reactor using any of the three fuels listed in Table 3.1.

A simple algebraic process will obtain an approximate expression for the stable period for large positive reactivities. The value of ω_0 will be assumed to be sufficiently large that $\omega_0 \gg \lambda_m$ where λ_m is the highest decay rate of any of the delayed neutron precursors. This condition holds in the case of uranium-235 for stable periods of the order of 10^{-2} sec or shorter, which requires that k be something like 1.02 or larger for a neutron lifetime of 10^{-4} sec.

Under these conditions, all of the quantities $(\omega_0 + \lambda_i)$ can be written as ω_0 in Eq. 3.11.

$$\frac{k-1}{k} = \frac{l \omega_0}{k} + \sum_{i=1}^{i=m} \frac{\omega_0 \beta_i}{\omega_0}$$

Upon simplification:

$$\frac{k-1}{k} = \frac{l \omega_0}{k} + \beta$$

And,

$$\omega_0 = \frac{k-1-k\beta}{l} = \frac{k(1-\beta)-1}{l} \quad 3.37$$

It should be noted that Eq. 3.37 is useful only if $[k(1-\beta)-1]$ is sufficiently positive that $\omega_0 \gg \lambda_m$. At the condition that $k = \frac{1}{1-\beta}$ Eq. 3.37 would give a value of zero for ω_0 , which is obviously in error.

When $k = \frac{1}{1-\beta}$, the reactor is prompt critical because the chain reaction can be maintained on prompt neutrons alone. This can be verified by rewriting Eq. 3.5, with the contribution of the delayed neutrons omitted.

$$\frac{dn}{dt} = \frac{n}{\ell} [k(1-\beta) - 1]$$

At $k = \frac{1}{1-\beta}$, $\frac{dn}{dt} = 0$, which indicates that the reactor is capable of operating on prompt neutrons alone at the prompt critical condition. The multiplication at prompt critical is sometimes stated as $(1+\beta)$ because

$$1 + \beta \cong \frac{1}{1-\beta}$$

Near and above a multiplication of prompt critical, the reactor dynamic behavior is almost entirely dependent upon the neutron lifetime, rather than on the decay rates of the precursors of delayed neutrons. This situation leads to considerable concern regarding the reactor safety system if the prompt critical condition is to be encountered.

3.7 Momentary Termination of an Excursion

Assume that the reactor is on the positive stable period T_0 produced by the multiplication k . The new value of multiplication, k' , required to momentarily halt this positive excursion will be determined. The situation is displayed in Fig. 3.11.

From Eq. 3.5, up to the time t_1 ,

$$\frac{dn}{dt} = \frac{n}{\ell} [k(1-\beta) - 1] + \sum_{i=1}^m \lambda_i C_i \quad 3.5$$

Under the assumption that k has been constant sufficiently long for the effects of the transient periods to become negligible, the stable period and the instantaneous period are virtually identical. Also, from the definition of the instantaneous period by Eq. 3.18

$$T_0 = T = \frac{n}{\frac{dn}{dt}}$$

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Fig. 3.11. Momentary Termination of a Positive Excursion.

Then,

$$\frac{1}{T_0} = \omega_0 = \frac{\frac{dn}{dt}}{n}$$

Thus, from Eq. 3.5

$$\frac{\frac{dn}{dt}}{n} = \omega_0 = \frac{k(1-\beta)-1}{\ell} + \frac{1}{n} \sum_{i=1}^{i=m} \lambda_i C_i$$

Or,

$$\ell \omega_0 = k(1-\beta)-1 + \frac{\ell}{n} \sum_{i=1}^{i=m} \lambda_i C_i \quad 3.38$$

After the step change from k to k' at the time t_1 , the rate of change of the neutron population is to be momentarily zero, although $k' < 1$.

Then, from Eq. 3.5 at $t = t_1$,

$$0 = \frac{n'}{\ell} [k'(1-\beta)-1] + \sum_{i=1}^{i=m} \lambda_i C_i'$$

Or,

$$\frac{\ell}{n'} \sum_{i=1}^{i=m} \lambda_i C_i' = - [k'(1-\beta)-1] \quad 3.39$$

At the time when the change from k to k' occurs, $n = n'$ and $C_i = C_i'$, therefore, at $t = t_1$,

$$\frac{\ell}{n} \sum_{i=1}^{i=m} \lambda_i C_i = - [k'(1-\beta)-1] \quad 3.40$$

Substitute Eq. 3.40 into Eq. 3.38.

$$\ell \omega_0 = k(1-\beta)-1 - [k'(1-\beta)-1]$$

Therefore,

$$k - k' = \frac{\ell \omega_0}{1-\beta} \quad 3.41$$

The difference between k and k' is extremely small for reactor conditions below prompt critical. For example, in the case of a uranium-235 light water moderated system, β is approximately 0.0075 and ℓ is about 10^{-4} sec. A period of 10 sec requires k to be approximately 1.003 from Fig. 3.3.

Here,

$$k - k' = \frac{(10^{-4})(10^{-1})}{1 - 0.0075} = 10^{-5}$$

Thus,

$$k' = 1.003 - 0.00001 = 1.00299$$

The multiplication must be reduced further to compensate for the gradual increase in the emission rate of delayed neutrons if the excursion is to be terminated permanently. In order to hold $\frac{dn'}{dt} = 0$ indefinitely, k' must be reduced in a manner that allows n' to remain constant at all times after t , in Eq. 3.39.

The preceding analysis is not limited to positive excursions in reactor power, Eq. 3.41 is valid for both positive and negative periods. Naturally, the multiplication is increased, rather than reduced, to momentarily stop a power decrease.

In case the reactor is above prompt critical, the approximate value of k' to momentarily terminate the positive excursion can be found quite easily. Substitute Eq. 3.37 into Eq. 3.41

$$k - k' = \frac{\ell[k(1-\beta)-1]}{\ell(1-\beta)} = k - \frac{1}{1-\beta}$$

Therefore,

$$k' = \frac{1}{1-\beta}$$

3.42

A reduction of the multiplication to prompt critical will momentarily reduce the rate of power rise to zero under these conditions.

4. KINETIC EQUATIONS OF A THERMAL REACTOR

The value of n has been assumed to be the total population of neutrons of all energies within the reactor. The assumption has also been made that the delayed neutrons were released at fission energies, when actually the delayed neutrons originate at energies considerably below the average energy of the prompt neutrons. Under these assumptions, C_i is the total population of the λ_{ih} group of precursors within the reactor. However, if desired, n may be taken as the population of neutrons within any desired energy range. This concept has been discussed previously in Section 2.

Suppose that n of Eq. 3.5 is taken as the number of thermal neutrons in the reactor. Then, the C_i of Eq. 3.6 is no longer the total population of a group of precursors, but C_i now becomes the number of precursors that would be needed if the delayed neutrons produced by the decay of these precursors were released directly as thermal neutrons. Obviously, the number of thermal neutrons existing within the reactor is smaller than the total population by some proportionality factor. Similarly, the number of precursors that would be needed if the delayed neutrons were released directly into the thermal group is smaller than the actual population of precursors by the same proportionality factor. Eq. 3.6 indicates the relation between the population of precursors to be considered and the population of the neutrons within the energy range selected. Eqs. 3.5 and 3.6 remain valid provided that new definitions of n and C_i are applied.

If the actual total precursor population together with thermal neutrons only are considered, the actual rate of production of thermal neutrons may be written as follows:

$$\left[\begin{array}{c} \text{rate of production} \\ \text{of} \\ \text{thermal neutrons} \end{array} \right] = \left[\begin{array}{c} \text{rate of production} \\ \text{of prompt} \\ \text{thermal neutrons} \end{array} \right] + \left[\begin{array}{c} \text{rate of production} \\ \text{of delayed} \\ \text{thermal neutrons} \end{array} \right] =$$

$$= (1 - \beta) \nu V \phi \sum_{ff} \epsilon_p P_s + \rho P_s \sum_{i=1}^{i=m} \lambda_i C_i$$

The symbols have been defined in Sections 1.2, 3.2 and 3.3.

The decay of precursors of delayed neutrons is assumed to release neutrons at fission energy; therefore, the delayed neutrons reaching thermal energies are reduced by the resonance escape probability and the non-leakage probability during slowing down.

Therefore,

$$\begin{aligned} \frac{dn_{th}}{dt} &= \left[\begin{array}{c} \text{rate of} \\ \text{production of} \\ \text{thermal neutrons} \end{array} \right] - \left[\begin{array}{c} \text{rate of} \\ \text{loss of} \\ \text{thermal neutrons} \end{array} \right] \\ &= (1-\beta) \nu V \phi \Sigma_{ff} \epsilon p P_S + p P_S \sum_{i=1}^{i=m} \lambda_i C_i - \frac{V \phi \Sigma_a}{P_{th}} \\ &= \left[\frac{(1-\beta) \nu V \phi \Sigma_{ff} \epsilon p P_S P_{th}}{V \phi \Sigma_a} - 1 \right] \frac{V \phi \Sigma_a}{P_{th}} + p P_S \sum_{i=1}^{i=m} \lambda_i C_i \end{aligned}$$

Use the definition of K from Eq. 1.8 and the definition of $\frac{V \phi \Sigma_a}{P_{th}}$ from Eq. 1.11.

Thus,

$$\frac{dn_{th}}{dt} = \left[(1-\beta)K - 1 \right] \frac{n_{th}}{\ell_{th}} + p P_S \sum_{i=1}^{i=m} \lambda_i C_i \quad 4.1$$

Compare Eq. 4.1 to Eq. 3.5.

The expressions for the precursors of delayed neutrons are as follows, considering the total population of a group of precursors:

$$\begin{aligned} \frac{dC_i}{dt} &= \left[\begin{array}{c} \text{rate of} \\ \text{formation} \end{array} \right] - \left[\begin{array}{c} \text{rate of} \\ \text{decay} \end{array} \right] \\ &= \nu V \phi \Sigma_{ff} \epsilon \beta_i - \lambda_i C_i \end{aligned}$$

Note that $V \phi \Sigma_{ff} \epsilon$ is the total number of fissions per second occurring within the reactor.

Now,

$$\begin{aligned} \lambda V \phi \Sigma_{ff} \in \beta_i &= \left(\frac{\lambda V \phi \Sigma_{ff} \epsilon_p P_S P_{th}}{V \phi \Sigma_a} \right) \frac{V \phi \Sigma_a}{p P_S P_{th}} \beta_i \\ &= k \frac{V \phi \Sigma_a}{P_{th}} \frac{\beta_i}{p P_S} \\ &= k \frac{n_{th}}{l_{th}} \frac{\beta_i}{p P_S} \end{aligned}$$

Therefore,

$$\frac{dC_i}{dt} = \frac{k n_{th} \beta_i}{l_{th} p P_S} - \lambda_i C_i \quad 4.2$$

Compare Eq. 4.2 to Eq. 3.6.

The derivation of Eqs. 4.1 and 4.2 did not include the slowing down time in the neutron life span. Neutrons which reached thermal energies were properly taken as being fewer than those released either by fission or by the decay of precursors because of the resonance capture probability and the leakage probability during slowing down. However, these thermal neutrons were assumed to drop instantly from their emitted energies to thermal energies.

5. REACTOR WITH AN INDEPENDENT NEUTRON SOURCE

5.1 Neutron Sources

One of the sources commonly used in reactors is composed of beryllium and polonium-210 in which the high-energy alpha particles emitted by the polonium produce $\text{Be}(\alpha, n)\text{C}$ reactions. The actual strength of the source in neutrons per second is a function of the source efficiency as well as the activity of the polonium. The source strength is usually given in curies, one curie being 3.7×10^{10} disintegrations per second of the polonium. An activity of one curie will produce an emission of 2×10^6 to 2.4×10^6 neutrons per second. Polonium-210 has a half life of 138.4 days, consequently the rate of neutron release will decrease accordingly. Polonium-210 is obtained by the irradiation of bismuth-209 in a reactor.

Polonium-beryllium sources with initial activities of 50 curies (and perhaps higher) may be obtained. A 50 curie source, without container, can be fabricated as a cylinder 0.7 inches in diameter and 0.7 to 1.0 inches in height. The container is wrought iron, nickel plated. The density of the Po - Be mixture alone is about 5 grams/cm³. It should be pointed out that polonium and certain forms of beryllium are extreme health hazards, especially when ingested. Therefore, the source must be completely sealed.

Another common reactor neutron source is composed of beryllium and antimony-124. Neutrons are released by the high-energy gamma photons emitted from the antimony which have (γ, n) reactions with the beryllium. Antimony-124, having a 60 day half-life, is produced by the irradiation of stable antimony-123 in a reactor. The neutron yield is approximately 2×10^6 neutrons per second for one curie. Source strengths of 40 curies are possible. This type of source is built up in strength by operation of the reactor in which it is placed, in contrast to polonium-beryllium which continues to decrease in strength whether or not it is subjected to nuclear radiation.

The decay of natural uranium also provides a small neutron source of approximately 15 neutrons per second for each kilogram of uranium.

A neutron source sometimes used in laboratories is composed of radium and beryllium. The high-energy alpha emission of the decay products of radium cause neutrons to be released from the beryllium. At equilibrium, such a source decays with the 1620 year half life of radium-226. Refer to the Reactor Handbook on Physics, USAEC - McGraw-Hill, 1955.

Reactors containing beryllium or deuterium will have another source of neutrons as an inherent characteristic. After the reactor has been operated at power, the decay of fission products will release gamma radiation having sufficient energy to produce (γ, n) reactions in the beryllium or deuterium. This source is not available for the initial startup, and will decay after a long shutdown period. Replacement of the fuel will also eliminate this source.

The apparent delayed neutron contribution will be somewhat larger in reactors containing beryllium or deuterium than listed in Table 3.1. The gamma radiation from the decay of fission products immediately following their formation will cause the release of neutrons from the beryllium or deuterium. These photoneutrons will lag the fission neutrons in time and will affect the behavior of the reactor in a manner similar to that of the delayed neutrons. The decay constant and yield of each particular specie of fission product emitting gamma energies sufficient to cause (γ, n) reactions in the beryllium will determine the equivalent delayed neutron emission due to the beryllium. There will be many such equivalent delayed neutron groups. The inhour formula, Eqs. 3.12, 3.14, 3.15 or 3.17, cannot be accurately applied to a reactor containing beryllium.

5.2 Subcritical Multiplication

The effect of adding an independent source of neutrons to the volume under consideration will be examined. The source neutrons are assumed to be spread out in both energy and space in the same manner as the neutrons from fissions. Eq. 3.5 becomes:

$$\frac{dn}{dt} = \left[\frac{n}{\ell} k - 1 \right] - \frac{kn\beta}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_i + S \quad 5.1$$

The source S is in neutrons per second. The source does not contribute directly to the precursors of delayed neutrons, thus Eq. 3.6 is applicable.

$$\frac{dC_i}{dt} = \frac{kn\beta_i}{\ell} - \lambda_i C_i \quad 3.6$$

Under steady state conditions, K must be somewhat less than unity, and also, the precursor concentrations must be constant; $\frac{dC_i}{dt} = 0$.

Then, from Eq. 3.6,

$$\frac{k n \beta}{\ell} = \sum_{i=1}^{i=m} \lambda_i C_i, \text{ and Eq. 5.1 becomes } \frac{dn}{dt} = \frac{n}{\ell} [k-1] + S.$$

Now, since the sub-critical reactor is at steady state, $\frac{dn}{dt} = 0$, and

$$\frac{n}{\ell} = \frac{S}{1-k} \quad 5.2$$

or,

$$n = S \ell \frac{1}{1-k} \quad 5.3$$

Note that the assumption has been made that the neutron lifetime is not affected by changes in multiplication.

The quantity $\frac{n}{\ell}$ is the total neutrons per second being released in the reactor. The quantity S is a constant; therefore, $\frac{1}{1-k}$ represents a multiplication of the source. This factor of $\frac{1}{1-k}$ is referred to as the subcritical multiplication of the reactor. (A discussion of subcritical multiplication is given in Glasstone and Edlund, pages 220-222).

In critical experiments, the counting rate of an instrument such as a fission chamber is observed and a plot made of the reciprocal of the counts per second against the mass of fuel in the reactor. The counting rate is proportional to n ; thus, the reciprocal counting rate is proportional to $(1-k)$ from Eq. 5.3. In Fig. 5.1 the intersection of the curve with the abscissa should yield the mass required to go critical. Many factors combine to limit the extent to which this type of data can be extrapolated; however, the information obtained enables one to safely proceed with the experiment. See Schultz, pages 217-220. Note that $k=1$ does not define the steady state condition with a source present.

5.3 Delayed Neutrons as a Source

The concept of the delayed neutrons as a source of neutrons for an imaginary reactor that has no delays is helpful as a qualitative approach to reactor kinetics. Think of the decay of precursors as an external source supplying neutrons, and of the multiplication of this imaginary reactor as $k - \beta$, where k is the multiplication of the actual reactor. Then, at steady state, $k=1$, and $n = n_0$; and the source is $\beta \frac{n_0}{\ell}$ neutrons per second.

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Fig. 5.1. Approach to Criticality.

Now, if the K of the actual reactor is suddenly increased to $(1 + \delta K)$, with δK less than β , the new multiplication of the imaginary reactor remains less than unity, and the jump is finite. The new fictional multiplication is $(1 + \delta K - \beta)$. Assume the source remains constant.

Substitute the expressions for the source and multiplication of this fictional reactor into Eq. 5.3, using n_1 for the new neutron population after the jump rise.

$$n = \beta \frac{n_0}{\ell} \ell \frac{1}{1 - (1 + \delta K - \beta)}$$

$$\frac{n_1}{n_0} = \frac{\beta}{-\delta K + \beta} = \frac{\beta}{\beta - \delta K} \quad 5.4$$

Compare Eq. 5.4 to Eq. 3.31.

After the jump, there will be an increase in the precursor population of the actual reactor, followed by a gradual increase in decay rate. The fictional source therefore begins to increase, producing the continuous exponential rise in neutron population. Notice that the imaginary reactor has remained subcritical throughout this variation.

The same type of argument can be used to describe the effect of a sudden decrease in K . Qualitative answers can be obtained for a change in K while the reactor is still in a transient caused by a previous change in reactivity.

5.4 Range of Neutron Level

Consider a reactor into which a source has been inserted. The range of power level from shutdown to full power can easily be estimated. Eq. 5.2 is used to determine the rate of release of neutrons at the shutdown condition. The rate of release of neutrons at full power is determined from the fissions per watt-second and the neutrons released per fission.

From Eq. 5.2, the neutrons released per second while shutdown may be written as $\frac{S}{1 - K_s}$, where K_s is the value of K shutdown.

Take 3×10^{10} fissions/watt-sec., and 2.5 neutrons released per fission.

Then,

$$\left[\begin{array}{l} \text{neutrons released} \\ \text{per second at} \\ \text{power} \end{array} \right] = (\text{power in watts}) (3 \times 10^{10})(2.5)$$

$$= (7.5 \times 10^{10})(\text{power in watts})$$

The range of flux level, and of power level, will be approximately the same as the range in neutron release rate. Thus,

$$\begin{aligned} \text{Range} &= \frac{\text{neutron release rate at power}}{\text{neutron release rate at shutdown}} \\ &= \frac{(7.5 \times 10^{10})(\text{power in watts})}{\frac{S}{1 - k_s}} \\ &= (\text{power in watts}) \frac{(1 - k_s)(7.5 \times 10^{10})}{S} \end{aligned} \quad 5.5$$

Take as an example a source of 10^7 neutrons/sec, a full power of 100 kw, and a shutdown k of 0.95. Then,

$$\text{Range} = \frac{(10^5)(1 - .95)(7.5 \times 10^{10})}{10^7} = 3.75 \times 10^7$$

As another example, consider a natural uranium reactor. Here, spontaneous fission contributes about 15 neutrons/kg-sec. The neutron source due to 40 tons of uranium may be computed.

$$S = 40 \times \frac{2000}{2.2} \times 15 = 5.45 \times 10^5 \text{ neutron/sec}$$

If the reactor runs at 4000 kw, and has a shutdown k of 0.95,

$$\text{Range} = \frac{(4 \times 10^6)(1 - .95)(7.5 \times 10^{10})}{5.45 \times 10^5} = 2.75 \times 10^{10}$$

A range of 10^{10} is a substantial number of decades to measure. Refer to Glasstone, pages 364-366 and Article 5.112; and to Schultz, pages 187-190.

6. FISSION PRODUCT POISONING

The effect of fission product buildup plays an important role in reactor control, especially in high-flux reactors having highly enriched fuel and low absorption materials. In the case of xenon-135 and samarium-149, particular attention must be given to the very large capture cross sections and the time behavior. This subject is discussed in Glasstone and Edlund, Chapter XI, pages 315-316 and pages 329-339; in Glasstone pages 263-270; and in Schultz pages 248-250, 23-26, 54-58 and 237.

6.1 Reactivity Change Due to a Poison

The change in the multiplication of a thermal reactor due to the introduction of a neutron absorbing poison can be examined by means of the thermal utilization term. Take f as the thermal utilization term without poison, and f' as the term with poison.

$$f = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{ae}} \quad 6.1$$

$$f' = \frac{\Sigma_{af}}{\Sigma_{af} + \Sigma_{ae} + \Sigma_p} \quad 6.2$$

Where, Σ_{af} = macroscopic absorption cross section of the fuel in cm^{-1}

Σ_{ae} = macroscopic absorption cross section of the reactor, not including fuel or poison, in cm^{-1}

Σ_p = macroscopic absorption cross section of the poison in cm^{-1}

The addition of a poison to a thermal reactor has little effect on the multiplication except through the thermal utilization term. Assume that the multiplication was unity before addition of the poison.

$$\frac{k-1}{k} = \frac{f'-f}{f'} = \rho_p \quad 6.3$$

Therefore, the reactivity after addition of the poison can be expressed by substituting Eqs. 6.1 and 6.2 into Eq. 6.3.

$$\rho_p = - \frac{\Sigma_p}{\Sigma_{af} + \Sigma_{ae}} \quad 6.4$$

Note that the effect of the poison is less for "dirty" reactors which normally have high absorption cross sections than for "clean" reactors having low values of Σ_{ae} .

A useful quantity called the poisoning, P , is defined by Eq. 6.5.

$$P = \frac{\Sigma_p}{\Sigma_{af}} \quad 6.5$$

The poisoning is equivalent to the absolute value of the reactivity change in a reactor in which all of the previous neutron absorptions had been in the fuel only. Then, Eq. 6.5 expresses the maximum change in reactivity that can be caused by a neutron absorbing poison.

6.2 Xenon Concentration

Xenon-135 has a capture cross section for thermal neutrons of about 3.5×10^6 barns; therefore, any appreciable amount will have a large effect on the reactivity of a thermal reactor. Tellurium-135 is formed in about 5.6% of the slow neutron fissions of uranium-235; this undergoes a series of stages of negative beta decay through iodine-135, xenon-135, and cesium-135 to barium-135, as follows:

In addition, xenon-135 is formed directly in about 0.3% of the fissions.

Since the half life of tellurium-135 is very much smaller than that of its daughter iodine-135, the analysis is simplified by assuming that iodine-135 is produced directly from fission. The expressions for the iodine concentration may be written as Eq. 6.6.

$$\frac{dI}{dt} = -\lambda_I I - \sigma_I \phi I + \gamma_I \Sigma_{ff} \phi \quad 6.6$$

- I = concentration of iodine-135 nuclei per cm^3
 λ_I = decay constant for iodine-135, $2.9 \times 10^{-5} \text{ sec}^{-1}$
 σ_I = microscopic cross-section of iodine-135 for capture of thermal neutrons, $7 \times 10^{-24} \text{ cm}^2$
 ϕ = thermal neutron flux, neutrons per $\text{cm}^2\text{-sec}$
 γ_I = fractional yield of iodine-135 (tellurium-135) from fission, 0.056
 Σ_{ff} = macroscopic fission cross section of the uranium-235 in the reactor, cm^{-1}

The second term is very small compared to the first term and can be neglected for values of ϕ less than 10^{15} or 10^{16} neutrons per $\text{cm}^2\text{-sec}$.

Then,

$$\frac{dI}{dt} = -\lambda_I I + \gamma_I \Sigma_{ff} \phi \quad 6.7$$

At steady state, $\phi = \phi_0$ and $\frac{dI}{dt} = 0$. Solve for I_0 , the equilibrium concentration of iodine-135.

$$I_0 = \frac{\gamma_I \Sigma_{ff} \phi_0}{\lambda_I} \quad 6.8$$

The equation describing the concentration of xenon-135 may be written as Eq. 6.9.

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_{ff} \phi - \lambda_X X - \sigma_X \phi X \quad 6.9$$

- X = concentration of xenon-135 nuclei per cm^3
 γ_X = fractional yield of xenon-135 from fission, 0.003
 λ_X = decay constant for xenon-135, $2.1 \times 10^{-5} \text{ sec}^{-1}$
 σ_X = microscopic cross section of xenon-135 for capture of thermal neutrons, $3.5 \times 10^{-18} \text{ cm}^2$

At steady state, $\phi = \phi_0$ and $\frac{dX}{dt} = 0$. Solve for X_0 , the equilibrium concentration of xenon-135.

$$X_0 = \frac{\lambda_I I_0 + \gamma_X \Sigma_{ff} \phi_0}{\lambda_X + \sigma_X \phi_0}$$

Introduce I_0 from Eq. 6.8.

$$X_0 = \frac{(\sigma_I + \sigma_X) \Sigma_{ff} \phi_0}{\lambda_X + \sigma_X \phi_0} \quad 6.10$$

It can be seen in Eq. 6.10 that the xenon disappears by two means; (1) the natural decay term $\lambda_X X$, and (2) the absorption term $\sigma_X \phi X$. The equilibrium value of the xenon poisoning, P_{X0} , can be found by means of Eqs. 6.5 and 6.10.

$$P_{X0} = \frac{\Sigma_X}{\Sigma_{af}} = \frac{\sigma_X \Sigma_{ff} \phi_0 (\sigma_I + \sigma_X)}{\Sigma_{af} (\lambda_X + \sigma_X \phi_0)} \quad 6.11$$

At values of thermal flux exceeding 10^{14} neutrons per cm^2 -sec, $\sigma_X \phi_0 \gg \lambda_X$, and the maximum upper limit can be found.

$$P_{XL} = \frac{\Sigma_{ff}}{\Sigma_{af}} (\sigma_I + \sigma_X)$$

For uranium-235, $\frac{\Sigma_{ff}}{\Sigma_{af}} = 0.84$.

$$P_{XL} = 0.05$$

Thus, the maximum possible decrease in reactivity due to the steady state xenon concentration cannot exceed 5%.

The next point to observe is the buildup of xenon following a reactor shutdown. Due to the fact that in a high-flux reactor a large part of the xenon-135 is being destroyed by the flux and due to the iodine-135 equilibrium concentration, the concentration of xenon-135 increases following a shutdown; that is, the iodine-135 continues to decay to xenon-135 which is not being destroyed as rapidly due to the small flux after shutdown. The steady-state concentration is troublesome; however, the increase after shutdown is an item of real concern.

The equation for the xenon concentration after shutdown of the reactor will be derived on the assumption that equilibrium conditions had been attained before shutdown. If the neutron flux were reduced to zero, the equation

for the iodine concentration can be stated as Eq. 6.12.

$$\frac{dI}{dt} = -\lambda_I I \quad 6.12$$

This equation can be solved and the initial condition that $I = I_0$ at $t = 0$ employed.

$$I = I_0 e^{-\lambda_I t} \quad 6.13$$

The expression for the xenon concentration at zero flux can be obtained from Eqs. 6.9 and 6.13.

$$\frac{dX}{dt} + \lambda_X X = \lambda_I I_0 e^{-\lambda_I t} \quad 6.14$$

This type of linear first order differential equation can be solved by multiplying both sides of the equation by an integrating factor which will allow the result to be integrated as an exact differential equation.

The integrating factor in this case is $e^{\int \lambda_X dt} = e^{\lambda_X t}$

$$e^{\lambda_X t} \frac{dX}{dt} + \lambda_X X e^{\lambda_X t} = \lambda_I I_0 e^{(\lambda_X - \lambda_I)t} \quad 6.15$$

The left-hand side of Eq. 6.15 is an exact derivative.

$$\frac{d(X e^{\lambda_X t})}{dt} = e^{\lambda_X t} \frac{dX}{dt} + \lambda_X X e^{\lambda_X t} \quad 6.16$$

Substitute Eq. 6.16 into Eq. 6.15, and integrate

$$X e^{\lambda_X t} = \frac{\lambda_I}{\lambda_X - \lambda_I} I_0 e^{(\lambda_X - \lambda_I)t} + k_1$$

Use the initial condition that $X = X_o$ at $t = 0$, and rearrange.

$$X = \frac{\lambda_I}{\lambda_X - \lambda_I} I_o e^{-\lambda_I t} + (X_o - \frac{\lambda_I}{\lambda_X - \lambda_I} I_o) e^{-\lambda_X t} \quad 6.17$$

The time, t_M , after shutdown at which the xenon concentration has reached its maximum value will be obtained from Eq. 6.17.

$$\frac{dX}{dt} = 0 = \lambda_I I_o \left(\frac{\lambda_I}{\lambda_X - \lambda_I} \right) e^{-\lambda_I t_M} - \lambda_X \left(X_o - \frac{\lambda_I}{\lambda_X - \lambda_I} I_o \right) e^{-\lambda_X t_M} \quad 6.18$$

Divide Eq. 6.18 by $\lambda_I I_o \left(\frac{\lambda_I}{\lambda_X - \lambda_I} \right) e^{-\lambda_X t_M}$, and rearrange.

$$\frac{e^{-\lambda_I t_M}}{e^{-\lambda_X t_M}} = \frac{\lambda_X}{\lambda_I} \left[1 - \left(\frac{\lambda_X - \lambda_I}{\lambda_I} \right) \left(\frac{X_o}{I_o} \right) \right] \quad 6.19$$

Write $\frac{e^{-\lambda_I t_M}}{e^{-\lambda_X t_M}}$ as $e^{(\lambda_X - \lambda_I) t_M}$, take the logarithm of Eq. 6.19, and rearrange.

$$t_M = \frac{1}{\lambda_X - \lambda_I} \ln \frac{\lambda_X}{\lambda_I} \left[1 + \left(\frac{\lambda_I - \lambda_X}{\lambda_I} \right) \left(\frac{X_o}{I_o} \right) \right] \quad 6.20$$

The xenon poisoning, P_X , after shutdown can be computed by means of Eqs. 6.5, 6.8, 6.10, and 6.17.

$$P_X = \phi_o \sigma_X \left(\frac{\sum_{ff}}{\sum_{af}} \right) \left[\frac{\delta_I}{\lambda_X - \lambda_I} e^{-\lambda_I t} + \left(\frac{\delta_I + \delta_X}{\lambda_X + \sigma_X \phi_o} - \frac{\delta_I}{\lambda_X - \lambda_I} \right) e^{-\lambda_X t} \right] \quad 6.21$$

All of the numerical values of the constants are known, since $\frac{\Sigma_{ff}}{\Sigma_{af}}$ is 0.84 for uranium-235. Curves illustrating the xenon poisoning after shutdown have been computed and plotted in Fig. 6.1. The time at the maximum concentration has been indicated in each case. An excess multiplication of approximately 0.5 would be needed in a highly-enriched "clean" reactor in order to over-ride the xenon poisoning under all conditions if the average thermal flux were as large as 2×10^{14} neutrons per $\text{cm}^2\text{-sec}$. This would require that the control system have the capacity to raise the reactor multiplication from unity to 1.5 at the time the reactor is started. The excess multiplication required probably limits the average thermal flux in the fuel to something less than 2×10^{14} neutrons per $\text{cm}^2\text{-sec}$ if restart is to be necessary at any time.

6.3 Samarium Concentration

The effect of samarium-149 is not usually serious during operation of the reactor although it does offer a limitation in the fact that it is stable and does not decay after shutdown. Samarium-149 is produced by the following beta decay chain:

The neodymium-149 is produced in 1.4% of the fissions of uranium-235 in a thermal reactor, but the decay to promethium-149 is very short compared to the decay of promethium-149 to samarium-149. Promethium-149 will be assumed to be formed directly from fission.

$$\frac{dP_m}{dt} = -\lambda_p P_m + \sigma_p \Sigma_{ff} \phi \quad 6.22$$

$$\frac{dS_m}{dt} = \lambda_p P_m - \sigma_s \phi S_m \quad 6.23$$

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Fig. 6.1. Xenon Poisoning After Shutdown.

- P_m = concentration of promethium-149 nuclei per cm^3
 λ_p = decay constant for promethium-149, $4.1 \times 10^{-6} \text{ sec}^{-1}$
 γ_p = fractional yield of promethium-149 (neodymium-149) from fission, 0.014
 S_m = concentration of samarium-149 nuclei per cm^3
 σ_s = microscopic cross section of samarium-149 for capture of thermal neutrons, $5.3 \times 10^{-20} \text{ cm}^2$

The equilibrium concentrations of samarium-149, S_{m_0} , and of promethium-149, P_{m_0} , can be found from Eqs. 6.22 and 6.23 by taking both $\frac{dP_m}{dt}$ and $\frac{dS_m}{dt}$ as zero.

$$P_{m_0} = \frac{\gamma_p}{\lambda_p} \Sigma_{ff} \phi_0 \quad 6.24$$

$$S_{m_0} = \frac{\gamma_p}{\sigma_s} \Sigma_{ff} \quad 6.25$$

The equilibrium poisoning, P_{s_0} , of the samarium can be found from Eqs. 6.5 and 6.25

$$P_{s_0} = \gamma_p \frac{\Sigma_{ff}}{\Sigma_{af}} \quad 6.26$$

Numerically,

$$P_{s_0} \cong 0.012$$

Thus, the steady state reduction in reactivity due to the samarium-149 will be a constant no larger than 1.2% regardless of the reactor power level.

The buildup of samarium after shutdown can be found from Eqs. 6.22 and 6.23 by assuming the flux to be zero and taking the initial values of promethium and samarium as P_{m_0} and S_{m_0} respectively.

$$P_m = P_{m_0} e^{-\lambda_p t} \quad 6.27$$

Then,

$$\frac{dS_m}{dt} = \lambda_p P_{m_0} e^{-\lambda_p t} \quad 6.28$$

Integrate Eq. 6.28

$$S_m = -P_{m_0} e^{-\lambda_p t} + k_1$$

At $t = 0$, $S_m = S_{m_0}$; then $k_1 = P_{m_0} + S_{m_0}$.

Therefore,

$$S_m = S_{m_0} + P_{m_0} (1 - e^{-\lambda_p t}) \quad 6.29$$

The samarium poisoning, P_s , after shutdown can be determined by means of Eqs. 6.5, 6.24, 6.25, and 6.29.

$$P_s = \frac{\Sigma_{ff}}{\Sigma_{af}} \left[\gamma_p + \frac{\sigma_p \phi_0}{\lambda_p} (1 - e^{-\lambda_p t}) \right] \quad 6.30$$

Eq. 6.30 indicates that the poisoning gradually builds up and approaches the numerical value, $P_{s\infty}$, stated in Eq. 6.31.

$$P_{s\infty} = 0.012 + 1.5 \times 10^{-16} \phi_0 \quad 6.31$$

This maximum value of the poisoning is only 0.042 for an average reactor thermal flux of 2×10^{14} neutrons per cm^2 -sec. Thus, the effect of samarium as a poison is small compared to the effect of xenon.

6.4 Reactor Instability Due to Fission Product Poisoning

Fission product poisoning can lead to a serious dynamic control problem, especially in a "clean" reactor operating near or above a thermal flux of 10^{14} neutrons per cm^2 -sec. A reduction of the flux, brought about by the

control system, will be followed by an initial increase in the poisoning of both xenon and samarium, and the control system must act to prevent a shutdown of the reactor. Similarly, an increase in the flux to a new higher operating level will be followed by an initial reduction in the poisoning, and the control system must prevent further power increases. Thus, the change in poisoning tends to cause the reactor power to continue to change in the direction of the original change, and this tendency must be countered by the control system.

In order for the instability to be present, the initial rate of change of the poisoning must be opposite in sign to the change of flux. The rate of change of xenon concentration can be obtained from Eq. 6.9. Assume that equilibrium conditions have been reached, and that the thermal flux is changed from ϕ_0 to ϕ_1 at the time t_0 . The equation for the rate of change of xenon concentration at the time t_0 will be written, using the relationships of Eq. 6.9.

$$\frac{dX}{dt_0} = (\lambda_I I_0 - \lambda_X X_0) - \phi_1 (\sigma_X X_0 - \gamma_X \Sigma_{ff}) \quad 6.32$$

Also, from Eq. 6.9, the following equation can be written to describe the equilibrium conditions prior to the change from ϕ_0 to ϕ_1 .

$$0 = (\lambda_I I_0 - \lambda_X X_0) - \phi_0 (\sigma_X X_0 - \gamma_X \Sigma_{ff}) \quad 6.33$$

Eq. 6.34 can be obtained by combining Eqs. 6.32 and 6.33.

$$\frac{dX}{dt_0} = (\phi_0 - \phi_1) (\sigma_X X_0 - \gamma_X \Sigma_{ff}) \quad 6.34$$

If the quantity $(\sigma_X X_0 - \gamma_X \Sigma_{ff})$ is positive, the initial change in xenon poisoning will be opposite in sign to the change in flux and the xenon poisoning tends to create an instability. The value of X_0 is a function of ϕ_0 , and the value of ϕ_{0X} at which the xenon poisoning tends to create instability

can be found by means of Eq. 6.35

$$\sigma_x X_{ox} - \gamma_x \Sigma_{ff} = 0 \quad 6.35$$

The xenon concentration X_{ox} occurs at the corresponding thermal flux ϕ_{ox} . Substitute Eq. 6.10 into Eq. 6.35, use X_{ox} for X_o , and ϕ_{ox} for ϕ_c , and solve for ϕ_{ox} .

$$\phi_{ox} = \frac{\lambda_x \sigma_x}{\sigma_x \gamma_I} \quad 6.36$$

Numerically, $\phi_{ox} = 3.22 \times 10^{11}$ neutrons per $\text{cm}^2\text{-sec}$.

Therefore, the xenon concentration tends to create instability at a thermal neutron flux greater than 3.22×10^{11} neutrons per $\text{cm}^2\text{-sec}$, but the magnitude of this instability is probably not significant at flux levels below 10^{13} neutrons per $\text{cm}^2\text{-sec}$.

The effect of samarium poisoning always tends to produce instability, but the effect is relatively small compared to that of xenon. The initial rate of change of samarium after a change in flux can be found by means of Eqs. 6.23, 6.24, and 6.25.

$$\frac{dS_m}{dt_o} = \gamma_p \Sigma_{ff} (\phi_o - \phi_I) \quad 6.37$$

The assumption has been made that equilibrium conditions had been attained before the flux was changed. Note that the initial rate of change of samarium poisoning is opposite in sign to the change in flux.

The preceding analysis merely indicates that the tendency toward instability due to fission product poisoning exists under certain conditions. Further analysis is necessary to determine the extent of the problem.

7. TEMPERATURE COEFFICIENT OF REACTIVITY

Glasstone and Edlund contains a brief discussion of reactivity temperature coefficients, pages 339-344. Also, see Glasstone, pages 255-261.

The effect of a temperature coefficient to be discussed in Sections 7.1 and 7.2 is assumed to be that of a single coefficient associated with the mean temperature of the fuel. The mean temperature of the fuel is a rather nebulous thing, and no single temperature or temperature coefficient actually exists. For example, a thermal reactor fuel containing both uranium-235 and uranium-238 will exhibit a Doppler broadening of the uranium-238 absorption resonance which depends on the fuel temperature. However, the effect of this Doppler broadening will depend upon the energy spectrum of the neutrons incident upon the uranium-238, and this spectrum is a function of the moderator temperature. The various partial coefficients which combine to form the total coefficient may vary in magnitude and sign, and they vary differently with respect to time and reactor power.

A heterogeneous reactor such as a graphite reactor with solid fuel cooled by light water in tubes around the fuel is an interesting case. The fuel may contribute to a negative coefficient which acts essentially instantly on heating, the cooling water may contribute to a positive coefficient slightly slower, and the contribution of the graphite to the overall coefficient may again be negative but on a much slower time scale due to the large heat capacity and the small amount of heat released in the graphite in comparison with that in the fuel. In such a reactor, it is interesting to note that if the flow of coolant is interrupted it is quite possible that a dangerous situation could be brought about by boiling in the coolant tubes and the consequent increase in reactivity as coolant is expelled; such a situation is to be avoided in the design if at all possible. A negative temperature coefficient is desired to provide stability.

7.1 Reactor with Constant Heat Removal Rate

The following special case is taken from notes by A. M. Weinberg and will serve to illustrate the effect of the temperature coefficient. Consider

a bare, homogeneous reactor. Think of the reactor as a feedback system in which a change in reactivity will change the fission rate, which in turn changes the temperature and density of the reactor; this makes a change in the reactivity, and the loop is completed. As usual, in such a situation, the question is one concerning stability. For simplicity, neglect the delayed neutrons which, for most cases, exert a damping action. The heat removal rate is P_L and is constant, k changes linearly with the temperature of the reactor. The following symbols will be used:

t = time

P_R = average instantaneous power produced by reactor

T_1 = steady-state temperature

T = average instantaneous temperature of reactor above T_1

P_L = heat removal rate

Q = heat content of the system

S = thermal capacity of the system per degree of temperature

$$\text{Then, } \frac{dQ}{dt} = S \frac{dT}{dt} = P_R - P_L \quad 7.1$$

$$\text{Also, } \delta k = \delta k_0 + \gamma T = \text{net excess } k \quad 7.2$$

Where, γ = temperature coefficient of reactivity per degree of temperature.

δk_0 = excess k which is put into the reactor by extraneous means at time $t = 0$.

The reactor has been operating at $k = 1$.

From Eq. 1.4,

$$\frac{dn}{dt} = \frac{k-1}{l} n = \frac{\delta k}{l} n$$

Now, if the power is proportional to the neutron population,

$$\frac{dP_R}{dt} = \frac{\delta k}{l} P_R = \frac{(\delta k_0 + \gamma T) P_R}{l} \quad 7.3$$

Next, divide Eq. 7.3 by Eq. 7.1

$$\frac{dP_R}{S dt} = \frac{P_R (\delta k_0 + \gamma T)}{l (P_R - P_L)} \quad 7.4$$

Rearrange Eq. 7.4

$$\frac{P_R - P_L}{P_R} dP_R = \frac{S}{l} (\delta k_0 + \gamma T) dT$$

$$\text{or, } dP_R - P_L \frac{dP_R}{P_L} = \frac{S}{\ell} (\delta K_0 + \gamma T) dT$$

Integrate and use the boundary conditions that $P_R = P_L$ and $T = 0$ at steady-state.

$$(P_R - P_L) - P_L [\ln(P_R) - \ln(P_L)] = \frac{S}{\ell} (\delta K_0 T + \gamma \frac{T^2}{2})$$

Divide by P_L , and rearrange $[\ln(P_R) - \ln(P_L)]$

$$\frac{P_R}{P_L} - 1 - \ln \frac{P_R}{P_L} = \frac{S}{P_L \ell} (\delta K_0 T + \gamma \frac{T^2}{2}) \quad 7.5$$

The exact behavior will depend on the solution of the equation for the condition at hand; however, the following situations are possible:

For $\gamma > 0$, (positive temperature coefficient) the system is always unstable, i.e., for $T > 0$, P_R increases and the system is regenerative.

For $\gamma < 0$, (negative temperature coefficient), the system will oscillate. From Eq. 7.1, $P_R = P_L$ when $\frac{dT}{dt} = 0$. These points define the maxima and minima.

For $P_R = P_L$ in Eq. 7.5,

$$\delta K_0 T + \frac{\gamma T^2}{2} = 0$$

Obviously, this equation is satisfied if $T = 0$. Also, the equation may be written,

$$\delta K_0 + \frac{\gamma T}{2} = 0$$

Then,

$$T = -\frac{2\delta K_0}{\gamma}$$

$$\text{Thus, } T_{max} = T_i - \frac{2\delta K_0}{\gamma} \quad 7.6$$

$$\text{and, } T_{min} = T_i \quad 7.7$$

Now consider a slow increase in K with γ being negative; by the time δK_0 has been put into the system, the effect of the temperature rise will just balance the increase in K .

Then, $-\delta T = \delta k_0$

$$T_{rise} = -\frac{\delta k_0}{\delta} \quad 7.8$$

7.2 Power Reactor with a Negative Temperature Coefficient

A qualitative discussion of the behavior of a power reactor having a negative temperature coefficient of reactivity follows:

7.2.1 Step Insertion of a Positive Reactivity. (Refer to Fig. 7.1)

The step insertion at time t_1 of a positive reactivity δk_0 into a reactor operating at steady state places the reactor on a positive period.

At time t_1 , $P_R = P_L$; therefore, $S \frac{dT}{dt} = 0$, and $\frac{dT}{dt} = 0$ by Eq. 7.1. Thus although the reactor power begins to rise instantly at t_1 , the temperature remains constant for the first instant. As the power increases, P_L being held constant, $\frac{dT}{dt}$ increases and the temperature begins to rise. Since δ is negative, as the temperature rises, the reactivity decreases and the reactor period increases, as indicated by the continuously decreasing slope of the plot of power against time.

At time t_2 , P_R has its maximum value; therefore, from Eq. 7.1, $\frac{dT}{dt}$ must also be a maximum. Since P_R is still greater than P_L , the temperature continues to rise, producing a negative reactivity.

At some time t_3 , P_R becomes equal to P_L , $\frac{dT}{dt}$ becomes equal to zero and the temperature reaches its maximum value and begins to decrease. The maximum possible value of the temperature rise is given by Eq. 7.6.

In general, the step insertion of a positive reactivity will cause the temperature to overshoot its final value, then to approach the final value with oscillations of decreasing amplitude. The reactor power will oscillate with decreasing amplitude about its original value and eventually will become constant at the original value. Usually, there are no more than one or two oscillations. As δ is increased in the negative sense, the frequency of oscillation increases while the power undershoot and temperature overshoot decrease.

7.2.2 Step Increase in Load Power Demand (Refer to Fig. 7.2)

A step increase, at time, t_1 , in the load power demand P_L makes both $(P_R - P_L)$ and $\frac{dT}{dt}$ negative, from Eq. 7.1. The reactor temperature begins to decrease instantly, causing the reactivity to become positive, thus placing the reactor on a positive period. As the reactor power output increases, $\frac{dT}{dt}$ decreases until time t_2 , when $P_R = P_L$, and both $\frac{dT}{dt}$ and $\frac{d(\delta k)}{dt}$ are zero.

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Fig. 7.1. Step Insertion of Reactivity.

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Fig. 7.2. Step Increase in Load Power Demand.

Since the reactivity is still positive, the power output continues to rise, and the temperature begins to increase because the reactor power exceeds the load power. The reactivity begins to decrease until, at some time t_3 , the reactivity becomes zero and the temperature is equal to its original value. At this point, the reactor goes on a negative period. The power output begins to decrease, but the temperature continues to increase, since P_R is greater than P_L . At time t_4 , $P_R = P_L$ and $\frac{dT}{dt} = 0$, therefore the maximum temperature and minimum reactivity occur at this point. Power, temperature, and reactivity, oscillate and eventually P_R stabilizes at the new value of P_L , while temperature stabilizes at its original value.

7.2.3 Gradual Changes in Load Power Demand and Rod Position

In the previous discussion, only the average temperature of the reactor has been considered. To determine the effects of changes in load power and rod position, the maximum and minimum temperatures of the coolant or working fluid must also be considered. Refer to Fig. 7.3.

A. For a constant flow rate, $(T_{\max} - T_{\min}) = \Delta T$ is proportional to P_L ; therefore, a gradual increase in P_L requires a gradual increase in ΔT from ΔT_1 to some greater value ΔT_2 . Since there is no motion of the control rod, the average temperature T_{mean} is unchanged during the load increase.

B. In the case of gradual rod withdrawal, assuming P_L to be constant ΔT will remain constant. The average reactor temperature, T_{mean} , will rise, however, in order to cancel the positive reactivity produced by rod withdrawal. The temperatures $T_{\max}$ and $T_{\min}$ of the coolant must increase by the same amount in order to maintain the same value of ΔT . Note that in order to maintain the heat removal rate P_L , constant as the maximum temperature of the coolant increases, appropriate adjustment of the heat removal device is necessary. Also, note that the rod is the means of controlling the mean temperature of the reactor. Refer to Eq. 7.8.

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Fig. 7.3. Gradual Changes in Reactor Operation.

8. START-UP TRANSIENTS

Consider a reactor having no temperature coefficient of reactivity in which the only method of changing K is by insertion or withdrawal of neutron absorbing rods. In the design of the safety and control system the question often arises; how fast is it safe to withdraw these rods during a start-up? This is an important question as it may be desirable to have a rapid start in order to beat the xenon buildup as well as for general economy of operation.

8.1 Reactor Safety During Start-up

The continuous withdrawal of the control rods will cause progressively shorter periods as well as progressively higher power levels. Start-up transients of a reactor using uranium-235 for two different rates of increase in K are shown in Fig. 8.1. These curves were taken from analog computer results reported in WAPD-TM-1, "Subpower Range Start-up Transients," by E. G. Good, March 1956. In each case, the neutron lifetime was 10^{-4} sec, and the initial value of K was 0.90. In the first case, the rate of increase in K was 10^{-3} per second and in the second case, the rate was 10^{-4} per second.

In both cases, the reactor became critical at a power level about 70 times greater than the source level. Thus, the power at the instant K is unity is probably within the milliwatt to watt range. If the source level were one milliwatt, the power would rise to 1000 megawatts only 9 seconds after the reactor was critical with a rate of increase in k of 10^{-3} per second. When the rate of increase in K is reduced to one-tenth the former rate, the power requires 60 seconds after critical in order to rise to 10 megawatts. In the first case, the reactor period is less than 70 milliseconds, and is longer than one-half second for the slower rod withdrawal. Note also that with the fast rod withdrawal, the reactor became prompt critical only 7.5 seconds after becoming critical, whereas the reactor was always below prompt critical for the slower case. The requirements of a safety system are far more stringent in the case of the faster rod withdrawal. If the safety system were to begin to insert the control rods at a power level 10 decades above the source level, in the case of the faster rod withdrawal, the power could continue to rise until the reactor multiplication was reduced

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Fig. 8.1. Reactor Startup Transients.

to approximately prompt critical. Refer to Section 3.7.

Note that the contribution of the delayed neutron precursors is quite small by the time the reactor is above prompt critical. The precursors having the longer half lives suffer the greatest relative decrease in effectiveness.

A computing machine offers one of the most powerful tools in analyzing the transient behavior of the reactor during start-up accident. The rate of insertion of the control rods for safety action, together with any change of K due to temperature increases should be taken into account. An analog computer was used to obtain the curves in Figs. 8.2 and 8.3.

Fig. 8.2 illustrates, at least qualitatively, a start-up accident of a 700 Mw gas-cooled power reactor. These curves were taken from ORNL-2676, "Gas-Cooled Reactor Project Semiannual Progress Report for Period Ending December 31, 1958." The rod withdrawal was assumed to continue indefinitely, increasing K at the rate of 10^{-4} per second. The power excursion was limited only by the temperature rise of the fuel acting through the negative temperature coefficient. No heat was removed from the fuel during the excursion. Although the power did not rise to an excessive value, the control rod withdrawal must be stopped to prevent the melting of the fuel.

The curves in Fig. 8.3 indicate the reactor power level during the safety action of a swimming-pool type reactor. The reactor was assumed to be on a period of 23 milliseconds with negligible delayed neutron contribution at the instant of the start of safety insertion of the control rods. The non-linear relationship between the rod position and K has been taken into account in this analysis. Refer to Fig. 8.4. The rods were assumed to be initially fully withdrawn, full rod stroke was taken to be 22 inches. The curves in Figs. 8.3 and 8.4 were taken from ORNL-2318, "Reactor Safety System Response," and are not limited in application to the start-up accident.

8.2 Lower Bound of the Reactor Period In A Start-up Accident

The following approximation of the lower bound of the reactor period during a start-up accident is obtained from work by H. W. Newson. The value of the shortest possible period the reactor can reach during continuous rod withdrawal is to be determined.

Assume that K is increasing linearly with time and that the reactor is considerably above prompt critical when the neutron level n_1 is reached.

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Fig. 8.2. Startup Accident with no Heat Removal.

Fig. 8.3. Neutron Flux Excursion for Specific Rod Insertion Accelerations.

Fig. 8.4. Control Rod Worth.

The analysis concerns the reactor at neutron levels sufficiently high that the source term can be neglected. Start with Eq. 3.5.

$$\frac{dn}{dt} = [k(1-\beta) - 1] \frac{n}{\ell} + \sum_{i=1}^{i=m} \lambda_i C_i \quad 3.5$$

Next, define an expression containing a constant R which relates the reactor multiplication and time.

$$Rt = k(1-\beta) - 1 \quad 8.1$$

The reactor is prompt critical at $t=0$ in Eq. 8.1 because $k = \frac{1}{1-\beta}$ at that instant. Also,

$$R = (1-\beta) \frac{dk}{dt} \cong \frac{dk}{dt}$$

Therefore, R is directly proportional to the rate of increase of K and has the units of reciprocal time. The value of R will be approximately proportional to the withdrawal velocity of the control rods, because the change in K is almost linearly proportional to the rod position except near the upper and lower limits of the rod stroke; refer to Fig. 8.4. Substitute Eq. 8.1 into Eq. 3.5, and divide by n .

$$\frac{\frac{dn}{dt}}{n} = \frac{Rt}{\ell} + \frac{1}{n} \sum_{i=1}^{i=m} \lambda_i C_i \quad 8.2$$

From Eq. 8.2,

$$\frac{\frac{dn}{dt}}{n} > \frac{Rt}{\ell}$$

Integrate this inequality, and use the boundary conditions that $n = n_p$ at $t = 0$, corresponding to prompt critical; and that $n = n_1$, at $t = t_1$. The value of n_1 is larger than n_p by at least one or two decades.

$$\ln \frac{n_1}{n_p} > \frac{Rt_1^2}{2\ell} \quad 8.3$$

At t_1 , the contribution due to the decay of precursors of delayed neutrons should be negligible compared to the prompt neutrons from fission, refer

to Fig. 8.1. Then, from Eq. 8.2,

$$\frac{\frac{dn_1}{dt}}{n_1} = \frac{R t_1}{l} \quad 8.4$$

The instantaneous reactor period, T_1 , at the time t_1 , can be determined by means of Eqs. 3.18 and 8.4.

$$\frac{1}{T_1} = \frac{\frac{dn_1}{dt}}{n_1} = \frac{R t_1}{l}$$

Solve for t_1 .

$$t_1 = \frac{l}{R T_1} \quad 8.5$$

Substitute Eq. 8.5 into Eq. 8.3

$$\ln \frac{n_1}{n_p} > \frac{l}{2 R t_1^2}$$

Rearrange, and take the square root of both sides.

$$T_1 > \sqrt{\frac{l}{2 R \ln \frac{n_1}{n_p}}} \quad 8.6$$

The actual value of $\frac{n_1}{n_p}$ is probably not known, but the value of $\frac{n_1}{n_0}$ can usually be estimated. The neutron level before start-up is n_0 .

$$T_1 > \sqrt{\frac{l}{2 R \ln \frac{n_1}{n_0}}} \quad 8.7$$

The inequality represented by Eq. 8.7 is greater than that in Eq. 8.6. The value of the right hand side of either Eq. 8.6 or Eq. 8.7 is smaller than the actual reactor period at the instant the power level n_1 is reached. (Refer to Glasstone, page 352, Eq. (6.116.1). It must be emphasized that this derivation should be used only for a rough estimate, and then only for multiplication increase rates no smaller than 10^{-3} per second and periods no longer than 0.1 second.

A short description of the safety considerations to be taken at start-up are given in Glasstone, pages 352 and 364-367, and in Schultz, pages 226-236.

The following references also include material on the start - up accident:

WAPD-13, "Solutions of the Pile Kinetic Equations when the Reactivity is a Linear Function of Time", by Sylvan Wallach.

Nucleonics, Vol. 5, July, 1949, page 61, "Derivation and Integration of the Pile Kinetic Equations", by H. Hurwitz, Jr.

AECD-3163, "Automatic Control of Power Reactors", by M. A. Schultz

Nucleonics, Vol. 12, No. 3, March, 1954, page 57, "Safeguard Considerations for Nuclear Power Reactors", by H. Hurwitz, Jr.

ORNL-2318, "Reactor Safety System Response"

WAPD-TM-1, "Subpower Range Start-up Transients", by E. G. Good, 1956

AERE RE/R 1486, "Linear Increases in Pile Reactivity", by T. E. Burnup
1954

9. MISCELLANEOUS CONSTANTS AND TERMS RELATED TO REACTOR CONTROL

9.1 Rod Calibration

The worth of control rod can be determined by an experimental means, as described below.

Assume that the rod to be calibrated is one of several in a particular reactor. The reactor is brought critical with the selected rod at a desired position. The power level should be several decades below the point that will affect the temperature in order that any temperature coefficient of reactivity will not affect the results. The rod to be calibrated is then withdrawn a slight amount to put the reactor on a fairly long stable period. After the elapse of sufficient time to allow the stable period to be determined by the nuclear instruments, the reactor is shut down. The maximum power must be limited in order to produce no temperature rise. A power rise of about two decades on periods at 15 to 300 seconds should be satisfactory. The period is obtained from the logarithmic plot of power against time.

The process is repeated with the rod to be calibrated at a new position. when the reactor is again brought critical. Naturally the other rods are at different positions, too.

The experimental data recorded are the rod positions before and after the rod movement, and the stable period resulting from the small withdrawal. The positive reactivity inserted by the rod movement can be determined by means of the inhour formula. A curve showing the relationship of stable period to positive reactivity will be helpful. The $\frac{\delta k}{k}$ per unit of rod motion, and the average of the rod position before and after each movement are computed. This reactivity per unit of rod motion is plotted against the average value of each position to obtain a differential plot of reactivity change. Refer to Figure 9.1. The curve of reactivity against rod position can be obtained by integration of the area under the curve in Fig. 9.1 as shown in Fig. 9.2.

A certain amount of error is introduced by the change in positions of the other rods as the rod being calibrated is set in a new position each time the reactor is brought critical.

Refer to ORNL-1682, "Reactivity Measurements with The Bulk Shielding Reactor", by R. G. Cochran, J. L. Meem, T. E. Cole, and E. B. Johnson.

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Fig. 9.1. Differential Rod Worth.

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Fig. 9.2. Rod Worth.

Usually, the worth of one control rod is quite small, having a total value of the order of 0.05 in terms of $\frac{\delta K}{K}$ in Fig. 9.2. In such a case, there is an insignificant difference between rod worth stated in terms of δK or in terms of reactivity. However, there may be a considerable difference between reactivity and δK for the complete rod system of a reactor. For example, it is difficult to obtain an accurate meaning from the statement that the worth of all the rods taken together is 20%. If K were unity when all the rods were fully inserted, their reactivity worth could be taken as $\frac{0.20}{1.20}$. On the other hand, if K were unity when all the rods were fully withdrawn, their reactivity worth could be taken as $\frac{0.20}{0.80}$. Unqualified numerical values of rod worth should be taken as meaning the approximate change in reactor multiplication brought about by changing the rod position, whether the worth is stated in terms of $\frac{\delta K}{K}$ or δK .

9.2 Measurement of The Temperature Coefficient of Reactivity

Once a control rod has been calibrated, the over-all temperature coefficient of reactivity can be measured. Some means of varying the average temperature of the reactor is necessary. Proper manipulation of the nuclear power and of the heat removal system may provide this means, or auxiliary heat may be necessary.

The reactor is brought critical at a known temperature and the position of a calibrated rod is recorded. The temperature is changed to a new known value and the reactor is again made critical by adjusting only the calibrated rod, leaving the other rods fixed in position. The change in reactivity introduced by the change in rod position is determined from the rod calibration. This reactivity change, divided by the temperature change, is the temperature coefficient of reactivity.

Considerable care should be exercised in producing the temperature change. A reduction in temperature is safer than an increase. A rise in temperature would start a positive power excursion in a reactor having a positive temperature coefficient, which in turn, would raise the temperature and continue the excursion until the reactor multiplication were reduced by an independent means. If the reactor had a positive temperature coefficient a reduction in temperature would cause a downward power excursion; if the temperature coefficient were negative, the reduction in temperature would cause a power rise, but the negative temperature coefficient itself would limit the excursion when the reactor temperature started to increase. Reducing

the temperature slowly, while simultaneously holding the reactor flux constant by withdrawing the calibrated control rod is one fairly safe method of performing the experiment. The safety system should provide for the safety of the reactor even with a continuous withdrawal of the calibrated control-rod at its maximum possible speed.

9.3 Coefficients Related to Multiplication Factor

The multiplication factor K is actually a function of many variables, some of which are interdependent. For example, the multiplication will be a function of control rod displacement, of fuel and moderator temperature, and of xenon and samarium poisoning, to mention only three of the more common variables of concern to reactor control. Of these three, the control rod displacement is the only independent variable, both temperature and fission product poisoning are dependent upon the reactor power and past history of the operation of the reactor. It has already been pointed out in Section 6.4 that the fission product poisoning can vary with time although the reactor power is held constant.

The variables which affect the multiplication factor are often called "coefficients", such as temperature coefficient and power coefficient. These terms must be used with care and properly defined in order to convey the intended meaning. It should be noted that these coefficients can be taken to be constant only over a limited range of reactor operation.

9.4 Units of Reactivity

Two terms often used in speaking of reactor kinetics are: (1) in-hours, and (2) dollars and cents. The first term is that amount of reactivity that will produce a period of one hour. The second term is defined in terms of the delayed neutron fraction; one dollar in reactivity is sufficient to allow the reactor to be self-sustaining without the delayed neutrons. Therefore, one dollar of reactivity will make the reactor prompt critical. Of course, one cent is equal to 0.01 dollar.

10. CIRCULATING FUEL REACTORS

The reactors previously considered had stationary fuel, thus, the delayed neutrons were released at the point where their precursors were formed inside the reactor. When the fuel circulates through the reactor, through a heat exchanger, and through the interconnecting piping, part of the delayed neutrons are released outside the reactor core. Those precursors which decay outside the reactor contribute no neutrons to help sustain the nuclear reactions. This condition can be analytically described by a modification of the kinetics equations to account for the loss of part of the delayed neutrons. The method to be described for accounting for this reduction in the delayed neutron contributions is based upon unpublished work by E. R. Mann.

10.1 Delayed Neutron Precursors in a Circulating Fuel Reactor

The fraction of the precursors of delayed neutrons that are useful to the circulating fuel reactor under steady state conditions will be computed. The ratio expressed in Eq. 10.1 defines the fraction of the delayed neutrons which are being released inside the reactor core.

$$\alpha_i = \frac{\lambda_i C_i}{\lambda_i C_{T_i}} \quad 10.1$$

where,

C_i = number of precursors of the i_{th} group inside the reactor core.

C_{T_i} = total number of precursors of the i_{th} group in existence, both inside and outside the reactor

The quantity $\lambda_i C_i$ is the rate of release of a typical group of delayed neutrons inside the reactor, and $\lambda_i C_{T_i}$ is the total rate of release of the same group of delayed neutrons throughout the fuel system. The symbol α_i represents the fraction of these delayed neutrons that are useful to the nuclear reactions; this fraction will be called the contribution factor.

In order to evaluate Eq. 10.1, it will be necessary to examine the variation of the concentration of precursors in a unit volume of fuel as this unit volume circulates around the fuel loop. Refer to Fig. 10.1. When this unit volume of fuel is inside the reactor, the rate of production of a typical group of precursors is $p \beta_i$, where p is the rate of release

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Fig. 10.1. Delayed Neutron Precursors in a Unit Volume of Fuel of a Circulating Fuel Reactor.

of potential neutrons per unit volume. Thus, p is proportional to the fission rate per unit volume. This fission rate per unit volume is assumed to be constant within the reactor core; therefore, $p \beta_i$ is constant at all times the unit volume of fuel in question is inside the reactor. The rate of decay of these precursors per unit volume of fuel is $\lambda_i \theta_i$ with θ_i being the number of precursors in this particular unit volume inside the reactor. This decay goes on whether the fuel is inside or outside the reactor. When the fuel is outside, there is no production of precursors; the decay rate is $\lambda_i \theta_i$, with θ_i being the number of precursors in this particular unit volume outside the reactor.

The total rate of production of the i th group of precursors is $p V \beta_i$, based on the preceding discussion. The volume of the reactor core is V ; thus, the total volume of the fuel system will exceed V . At steady state conditions, the total decay rate must equal the total production rate.

$$\lambda_i C_{Ti} = p V \beta_i \quad 10.2$$

The decay rate of all of the i th group of precursors inside the reactor core is $\lambda_i V \bar{\theta}_i$. The symbol $\bar{\theta}_i$ represents the average concentration of this precursor group inside the reactor core.

$$\lambda_i C_i = \lambda_i V \bar{\theta}_i \quad 10.3$$

Substitute Eqs. 10.2 and 10.3 into Eq. 10.1

$$\alpha_i = \frac{\lambda_i \bar{\theta}_i}{p \beta_i} \quad 10.4$$

The expression for $\bar{\theta}_i$ will be obtained.

During the time the unit volume of fuel is passing through the reactor core, Eq. 10.5 may be applied.

$$\frac{d\theta_i}{dt} = p \beta_i - \lambda_i \theta_i \quad 10.5$$

This equation can be solved using the boundary conditions in Fig. 10.1.

$$p \beta_i - \lambda_i \theta_i = k_1 e^{-\lambda_i t}$$

At $t = 0$, $\theta_i = M$ from Fig. 10.1; therefore,

$$k_1 = p \beta_i - \lambda_i M$$

Then, $p\beta_i - \lambda_i \theta_i = p\beta_i e^{-\lambda_i t} - \lambda_i M e^{-\lambda_i t}$

Inside the reactor,

$$\theta_i = \frac{p\beta_i}{\lambda_i} (1 - e^{-\lambda_i t}) + M e^{-\lambda_i t} \quad 10.6$$

Rearrange Eq. 10.6

$$\theta_i = \frac{p\beta_i}{\lambda_i} + (M - \frac{p\beta_i}{\lambda_i}) e^{-\lambda_i t} \quad 10.7$$

The average precursor concentration inside the core is described by Eq. 10.8.

$$\bar{\theta}_i = \frac{1}{\tau_i} \int_0^{\tau_i} \theta_i dt \quad 10.8$$

Substitute Eq. 10.7 into Eq. 10.8 and solve

$$\bar{\theta}_i = \frac{1}{\tau_i} \left[\frac{p\beta_i}{\lambda_i} t - \frac{1}{\lambda_i} \left(M - \frac{p\beta_i}{\lambda_i} \right) e^{-\lambda_i t} \right] \Big|_0^{\tau_i}$$

Make use of the upper and lower limits.

$$\bar{\theta}_i = \frac{p\beta_i}{\lambda_i} + \frac{1}{\lambda_i \tau_i} \left(\frac{p\beta_i}{\lambda_i} - M \right) \left(e^{-\lambda_i \tau_i} - 1 \right) \quad 10.9$$

In order to evaluate Eq. 10.9, the expression for M must be determined. An expression for the precursor concentration outside the reactor will be needed.

While the unit volume is outside the reactor, there is no production of precursors, and Eq. 10.10 is applicable.

$$\frac{d\theta_i}{dt} = -\lambda_i \theta_i' \quad \begin{matrix} 10.10 \\ 10.10 \end{matrix}$$

The solution of Eq. 10.10 follows, using the symbols in Fig. 10.1

$$\theta_i' = k_2 e^{-\lambda_i t}$$

At $t = \tau_1$, $\theta_i' = N$.

Then, outside the reactor

$$\theta_i' = N e^{-\lambda_i (t - \tau_1)} \quad \begin{matrix} 10.11 \\ 10.11 \end{matrix}$$

At $t = \tau_2$, $\theta_i' = M$ in Fig. 10.1

Therefore, from Eq. 10.11,

$$M = N e^{-\lambda_i(\tau_2 - \tau_1)} = N e^{-\lambda_i \tau_2}$$

$$\text{or, } N = M e^{\lambda_i \tau_2} \quad 10.12$$

The number of precursors per unit volume of fuel leaving the reactor is N_i therefore, from Fig. 10.1, at $t = \tau_1$, $\bar{\theta}_i = N$ in Eq. 10.6

$$N = \frac{p\beta_i}{\lambda_i} \left(1 - e^{-\lambda_i \tau_1} \right) + M e^{-\lambda_i \tau_1} \quad 10.13$$

Substitute Eq. 10.12 into Eq. 10.13 and solve for M .

$$M = \frac{p\beta_i}{\lambda_i} \left(\frac{1 - e^{-\lambda_i \tau_1}}{e^{\lambda_i \tau_2} - e^{-\lambda_i \tau_1}} \right) \quad 10.14$$

Now that M has been evaluated, substitute Eq. 10.14 into Eq. 10.9

$$\begin{aligned} \bar{\theta}_i &= \frac{p\beta_i}{\lambda_i} + \frac{1}{\lambda_i \tau_1} \left[\frac{p\beta_i}{\lambda_i} - \frac{p\beta_i}{\lambda_i} \left(\frac{1 - e^{-\lambda_i \tau_1}}{e^{\lambda_i \tau_2} - e^{-\lambda_i \tau_1}} \right) \left(e^{-\lambda_i \tau_1} - 1 \right) \right] \\ &= \frac{p\beta_i}{\lambda_i} \left\{ 1 + \frac{1}{\lambda_i \tau_1} \left[1 - \left(\frac{1 - e^{-\lambda_i \tau_1}}{e^{\lambda_i \tau_2} - e^{-\lambda_i \tau_1}} \right) \left(e^{-\lambda_i \tau_1} - 1 \right) \right] \right\} \end{aligned}$$

Simplify the term inside the square brackets through the use of a common denominator, and rearrange.

$$\bar{\theta}_i = \frac{p\beta_i}{\lambda_i} \left[1 - \frac{1}{\lambda_i \tau_1} \left(\frac{e^{\lambda_i \tau_2} - 1}{e^{\lambda_i \tau_2} - e^{-\lambda_i \tau_1}} \right) \left(1 - e^{-\lambda_i \tau_1} \right) \right] \quad 10.15$$

Substitute Eq. 10.15 into Eq. 10.4.

$$\alpha_i = 1 - \frac{1}{\lambda_i \tau_1} \left(\frac{e^{\lambda_i \tau_2} - 1}{e^{\lambda_i \tau_2} - e^{-\lambda_i \tau_1}} \right) \left(1 - e^{-\lambda_i \tau_1} \right) \quad 10.16$$

This fraction, α_i will be used in setting up the kinetic equations of the circulating fuel reactor.

The values for α_i in Eq. 10.16 are exact only at steady state because the computation assumed that the precursor concentration at the inlet and at the outlet of the reactor core does not vary with time. However, these

α_i 's are a fairly good approximation during transients having reactor periods of large magnitude.

A family of curves has been drawn in Fig. 10.2 using the ratio $\frac{\tau_2}{\tau_1}$ as a parameter, with α_i plotted against $\lambda_i \tau_1$. To obtain these curves, Eq. 10.16 is rewritten as follows:

$$\alpha_i = 1 - \frac{1}{\lambda_i \tau_1} \left[\frac{e^{\lambda_i \tau_1 (\frac{\tau_2}{\tau_1})} - 1}{e^{\lambda_i \tau_1 (\frac{\tau_2}{\tau_1})} - e^{-\lambda_i \tau_1}} \right] (1 - e^{-\lambda_i \tau_1}) \quad 10.17$$

Now, if X is small, $e^X = 1 + X$, and Eq. 10.17 can be simplified when the exponents are small.

$$\alpha_i = 1 - \frac{\tau_2}{\tau_2 - \tau_1}$$

Then,

$$\alpha_i = \frac{\tau_1}{\tau_2} \quad 10.18$$

This is the asymptotic value of α_i as the circulation rate becomes high and τ_1 becomes small, or for the cases when λ_i is small.

10.2 Kinetic Equations of a Circulating Fuel Reactor

The kinetic equations of circulating fuel reactor are similar to those for the stationary fuel case. However, it will be necessary to recognize that only a portion of the precursors of delayed neutrons which are produced are useful to the reactor.

The number of neutrons inside the reactor core is n ; therefore $\frac{kn}{\ell}$ is the rate of release of potential neutrons, as defined in Section 3. Note that the quantity pV in Eq. 10.2 is identical to the quantity $\frac{kn}{\ell}$. The definition of the multiplication factor, k , is the same as that given in Section 3. The quantity $\frac{kn B_i}{\ell}$ is the rate of formation of the i_{th} group of precursors of delayed neutrons. The kinetic equations for the precursors can be stated in the form of Eq. 10.19.

$$\frac{dC_{Ti}}{dt} = \frac{kn B_i}{\ell} - \lambda_i C_{Ti} \quad 10.19$$

Fig. 10.2. Delayed Neutron Contribution Factor for Circulating Fuel Systems.

The quantity $\frac{dC_{Ti}}{dt}$ is the total rate of change of all of the precursors of the i th group within the system, not the rate of change of precursors in a small volume of fuel flowing around the fuel loop. Since the nuclear reactions can make use of only those precursors inside the reactor core, the precursor kinetic equations will be written in terms of C_i rather than in terms of C_{Ti} . The relation between C_{Ti} and C_i can be obtained from the definition of α_i in Eq. 10.1. Multiply Eq. 10.19 by α_i and use the relationship in Eq. 10.1 with the assumption that α_i does not vary during a transient.

$$\frac{dC_i}{dt} = \frac{k\alpha_i B_i}{\ell} - \lambda_i C_i \quad 10.20$$

The kinetic equation of the neutron population can be written in terms of the precursors within the reactor core. It will be identical to Eq. 3.5

$$\frac{dn}{dt} = \left[k(1-\beta) - 1 \right] \frac{n}{\ell} - \sum_{i=1}^{i=m} \lambda_i \alpha_i C_{Ti} \quad 3.5$$

Eq. 3.5 and Eq. 10.20 will be referred to as the kinetic equations of the circulating fuel reactor.

If the kinetic equation of the neutron population were written in terms of the total precursor population, the equation would appear as follows:

$$\frac{dn}{dt} = \left[k(1-\beta) - 1 \right] \frac{n}{\ell} - \sum_{i=1}^{i=m} \lambda_i \alpha_i C_{Ti} \quad 10.21$$

Eq. 10.21 together with Eq. 10.19 will give the same result as Eqs. 3.5 and 10.20 in the case of a step change in k with constant flow. However, Eq. 10.21 will lead to the erroneous conclusion that the rate of release of delayed neutrons into the reactor core will instantly jump if the circulation rate were suddenly reduced. Actually, the number of precursors inside the reactor core will increase gradually, and the rate of release of delayed neutrons into the reactor core has the same value immediately after a step change in flow as immediately before the change. In this respect, Eq. 3.5 used with Eq. 10.20 will provide a better approximation of the transient introduced by a change in flow.

A graphical representation of the loss and gain of neutrons is indicated in Fig. 10.3. The various rates of production and loss are the same as

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Fig. 10.3. Neutron Population Loss and Gain in a Circulating Fuel Reactor.

those for Fig. 3.1, except for the addition of two new terms. The quantity $\frac{kn\alpha_i\beta_i}{\ell}$ represents the rate of production of precursors which decay inside the reactor and $(1-\alpha_i)\frac{kn\beta_i}{\ell}$ represents the rate of production of precursors which decay outside the reactor.

10.3 Multiplication Factor of a Circulating Fuel Reactor at Steady State

The fact that only a portion of the precursors of delayed neutrons which are formed release delayed neutrons that are useful to the reactor will require that the multiplication factor exceed unity in order to hold the reactor at steady state. An expression for this necessary value of k will be derived in the following analysis.

Take $\frac{dC_i}{dt} = 0$, and employ Eq. 10.20

$$\sum_{i=1}^{i=m} \lambda_i C_i = \frac{kn}{\ell} \sum_{i=1}^{i=m} \alpha_i \beta_i \quad 10.22$$

Substitute Eq. 10.22 into Eq. 3.5 and take $\frac{dn}{dt} = 0$.

$$0 = \frac{n}{\ell} [k(1-\beta) - 1] + \frac{kn}{\ell} \sum_{i=1}^{i=m} \alpha_i \beta_i$$

Divide by $\frac{n}{\ell}$ and solve for k .

$$0 = k(1-\beta + \sum_{i=1}^{i=m} \alpha_i \beta_i) - 1$$

Using $\beta = \sum_{i=1}^{i=m} \beta_i$,

$$k[1 - \sum_{i=1}^{i=m} \beta_i(1-\alpha_i)] = 1$$

$$\begin{aligned} \text{or } k &= \frac{1}{1 - \sum_{i=1}^{i=m} \beta_i(1-\alpha_i)} \\ &\cong 1 + \sum_{i=1}^{i=m} \beta_i(1-\alpha_i) \end{aligned} \quad 10.23$$

If all α_i 's are equal, and if Eq. 10.17 may be applied,

$$\begin{aligned} k &\cong 1 + \beta(1-\alpha) \\ &\cong 1 + \beta \left(1 - \frac{\tau_i}{\tau_d}\right) \end{aligned}$$

$$\text{or } k \cong 1 + \beta \frac{\tau_i}{\tau_d}$$

The term "critical" cannot be synonymous with $k = 1$ in the circulating fuel reactor because both k and the circulation rate affect the condition at which the reactor will sustain itself at a constant power level. If the reactor were initially at a steady state power level with the fuel not circulating, the system could be considered to be critical. However, starting the circulation of the fuel would then throw the reactor into a downward transient, and the system is no longer critical. If the reactor were initially at steady state with the fuel in circulation, stopping the flow would produce a transient that would raise the power level until counteracted by an independent reduction in k . In addition, the term "multiplication" must be used with care, if used at all, in connection with circulating fuel reactors.

10.4 Solution of The Kinetic Equations of Circulating Fuel Reactors

The kinetics of the circulating fuel reactor may be examined by means of Eqs. 3.5 and 10.20. Take the case of step changes in multiplication and in fuel circulation rate. New values are α_i and k , previous steady state values are α_i'' and k'' . The previous steady state neutron population is n_0 , and the previous steady state precursor population is C_{i0} .

$$\frac{dn}{dt} = \frac{n}{\ell} \left[k(1 - \beta) - 1 \right] + \sum_{i=1}^m \lambda_i C_i \quad 3.5$$

$$\frac{dC_i}{dt} = \frac{k n \alpha_i B_i}{\ell} - \lambda_i C_i \quad 10.20$$

Solutions:

$$n = \sum_{j=0}^{j=m} A_j e^{\omega_j t} \quad 10.24$$

$$C_i = \sum_{j=0}^{j=m} C_{ij} e^{\omega_j t} \quad 10.25$$

Substitute Eq. 10.25 and Eq. 10.24 into Eq. 10.20, solve for C_{ij} , as in the case of Eq. 3.9.

$$C_{ij} = \frac{k A_j B_i \alpha_i}{\ell (\omega_j + \lambda_i)} \quad 10.26$$

Substitute Eq. 10.25 and Eq. 10.24 into Eq. 3.5, and rearrange as for Eq. 3.10

$$\omega_j A_j = \frac{k[(1-\beta)-1]}{\ell} A_j + \sum_{i=1}^{i=m} \lambda_i C_{ij} \quad 10.27$$

Substitute Eq. 10.26 into Eq. 10.27, divide by A_j , multiply by ℓ ;
substitute $\beta = \sum_{i=1}^{i=m} \beta_{ij}$ and rearrange as in the derivation of Eq. 3.12.

$$\ell \omega_j = (k-1) + k \sum_{i=1}^{i=m} \frac{\lambda_i \beta_{ij} \alpha_i - \lambda_i \beta_{ij} - \omega_j \beta_{ij}}{\omega_j + \lambda_i}$$

or

$$\frac{k-1}{k} = \frac{\ell \omega_j}{k} + \sum_{i=1}^{i=m} \frac{\beta_{ij} [\omega_j + \lambda_i (1 - \alpha_i)]}{\omega_j + \lambda_i} \quad 10.28$$

Use $\rho = \frac{k-1}{k}$ and $T_j = \frac{1}{\omega_j}$

$$\rho = \frac{\ell}{T_j k} + \sum_{i=1}^{i=m} \frac{\beta_{ij} [1 + T_j \lambda_i (1 - \alpha_i)]}{1 + T_j \lambda_i} \quad 10.29$$

Note that $\rho \neq 0$ at steady state.

To determine the values of A_j ;

From Eq. 10.26 and Eq. 10.25

$$C_i = \frac{k \beta_i \alpha_i}{\ell} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} e^{\omega_j t} \quad 10.30$$

Assume steady state up to $t = 0$, then from Eq. 10.20 using k'' and α_i''

$$\frac{dC_i}{dt} = 0 = \frac{k'' n_0 \beta_i \alpha_i''}{\ell} - \lambda_i C_{i0}$$

or,

$$C_{i0} = \frac{k'' \beta_i \alpha_i''}{\lambda_i \ell} n_0 \quad 10.31$$

Set Eq. 10.30 equal to Eq. 10.31 at $t = 0$, solve for n_0 .

$$n_0 = \frac{k \alpha_i}{k'' \alpha_i''} \sum_{j=0}^{j=m} \frac{\lambda_i A_j}{\omega_j + \lambda_i} \quad 10.32$$

Also, from Eq. 10.24

$$n_0 = \sum_{j=0}^{j=m} A_j \quad 10.33$$

Next, assume that there are step changes to k' and α'_i at the time t_1 .

The resultant neutron and precursor populations are n' and C'_i , respectively.

The symbol A'_j will be used for $t \geq t_1$.

From Eq. 10.26 and Eq. 10.25 for $t \geq t_1$, using k' and α'_i :

$$C'_i = \frac{k' \beta_i \alpha'_i}{\ell} \sum_{j=0}^{j=m} \frac{A'_j}{\omega'_j + \lambda_i} e^{\omega'_j(t-t_1)} \quad 10.34$$

Also, from Eq. 10.24 for $t \geq t_1$;

$$n' = \sum_{j=0}^{j=m} A'_j e^{\omega'_j(t-t_1)} \quad 10.35$$

At $t = t_1$, Eq. 10.30 equals Eq. 10.34

$$\sum_{j=0}^{j=m} \frac{A'_j}{\omega'_j + \lambda_i} = \frac{k \alpha_i}{k' \alpha'_i} \sum_{j=0}^{j=m} \frac{A_j}{\omega_j + \lambda_i} e^{\omega_j t_1} \quad 10.36$$

Also, at $t = t_1$, Eq. 10.24 equals Eq. 10.35

$$\sum_{j=0}^{j=m} A'_j = \sum_{j=0}^{j=m} A_j e^{\omega_j t_1} \quad 10.37$$

The equations for the circulating fuel reactor are more general than those of the stationary fuel reactor. Eqs. 10.29 and 3.15 are equal for an α_i of unity. Eqs. 10.32 and 3.24 are equal for k'' , α''_i , and α_i each equal to unity. Similarly, Eqs. 10.36 and 3.28 may be compared.

In the case of Eq. 10.29,

$$\rho \rightarrow \sum_{i=1}^{i=m} \beta_i (1 - \alpha_i) \text{ when } T_j = \infty$$

Then,
$$\frac{k-1}{k} = \sum_{i=1}^{i=m} \beta_i (1 - \alpha_i)$$

$$\text{or, } k = \frac{1}{1 - \sum_{i=1}^{i=m} \beta_i (1 - \alpha_i)}$$

This is the value of k required to hold the reactor at steady state.

Compare to Eq. 10.22.

10.5 Initial Neutron Level Jump of Circulating Fuel Reactors

The approximate size of the jump in neutron level may be determined for a step change in multiplication. The multiplication constant changes from k'' to k , α_i remains constant.

From Eq. 10.20 at steady state:

$$\begin{aligned} \lambda_i C_i &= \frac{k'' n_0 \beta_i \alpha_i}{l} \\ \text{or } \sum_{i=1}^{i=m} \lambda_i C_i &= \frac{k'' n_0}{l} \sum_{i=1}^{i=m} \beta_i \alpha_i \end{aligned} \quad 10.38$$

Assume that the precursor concentration remains constant for the moment.

Then, Eq. 3.5 becomes:

$$\frac{dn}{dt} = \frac{n}{l} [(k-1) - k\beta] + \frac{k'' n_0}{l} \sum_{i=1}^{i=m} \beta_i \alpha_i \quad 10.39$$

If $k\beta > (k-1)$, $\frac{dn}{dt}$ will approach zero as n increases.

or, $\frac{dn}{dt} = 0$ at $n = n_1$

$$\begin{aligned} \text{Then, } 0 &= \frac{n_1}{l} [(k-1) - k\beta] + \frac{k'' n_0}{l} \sum_{i=1}^{i=m} \beta_i \alpha_i \end{aligned} \quad \begin{array}{l} 10.40 \\ 10.40 \end{array}$$

$$\text{or, } \frac{n_1}{n_0} = \frac{k'' \sum_{i=1}^{i=m} \beta_i \alpha_i}{k\beta - (k-1)} = \frac{k'' \sum_{i=1}^{i=m} \beta_i \alpha_i}{k\beta - \delta k} \quad 10.41$$

$$\text{or, } \frac{n_1}{n_0} \approx \frac{\sum_{i=1}^{i=m} \beta_i \alpha_i}{\beta - \delta k} \quad 10.42$$

Compare Eq. 10.42 to Eq. 3.32

If all α_i for all groups are equal:

$$\frac{n_1}{n_0} \cong \frac{\beta\alpha}{\beta - \delta k} \quad 10.43$$

Note that $k'' \neq 1$ and that $\delta k = k - 1$. Thus, δk is not the step change from the steady state value of k'' . In order to hold the circulating fuel reactor at steady state, k'' is larger than unity, and a given increase in multiplication will make the quantity $(\beta - \delta k)$ smaller than for the same increase in multiplication for a stationary fuel reactor. Therefore, the jump will be larger when the fuel is circulating than for the stationary fuel case, even though α_i is less than unity.

A step change in circulation rate will not produce a jump in power level. The value of C_i begins to change because $\frac{dC_i}{dt} \neq 0$ for a change in flow, refer to Eq. 10.20. However, since the value of C_i does not change at the instant of the step change in α_i , the value of $\frac{dn}{dt}$ does not suffer an instantaneous change, but changes only gradually, and n does not jump. The value of $\frac{dn}{dt}$ remains constant at the value just previous to the step change in α_i , providing another boundary condition. Eq. 3.5 contains no factor which changes at the instant of a step change in fuel flow.

Other material on the kinetics of circulating fuel reactor is available in the following references:

Journal of Applied Physics, Vol. 25, No. 6, June, 1954, page 702, "Kinetics of the Circulating Fuel Reactor", by W. K. Ergen

BNL-334 "Theory of Low Power Kinetics of Circulating Fuel Reactors with Several Groups of Delayed Neutrons", by J. A. Fleck, Jr.

Nucleonics, Vol. 12, No. 10, October, 1954, page 52, "Kinetics of Circulating Reactors at Low Power", by J. A. Fleck, Jr.

10.6 Xenon Poisoning

One aspect of the circulating fuel reactor is the possibility of the continuous removal of xenon during operation. Xenon is a gas above -107.1°C at standard pressure, thus it is possible for the xenon to escape from the fuel under the proper conditions. The release rate will be directly proportional to the concentration, and will appear as another loss term in the differential equation, Eq. 6.9. The equilibrium value will

be lower than that from Eq. 6.10. Refer to "The Aircraft Reactor Experiment - Physics", by W. K. Ergen, A. D. Callihan, C. B. Mills, and Dunlap Scott, Nuclear Science and Engineering, Volume 2, pages 826-840, 1957.

11. ANALOG COMPUTING

The analysis of the dynamic behavior of a reactor system will require a computing machine except for the most elementary cases. An analog computer provides a relatively simple method of examining the control problem of a reactor system while the design of the plant is in progress. The system analysis will then provide the basic information needed for the design of the control and safety systems, and the effects of the control and safety actions can be examined.

The computer, or simulator, to be discussed here is described in more detail in ORNL-2405, ORNL Reactor Controls Analog Facility, Operation Manual by F. P. Green. It is also discussed by E. R. Mann in a paper, "Electronic Analog Devices for Design of Reactor Controls", published by the Institute of Radio Engineers in IRE Transactions of the Professional Group on Nuclear Science, Volume NS-3, Number 2, March 1956.

An analog computer is made up of components which basically function as operators in the mathematical sense. For example; a variable which is a function of time is to be added to another function of time, or multiplied by another variable, or integrated with respect to time. Only a few of the operations that can be performed by an analog facility will be described.

11.1 Solution of a Given Mathematical Expression

As a specific example, the following relationship is to be considered:

$$y = 3 \frac{dx}{dt} + 5x - 0.6 \int x dt \quad 11.1$$

The variable y is an independent function of time, the variable x is to be determined as y varies. A block diagram to specify the operators required to solve for x is to be set up. The first step is to solve for the highest ordered differential.

$$3 \frac{dx}{dt} = y - 5x + 0.6 \int x dt \quad 11.2$$

The first operator may be drawn as a block and marked "sum" as in Figure 11.1. This summing operator has three input variables, which when algebraically added provide an output of $+ 3 \frac{dx}{dt}$.

Next, if the proper scale change is performed on $+ 3 \frac{dx}{dt}$, $+ \frac{dx}{dt}$ is obtained. This $+ \frac{dx}{dt}$ is integrated to obtain $+ x$, and a scale change with a sign inversion is performed to obtain $-5x$.

Fig. 11.1. Computer Operations for Solution of $y = 3 \frac{dx}{dt} + 5x - 0.6 \int x dt$.

Also, $+X$ may be integrated again and a scale change taken to obtain $+0.6 \int X dt$. Now, all the inputs to the first summing operator have been obtained, and the dependent variable X has been determined by the solution of Eq. 11.2. The complete solution by means of the operators is drawn in Fig. 11.1.

In order to obtain the solution on the computer, the initial conditions must be known. In other words, the values of X and of $\int X dt$ must be established and set into the facility at the condition for $t=0$. In general, the initial conditions must be known whenever an integration is performed.

Another point that must be considered is that a device to vary y as a function of time must be provided.

In the ORNL Analog Facility, the input and output variables are voltages, the maximum allowable value being 100 volts, either positive or negative. These voltages are measured with the frame of the machine being a common point, thus they are measured with respect to ground. Also, in the ORNL computer, each operation on a variable results in a change of sign. This means that the integration of $+\frac{dX}{dt}$ will result in $-X$, and the sum $+y + 0.6 \int X dt - 5X$ will produce $-3\frac{dX}{dt}$ rather than $+3\frac{dX}{dt}$. The actual block diagram used in setting up the computer is called a computer diagram, and the diagram to solve the specific example taken above will be developed.

First, it will be necessary to study the symbols used in the diagram to represent the operations to be performed.

11.2 Simple Operators

It should be noted that each of the operators to be described employ a high-gain chopper-stabilized electronic amplifier as the basic unit.

11.2.1 Amplifier (Inverter and Scale Changer)

The circuit of a simple amplifier is indicated in Fig. 11.2. The resistor R_3 is inside the amplifier, the ratio of $\frac{V_2}{V_3}$ is 10^6 to 10^8 , V_2 is the output voltage, V_1 is the input voltage to the electronic amplifier. The gain of the complete circuit is $\frac{V_2}{V_1}$. Note the polarity reversal between V_1 and V_2 . The voltage and current equations may be simply stated:

$$V_1 = V_3 + i_1 R_1 \quad 11.3$$

$$V_3 + V_2 = i_2 R_2 \quad 11.4$$

$$V_3 = i_3 R_3 = (i_1 - i_2) R_3 \quad 11.5$$

Now, if $\frac{V_3}{V_2} = 10^{-8}$, $V_2 = i_2 R_2$, since V_3 is negligible in Eq. 11.4. The normal range of V_2 is from -100 volts to 100 volts; V_3 has a value of 10^{-6} volts when V_2 has the maximum value of 100 volts. If V_1 has a value of only 1 volt under these conditions, V_3 is negligible compared to V_1 in Eq. 11.3, and thus $V_1 = i_1 R_1$. Substitute $i_1 = \frac{V_1}{R_1}$ and $i_2 = \frac{V_2}{R_2}$ into the equation for V_3 , Eq. 11.5.

$$V_3 = \frac{V_1 R_3}{R_1} - \frac{V_2 R_3}{R_2}$$

Divide by V_2 :

$$\frac{V_3}{V_2} = \frac{V_1 R_3}{V_2 R_1} - \frac{R_3}{R_2}$$

Now $V_3 \ll V_2$; therefore,

$$\frac{V_1 R_3}{V_2 R_1} = \frac{R_3}{R_2}$$

Divide by R_3 and rearrange:

$$\frac{V_2}{V_1} = \frac{R_2}{R_1} \quad 11.6$$

The amplifier acts to reduce V_3 to as small a value as possible. For computer work, the circuit of Fig. 11.2 is symbolized as in Fig. 11.3. Here, the path for i_3 has been omitted from the circuit and V_3 is taken as zero. The ground connection has been omitted because all voltages are measured with respect to ground. The equation relating V_1 and V_2 is simple.

$$+ a V_1 - b V_2 = 0 \quad 11.7$$

or,

$$\frac{V_2}{V_1} = \frac{a}{b}$$

Note that the ratio $\frac{R_2}{R_1}$ in Eq. 11.6 has been replaced by the ratio $\frac{a}{b}$. In fact, the input coefficient a is proportional to the reciprocal of the resistance R_1 , and the input coefficient b is proportional to the reciprocal of the resistance R_2 . Certain values of the coefficients and the resistances are stated in Table 11.1

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Fig. 11.2. Simple Amplifier.

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Fig. 11.3. Computer Symbol for a Simple Amplifier.

TABLE 11.2 AMPLIFIER INPUT COEFFICIENTS

Input Coefficient (Units)	Input Resistance (Megohms)
1	1
5	0.2
10	0.1

The sum of several voltages is taken by the circuit indicated in Fig. 11.4. Again, the input coefficients, a , b , or C , are proportional to the reciprocals of the resistances employed. The basic mathematical relation is stated in Eq. 11.8.

$$+bV_1 + cV_2 - aV_3 = 0 \quad 11.8$$

or,

$$V_3 = \frac{b}{a} V_1 + \frac{c}{a} V_2$$

Here, it has been assumed that the two input voltages are V_1 and V_2 , and that both are positive with respect to ground. Since the operator inverts the polarity, the output voltage is negative. The letters a , b , and C indicate the numerical value of the coefficient of the scale change obtained in this summation. The numerical values of these coefficients available are generally 1, 5, and 10, although 2 and 20 can usually also be obtained. It should be noted that there are actually three input voltages, namely $+V_1$, $+V_2$, and $-V_3$. One useful way of thinking of the action involved here is to consider the device as a mechanism which adjusts the output voltage V_3 , in such a manner that the sum $+bV_1 + cV_2 - aV_3$ is equal to zero.

More than two variables may be added, if desired.

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Fig. 11.4. Summation of Several Voltages.

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Fig. 11.5. Integrator.

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Fig. 11.6. Computer Symbol for an Integrator.

11.2.2 Integrator

Integration can be obtained by replacing the resistor R_2 with a capacitor C ; thus, the circuit in Fig. 11.2 is changed to that in Fig. 11.5. On the basis that both i_3 and v_3 are negligible, the mathematical relations are stated as follows:

$$i_1 = \frac{V_1}{R_1} \quad 11.9$$

$$i_2 = C \frac{dv_2}{dt} \quad 11.10$$

$$i_1 = i_2 \quad 11.11$$

Substitute Eqs. 11.9 and 11.10 into Eq. 11.11

$$\frac{V_1}{R_1} - C \frac{dv_2}{dt} = 0 \quad 11.12$$

or,

$$v_2 = \frac{1}{R_1 C} \int V_1 dt$$

The computer symbol for the integrator is drawn in Fig. 11.6. The mathematical relationship is expressed in Eq. 11.13.

$$+ a V_1 - b \frac{dv_2}{dt} = 0 \quad 11.13$$

or,

$$v_2 = \frac{a}{b} \int V_1 dt$$

Again, a is proportional to the reciprocal of the resistance, with the values in Table 11.1. The coefficient b is directly proportional to the value of the capacitance; in fact, this coefficient is the numerical value of the capacitance in microfarads. Note the reversal in sign as in the summation operation. The coefficient a on the input is available as in the summing amplifier and may have the numerical values given above. The coefficient b written outside the triangle may have the values of 0.1, 1, or 10, with 1 being the customary value. The terminal marked IC is used in case an initial condition other than zero volts is required from the integrator. In other words, at the instant in time that the system operation is to commence, the voltage V_2 is to have a certain

specific value other than zero, and this value with sign reversed as in the case of an input voltage is written at the place indicated by the symbols IC. In many applications, an initial condition is zero, and the terminal marked IC is then omitted.

It is possible to algebraically sum as well as integrate in one operation. Refer to Fig. 11.7, in which a second input voltage V_3 is applied. The basic relation is

$$+aV_1 + CV_3 - b \frac{dV_2}{dt} \quad 11.14$$

11.2.3 First Order Linear Operator (First Order Lag)

Actually this operation is identical to the combination of summation and integration shown in Fig. 11.7. If $-V_2$ in Fig. 11.7 were connected to the terminal where V_3 is applied, the operator would appear as in Fig. 11.8; and V_3 is replaced by $-V_2$ in the mathematical relations. The basic relation is:

$$+aV_1 - CV_2 - b \frac{dV_2}{dt} \quad 11.15$$

$$\text{or } V_1 = \frac{C}{a} V_2 + \frac{b}{a} \frac{dV_2}{dt}$$

$$\text{or } V_2 = \frac{1}{b} \int (aV_1 - CV_2) dt$$

The basic relation is that of the first order linear differential equation having constant coefficients. The output voltage V_2 does not instantaneously follow a change of the input voltage V_1 ; in other words, a lag is introduced. Due to the fact that the differential equation is of first order, this operation is sometimes called first order lag. When steady-state is reached with V_1 being constant, the voltage V_2 will also become constant.

11.3 Analog Calculation

11.3.1 Computer Diagram for Equation

For simplicity, it will be assumed that any desired constant value of scale change is available. The diagram will be developed in the same manner as that for the required functions previously drawn in block form in Fig. 11.1

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Fig. 11.7. Summing and Integrating.

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Fig. 11.8. First Order Linear Operator.

The first operator (amplifier 1) is to have input voltages of $+y$, $+0.6 \int x dt$, and $-5x$, and the output voltage is to be proportional to $\frac{dx}{dt}$. Refer to Fig. 11.9.

The mathematical expression for the sum being taken is

$$3 \frac{dx}{dt} = +y + 0.6 \int x dt - 5x$$

consequently the output of the summation operator is $-3 \frac{dx}{dt}$ because of the inherent reversal in sign. The basic relation for the operation is

$$+y + 0.6 \int x dt - 5x - 3 \frac{dx}{dt} = 0$$

which is the desired equation.

The next operator (amplifier 2) is to have an input of $-3 \frac{dx}{dt}$ and an output voltage of $+x$. Clearly, the operator must be an integrator. Note the scale factor of $1/3$ on the input voltage.

In addition, amplifier 3 is used to change the sign x and also raise the value of x by a factor of 5.

Next, the quantity $+0.6 \int x dt$ is obtained by first integrating with amplifier 4; then raising this integrated output by a factor of 0.6 and reversing the sign by means of amplifier 5.

All of the dependent variables required by the summation operator have now been generated, and the complete diagram is drawn in Fig. 11.9. The diagram of Fig. 11.9 requires five amplifiers; it is possible to reduce this number by combining the operations of summation and integration. It is possible to obtain an output proportional to x ; however, $\frac{dx}{dt}$ cannot be measured. Refer to Fig. 11.10. The coefficient associated directly with the integration is unity. The summation coefficients are those which are required to satisfy the equation. The basic relation of the first operator (amplifier 1) in Fig. 11.10 follows:

$$+\frac{1}{3}y - \frac{5}{3}x + 0.2 \int x dt - 1 \frac{dx}{dt} = 0$$

Rearrange, and multiply by 3:

$$y = 5x + 3 \frac{dx}{dt} - 0.6 \int x dt$$

This is the original equation.

The quantity of $+0.2 \int x dt$ is obtained by means of the second

Fig. 11.9. Computer Diagram for Solution of $y = 3 \frac{dx}{dt} + 5x - 0.6 \int x dt$.

Fig. 11.10. Alternate Computer Diagram for Solution of $y = 3 \frac{dx}{dt} + 5x - 0.6 \int x dt$.

operator. A unity gain inverter may be used to obtain $+X$ from $-X$.

Note that the factor of $1/5$ is introduced in the input to the second integrator, rather than in the output. This reduction before integration is to limit the amplifier output to a maximum value less than 100 volts. It should also be noted that the feedback connection from the output of a summing operator back to the input is often omitted on a computer diagram when the summation scale factor on the output voltage is unity. This connection has been omitted in the inverter to obtain $+X$ in Fig. 11.10. In addition, the omission of the value of the summation scale factor on any input is interpreted to mean unity.

The values of the scale coefficients of the summation in Fig. 11.10 are not available on the amplifiers. Obviously, the fixed values previously stated are hardly suitable for a wide variety of applications, consequently a continuously adjustable element is available to be added into the circuit. These elements are potentiometers, sometimes called "pots", which can be adjusted to reduce the respective voltage to any desired fraction of the input voltage to the pot. The phrase "voltage divider" aptly describes the function of one of these devices. When a pot is added to the input circuit of a summation amplifier, the actual scale change is equal to the product of the voltage ratio due to the pot and the coefficient of the amplifier input. Note that the pot ratio cannot exceed unity. In order to achieve an input coefficient of $5/3$, a pot ratio of 0.333 together with a fixed coefficient of 5 could be used.

The diagram of Fig. 11.10 is redrawn in Fig. 11.11 showing the pots as circles, with the pot voltage ratios written beside the circles and the identification numbers written inside the circles.

The initial conditions are indicated in Fig. 11.11. No initial condition is shown for $\int X dt$ because the initial value is zero volts. Pot 3 provides $+5.0$ volts to the integrator to produce -5.0 volts as the initial value of $-X$. Note the reversal in sign between the output and the initial condition as in the case of the relation between the output and input voltages.

The source of $+100$ volts is provided by the power supply which is part of the analog facility. A source of -100 volts is also provided.

The voltage $-X$ is not converted to $+X$ because the indicator of

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Fig. 11.11. Final Diagram for Solution of $y = 3 \frac{dx}{dt} + 5x - 0.6 \int x dt$.

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Fig. 11.12. Computer Diagram for Obtaining y.

this voltage is equipped with a polarity reversing switch in order that either positive or negative voltages may be indicated in the up-scale direction.

It should be pointed out that the pot settings read on the potentiometer dials are not identical to the pot voltage ratios obtained. The current which flows through the pot resistance will reduce the voltage ratio to a value lower than the actual setting of the dial. Consequently, the pots are adjusted to give the desired voltage ratio, as determined by voltage measurements, after the circuits are connected. The setting of the pot 3 in Fig. 11.11 is not shown because this pot is to be set to supply + 5.0 volts, rather than to provide a voltage ratio from a varying voltage source.

11.3.2 Actual Solution

The initial linear increase in y was obtained by integrating a constant voltage of 1.42 volts, as indicated in Fig. 11.12. After this voltage had been applied to the integrator for 14.3 seconds, the switch in the output of the integrating amplifier was thrown to obtain a constant value of -11.2 volts. After 13.3 seconds, the switch was thrown to the center position, which held y at zero volts.

Two of the pot voltage ratios in Fig. 11.12 are blank because the actual settings were unimportant, the desired voltage being the important consideration. Two potentiometers in cascade, or a "pot on a pot" were used in obtaining the low value of -1.42 volts from the source of -100 volts in order to avoid taking this rather large reduction on one pot.

The voltage y out of the diagram of Fig. 11.12 was applied as the input of Fig. 11.11. The voltages y and x were recorded on strip-chart recorders and redrawn for Fig. 11.13. The initial conditions taken were + 5.0 volts for + x and zero volts for $\int x dt$, as previously stated.

The actual process followed in obtaining a solution of the differential equation is described in the succeeding paragraphs. The solution is shown in Fig. 11.13, together with the variation in y for which this solution was obtained. This particular variation of the independent variable was taken in order that it could be expressed mathematically, if desired.

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Fig. 11.13. Analog Solution of a Mathematical Equation.

11.4 Additional Computer Operators

11.4.1 Function Generators

It is often necessary to produce a voltage V_2 which is dependent directly upon the value of V_1 , but is not time dependent. For example, the variable $W^{0.8}$ may be needed, with W known. In other cases, the relationship between the desired variable and the independent variable is available only as a graph.

The function generator produces the desired variable with the known independent variable as the input. Refer to Fig. 11.14. The function generator is set up to obtain the desired relation between the output and input voltages.

11.4.2 Multiplication

1. Electronic Multiplier

The electronic multipliers in the ORNL Analog Facility depend upon function generators for their operation. First, the sum and the difference of the two variables in question are taken by means of summing and inverting operators. Next, the square of this sum and of this difference are individually obtained by function generators. The difference of the squares, obtained by another summation, provides a voltage proportional to the product. More information is given in Fig. 21, page 35 of ORNL-2405. The symbol is given in Fig. 11.15. The output is $-\frac{xy}{100}$ when the inputs are x and y . The multiplier operates with either polarity input.

2. Servo Multiplier

The basic element in the servo multiplier is a potentiometer, or pot, used as a voltage divider. Refer to Fig. 11.16. One of the input variables is the voltage V_1 , the other is the position α of the sliding contact. The output voltage V_o will be proportional to the product $V_1 \alpha$, α being expressed as a fraction of the total distance from the common ground point to the upper end of the resistance. Both polarities of V_1 are required if V_o is to change sign; α may be either positive or negative to place the sliding contact in either the upper or lower half of the resistance, producing either positive or negative signs of V_o .

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Fig. 11.14. Function Generator.

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Fig. 11.15. Electronic Multiplier.

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Fig. 11.16. Voltage Divider as a Multiplier.

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Fig. 11.17. Servo Multiplier.

Since voltages are to be multiplied, it is necessary that α be continuously directly proportional to one of the input voltages impressed upon the multiplier. This proportionality is produced by an electro-mechanical servo system utilizing an electric motor to position the potentiometer shaft. The input X in Fig. 11.17 is the voltage to the servo which positions the sliding contact.

Either polarity of output voltage is available by interchanging the polarities of the voltages $+y$ and $-y$ applied to the potentiometer terminals. Also, the servo motor positions the shafts of several pots; consequently additional voltages may be multiplied by X . This is useful when the products of X and z as well as of X and y must be obtained. The ability of the servo positioner to follow a rapidly varying input voltage limits the frequency of X to a maximum of approximately two cycles per second. There is no such limitation on the rate of change of the voltage applied to the potentiometer resistance. In cases where X does not change sign, it is possible to operate the multiplier with only one polarity of voltage applied to the potentiometer resistance.

11.4.3 Division

The process of division will be considered as the inverse of multiplication. The voltage z is to be obtained.

$$\begin{aligned} \text{If, } z &= \frac{X}{y} \\ \text{then } X &= zy \end{aligned}$$

In this last equation, X could be found by multiplying z by y ; however, z is not known but is desired, and both X and y are known. Then, the problem is to force the computer to generate a voltage V that when multiplied by y will produce a voltage equal to the known value of X . If this voltage V can be generated such that

$$Vy = X$$

then, obviously,

$$z = V$$

and the operation of division is accomplished.

In this application, it is necessary to use an amplifier to obtain the maximum voltage amplification possible. The output voltage is not applied as one of the input voltages. Refer to Fig. 11.18.

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Fig. 11.18. Amplifier Connected for Maximum Gain.

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Fig. 11.19. Divider, Positive Denominator.

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Fig. 11.20. Divider, Negative Denominator.

Either a or b in Fig. 11.18 may represent this application. With the amplification available, the actual net input must be near zero in order to maintain an output between $+ 100$ and $- 100$ volts. The only practical mathematical relation possible is

$$+av_1 - bv_2 = 0 \quad 11.16$$

The value of V_o is indeterminable as shown here; therefore, the circuit external to that indicated in Fig. 11.18 must be used to make this mathematical relation possible by making use of the output voltage V_o . The two input voltages must be opposite in sign.

The diagram for the basic operation of division is drawn in Fig. 11.19. Assume that both the voltages y and x have positive values. Further, assume that the voltage V is available as a positive quantity to be applied as one input to the multiplier. The output of the multiplier is $-\frac{Vy}{100}$ in this case since the multiplier produces a change in voltage sign. This negative product is compared to the input voltage x by means of the extremely high gain amplifier illustrated in Fig. 11.18. The output of this comparison amplifier is reversed in sign by the inverter amplifier which supplies the positive voltage V back to the multiplier. The high gain comparison amplifier requires that

$$x - \frac{Vy}{100} = 0$$

in order that the voltage V is finite. Therefore, $V = 100 \frac{x}{y}$. The desired voltage was the quotient $\frac{x}{y}$; therefore, the division operation has been produced. All voltages must be limited to a maximum value of ± 100 . It has been assumed that both input coefficients of the comparison amplifier were unity.

Another way of considering the operation of this circuit is to imagine that both x and y are simultaneously applied. The comparison amplifier and sign inverter will produce a positive voltage V . This positive voltage, together with the other positive input y to the multiplier will produce a negative voltage as one of the inputs to the comparison amplifier. This last negative input will reduce the numerical value of V until the output of the multiplier is sufficient to balance the voltage x .

The preceding diagram is applicable for either positive or negative values of the numerator of the quotient, which is x in this example.

It is applicable only for positive values of the denominator y . The diagram of Fig. 11.20 is for negative denominators, and for either positive or negative numerators.

This inverse operation of multiplication to obtain division can be applied by means of either the electronic multiplier or the servo multiplier. Note the signs of the voltages associated with the multiplier. However, other methods of obtaining quotients by use of the servo multiplier are described in ORNL-2405.

11.4.4 Function Delay

One commonly encountered situation in analog computing involves transport lag. For example, consider a pipe carrying a fluid. At equilibrium the temperature of the fluid leaving the downstream end of the pipe is the same as that entering at the other end. However, if the temperature of the fluid entering the pipe changes, there will be a time lapse before this temperature change can be observed at the downstream end.

The symbol for a function delay, or transport lag, is drawn in Fig. 11.21. The symbol τ represents the time delay. The output voltage V_o is delayed τ seconds later than the input voltage V_i . In addition the sign of V_o will be reversed, and the magnitude reduced to one-half in the ORNL Facility.

$$V_o(t) = -\frac{1}{2} V_i(t - \tau) \quad 11.17$$

The above expression indicates the mathematical relation. The symbol (t) means "a function of time", and $(t - \tau)$ means "a function of time delayed by the time interval τ ". The delay time is adjustable, but is not variable while the circuit is in operation.

It is a relatively simple matter to employ an inverter with a voltage gain of two following the function delay device in order to obtain the original voltage polarity as well as the original voltage magnitude at steady state. In the usual case, this extra simplifier and circuit is included in the symbol for the function delay in Fig. 11.21

11.4.5 Explicit Circuits

The computer diagram symbols do not always suffice to prescribe the computer operations. It is sometimes necessary to draw the explicit

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Fig. 11.21. Transport Lag.

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Fig. 11.22. Direct Differentiation.

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Fig. 11.23. Inverse Integration.

circuit, showing the electrical circuit elements in detail. An outstanding example of this situation is the use of passive elements to simulate that portion of a nuclear reactor system represented by the kinetic equations relating nuclear power and multiplication. Refer to Section 11.5.

In cases in which neither the explicit circuit nor the general symbol for an operation is known, one can specify the operation to be performed by drawing a box and stating the necessary operation. The operations specified in Fig. 11.1 are examples of this method, which may be especially useful when complicated operations must be carried out.

11.4.6 Differentiation

The process of differentiation with respect to time should be attempted only in cases of absolute necessity. In spite of the excellence of the amplifiers and circuit components of the Analog Facility, there will be a certain amount of noise in the voltage to be differentiated. A source of these small, fast, and undesired voltage variations may be a multiplier or a function generator. This noise may be especially troublesome at low voltage levels. The differentiation of this noisy voltage will yield a much noisier voltage which may be almost unusable. The example of the solution to the mathematical expression intentionally avoided the process of differentiation.

1. Direct Differentiation

A relatively simple way of obtaining the time-derivative of a voltage is shown in the explicit circuit of Fig. 11.22. The equations follow, on the basis that the input current and voltage are negligible.

$$\begin{aligned} i_1 &= i_2 \\ C \frac{dv_1}{dt} &= \frac{v_2}{R} \\ \text{Therefore,} \quad v_2 &= RC \frac{dv_1}{dt} \end{aligned} \quad 11.18$$

2. Differentiation as the Inverse of Integration

An integrator and comparison amplifier can be used to produce the time-derivative in much the same manner as the division process was carried out as the inverse of multiplication. The symbols of Fig. 11.23 illustrates this procedure.

Assume, for the moment, that the input voltage y is positive in sign. When this positive voltage is applied to the comparison amplifier, the output voltage is negative. The inverter then applies a positive voltage to the integrator input, which in turn supplies a negative voltage to the comparison amplifier. The difference between the two voltages to the input of the comparison amplifier is very nearly zero; consequently, the output of the integrator is taken as equal to the original input voltage y .

Then,

$$y = \int x dt$$

or,

$$x = \frac{dy}{dt}$$

Thus, the time-derivative is taken.

This type of differentiation does not eliminate the problem of differentiating noisy voltages.

11.5 Simulation of a Reactor

The basic process of simulating a reactor requires the computer to simultaneously solve the neutron equation, Eq. 3.5 or Eq. 5.1, and the equation for each precursor of delayed neutrons, Eq. 3.6 or Eq. 10.20. In many cases, the effect of a temperature coefficient of reactivity must be included.

A simplified computer diagram for a reactor with a negative temperature coefficient in the fuel and six delay groups is drawn in Fig. 11.24. The amplifier input coefficients have been omitted in the interest of simplicity. Each of the first order linear operators using amplifiers 3, 4, 5, 6, 7, and 8 supplies a voltage representative of the production rate of one group of delayed neutrons. For example, pot 5 is adjusted to provide a voltage to amplifier 3 which is proportional to $-\frac{kn}{\ell} \beta, \alpha,$. The output of the first order linear operator will be $+\lambda, C_1$; the equation for amplifier 3 follows, provided that the proper coefficients are employed.

$$-\frac{kn}{\ell} \beta, \alpha, + \lambda, C_1 + \frac{dC_1}{dt} = 0 \quad 11.19$$

Amplifiers 9 and 10 are used as inverters and scale changers, and to reduce the number of connections necessary on amplifiers 1 and 2. The sum of the outputs of amplifiers 9, 6, 7, and 8 represents the total rate of

production of delayed neutrons.

A voltage representing the production rate of prompt neutrons is obtained from pot 2, and a voltage proportional to the independent neutron source is taken from pot 1. In order to provide the initial conditions for start-up, a source is necessary in the simulator as well as in the reactor. The equation for amplifier 1 can be written as follows, again, by employing the proper coefficients.

$$-\frac{kn}{\ell}(1-\beta) - \sum_{i=1}^{i=6} \lambda_i C_i - S + \frac{n}{\ell} + \frac{dn}{dt} = 0 \quad 11.20$$

This equation is identical to that of a reactor having a neutron source, Eq. 5.1.

The means of adjusting the value of k is introduced between amplifiers 1 and 2. The servo multiplier takes the value of $-(k-1)$, from amplifier 15 and supplies the product of $\frac{n}{\ell}$ and $(k-1)$ to amplifier 2. Amplifiers 11 and 12 serve as scale changers and inverters to provide the proper voltage across the voltage divider of the servo multiplier. The output of amplifier 2 is the sum of $\frac{n}{\ell}$ and $(k-1)\frac{n}{\ell}$. Therefore, k is properly introduced.

Amplifier 15 takes the sum of the effects on the change in k due to the temperature change and the control rod position change. The value of the temperature coefficient is set into pot 30, and pot 29 changes the control rod position.

The fuel temperature is obtained by integrating the difference between the rate of heat production in the fuel and the rate of heat removal from the fuel. The setting of pot 25 represents the heat removal rate, and the heat production rate is taken as proportional to the neutron population. The fuel temperature is then proportional to the output of amplifier 13. Amplifier 14 is merely an inverter.

It is possible to replace amplifiers 3, 4, 5, 6, 7, 8, 9, and 10 with a passive network as discussed in ORNL-1632, "Oak Ridge National Laboratory Reactor Controls Computer", by J. J. Stone and E. R. Mann. Refer to Fig. 11.25 for the explicit circuit; and to Fig. 47, page 58 of ORNL-2405 for a detailed description of this passive network. The voltages across capacitors C_1 through C_6 in Fig. 11.24 are proportional to the number

Fig. 11.25. Nuclear Reactor Simulator Using Passive Elements in the Delayed Neutron Groups.

of precursors in the reactor. Means of charging these capacitors to produce a desired initial condition are provided in the ORNL Facility.

11.6 Simulation of a Complete Reactor System

The simulation of a reactor system should help define the control problem. The possibility of controlling certain variables can be studied, and the performance required of a control system can be examined. In some instances, it may be found impossible to obtain control through a method which seemed appropriate until carefully examined. There is very little to be gained by using the simulator to exhibit variations which are already known.

The simulation of the relatively simple system to be described here was undertaken for two reasons; (1) the process of setting up the simulator would serve as an example of analog computation, and (2) the results would indicate the actual behavior to be expected of a certain type of power reactor system.

11.6.1 Reactor System

The simplified flow diagram of the reactor system which was simulated is drawn in Fig. 11.26. The system was composed of a circulating fuel reactor, a simple counter-flow heat exchanger coupling the fuel and coolant loops, and a single coolant loop from which the heat was removed by a steam generator. The steam was produced at 800°F, the design point heat load was 350 megawatts. The fluids were referred to as "buttermilk" to indicate that this was obviously a hypothetical reactor system, simulated as a particular example of analog computing. The delayed neutrons were those of uranium-235.

The symbols in the discussion are defined as follows:

- T_{fi} - reactor fuel inlet temperature or heat exchanger fuel outlet temperature
- T_{fr} - reactor fuel mean temperature
- T_{fo} - reactor fuel outlet temperature or heat exchanger fuel inlet temperature
- T_{fh} - heat exchanger fuel mean temperature
- T_{he} - heat exchanger metal mean temperature
- T_{ci} - heat exchanger coolant inlet temperature
- T_{ch} - heat exchanger coolant mean temperature

	REACTOR	FUEL SIDE, HEAT EXCHANGER	METAL, HEAT EXCHANGER	COOLANT SIDE, HEAT EXCHANGER	COOLANT SIDE, STEAM GENERATOR
TRANSIT TIME	5.0 sec	6.67 sec		10.0 sec	12.5 sec, COOLANT 15.0 sec, EQUIVALENT
RATE OF TEMPERATURE CHANGE AT 350 Mw	60 °F/sec	45 °F/sec	150 °F/sec	30 °F/sec	24 °F/sec, COOLANT ONLY 20 °F/sec, COOLANT AND STEAM GENERATOR METAL

TEMPERATURE COEFFICIENT OF REACTIVITY = $-2.89 \times 10^{-5}/^{\circ}\text{F}$.

Fig. 11.26. Elementary Flow Diagram and Data; ORSORT Buttermilk Reactor.

- T_{co} - heat exchanger coolant outlet temperature
- T_{gi} - steam generator coolant inlet temperature
- T_{cg} - steam generator coolant mean temperature
- T_{go} - steam generator coolant outlet temperature
- T_s - steam temperature
- P_r - reactor heat production rate
- P_f - fuel to heat exchanger metal heat transfer rate
- P_c - heat exchanger metal to coolant heat transfer rate
- P_s - steam heat removal rate
- τ - transit time of coolant in piping from heat exchanger to steam generator and in return from steam generator to heat exchanger

There was no transport lag of the fuel between the reactor and heat exchanger because these pipe lines would probably be short to limit the volume to be shielded as well as to hold down the fuel inventory. Consequently, T_{fi} is both the heat exchanger outlet and reactor inlet temperature, and T_{fo} is the heat exchanger inlet as well as the reactor outlet temperature.

The design point temperatures are listed on the flow diagram. The average rate of change of the temperature of the fluid passing through a region where heat is added or removed is obtained by dividing the temperature change by the transit time through this region. The temperature change is the difference between the inlet and outlet temperatures. The average rate of change of the temperature of a solid is determined on the basis that the entire design point power is being stored in the solid as heat energy.

The negative temperature coefficient of reactivity was that of the fuel alone. It was assumed that the temperature coefficient contributed by any moderator would be small, and that in addition, the temperature change of such a moderator would be small during a transient.

The fluids were assumed to circulate at constant velocity. The energy put into the system by any necessary pumps was neglected. Heat loss by convection or radiation was also neglected.

The computer diagram for simulating the reactor system is drawn in Fig. 11.27. A power of 350 megawatts was represented by 70 volts of potential from the reactor, and by 14 volts in the heat transfer portions of the system. Zero volts meant zero power in every case. A temperature of 500°F was analogous to zero volts, and 1500°F was represented by 100.

Fig. 11.27. Elementary Computer Diagram for Simulation of ORSORT Buttermilk Reactor.

volts, giving a ratio of 10°F per volt. A suppressed temperature scale was possible because the temperature scale itself is arbitrary. The time scale was unity, one second of time on the reactor being also one second of time on the computer.

The identification numbers of the amplifiers are written inside the triangles which represent the amplifiers in Fig. 11.27. The potentiometers are identified by the numbers inside the circles, the desired voltage ratios being written outside the circles. The symbols for the variables of power and temperature with their design point voltage values are shown in the drawing.

11.6.2 Nuclear Reactor

The process of simulating the nuclear portion of a circulating fuel reactor having a negative temperature coefficient of reactivity was carried out in a manner similar to that described in Section 11.5, using passive elements. Amplifiers 39 and 40 were connected to the circuit elements which included the source, the prompt neutrons, and the delayed neutrons. The symbol G on the input of amplifier 39 means that the reactor network is connected directly to the grid of the amplifier input tube, and that no input coefficients other than those in the reactor network were used. The contributions of the precursors of the delayed neutrons were computed by the method described in Section 10. Refer to Table 11.2 for the numerical values.

The values of K to hold the reactor at steady state was computed by Eq. 10.23 to be 1.00328. The value of K found on the computer was 1.00326.

Once the circuit was in operation, the source voltage was removed to eliminate the effect of a source on the transients.

A five-to-one reduction of the voltage representing nuclear power was obtained by amplifier 30, and a polarity reversal of this voltage was produced by amplifier 31. The positive and negative values of P were applied to the potentiometer resistance of servo multiplier 3. The maximum output voltage of this multiplier was one-fifth of that supplied amplifier 40 directly by amplifier 39. Since the coefficient of the input of amplifier 40 from the multiplier was one-tenth of that of the input from amplifier 39, the actual maximum excess nuclear multiplication of the reactor available from the potentiometer of multiplier 3 was one-fiftieth, or 0.02.

TABLE 11.2

CONTRIBUTIONS OF DELAYED NEUTRONS, ORSORT
BUTTERMILK REACTOR

Delay Group	$\beta_i \times 10^5$	$\lambda_i (\text{sec}^{-1})$	α_i	$\beta_i \alpha_i \times 10^5$
1	26	0.0125	0.429	11.2
2	170	0.0314	0.431	73.4
3	210	0.154	0.453	95.3
4	240	0.453	0.622	149.2
5	84	1.61	0.876	73.6
6	25	14.0	0.986	24.6

temperature of the fuel in the reactor core, would be $60^{\circ}\text{F}/\text{sec}$. This value of $60^{\circ}\text{F}/\text{sec}$ is determined by the relationship between the temperatures T_{fo} and T_{fr} at design point conditions. The transit time of the fuel in the reactor is 5.0 seconds, and the temperature rises 300°F ; therefore, the average rate of rise of the moving fuel in transit is $60^{\circ}\text{F}/\text{sec}$. This means that a step rise in reactor power from zero to design point would produce a rate of rise of $60^{\circ}\text{F}/\text{sec}$ in each unit volume of the fuel. Thus, the initial rate of increase of the mean temperature is also $60^{\circ}\text{F}/\text{sec}$; however, if the fuel continued to enter the reactor at a constant temperature, the mean temperature, T_{fr} , would asymptotically approach a constant value 150°F higher than T_{fi} .

The simulator provided an initial rate of increase in T_{fr} of 6 volts/sec, corresponding to a temperature rise of $60^{\circ}\text{F}/\text{sec}$. At steady state design point, T_{fr} was 50 volts, corresponding to 1000°F , with T_{fi} being 35 volts, or 850°F . The mathematical relation pertaining to amplifier 1 is stated as follows:

$$-(0.429)(1) P_r - (0.400)(1) T_{fi} + (0.400)(1) T_{fr} + (1) \frac{dT_{fr}}{dt} = 0$$

Note that the coefficient of the rate of change of T_{fr} is unity. Assuming that T_{fi} and T_{fr} are equal, and that P_r makes a step increase from zero to 14 volts,

$$(0.429)(14) + \frac{dT_{fr}}{dt} = 0$$

and,

$$\frac{dT_{fr}}{dt} = 6 \text{ volts/sec.}$$

at steady state,

$$-(0.429)(14) - (0.400) T_{fi} + (0.400) T_{fr} = 0$$

$$T_{fr} - T_{fi} = \frac{(0.429)(14)}{0.400} = \frac{6}{0.4}$$

$$= 15 \text{ volts}$$

The preceding mathematics merely indicates that the pot voltage ratios were correct. The necessary ratios were computed by making use of the initial rise in T_{fr} of 6 volts/sec, together with the value of P_r of 14 volts to compute the ratio of 0.429 for pot 60. Once this ratio was

The output of amplifier 32 was the summation of the effects on the multiplication factor due to shim rods, the regulating rod, and the mean temperature of the reactor fuel. A change in nuclear multiplication of 0.01 could be obtained with a 50 volt change of the voltage from pot 58 because 100 volts applied to the input of multiplier 3 produced the maximum output of the multiplier into amplifier 40. Thus, a change of 50 volts of the regulating rod was equivalent to a change in K of 0.01. An increase in T_{fr} by 346°F , or 34.6 volts, would counteract this change in K by the rod, as computed by the voltage relations of amplifier 32.

$$(0.289)(5) \Delta T_{fr} = (1)(50)$$

$$\Delta T_{fr} = \frac{50}{(5)(0.289)} = 34.6$$

The symbol ΔT_{fr} means change in T_{fr} . It should be noted that the voltage ratio of 0.289 of pot 13 bore a direct numerical relation to the temperature coefficient. A temperature change of 346°F to produce a change in K of 0.01 means a temperature coefficient of reactivity of $2.89 \times 10^{-5}/^{\circ}\text{F}$. The negative sign of this coefficient was obtained from the polarities of the voltages representing temperature and rod position. Pot 63, representing the shim rods, was used in order that the regulating rod pot could be set at 50 volts at design point.

Note that the feedback connection of amplifier 32 has been omitted because the coefficient was unity. All feedback connections have been omitted on summing amplifiers when the coefficient is unity.

11.6.3 Fuel Loop

The fuel loop was simulated by amplifiers 1, 2, 3, and 4. These four amplifiers were connected as first order linear operators. Refer to Fig. 11.8. The relations concerning amplifier 1 will be studied in detail.

Amplifier 1 and its associated circuit represent the lower half of the reactor, with the fuel entering at the temperature T_{fi} , heat being added at the rate of P_r , and the fuel reaching a mean temperature of T_{fr} before leaving this portion of the reactor. If P_r were zero, then obviously T_{fi} and T_{fr} would be equal. Next, if P_r were suddenly step increased to the design point value, with T_{fi} remaining constant at its original value, the initial rate of increase in the temperature T_{fr} , the mean

known, the ratio of 0.400 of each of pots 1 and 2 was computed to produce a 15 volt difference between T_{fi} and T_{fr} . The ratios of pots 1 and 2 must be identical in order that the temperatures T_{fi} and T_{fr} be equal at steady state if R_f were zero.

The circuit associated with amplifier 2 was the same as that with amplifier 1 except for the reversal in signs of the voltages. The initial rates of increase in T_{fo} and T_{fr} will be identical following a step increase in R_f ; however, T_{fr} will rise in value if T_{fi} remains constant, eventually forcing T_{fo} to rise 30 volts above T_{fi} at design point.

The situation regarding amplifiers 3 and 4 was similar to that of amplifiers 1 and 2, except that the heat was being removed, thus lowering the fluid temperature. A step increase in R_f of 14 volts will produce an initial rate of decrease in T_{fh} of 4.5 volts/sec if T_{fo} remains constant. The ratio of pot 12 was computed to be 0.321 from this data. The ratio 0.300 of both pots 5 and 6 was computed from the requirement that T_{fh} must be 15 volts lower than T_{fo} at the steady state design point condition.

The circuit of amplifier 4 was identical to that of amplifier 3 except for the voltage signs.

There is an interesting relationship in these circuits. In the case of amplifier 1, the quotient obtained by dividing the coefficient of the rate of change of T_{fr} by the summation coefficient of T_{fr} is 2.5, which is the time in seconds for the fuel to pass half way through the reactor. The same situation applies to the coefficients on the output voltage of amplifier 2. In the case of amplifiers 3 or 4, this quotient is 3.33, again, the time in seconds for the fuel to pass through the section of the heat exchanger represented by the circuit. If the coefficient of the integrator had been 0.1 rather than 1 for amplifier 1, pots 60, 2, and 1 would have ratios of 0.0429, 0.0400, and 0.0400, respectively. The quotient obtained by dividing the coefficient of the integrator by the summation coefficient of the output will be equal to the transit time of the fluid in that portion of the system represented by the circuit.

11.6.4 Heat Exchanger

The rate of transfer of heat between a fluid and the metal of the heat exchanger was taken as being directly proportional to the difference

of the mean temperatures of the metal and the fluid. The mean temperature of the metal was taken to be proportional to the integral of the difference between the power being absorbed by the metal and the power being removed from the metal. Thus:

$$P_f \sim T_{fh} - T_{he}$$

$$P_c \sim T_{he} - T_{ch}$$

$$T_{he} \sim \int (P_f - P_c) dt$$

or,

$$\frac{dT_{he}}{dt} \sim P_f - P_c$$

The symbol $\sim$ means "proportional to".

The metal of the heat exchanger was assumed to rise at the rate of 150°F/sec if all of the 350 megawatts of power were being stored in the metal. Thus, a difference between P_c and P_f of 14 volts was to cause T_{he} to rise at the rate of 15 volts/sec. The equation for amplifier 7 may be stated as follows:

$$-(0.214)(5)P_f + (0.214)(5)P_c + (1)\frac{dT_{he}}{dt} = 0$$

or,

$$\frac{dT_{he}}{dt} = 1.07(P_f - P_c)$$

Now, if

$$P_f - P_c = 14 \text{ volts}$$

$$\frac{dT_{he}}{dt} = (1.07)(14) = 15 \text{ volts/sec.}$$

The ratio of 0.214 on pots 11 and 18 to obtain a total summation coefficient of 1.07 for P_f and P_c was determined by setting $\frac{dT_{he}}{dt}$ at 15 volts/sec and computing the coefficient for the difference of P_f and P_c .

The difference of 50°F between the mean temperature of the fuel and the mean temperature of the metal was to force 350 megawatts of power into the heat exchanger metal from the fuel. The 5 volt difference between T_{fh} and T_{he} at design point was to produce 14 volts as the value of P_f . The expression for the conditions of amplifier 5 are written as follows:

$$+ (0.560)(5) T_{fh} - (0.560)(5) T_{he} - (1) P_f = 0$$

or,

$$P_f = 2.8 (T_{fh} - T_{he})$$

Obviously, the coefficient of the difference between T_{fh} and T_{he} had to be 2.8 if a 5 volt difference is to produce 14 volts as P_f . Thus, the ratios of pots 9 and 16 were 0.560. A similar relation was applied to amplifier 9 which computed P_c , requiring the ratios of pots 15 and 25 to be 0.560, also.

On the basis that $-P_f$ and $+P_c$ were available, amplifier 7 computed T_{he} by integrating the difference between the heat input and output rates of the metal. Amplifier 8 then produced $-T_{he}$. Amplifier 5 computed $-P_f$ from $+T_{fh}$ and $-T_{he}$; amplifier 6 then gave $+P_f$. Assuming that $+T_{ch}$ was available, amplifier 9 computed $+P_c$ from $+T_{ch}$ and $-T_{he}$, then amplifier 10 produced $-P_c$.

An approximation of the somewhat complex heat exchanger may be set up using only two amplifiers. The values of P_f and P_c are almost identical except during fast transients. If they are taken as being equal and called P , then this value of P is proportional to the difference between the temperatures T_{fh} and T_{ch} . A circuit to perform these operations was set up as drawn in Fig. 11.28. In order to obtain a negative value of T_{ch} , the polarities of the voltages in the coolant loop and steam generator were reversed by reversing the polarity of the steam load voltage.

The reactor transients in the simulation using the simplified approximate heat exchanger were practically identical to those using the system drawn in Fig. 11.27. The variables of reactor power and temperatures, and of steam power and temperature differed only slightly from those in Figs. 11.29, 11.30, 11.31, 11.32, and 11.33, which were obtained using the more complete heat exchanger. No heat storage in the metal was considered. The effect of heat storage may be at least partially simulated by introducing first order lag into the circuit of amplifier 17 in Fig. 11.28.

11.6.5 Coolant Loop

The circuit for simulating the coolant loop was set up in a manner similar to that for the fuel loop. The initial rate of increase

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Fig. 11.28. Computer Diagram for a Simplified Heat Exchanger.

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Fig. 11.29. Load Increase; Simulation of ORSORT Buttermilk Reactor.

Fig. 11.30. Rod Insertion; Simulation of ORSORT Buttermilk Reactor.

Fig. 11.31. Rod Withdrawal; Simulation of ORSORT Buttermilk Reactor.

Fig. 11.32. Load Decrease; Simulation of ORSORT Buttermilk Reactor.

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Fig. 11.33. Load Drop; Simulation of ORSORT Buttermilk Reactor.

of T_{ch} from amplifier 14 at the rate of 3 volts/sec, when a step change in P_e of 14 volts was applied, required that pot 20 have a voltage ratio of 0.214. In order that the steady state value of T_{ch} was 15 volts higher than T_{ci} at design point, and equal when P_e was zero, the ratios of pots 50 and 28 were 0.200. Pots 19, 27, and 22 of amplifier 11 were set with the same ratios as pots 20, 50 and 28, respectively.

A function delay of 5 sec. was connected to simulate the transport lag in each section of the piping between the heat exchanger and the steam generator. An extra amplifier was included in each function delay to give the proper sign and value to the output voltage. Refer to the description of Function Delay in Section 11.4.4.

The heat removal from the coolant in the steam generator was simulated by the circuits of amplifiers 12 and 13. The actual transit time of the coolant through the steam generator was 12.5 sec, giving a coolant rate of decrease in temperature of $24^{\circ}\text{F}/\text{sec}$. However, in an attempt to account for the heat capacity of the steam generator, the simulator was set up with a $20^{\circ}\text{F}/\text{sec}$ rate of decrease in temperature. The heat capacity of the steam generator was not set up as a separate circuit. Pot 56, with a voltage ratio of 0.143, produced the initial rate of decrease in T_{cg} of 2 volts/sec when P_g was step increased to 14 volts. The ratios of 0.133 in pots 48 and 24 set up the steady state conditions. Pots 54, 23, and 26 were identical to pots 56, 48, and 24.

11.6.6 Steam Generator

A complete simulation of the steam generator was not attempted. The voltage from pot 15 determined the steam load. This load was assumed to be constant at the value set by pot 15 regardless of the steam temperature T_g . The steam load P_g from amplifiers 27 and 28 produced the required temperature drop in the coolant passing through the steam generator at the design point condition.

The steam temperature was assumed to be associated with the inlet temperature of the coolant into the steam generator. This assumption was reasonable on the basis of counter flow of the steam and coolant. At design point, the difference between the steam temperature and the coolant inlet temperature was 250°F , or 25 volts. This difference was taken as proportional to the steam load. Pot 55 was set for a voltage ratio of 0.357

in order that amplifier 29 would compute the steam temperature in accord with these assumptions.

11.6.7 Simulation Results

The results of varying the steam load and rod position are shown in Figs. 11.29, 11.30, 11.31, 11.32, and 11.33.

The ramp changes in load and rod reactivity were considerably faster than would be employed in a power plant of this type. Note that a change in the rod δK of 0.005 produced a change of 173°F in the reactor mean temperature, which agrees with the specified temperature coefficient. The step drop in load was simulated to approximate the variation caused by the loss of electrical load in case the circuit breaker connecting the electric generator to the transmission line were to open.

A rod δK value of zero was selected as the datum plane that caused a reactor fuel mean temperature of 1000°F .

11.6.8 Period and Log N

The computation of the reactor period and the logarithm of the power were not important to the system studied. This portion of the simulation is included here as an example of one of the types of calculations that an analog computer can perform.

The principle employed was that of dividing $\frac{dn}{dt}$ by n to obtain a voltage proportional to the reciprocal period. The voltage proportional to $\frac{dn}{dt}$ was then integrated with respect to time to produce a voltage proportional to the logarithm of n .

The explicit circuit is drawn in Fig. 11.34. The time derivative of P , or n , was taken by amplifier 36. Amplifier 38 merely reversed the sign of the voltage P . Pot 69 was connected to provide a certain amount of smoothing of the differentiated voltage. Without this 6000 ohms of damping resistance, the output of amplifier 36 was quite noisy. Amplifier 34 together with servo multiplier 4 computed the quotient of $\frac{dn}{dt}$ divided by n . When the period was 3 sec, the output of amplifier 34 was -100 volts. When the period was infinite, this output was zero. The servo multiplier positioned the slider of the pot having the 100 volt reference to obtain a voltage from this slider equal to the input voltage P . Since the slider of the multiplier pot in the output of amplifier 34 was also being positioned, the coefficient of the output voltage of amplifier 34

Fig. 11.34. Explicit Circuit for Period and Log N for Simulation of ORSORT Buttermilk Reactor.

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was continuously varied by the servo multiplier. The output of amplifier 34 may be stated as follows:

$$\left[\begin{array}{c} \text{Output of} \\ \text{Amplifier 34} \end{array} \right] = - \frac{\left[\begin{array}{c} \text{Output of} \\ \text{Amplifier 36} \end{array} \right]}{\left[\begin{array}{c} \text{Voltage ratio of} \\ \text{Multiplier pots} \end{array} \right]}$$

The multiplier pot voltage ratio was 0.700 at the condition taken in the circuit diagram, causing the output of amplifier 34 to be -100 volts.

Amplifier 16 computed the logarithm of the power by integrating the output of amplifier 34. Pot 73 was set to have an actual output of -3.623 volts when the output of amplifier 34 was -100 volts. This value of -3.623 volts would increase the output of amplifier 16 by 25 volts after a time lapse of 6.9078 sec, which would cause the log power recorder to rise one decade. A 3 sec period will produce one decade of power change in 6.9078 sec. In order that the scale of the log power recorder was properly indexed, the output of pot 36 was applied to amplifier 16 to set the recorder at the desired value corresponding to the reactor power. The voltage from pot 36 was zero during the recording of a transient.

The output of amplifier 34 was supplied to amplifier 19 in order that the standard period chart paper could be used on the period recorder. The voltage from pot 37 properly indexed the recorder to indicate an infinite period when the output of amplifier 34 was zero. The circuit resistances provided that the recorder indicated full scale when the period was 3 sec. A capacitor to smooth out the rapid variations of the output of amplifier 34 was necessary, especially at low values of reactor power.

The results of a step increase in nuclear multiplication are described by the curves in Fig. 11.35. The reactor temperature coefficient was eliminated, and the delayed neutron precursors were set up for stationary uranium-235. Note the noisy indication of reactor period at low power levels. Also, note that the log power curve is asymptotically approaching a straight line as the period becomes longer and approaches the asymptotic value of 22.7 sec.

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Fig. 11.35. Step Increase in δk of 0.00198, Stationary Fuel, No Temperature Coefficient; Simulation of ORSORT Buttermilk Reactor.

11.7 Instantaneous Loss of Fuel Flow

Upon the instantaneous loss of the flow of the fuel, the initial rate of increase of the mean fuel temperature is 60°F per second because the nuclear power of 350 megawatts will produce this rate of temperature increase. The rise in temperature will act through the negative temperature coefficient of reactivity to reduce the power, but the temperature will continue to rise.

The power will asymptotically approach zero and the temperature will increase at a decreasing rate. The temperature will never decrease because there is no loss of heat from the reactor under the assumed conditions.

When the fuel flow ceases, the contributions of the delayed neutrons would reach their normal stationary fuel values if the reactor could be operated at steady state with no flow. The original value of k with circulating fuel was 1.00328. An increase in the mean fuel temperature of 113.5°F would cancel the excess k by virtue of the negative fuel temperature coefficient of $2.89 \times 10^{-5}/^{\circ}\text{F}$. However, there is no mechanism for heat removal, thus the only steady state value of power is zero with no flow.

11.7.1 Simulator Connections

The elementary computer diagram in Fig. 11.36 for simulating the loss of fuel flow incorporates the nuclear reactor, the means for generating the mean fuel temperature, and the means for control of the nuclear reaction by the negative temperature coefficient and control rod. The diagram is similar to a portion of the complete system in Fig. 11.27. The source was zero at the initial conditions. The delayed neutron network was initially set up to simulate the steady state condition for the circulating fuel. At the start of the transient, this network was switched to produce the precursors of delayed neutrons at the same rate as for stationary fuel. There are two pots for each delayed neutron group. One pot sets the initial value of $P_0 \beta_i \alpha_i \times 10^3$, P_0 being the initial power level which was 70 volts. The second pot for each group of delayed neutrons sets the value of $\beta_i \alpha_i \times 10^5$ for the new condition of operation. The pot settings are listed in Table 11.3. The values of $\beta_i \alpha_i \times 10^5$ are taken from Table 11.2. It should be noted that the value of α_i in the operating settings was unity; thus, the

TABLE 11.3

DELAYED NEUTRON CONTRIBUTIONS, SIMULATION OF LOSS OF FLOW

Delay Group	Initial Condition Settings	Operating Settings
	$\rho, \beta, \alpha_i \times 10^3$	$\beta, \alpha_i \times 10^5$
1	7.8	26
2	51.4	170
3	66.7	210
4	104.5	240
5	51.6	84
6	17.2	25

operating setting are actually $\beta_i \times 10^5$. The control rods, pot 73, was adjusted to obtain + 70 volts at the output of amplifier 39 under the initial conditions.

Amplifier 38 was connected as an integrator to compute the rise in fuel temperature, with an initial value of zero volts. Pot 60 was set to provide a voltage ratio of 0.429 in order that the output of amplifier 38 would rise 6 volts per second when the output of amplifier 39 was +70 volts. Thus, the fuel mean temperature rise would be $60^\circ\text{F}/\text{sec}$ at a nuclear power of 350 megawatts. A voltage rise of 34.6 volts from amplifier 38 would reduce K by 0.01, which is equivalent to the effect of a temperature rise of 346°F . Refer to Section 11.6.2.

11.7.2 Operation

The transient was started by switching the delayed neutrons circuits from their initial condition settings to their operating settings. Amplifier 38 was switched into the circuit as an integrator to indicate the rise in fuel mean temperature. The results are shown in Fig. 11.37.

11.7.3 Comparison of Circulating Fuel and Stationary Fuel

A second transient was obtained under the assumption that the fuel was stationary, but had the heat capacity and temperature coefficient of the case with circulating fuel. The heat would be removed by some sort of circulating coolant which would not affect the delayed neutrons.

A sudden loss of heat removal was examined. The power was found to decrease somewhat more slowly than for the case of an abrupt loss of fuel flow in the circulating fuel reactor. The temperature rise was somewhat faster than with circulating fuel. The sudden loss of fuel flow in the circulating fuel case tends to place the reactor on a positive period because of the additional delayed neutrons which had previously been produced external to the reactor. However, the initial levels of the precursors of the delayed neutrons are low compared to the stationary fuel case, and this reactor could be placed on a short negative period by less reduction in K with circulating fuel than with stationary fuel. Refer to Section 10.

A reactor system having a different fuel heat capacity, temperature coefficient, or fuel circulation rate may not exhibit this particular relation between the stationary fuel and circulating fuel cases. No general conclusions should be drawn from this one example; each system must be individually examined.

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Fig. 11.37. Loss of Fuel Flow; Simulation of ORSORT Buttermilk Reactor.

12. NUCLEAR REACTOR INSTRUMENTATION

The instrumentation to be described will be limited to that peculiar to the control of nuclear reactors. The temperature, or rate-of-change of the temperature, of certain components of any reactor are surely to be of concern to the proper operation and safety of the plant; however, a knowledge of the fission power is imperative because the nuclear power is one of the important factors in determining the time behavior of the temperature. Refer to Figures 12.1 and 12.2. Although the measurement of such variables as temperature, flow, and pressure are extremely important to reactor control, this type of instrumentation will not be discussed since these variables are common to many commercial non-nuclear applications.

An adequate start-up procedure will probably require the measurement of the reactor flux over a wide range. Period information may be important in order that adequate control and safety equipment can be applied. A knowledge of the reactor flux is essential to the operation of a research reactor wherein the flux is the prime quantity produced.

Reference material on instrumentation can be found in Glasstone, Chapters 5 and 6, and in Schultz, Chapters 5 and 7.

The discussion of instrumentation in this section of these notes is a minimum and is not meant to be all inclusive nor exhaustive. The neutron sensors and circuits used in the control of some of the research reactors at the Oak Ridge National Laboratory will be described briefly.

12.1 Neutron Sensors for Control and Safety

The nuclear instruments collectively must cover an operating range of the order of 10 decades. A single sensor can hardly operate successfully over such a wide range; consequently, separate sensors are used to cover different portions of the total flux range.

12.1.1 Fission Chamber

At low values of power in the range of 10^{-6} to 10^{-10} of full power, the thermal flux available at the sensor cannot produce a continuous indication because of the small number of neutrons which fall into the chamber each second. Counting techniques must be employed.

A schematic diagram of a fission chamber and the associated electrical circuit in a counting channel is shown in Figure 12.3. The active coating

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Fig. 12.1. Example of Reactor Nuclear Instrumentation.

Fig. 12.2. Example of Reactor Process Instrumentation.

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Fig. 12.3. Fission Chamber Circuit.

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Fig. 12.4. Compensated Ion Chamber Circuit.

on the inside surface of the chamber is uranium-oxide to a thickness of one to two milligrams per cm^2 . Usually, highly enriched uranium is employed. The space between the electrodes is filled with argon or nitrogen, or a mixture of argon and nitrogen, at a pressure of one to two atmospheres. This gas may be sealed in the chamber, or it may be introduced by means of a continuous flow.

When thermal neutrons fall into the chamber and cause fission of the uranium-235, some of the fission fragments escape from the solid surface with sufficient velocity to create ion-pairs in the gas. The free electrons produced are collected on the positive electrode in a time duration of approximately one microsecond, which causes a small reduction in the voltage across the chamber. The collection of the heavier slower moving positive ions on the negative electrode, together with the time required to recharge the chamber, will cause the slight reduction in chamber voltage to continue for a longer time. The dc potential applied between the chamber electrodes is of the order of 300 volts.

Alpha particles from the naturally radioactive uranium within the chamber and gamma photons from the decay of fission products within the reactor also produce ion-pairs in the gas inside the fission chamber. However, the voltage pulses produced by these alpha particles and gamma photons are much smaller than those caused by the very much higher energy fission products released from the uranium coating. This large difference in pulse height allows the pulse amplifier circuit to discriminate against the gamma photons and alpha particles. The pulse height selector in the amplifier is adjusted to permit only the large pulses caused by neutrons to be counted. It should be noted that the voltage pulses will not all have the same height and shape because the ion pairs will be formed within different areas inside the chamber.

It is possible for the gamma photons to "pile up" and produce a single pulse sufficiently large to be counted. In addition, a very large number of gamma photons per second can produce a continuous current through the chamber and lower the chamber voltage due to the voltage drop across the resistor in series with the dc power supply.

The capacitance of the cable attached between the chamber and the first stage of amplification reduces the size of the voltage pulse received at the pulse amplifier. In addition to the greater voltage gain in the pulse amplifier that is necessary when the cable is long, the signal-to-noise ratio

becomes poorer and the useful sensitivity of the chamber may decrease¹.

The sensitivity of the ORNL Q-1092 fission chamber is approximately 0.06 counts per second at a thermal flux of one neutron per cm^2 -sec. The uranium-235 coating of this chamber is on the inside surface of a cylinder three inches long and one inch inside diameter. The positive electrode is a rod, 0.04 inches in diameter.

It is possible to operate fission chambers to provide a continuous current rather than pulses. The neutron flux must be sufficient and the proper external electrical circuits must be employed.

Fission chambers are usually withdrawn and shielded as the reactor power increases. This accomplishes two objectives, (1) the uranium is not consumed as rapidly, thus the useful chamber life is extended, and (2) the chambers continue to provide neutron flux information.

12.1.2 Compensated Chamber

Compensated ion chambers are used within the range of 10^{-6} to 3 times normal reactor power. The signal provided by this chamber is a continuous current, rather than distinct pulses.

Discrimination against gamma photons is accomplished by the bridge circuit in Fig. 12.4. The chamber actually contains two separate gas-filled volumes; the gas in both volumes is ionized by gamma radiation and allows a corresponding current to flow. Both surfaces of one volume are coated with boron-10; therefore, the gas in this volume is also ionized by the alpha particles and lithium-7 nuclei emitted by the exothermic reaction between the boron and the incident neutrons. On the average, approximately 30% of the ion pairs are formed by the lithium-7 nuclei, and 70% are produced by the alpha particles².

When the chamber volumes and voltages are properly adjusted for a given gamma flux distribution, the gamma-induced currents of the two volumes are nearly identical, and the signal current is that due to neutrons only.

The volume of the section of the chamber not coated with boron is made adjustable in the ORNL design. Perfect compensation is almost impossible to achieve because of variations in the gamma flux distribution; in addition, there is virtually no means of determining whether the chamber is properly compensated. One method of obtaining the data for compensating the chamber

1. "Characteristics of Fission Chambers," J. L. Anderson, Reactor Controls Department, Oak Ridge National Laboratory.
2. J. C. Gundlach, Oak Ridge National Laboratory, unpublished

is to shut down the reactor and adjust the chamber voltages and volume with the chamber in place in a flux field having negligible neutrons compared to gamma photons. The chamber is usually under-compensated rather than over-compensated in order to prevent a reversal of the signal current.

The structural material of the electrodes of the ORNL Q-1045 compensated chamber is graphite, the outer casing is magnesium. Every effort is made to use materials that will not become radioactive in order to eliminate signal currents due to activity in the chamber itself. The outside body of the chamber is approximately 36 inches long and 4 inches in diameter. The active portion of the chamber is only about 4 inches long, and the remainder of the body is filled with graphite. This graphite fills the space directly back of the sensitive region of the chamber, thus serving as a reflector and scatterer. The neutron sensitivity of this chamber is approximately 8×10^{-15} amperes for a unit thermal flux of one neutron per $\text{cm}^2\text{-sec}$; the uncompensated gamma sensitivity is approximately 2.5×10^{-12} amperes per R/hr. The useful neutron flux range of the chamber is from 10^2 to 10^{10} neutrons per $\text{cm}^2\text{-sec}$, depending upon the intensity of the gamma flux.

Compensated chambers are usually filled with nitrogen at one atmosphere pressure, and may be either sealed or have a continuous gas flow.

12.1.3 Uncompensated Chamber

An uncompensated parallel-circular-plate ion chamber is used at ORNL in the reactor safety system to initiate emergency shutdown at an excessive neutron flux level. No discrimination against gamma photons is employed; thus, the range of this chamber is restricted to two or three decades around the normal operating level. The chamber and circuit are shown schematically in Figure 12.5.

The electrodes of the ORNL Q-975 PCP Chamber are graphite³, coated with boron-10 to a thickness of approximately 0.3 milligrams per cm^2 . The gas is nitrogen at one atmosphere of pressure, either sealed or with continuous flow.

The chamber body is approximately 33 inches long and 3 inches in diameter with the active section being about 6 inches long. The main body is filled with graphite to scatter the neutrons and to eliminate leaving a void back of the sensitive portion of the chamber. The neutron sensitivity of this chamber is approximately 5×10^{-15} amperes per unit thermal flux of one neutron per $\text{cm}^2\text{-sec}$; the gamma sensitivity is approximately 2.5×10^{-12} amperes per R/hr.

3. ORNL-1080, "The Neutron Sensitive PCP Ionization Chamber", R. K. Abele and J.C. Gundlach

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Fig. 12.5. PCP Chamber Circuit.

The useful neutron flux range is from 10^7 to 10^{10} neutrons per $\text{cm}^2\text{-sec}$.

12.1.4 Thermopile

The elementary form of another type of neutron sensor consists of two thermocouples connected in electrical opposition, but with one thermocouple attached to a small quantity of boron-10 or uranium-235. The electrical connection eliminates an output signal when both thermocouples are at the same temperature. However, the thermocouple attached to the boron or uranium will be at a higher temperature in the presence of a neutron flux. Consequently, the output signal is a function of neutron flux.

Several thermocouples are usually employed to obtain sufficient output potential to be useable. Thermopiles are relatively much smaller in volume than chambers, but they are limited in range and in speed of response.

12.2 Nuclear Instrumentation Circuits

12.2.1 Log Diode

The logarithmic element used in the ORNL logarithmic amplifiers is a type 9004 diode operated in the region where the kinetic energy of the electrons emitted from the cathode produce the potential difference between the cathode and plate. Refer to Figure 12.6.⁴ In order to reduce the electron flow from the cathode to the plate to a very low value, the plate must be made quite negative with respect to the cathode. This is due to the fact that some of the electrons emitted by the cathode have several electron volts of kinetic energy and can overcome the opposing electric field. More electrons have sufficient energy to reach the plate when the plate is less negative with respect to the cathode.

The relationship between the plate current and plate voltage is approximately logarithmic over a current range of 10^{-12} to 10^{-4} amperes. The plate potential rises approximately 0.2 volt per decade of current increase.

Although this logarithmic relationship between the tube current and plate voltage is present over a wide current range, the actual numerical value of the plate voltage is a function of the cathode surface temperature and electron emission characteristics. At a given plate current, an increase in the cathode temperature will increase the plate voltage in the negative direction. Mechanical shock will also change the plate voltage in an erratic manner. Some of these diodes exhibit a plate voltage drift during the life of the tube.

Fig. 12.6. Example of Logarithmic Characteristics of the Type 9004 Diode.

12.2.2 Count Rate Channel

Refer to Figure 12.7. This channel is the only one in operation during the low power range of the reactor. The scaller is used during critical experiments when the counting rate is below one count per second and the statistical variations are relatively large. For a normal startup, the counting rate should be at least as high as two counts per second and the Log Count Rate Recorder should be in operation.

Pulses from an output of the A7 Linear Amplifier are supplied to the Log Count Rate Amplifier. These pulses trigger a univibration which in turn supplies pulses of constant height (approximately 230 volts) and constant duration (approximately 15 microseconds) to a circuit that smooths the constant charge pulses and takes the logarithm of the resulting current. The diode described in Section 12.2.1 is used to obtain the logarithmic relation. The ORNL count rate channel has an upper limit of 10^4 counts per sec.

Considerable filtering of the diode voltage is required in order to eliminate the effects of statistical fluctuations in the counting rate and to smooth out the pulses to obtain a useable signal for the Period Recorder. This filtering increases the effective time lag between the indicators and the sensor. The count rate channel is not capable of following rapid changes in the neutron flux variation, and the information from this channel must be weighted accordingly.

It is possible to obtain period information of a sort, by taking the derivative of the voltage across the diode.

12.2.3 Linear Flux Channel

A d-c amplifier connected to a compensated ion chamber will provide a linear indication of the thermal flux at the chamber. This amplifier is provided with a large number of separate full-scale current ranges that are manually selected.

The signal for the automatic flux controller of the swimming-pool type reactors is obtained from this channel.

12.2.4 Log N and Period Safety Channel

The logarithmic diode described in Section 12.2.1 is used to derive a signal proportional to the logarithm of the flux. This tube is connected as the current detector in Figure 12.4, with the plate of the tube connected

Fig. 12.7. Nuclear Instrument Block Diagram.

directly to the compensated chamber signal lead. Additional amplification of the voltage across the diode is obtained as necessary in the Log N Amplifier. A long cable connected between the chamber and diode will introduce capacitance which will seriously affect the ability of the channel to respond to a fast flux transient. During a fast excursion, the Log N indicator will lag behind the actual change and the period indicator will not show the correct instantaneous reactor period.

The signal for the Period Recorder is obtained by differentiating this voltage that is proportional to the logarithm of the flux. The resulting signal voltage is proportional to the reciprocal of the period.

A signal to the safety system is also provided by the Log N Amplifier. The Period Amplifier differentiates the logarithmic signal from the Log N Amplifier to derive a signal suitable for use by a Sigma Amplifier.

12.2.5 Safety Channels

The control rods of a swimming-pool type reactor are suspended from magnets which are supplied by the Magnet Amplifiers. A scram is produced by the reduction of the magnet current and the consequent release of the rods.

The output currents of the Magnet Amplifiers are determined by the Sigma Bus voltage. The Sigma Bus connects the output terminals of all the Sigma Amplifiers to the input terminals of all the Magnet Amplifiers. The magnet current decreases as the Sigma Bus voltage increases above 34 volts, and the current also decreases when the Sigma Bus voltage is reduced below 34 volts. Therefore, a short-circuit between the Sigma Bus and ground will scram the reactor.

Any one of the Sigma Amplifiers is capable of raising the Sigma Bus voltage and scrambling the reactor. This portion of the safety system is sometimes referred to as an "auction circuit".

The Sigma Amplifiers in the Level Safety Channels supply 37 volts to the Sigma Bus when the reactor flux is zero. At a power level of 150% normal, the PCP chamber current is approximately 41 microamperes, which raises the Sigma Bus to about 45 volts and releases the control rods. Two or more Level Safety Channels are employed for reliability.

A signal from the Period Amplifier rather than from an uncompensated chamber is supplied the Sigma Amplifier in the Period Safety Channel. This

Sigma Amplifier scrams the reactor at an input signal corresponding to a period of approximately one second.

Considerable care is taken to insure that the safety channels are in proper working condition. Each Sigma Amplifier is provided with a monitoring system to determine whether the amplifier voltages are present and the amplifier is functioning properly. In addition, the PCP chamber voltages are monitored through additional connections to the chamber. A malfunction detected by the monitoring system produces an immediate annunciation to call the reactor operator's attention to the situation. In some cases, the monitoring system is also connected to scram reactor.

The ORNL PCP chambers have several terminals arranged to allow the monitoring system to determine whether the proper chamber voltage is present. Fission chambers, or uncompensated chambers having only one terminal, are not used by ORNL because this monitoring system cannot be applied without additional terminals.

13. REACTOR SAFETY AND CONTROL SYSTEMS

A control system provides the means of operating the reactor plant at the conditions the plant is designed to fulfill. If a plant is to be operated over a given power range with power changes to be made no faster than a specified rate, then the control system should provide the means of operating the plant over the required range of power and within the required rate of change. Naturally, the control system is limited to the mechanisms provided by the design.

The purpose of the safety system is to return the plant to a safe condition when an unsafe condition has occurred. Again, the safety system must operate with the mechanisms available in the design, and it is entirely possible that the safety system cannot cope with every type of malfunction or accident. The following material, excerpted from ORNL CF-53-3-241, A PHILOSOPHY OF REACTOR CONTROL, by W. H. Jordan, is one example of a situation which reduces the effectiveness of a safety system:

So much emphasis has been placed on the measurement of neutron level that the importance of maintaining a cooling rate that corresponds to the neutron level is sometimes overlooked. For example, the design may call for a multiplicity of devices for scrambling the reactor when a given neutron level is exceeded, but have only one or two relays that operate at reduced coolant flow. Or one may decide to operate the reactor at low power with reduced coolant flow without reducing the neutron scram level to correspond. While these are obvious considerations they are perhaps violated more frequently than any other safety precaution.

The designer of a control system or of a safety system must constantly guard against the use of precedence as a substitute for analysis. Each type of reactor must be considered separately. Control and safety systems which have apparently been satisfactory for a particular reactor may be unsatisfactory in another reactor which appears to be similar. For example, a change in a design to uranium of low enrichment may introduce a negative temperature coefficient in the fuel due to the Doppler broadening; refer to Section 7. This fuel temperature coefficient is highly desirable from the standpoint of reactor safety; on the other hand, the operation of the reactor may require the control system to handle fairly large amounts of reactivity in order to hold a given temperature constant because of this same Doppler effect. Only an adequate safety analysis can determine whether the example cited above will create an unsafe condition. Every reactor plant must be

examined for possible interference between the control system and safety system.

An employment of a "Manual Scram" actuated by the human operator as a last ditch safety measure must be carefully analyzed. The operator can take three kinds of action: (1) no action, (2) the correct action, or (3) the wrong action. Dependence on the operator to correctly assimilate several pieces of information and take the proper safety action under the stress of an impending reactor incident cannot be classed as a reactor safety system. The question could properly be asked, "How can the operator be expected to find the correct solution to a safety problem in a few seconds when the designers did not find the answer in several months?"¹

Two approaches to the basic design of a control system are possible. One approach is to attempt to force the plant to operate as the design specifies. An opposite approach is to allow the plant to follow the natural behavior inherent in its design. The first approach will probably require a more expensive and complicated control system, but the second approach will require that the reactor design provide a stable and inherently safe plant. The first method could conceivably operate a power reactor at its optimum condition at all times, and utilize the reactor to the utmost; although the problem of reactor safety may be very difficult. It is worth noting that a reactor operating at a temperature only slightly below the damaging temperature will be vulnerable to many types of accidents. The second method will probably require that the reactor normally operate at lower than the maximum safe fuel temperature, in order to provide a margin for natural control. The first method places a premium on reliable instruments and control hardware, while the second approach requires a minimum of reliability of the instruments and hardware. No attempt will be made here to reconcile these two approaches, but safety and reliability of the complete plant together with the costs of construction and operation must always be considered.

13.1 Basic Control Criteria

The following basic rules are suggested as the criteria for the formulation of a control system:

1. E. R. Mann, ORNL, private communication

1. The control system should be as slow in response as the system requirements will allow.
2. The control system should rely as much as possible on the self-regulating features of the system being controlled with a minimum of controlling variables.
3. If possible, the control should be simple, but reliability is the primary consideration. A large lead-band in the controller to allow the system to stabilize itself and thus minimize the necessary control action is desirable, but not essential.

The following suggested criteria for the control rod actuator has been found quite worthwhile at ORNL:

1. The actuator should be positive in producing rod motion. This requires the ability to apply force to the control in both directions, an impossibility when the rods are suspended on chains or cables.
2. The position indication of the rod should be positive. There should be no guesswork concerning the direction and magnitude of the rod travel under either normal or emergency conditions.
3. A failure of one actuator should not jeopardize operation of other rod actuators.
4. Each actuator mechanism should be amenable to inspection and repair.

The actual application of control rod drives sometimes requires that one or more of these desired characteristics cannot be attained. In particular, when the rod drive mechanism cannot apply a positive force to the rod in both directions, the position indicator becomes of major importance.

Refer to ORNL-2090, A PHILOSOPHY OF CONTROL-SYSTEM DESIGN, by E. P. Epler for a further discussion of control system criteria.

13.2 Logic Diagrams

An information block diagram or logic diagram is one manner of graphically describing a control system, or a safety system, in its basic form. An example of such a diagram for the control of a regulating rod in a reactor is shown in Figure 13.1. This diagram is not intended to be an example of the correct manner of operating a regulating rod, but is merely an example of the use of an information diagram. A control system, or safety system, is essentially concerned with the handling of information which is obtained from sensors throughout the reactor plant. The information from these sensors must be

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Fig. 13.1. Example of Regulating Rod Control Logic Diagram.

assimilated and certain control measures taken. In this respect, the control system performs much like a computing machine in determining whether to prevent, produce, or continue any given control action. The use of a block diagram to describe the computing process allows the control system designer to concentrate upon the logic of the system, rather than requiring him to think in terms of particular pieces of physical devices.

The statements inside the single-bordered boxes in Figure 13.1 are conditions which will pass control information when the condition is fulfilled. Rod motion can be obtained only when the control information is transmitted to the actuator. The neutron flux should be measured before withdrawal of the regulating rod in the example in Figure 13.1; therefore, one condition that must be satisfied before withdrawal of the rod is that the counting rate of the fission chamber channel exceed two counts per second. In addition, rod withdrawal is not to be permitted whenever the period is shorter than 20 seconds. Other permissive conditions are also indicated. The statements inside the double-bordered boxes are conditions that must be established manually by the operator. He may set up any given manual condition at any time, but no action results unless the information path is completed and the information is transmitted to the actuating mechanism. The horizontal dashed lines mean that these manual conditions are selected on a common device, such as a multi-position switch. For example, a manual selector switch can be in either of the three positions -- "Withdraw", "off", or "Insert" -- but only one selection is possible at any given time. The "Manual-Servo" selector provides for either automatic or manual control of the regulating rod subject to the other permissive restrictions shown in the diagram. The small arrow pointing away from the "Servo" box means that this selector will automatically return to the "Manual" condition whenever the information path to the "Servo" box is broken, such as by changing the "Withdraw-Off-Insert" selector from the "Off" condition. Automatic control is possible only if all the necessary permissive conditions are satisfied and the operator has manually selected "Servo" control.

Rod insertion by manual selection is possible at any time in the example in Figure 13.1, and in addition, automatic insertion will occur provided certain conditions arise. A portion of the conditions which provide automatic insertion are opposite to the requirements necessary for rod

withdrawal. In order to permit withdrawal, several conditions must be satisfied simultaneously, but automatic insertion occurs if any one of four conditions arises.

After the control system is basically described, the necessary physical hardware must be selected. The proper use of sensors, instruments, relays, actuators, and related equipment is essential in order to provide a thoroughly adequate and reliable control system.

13.3 Reactivity Changes Not Caused by Rod Motion

The material in this section was gathered from conference between E. P. Epler, S. H. Hanauer, W. H. Jordan, and C. S. Walker in January, 1957.

Some reactors have been constructed with large amounts of reactivity controlled by rod motion. In the MTR, for example, the rods have a worth in excess of 40% δk . On the other hand, it may be difficult or undesirable to install rods of any sizeable worth; indeed, with some types of reactors, the rods may be omitted altogether. The natural uranium-graphite reactors commonly have less than 5% δk controlled by their rods; in some homogeneous and other liquid-fuel reactors, no rods are installed and the reactor is shimmed with fuel enrichment and regulated by exploiting a large negative temperature coefficient.

The δk available for control in all rods, and the loaded excess reactivity of the assembly (rods fully withdrawn), together determine the δk margin by which the reactor is shut down when the rods are fully inserted. In the swimming-pool type reactor, for example, the rods control about 7% δk . Using the rule of thumb that one may load until the excess reactivity equals half the worth of the rods, this reactor may be loaded with about 3% excess k . The shutdown k will then be 0.96, and the shutdown margin, 4%. In the MTR, the shutdown margin is greater than 15%. In a natural uranium-graphite reactor, on the other hand, the shutdown margin may be as low as 1%, and the uncertainties in fuel loading, reactivity of experiments, and temperature effects, may be as large as this shutdown margin.

13.3.1 Fuel Loading (The Critical Experiment)

The loading of fuel into a reactor naturally falls into two categories:

1. A liquid fuel loaded through a pipe and throttling valve under the control of the operator, is an example of continuous loading. Another example

of a loading process which is practically continuous is the use of natural uranium slugs, thousands of which are loaded one-by-one into long channels.

2. At the other extreme, the fuel of some reactors is divided into a small number of indivisible fuel elements, sometimes as few as 15 or 20 of them. The addition of fuel, and hence the increase in reactivity is therefore quantized; and there is a minimum and often large step which the operator must reckon with at each stage of the loading.

1. Continuous Loading:

Loading a reactor in the continuous mode is done by loading fuel at some constant rate until the instruments indicate a substantial subcritical rise. The loading proceeds more and more cautiously until the reactor is critical. One way of achieving this is to withdraw the rods fully between successive small fuel additions. This is not quantized fuel loading, since the additions may be as small as desired. The maximum rate at which the fuel may be loaded is limited first by consideration of the startup accident wherein the fuel addition is equivalent to rod withdrawal, and second, by operational convenience. The rate allowed by the startup-accident calculation may be altogether too fast for an orderly approach to criticality.

In order to cope with a startup accident, means must be provided to decrease the multiplication. Any control rods must be withdrawn a substantial amount before fuel addition is begun, so that they may be inserted to control an unsafe condition arising from errors in the loading procedure or breakdown in the machinery.

2. Quantized Loading:

This discussion will start with a partly loaded lattice, to which it is desired to load the next fuel element. The rods are first fully withdrawn using standard startup procedure, and the counting rate determined. The rods are then fully inserted, and the counting rate determined for this condition. From these counting rates, the approximate worth of the control rods may be determined in terms of fuel mass by reading from the curve of inverse counting-rate versus mass of fuel the amount of fuel whose removal would give the same decrease in counting rate as inserting the rods, refer to Figure 5.1. The operator next considers the quantity of fuel in the next fuel element to be loaded, in order to assure himself that its addition will not

result in the reactor reaching critical with the rods near the bottom. This implies that no fuel elements shall have a reactivity worth as much as half the reactivity worth of the control rods. Although the determination of control-rod worth need not be done before each fuel addition, the operator must at all times know the rod worth as compared to the fuel element worth.

The operator having determined that loading the next element is safe, it is loaded under his direction. The two related problems will next be considered; how shall the fuel element be loaded, and where shall the control rods be during this operation?

Assume that the fuel element is loaded with the rods partly withdrawn, and, through some error, the reactor is made supercritical by this fuel addition with the rods in this position. The power level will rise until the safety system releases the rods. Assume that the rods are then fully inserted, and that the reactor is driven subcritical. The power burst required to initiate the action of the safety system would not have occurred if the rods had been fully inserted before the step increase in multiplication took place. This power burst caused an unnecessary hazard to the reactor system and to the operating personnel. In addition, the partial withdrawal of the rods has increased the probability of a given fuel addition producing a supercritical condition.

In the case of reactors having only a small amount of reactivity in the rods, the operator may have no confidence in his determination of the relative worths of rods and fuel element. In this case, he will properly withdraw the rods part way before each fuel addition, and he must load the fuel element slowly and reversibly. If the control rods are inserted by the safety system the new fuel element must be removed immediately. It is conceivable that direct human handling by skilled technicians might accomplish this, but what is really attempted here is an approach to continuous loading, where the loading rate must be controlled, because the operator has lost confidence that the conditions for quantized loading are fulfilled. Under these circumstances, when the insertion of fuel must be stopped or reversed to avoid a supercritical condition, the fuel should be loaded by machinery which inherently limits the rate of fuel insertion to a safe value. This machinery should provide a means of quickly removing a satisfactory quantity of fuel.

The control system should include the fuel loading process as well as the rod control. The loading should be under the control of the reactor operator at the control board at all times.

13.3.2 Unscheduled Reactivity Changes

The determination of whether the rods should be partially withdrawn in a reactor that is shutdown must be obtained from safety considerations. The probability that the reactor may become critical when the rods are fully inserted must be evaluated. It is extremely improbable that a reactor having the shutdown margin of the MTR can be made critical with the rods fully inserted due to any credible non-catastrophic accident. On the other hand, other types of reactors may become supercritical with the rods fully inserted as a result of what might appear to be a minor occurrence or accident. Examples of this are loss of light-water coolant from a natural uranium-graphite reactor, a change in an in-pile experiment, or leakage of fuel from a region of low importance to a region of higher importance in a circulating fuel reactor. A reactor with a small shutdown margin is especially susceptible. Each reactor must be examined to determine the safest shut down position of the rods.

Consider an unscheduled increase in multiplication occurring at a constant rate, with the rods initially partly withdrawn. The incident is illustrated in Figure 13.2. The increase in multiplication causes the reactor power to initiate a rod drop at the time t_1 due to either a high power level or a short period. The reactor is thus made subcritical immediately after the time t_1 . Now, the time interval between t_1 and t_2 is available for further corrective action to prevent another recurrence of a supercritical condition at the time t_2 .

If an alert operator had been watching a reliable startup instrument, he could have detected the start of the incident long before the time t_1 . Many reactor operators choose to use the safety system and the partly withdrawn rods as a more reliable detector which works even in the absence of an operator. Usually, instead of partly withdrawing all rods, a few rods, designated "Safety Rods", are fully withdrawn for this protection, and the

δk which they control has been named "spare negative k ". These rods should be withdrawn immediately after shutdown, and their automatic insertion by the safety system should be understood to indicate an urgent need to

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Fig. 13.2. Application of Spare Negative Reactivity.

dismantle part of the reactor so as to decrease the multiplication before the time t_2 , the time when the reactor will go supercritical again.